Inverse

Single Correct Answer Type

1. If x, y, z are natural numbers such that $\cot^{-1}x+cot^{-1}y=\cot^{-1}z$ then the number of ordered triplets (x,y,z) that satisfy the equation is _______

A) 0 B) 1 C) 2 D) Infinite solutions

Key. D

Sol. $x=1:y=1$ is not a solution of the given equation

Suppose $(x,y)\neq(1,1)$

Then $\dfrac{\frac{1}{x}+\frac{1}{y}}{1-\frac{1}{xy}}=\dfrac{1}{z}$ $\Rightarrow x(z-y)=-(1+yz)$ $\Rightarrow x=-\dfrac{(1+yz)}{z-y}$

If $y=n+1:z=n$ then $x=n^2+n+1$

All such numbers are solutions

2. The number of solutions of the equation $\tan^{-1}\left(\dfrac{x}{1-x^2}\right)+\tan^{-1}\left(\dfrac{1}{x^3}\right)=\dfrac{3\pi}{4}$ belonging to the interval $(0,1)$ is

A) 0 B) 1 C) 2 D) 3

Key: A

Hint: $\because x\in(0,1)\dfrac{x}{1-x}>0;\dfrac{1}{x^3}>0\ \&\ \dfrac{x}{1-x^2}.\dfrac{1}{x^3}>1$

$$\therefore \tan^{-1}\frac{x}{1-x^2}+\tan^{-1}\frac{1}{x^3}=\pi+\tan^{-1}\left(\frac{\frac{x}{1-x^2}+\frac{1}{x^3}}{1-\frac{x}{1-x^2}.\frac{1}{x^3}}\right)=\pi+\tan^{-1}\left(\frac{-1}{x}\right)=\frac{3\pi}{4}$$

$\Rightarrow x=1(\text{not possible})$

3. Number of common points for the curves $\mathrm{y}=\sin^{-1}(2\mathrm{x})+\tan^{-1}\left(\dfrac{1}{[2\mathrm{x}]}\right)+2$ and $\mathrm{y}=\cos^{-1}(2\mathrm{x}+5)+1$ is (where [.] denotes greatest integer function)

(A) 0 (B) 1 (C) 3 (D) 4

Key: A

Hint: As domain of first function is $\left[-\dfrac{1}{2},0\right)\cup\left\{\dfrac{1}{2}\right\}$ and domain of 2^{nd} function $[-3,-2]$ there is no common point

4. If $\sin^{-1}\left(\sin\dfrac{33\pi}{7}\right)+\cos^{-1}\left(\cos\dfrac{46\pi}{7}\right)+\tan^{-1}\left(-\tan\dfrac{13\pi}{8}\right)+\cot^{-1}\left(\cot\left(-\dfrac{19\pi}{8}\right)\right)=\dfrac{\mathrm{a}\pi}{\mathrm{b}}$

Where a and b are in their lowest form, then (a + b) equal to

(A) 17 (B) 20 (C) 23 (D)None of these

Key: B

Hint: On solving, we get $\frac{a\pi}{b}=\frac{13\pi}{7}\Rightarrow 13+7=20$.

5. If 0 < x < 1, the number of solutions of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$ is

(a) 0 (b) 1 (c) 2 (d) 3

Key: b

Hint: The given equation can be written as

$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$\Rightarrow \frac{2x}{2-x^2}=\frac{2x}{1+3x^2} \Rightarrow x+3x^3=2x-x^3$

$\Rightarrow 4x^3-x=0 \Rightarrow x(4x^2-1)=0$

$\Rightarrow$ x = 0, $x=\pm\frac{1}{2}$. Thus x = $\frac{1}{2}$

6. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$ then $x^2+y^2+z^2-xy-yz-zx$ equals to

A) 0 B) 1 C) 2 D) 3

Key. A

Sol. Since maximum value of $\cos^{-1}x$ is π

Therefore, $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi \Rightarrow \cos^{-1}x=\cos^{-1}y=\cos^{-1}z=\pi$

$\Rightarrow x=y=z=-1 \Rightarrow x^2+y^2+z^2-xy-yz-zx=0$

7. $\sum_{K=1}^{\infty}Tan^{-1}\left(\frac{1}{2K^2}\right)=\theta$, then $Tan\theta$ = _______

(1) 0 (2) 1 (3) $\sqrt{3}$ (4) ∞

Key. 2

Sol. $Tan^{-1}\left[\frac{(2K+1)-(2K-1)}{1+(2K+1)(2K-1)}\right]=Tan^{-1}(2K+1)-Tan^{-1}(2K-1)$

Expanding Σ we get, $Tan^{-1}1$

8. If $x\geq 1$, then $2Tan^{-1}x+Sin^{-1}\left(\frac{2x}{1+x^2}\right)=$ __________

(1) $4Tan^{-1}x$ (2) 0 (3) $\frac{\pi}{2}$ (4) π

Key. 4

Sol. When $x\geq 1$; $Sin^{-1}\frac{2x}{1+x^2}=\pi-2Tan^{-1}x$

9. $Cos^{-1}\ Cos(12)-Sin^{-1}(Sin12)$ = ________

(1) 0 (2) π (3) $8\pi-24$ (4) $8\pi+24$

Key. 3

Sol. $Cos^{-1}Cos(4\pi-12)-Sin^{-1}Sin(12-4\pi)$

10. If $Sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}.....\infty\right)+Cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}.....\infty\right)=\frac{\pi}{2}$ for $0<|x|<\sqrt{2}$.

Then x = ________

(1) $\frac{1}{2}$ (2) 1 (3) $-\frac{1}{2}$ (4) -1

Key. 2

Sol. $Sin^{-1}x+Cos^{-1}x=\frac{\pi}{2}$

$\therefore x-\frac{x^2}{2}+\frac{x^3}{4}.....\infty=x^2-\frac{x^4}{2}+\frac{x^6}{4}.....\infty$

11. The number of solutions of equation $Sin^{-1}\left(|x^2-1|\right)+Cos^{-1}\left(|2x^2-5|\right)=\frac{\pi}{2}$ is ______

(1) 1 (2) 0 (3) 3 (4) 2

Key. 4

Sol. $|x^2-1|=|2x^2-5|\Rightarrow x=\pm\sqrt{2}$

12. The number of real solutions of $Tan^{-1}\sqrt{x(x+1)}+Sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is _______

(1) 0 (2) 1 (3) 2 (4) infinite

Key. 3

Sol. $Cos^{-1}\frac{1}{\sqrt{x^2+x+1}}+Sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}\Rightarrow x^2+x+1=1$

13. $\sin^{-1}(\sin 5)>x^2-4x$ holds if

A) $x<2-\sqrt{9-2\pi}$ B) $-1<x<5$

C) $x\in(-\infty,-1)\cup(5,\infty)$ D) $x\in(2-\sqrt{9-2\pi},2+\sqrt{9-2\pi})$

Key. D

Sol. $\because \frac{3\pi}{2}<5<\frac{5\pi}{2}$ $\therefore \sin^{-1}(\sin 5)=5-2\pi$

Given $\sin^{-1}(\sin 5)>x^2-4x\Rightarrow x^2-4x<5-2\pi$

$\Rightarrow x^2-4x+(2\pi-5)<0$

Roots of $x^2-4x+2\pi-5=0$ are $2\pm\sqrt{9-2\pi}$

$\therefore x^2-4x+2\pi-5<0$

$\Rightarrow 2-\sqrt{9-2\pi}<x<2+\sqrt{9-2\pi}$

14. The value of $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)$ is

1) $\frac{14}{15}$ 2) $-\frac{14}{15}$ 3) $\frac{13}{12}$ 4) $\frac{12}{13}$

Key. 1

Sol. $\sin\left(\tan^{-1}\frac{3}{4}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)=\sin\left(\sin^{-1}\frac{3}{5}\right)+\cos\left(\cos^{-1}\frac{1}{3}\right)=\frac{3}{5}+\frac{1}{3}=\frac{14}{15}$

15. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

1) zero 2) one 3) two 4) infinite

Key. 3

Sol. $x(x+1)\geq 0$ and $x^2+x+1\leq 1\Rightarrow x(x+1)\leq 0$

$\therefore x=0$ or – 1

16. If $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right)<\pi-3$ then x satisfies

1) $|x|>1$ 2) $|x|<1$ 3) $|x|\leq 1$ 4) $|x|\geq 1$

Key. 2

Sol. $\sin^{-1}\left(\sin\left(\pi-\left(2+\frac{2}{1+x^2}\right)\right)\right)<\pi-3\Rightarrow \pi-2-\frac{2}{1+x^2}<\pi-3$

$\Rightarrow |x|<1$

17. The value of 'a' for which $ax^2+\sin^{-1}\left(x^2-2x+2\right)+\cos^{-1}\left(x^2-2x+2\right)=0$ has a real solution is

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$ 3) $\frac{2}{\pi}$ 4) $-\frac{2}{\pi}$

Key. 2

Sol. Clearly x = 1 is the only solution $\Rightarrow a+\frac{\pi}{2}=0\Rightarrow a=-\frac{\pi}{2}$

18. Given $0\leq x\leq\frac{1}{2}$ then the value of $\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$ is

1) – 1 2) 1 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

Key. 2

Sol. Put $x=\sin\theta\Rightarrow\tan\left[\sin^{-1}\left(\sin\left(\theta+\frac{\pi}{4}\right)\right)-\theta\right]=1$

19. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ then $\frac{\sum_{r=1}^{2}\left(x^{100r}+y^{103r}\right)}{\sum x^{201}y^{201}}=$

1) 0 2) 2 3) 4 4) $\frac{4}{3}$

Key. 4

Sol. x = y = z = 1

$\therefore$ Required value = $\frac{4}{3}$

20. Sum to infinite terms of the series

$\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)+.........$ is

1) $\frac{\pi}{4}$ 2) $\tan^{-1}2$ 3) $\tan^{-1}3$ 4) $\cot^{-1}3$

Key. 2

Sol.

$$T_n=\cot^{-1}\left(n^2+\frac{3}{4}\right)=\tan^{-1}\left(\frac{1}{n^2+\frac{3}{4}}\right)=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$$

$$S_n=\sum_{n=1}^{n}t_n=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\frac{1}{2}\Rightarrow S_\infty=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}=\cot^{-1}\frac{1}{2}=\text{Tan}^{-1}2$$

21. If α is the only real root of the equation $x^3+bx^2+cx+1=0$ (b<c) then the value of $\tan^{-1}\alpha+\tan^{-1}\left(\frac{1}{\alpha}\right)$ is

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$

3) 0 4) can not be determined

Key. 2

Sol. $f(0)=1,\ f(-1)=b-c<0\Rightarrow\alpha\in(-1,0)\Rightarrow\alpha<0$

$$\therefore\tan^{-1}(\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)=\tan^{-1}(\alpha)-\pi+\cot^{-1}\alpha=-\pi+\frac{\pi}{2}=-\frac{\pi}{2}$$

22. If the mapping f(x) = ax + b, a>0 maps [- 1,1] onto [0, 2] then $\cot\left[\cot^{-1}7+\cot^{-1}8+\cot^{-1}18\right]$ is equal to

1) f(-1) 2) f(0) 3) f(1) - 1 4) f(1) + 1

Key. 4

Sol. $f(-1)=0,\ f(1)=2\Rightarrow -a+b=0$

$a+b=2$

$\Rightarrow b=1,\ a=1\Rightarrow f(x)=x+1$

$$\cot\left[\text{Tan}^{-1}\frac{1}{7}+\text{Tan}^{-1}\frac{1}{8}+\text{Tan}^{-1}\frac{1}{18}\right]=\cot\left[\text{Tan}^{-1}\left(\frac{15}{55}\right)+\text{Tan}^{-1}\left(\frac{1}{18}\right)\right]$$

$$=\cot\left[\text{Tan}^{-1}\left(\frac{65}{195}\right)\right]=\cot\left[\text{Tan}^{-1}\left(\frac{1}{3}\right)\right]=3=f(2)$$

23. If $\cot^{-1}\left(\frac{n}{\pi}\right)>\frac{\pi}{6},\ n\in N$ then the maximum value of n is

1) 1 2) 5 3) 9 4) 6

Key. 2

Sol. $\frac{n}{\pi}<\cot\left(\frac{\pi}{6}\right)<\sqrt{3}\Rightarrow n<\sqrt{3}\pi<5.34$

24. The number of solutions for the equation $\cos^{-1}(1-x)+m\cos^{-1}x=\frac{n\pi}{2}$ where m > 0, $n\leq 0$ is

1) 0 2) 1 3) 2 4) Infinite

Key. 1

Sol. L.H.S is always +ve and R.H.S is negative.

There is no solution.

25. The value of $\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right]$ is

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

Key. 1

Sol. $\sin^{-1}\left[\cot\left(\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)+\cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}\sqrt{2}\right)\right]=\sin^{-1}[0]=0$

26. Number of solutions of the equation $\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}$ is

1) Infinite 2) 1 3) 0 4) 2

Key. 2

Sol. $\frac{\pi^2}{4}-2\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)=\frac{5\pi^2}{8}\Rightarrow\tan^{-1}x=\frac{-\pi}{4},\frac{3\pi}{4}$

27. The value of $\tan^2\left(\sec^{-1}3\right)+\cot^2\left(\operatorname{cosec}^{-1}4\right)$ is

1) 20 2) 21 3) 23 4) 25

Key. 3

Sol. Let $\sec^{-1}3=\alpha,\ \operatorname{cosec}^{-1}4=\beta\Rightarrow\tan^2\alpha+\cot^2\beta=9-1+16-1=23$

28. If $x=\sin\left(2\tan^{-1}2\right),\ y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$ then

1) x = 1 – y 2) $x^2 = 1 - y$ 3) $x^2 = 1 + y$ 4) $y^2 = 1 - x$

Key. 4

Sol. Let $\tan^{-1}2=\alpha \Rightarrow x=\sin 2\alpha=\frac{4}{5}$

$$y=\sin\left(\frac{\beta}{2}\right)=\sqrt{\frac{1-\cos\beta}{2}}=\sqrt{\frac{1-\frac{3}{5}}{2}}=\frac{1}{\sqrt{5}}$$

29. If $\cos^{-1}(\frac{x}{3})+\cos^{-1}(\frac{y}{2})=\frac{\theta}{2}$ then $4x^2-12xy\cos\frac{\theta}{2}+9y^2=$

1) 36 2) 36 – 36 cos θ 3) 18 – 18 cos θ 4) 18 + 18 cos θ

Key. 3

Sol. $\cos\frac{\theta}{2}=\frac{xy}{6}-\sqrt{\left(1-\frac{x^2}{9}\right)\left(1-\frac{y^2}{4}\right)} \Rightarrow \left(\cos\frac{\theta}{2}-\frac{xy}{6}\right)^2=1-\frac{x^2}{9}-\frac{y^2}{4}+\frac{x^2y^2}{36}$

30. The value of $\sin\left(\cot^{-1}(\cos(\tan^{-1}x))\right)$ is equal to

1) $\sqrt{\frac{x^2+1}{x^2+2}}$ 2) $\sqrt{\frac{x+2}{x^2+1}}$

3) $\sqrt{\frac{x^2-1}{x^2+2}}$ 4) $\sqrt{\frac{x^2+2}{x^2+1}}$

Key. 1

Sol. $\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+\cos^2\theta}}\right)\right)=\frac{1}{\sqrt{1+\cos^2\theta}}=\frac{\sec\theta}{\sqrt{1+\sec^2\theta}}=\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

31. If $\sin^{-1}\left(a-\frac{a^2}{3}+\frac{a^3}{9}-...........\infty\right)+\cos^{-1}\left(1+b+b^2+..........\infty\right)=\frac{\pi}{2}$ then

1) a = -3, b = 1 2) a = 1, b = $\frac{-1}{3}$

3) $a=\frac{1}{6}, b=\frac{1}{2}$ 4) $a=\frac{1}{6}, b=\frac{1}{3}$

Key. 2

Sol. $\frac{a}{1+\frac{a}{3}}=\frac{1}{1-b} \Rightarrow \frac{3a}{a+3}=\frac{1}{1-b}$

There are infinitely many solutions but option 'b' satisfies.

32. The value of $\cos^{-1}(\cos 10)=$

1) 10 2) $4\pi-10$ 3) $2\pi+10$ 4) $2\pi-10$

Key. 2

Sol. $\cos^{-1}(\cos 10) = \cos^{-1}(\cos(4\pi - 10)) = 4\pi - 10$

33. The sum to infinite terms of the series $\operatorname{cosec}^{-1}\sqrt{10} + \operatorname{cosec}^{-1}\sqrt{50} + \operatorname{cosec}^{-1}\sqrt{170} + \ldots\ldots\ldots + \operatorname{cosec}^{-1}\sqrt{(n^2+1)(n^2+2n+2)}$ is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

Key. 1

Sol. Given series = $\cot^{-1}(3) + \cot^{-1}(7) + \cot^{-1}(13) + \cot^{-1}(21) + \ldots\ldots$

$$= \operatorname{Tan}^{-1}\left(\frac{1}{3}\right) + \operatorname{Tan}^{-1}\left(\frac{1}{7}\right) + \operatorname{Tan}^{-1}\left(\frac{1}{13}\right) + \operatorname{Tan}^{-1}\left(\frac{1}{21}\right) + \ldots\ldots\ldots$$

$$= \sum_{n=1}^{\infty} \operatorname{Tan}^{-1}\left(\frac{1}{1+n(n+1)}\right) = \frac{\pi}{4}$$

34. Indicate the relation which is true?

1) $\tan\left|\tan^{-1} x\right| = |x|$ 2) $\cot\left|\cot^{-1} x\right| = |x|$

3) $\tan^{-1}|\tan x| = |x|$ 4) $\sin\left|\sin^{-1} x\right| = |x|$

Key. 1

Sol. $\left|\tan^{-1} x\right| = \tan^{-1} x$ if $= 0 \le x < \frac{\pi}{2}$

$= -\tan^{-1} x$ if $-\frac{\pi}{2} < x < 0$

but $\tan^{-1}|x| = \tan^{-1} x$ if x > 0

$= \tan^{-1} x$ if x < 0

$\therefore \left|\tan^{-1} x\right| = \tan^{-1}|x| \Rightarrow \tan\left(\tan^{-1}(|x|)\right) = |x|$

35. Identify the pair of functions which are not identical

1) $y = \tan(\cos^{-1} x); y = \frac{\sqrt{1-x^2}}{x}$ 2) $y = \tan(\cot^{-1} x); y = \frac{1}{x}$

3) $y = \sin(\text{arc tanx}); y = \frac{x}{\sqrt{1+x^2}}$ 4) $y = \cos(\tan^{-1}\text{x}); y = \sin(\tan^{-1} x)$

Key. 4

Sol. Conceptual

36. If $\cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha} = x$ then sinx =

1) $\tan^2\left(\frac{\alpha}{2}\right)$ 2) $\cot^2\left(\frac{\alpha}{2}\right)$ 3) $\tan\alpha$ 4) $\cot\left(\frac{\alpha}{2}\right)$

Key. 1

Sol. Let $\cot^{-1}\sqrt{\cos\alpha}=\theta \Rightarrow \sqrt{\cos\alpha}=\cot\theta$

$$\theta-\tan^{-1}(\cot\theta)=x \Rightarrow \theta-\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\theta\right)\right)=x$$

$$\Rightarrow 2\theta-\frac{\pi}{2}=x$$

$$\Rightarrow \sin x=-\cos 2\theta=-\frac{1-\frac{1}{\cos\alpha}}{1+\frac{1}{\cos\alpha}}=\tan^2\left(\frac{\alpha}{2}\right)$$

37. If $x\geq 1$ then $2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)=$

1) $4\tan^{-1}x$ 2) 0 3) $\frac{\pi}{2}$ 4) π

Key. 4

Sol. If $x\geq 1$ then $2\tan^{-1}x=\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\tan^{-1}\left(\frac{2x}{x^2-1}\right)$$

38. The root of the equation $\tan^{-1}\left(\frac{x-1}{x+1}\right)+\tan^{-1}\left(\frac{2x-1}{2x+1}\right)=\tan^{-1}\left(\frac{23}{36}\right)$ is

1) $-\frac{3}{8}$ 2) $-\frac{1}{2}$ 3) $\frac{3}{4}$ 4) $\frac{4}{3}$

Key. 4

Sol. $\frac{2x^2-1}{3x}=\frac{23}{36}\Rightarrow x=\frac{4}{3},\frac{-3}{8}$ but x can not be negative.

39. Find maximum value of x for which $2\tan^{-1}x+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is independent of x.

Key. 0

Sol. Let x = tanθ $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$2\theta+\cos^{-1}\cos 2\theta$

(i) $0\leq 2\theta<\frac{\pi}{2}.2$

$\cos^{-1}\cos 2\theta=2\theta$

So given expression = 4θ = $4\tan^{-1}x$

(ii) $-\frac{\pi}{2}<\theta\le 0 \quad \Rightarrow -\pi<2\theta\le 0$

$\cos^{-1}\cos 2\theta=-2\theta$

So, given expression becomes independent of θ.

For $-\frac{\pi}{2}<\theta\le 0 \quad \Rightarrow -\infty<x\le 0$

40. $\sin^{-1}|\cos x|-\cos^{-1}|\sin x|=a$ has at least one solution iff $a\in$

a) $\{0\}$ b) $\left[0,\frac{\pi}{2}\right]$ c) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ d) $(0,\pi)$

Key. A

Sol. $\sin^{-1}|\cos x|-\cos^{-1}|\sin x|=\frac{\pi}{2}-\cos^{-1}|\cos x|-\frac{\pi}{2}+\sin^{-1}|\sin x|=a$

$\Rightarrow \sin^{-1}|\sin x|-\cos^{-1}|\cos x|=a$

$\Rightarrow a=0\forall x$

41. If x_1,x_2,x_3,x_4 are the roots of the equation $x^4-x^3\sin 2\beta+x^2\cos 2\beta-x\cos\beta-\sin\beta=0,$ then $\sum_{i=1}^{4}\tan^{-1}x_i$ is equal to

a) β b) $\frac{\pi}{2}-\beta$ c) $\pi-\beta$ d) $-\beta$

Key. B

Sol. $\sum_{i=1}^{4}\tan^{-1}x_i=\tan^{-1}\left(\frac{\sum x_1-\sum x_1x_2x_3}{1-\sum x_1x_2+\pi x_i}\right)=\tan^{-1}\left(\frac{\sin 2\beta-\cos\beta}{1-\cos 2\beta-\sin\beta}\right)$

$=\tan^{-1}\left\{\frac{\cos\beta(2\sin\beta-1)}{\sin\beta(2\sin\beta-1)}\right\}=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\beta\right)\right)=\frac{\pi}{2}-\beta$

42. The least and greatest values of $\left(\sin^{-1}x\right)^3+\left(\cos^{-1}x\right)^3$ are respectively : $x\in[-1,1]$

a) $-\frac{\pi}{2},\frac{\pi}{2}$ b) $-\frac{\pi^3}{8},\frac{\pi^3}{8}$ c) $\frac{\pi^3}{32},\frac{7\pi^3}{8}$ d) $\frac{\pi^3}{32},\frac{\pi^3}{8}$

Key. C

Sol. $\left(\sin^{-1}x+\cos^{-1}x\right)^3-3\sin^{-1}x\cos^{-1}x\left(\sin^{-1}x+\cos^{-1}x\right)$

$=\frac{\pi^3}{8}-\frac{3\pi}{2}\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)$

$=\frac{\pi^3}{8}+\frac{3\pi}{2}\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\right]-\frac{3\pi^3}{32}=\frac{\pi^3}{32}+\frac{3\pi}{2}\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\left(\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\le\left(\frac{3\pi}{4}\right)^2\right)$

L.V.= $\frac{\Pi^3}{32}$ & $G.V = \frac{7\Pi^3}{8}$

43. The area bounded by the identity curve in the first quadrant, y = 0 and $x = \sin^{-1}\left(a^4+1\right)+\cos^{-1}\left(a^4+1\right)-\tan^{-1}\left(a^4+1\right)$ is

a) $\frac{\pi^2}{8}-\frac{a^2}{4}$ b) $\frac{\pi^2}{32}+\frac{a^2}{4}$ c) $\frac{\pi^2}{16}+\frac{a^2}{2}$ d) $\frac{\pi^2}{8}-\frac{a^2}{4}$

Key. B

Sol. x is defined if $a^4 = 0 \Rightarrow a = 0$.

$\Rightarrow x = \frac{\pi}{2}+0-\frac{\pi}{4}=\frac{\pi}{4}$.

Area = $\frac{1}{2}\frac{\pi}{4}\frac{\pi}{4}=\frac{\pi^2}{32}$ sq. units

44. Let $I_1 = \frac{\pi^2}{16}+\sqrt{2}, I_2 = \left(\tan^{-1}\left(\frac{1}{e}\right)\right)^2+\frac{2e}{\sqrt{e^2+1}}$, $I_3 = \left(\tan^{-1}e\right)^2+\frac{2}{\sqrt{e^2+1}}$, then which of the following is true.

a) $I_1 < I_2 < I_3$ b) $I_2 < I_1 < I_3$ c) $I_1 < I_3 < I_2$ d) $I_3 < I_2 < I_1$

Key. B

Sol. Consider the function $f(x) = \left(\tan^{-1}x\right)^2+\frac{2}{\sqrt{x^2+1}}$ then $f^1(x) > 0 \ \forall \ x \in (0,\infty)$

$\Rightarrow f\left(\frac{1}{e}\right) < f(1) < f(e) \Rightarrow I_2 < I_1 < I_3$

45. If 0 < x < 1, then $\sqrt{1+x^2}$ $[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x\}^2 - 1]^{1/2}$ =

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Key. C

Sol. Exp. $= \sqrt{1+x^2}\left[\left\{x.\cos\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right)+\sin\left(\sin^{-1}\frac{1}{1+x^2}\right)\right\}^2-1\right]^{1/2}$

$= \sqrt{1+x^2}\left[\left\{x.\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right\}^2-1\right]^{1/2}$

$= \sqrt{1+x^2}[1+x^2-1]^{1/2} = x\sqrt{1+x^2}$

46. The value of 'a' for which $ax^2+\sin^{-1}(x^2-2x+2)+\cos^{-1}(x^2-2x+2)=0$ has a real solution , is

a) $-\frac{2}{\pi}$ b) $\frac{2}{\pi}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$

Key. C

Sol. Here, $x^2-2x+2=(x-1)^2+1\ge 1$

But $-1\le\left(x^2-2x+2\right)\le 1$

Which is possible only when $x^2-2x+2=1$

$\therefore x=1$

Then, $a(1)^2+\sin^{-1}(1)+\cos^{-1}(1)=0$

$\Rightarrow a+\frac{\pi}{2}+0=0$

$\therefore a=-\frac{\pi}{2}$

47. The value of tan $\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right)-\frac{\pi}{2}\right)\right\}$ is

a) $\frac{2}{3\sqrt{5}}$ b) $\frac{2}{3}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$

Key. A

Sol. $\tan\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right)-\frac{\pi}{2}\right)\right\}=\tan\left\{\pi-\cos^{-1}\left(\frac{2}{7}\right)-\frac{\pi}{2}\right\}$

$=\tan\left\{\frac{\pi}{2}-\cos^{-1}\left(\frac{2}{7}\right)\right\}=\tan\left\{\sin^{-1}\left(\frac{2}{7}\right)\right\}=\tan\tan^{-1}\left(\frac{2}{3\sqrt{5}}\right)=\frac{2}{3\sqrt{5}}$

48. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{4\pi}{3}$

Key. A

Sol. $\because \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)+\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)=\frac{2\pi}{3}+\left(\pi-\frac{2\pi}{3}\right)=\pi$

49. The range of the function $f(x)=2\tan^{-1}\left(\frac{1+x}{1-x}\right)+\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

A) $\{\pi\}$ B) $\{-\pi,\pi\}$ C) $(-\pi,\pi]$ D) $[-\pi,\pi]$

Key. D

Sol.
$$f(x)=\begin{cases}\pi+4Tan^{-1}x & ,x<0\\ \pi & ,0\le x<1\\ -\pi & ,x>1\end{cases}$$

50. If domain of the function $f\left(x\right)=\cos^{-1}\frac{x^3}{2}+\sqrt{\sin^{-1}\log_2\left(x+1\right)}$ is [a,b], then a + 2b =

(A) 0 (B) 1 (C) 2 (D) 3

Key. C

Sol. $-1\le\frac{x^3}{2}\le1$ $0\le\log_2\left(x+1\right)\le1$

$\Rightarrow-\sqrt[3]{2}\le x\sqrt[3]{2}\Rightarrow0\le x\le1$

51. If $y=\cot^{-1}\left(\sqrt{\cos2\theta}\right)-\tan^{-1}\left(\sqrt{\cos2\theta}\right)$ then $\sin y$ =

a) $\tan^2\theta$ b) $\cot^2\theta$

c) $\tan\theta$ d) $\cot\theta$

Key. A

Sol. $\frac{\pi}{2}-y=2\tan^{-1}\left(\sqrt{\cos^2\theta}\right)\Rightarrow\sin y=\cos\left(\frac{\pi}{2}-y\right)=\tan^2\theta$

52. If $\alpha,\beta>0$ then $\frac{\alpha^3}{2}\cos ec^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)=$

a) $\left(\alpha-\beta\right)\left(\alpha^2+\beta^2\right)$ b) $\left(\alpha+\beta\right)\left(\alpha^2-\beta^2\right)$ c) $\left(\alpha+\beta\right)\left(\alpha^2+\beta^2\right)$ d) 0

Key. C

Sol. Let $\frac{1}{2}\tan^{-1}\left(\frac{\alpha}{\beta}\right)=\theta;\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\alpha}\right)=\frac{\pi}{4}-\theta$

G.E = $\frac{\alpha^3}{2\sin^2\theta}+\frac{\beta^2}{2\cos^2\left(\frac{\pi}{4}-\theta\right)}=\frac{\alpha^3}{1-\cos^2\theta}+\frac{\beta^3}{1+\sin^2\theta}$

$$=\frac{\alpha^3\left(\sqrt{\alpha^2+\beta^2}\right)}{\sqrt{\alpha^2+\beta^2-\beta}}+\frac{\beta^3\sqrt{\alpha^2+\beta^2}}{\sqrt{\alpha^2+\beta^2+\alpha}}=\left(\alpha+\beta\right)\left(\alpha^2+\beta^2\right)$$

53. The value of the expression $\sin^{-1}\left(\sin\frac{22\pi}{7}\right)+\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\tan^{-1}\left(\tan\frac{5\pi}{7}\right)+\sin^{-1}\left(\cos2\right)$ is

A) $\frac{17\pi}{42}-2$ B) -2 C) $\frac{-\pi}{21}-2$ D) none of these

Key. A

Sol. $\sin^{-1}\sin\left(\frac{22\pi}{7}\right)=\sin^{-1}\sin\left(3\pi+\frac{\pi}{7}\right)=-\frac{\pi}{7}$

$\cos^{-1}\cos\left(\frac{5\pi}{3}\right)=\cos^{-1}\cos\left(2\pi-\frac{\pi}{3}\right)=\frac{\pi}{3}$

$\tan^{-1}\tan\left(\frac{5\pi}{7}\right)=\tan^{-1}\tan\left(\pi-\frac{2\pi}{7}\right)=-\frac{2\pi}{7}$

$\sin^{-1}\cos(2)=\frac{\pi}{2}-\cos^{-1}\cos 2=\frac{\pi}{2}-2$

$\therefore$ Required Value $=-\frac{\pi}{7}+\frac{\pi}{3}-\frac{2\pi}{7}+\frac{\pi}{2}-2=\frac{(-18+35)\pi}{42}-2=\frac{17\pi}{42}-2$

54. If $\sin^{-1}(x-1)+\cos^{-1}(x-3)+\tan^{-1}\left(\frac{x}{2-x^2}\right)=\cos^{-1}k+\pi$, then the value of K =

A) 1 B) $-\frac{1}{\sqrt{2}}$ C) $\frac{1}{\sqrt{2}}$ D) none of these

Key. C

Sol. $\sin^{-1}(x-1) \Rightarrow -1\le x-1\le 1 \Rightarrow 0\le x\le 2$

$\cos^{-1}(x-3) \Rightarrow -1\le x-3\le 1 \Rightarrow 2\le x\le 4$

$\tan^{-1}\left(\frac{x}{2-x^2}\right) \Rightarrow x\in R,\ x\ne\sqrt{2},-\sqrt{2}$

$\therefore$ $x=2$

$\sin^{-1}(2-1)+\cos^{-1}(2-3)+\tan^{-1}\frac{2}{2-4}=\cos^{-1}k+\pi$

$\Rightarrow$ $\sin^{-1}1+\cos^{-1}(-1)+\tan^{-1}(-1)=\cos^{-1}k+\pi$

$\frac{\pi}{2}+\pi-\frac{\pi}{4}=\cos^{-1}k+\pi$

$\Rightarrow$ $\cos^{-1}K=\frac{\pi}{4}$ $\Rightarrow$ $K=\frac{1}{\sqrt{2}}$

55. If $\sin^{-1}\sin(5)>x^2-4x$, then the number of possible integral values of x is

A) 1 B) 2 C) 3 D) none of these

Key. C

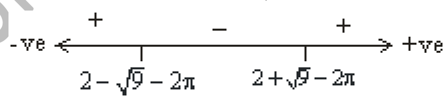
Sol. $\sin^{-1}\sin 5=\sin^{-1}\sin(5-2\pi)=5-2\pi$ $\left(\text{As}-\frac{\pi}{2}\le 5-2\pi\le\frac{\pi}{2}\right)$

$\therefore$ $\sin^{-1}\sin 5>x^2-4x$

$\Rightarrow$ $5-2\pi>x^2-4x$

$\Rightarrow$ $x^2-4x+2\pi-5<0$

Sign sum of $\left(x^2-4x+2\pi-5\right)$



$2-\sqrt{9-2\pi}<x<2+\sqrt{9-2\pi}$

Integral values of x are 1, 2, 3

Number of integral value of x = 3

56. If $x\in[-1,0)$, then $\cos^{-1}(2x^2-1)-2\sin^{-1}x=$

A) $-\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) -2π

Key. B

Sol. $\cos^{-1}\left(2x^2-1\right)=2\pi-2\cos^{-1}x$ (as $x<0$)

$\cos^{-1}\left(2x^2-1\right)-2\sin^{-1}x=2\pi-2\cos^{-1}x-2\sin^{-1}x$

$=2\pi-2\left(\cos^{-1}x+\sin^{-1}x\right)$

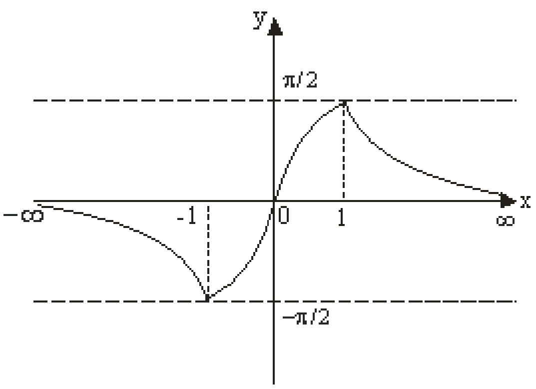
$=2\pi-2\frac{\pi}{2}=\pi$

57. α_1 and α_2 satisfies $\sin^{-1}\frac{2x}{1+x^2}=\tan^{-1}\frac{2x}{1-x^2}$ and $\left|\alpha_1-\alpha_2\right|<K$, for all α_1 and α_2 then k is equal to

A) 1 B) 3/2 C) 2 D) none of these

Key. C

Sol. Graph $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

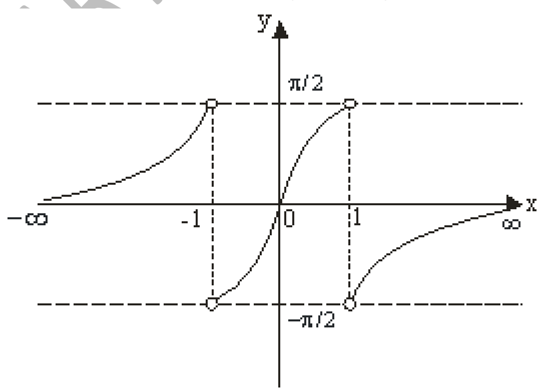


Graph of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

From graph one can say

$-1<x<1 \Rightarrow -1<\alpha_1<1$

$-1<\alpha_2<1 \Rightarrow \left|\alpha_1-\alpha_2\right|<2$



58. The solution of the inequality $\log_{1/2}\sin^{-1}x>\log_{1/2}\cos^{-1}x$ is

A) $x \in \left[0, \frac{1}{\sqrt{2}}\right]$ B) $x \in \left(\frac{1}{\sqrt{2}}, 1\right]$ C) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$ D) None of these

Key. C

Sol. $\log_{1/2} \sin^{-1} x > \log_{1/2} \cos^{-1} x$

$\Leftrightarrow \cos^{-1} x > \sin^{-1} x, \quad 0 < x < 1$

$\Leftrightarrow \cos^{-1} x > \frac{\pi}{2} - \cos^{-1} x, \ 0 < x < 1$

$\Leftrightarrow \cos^{-1} x > \frac{\pi}{4}, \ 0 < x < 1$

$\Leftrightarrow 0 < x < \frac{1}{\sqrt{2}}$

59. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is

A) $\frac{6}{17}$ B) $\frac{7}{16}$ C) $\frac{16}{7}$ D) none of these

Key. D

Sol. Since $\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

$\therefore \quad \tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right] = \tan\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right] = \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4}.\frac{2}{3}} = \frac{17}{6}$

60. If $\tan^{-1}\left(\sqrt{\cos\alpha + 1}\right) + \cot^{-1}\left(\sqrt{\cos\alpha + 1}\right) = \mu$ (where $\alpha \neq n\pi + \frac{\pi}{2}, n \in I$), then $\sin\mu$ is equal to

A) $\tan^2\alpha$ B) $\tan 2\alpha$ C) $\sec^2\alpha - \tan^2\alpha$ D) $\cos^2\frac{\alpha}{2}$

Key. C

Sol. Since $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \ \forall x \in R$,

$\therefore \quad \mu = \frac{\pi}{2} \quad \Rightarrow \quad \sin\mu = 1 = \sec^2\alpha - \tan^2\alpha$

61. Range of the function $f(x) = \cos^{-1}(-\{x\})$, where {.} is fractional part function, is

A) $\left(\frac{\pi}{2}, \pi\right)$ B) $\left(\frac{\pi}{2}, \pi\right]$ C) $\left[\frac{\pi}{2}, \pi\right)$ D) $\left(0, \frac{\pi}{2}\right]$

Key. C

Sol. $\therefore \quad \frac{\pi}{2} \leq \cos^{-1}(-x\{x\}) < \pi$

$\therefore$ the range is $\left[\frac{\pi}{2}, \pi\right)$

62. The sum of solutions of the equation $2\sin^{-1}\sqrt{x^2 + x + 1} + \cos^{-1}\sqrt{x^2 + x} = \frac{3\pi}{2}$ is

A) 0 B) – 1 C) 1 D) 2

Key. B

Sol. $0 \le x^2 + x + 1 \le 1$ and $0 \le x^2 + x \le 1$

$\therefore$ x = - 1, 0

For x = - 1

L.H.S $= 2\sin^{-1}1 + \cos^{-1}0 = \frac{3\pi}{2}$

$\therefore$ x = - 1 is a solution

for x = 0 L.H.S $= 2\sin^{-1}1 + \cos^{-1}0 = \frac{3\pi}{2}$

$\therefore$ x = 0 is a solution

$\therefore$ sum of the solution = - 1

63. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then the value of $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)}$ will be

A) 2abc B) abc C) $\frac{1}{2}$abc D) $\frac{1}{3}$abc

Key. A

Sol. Let $\sin^{-1} a = A$,

$\sin^{-1} b = B$

$\sin^{-1} c = C$

$\therefore$ sin A = a, sin B = b, sin C = c

and A + B + C = π, then

sin 2A + sin 2B + 2C = 4 sin A sin B sin C

Now $a\sqrt{(1-a^2)} + b\sqrt{(1-c^2)} + c\sqrt{(1-c^2)}$ = sin A cos A + sin B cos B + sin C cos C

$= \frac{1}{2}$ [sin 2A + sin 2B + sin 2C] = 2 sin A sin B sin C = 2abc

64. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$, then x =

A) $\frac{3\pi}{4}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) None of these

Key. B

Sol. We have $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow$ $\tan(2\tan^{-1}\cos x) = 2 \operatorname{cosec} x$

$\Rightarrow$ $\frac{2\cos x}{1-\cos^2 x} = 2\operatorname{cosec} x$ $\Rightarrow$ $\frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x$

$\Rightarrow$ $\sin x = \cos x$ $\Rightarrow$ $x = \frac{\pi}{4}$.

65. $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] =$

A) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$ B) $\frac{\pi}{4} + \cos^{-1}x^2$ C) $\frac{\pi}{4} - \cos^{-1}x^2$ D) $\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$

Key. A

Sol. $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$ (putting $x^2=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x^2$)

$=\tan^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right)=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)=\tan^{-1}\tan\left(\frac{\pi}{4}+\theta\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$

66. If x_1, x_2, x_3, x_4 are roots of the equation $x^4-x^3\sin 2\beta+x^2\cos 2\beta-x\cos\beta-\sin\beta=0$, then $\tan^{-1}x_1+\tan^{-1}x_2+\tan^{-1}x_3+\tan^{-1}x_4=$

A) β B) $\frac{\pi}{2}-\beta$ C) $\pi-\beta$ D) $-\beta$

Key. B

Sol. We have

$\Sigma x_1=\sin 2\beta,\ \Sigma x_1x_2=\cos 2\beta, \Sigma x_1x_2x_3=\cos\beta$

and $x_1x_2x_3x_4=-\sin\beta$

$\therefore \quad \tan^{-1}\left(\frac{\Sigma x_1-\Sigma x_1x_2x_3}{1-\Sigma x_1x_2+x_1x_2x_3x_4}\right)=\tan^{-1}\left(\frac{\sin 2\beta-\cos\beta}{1-\cos 2\beta-\sin\beta}\right)$

$=\tan^{-1}\left(\frac{(2\sin\beta-1)\cos\beta}{\sin\beta(2\sin\beta-1)}\right)=\tan^{-1}(\cot\beta)=\tan^{-1}\left[\tan\left(\frac{\pi}{2}-\beta\right)\right]=\frac{\pi}{2}-\beta$.