Hyperbola

Single Correct Answer Type

1. A line drawn through the point P (-1, 2) meets the hyperbola $xy = c^2$ at the points A and B. (points A and B lie on same side of P) and Q is a point on AB such that PA, PQ and PB are in H.P then locus of Q is

A. $x-2y=2c^2$ B. $2x-y=2c^2$ C. $2x+y+2c^2=0$ D. $x+2y=2c^2$

Key. B

Sol. Locus of Q is $S_1 = 0$

$2x-y=2c^2$

2. If the asymptote of the hyperbola $(x+y+1)^2-(x-y-3)^2=5$ cut each other at A and the coordinate axis at B and C then radius of circle passing through the points A,B,C is

A. 3 B. $\frac{\sqrt{5}}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\sqrt{3}$

Key. B

Sol. Centre of rectangular hyperbola = (1,-2)

So equation of asymptotes are x = 1, y = -2

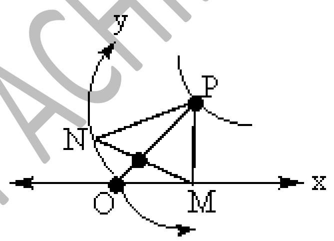
So radius of circle $=\frac{\sqrt{5}}{2}$

3. PM and PN are the perpendiculars from any point P on the rectangular hyperbola xy = 8 to the asymptotes. If the locus of the mid point of MN is a conic, then the least distance of (1, 1) to director circle of the conic is

A. $\sqrt{3}$ B. $\sqrt{2}$ C. $2\sqrt{3}$ D. $2\sqrt{5}$

Key. B

Sol. OMPN is rectangle.



$P=(Ct,\frac{c}{t})$

Mid point $=\left(\frac{ct}{2},\frac{c}{2t}\right)=(x,y)$ $\therefore cy=\frac{c^2}{4}\Rightarrow e=\sqrt{2}$

4. A hyperbola passing through origin has 3x – 4y – 1=0 and 4x – 3y – 6 = 0 as its asymptotes. Then the equations of its transverse and conjugate axes are

A) x – y – 5 = 0 and x + y + 1 = 0 B) x – y = 0 and x + y + 5 = 0

C) x + y – 5 = 0 and x – y – 1 = 0 D) x + y – 1 = 0 and x – y – 5 = 0

Key. C

Sol. Transverse and conjugate axes are the bisectors of the angle between asymptotes.

$$\frac{3x-4y-1}{5}=\pm\left(\frac{4x-3y-6}{5}\right) \text{ etc......}$$

5. If the asymptotes of the hyperbola $(x+y+1)^2-(x-y-3)^2=5$ cuts each other at A and the coordinate axes at B and C, then radius of the circle passing through the points A, B, C is

A) 3 B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{3}$

Key. B

Sol. (B) Centre of rectangular hyperbola (1, –2)

So equation of asymptotes are x = 1, y = –2

So radius of circle = $\frac{\sqrt{5}}{2}$

6. If a chord joining P($a\text{Sec}\theta, a\tan\theta$), Q($a\text{Sec}\alpha, a\tan\alpha$) on the hyperbola $x^2-y^2=a^2$ is the normal at P,then $\text{Tan}\,\alpha =$

A) $\text{Tan}\theta(4\sec^2\theta+1)$ B) $\text{Tan}\theta(4\sec^2\theta-1)$ C) $\text{Tan}\theta(2\text{Sec}^2\theta-1)$ D) $\text{Tan}\theta(1-2\text{Sec}^2\theta)$

Key. B

Sol. Slope of chord joining P and Q = slope of normal at P

$\frac{\text{Tan}\alpha-\text{Tan}\theta}{\sec\alpha-\sec\theta}=-\frac{\text{Tan}\theta}{\sec\theta} \Rightarrow \text{Tan}\alpha-\text{Tan}\theta=-k\text{Tan}\theta$ and $\sec\alpha-\sec\theta=k\sec\theta$

$\therefore (1-k)Tan\theta = Tan\alpha \rightarrow 1.\ (1+k)\sec\theta=\sec\alpha \rightarrow 2.$

$[(1+k)\sec\theta]^2-[(1-k)\text{Tan}\theta]^2 = \sec^2\alpha-\text{Tan}^2\alpha$

$\Rightarrow k=-2(\sec^2\theta+\text{Tan}^2\theta) = -4\sec^2\theta+2$

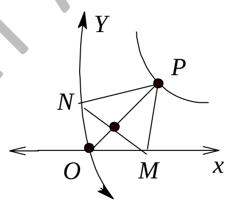
From (1) $Tan\alpha = Tan\theta\ (1+4\sec\theta^2-2)=\text{Tan}\theta(4\sec\theta^2-1)$.

7. PM and PN are the perpendiculars from any point P on the rectangular hyperbola $xy=c^2$ to the asymptotes. If the locus of the mid point of MN is a conic, then its eccentricity is

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{\sqrt{2}}$

Key. B

Sol. OMPN is rectangle.



$P=\left(Ct,\frac{c}{t}\right)$

Mid point = $\left(\frac{ct}{2},\frac{c}{2t}\right)=(x,y)$ $\therefore xy=\frac{c^2}{4} \Rightarrow e=\sqrt{2}$

8. A variable straight line of slope 4 intersects the hyperbola xy = 1 at two points. The locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

A) $16x^2+10xy+y^2=2$ B) $16x^2-10xy+y^2=2$

C) $16x^2+10xy+y^2=4$ D) $16x^2-10xy+y^2=4$

Key. A

Sol. Let P(h, k)

y - k = 4(x - h) --- (1)

Let it meets xy = 1 ----(2) at A (x_1, y_1) and B (x_2, y_2)

$$x_1+x_2=\frac{4h-k}{4}, x_1x_2=-\frac{1}{4} \text{Also} \Rightarrow \therefore \frac{2x_1+x_2}{3}=h \Rightarrow x_1=\frac{8h+k}{4}, x_2=\frac{2h+k}{2}$$

$$\Rightarrow 16x^2+10xy+y^2=2$$

9. The length of the transverse axis of the hyperbola $9x^2-16y^2-18x-32y-151=0$ is

1) 8 2) 4 3) 6 4) 2

Key. 1

Sol. Given hyperbola is $\frac{(x-1)^2}{16}-\frac{(y+1)^2}{9}=1$

Length of the transverse axis is 2a=8.

10. The equation of a hyperbola , conjugate to the hyperbola $x^2+3xy+2y^2+2x+3y=0$ is

1) $x^2+3xy+2y^2+2x+3y+1=0$ 2) $x^2+3xy+2y^2+2x+3y+2=0$

3) $x^2+3xy+2y^2+2x+3y+3=0$ 4) $x^2+3xy+2y^2+2x+3y+4=0$

Key. 2

Sol. Let $H=x^2+3xy+2y^2+2x+3y=0$ and C=0 is its conjugate. Then C + H=2A, where A=0 is the combined equation of asymptotes. Equation of asymptotes is $x^2+3xy+2y^2+2x+3y+\lambda=0$, where $\Delta=abc+2fgh-af^2-bg^2-ch^2=0 \Rightarrow \lambda=1$

$\therefore C=2\left(x^2+3xy+2y^2+2x+3y+1\right)-\left(x^2+2y^2+3xy+2x+3y\right)$

$\Rightarrow$ equation of conjugate hyperbola is $x^2+3xy+2y^2+2x+3y+2=0$

11. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB is an equilateral triangle O being the origin, then the eccentricity of the hyperbola satisfies

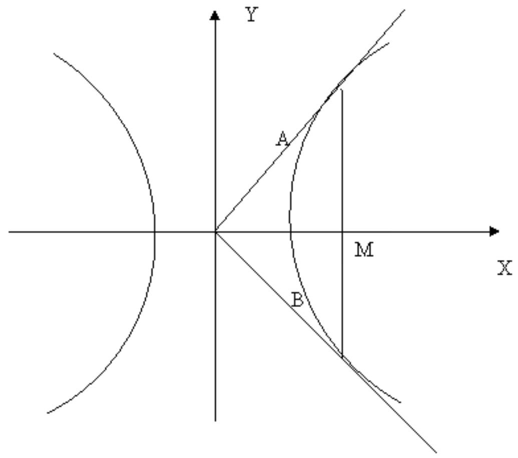
1) $e>\sqrt{3}$ 2) $1<e<\frac{1}{\sqrt{3}}$ 3) $e=\frac{2}{\sqrt{3}}$ 4) $e>\frac{2}{\sqrt{3}}$

Key. 4

Sol. Let the length of the double ordinate be 2ℓ

$\therefore$ AB=2ℓ and AM=BM=ℓ

Clearly ordinate of point A is ℓ.



The abscissa of the point A is given by

$$\frac{x^2}{a^2}-\frac{l^2}{b^2}=1 \Rightarrow x=\frac{a\sqrt{b^2+l^2}}{b}$$

$\therefore$ A is $\left(\frac{a\sqrt{b^2+l^2}}{b}, l\right)$

Since ΔOAB is equilateral triangle, therefore

OA=AB=OB=$2l$

Also, $OM^2+AM^2=OA^2 \therefore \frac{a\left(b^2+l^2\right)}{b}+l^2=4l^2$

We get $l^2=\frac{a^2b^2}{3b^2-a^2}$

Since $l^2>0 \therefore \frac{a^2b^2}{3b^2-a^2}>0 \Rightarrow 3b^2-a^2>0$

$$\Rightarrow 3a^2\left(e^2-1\right)-a^2>0 \Rightarrow e>\frac{2}{\sqrt{3}}$$

12. If the line $5x+12y-9=0$ is a tangent to the hyperbola $x^2-9y^2=9$, then its point of contact is

1) (-5,4/3) 2) (5,-4/3) 3) $(3,-1/2)$ 4) (5,4/3)

Key. 2

Sol. Common Point

13. Any chord passing through the focus $(ae,0)$ of the hyperbola $x^2 - y^2 = a^2$ is conjugate to the line

1) $ex - a = 0$ 2) $ae + x = 0$ 3) $ax + e = 0$ 4) $ax - e = 0$

Key. 1

Sol. $S_1 = 0$

14. Number of points from where perpendicular tangents to the curve $\frac{x^2}{16} - \frac{y^2}{25} = 1$ can be drawn, is:

1) 1 2) 2 3) 0 4) 3

Key. 3

Sol. Director circle is set of points from where drawn tangents are perpendicular in this case $x^2 + y^2 = a^2 - b^2$ (equation of director circle)i.e., $x^2 + y^2 = -9$ is not a real circle so there is no points from where tangents are perpendicular.

15. $x^2 - y^2 + 5x + 8y - 4 = 0$ represents

1) Rectangular hyperbola 2)Ellipse

3) Hyperbola with centre at (1,1) 4)Pair of lines

Key. 1

Sol. $\Delta \neq 0,\ x^2 - ab > 0, a + b = 0$

16.

1) $(2\sqrt{2}, 2\sqrt{2}), (-2\sqrt{2}, -2\sqrt{2})$ 2) $(-3\sqrt{2}, -3\sqrt{2}), (3\sqrt{2}, 3\sqrt{2})$

3) $(2\sqrt{2}, -2\sqrt{2}), (-2\sqrt{2}, 2\sqrt{2})$ 4) (-2. 2)

Key. 1

Sol. foci of $xy = c^2$ is $(\pm c\sqrt{2}, \pm c\sqrt{2})$

17. Which of the following is INCORRECT for the hyperbola $x^2 - 2y^2 - 2x + 8y - 1 = 0$

1) Its eccentricity is $\sqrt{2}$ 2) Length of the transverse axis is $2\sqrt{3}$

3) Length of the conjugate axis is $2\sqrt{6}$ 4) Latus rectum $4\sqrt{3}$

Key. 1

Sol. The equation of the hyperbola is $x^2-2y^2-2x+8y-1=0$

Or $(x-1)^2-2(y-2)^2+6=0$

Or $\frac{(x-1)^2}{-6}+\frac{(y-2)^2}{3}=1;$ or $\frac{(y-2)^2}{3}-\frac{(x-1)^2}{6}=1\rightarrow 1$

Or $\frac{Y^2}{3}-\frac{X^2}{6}=1$, where X = x -1 and Y = y - 2 $\rightarrow 2$

$\therefore$ the centre=(0,0)in the X-Y coordinates.

$\therefore$ the centre=(1,2)in the x-y coordinates .using $\rightarrow 2$

If the transverse axis be of length 2a, then a= $\sqrt{3}$,since in the equation (1) the transverse axis is parallel to the y-axis.

If the conjugate axis is of length 2b,then b = $\sqrt{6}$

But $b^2=a^2\left(e^2-1\right)$

$\therefore$ $6=3\left(e^2-1\right),\therefore e^2=3$ or $e=\sqrt{3}$

The length of the transverse axis = $2\sqrt{3}$

The length of the conjugate axis = $2\sqrt{6}$

Latus rectum $4\sqrt{3}$

18. If the curve $xy=R^2-16$ represents a rectangular hyperbola whose branches lies only in the quadrant in which abscissa and ordinate are opposite in sign but not equal in magnitude, then

1) $|R|<4$ 2) $|R|\geq 4$ 3) $|R|=4$ 4) $|R|=5$

Key. 1

Sol. conceptual

19. If the line ax + by + c=0 is a normal to the curve xy=1,then

1) a > 0, b > 0 2) a < 0,b < 0 3) a < 0,b > 0 4) a=b=1

Key. 3

Sol. Slope of the line $\frac{-a}{b}$ is equal to slope of the normal to the curve.

∴ either a > 0 & b < 0 (or) a < 0 & b > 0.

20. The equation of normal at $\left(at, \frac{a}{t}\right)$ to the hyperbola $xy = a^2$ is

1) $xt^3 - yt + at^4 - a = 0$
2) $xt^3 - yt - at^4 + a = 0$
3) $xt^3 + yt + at^4 - a = 0$
4) $xt^3 + yt - at^4 - a = 0$

Key. 2

Sol. Equation of tangent is $s_1 = 0$ normal is $\perp^r$ to tangent and passing through $\left(at, \frac{a}{t}\right)$ is $xt^3 - yt - at^4 + a = 0$

21. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ to its asymptotes is equal to:

1) $\frac{6}{5}$
2) $\frac{36}{13}$
3) Depending on θ
4) $\frac{5}{6}$

Key. 2

Sol. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is equal to $\frac{a^2b^2}{a^2 + b^2}$

22. The foot of the perpendicular from the focus to an asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

1) (ae , be)
2) (a/e,b/e)
3) (e/a,e/b)
4) (be,ae)

Key. 2

Sol. Focus S=(ae,0) Equation of one asymptote is bx-ay=0

Let (h,k) be the foot of the perpendicular from s to bx-ay=0

Then $\frac{h-ae}{b} = \frac{k-0}{-a} = \frac{-abe}{a^2+b^2} \Rightarrow \frac{h-ae}{b} = \frac{-abe}{a^2e^2} \;\&\; \frac{k}{-a} = \frac{-abe}{a^2e^2}$

On simplification, we get h=a/e, k=b/e

Foot of the perpendicular is (a/e,b/e)

23. The area of the triangle formed by the asymptotes and any tangent to the hyperbola $x^2 - y^2 = a^2$

1) $4a^2$
2) $3a^2$
3) $2a^2$
4) a^2

Key. 4

Sol. Equation of any tangent to $x^2 - y^2 = a^2$

i.e. $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$ is $\frac{x}{a}\sec\theta - \frac{y}{a}\tan\theta = 1 \rightarrow (1)$

$or\ x\sec\theta - y\tan\theta = a$

equation of other two sides of the triangle are

x-y=0..(2) x + y=0(3)

The two asymptotes of the hyperbola $x^2 - y^2 = a^2$

Are x-y=0 and x + y=0)

Solving (1) (2) and (3) in pairs the coordinates of the vertices of the triangle are (0,0)

$\left(\frac{a}{\sec\theta + \tan\theta}, \frac{a}{\sec\theta + \tan\theta}\right)$

And $\left(\frac{a}{\sec\theta - \tan\theta}, \frac{-a}{\sec\theta - \tan\theta}\right) -$

Area of triangle = $\frac{1}{2}\left|\frac{a^2}{\sec^2\theta - \tan^2\theta} + \frac{a^2}{\sec^2\theta - \tan^2\theta}\right|$

$\frac{1}{2}\left(a^2 + a^2\right) \qquad \because \sec^2\theta - \tan^2\theta = 1$

$= a^2$

24. The foot of the normal $3x + 4y = 7$ to the hyperbola $4x^2 - 3y^2 = 1$ is

1) $(1,1)$ 2) $(1,-1)$ 3) $(-1,1)$ 4) $(-1,-1)$

Key. 1

Sol. Since the point $(1,1)$ lies on the normal and hyperbola it is the foot of the normal

25. Tangent at the point $\left(2\sqrt{2},3\right)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ meet its asymptotes at A and B, then area of the triangle OAB, O being the origin is

1) 6 sq. units 2) 3 sq. units 3) 12 sq. units 4) 2 sq. units

Key. 1

Sol. Since area of the Δ formed by tangent at any point lying on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and its asymptotes is always constant and is equal to ab. Therefore, required area is 2 X 3=6 square units.

26. Eccentricity of hyperbola $\frac{x^2}{k}+\frac{y^2}{k}=1(k<0)$ is :

1) $\sqrt{1+k}$ 2) $\sqrt{1-k}$ 3) $\sqrt{1+\frac{1}{k^2}}$ 4) $\sqrt{1-\frac{1}{k}}$

Key. 4

Sol. Given equation can be rewritten as $\frac{y^2}{k^2}-\frac{x^2}{(-k)}=1(-k>0)$

$e^2=1+\frac{(-k)}{k^2}=1-\frac{1}{k}\Rightarrow e=\sqrt{1-\frac{1}{k}}$

27. If the circle $x^2+y^2=a^2$ intersect the hyperbola $xy=c^2$ in four points $P(x_1,y_1),Q(x_2,y_2),R(x_3,y_3),S(x_4,y_4)$ then which of the following does not hold

1) $x_1+x_2+x_3+x_4=0$ 2) $x_1x_2x_3x_4=y_1y_2y_3y_4=c^4$

3) $y_1+y_2+y_3+y_4=0$ 4) $x_1+y_2+x_3+y_4=0$

Key. 4

Sol. $x^2+\frac{c^4}{x^2}=a^2$ $\Rightarrow \mathbf{x^4-a^2x^2+c^4=0}$, 4th option does not hold

28. If a normal to the hyperbola x y = c^2 at $\left(ct_1,\frac{c}{t_1}\right)$ meets the curve again at $\left(ct_2,\frac{c}{t_2}\right)$, then:

1) $t_1t_2=-1$ 2) $t_2=-t_1-\frac{2}{t_1}$ 3) $t_2^3t_1=-1$ 4) $t_1^3t_2=-1$

Key. 4

Sol. Equation of normal at $\left(ct_1,\frac{c}{t_1}\right)$ is

$t_1^3x-t_1y-ct_1^4+c=0$

It passes through $\left(ct_2,\frac{c}{t_2}\right)$

Ie., $t_1^3.ct_2-t_1.\frac{c}{t_2}-ct_1^4+c=0$

$\Rightarrow(t_1-t_2)(t_1^3t_2+1)=0$

$\Rightarrow t_1^3t_2=-1$

29. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola xy=c^2 is

1) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ 2) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$ 3) $\frac{y}{x_1+x_2}+\frac{x}{y_1+y_2}=1$ 4) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

Key. 1

Sol. Mid point of the chord is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

The equation of the chord in terms of its mid-point is $s_1 = s_{11}$

30. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then $CP^2+CQ^2+CR^2+CS^2=$

1) r^2 2) $2r^2$ 3) $3r^2$ 4) $4r^2$

Key. 4

Sol. $CP=CQ=CR=CS=r$

31. The product of focal distances of the point (4,3) on the hyperbola $x^2-y^2=7$ is

1) 25 2) 12 3) 9 4) 16

Key. 1

Sol. $e=\sqrt{2},\ sp.s'p=(ex_1+a)(ex_1-a)=25$

32. Let $y=4x^2$ & $\frac{x^2}{a^2}-\frac{y^2}{16}=1$ intersect iff

1) $|a|\le\frac{1}{\sqrt{2}}$ 2) $a>\frac{1}{\sqrt{2}}$ 3) $a>-\frac{1}{\sqrt{2}}$ 4) $a>\sqrt{2}$

Key. 1

Sol. $y=4x^2$ & $\frac{1}{4}y=x^2$

Using $\frac{1}{4a^2}y-\frac{y^2}{16}=1$

$\Rightarrow 4y-a^2y^2=16a^2$

$\Rightarrow a^2y^2-4y+16a^2=0$

$\Rightarrow D \geq 0$ for intersection of two curves

$\Rightarrow 16 - 4a^2(16a^2) \geq 0$

$\Rightarrow 1 - 4a^4 \geq 0$

$\Rightarrow (2a^2) \leq 1$

$\Rightarrow |\sqrt{2}a| \leq 1 \Rightarrow -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

33. If angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 45°, then value of eccentricity e is

1) $\sqrt{4 \pm 2\sqrt{2}}$ 2) $\sqrt{4 + 2\sqrt{2}}$ 3) $\sqrt{4 - 2\sqrt{2}}$ 4) $\sqrt{4 - 3\sqrt{2}}$

Key. 3

Sol. $2\tan^{-1}\frac{b}{a} = 45^\circ \Rightarrow \frac{b}{a} = \tan 22^\circ = \frac{a^2(e^2-1)}{a^2} = (\sqrt{2}-1)^2$

$\Rightarrow e^2 - 1 = 3 - 2\sqrt{2} \Rightarrow e = \sqrt{4 - 2\sqrt{2}}$.

34. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

1) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ 2) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

3) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ 4) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Key. 1

Sol. Equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$. Its eccentricity is $e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Coordinates of foci are $(\pm 1, 0)$.

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a = \sin\theta$

Also, $ae_1 = 1 \Rightarrow e_1 = \operatorname{cosec}\theta$

$\therefore b^2 = a^2(e_1^2 - 1) = 1 - \sin^2\theta = \cos^2\theta$

Equation of the hyperbola is thus $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

35. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

1) Equation of ellipse is $x^2 + 2y^2 = 1$
2) the foci of ellipse are $(\pm 1, 0)$
3) equation of ellipse are $x^2 + 2y^2 = 4$
4) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Key. 2

Sol. If two concentric conics intersect orthogonally then they must be confocal, so ellipse and hyperbola will be confocal

$\Rightarrow (\pm ae, 0) \equiv (\pm 1, 0)$

[foci of hyperbola are $(\pm 1, 0)$]

36. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x axis at $(9,0)$, then the eccentricity of the hyperbola is:

1) $\sqrt{\frac{5}{2}}$ 2) $\sqrt{\frac{3}{2}}$ 3) $\sqrt{2}$ 4) $\sqrt{3}$

Key. 2

Sol. Normal at (6,3) is

$$\frac{a^2x}{6} + \frac{b^2y}{3} = a^2 + b^2,$$

$$\Rightarrow \frac{9a^2}{6} = a^2 + b^2 \Rightarrow \frac{3}{2} = 1 + \frac{b^2}{a^2}$$

$$\therefore \quad \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow e^2 - 1 = \frac{1}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$$

37. For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in 'α'

1) abscissae of vertices
2) abscissae of foci
3) Eccentricity
4) directrix

Key. 2

Sol. Hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

Coordinates of vertices are $(\pm\cos\alpha, 0)$, eccentricity of the hyperbola is $e = \sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}} = |\sec\alpha|$

$\therefore$ Coordinates of foci are thus $(\pm 1, 0)$, which are independent of α.

Directrix is $x = \pm\cos^2\alpha$

38. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

Key. D

Sol. $y^2 = 8k, xy = -1$

Let $P\left(t, \frac{-1}{t}\right)$ be any point on xy = - 1

Equation of the tangent to $xy = -1$ at $P\left(t, \frac{-1}{t}\right)$ is

$$\frac{xy_1 + yx_1}{2} = -1$$

$$\frac{-x}{t} + yt = -2$$

$$y = \frac{x}{t^2} + \left(\frac{-2}{t}\right) \ldots\ldots\ldots\ldots\ldots(1)$$

If (1) is tangent to the parabola $y^2 = 8x$ then

$$\frac{-2}{t} = \frac{2}{1/t^2} \Rightarrow t^3 = -1$$

$$t = -1$$

$\therefore$ Common tangent is y = x+2

39. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity *e* of the hyperbola, satisfies

(a) $1 < e < 2/\sqrt{3}$ (b) $e = 2/\sqrt{3}$ (c) $e = \sqrt{3}/2$ (d) $e > 2/\sqrt{3}$

Key. D

Sol. If OPQ is equilateral triangle then OP makes 30^0 with x-axis.

$\left(\frac{\sqrt{3}r}{2}, \frac{r}{2}\right)$ ties on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow r^2 = \frac{16a^2b^2}{12b^2 - 4a^2} > 0$$

$$\Rightarrow 12b^2 - 4a^2 > 0 \Rightarrow \frac{b^2}{a^2} > \frac{4}{12}$$

$$e^2 - 1 > \frac{1}{3}$$

$$e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$$

40. The locus of a point, from where tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ contain an angle of 45^0, is

(A) $\left(x^2+y^2\right)+a^2\left(x^2-y^2\right)=4a^2$ (B) $2\left(x^2+y^2\right)+4a^2\left(x^2-y^2\right)=4a^2$

(C) $\left(x^2+y^2\right)^2+4a^2\left(x^2-y^2\right)=4a^4$ (D) $\left(x^2+y^2\right)^2+a^2\left(x^2-y^2\right)=a^4$

Key. C

Sol. Equation of tangent to the hyperbola : $y=mx\pm\sqrt{m^2a^2-a^2}$

$\Rightarrow$ Let $P\left(x_1,y_1\right)$ be locus

$\Rightarrow y-mx=\pm\sqrt{m^2a^2-a^2}$

S.B.S

$\Rightarrow m^2\left(x_1^2-a^2\right)-2y_1x_1m+y_1^2+a^2=0$

$m_1+m_2=\frac{2x_1y_1}{x_1^2-a^2}$; $m_1m_2=\frac{y_1^2+a^2}{x_1^2-a^2}$

$\tan 45^0=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\Rightarrow\left(1+m_1m_2\right)^2=\left(m_1-m_2\right)^2=\left(m_1+m_2\right)^2-4m_1m_2$

$\Rightarrow\left(1+\frac{y_1+a^2}{x_1^2-a^2}\right)=\left(\frac{2x_1y_1}{x_1^2-a^2}\right)-4\left(\frac{y_1^2+a^2}{x_1^2-a^2}\right)$

41. If a circle cuts the rectangular hyperbola xy=1 in 4 points $\left(x_r,y_r\right)$ where r =1,2,3,4. Then ortho centre of triangle with vertices at $\left(x_r,y_r\right)$ where r=1,2,3 is

1. $\left(x_4,y_4\right)$ 2. $\left(-x_4,-y_4\right)$ 3. $\left(-x_4,+y_4\right)$ 4. $\left(+x_4,-y_4\right)$

Key. 2

Sol. $xy=1$ cuts the circle in 4-points then $x_1x_2x_3x_4=1,\ y_1y_2y_3y_4=1$

Ortho centre of triangle with vertices $\left(x_1,y_1\right)\left(x_2,y_2\right)\left(x_3,y_3\right)$

Ie $\left(\frac{-1}{x_1x_2x_3},-\left(y_1y_2y_3\right)^{-1}\right)$

$-\left(-x_4,-y_4\right)$

42. A hyperbola passing through origin has 3x – 4y – 1=0 and 4x – 3y – 6 = 0 as its asymptotes. Then the equations of its transverse and conjugate axes are

A) x – y – 5 = 0 and x + y + 1 = 0 B) x – y = 0 and x + y + 5 = 0
C) x + y – 5 = 0 and x – y – 1 = 0 D) x + y – 1 = 0 and x – y – 5 = 0

Key. C

Sol. Transverse and conjugate axes are the bisectors of the angle between asymptotes.

$\frac{3x-4y-1}{5}=\pm\left(\frac{4x-3y-6}{5}\right)$ etc......

43. If the asymptotes of the hyperbola $\left(x+y+1\right)^2-\left(x-y-3\right)^2=5$ cuts each other at A and the coordinate axes at B and C, then radius of the circle passing through the points A, B, C is

A) 3 B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{3}$

Key. B

Sol. Centre of rectangular hyperbola (1, –2)

So equation of asymptotes are x = 1, y = –2

So radius of circle = $\frac{\sqrt{5}}{2}$

44. If a chord joining P($a\text{Sec}\theta, a\tan\theta$), Q($a\text{Sec}\alpha, a\tan\alpha$) on the hyperbola $x^2 - y^2 = a^2$ is the normal at P,then $\text{Tan}\alpha =$

A) $\text{Tan}\theta(4\sec^2\theta+1)$ B) $\text{Tan}\theta(4\sec^2\theta-1)$ C) $\text{Tan}\theta(2\text{Sec}^2\theta-1)$ D) $\text{Tan}\theta(1-2\text{Sec}^2\theta)$

Key. B

Sol. Slope of chord joining P and Q = slope of normal at P

$\frac{\text{Tan}\alpha-\text{Tan}\theta}{\sec\alpha-\sec\theta} = -\frac{\text{Tan}\theta}{\sec\theta} \Rightarrow \text{Tan}\alpha-\text{Tan}\theta = -k\text{Tan}\theta$ and $\sec\alpha-\sec\theta = k\sec\theta$

$\therefore (1-k)Tan\theta = Tan\alpha \to 1.\ (1+k)\sec\theta = \sec\alpha \to 2.$

$[(1+k)\sec\theta]^2 - [(1-k)\text{Tan}\theta]^2 = \sec^2\alpha - \text{Tan}^2\alpha$

$\Rightarrow k = -2\left(\sec^2\theta + \text{Tan}^2\theta\right) = -4\sec^2\theta + 2$

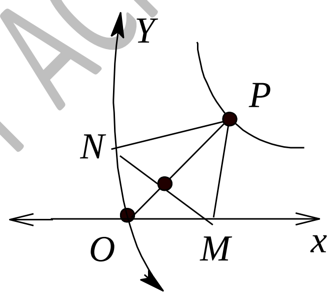
From (1) $Tan\alpha = Tan\theta\ \left(1+4\sec\theta^2-2\right) = \text{Tan}\theta\left(4\sec\theta^2-1\right)$.

45. PM and PN are the perpendiculars from any point P on the rectangular hyperbola $xy = c^2$ to the asymptotes. If the locus of the mid point of MN is a conic, then its eccentricity is

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{\sqrt{2}}$

Key. B

Sol. OMPN is rectangle.



$P = \left(Ct, \frac{c}{t}\right)$

Mid point = $\left(\frac{ct}{2}, \frac{c}{2t}\right) = (x, y)$ $\therefore xy = \frac{c^2}{4} \Rightarrow e = \sqrt{2}$

46. A variable straight line of slope 4 intersects the hyperbola xy = 1 at two points. The locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

A) $16x^2 + 10xy + y^2 = 2$ B) $16x^2 - 10xy + y^2 = 2$

C) $16x^2+10xy+y^2=4$ D) $16x^2-10xy+y^2=4$

Key. A

Sol. Let P(h, k)

y - k = 4(x - h) --- (1)

Let it meets xy = 1 ----(2) at A (x_1, y_1) and B (x_2, y_2)

$$x_1+x_2=\frac{4h-k}{4}, x_1x_2=-\frac{1}{4} \text{ Also } \Rightarrow \therefore \frac{2x_1+x_2}{3}=h \Rightarrow x_1=\frac{8h+k}{4}, x_2=\frac{2h+k}{2}$$

$$\Rightarrow 16x^2+10xy+y^2=2$$

47. From a point *P* on the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ straight lines are drawn parallel to the asymptotes of the hyperbola. Then the area of parallelogram formed by the asymptotes and the two lines through P is

A) dependent on coordinates of P B) 4 C) 6 D) $8\sqrt{2}$

Key. B

Sol. Area of parallelogram is $\frac{ab}{2}=\frac{4\times 2}{2}=4$

48. The eccentricity of the conic defined by $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$

A) 5/2 B) 5/3 C) $\sqrt{2}$ D) $\sqrt{11}/3$

Key. B

Sol. Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.

$2ae=5$ and $2a=3$ $\therefore e=5/3$

49. The asymptotes of a hyperbola are $3x-4y+2=0$ and $5x+12y-4=0$. If the hyperbola passes through the point (1, 2) then slope of transverse axis of the hyperbola is

A) 6 B) −7/2 C) −8 D) 1/8

Key. C

Sol. Axes of hyperbola are bisectors of angles between asymptotes.

50. If P is a point on the rectangular hyperbola $x^2 - y^2 = a^2$, C being the center and S, S' are two foci, then $SP.S'P$ =

a) 2 b) $(CP)^2$ c) $(CS)^2$ d) $(SS')^2$

Key. B

Sol. Let P = (a sec θ, a tan θ), S_1 S^1 = $\left(\pm a\sqrt{2},0\right)$

SP= $a\left(\sqrt{2}\sec\theta-1\right)$, S^1 P = $a\left(\sqrt{2}\sec\theta+1\right)$

SP-S^1P = a^2 ($\sec^2\theta$ + $\tan^2\theta$) = CP^2

51. An equation of common tangent to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is

a) 2x – y + 1 = 0 b) x – y + 2 = 0 c) x + y + 2 = 0 d) 2x + y – 1 = 0

Key. A

Sol. Let m be the slope of the common tangent

$$\therefore \frac{2}{m} = \sqrt{m^2 - 3} \Rightarrow m = \pm 2$$

Equation of common tangents are y = 2x+1 or y = -2x-1

52. Let P(θ), Q(φ) be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ satisfying θ + φ = π/2. If (h, k) be the point of intersection of normals at P and Q, then k is equal to

a) $\frac{a^2 + b^2}{a}$ b) $-\frac{a^2 + b^2}{a}$ c) $-\frac{a^2 + b^2}{b}$ d) $\frac{a^2 + b^2}{b}$

Key. C

Sol. Solving the normals at θ, φ and using $\theta + \varphi = \frac{\pi}{2}$

53. A chord of the hyperbola $x^2 - 2y^2 = 1$ is bisected at the point (–1, 1). Then the area of the triangle formed by the chord and the coordinate axes is

a) 1 b) 2 c) 1/2 d) 1/4

Key. D

Sol. Equation of the chord as $S_1 = S_{11}$ = Req Area $\frac{1}{4}$

54. The angle of intersection between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2 - k^2} - \frac{y^2}{k^2 - b^2} = 1, (a > k > b > 0)$ is

a) $\tan^{-1}\left(\frac{b}{a}\right)$ b) $\tan^{-1}\left(\frac{b}{ka}\right)$

c) $\tan^{-1}\left(\frac{a}{kb}\right)$ d) None of these

Key. D

Sol. Confocal ellipse and hyperbola cut at right angles

55. Let A is the number of tangents drawn from a point on the asymptote of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (except origin) to the hyperbola itself. B is the number of normals which can be drawn from centre of $xy = c^2$ to the $xy = c^2$. C is the maximum number of normals which can be drawn from a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. D is the number of tangent common to both branches of a hyperbola. Then number of normals which can be drawn from the point (ABD, BC) to $y^2 - 48y - 4x + 616 = 0$ is (If A =3, B = 5, C = 4 then ABC = 354)

a) 1 b) 0 c) 2 d) 3

Key. D

Sol. A – 1, B = 2, C = 4, D = 0

From (120, 24) we can draw 3 normals to

$(y - 24)^2 = 4(x - 10)$ since (x – 10) > 2

56. If the normal at P(8, 2) on the curve xy = 16 meets the curve again at Q. Then angle subtended by PQ at the origin is

a) $\tan^{-1}\left(\frac{15}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{15}\right)$

c) $\tan^{-1}\left(\frac{261}{55}\right)$ d) $\tan^{-1}\left(\frac{55}{261}\right)$

Key. A

Sol. If a normal cuts the hyperbola at point $\left(t, \frac{1}{t}\right)$ meets the curve again at $\left(ct^1, \frac{C}{t^1}\right)$ then $t^3t^1 = -1$

57. A triangle is inscribed in the curve $xy = c^2$ and two of its sides are parallel to $y + m_1x = 0$ and $y + m_2x = 0$. Then the third side touches the hyperbola

a) $4m_1m_2xy = c^2(m_1 + m_2)^2$ b) $m_1m_2xy = c^2(m_1 + m_2)$

c) $2m_1m_2xy = c^2(m_1 + m_2)^2$ d) $4m_1m_2xy = c^2(m_1 - m_2)^2$

Key. A

Sol. $m(AC) = \frac{-1}{t_1t_3} = -m_1, m(BC) = -m_2 = \frac{-1}{t_2t_3}, m_1m_2 = \frac{1}{t_3^2.t_1t_2}$

$m_1 + m_2 = \frac{1}{t_3}\left(\frac{t_2 + t_1}{t_1t_2}\right)$ Compare chord $Ab = x + yt_1t_2 = c(t_1 + t_2)$ with $\frac{x}{t} + y + = 2k$

58. Let a hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

a) The equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$ b) The equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

c) Focus of hyperbola is (5, 0) d) vertex of hyperbola is $\left(5\sqrt{3}, 0\right)$

Key. C

Sol. Conceptual

59. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2x} - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, Then area of triangle ABC is

a) $\sqrt{\frac{3}{2}} + 1$ b) $1 - \sqrt{\frac{2}{3}}$

c) $1 + \sqrt{\frac{2}{3}}$ d) $\sqrt{\frac{3}{2}} - 1$

Key. D

Sol. Area $= \frac{1}{2}a(e-1) \times \frac{b^2}{a} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{2}} = \sqrt{\frac{3}{2}} - 1$

60. If the equation to the hyperbola is $3x^2-5xy-2y^2+5x+11y-8=0$ then equation to the conjugate hyperbola is

a) $3x^2-5xy-2y^2+5x+11y-16=0$

b) $3x^2-5xy-2y^2+5x+11y-12=0$

c) $3x^2-5xy-2y^2+5x+11y-4=0$

d) $3x^2-5xy-2y^2+5x+11y-20=0$

Key. A

Sol. $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ be the equation to the pair of asymptotes then c = -12. And hence equation to the conjugate hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$

61. Locus of the mid points of the chords of the hyperbola $x^2-y^2=a^2$ that touch the parabola $y^2=4ax$ is

(A) $x^2(x-a)=y^3$ (B) $y^2(x-a)=x^3$

(C) $x^3(x-a)=y^2$ (D) $y^3(x-a)=x^2$

Key. B

Sol. Let the mid point = (*h, k*)

$\therefore$ equation of the chord $xh-yk=h^2-k^2$

$yk=xh+(k^2-h^2)$

$y=\frac{xh}{k}+\frac{(k^2-h^2)}{k}$

$\frac{k^2-h^2}{k}=\frac{ak}{h}$

$\Rightarrow k^2h-h^3=ak^2 \quad \Rightarrow k^2(h-a)=h^3 \quad \therefore x^3=y^2(x-a)$

62. If e is the eccentricity of of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and θ is the angle between the asymptotes. The $\cos\theta/2$ is equal to

(A) $\sqrt{e}$ (B) $\frac{e}{1+e}$

(C) $\frac{1}{\sqrt{e}}$ (D) $\frac{1}{e}$

Key. D

Sol. $\tan\frac{\theta}{2}=\frac{b}{a}$

$\cos\frac{\theta}{2}=\frac{a}{\sqrt{a^2+b^2}}=\frac{1}{\sqrt{1+\frac{b^2}{a^2}}}=\frac{1}{e}.$

63. Locus of the midpoints of the chords of the hyperbola $x^2-y^2=a^2$ that touch the parabola $y^2=4ax$ is

A) $x^2(x-a)=y^3$ B) $y^2(x-a)=x^3$ C) $x^3(x-a)y^2$ D) $y^3(x-a)x^2$

Key. B

Sol. let the mid point (h,k) equation of the chord is $xh - yk = h^2 - k^2$

$$y = \frac{xh}{k} + \frac{\left(k^2 - h^2\right)}{k};\ \frac{\left(k^2 - h^2\right)}{k} = \frac{ak}{h} \Rightarrow k^2\left(h - a\right) = h^3 \Rightarrow x^3 = y^3\left(x - a\right)$$

64. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1 - \sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}} - 1$

(C) $1 + \sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}} + 1$

Key. B

Sol. $x^2 - 2\sqrt{2}x - 2(y^2 + 2\sqrt{2}y) = 6$

$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 - 2 + 4 = 6$

$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4$

$\Rightarrow \frac{(x-\sqrt{2})^2}{4} - \frac{(y+\sqrt{2})^2}{2} = 1$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow 2 = 4(e^2 - 1)$

$\Rightarrow e^2 - 1 = 1/2$

$e = \sqrt{3}/2$

area = $\frac{1}{2}$ (ae – a) × b^2/a

= (e – 1) = $\left(\sqrt{\frac{3}{2}} - 1\right)$

65. The equations of the transverse and conjugate axes of a hyperbola respectively are $x + 2y - 3 = 0$ and $2x - y + 4 = 0$ and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is

(A) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$ (B) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

(C) $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$ (D) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

Key. B

Sol. The equation of the hyperbola is

$$\frac{\left(\frac{|2x-y+4|}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{\left(\frac{|x+2y-3|}{\sqrt{5}}\right)^2}{\left(\frac{2}{\sqrt{3}}.\frac{1}{2}\right)^2} = 1$$

66. If $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\cos^2\frac{\theta_1+\theta_2}{2} = \lambda \cos^2\frac{\theta_1-\theta_2}{2}$, where λ is equal to

(A) $\frac{a^2+b^2}{a^2}$ (B) $\frac{a^2+b^2}{b^2}$

(C) $\frac{a^2+b^2}{ab}$ (D) $\frac{a^2+b^2}{2ab}$

Key. A

Sol. Equation of any chord joining the points $P(\theta_1)$ and $Q(\theta_2)$ is,

$\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\frac{y}{b}$

$\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1+\theta_2}{2}\right)$. If it passes through (ae , 0), then

$\Rightarrow e^2\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)=\cos^2\left(\frac{\theta_1+\theta_2}{2}\right)$

$\Rightarrow \lambda = e^2 = 1 + \frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$

67. If the normal at the points P_i (x_i, y_i), i = 1 to 4 on the hyperbola $xy = c^2$ are concurrent at the point Q(h, k), then $\frac{(x_1+x_2+x_3+x_4)(y_1+y_2+y_3+y_4)}{x_1x_2x_3x_4}$ is equal to

(A) $\frac{hk}{c^4}$ (B) $\frac{h^2k^2}{c^6}$

(C) $\frac{\sqrt{|hk|}}{c^3}$ (D) $-\frac{hk}{c^4}$

Key. D

Sol. Equation of normal at any point $P(ct, \frac{c}{t})$ on xy

$= c^2$, is $xt^3 - yt - ct^4 + c = 0$

If it passes through Q (h, k), then

$ct^4 - ht^3 + kt - c = 0$

If it's roots are t_1, t_2, t_3 and t_4, then

$t_1 + t_2 + t_3 + t_4 = h/c$

$\Rightarrow ct_1 + ct_2 + ct_3 + ct_4 = h \Rightarrow \Sigma x_i = h, \Sigma t_1 t_2 t_3 = -\frac{k}{c}, t_1 t_2 t_3 t_4 = -1$

$\Rightarrow (ct_1)(ct_2)(ct_3)(ct_4) = -c^4 \Rightarrow \Sigma \frac{c}{t_i} = k \Rightarrow \Sigma y_i = k$ and $x_1x_2x_3x_4$

$= -c^4 \Rightarrow \frac{\Sigma x_i \Sigma y_i}{x_1x_2x_3x_4} = -\frac{hk}{c^4}$

68. A tangent to the hyperbola $y = \frac{x+9}{x+5}$ passing through the origin is

(A) x + 25y = 0 (B) 5x + y = 0

(C) 5x – y = 0 (D) x – 25y = 0

Key. A

Sol. $y=\frac{x+9}{x+5}=1+\frac{4}{x+5}$

$\frac{dy}{dx}$ at $(x_1, y_1) = \frac{-4}{(x_1+5)^2}$

Equation of tangent

$$y - y_1 = \frac{-4}{(x_1+5)^2}(x - x_1)$$

$$y - 1 - \frac{4}{x_1+5} = \frac{-4}{(x_1+5)^2}\cdot(x - x_1)$$

Since it passes through (0, 0)

$(x_1 + 5)^2 + 4(x_1 + 5) + 4x_1 = 0$

$x_1 = -15$ or $x_1 = -3$. So equation are

x + 25 y = 0 or, x + y = 0.

69. The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$

(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Key. B

Sol. Equation of tangent at point P(θ)

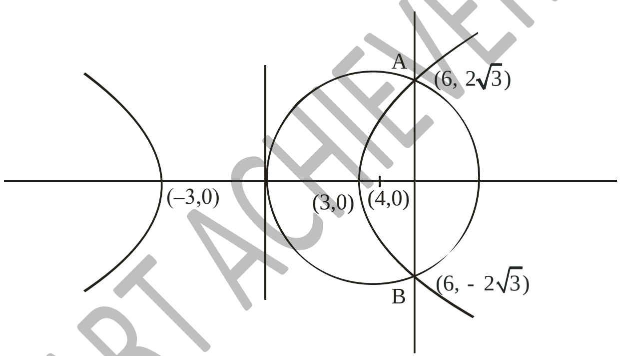
$$\frac{x\sec\theta}{3} - \frac{y\tan\theta}{2} - 1 = 0 \qquad \text{.....(i)}$$

since eq. (i) will be a tangent to the circle

$$\therefore \frac{\frac{4\sec\theta}{3} - 1}{\sqrt{\frac{\sec^2\theta}{9} + \frac{\tan^2\theta}{4}}} = 4$$

by solving it we will get

$$2x - \sqrt{5}y + 4 = 0$$



70. There is a point P on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ such that its distance to the right directrix is the average of its distance to the two foci. Let the x-coordinate of P be $\frac{m}{n}$ with m and n being integers, (n > 0) having no common factor except 1. Then n – m equals

(A) 59 (B) 69 (C) -59 (D) -69

Key. B

Sol. It turns out that P has to be on the left branch. x-coordinate is found to be $-64/5$

71. The reflection of the hyperbola $xy=1$ in the line $y=2x$ is the curve $12x^2+r\,xy+s\,y^2+t=0$ then the value of $'r'$ is

a) – 7 b) 25 c) – 175 d) 90

Key. A

Sol. The reflection of (α,β) in the line $y=2x$ is

$(\alpha_1,\beta_1)=\left(\frac{4\beta-3\alpha}{5},\frac{4\alpha+3\beta}{5}\right)$ = $\alpha_1\beta_1=1$

$\Rightarrow 12\alpha^2-7\alpha\beta-12\beta^2+25=0$

72. Chords of the parabola $y^2=4x$ touch the hyperbola $x^2-y^2=1$. The locus of the point of intersection of the tangents drawn to the parabola at the extremities of such chords is

a) a circle b) a parabola

c) an ellipse d) a rectangular hyperbola

Key. C

Sol. The chord of contact $y\,y_o=2\left(x+x_o\right)$ of the point $P\left(x_o,y_o\right)$ w.r.t the parabola is tangent to the hyperbola $x^2-y^2=1$ *iff* $2x_0^2+y_0^2=4$. Locus of P is the ellipse $2x^2+y^2=4$

73. A chord of the hyperbola $x^2 – 2y^2 = 1$ is bisected at the point (–1, 1). Then the area of the triangle formed by the chord and the coordinate axes is

a) 1 b) 2 c) $\frac{1}{2}$ d) 1/4

Key. D

Sol. Equation of the chord as $S_1 = S_{11}$ = Req Area $\frac{1}{4}$

74. A pair of tangents with inclinations α,β are drawn from an external point P to the parabola $y^2=16x$. If the point P varies in such a way that $\tan^2\alpha+\tan^2\beta=4$ then the locus of P is a conic whose eccentricity is

A) $\frac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\frac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1=\tan\alpha, m_2=\tan\beta$, Let $P=(h,k)$

m_1,m_2 are the roots of $K=mh+\frac{4}{m}\Rightarrow hm^2-Km+4=0$

$m_1+m_2=\frac{K}{h};\quad m_1m_2=\frac{4}{h}$

$m_1^2+m_2^2=\frac{K^2}{h^2}-\frac{8}{h}=4$

Locus of P is $y^2-8x=4x^2\Rightarrow y^2=4(x+1)^2-4\Rightarrow\frac{(x+1)^2}{1}-\frac{y^2}{4}=1$

75. From a point *P* on the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ straight lines are drawn parallel to the asymptotes of the hyperbola. Then the area of parallelogram formed by the asymptotes and the two lines through P is

A) dependent on coordinates of P B) 4 C) 6 D) $8\sqrt{2}$

Key. B

Sol. Area of parallelogram is $\frac{ab}{2}=\frac{4\times 2}{2}=4$

76. The asymptotes of a hyperbola are $3x-4y+2=0$ and $5x+12y-4=0$. If the hyperbola passes through the point (1, 2) then slope of transverse axis of the hyperbola is

A) 6 B) –7/2 C) –8 D) 1/8

Key. C

Sol. Axes of hyperbola are bisectors of angles between asymptotes.

77. Locus of the midpoints of the chords of the hyperbola $x^2-y^2=a^2$ that touch the parabola $y^2=4ax$ is

A) $x^2(x-a)=y^3$ B) $y^2(x-a)=x^3$ C) $x^3(x-a)y^2$ D) $y^3(x-a)x^2$

Key. B

Sol. let the mid point (h,k) equation of the chord is $xh-yk=h^2-k^2$

$$y=\frac{xh}{k}+\frac{(k^2-h^2)}{k};\ \frac{(k^2-h^2)}{k}=\frac{ak}{h}\Rightarrow k^2(h-a)=h^3\Rightarrow x^3=y^3(x-a)$$

78. Two distinct tangents can be drawn from the point $(\alpha,2)$ to different branches of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1,$ if 'α' belongs to

A) $\left(\frac{-3}{2},\frac{5}{2}\right)$ B) $\left(\frac{-5}{2},\frac{5}{2}\right)$ C) $\left(\frac{-7}{2},\frac{7}{2}\right)$ D) $\left(\frac{-3}{2},\frac{3}{2}\right)$

Key. D

Sol. The point on the line $y=2$ that should lie between the asymptotes where the curve do not exist.Equation of asymptotes are $4x=\pm 3y$. The point of intersection of $y=2$ with asymptotes are $x=\pm\frac{3}{2}$

$\therefore\ \frac{-3}{2}<\alpha<\frac{3}{2}$

79. A hyperbola passing through origin has $3x-4y-1=0$ and $4x-3y-6=0$ as its asymptotes. Then the equation of its transverse axis is

A) $x-y-5=0$ B) $x+y+1=0$

C) $x+y-5=0$ D) $x-y-1=0$

Key. A

Sol. Asymptotes are equally inclined to the axes of hyperbola. Find the bisector of the asymptotes which bisects the angle containing the origin.

80. A hyperbola has centre $'C'$ and one focus at $P(6,8)$. If its two directrices are $3x+4y+10=0$ and $3x+4y-10=0$ then $CP=$

A) 14 B) 8 C) 10 D) 6

Key. A

Sol. $\frac{2a}{e}=4 \Rightarrow a=2e,\ P$ is nearest to $3x+4y-10=0$

$\Rightarrow ae-\frac{a}{e}=8 \Rightarrow e=\sqrt{5},\ a=2\sqrt{5}$

$CP=ae=10$

81. If a variable tangent to the circle $x^2+y^2=1$ intersects the ellipse $x^2+2y^2=4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

a) eccentricity is $\frac{\sqrt{3}}{2}$ b) eccentricity is $\frac{\sqrt{5}}{2}$

c) latus -rectum is of length 2 units d) foci are $\left(\pm 2\sqrt{5},0\right)$

Key: A,C

Hint: A tangent to the circle $x^2+y^2=1\ is\ x\cos\theta+y\sin\theta=1. R(x_o,y_o)$ is the point of intersection of the tangents to the ellipse at P and Q $\Leftrightarrow x\cos\theta+y\sin\theta=1$ and $x_o x+2y_o y=4$ represent the same line

$\Leftrightarrow x_o=4\cos\theta and\ y_o=2\sin\theta$

$\Leftrightarrow \frac{x_0^2}{16}+\frac{y_0^2}{4}=1.$ Hence, locus of P is the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$

82. A variable straight line with slope $m(m\neq 0)$ intersects the hyperbola xy=1 at two distinct points .Then the locus of the point which divides the line segment between these two points in the ratio 1:2 is

(A) An ellipse (B) A hyperbola (C) A circle (D) A parabola

Key: B

Hint: Let the points of intersection be $\left(t_1,\frac{1}{t_1}\right)\left(t_2,\frac{1}{t_2}\right)$.given $m=-\frac{1}{t_1t_2}\ or\ t_1t_2=-\frac{1}{m}$

also by section formula ,

solving for t_1,t_2 and eliminating them gives $2m^2x^2+5mxy+2y^2=m$ which is always a hyperbola as

$\frac{25m^2}{4}-4m^2=\frac{9m^2}{4}>0,\forall m\neq 0$

83. A tangent to the parabola $x^2 = 4ay$ meets the hyperbola $x^2-y^2=a^2$ at two points P and Q, then midpoint of P and Q lies on the curve

a) $y^3=x(y-a)$ b) $y^3=x^2(y-a)$

c) $y^2=x^2(y-a)$ d) $y^2=x^3(a-y)$

Key: B

Hint: Equation of tangent to parabola $y = mx - am^2 ....(1)$ equation of chord of hyperbola whose midpoint is (h, k) is $hx - ky = h^2 - k^2 ......(2)$ form (1) and (2)

$$\frac{m}{h} = \frac{1}{k} = \frac{am^2}{h^2 - k^2} \Rightarrow k^3 = h^2(k - a)$$

84. The equation of a tangent to the hyperbola $3x^2 - y^2 = 3$, parallel to the line $y = 2x + 4$ is

(A) y = 2x + 3 (B) y = 2x + 1

(C) y = 2x + 4 (D) y = 2x + 2

Key. B

Sol. $3x^2 - y^2 = 3, \frac{x^2}{1} - \frac{y^2}{3} = 1$

Equation of tangent in terms of slope.

$y = mx \pm \sqrt{(m^2 - 3)}$

Here, m = 2,

then $y = 2x \pm 1$

85. A circle cuts the X-axis and Y-axis such that intercept on X-axis is a constant a and intercept on Y-axis is a constant b. Then eccentricity of locus of centre of circle is

1. 1 2. $\frac{1}{2}$ 3. $\sqrt{2}$ 4. $\frac{1}{\sqrt{2}}$

Key. 3

Sol. Locus of centre of circle is a rectangular hyperbola hence its eccentricity is $\sqrt{2}$

86. If a circle cuts the rectangular hyperbola xy=1 in 4 points (x_r, y_r) where r =1,2,3,4. Then ortho centre of triangle with vertices at (x_r, y_r) where r=1,2,3 is

1. (x_4, y_4) 2. $(-x_4, -y_4)$

3. $(-x_4, +y_4)$ 4. $(+x_4, -y_4)$

Key. 2

Sol. $xy = 1$ cuts the circle in 4-points then $x_1x_2x_3x_4 = 1,\ y_1y_2y_3y_4 = 1$

Ortho centre of triangle with vertices $(x_1, y_1)(x_2, y_2)(x_3, y_3)$

Ie $\left(\frac{-1}{x_1x_2x_3}, -(y_1y_2y_3)^{-1}\right)$

$= (-x_4, -y_4)$

87. The locus of a point, from where tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ contain an angle of 45^0, is

(A) $(x^2 + y^2) + a^2(x^2 - y^2) = 4a^2$ (B) $2(x^2 + y^2) + 4a^2(x^2 - y^2) = 4a^2$

(C) $(x^2 + y^2)^2 + 4a^2(x^2 - y^2) = 4a^4$ (D) $(x^2 + y^2)^2 + a^2(x^2 - y^2) = a^4$

Key. C

Sol. Equation of tangent to the hyperbola : $y = mx \pm \sqrt{m^2a^2 - a^2}$

$\Rightarrow$ Let $P(x_1, y_1)$ be locus

$\Rightarrow y - mx = \pm\sqrt{m^2a^2 - a^2}$

S.B.S

$\Rightarrow m^2\left(x_1^2 - a^2\right) - 2y_1x_1m + y_1^2 + a^2 = 0$

$m_1 + m_2 = \dfrac{2x_1y_1}{x_1^2 - a^2}$; $m_1m_2 = \dfrac{y_1^2 + a^2}{x_1^2 - a^2}$

$\tan 45^0 = \left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right|$

$\Rightarrow (1 + m_1m_2)^2 = (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$\Rightarrow \left(1 + \dfrac{y_1 + a^2}{x_1^2 - a^2}\right) = \left(\dfrac{2x_1y_1}{x_1^2 - a^2}\right) - 4\left(\dfrac{y_1^2 + a^2}{x_1^2 - a^2}\right)$

88. If PQ is a double ordinate of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity *e* of the hyperbola, satisfies

(a) $1 < e < 2/\sqrt{3}$ (b) $e = 2/\sqrt{3}$ (c) $e = \sqrt{3}/2$ (d) $e > 2/\sqrt{3}$

Key. D

Sol. If OPQ is equilateral triangle then OP makes 30^0 with x-axis.

$\left(\dfrac{\sqrt{3}r}{2}, \dfrac{r}{2}\right)$ ties on hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\Rightarrow r^2 = \dfrac{16a^2b^2}{12b^2 - 4a^2} > 0$

$\Rightarrow 12b^2 - 4a^2 > 0 \Rightarrow \dfrac{b^2}{a^2} > \dfrac{4}{12}$

$e^2 - 1 > \dfrac{1}{3}$

$e^2 > \dfrac{4}{3} \Rightarrow e > \dfrac{2}{\sqrt{3}}$

89. Consider a hyperbola xy=4 and a line 2x+y=4. Let the given line intersect the x-axis at R. If a line through 'R' intersects the hyperbola at S and T. The minimum value of $RS \times RT$ is

A) 24 B) 16 C) 8 D) 4

Key. C

Sol. S, T = $(2 + r\cos\theta, 0 + r\sin\theta)$

$r^2\cos\theta\sin\theta + 2\sin\theta - 4 = 0$

RS.RT = $\dfrac{4}{\sin\theta\cos\theta} = \dfrac{8}{\sin 2\theta} \geq 8$

90. The normal at 'P' on a hyperbola of eccentricity 'e' intersects its transverse and conjugate axes at L and M respectively. If the locus of the mid point of LM is a hyperbola then its eccentricity is

A) $\dfrac{e+1}{e-1}$ B) $\dfrac{e}{\sqrt{e^2-1}}$ C) e D) $\dfrac{2e}{\sqrt{e^2-1}}$

Key. B

Sol. Normal : $ax\cos\theta + by\cot\theta = a^2 + b^2$

$$L = \left(\frac{a^2+b^2}{a}\sec\theta, 0\right),\ M = \left(0, \frac{a^2+b^2}{b}\tan\theta\right)$$

Locus is $\dfrac{x^2}{\frac{a^2e^2}{4}} - \dfrac{y^2}{\frac{a^2e^2}{4b^2}} = 1$

$$e_1 = \frac{e}{\sqrt{e^2-1}}$$

91. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2x} - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, Then area of triangle ABC is

a) $\sqrt{\frac{3}{2}}+1$ b) $1-\sqrt{\frac{2}{3}}$ c) $1+\sqrt{\frac{2}{3}}$ d) $\sqrt{\frac{3}{2}}-1$

Key. D

Sol. Area $= \frac{1}{2}a(e-1)\times\frac{b^2}{a} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}} = \sqrt{\frac{3}{2}}-1$

92. If the equation to the hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$ then equation to the conjugate hyperbola is

a) $3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$

b) $3x^2 - 5xy - 2y^2 + 5x + 11y - 12 = 0$

c) $3x^2 - 5xy - 2y^2 + 5x + 11y - 4 = 0$

d) $3x^2 - 5xy - 2y^2 + 5x + 11y - 20 = 0$

Key. A

Sol. $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ be the equation to the pair of asymptotes then c = -12. And hence equation to the conjugate hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$

93. A tangent to the circle $x^2 + y^2 = 4$ intersects the hyperbola $x^2 - 2y^2 = 2$ at P and Q. If locus of mid–point of PQ is $(x^2 - 2y^2)^2 = \lambda\,(x^2 + 4y^2)$; then λ equals

(A) 2 (B) 4

(C) 6 (D) 8

Key. B

Sol. Equation of chord of hyperbola $\frac{x^2}{2} - \frac{y^2}{1} = 1$, whose mid–point is (h, k) is

$$\frac{hx}{2} - ky = \frac{h^2}{2} - \frac{k^2}{1}$$

It is tangent to the circle $x^2 + y^2 = 4$, then $\left|\dfrac{\frac{h^2}{2}-k^2}{\sqrt{\frac{h^2}{4}+k^2}}\right| = 2$

$\Rightarrow \left(\frac{h^2}{2}-k^2\right)^2 = 4\left(\frac{h^2}{4}+k^2\right) \Rightarrow (x^2-2y^2)^2 = 4(x^2+4y^2) \Rightarrow \lambda = 4.$

94. Length of latusrectum of the conic satisfying the differential equation $xdy+ydx=0$ and passing through the point (2, 8) is

A) $4\sqrt{2}$ B) 8 C) $8\sqrt{2}$ D) 16

Key. C

Sol. $\frac{dy}{y}+\frac{dx}{x}=0 \Rightarrow xy=16$

$\therefore\ y=-x$ is conjugate axis centre is (0, 0).

Vertices are (4, 4), $(-4,-4)$. $e=\sqrt{2}$

Length of transverse axis = $8\sqrt{2}=2a$

L.R = $2a(e^2-1)$

95. From a point *P* on the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ straight lines are drawn parallel to the asymptotes of the hyperbola. Then the area of parallelogram formed by the asymptotes and the two lines through P is

A) dependent on coordinates of P B) 4 C) 6 D) $8\sqrt{2}$

Key. B

Sol. Area of parallelogram is $\frac{ab}{2}=\frac{4\times 2}{2}=4$

96. The eccentricity of the conic defined by $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$

A) 5/2 B) 5/3 C) $\sqrt{2}$ D) $\sqrt{11}/3$

Key. B

Sol. Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.

$2ae=5$ and $2a=3$ $\therefore e=5/3$

97. The asymptotes of a hyperbola are $3x-4y+2=0$ and $5x+12y-4=0$. If the hyperbola passes through the point (1, 2) then slope of transverse axis of the hyperbola is

A) 6 B) –7/2 C) –8 D) 1/8

Key. C

Sol. Axes of hyperbola are bisectors of angles between asymptotes.

98. A triangle is inscribed in the curve $xy=c^2$ and two of its sides are parallel to $y+m_1x=0$ and $y+m_2x=0$. Then the third side touches the hyperbola

a) $4m_1m_2xy=c^2(m_1+m_2)^2$ b) $m_1m_2xy=c^2(m_1+m_2)$

c) $2m_1m_2xy=c^2(m_1+m_2)^2$ d) $4m_1m_2xy=c^2(m_1-m_2)^2$

Key. A

Sol. $m(AC)=\frac{-1}{t_1t_3}=-m_1, m(BC)=-m_2=\frac{-1}{t_2t_3}, m_1m_2=\frac{1}{t_3^2.t_1t_2}$

$$m_1+m_2=\frac{1}{t_3}\left(\frac{t_2+t_1}{t_1t_2}\right)$$ Compare chord $Ab = x+yt_1t_2 = c\left(t_1+t_2\right)$ with $\frac{x}{t}+y+=2k$

99. Let a hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

a) The equation of hyperbola is $\frac{x^2}{16}-\frac{y^2}{9}=1$ b) The equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

c) Focus of hyperbola is (5, 0) d) vertex of hyperbola is $\left(5\sqrt{3},0\right)$

Key. C

Sol. Conceptual

100. The length of the transverse axis of the hyperbola $9x^2-16y^2-18x-32y-151=0$ is

1) 8 2) 4 3) 6 4) 2

Key. 1

Sol. Given hyperbola is $\frac{(x-1)^2}{16}-\frac{(y+1)^2}{9}=1$

Length of the transverse axis is 2a=8.

101. The equation of a hyperbola , conjugate to the hyperbola $x^2+3xy+2y^2+2x+3y=0$ is

1) $x^2+3xy+2y^2+2x+3y+1=0$ 2) $x^2+3xy+2y^2+2x+3y+2=0$

3) $x^2+3xy+2y^2+2x+3y+3=0$ 4) $x^2+3xy+2y^2+2x+3y+4=0$

Key. 2

Sol. Let $H=x^2+3xy+2y^2+2x+3y=0$ and C=0 is its conjugate. Then C + H=2A, where A=0 is the combined equation of asymptotes. Equation of asymptotes is $x^2+3xy+2y^2+2x+3y+\lambda=0$, where $\Delta=abc+2fgh-af^2-bg^2-ch^2=0\Rightarrow\lambda=1$

$\therefore C=2\left(x^2+3xy+2y^2+2x+3y+1\right)-\left(x^2+2y^2+3xy+2x+3y\right)$

$\Rightarrow$ equation of conjugate hyperbola is $x^2+3xy+2y^2+2x+3y+2=0$

102. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB is an equilateral triangle O being the origin, then the eccentricity of the hyperbola satisfies

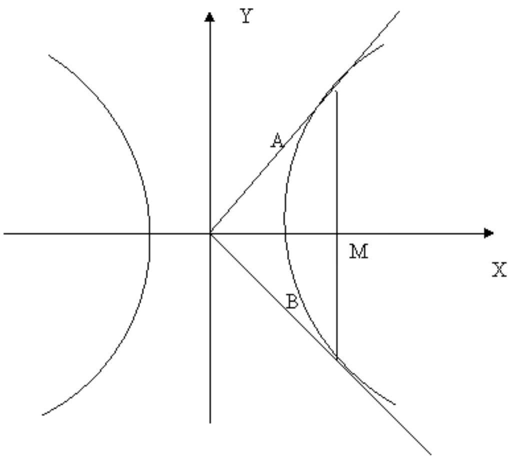
1) $e > \sqrt{3}$ 2) $1 < e < \frac{1}{\sqrt{3}}$ 3) $e = \frac{2}{\sqrt{3}}$ 4) $e > \frac{2}{\sqrt{3}}$

Key. 4

Sol. Let the length of the double ordinate be 2ℓ

$\therefore$ AB=2ℓ and AM=BM=ℓ

Clearly ordinate of point A is ℓ.



The abscissa of the point A is given by

$$\frac{x^2}{a^2} - \frac{l^2}{b^2} = 1 \Rightarrow x = \frac{a\sqrt{b^2 + l^2}}{b}$$

$\therefore$ A is $\left(\frac{a\sqrt{b^2 + l^2}}{b}, l\right)$

Since ΔOAB is equilateral triangle, therefore

OA=AB=OB=$2l$

Also, $OM^2 + AM^2 = OA^2 \therefore \frac{a\left(b^2 + l^2\right)}{b} + l^2 = 4l^2$

We get $l^2 = \frac{a^2 b^2}{3b^2 - a^2}$

Since $l^2 > 0 \therefore \frac{a^2 b^2}{3b^2 - a^2} > 0 \Rightarrow 3b^2 - a^2 > 0$

$$\Rightarrow 3a^2\left(e^2 - 1\right) - a^2 > 0 \Rightarrow e > \frac{2}{\sqrt{3}}$$

103. If the line $5x+12y-9=0$ is a tangent to the hyperbola $x^2-9y^2=9$, then its point of contact is

1) (-5,4/3) 2) (5,-4/3) 3) $(3,-1/2)$ 4) (5,4/3)

Key. 2

Sol. Common Point

104. Any chord passing through the focus $(ae,0)$ of the hyperbola $x^2-y^2=a^2$ is conjugate to the line

1) $ex-a=0$ 2) $ae+x=0$ 3) $ax+e=0$ 4) $ax-e=0$

Key. 1

Sol. $S_1=0$

105. Number of points from where perpendicular tangents to the curve $\frac{x^2}{16}-\frac{y^2}{25}=1$ can be drawn, is:

1) 1 2) 2 3) 0 4) 3

Key. 3

Sol. Director circle is set of points from where drawn tangents are perpendicular in this case $x^2+y^2=a^2-b^2$ (equation of director circle)i.e., $x^2+y^2=-9$ is not a real circle so there is no points from where tangents are perpendicular.

106. $x^2-y^2+5x+8y-4=0$ represents

1) Rectangular hyperbola 2)Ellipse

3) Hyperbola with centre at (1,1) 4)Pair of lines

Key. 1

Sol. $\Delta \neq 0,\ x^2-ab>0,\ a+b=0$

107. Coordinates if foci of the hyperbola xy=4 are

1) $(2\sqrt{2},2\sqrt{2}),(-2\sqrt{2},-2\sqrt{2})$ 2) $(-3\sqrt{2},-3\sqrt{2}),(3\sqrt{2},3\sqrt{2})$

3) $(2\sqrt{2},-2\sqrt{2}),(-2\sqrt{2},2\sqrt{2})$ 4) (-2. 2)

Key. 1

Sol. foci of $xy=c^2$ is $(\pm c\sqrt{2},\pm c\sqrt{2})$

108. Which of the following is INCORRECT for the hyperbola $x^2-2y^2-2x+8y-1=0$

1) Its eccentricity is $\sqrt{2}$ 2) Length of the transverse axis is $2\sqrt{3}$

3) Length of the conjugate axis is $2\sqrt{6}$ 4) Latus rectum $4\sqrt{3}$

Key. 1

Sol. The equation of the hyperbola is $x^2-2y^2-2x+8y-1=0$

Or $(x-1)^2-2(y-2)^2+6=0$

Or $\frac{(x-1)^2}{-6}+\frac{(y-2)^2}{3}=1;$ or $\frac{(y-2)^2}{3}-\frac{(x-1)^2}{6}=1\rightarrow 1$

Or $\frac{Y^2}{3}-\frac{X^2}{6}=1$, where X = x -1 and Y = y - 2 $\rightarrow 2$

$\therefore$ the centre=(0,0)in the X-Y coordinates.

$\therefore$ the centre=(1,2)in the x-y coordinates .using $\rightarrow 2$

If the transverse axis be of length 2a, then a= $\sqrt{3}$,since in the equation (1) the transverse axis is parallel to the y-axis.

If the conjugate axis is of length 2b,then b = $\sqrt{6}$

But $b^2=a^2\left(e^2-1\right)$

$\therefore$ $6=3\left(e^2-1\right), \therefore e^2=3$ or $e=\sqrt{3}$

The length of the transverse axis = $2\sqrt{3}$

The length of the conjugate axis = $2\sqrt{6}$

Latus rectum $4\sqrt{3}$

109. If the curve $xy=R^2-16$ represents a rectangular hyperbola whose branches lies only in the quadrant in which abscissa and ordinate are opposite in sign but not equal in magnitude, then

1) $|R|<4$ 2) $|R|\geq 4$ 3) $|R|=4$ 4) $|R|=5$

Key. 1

Sol. Conceptual

110. Assertion: The pair of asymptotes of $\frac{x^2}{10}-\frac{y^2}{4}=1$ and the pair of asymptotes of $\frac{x^2}{10}-\frac{y^2}{4}=-1$ coincide.

Reason : A hyperbola and its conjugate hyperbola possess the same pair of asymptotes

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true R is false

4) A is false R is true

Key. 1

Sol. Conceptual

111. If the line ax + by + c=0 is a normal to the curve xy=1,then

1) a > 0, b > 0 2) a < 0,b < 0 3) a < 0,b > 0 4) a=b=1

Key. 3

Sol. Slope of the line $\frac{-a}{b}$ is equal to slope of the normal to the curve.

$\therefore$ either a > 0 & b < 0 (or) a < 0 & b > 0.

112. The equation of normal at $\left(at, \frac{a}{t}\right)$ to the hyperbola $xy = a^2$ is

1) $xt^3 - yt + at^4 - a = 0$ 2) $xt^3 - yt - at^4 + a = 0$

3) $xt^3 + yt + at^4 - a = 0$ 4) $xt^3 + yt - at^4 - a = 0$

Key. 2

Sol. Equation of tangent is $s_1 = 0$ normal is $\perp^r$ to tangent and passing through

$\left(at, \frac{a}{t}\right)$ is $xt^3 - yt - at^4 + a = 0$

113. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ to its asymptotes is equal to:

1) $\frac{6}{5}$ 2) $\frac{36}{13}$ 3) Depending on θ 4) $\frac{5}{6}$

Key. 2

Sol. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is equal to $\frac{a^2b^2}{a^2 + b^2}$

114. The foot of the perpendicular from the focus to an asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

1) (ae , be) 2) (a/e,b/e) 3) (e/a,e/b) 4) (be,ae)

Key. 2

Sol. Focus S=(ae,0) Equation of one asymptote is bx-ay=0

Let (h,k) be the foot of the perpendicular from s to bx-ay=0

Then $\frac{h - ae}{b} = \frac{k - 0}{-a} = \frac{-abe}{a^2 + b^2} \Rightarrow \frac{h - ae}{b} = \frac{-abe}{a^2e^2} \,\&\, \frac{k}{-a} = \frac{-abe}{a^2e^2}$

On simplification, we get h=a/e, k=b/e

Foot of the perpendicular is (a/e,b/e)

115. The area of the triangle formed by the asymptotes and any tangent to the hyperbola $x^2 - y^2 = a^2$

1) $4a^2$ 2) $3a^2$ 3) $2a^2$ 4) a^2

Key. 4

Sol. Equation of any tangent to $x^2 - y^2 = a^2$

i.e. $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$ *is* $\frac{x}{a}\sec\theta - \frac{y}{a}\tan\theta = 1 \rightarrow (1)$

or $x\sec\theta - y\tan\theta = a$

equation of other two sides of the triangle are

x-y=0..(2) x + y=0(3)

The two asymptotes of the hyperbola $x^2 - y^2 = a^2$

Are x-y=0 and x + y=0)

Solving (1) (2) and (3) in pairs the coordinates of the vertices of the triangle are (0,0)

$\left(\frac{a}{\sec\theta + \tan\theta}, \frac{a}{\sec\theta + \tan\theta}\right)$

And $\left(\frac{a}{\sec\theta - \tan\theta}, \frac{-a}{\sec\theta - \tan\theta}\right) -$

Area of triangle = $\frac{1}{2}\left|\frac{a^2}{\sec^2\theta - \tan^2\theta} + \frac{a^2}{\sec^2\theta - \tan^2\theta}\right|$

$\frac{1}{2}\left(a^2 + a^2\right) \quad \because \sec^2\theta - \tan^2\theta = 1$

$= a^2$

116. The foot of the normal $3x + 4y = 7$ to the hyperbola $4x^2 - 3y^2 = 1$ is

1) $(1,1)$ 2) $(1,-1)$ 3) $(-1,1)$ 4) $(-1,-1)$

Key. 1

Sol. Since the point $(1,1)$ lies on the normal and hyperbola it is the foot of the normal

117. Tangent at the point $\left(2\sqrt{2},3\right)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ meet its asymptotes at A and B, then area of the

triangle OAB, O being the origin is

1) 6 sq. units 2) 3 sq. units 3) 12 sq. units 4) 2 sq. units

Key. 1

Sol. Since area of the Δ formed by tangent at any point lying on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and its asymptotes is always constant and is equal to ab. Therefore, required area is 2 X 3=6 square units.

118. Eccentricity of hyperbola $\frac{x^2}{k}+\frac{y^2}{k}=1(k<0)$ is :

1) $\sqrt{1+k}$ 2) $\sqrt{1-k}$ 3) $\sqrt{1+\frac{1}{k^2}}$ 4) $\sqrt{1-\frac{1}{k}}$

Key. 4

Sol. Given equation can be rewritten as $\frac{y^2}{k^2}-\frac{x^2}{(-k)}=1(-k>0)$

$e^2=1+\frac{(-k)}{k^2}=1-\frac{1}{k}\Rightarrow e=\sqrt{1-\frac{1}{k}}$

119. If the circle $x^2+y^2=a^2$ intersect the hyperbola $xy=c^2$ in four points $P(x_1,y_1),Q(x_2,y_2),R(x_3,y_3),S(x_4,y_4)$ then which of the following does not hold

1) $x_1+x_2+x_3+x_4=0$ 2) $x_1x_2x_3x_4=y_1y_2y_3y_4=c^4$

3) $y_1+y_2+y_3+y_4=0$ 4) $x_1+y_2+x_3+y_4=0$

Key. 4

Sol. $x^2+\frac{c^4}{x^2}=a^2$ $\Rightarrow \mathbf{x^4-a^2x^2+c^4=0}$, 4th option does not hold

120. If a normal to the hyperbola x y = c² at $\left(ct_1,\frac{c}{t_1}\right)$ meets the curve again at $\left(ct_2,\frac{c}{t_2}\right)$, then:

1) $t_1t_2=-1$ 2) $t_2=-t_1-\frac{2}{t_1}$ 3) $t_2^3t_1=-1$ 4) $t_1^3t_2=-1$

Key. 4

Sol. Equation of normal at $\left(ct_1,\frac{c}{t_1}\right)$ is

$t_1^3x-t_1y-ct_1^4+c=0$

It passes through $\left(ct_2,\frac{c}{t_2}\right)$

Ie., $t_1^3.ct_2 - t_1.\frac{c}{t_2} - ct_1^4 + c = 0$

$\Rightarrow (t_1 - t_2)(t_1^3 t_2 + 1) = 0$

$\Rightarrow t_1^3 t_2 = -1$

121. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola xy=c^2 is

1) $\frac{x}{x_1+x_2} + \frac{y}{y_1+y_2} = 1$ 2) $\frac{x}{x_1-x_2} + \frac{y}{y_1-y_2} = 1$ 3) $\frac{y}{x_1+x_2} + \frac{x}{y_1+y_2} = 1$ 4) $\frac{x}{y_1-y_2} + \frac{y}{x_1-x_2} = 1$

Key. 1

Sol. Mid point of the chord is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The equation of the chord in terms of its mid-point is $s_1 = s_{11}$

122. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then $CP^2 + CQ^2 + CR^2 + CS^2 =$

1) r^2 2) $2r^2$ 3) $3r^2$ 4) $4r^2$

Key. 4

Sol. $CP = CQ = CR = CS = r$

123. The product of focal distances of the point (4,3) on the hyperbola $x^2 - y^2 = 7$ is

1) 25 2) 12 3) 9 4) 16

Key. 1

Sol. $e = \sqrt{2},\ sp.s'p = (ex_1 + a)(ex_1 - a) = 25$

124. Let $y = 4x^2$ & $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ intersect iff

1) $|a| \le \frac{1}{\sqrt{2}}$ 2) $a > \frac{1}{\sqrt{2}}$ 3) $a > -\frac{1}{\sqrt{2}}$ 4) $a > \sqrt{2}$

Key. 1

Sol. $y = 4x^2$ & $\frac{1}{4}y = x^2$

Using $\frac{1}{4a^2}y - \frac{y^2}{16} = 1$

$\Rightarrow 4y - a^2y^2 = 16a^2$

$\Rightarrow a^2y^2 - 4y + 16a^2 = 0$

$\Rightarrow D \geq 0$ for intersection of two curves

$\Rightarrow 16 - 4a^2\left(16a^2\right) \geq 0$

$\Rightarrow 1 - 4a^4 \geq 0$

$\Rightarrow \left(2a^2\right) \leq 1$

$\Rightarrow \left|\sqrt{2}a\right| \leq 1 \Rightarrow -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

125. If angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 45°, then value of eccentricity e is

1) $\sqrt{4 \pm 2\sqrt{2}}$ 2) $\sqrt{4 + 2\sqrt{2}}$ 3) $\sqrt{4 - 2\sqrt{2}}$ 4) $\sqrt{4 - 3\sqrt{2}}$

Key. 3

Sol. $2\tan^{-1}\frac{b}{a} = 45^\circ \Rightarrow \frac{b}{a} = \tan 22^\circ = \frac{a^2\left(e^2-1\right)}{a^2} = \left(\sqrt{2}-1\right)^2$

$\Rightarrow e^2 - 1 = 3 - 2\sqrt{2} \Rightarrow e = \sqrt{4 - 2\sqrt{2}}$.

126. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

1) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ 2) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

3) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ 4) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Key. 1

Sol. Equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$. Its eccentricity is $e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Coordinates of foci are $(\pm 1, 0)$.

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a = \sin\theta$

Also, $ae_1 = 1 \Rightarrow e_1 = \operatorname{cosec}\theta$

$\therefore b^2 = a^2\left(e_1^2 - 1\right) = 1 - \sin^2\theta = \cos^2\theta$

Equation of the hyperbola is thus $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

127. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

1) Equation of ellipse is $x^2 + 2y^2 = 1$
2) the foci of ellipse are $(\pm 1, 0)$
3) equation of ellipse are $x^2 + 2y^2 = 4$
4) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Key. 2

Sol. If two concentric conics intersect orthogonally then they must be confocal, so ellipse and hyperbola will be confocal

$\Rightarrow (\pm ae, 0) \equiv (\pm 1, 0)$

[foci of hyperbola are $(\pm 1, 0)$]

128. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x axis at $(9,0)$, then the eccentricity of the hyperbola is:

1) $\sqrt{\frac{5}{2}}$
2) $\sqrt{\frac{3}{2}}$
3) $\sqrt{2}$
4) $\sqrt{3}$

Key. 2

Sol. Normal at (6,3) is

$\frac{a^2x}{6} + \frac{b^2y}{3} = a^2 + b^2$,

$\Rightarrow \frac{9a^2}{6} = a^2 + b^2 \Rightarrow \frac{3}{2} = 1 + \frac{b^2}{a^2}$

$\therefore \quad \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow e^2 - 1 = \frac{1}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$

129. For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in 'α'

1) abscissae of vertices
2) abscissae of foci
3) Eccentricity
4) directrix

Key. 2

Sol. Hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

Coordinates of vertices are $(\pm\cos\alpha, 0)$, eccentricity of the hyperbola is $e = \sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}} = |\sec\alpha|$

$\therefore$ Coordinates of foci are thus $(\pm 1, 0)$, which are independent of α.

Directrix is $x = \pm\cos^2\alpha$

130. A ray emanating from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8, then the equation of the reflected ray after first reflection is (P lies in first quadrant)

A) $\sqrt{3}x - y + 7 = 0$ B) $3\sqrt{3}x - 13y + 15\sqrt{3} = 0$

C) $\sqrt{3}x + y - 14 = 0$ D) $3\sqrt{3}x + 13y - 15\sqrt{3} = 0$

Key. B

Sol. foci = $(\pm 5, 0)$

$\therefore$ Equation of reflected ray after first reflection passes through $P, S^1; P = (8, 3\sqrt{3}), S^1 = (-5, 0)$

131. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola then eccentricity e of the hyperbola satisfies

A) $1 < e < \frac{2}{\sqrt{3}}$ B) $e = \frac{2}{\sqrt{3}}$ C) $e = \frac{\sqrt{3}}{2}$ D) $e > \frac{2}{\sqrt{3}}$

Key. D

Sol. Let $PQ = 2\ell$

$\ell^2 = \frac{a^2b^2}{3b^2 - a^2} > 0 \Rightarrow e > \frac{2}{\sqrt{3}}$

132. $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\theta}{2}\tan\frac{\phi}{2}$ is

A) $\frac{e-1}{e+1}$ B) $\frac{1-e}{1+e}$ C) $\frac{1+e}{1-e}$ D) $\frac{e+1}{e-1}$

Key. B or C

Sol. Conceptual Question

133. If a variable straight line $x\cos\alpha + y\sin\alpha = P$, which is a chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 (b > a)$, subtend a right angle at the centre of the hyperbola, then it always touches a fixed circle whose radius is

A) $\frac{ab}{\sqrt{b-2a}}$ B) $\frac{a}{\sqrt{a-b}}$ C) $\frac{ab}{\sqrt{b^2-a^2}}$ D) $\frac{ab}{b\sqrt{b+a}}$

Key. C

Sol. Making homogeneous equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with the help of $x\cos\alpha + y\sin\alpha = P$

$(\ )x^2 + (\ )y^2 = 0$

$\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{P^2} \Rightarrow P = \frac{ab}{\sqrt{b^2 - a^2}}$

P is the length of perpendicular drawn from (0, 0) to $x\cos\alpha + y\sin\alpha = P$

Radius $= P = \dfrac{ab}{\sqrt{b^2 - a^2}}$

134. The normal at P to a hyperbola of eccentricity e, intersects its transverse and conjugate axes at L and M respectively. If locus of the mid point of LM is a hyperbola, then eccentricity of the hyperbola is

A) $\dfrac{e+1}{e-1}$ B) $\dfrac{e}{\sqrt{e^2-1}}$ C) e D) $\dfrac{2e}{\sqrt{e^2-1}}$

Key. B

Sol. $N_P : ax\cos\theta + by\cot\theta = a^2 + b^2$

$L\left(\dfrac{a^2+b^2}{a}\sec\theta, 0\right)$

$M\left(0, \dfrac{a^2+b^2}{b}\tan\theta\right)$

Locus is $\dfrac{x^2}{\left(\dfrac{a^2+b^2}{2a}\right)^2} - \dfrac{y^2}{\left(\dfrac{a^2+b^2}{2b}\right)^2} = 1 \Rightarrow e_1 = \dfrac{e}{\sqrt{e^2-1}}$

135. If e is the eccentricity of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and θ is the angle between the asymptotes, then $\cos\dfrac{\theta}{2}$ is equal to

A) $\dfrac{1-e}{e}$ B) $\dfrac{2}{e} - e$ C) $\dfrac{1}{e}$ D) $\dfrac{2}{e}$

Key. C

Sol. $\theta = 2\tan^{-1}\dfrac{b}{a} \Rightarrow \tan\dfrac{\theta}{2} = \dfrac{b}{a}$

$\cos\dfrac{\theta}{2} = \dfrac{a}{\sqrt{a^2+b^2}} = \dfrac{1}{\sqrt{1+\dfrac{b^2}{a^2}}} = \dfrac{1}{e}$

136. Area of triangle formed by the lines $x - y = 0, x + y = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is

A) $|a|$ B) $\dfrac{1}{2}|a|$ C) a^2 D) $\dfrac{1}{2}a^2$

Key. C

Sol. Any tangent to $x^2 - y^2 = a^2$ is $x\sec\phi - y\tan\phi = a$

Area $= |a|$

137. The locus of the point of intersection of the line $\sqrt{3}x - y - 4\sqrt{3}K = 0$ and $\sqrt{3}Kx + Ky - 4\sqrt{3} = 0$ is a hyperbola of eccentricity is

A) 1 B) 2 C) 2.5 D) $\sqrt{3}$

Key. B

Sol. $K = \dfrac{\sqrt{3}x - y}{4\sqrt{3}} = \dfrac{4\sqrt{3}}{\sqrt{3}x - y}$

$\Rightarrow 3x^2 - y^2 = 48 \Rightarrow \dfrac{x^2}{16} - \dfrac{y^2}{48} = 1$

$48 = 16(e^2 - 1) \Rightarrow e = 2$

138. The locus of the middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is

A) $3x - 4y = 4$ B) $3y - 4x + 4 = 0$ C) $4x - 4y = 3$ D) $3x - 4y = 2$

Key. A

Sol. Let locus be $P(h, k)$, $T = S_1$

$3hx - 2ky + 2(x + h) - 3(k + y) = 3h^2 - 2k^2 + 4h - 6k$

Slope $= \dfrac{3h + 2}{2k + 3} = 2 \Rightarrow 3x - 4y = 4$

139. From a point P(1, 2) pair of tangent's are drawn to a hyperbola 'H' in which one tangent to each arm of hyperbola. Equation of asymptotes of hyperbola H are $\sqrt{3}x - y + 5 = 0$ & $\sqrt{3}x + y - 1 = 0$ then eccentricity of 'H' is

A) 2 B) $\dfrac{2}{\sqrt{3}}$ C) $\sqrt{2}$ D) $\sqrt{3}$

Key. B

Sol. Since $c_1c_2\left(a_1a_2 + b_1b_2\right) < 0$

$\therefore$ origin lies in acute angle

P(1, 2) lies in obtuse angle

Acute angle between the asymptotes is $\dfrac{\pi}{3}$

$\therefore$ $e = \sec\dfrac{\theta}{2} = \sec\dfrac{\pi}{6} = \dfrac{2}{\sqrt{3}}$

140. If a variable line has its intercepts on the co-ordinates axes e, e', where $\dfrac{e}{2}, \dfrac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2 + y^2 = r^2$, where r =

A) 1 B) 2 C) 3 D) can not be decided

Key. B

Sol. Since $\dfrac{e}{2}$ and $\dfrac{e'}{2}$ are eccentricities of a hyperbola and its conjugate

$\therefore$ $\dfrac{4}{e^2} + \dfrac{4}{e'^2} = 1$

i.e. $4 = \dfrac{e^2 e'^2}{e'^2 + e'^2}$

line passing through the points (e, 0) and $(0, e')$ $e'x + ey - ee' = 0$

it is tangent to the circle $x^2 + y^2 = r^2$

$\therefore$ $\dfrac{ee'}{\sqrt{e^2 + e'^2}} = r$

$\therefore$ $r^2 = \dfrac{e^2 e'^2}{e^2 + e'^2} = 4$

$\therefore$ r = 2

141. If angle between asymptote's of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 120° and product of perpendiculars drawn from foci upon its any tangent is 9, then locus of point of intersection of perpendicular tangents of the hyperbola can be –

A) $x^2+y^2=6$ B) $x^2+y^2=9$ C) $x^2+y^2=3$ D) $x^2+y^2=18$

Key. D

Sol. $b^2=9$

$\frac{b}{a}=\tan 30°=\frac{1}{\sqrt{3}}$

$\therefore$ $a^2=3b^2=27$

$\therefore$ Required locus is director circle of the hyperbola & which is $x^2+y^2=27-9,\ x^2+y^2=18$

If $\frac{b}{a}=\tan 60°$ is taken then

$$a^2=\frac{b^2}{3}=\frac{9}{3}=3.$$

$\therefore$ Required locus is $x^2+y^2=3-9=-6$ which is not possible.

142. 'C' be a curve which is locus of point of intersection of lines x = 2 + m and my = 4 – m. A circle $s\equiv(x-2)^2+(y+1)^2=25$ intersects the curve C at four points P,Q,R and S. If O is centre of the curve 'C' then $OP^2+OQ^2+OR^2+OS^2$ is

A) 50 B) 100 C) 25 D) 25/2

Key. B

Sol. x – 2 = m

$y+1=\frac{4}{m}$

$\therefore$ (x – 2) (y + 1) = 4

$\Rightarrow$ XY = 4, where X = x – 2, Y = y + 1

$S\equiv(x-2)^2+(y+1)^2=25$

$\Rightarrow$ $X^2+Y^2=25$

Curve 'C' & circle S both are concentric

$\therefore$ $OP^2+OQ^2+OR^2+OS^2=4r^2=4.25=100$

143. The combined equation of the asymptotes of the hyperbola $2x^2+5xy+2y^2+4x+5y=0$ is

A) $2x^2+5xy+2y^2+4x+5y+2=0$ B) $2x^2+5xy+2y^2+4x+5y-2=0$

C) $2x^2+5xy+2y^2=0$ D) none of these

Key. A

Sol. Let the equation of asymptotes be

$2x^2+5xy+2y^2+4x+5y+\lambda=0$ …(1)

This equation represents a pair of straight lines therefore

$abc+2fgh-af^2-bg^2-ch^2=0$

$\therefore$ $4\lambda+25-\frac{25}{2}-8-\lambda\frac{25}{4}=0$ $\Rightarrow$ $-\frac{9\lambda}{4}+\frac{9}{2}=0$

$\Rightarrow$ $\lambda=2$

Putting the value of λ in (i), we get $2x^2+5xy+2y^2+4x+5y+2=0$ this is the equation of the asymptotes.

144. If $\alpha+\beta=3\pi$ then the chord joining the points α and β for the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through

A) focus
B) centre
C) one of the end points of the transverse axis
D) one of the end points of the conjugates axis

Key. B

Sol. (i) Equation of chord joining α and β is

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right)-\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha+\beta}{2}\right)$$

$\therefore$ $\alpha+\beta=3\pi$

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{y}{b}=0$$

If passes through the centre (0, 0)

145. For a given non-zero value of m each of the lines $\frac{x}{a}-\frac{y}{b}=m$ and $\frac{x}{a}+\frac{y}{b}=m$ meets the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at a point. Sum of the ordinates of these points, is

A) $\frac{a\left(1+m^2\right)}{m}$
B) $\frac{b\left(1-m^2\right)}{m}$
C) 0
D) $\frac{a+b}{2m}$

Key. C

Sol. Ordinate of the point of intersection of the line $\frac{x}{a}-\frac{y}{b}=m$ and the hyperbola is given by

$\left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{a}-\frac{y}{b}+\frac{2y}{b}\right)=1$ i.e. $m\left(m+\frac{2y}{b}\right)=1$ i.e. $y=\frac{b\left(1-m^2\right)}{2m}$

Similarly ordinate of the point of intersection of the line $\frac{x}{a}+\frac{y}{b}=m$ and the hyperbola is given by

$y=\frac{b\left(m^2-1\right)}{2m}$ $\therefore$ Sum of the ordinates is 0.

146. The equation of the transverse axis of the hyperbola $\left(x-3\right)^2+\left(y+1\right)^2=\left(4x+3y\right)^2$ is

A) x + 3y = 0
B) 4x + 3y = 9
C) 3x – 4y = 13
D) 4x + 3y = 0

Key. C

Sol. $\left(x-3\right)^2+\left(y+1\right)^2=\left(4x+3y\right)^2$

$$\left(x-3\right)^2+\left(y+1\right)^2=25\left(\frac{4x+3y}{5}\right)^2$$

PS = 5PM

$\therefore$ directrix is 4x + 3y = 0 and focus (3, - 1)

So transverse axis has slope $=\frac{3}{4}$ and equation of transverse axis $y+1=\frac{3}{4}\left(x-3\right)$

$\Rightarrow$ 3x – 4y = 13

147. For which of the hyperbola we can have more than one pair of perpendicular tangents?

A) $\frac{x^2}{4}-\frac{y^2}{9}=1$
B) $\frac{x^2}{4}-\frac{y^2}{9}=-1$
C) $x^2-y^2=4$
D) xy = 4

Key. B

Sol. Locus of point of intersection of perpendicular tangents is director circle for $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ equation of director circle is $x^2+y^2=a^2-b^2$ which is real if a > b

$\Rightarrow$ B is correct answer.

148. From point (2, 2) tangents are drawn to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ then point of contact lie in

A) I & II quadrants B) I & IV quadrants C) I & III quadrants D) III & IV quadrants

Key. D

Sol. Equation of Asymplote are 4y – 3x = 0 and 4y + 3x = 0

Since point (2, 2) lies above the asymptotes 4y – 3x = 0,

Hence point of constant of pair of tangent are in III & IV quadrant

149. The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy=c^2$ is

A) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ B) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$

C) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ D) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

Key. A

Sol. Mid point is $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$\therefore$ equation of the chord to the hyperbola $xy=c^2$ whose midpoint is M, is $\frac{x}{\frac{x_1+x_2}{2}}=\frac{y}{\frac{y_1+y_2}{2}}=2$

$\Rightarrow \frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$

150. The locus of the foot of the perpendicular from the centre of the hyperbola $xy=c^2$ on a variable tangent is

A) $(x^2-y^2)^2=4c^2xy$ B) $(x^2+y^2)^2=2c^2xy$ C) $(x^2+y^2)=4x^2xy$ D) $(x^2+y^2)^2=4c^2xy$

Key. D

Sol. Equation of tangent at $P, \frac{x}{t}+ty=2c$.

or $x+t^2y=2ct$ …(i)

slope of tangent $=-\frac{1}{t^2}$

$\therefore$ equation of CM is $y=t^2\ x$ …(ii)

Squaring (i), $(x+t^2y)^2=4c^2t^2$

Using (ii), we get $\left(x+\frac{y^2}{x}\right)^2=4c^2+\frac{y}{x}\Rightarrow(x^2+y^2)=4c^2xy$

151. If $P(x_1,y_1), Q(x_2, y_2), R(x_3, y_3)$ & $S(x_4, y_4)$ are 4 concyclic points on the rectangular hyperbola $xy=c^2$, the co-ordinates of the orthocenter of the triangle PQR are

A) $(x_4,-y_4)$ B) (x_4,y_4) C) $(-x_4,-y_4)$ D) $(-x_4,y_4)$

Key. C

Sol. Let P,Q,R,S are $\left(ct, \frac{c}{t}\right)$

Where t is t_1, t_2, t_3, t_4 respectively let equation of circle is $x^2 + y^2 = r^2$

$\left(ct \frac{c}{t}\right)$ satisfy this equation

$\therefore \quad c^2t^2 + \frac{c^2}{t^2} - r^2 = 0$

$c^2t^4 - r^2t^2 + c^2 = 0$

Its roots are t_1, t_2, t_3, t_4

$t_1, t_2, t_3, t_4 = 1$ …(i)

Coordinates of orthocenter of ΔPQR are $\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$

$\Rightarrow \quad \left(-ct_4, -\frac{c}{t_4}\right)$ (using (i))

$\Rightarrow \quad (-x_4, -y_4)$

152. If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b) and $x^2 - y^2 = c^2$ cut at right angles then

A) $(x_4, -y_4)$ B) (x_4, y_4) C) $(-x_4, -y_4)$ D) $(-x_4, y_4)$

Key. C

Sol. Let P on the ellipse is $(a\cos\theta, b\sin\theta)$

Slope of tangent at P on the ellipse $m_1 = -\frac{b\cos\theta}{a\sin\theta}$

Slope of tangent at P on the hyperbola $x^2 - y^2 = c^2$, is

$m_2 = \frac{a\cos\theta}{b\sin\theta}$

Since these curves are intersecting at right angle

$\therefore \quad m_1m_2 = -1$

$-\frac{b}{a} \times \frac{\cos\theta}{\sin\theta} \times \frac{a}{b}\frac{\cos\theta}{\sin\theta} = -1 \Rightarrow \quad \tan^2\theta = 1$

$P(a\cos\theta, b\sin\theta)$ also lies on hyperbola

$\therefore \quad a^2\cos^2\theta - b^2\sin^2\theta = c^2$

$a^2 - b^2\tan^2\theta = c^2 + c^2\tan^2\theta$

$\Rightarrow \quad a^2 - b^2 = c^2 + c^2 \quad [\because \tan^2\theta = 1]$

$a^2 - b^2 = 2c^2$

153. If radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{(b')^2} = 1$ are 2r and r respectively and e_e and e_h be the eccentricities of the ellipse and the hyperbola respectively then

A) $2e_n^2 - e_e^2 = 6$ B) $e_e^2 - 4e_n^2 = 6$ C) $4e_n^2 - e_e^2 = 6$ D) none of these

Key. C

Sol. Equation of director circles of ellipse and hyperbola are respectively.

$x^2 + y^2 = a^2 + b^2$

and $x^2+y^2=a^2-b^2$

$a^2+b^2=4r^2$ …(1)

$a^2-b^2=r^2$ …(2)

So $2a^2=5r^2$

$a^2=\frac{5r^2}{2}$

$b^2=4r^2-\frac{5r^2}{2}$

$b^2=\frac{3r^2}{2}$

$e_n^2=1-\frac{b^2}{a^2}$

$\Rightarrow \quad e_n^2=1-\frac{3r^2}{2}\times\frac{2}{5r^2}=1-\frac{3}{5}=\frac{2}{5}$

$e_n^2=1+\frac{b^2}{a^2}$

$\Rightarrow \quad e_n^2=1+\frac{3}{5}=\frac{8}{5}$

So $\quad 4e_n^2-e_n^2=4\times\frac{8}{5}-\frac{2}{5}=\frac{30}{5}=6$

154. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ & the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide then the value of b^2 is

A) 4 B) 9 C) 16 D) none

Key. C

Sol. For ellipse $a^2=16$ $\Rightarrow$ $e=\sqrt{1-\frac{b^2}{a^2}}=\frac{\sqrt{25-b^2}}{5}$

$\Rightarrow \quad \text{focii}-(\pm ae,\ 0)=\left(\pm\sqrt{25-b^2},0\right)$

For hyperbola, $e=\sqrt{1+\frac{b^2}{a^2}}=\frac{5}{4}$

$\therefore \quad \text{focii}=(\pm ae,0)=(\pm 3,0)$

$\therefore \quad \sqrt{25-b^2}=3\Rightarrow b^2=16$

155. The tangent at any point $P(x_1,y_1)$ on the hyperbola $xy=c^2$ meets the co-ordinate axes at points Q & R. The circumcentre of ΔOQR has co-ordinates.

A) (0, 0) B) (x_1,y_1) C) $\left(\frac{x_1}{2},\frac{y_1}{2}\right)$ D) $\left(\frac{2x_1}{3},\frac{2y_1}{3}\right)$

Key. B

Sol. Tangent at $P(x_1,y_1)$ on $xy=c^2$ is

$\frac{x}{x_1}+\frac{y}{y_1}=2$

$\therefore \quad Q=(2x_1,0),\ R=(0,2y_1)$

Now OQR is a right Δ and QR is the hypotenuse.

$\therefore$ circumcentre = mid pt, of $QR = (x_1, y_1)$

156. The locus of the mid points of the chords passing through a fixed point (α, β) of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

A) a circle with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ B) an ellipse with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

C) a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ D) straight line passing through $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

Key. C

Sol. Let (h, k) be the mid point

$\therefore \quad T = S_1 \quad \Rightarrow \quad \frac{xh}{a^2} - \frac{yk}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$ …(i)

(1) passes through (α, β) so putting (α, β) in it

$\Rightarrow \quad \frac{\alpha x}{a^2} - \frac{\beta y}{b^2} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \Rightarrow \left(\frac{x^2}{a^2} - \frac{\alpha x}{a^2}\right) - \left(\frac{y^2}{b^2} - \frac{\beta y}{b^2}\right) = 0$

$\Rightarrow \quad \frac{\left(x - \frac{\alpha}{2}\right)^2}{a^2} - \frac{\left(y - \frac{\beta}{2}\right)^2}{b^2} + \frac{\alpha^2}{4a^2} - \frac{\beta^2}{4b^2} = 0$

Which is a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

157. If two conics $a_1x^2 + 2h_1xy + b_1y^2 = c_1$ and $a_2x^2 + 2h_2xy + b_2y^2 = c_2$ intersect in four concyclic points, then

A) $(a_1 - b_1)h_2 = (a_2 - b_2)h_1$ B) $(a_1 - b_1)h_1 = (a_2 - b_2)h_2$

C) $(a_1 + b_1)h_2 = (a_2 + b_2)h_1$ D) $(a_1 + b_1)h_1 = (a_2 + b_2)h_2$

Key. A

Sol. On removing xy terms by multiplying $a_1x^2 + 2h_1xy + b_1^2 = C_1$ by h_2 and $a_2x^2 + 2h_2xy + b_2y^2 = C_2$ by h_1 and subtracting we have

$(a_1h_2 - a_2h_1)x^2 + (b_1h_2 - b_2h_1)y^2 = C_1h_2 - C_2h_1$

Now this will represent a circle if coefficient of x^2 = coefficient of y^2

i.e. $a_1h_2 - a_2h_1 = b_1h_2 - b_2h_1$

i.e. $(a_1 - b_1)h_2 = (a_2 - b_2)h_2$

158. The transverse axis of a hyperbola is of length 2a and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2: 1, the equation of the hyperbola is

A) $4x^2 - 5y^2 = 4a^2$ B) $4x^2 - 5y^2 = 5a^2$ C) $5x^2 - 4y^2 = 4a^2$ D) $5x^2 - 4y^2 = 5a^2$

Key. D

Sol.

Clearly $\frac{2ae}{3} = a \quad \Rightarrow \quad e = \frac{3}{2}$

$\therefore \quad S = \left(\frac{3a}{2}, 0\right)$

Directrix is $x = \frac{2a}{3}$

$\therefore$ equation of hyperbola will be $\left(x-\frac{3a}{2}\right)^2+y^2=\frac{9}{4}\left(x-\frac{2a}{3}\right)^2$

Which reduces to $5x^2-4y^2=5a^2$