PHYSICS

The following question given below consist of an "Assertion" (A) and "Reason" (R) Type questions. Use the following Key to choose the appropriate answer.

(A) If both (A) and (R) are true, and (R) is the correct explanation of (A).

(B) If both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) If (A) is true but (R) is false.

(D) If (A) is false but (R) is true.

Q. 1 **Assertion:** If the radius of earth (assuming to be perfect sphere) is decreased keeping its mass constant, effective value of g will increase at poles while may increase or decrease at equator.

Reason : Value of g on the surface of earth is given by $g = \frac{GM}{R^2}$. **[B]**

Q.2 **Assertion :** At height h from ground and at depth h below ground, where h is approximately equal to 0.62 R, the value of g acceleration due to gravity is same.

Reason : Value of g decreases both sides, in going up and down.

Sol. **[B]**

$g_d = g_h$

$$g_s\left[1-\frac{h}{R}\right] = \frac{g_s}{\left(1+\frac{h}{R}\right)^2}$$

$$\left(1-\frac{h}{R}\right)\left(1+\frac{h}{R}\right)^2 = 1 \Rightarrow h \approx 0.62R$$

Q. 3 **Assertion :** Radius of circular orbit of a satellite is made two times, then its areal velocity will also become two times.

Reason : Areal velocity is given as

$$\frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m}.$$

Sol. **[D]**

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant.}$$

Q.4 **Assertion :** Radius of a circular orbit of a satellite is made two times, then its areal velocity will also becomes two times.

Reason : Areal velocity is given as

$\frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m}$ **[D]**

Q.5 **Assertion** : To obtain same change in the value of ‘g’, depth (d) below the surface of earth must be equal to twice the height (h) above the surface of earth.

Reason : ‘g’ changes less with depth than with height. **[C]**

Q.6 **Assertion** : Orbiting satellite or body has K.E. of always less than that of Potential energy.

Reason : For any bound state, the magnitude of potential energy is always twice that of kinetic energy (K.E.) **[A]**

Q.7 **Assertion** : A planet moves faster, when it is closer to the sun in its orbit and vice-versa.

Reason : Orbital velocity in an orbit of planet is constant. **[C]**

Q.8 **Assertion** :When radius of orbit of a satellite is made 4 times, its time period becomes 8 times.

Reason : Greater the height above the surface of earth, greater is the time period of revolution.

[C]

Q.9 **Assertion** : When a body is projected with velocity $V = V_0$ (where V_0 is orbital velocity) then path of the projectile is circular.

Reason : Gravitational force between body and the earth provides the centripetal force. **[B]**

Q.10 **Assertion :** An astronaut in an orbiting space station above the Earth experiences weightlessness.

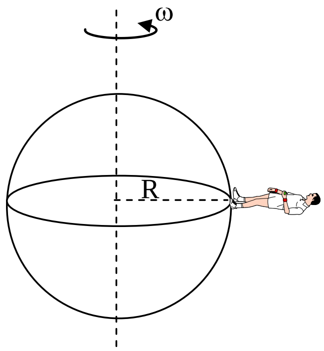
Reason : An object moving around the Earth under the influence of Earth’s gravitational force is in a state of ‘free-fall’. **[IIT – 2008]**

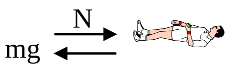
[A]

Q.11 **Assertion :** A person on equator of earth feels weightlessness.

Reason : Angular velocity of rotation of earth about its own axis is $\sqrt{\frac{g}{2R}}$, where R is the radius of earth. **[C]**

Sol.





$mg - N = m\omega^2 R$

$N = mg - m\omega^2 R$

For weightlessness means absence of normal reaction : N = 0

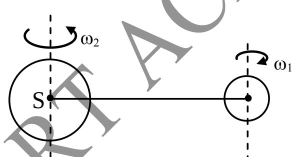
$N = 0 = mg - m\omega^2 R$

$\omega = \sqrt{\frac{g}{R}}$

Q.12 **Assertion :** Duration of sedrial day is smaller than solar day.

Reason : Earth rotates about its axis in opposite sense (ray clockwise) to that it revolves around sun (anticlockwise). **[A]**

Sol.



Let

ω_1 = Velocity of earth with respect to fixed star about its axis.

ω_2 = Angular velocity of earth about axis passing through sun

$\omega_{earth,\ sun} = \omega_1 - \omega_2 < \omega_1$

Q.13 **Assertion :** The comet does not obey Kepler's laws of planetary motion.

Reason : The comet does not have elliptical orbit. **[A]**

Q.14 **Assertion:** At height h from surface of earth and at depth h below the surface where h is approximately equal to 0.62 R (R is radius of earth) the value of acceleration due to gravity g is same.

Reason: Value of g decreases both sides in going up and down from earth's surface. **[B]**

Q.15 **Statement I :** Escape velocity of a tennis ball from the surface of earth is the same as the escape velocity of a cricket ball from the surface of earth.

Statement II : Escape velocity of a body is independent of the mass of the body. **[A]**

Q.16 **Statement I :** The weight of a body at the centre of earth is zero.

Statement II : The mass of a body decreases with increase in depth below the surface of earth. **[C]**

Q.17 **Statement I :** Earth continuously attracts the moon towards its centre and yet the moon does not move towards the centre of earth.

Statement II : The gravitational force of atteraction between the moon and earth provides the necessary centripetal force. **[D]**

Q.18 **Assertion :** At height h from ground and at depth h below ground, where h is approximately equal to 0.62 R, the value of g acceleration due to gravity is same.

Reason : Value of g decreases both sides, in going up and down.

(A) a (B) b

(C) c (D) d **[B]**

Q.19 **Assertion :** If radius of earth (assuming to be perfect sphere) is decreased keeping its mass constant, effective value of g will increase at poles while may increase or decrease at equator.

Reason : Value of g on the surface of earth is given by $g = \frac{GM}{R^2}$

(A) a (B) b

(C) c (D) d **[B]**

Q.20 **Assertion :** Radius of circular orbit of a satellite is made two times, then its areal velocity will also become two times.

Reason: Areal velocity is given as $\frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m}$.

(A) a (B) b

(C) c (D) d **[D]**

PHYSICS

Q.1 Density of a planet is two times the density of earth. Radius of this planet is half the radius of earth. Match, the following. (As compared to earth)

	Column I		**Column II**
(A)	Acceleration due to gravity on this planet's surface	**(P)**	Half
(B)	Gravitational potential on the surface	**(Q)**	Same
(C)	Gravitational potential at centre	**(R)**	Two times
(D)	Gravitational field strength at centre	**(S)**	Four times

Sol. **A $\rightarrow$ Q B $\rightarrow$ P**

C $\rightarrow$ P D $\rightarrow$ Q

$\rho_P = 2\rho_e$

and $R = \frac{1}{2} R_e$

(A) $g_P = \frac{4}{3}\pi G\rho R = g_e$

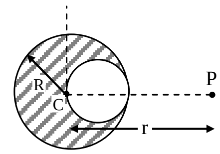
(B) $V_P = -\frac{GM}{R} = -gR = \frac{V_e}{2}$

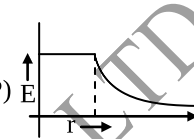
(C) $V = -\frac{3GM}{2R} = -\frac{3}{2}gR = \frac{V_e}{2}$

(D) $I = I_e = 0$

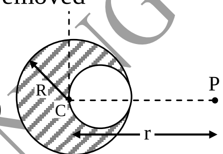
Q.2 Column I shows various mass distribution and Column II contains magnitude of gravitation field vs 'r' and magnitude of gravitation potential vs 'r' graphs. Where 'r' is distance from centre 'C'.

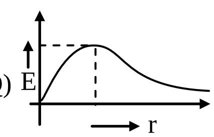
Column I **Column II**

(A) 

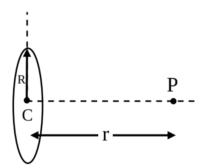
(P) 

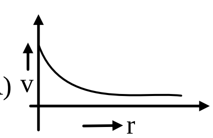
Solid sphere of radius 'R' from which sphere of radius 'R/2' is removed

(B) 

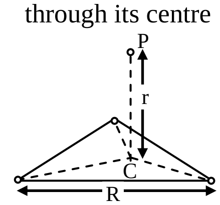
(Q) 

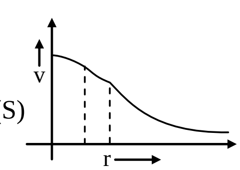
Cross section of long solid cylinder from which a cylinder of radius 'R/2' is removed

(C) 

(R) 

Ring of radius 'R', point 'P' is at the axis of ring perpendicular to ring and passing through its centre

(D) 

(S) 

There identical masses placed at the corner of equilateral triangle, Point 'P' lies of a line perpendicular to plane of triangle and passing through its centroid

Sol. **A$\rightarrow$P,S ; B$\rightarrow$ P ; C$\rightarrow$ Q,R ; D$\rightarrow$Q,R**

(A) $E = \frac{4\pi\rho G}{3}(R/2)$: $r \le R$

[ρ = density ,

R = Radius of bigger sphere]

$= \frac{GM}{r^2}$: $r > R$

$|V| = \frac{2\pi G\rho}{3}\left\{\frac{3R^2}{4} - (r - R/2)^2\right\}$: $r \le R$

$= \frac{GM}{r}$: $r > R$

(B) $E = 2\pi G\rho \cdot (R/2)$: $r \le R$

$= \frac{2\pi G\rho R^2}{r}$: $r > R$

(C) $E = GM \frac{r}{(R^2 + r^2)^{3/2}}$

$V = \frac{GM}{(R^2 + r^2)^{1/2}}$

(D) $E = G.3m \frac{r}{(\frac{R^2}{3} + r^2)^{3/2}}$

$V = \frac{G.3m}{(\frac{R^2}{3} + r^2)^{1/2}}$

Q.3

Column - I	**Column - II**
(A) Elliptical orbit of planet	(P) Kinetic energy conservation
(B) Circular orbit of satellite	(Q) Angular momentum conservation
(C) Escape velocity	(R) Independent of mass of particle/ satellite
(D) Orbital velocity	(S) $\sqrt{\frac{GM}{R}}$

Sol. **(A) → Q; (B) → P, Q; (C) → R; (D) → R, S**

Q.4 Density of a planet is two times the density of earth. Radius of this planet is half the radius of earth. Match, the following. (As compared to earth)

	Column I		**Column II**
(A)	Acceleration due to gravity on this planet's surface	**(P)**	Half
(B)	Gravitational potential on the surface	**(Q)**	Same
(C)	Gravitational potential at centre	**(R)**	Two times
(D)	Gravitational field strength at centre	**(S)**	Four times

Ans. $A \to Q$; $B \to P$; $C \to P$; $D \to Q$

Q.5 In elliptical orbit of a planet, as the planet moves from apogee position to perigee position, match the following table :

	Table I		**Table II**
(A)	speed of planet	**(P)**	remains same
(B)	distance of planet from centre of sun	**(Q)**	decreases
(C)	potential energy	**(R)**	increases
(D)	angular momentum about centre of sun	**(S)**	can not say

Ans. $A \to R$; $B \to Q$; $C \to Q$; $D \to P$

Q.6 On the surface of earth acceleration due to gravity is g and gravitational potential is V. Match the following:

Table-1	**Table-2**
(A) At height h = R, value of g	(P) decreases by a factor 1/4
(B) At depth h = R/2, value of g	(Q) decreases by a factor 1/2
(C) At height h = R, value of V	(R) increases by a factor 11/8
(D) at depth h = R/2, value of V	(S) increases by a factor 2
	(T) None

Ans. $A \to Q$; $B \to Q$; $C \to S$; $D \to T$

Q.7 A particle is projected from the surface of earth with speed v. Suppose it travels a distance x when its speed becomes v to $\frac{v}{2}$ and y when speed changes from $\frac{v}{2}$ to 0. Similarly, the corresponding times are suppose t_1 and t_2. Then:

Table-1	Table-2
(A) $\frac{x}{y}$	(P) 1
(B) $\frac{t_1}{t_{f2}}$	(Q) >1
	(R) < 1

Ans. $A \to R; B \to R$

Q.8 Density of a planet is tro times the density of earth. Radius of this planet is half. Match, the followings. (As compared to earth)

Table-1	Table-2
(A) Acceleration due to gravity on this planet's surface	(P) Half
(B) Gravitational potential on the surface	(Q) Same
(C) Gravitational potential at centre	(R) Two times
(C) Gravitational field strength at centre	(S) Four times

Ans. $A \to Q; B \to P;\ C \to S; D \to Q$

Q.9 In elliptical orbit of a planet, as the planet moves from apogee position to perigee position, match the following table:

Table-1	Table-2
(A) speed of planet	(P) remains same
(B) distance of planet from centre of sun	(Q) decreases
(C) potential energy	(R) increases
(D) angular momentum about centre of sun	(S) can not say

Ans. $A \to R; B \to Q;\ C \to Q; D \to P$

Q.10 Match the following:

Table-1	Table-2
(A) Kepler's first law	(P) $T^2 \propto r^3$
(B) Kepler's second law	(Q) areal velocity constant
(C) Kepler's third law	(R) orbit of planet is elliptical

Ans. $A \to R; B \to Q;\ C \to P$

Q.11 Match the following:

Table-1	Table-2
(A) Time period of an earth satellite in circular orbit	(P) Independent of of satellite
(B) Orbital velocity of satellite	(Q) Independent of radius of orbit
(C) Mechanical energy of mass of earth	(R) Independent
	(S) None

Ans. $A \to P; B \to P;\ C \to S$

Q.12 If earth decreases its rotational speed. Match the following:

Table-1	Table-2
(A) Value of g at pole	(P) will reamain same
(B) Value of g at equator	(Q) will increase
(C) Distance of geostationary satellite	(R) will decrease
(D) Energy of geostationary satellite	(S) can not say

Ans. $A \to P; B \to Q;\ C \to Q; D \to Q$

Q.13 Match the following (for a satellite in circular orbit)

Table-1	Table-2
(A) Kinetic energy	(P) $-\frac{GMm}{2r}$
(B) Potential energy	(Q) $\sqrt{\frac{GM}{r}}$
(C) Total energy	(R) $-\frac{GMm}{r}$
(D) Orbital velocity	(S) $\frac{GMm}{2r}$ W

Ans. $A \to S; B \to R;\ C \to P; D \to Q$

Q.14 Considering earth to be homogeneous sphere but keeping in mind its spin match the following :

Column-I	Column-II
(A) Acceleration due to gravity	(P) May change point to point
(B) Orbital angular momentum of earth as seen from different star	(Q) dependent upon direction of projection
(C) Escape velocity from the earth	(R) Remains constant
(D) Gravitational potential due to earth at a particular point	(S) Changes with time

Sol. **A $\rightarrow$ P ; B $\rightarrow$ S ; C $\rightarrow$ P,Q ; D $\rightarrow$ R**

Q.15 Let V and E denote the gravitational potential and gravitational field respectively at a point due to certain uniform mass distribution described in four different situation of column – I, then

	Column -I		Column -II
(A)	At centre of thin spherical shell	(P)	$E = 0$
(B)	At centre of solid sphere	(Q)	$E \neq 0$
(C)	At solid sphere has non concentric spherical cavity at the centre of the spherical cavity	(R)	$V \neq 0$
(D)	At centre of line joining two point masses of equal magnitude	(S)	$V = 0$

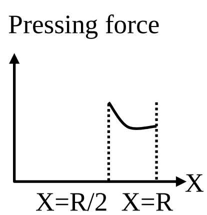
Sol.A $\rightarrow$ P, R B $\rightarrow$ P, R

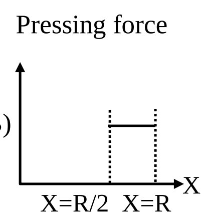
C $\rightarrow$ Q, R D $\rightarrow$ P, R

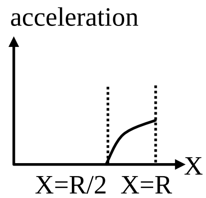
Answer are self explanatory

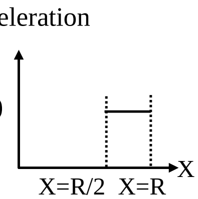
PHYSICS

Q.1 A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's centre. The wall of the funnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall and the acceleration of the particle varies with x (distance of the particle from the centre) according to:

(A) 

(B) 

(C) 

(D) 

[B, C]

Sol. Net force towards centre of earth

$= mg' = \frac{mgx}{R}$

Normal force N = mg′ sin θ

Thus pressing face $N = \frac{mgx}{R}\,\frac{R}{2X}$

$N = \frac{mg}{2}$ Constant and independent of X

tangential force F = ma = mg′ cos θ

$a = g' \cos\theta = \frac{gx}{R}\,\frac{\sqrt{\frac{R^2}{4} - X^2}}{X}$

$a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$

curve is parabolic and at X = R/2, a = 0

Q.2 At surface of earth potential energy of a particle is U and potential is V. Change in potential energy and potential at height h = R are suppose ΔU and ΔV. Then :

(A) ΔU = – U/2 (B) ΔU = U/2

(C) ΔV = V/2 (D) ΔV = – V/2

Sol. **[A,D]**

$U_i = -\frac{GMm}{R} = U$ (given)

$\Delta U = U_f - U_i = \frac{-GMm}{2R} + \frac{GMm}{R}$

$= \frac{GMm}{2R} = -\frac{U}{2}$

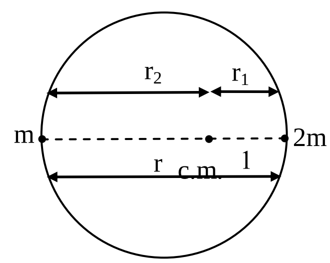
Same is the case with potential.

Q.3 A double star in a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to-

(A) $r^{3/2}$ (B) r

(C) $m^{1/2}$ (D) $m^{-1/2}$

Sol. **[A,D]**



$r_2 = \frac{2mr}{m+2m} = \frac{2r}{3}$

$T_2^2 = \frac{4\pi^2 r_2^3}{Gm}$

$T_2^2 = \frac{32\pi^2 r^3}{27Gm}$

$T_2 \propto r^{3/2}$, $T_2 \propto m^{-1/2}$

Q.4 Which of the following is correct ?

(A) Moon has no atmosphere

(B) The tangential acceleration of a planet is zero

(C) Gravitational force is a medium dependent force

(D) Gravitational force is a conservative force

[A,B,D]

Q.5 Total energy of planet of mass m moving around the sun along an elliptical orbit depends on–

(A) Mass of the sun

(B) Mass of the planet

(C) Semi major axis

(D) Independent of mass of the sun **[A,B,C]**

Q.6 An object is revolving around the earth at height h from earth's surface. Then–

(A) Its time period is independent of h

(B) Its time period depends on h

(C) Orbital velocity of an object depends on h

(D) If $h << R_e$ then the time period of an object around the earth is 24 hrs. **[B,C]**

Q.7 A satellite is to be stationed in an orbit such that it can be used for relay purposes (such a satellite is called a Geostationary satellite). The conditions such a should fulfill is/are–

(A) its orbit must lie in equatorial plane

(B) its sense of rotation must be from west to east

(C) its orbital radius must be 42400 km

(D) its orbit must not be elliptical and should be circular **[All]**

Q.8 A satellite is orbiting round the earth's surface in an orbit as close as possible to the surface of the earth. Then-

(A)The time period of revolution of satellite is independent of its mass and is minimum

(B)The orbital speed of satellite is maximum

(C)The total energy of the "earth plus satellite" system is minimum

(D)The total energy of the "earth plus satellite" system is maximum **[A,B,C]**

Q.9 A planet is revolving round the sun. Its distance from the sun at Apogee is r_A and that at Perigee is r_P. The mass of planet and sun is m and M respectively, v_A and v_P is the velocity of planet at Apogee and perigee respectively and T is the time period of revolution of planet round the sun. Then-

(A) $T^2 = \frac{\pi^2}{2GM}(r_A + r_P)^3$

(B) $T^2 = \frac{\pi^2}{2GM}(r_A + r_P)^3$

(C) $v_A r_A = v_P r_P$

(D) $v_A < v_P$; $r_A > r_P$ **[B,C,D]**

Q.10 A body is imparted a velocity v from the surface of the earth. If v_0 is orbital velocity and v_e be the escape velocity then for–

(A) $v = v_0$, the body follows a circular track around the earth

(B) $v > v_0$ but $< v_e$, the body follows elliptical path around the earth

(C) $v < v_0$, the body follows elliptical path and returns to surface of earth

(D) $v > v_e$, the body follows hyperbolic path and escapes the gravitational pull of the earth **[All]**

Q.11 Three point masses each of mass m are at the corners of an equilateral triangle of side ℓ. The system rotates about the centre of the triangle with the separation of masses not changing during rotation. If T is the time period of rotation then–

(A) $T \propto \ell^{3/2}$ (B) $T \propto \ell^{1/2}$

(C) $T \propto m^{1/2}$ (D) $T \propto m^{-1/2}$

[A,D]

Q.12 A body of mass m is projected vertically upwards with velocity V from earth's surface and attains height "h". Then–

(A) If $V = V_e$(where V_e is escape velocity) then $h = \infty$

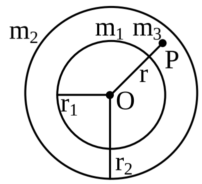
(B) If V is very small then $h = \frac{V^2}{2g}$

(C) If h = R, then projection kinetic energy $\frac{mgR}{2}$

(D) If h = R then total energy is negative

[All]

Q.13 Two concentric spherical shells have masses m_1 and m_2 and radii r_1 and r_2. Then–



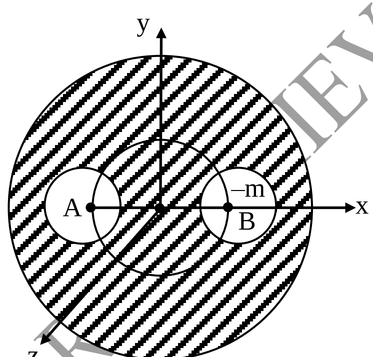
(A) Outer shell will have no contribution in gravitational field at point P

(B) Force on P is directed towards O

(C) Force on P is $\dfrac{Gm_1m_2}{r^2}$

(D) Force on P is $\dfrac{Gm_1m_3}{r^2}$ **[A,B,D]**

Q.14 A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres A(–2, 0, 0) and (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in figure, then – **[IIT-1993]**



(A) The gravitational force due to this object at the origin is zero

(B) The gravitational force at the point B(2, 0, 0) is zero

(C) The gravitational potential is the same at all points of circle $y^2 + z^2 = 36$

(D) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$ **[A,C,D]**

Q.15 The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively . Then- **[IIT-1994]**

(A) $\dfrac{F_1}{F_2} = \dfrac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(B) $\dfrac{F_1}{F_2} = \dfrac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(C) $\dfrac{F_1}{F_2} = \dfrac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(D) $\dfrac{F_1}{F_2} = \dfrac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$ **[A,B]**

Q. 16 For a satellite to be geostationary, which of the following are essential conditions ?

(A) It must always be stationed above the equator.

(B) It must rotate from west to east

(C) It must be about 36,000 km above the earth

(D) Its orbit must be circular, and not elliptical.

[A,B,C,D]

Q. 17 If a satellite orbits as close to the earth's surface as possible-

(A) its speed is maximum

(B) time period of its rotation is minimum

(C) the total energy of the earth plus satellite system is minimum

(D) the total energy of the earth plus satellite system is maximum **[A,B,C]**

Q.18 Three mass 'm' each are kept at corner of a equilateral triangle and are rotating under effect of mutual gravitational force -

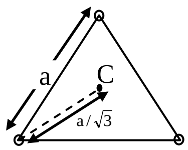
(A) Radius of circular path followed by mass is a/2

(B) Velocity of mass is $\sqrt{\dfrac{Gm}{a}}$

(C) Binding energy of system is $\dfrac{1.5Gm^2}{a}$

(D) Time period of mass is $\sqrt{\dfrac{\pi a^3}{2Gm}}$ **[B,C]**

Sol.



Distance of any mass from centre = $\frac{a}{\sqrt{3}}$

$\therefore$ Radius of circular path followed = $a/\sqrt{3}$

Mass is moving in circular path of radius $a/\sqrt{3}$

$\therefore \frac{mv^2}{(a/\sqrt{3})} = \frac{\sqrt{3}Gm^2}{a}$

$\Rightarrow v = \sqrt{\frac{Gm}{a}} \quad \therefore T = \frac{2\pi(a/\sqrt{3})}{v} = \sqrt{\frac{2\pi a^3}{3Gm}}$

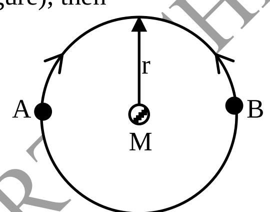
Total K.E. $= 3\left(\frac{1}{2}mv^2\right) = \frac{3}{2}\frac{Gm^2}{a}$

P.E. $= -3.\frac{Gm^2}{a}$

$\therefore$ Total energy $= -\frac{3}{2}.\frac{Gm^2}{a}$

$\therefore$ B.E. $= \frac{3}{2}\frac{Gm^2}{a}$

Q.19 Two satellites A and B of equal mass m_0, moving in the same circular orbit of radius r around the earth (mass M) but in opposite sense of rotation and therefore on a collision course (see figure), then -



(A) Total mechanical energy of the two satellites plus earth system before collision is $\frac{-2GMm}{r}$

(B) Total mechanical immediately after completely inelastic collision is $\frac{-2GMm}{r}$

(C) Subsequent motion of combined satellites mass is along straight line

(D) Subsequent motion of combined satellites mass is along circular orbit of reduced radius **[B,C]**

Sol. (A) T.E. $= 2\left[-\frac{GMm}{2r}\right] = \frac{-GMm}{r}$

(B) T.E. $= -\frac{GM(2m)}{r} = -\frac{2GMm}{r}$

(C) Velocity is zero so along straight line towards earth.

Q.20 Choose the correct statements -

(A) Acceleration due to gravity increases with increasing altitude

(B) Acceleration due to gravity decreases with increasing depth (assume the earth to be a sphere of uniform density)

(C) Acceleration due to gravity is independent of mass of the body

(D) The formula $-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is more accurate than the formula mg $(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of earth

Sol. **[B, D]**

Q.1 A body is projected up with a velocity equal to $\frac{3}{4}$ th of the escape velocity from the surface of the earth. The height it reaches from the centre of the earth is : (Radius of the earth = R) :

(A) $\frac{10R}{9}$ (B) $\frac{16R}{7}$

(C) $\frac{9R}{8}$ (D) $\frac{10R}{3}$

[B]

Sol. $v = \frac{3}{4}\ v_e$

$\text{K.E.} = \frac{1}{2}\ mv^2 = \frac{1}{2}\ m\left(\frac{3}{4}v_e\right)^2$

$= \frac{9}{32}\ mv_e^2$

$= \frac{9}{32}\ m\left(\frac{2GM}{R}\right)$

$\text{K.E} = \frac{9}{16}\ \frac{GMm}{R}$

$\text{P.E.} = -\frac{GMm}{R}$

Total energy = K.E. + P.E. = $-\frac{7}{16}\ \frac{GMm}{R}$

Let the height above the surface of earth be h; then P.E. = $-\frac{GMm}{h}$

Total energy = P.E. above earth's surface

$-\frac{7}{16}\ \frac{GMm}{R} = -\frac{GMm}{h}$

$\therefore \quad h = \frac{16R}{7}$

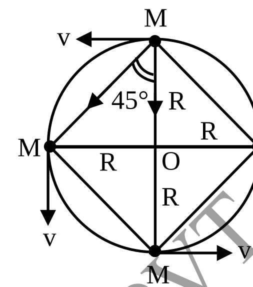
Q.2 Four particles of equal mass M move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is -

(A) $\frac{GM}{R}$ (B) $\sqrt{\left[2\sqrt{2}\ \frac{GM}{R}\right]}$

(C) $\sqrt{\left[\frac{GM}{R}(2\sqrt{2}+1)\right]}$ (D) $\sqrt{\left[\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)\right]}$

[D]

Sol. Gravitational force on each due to other three particles provides the necessary centripetal force.



$$\therefore\ \sqrt{2}\ \frac{GM^2}{(\sqrt{2}\,R)^2}\cos 45° + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$$

Simplifying it, we get

$$v = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$$

Q. 3 The gravitational field in a region is 10 N/kg $(\hat{i}-\hat{j})$. Then the work done by gravitational force to shift slowly a particle of mass 1kg from point (1m, 1m) to a point (2m, –2m) is -

(A) 10 J (B) –10J (C) –40J (D) + 40 J

Sol. **[D]**

$W_g = \vec{F}_g\,.\,\Delta\vec{S} = 10\,(\hat{i}-\hat{j}).(\hat{i}-3\,\hat{j})$

$= 10 + 30 = 40$ J

Q.4 Two bodies of masses m_1 and m_2 are initially at rest placed infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity when they are r distance apart is –

(A) $\sqrt{\frac{2G(m_1+m_2)}{r}}$ (B) $\sqrt{\frac{2G\,m_1 m_2}{(m_1+m_2)r}}$

(C) $\sqrt{\frac{G(m_1+m_2)}{r}}$ (D) $\sqrt{\frac{G\,m_1 m_2}{(m_1+m_2)r}}$

Sol. **[A]**

$m_1v_1 - m_2v_2 = 0$ by conservation of momentum

$\frac{1}{2}\ m_1v_1^2 + \frac{1}{2}\ m_2v_2^2 - \frac{G\,m_1 m_2}{r} = 0$

(by conservation of energy)

Also, $v_{rel.} = v_1 + v_2$

Q. 5 A projectile is fired vertically upwards from the surface of the earth with a velocity kv_e, where v_e is the escape velocity and $k < 1$. If R is the radius of the earth, the maximum height to which it will rise measured from the centre of earth will be - (Neglect air resistance)

(A) $\frac{1-k^2}{R}$ (B) $\frac{R}{1-k^2}$

(C) $R(1 - k^2)$ (D) $\frac{R}{1+k^2}$ **[B]**

Sol. Vertically upwards then let r is the maximum height so from conservation of energy

$$\frac{1}{2}\, m\, (kv_e)^2 + \left\{-\frac{GM_eM}{R}\right\} = 0 + \left\{-\frac{GM_eM}{r}\right\}$$

$$\Rightarrow -GM_eM/r = \frac{1}{2}\, m\, \left(k^2\left(2\frac{GM_e}{R}\right)\right\} - \frac{GM_eM}{R}$$

$$= -(1-k^2)\,\frac{GM_eM}{R} \Rightarrow r = \frac{R}{1-k^2}$$

Q. 6 A man weight W on the surface of the earth. What is his weight at a height equal to R ?

(A) W (B) $\frac{W}{2}$ (C) $\frac{W}{4}$ (D) $\frac{W}{8}$

[C]

Sol. $\frac{W_h}{W} = \frac{mg_h}{mg}$

$= \frac{g_h}{g}$

$= \frac{R^2}{(R+R)^2}$

$W_h = \frac{W}{4}$

Q. 7 If the distance between the earth and the sun becomes half its present value, the number of days in a year would have been -

(A) 64.5 (B) 129

(C) 182.5 (D) 730 **[B]**

Sol. $T = \left[\frac{1}{2}\right]^{\frac{3}{2}} \times 365$ days

$= \frac{1}{2\sqrt{2}} \times 365$ days

$= \frac{182.5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ days

$= 91.25 \times 1.414$ days

$= 129$ days

Q. 8 A particle is kept at rest at a distance R (Earth's radius) above the earth's surface. The minimum speed with which it should be projected so that it does not return is :

(A) $\sqrt{\frac{GM}{4R}}$ (B) $\sqrt{\frac{GM}{2R}}$

(C) $\sqrt{\frac{GM}{R}}$ (D) $\sqrt{\frac{2GM}{R}}$ **[C]**

Sol. using conservation of energy

$-\frac{GMm}{2R} + \frac{1}{2}mv^2 = 0$

or $mv^2 = \frac{GMm}{R}$

or $v = \sqrt{\frac{GM}{R}}$

Q. 9 Acceleration due to gravity at earth's surface is $10\ ms^{-2}$. The value of acceleration due to gravity at the surface of a planet of mass $\frac{1}{5}$th and radius $\frac{1}{2}$ of the earth is -

(A) $4\ ms^{-2}$ (B) $6\ ms^{-2}$

(C) $8\ ms^{-2}$ (D) $12\ ms^{-2}$ **[C]**

Sol. $g_p = \frac{GM_p}{R_p^2}$

$= G \times \frac{1}{5} \times Me \times \frac{4}{R_e^2}$

$= \frac{4}{5}\, g = 8\ ms^{-2}$

Q. 10 With what angular velocity the earth should spin in order that a body lying at 45° latitude may become weightless ?

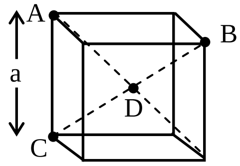
(A) $\sqrt{\frac{g}{R}}$ (B) $\sqrt{\frac{2g}{R}}$

(C) $2\sqrt{\frac{g}{R}}$ (D) None of these **[B]**

Sol. $0 = g - R\omega^2 \cos^2 45°$

$$\frac{R\omega^2}{2} = g \text{ or } \omega = \sqrt{\frac{2g}{R}}$$

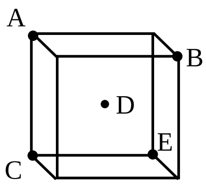
Q. 11 Four identical masses m each are kept at points A, B, C & D shown in figure. Gravitational force on mass at point D (body centre) is -



(A) $\frac{3Gm^2}{a^2}$ (B) $\frac{12Gm^2}{a^2}$

(C) $\frac{4Gm^2}{a^2}$ (D) $\frac{4Gm^2}{3a^2}$ **[D]**

Sol.



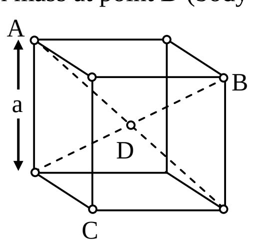
Let us put identical mass at E.
Due to symmetry net force on mass at 'D' is equal to zero.
$\therefore$ Required force = Force due to mass placed at E

$$= \frac{Gm^2}{\left(\sqrt{3}a/2\right)^2} = \frac{4\,Gm^2}{3a^2}$$

Q. 12 The total energy of a satellite is -
(A) Always positive
(B) Always negative
(C) Always zero
(D) +ve or –ve depending upon radius of orbit. **[B]**

Sol. T. E = $-\frac{GMm}{2r}$

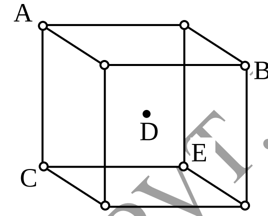
Q.13 Four identical masses m each are kept at points A, B, C & D shown in figure. Gravitational force on mass at point D (body centre) is -



(A) $\frac{3\,Gm^2}{a^2}$ (B) $\frac{12\,Gm^2}{a^2}$

(C) $\frac{4\,Gm^2}{a^2}$ (D) $\frac{4\,Gm^2}{3a^2}$ **[D]**

Sol. Let us put identical mass at E.
Due to symmetry net force on mass at 'D' is equal to zero.



$\therefore$ Required force=Force due to mass placed at E

$$= \frac{Gm^2}{(\sqrt{3}a/2)^2} = \frac{4Gm^2}{3a^2}$$

Q.14 There is a concentric hole of radius R in a solid sphere of radius 2R. Mass of remaining portion is M, then the gravitational potential at centre is

(A) $\frac{-5GM}{7R}$ (B) $\frac{-7GM}{14R}$

(C) $\frac{-3GM}{7R}$ (D) $\frac{-9\,GM}{14R}$ **[D]**

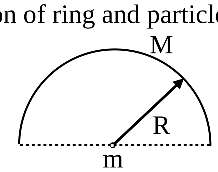
Sol. Potential at centre due to a solid sphere of radius r and mass m is

$$v = -\frac{3Gm}{2r}$$

Here required potential

$$= \begin{bmatrix}\text{Potential due to}\\ \text{sphere of radius } 2R\end{bmatrix} - \begin{bmatrix}\text{Potential due to}\\ \text{sphere of radius } R\end{bmatrix}$$

Q.15 A particle of mass m is placed on the centre of a fixed uniform semi-circular ring of radius R and mass M as shown. Then work required to displace the particle slowly from centre of ring to infinity is : (Assume only gravitational interaction of ring and particle)



(A) $\frac{GMm}{R}$ (B) $-\frac{GMm}{R}$

(C) $\frac{GMm}{\pi R}$ (D) $-\frac{GMm}{\pi R}$ **[A]**

Sol. $U_i = -\frac{GMm}{R}$ and $U_f = 0$

$\therefore \; W = \Delta U = \frac{GMm}{R}$

Q.16 A spherical hole is made in a solid sphere of radius R. The mass of the sphere before hollowing was M. The gravitational field at the centre of the hole due to the remaining mass is –

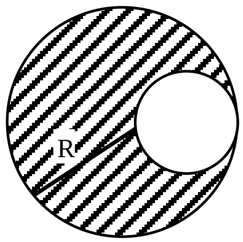


Fig.

(A) zero (B) $\frac{GM}{8R^2}$

(C) $\frac{GM}{2R^2}$ (D) $\frac{GM}{R^2}$ **[C]**

Sol. By the principle of superposition of fields

$\vec{E} = \vec{E}_1 + \vec{E}_2$

Here, $\vec{E}$ = net field at the centre of hole due to entire mass

$\vec{E}_1$ = field due to remaining mass

and $\vec{E}_2$ = field due to mass in hole = 0

$\therefore \; \vec{E}_1 = \vec{E} = \left(\frac{GM}{R^3}\right).r$

where $r = \frac{R}{2}$

$\therefore \; \vec{E} = \frac{GM}{2R^2}$

Q.17 The gravitational field due to a mass distribution is $E = \frac{A}{x^2}$ in x-direction. Here, A is a constant. Taking the gravitational potential to be zero at infinity, potential at **x** is –

(A) $\frac{2A}{x}$ (B) $\frac{2A}{x^3}$

(C) $\frac{A}{x}$ (D) $\frac{A}{2x^2}$ **[C]**

Sol. $V(x) = -\int_{\infty}^{x} E\,dx = -\int_{\infty}^{x} \frac{A}{x^2}\,dx = \frac{A}{x}$

Q.18 A body is projected from the surface of earth with a velocity 2 v_e where v_e is the escape velocity. The velocity of the body when it escapes the gravitational field of the earth is -

(A) $\sqrt{2}\, v_e$ (B) $\sqrt{3}\, v_e$

(C) $\sqrt{7}\, v_e$ (D) $\sqrt{11}\, v_e$

Sol. **[B]**

$v = v_e\sqrt{n^2 - 1}$ or $v = v_e\sqrt{2^2 - 1}$, $v = v_e\sqrt{3}$

Q.19 Two bodies of masses 10 kg and 100 kg are separated by a distance of 2m. The gravitational potential at the mid-point of the line joining the two bodies is :

(A) -7.3×10^{-7} J/kg (B) -7.3×10^{-8} J/kg

(C) -7.3×10^{-9} J/kg (D) -7.3×10^{-6} J/kg

[C]

Sol. Gravitational potential

$= -\frac{G \times 10}{1} - \frac{G \times 100}{1} = -110\,G$

$= -110 \times 6.67 \times 10^{-11}$ J kg^{-1}

$= -7.3 \times 10^{-9}$ J/kg

Q.20 For the earth escape velocity is 11.2 km/s. What will be the escape velocity of that planet whose mass and radius are four times those of earth ?

(A) 11.2 km/s (B) 44.8 km/s

(C) 2.8 km/s (D) 0.7 km/s

Sol. **[A]** $v_e = \sqrt{\frac{2GM_e}{R_e}}$ and $v_p = \sqrt{\frac{2GM_p}{R_p}}$

$\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e}.\frac{R_e}{R_p}} = \sqrt{4 \times \frac{1}{4}} = 1$

or $v_p = v_e = 11.2$ km/s

Q.21 The weight of an object in the coal mine, sea level, at the top of the mountain are W_1, W_2 and W_3 respectively, then-

(A) $W_1 < W_2 > W_3$ (B) $W_1 = W_2 = W_3$

(C) $W_1 < W_2 < W_3$ (D) $W_1 > W_2 > W_3$

[A]

Q.22 The height above surface of earth where the value of gravitational acceleration is one fourth of that at surface, will be-

(A) $R_e/4$ (B) $R_e/2$

(C) $3R_e/4$ (D) R_e **[D]**

Q.23 The magnitude of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then-

(A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(B) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 > R$ and $r_2 > R$

(C) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$ **[A]**

Q.24 The diameters of two planets are in ratio 4 : 1. Their mean densities have ratio 1 : 2. The ratio of 'g' on the planets will be-

(A) 1 : 2 (B) 1 : 4

(C) 2 : 1 (D) 4 : 1 **[C]**

Q.25 If the earth suddenly stops rotating, the value of g at any place will -

(A) remain same

(B) decrease

(C) increase

(D) none of these **[C]**

Q.26 If two bodies of mass M and m are revolving around the centre of mass of the system in circular orbits of radii R and r respectively due to mutual interaction. Which of the following formulae is applicable ?

(A) $\frac{GMm}{(R+r)^2} = m\omega^2 r$ (B) $\frac{GMm}{R^2} = m\omega^2 r$

(C) $\frac{GMm}{r^2} = m\omega^2 R$ (D) $\frac{GMm}{R^2 + r^2} = m\omega^2 r$

[A]

Q.27 The earth's radius is R and acceleration due to gravity at its surface is g. If a body of mass m is sent to a height of R/4 from the earth's surface, the minimum speed with which the body must be thrown to reach a height of R/4 above the surface of the earth is -

(A) $\sqrt{\frac{2gR}{5}}$ (B) $\sqrt{\frac{gR}{3}}$

(C) $\sqrt{gR}$ (D) $\sqrt{2gR}$

Sol. **[A]** $\frac{1}{2}mv^2 = \Delta U = \frac{1}{5}mgR$

$v = \sqrt{\frac{2gR}{5}}$

Q.28 A body of mass m rises to height h = R/5 from the earth's surface, where R is earth's radius. If g is acceleration due to gravity at earth's surface, the increase in potential energy is -

(A) $\frac{5}{6}$ mg/h (B) $\frac{1}{6}$ mgh

(C) $\frac{3}{5}$ mgh (D) $\frac{6}{7}$ mgh **[B]**

Q.29 The distance of Neptune and Saturn from the Sun are nearly 10^{13} m and 10^{12} m respectively. Their periodic times will be in the ratio -

(A) 10 (B) 100

(C) $10\sqrt{10}$ (D) 1000 **[C]**

Sol. $\frac{T_n^2}{T_s^2} = \frac{R_n^3}{R_s^3}$

$\frac{T_n}{T_s} = \left[\frac{10^{13}}{10^{12}}\right]^{3/2}$

$= \left(\sqrt{10}\right)^3 = 10\sqrt{10}$

Q.30 The time period of a satellite of Earth is 5 hours. If the separation between the Earth and the satellite is increased to 4 times the previous value, the new time period will become -

(A) 40 hours (B) 20 hours

(C) 10 hours (D) 80 hours **[A]**

Sol. $T^2 \propto r^3$; $T'^2 \propto (4r)^3$ or $T^2/T'^2 = 64$

or $T'/T = 8$

or $T' = 8 \times 5h = 40$ h

Q.31 A person brings a mass of 1kg from infinity to a point A. Initially the mass was at rest but it moves at a speed of 2m/s as it reaches A. The work done by the person on the mass is –3J. The potential at A is-

(A) –3J/kg (B) –2 J/kg
(C) –5 J/kg (D) none of these

[C]

Q.32 A planet is moving in an elliptical orbit. If T, V, E and L are respectively its kinetic energy, potential energy, total energy and magnitude of angular momentum, then which of the following statements is true ?

(A) T is conserved
(B) V is always positive
(C) E is always negative
(D) L is conserved but the direction of vector $\vec{L}$ will continuously change **[C]**

Q.33 Which statement is not true for artificial geostationary satellite ?

(A) It revolves round the earth in equatorial plane
(B) It revolves round the earth with in great circle
(C) It revolves round the earth with time period of 24 hours
(D) It revolves round the earth with a velocity of 8 kilometer/second **[D]**

Q.34 One satellite is revolving round the earth in an elliptical orbit. Its speed will-

(A) be same at all the points of orbit
(B) be maximum at the point farthest from the earth
(C) be maximum at the point nearest from the earth
(D) depend on mass of satellite **[C]**

Q.35 The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is-

(A) $\frac{3}{2}$ v (B) $\sqrt{\frac{3}{2}}$ v
(C) $\sqrt{\frac{2}{3}}$ v (D) $\frac{2}{3}$ v **[C]**

Q.36 The escape velocity from a planet is V. If its mass and radius becomes four and two times respectively, then the escape velocity will become -

(A) V (B) 2V
(C) 0.5V (D) $\sqrt{2}$ V **[D]**

Q.37 Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to $R^{-5/2}$, then T^2 is proportional to-

(A) R^3
(B) $R^{7/2}$
(C) $R^{5/2}$
(D) $R^{3/2}$ **[B]**

Q.38 A planet of mass m is moving in an elliptical path about the sun. Its maximum and minimum distances from the sun are r_1 and r_2 respectively. If M_s is the mass of sun then the angular momentum of this planet about the centre of sun will be -

(A) $\sqrt{\frac{2GM_s}{(r_1 + r_2)}}$
(B) $2GM_s m \sqrt{\frac{r_1 r_2}{(r_1 + r_2)}}$
(C) $m\sqrt{\frac{2GM_s r_1 r_2}{(r_1 + r_2)}}$

(D) $\sqrt{\dfrac{2GM_s m(r_1+r_2)}{r_1 r_2}}$ **[C]**

Q.39 Two artificial satellites whose masses are m_1 and m_2 are moving in circular orbits of radii r_1 and r_2 respectively if $r_1 > r_2$ then which of the following statements is true about the speeds v_1 and v_2 of the satellites ?

(A) $v_1 = v_2$ (B) $v_1 > v_2$

(C) $v_1 < v_2$ (D) $\dfrac{v_1}{r_1} = \dfrac{v_2}{r_2}$

[C]

Q.40 A body is projected from the surface of earth with a velocity $2v_e$ where v_e is the escape velocity. The velocity of the body when it escapes the gravitational field of the earth is :

(A) $\sqrt{2}v_e$ (B) $\sqrt{3}\,v_e$

(C) $\sqrt{7}\,v_e$ (D) $\sqrt{11}\,v_e$ **[B]**

Sol. $v = \sqrt{n^2-1}\ v_e$

or $v = \sqrt{2^2-1}\ v_e$

$v = \sqrt{3}\ v_e$

Q.41 The radius of earth is 6.4×10^6m and acceleration due to gravity at earth's surface is 9.8 m/s^2. The temperature required by the Oxygen molecules to escape from the earth's surface is- (universal gas constant R = 8.3 joule/mole - K)

(A) 1.59×10^5K (B) 15.9×10^5 K

(C) 159×10^5K (D) 0.159×10^4 K

[A]

Q.42 A satellite is revolving around earth in a circular orbit. The radius of orbit is half of the radius of the orbit of moon. Satellite will complete one revolution in -

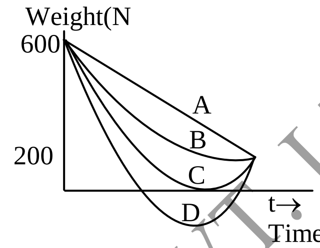
(A) $2^{-3/2}$ lunar month

(B) $2^{-2/3}$ lunar month

(C) $2^{3/2}$ lunar month

(D) $2^{2/3}$ lunar month **[A]**

Q.43 Imagine the acceleration due to gravity on earth is 10m/s^2 and on mars is 4 m/s^2. A traveller of mass 60kg goes from earth to mars by a rocket moving with constant velocity. If effect of other planets is assumed to be negligible, which one of the following graphs shown the variation of weight of traveller with time –



(A) A (B) B

(C) C (D) D **[C]**

Q.44 The velocity with which a projectile must be fired so that it escapes Earth's gravitation does not depend on -

(A) mass of the earth

(B) mass of the projectile

(C) radius of the projectile's orbit

(D) gravitational constant **[B]**

Sol. $v = \sqrt{\dfrac{2GM}{R}}$, v does not depends mass of the projectile.

Q.45 The escape velocity from the earth is about 11 km/s. The escape velocity from a planet having twice the radius and the same mean density as the earth is -

(A) 22 km/s (B) 11 km/s

(C) 5.5 km/s (D) 15.5 km/s **[A]**

Sol. $v = \sqrt{\dfrac{2GM}{R}} = \sqrt{2G\dfrac{4}{3}\pi R^2 d}$

$v \propto R\sqrt{d}$

$v = 2v_0$

Q.46 The kinetic energy needed to project a body of mass m from the earth's surface to infinity is -

(A) $\dfrac{1}{4}$ mg R (B) $\dfrac{1}{2}$ mg R

(C) mg R (D) 2mg R. **[C]**

Sol. $E_R - \frac{GMm}{R} = 0$

or $E_R = \frac{GMm}{R}$

or $E_R = \frac{gR^2m}{R}$

or $E_R = mgR$

Q.47 A body is projected up with a velocity equal to (3/4)th of the escape velocity from the surface of the earth. The height it reaches is-
(Radius of earth = R)

(A) $\frac{10R}{9}$ (B) $\frac{9}{7}R$ (C) $\frac{9}{8}R$ (D) $\frac{10R}{3}$

Sol. **[B]** Escape velocity for the earth is

$v_e = \sqrt{\frac{2GM}{R}}$

Given the velocity projection of the body

$= v = \frac{3}{4} v_e = \frac{3}{4}\sqrt{\frac{2GM}{R}}$

Total energy on the earth

= Total enery at maximum height h

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0 + \left(-\frac{GMm}{R+h}\right)$$

$$\frac{1}{2} m . \frac{9}{16} . \frac{2GM}{R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{9}{16} - 1 = -\frac{R}{R+h} \text{ or } -\frac{R}{R+h} = \frac{-7}{16}$$

$7R + 7h = 16 R$

$7h = 9R \Rightarrow h = \frac{9}{7}R$

Q.48 The masses and radii of Earth and Moon are M_1, R_1 and M_2, R_2 respectively. Their centre are at a distance d apart. The minimum speed with which a particle of mass m should be projected from a point mid-way between the two centres so as to escape to infinity is :

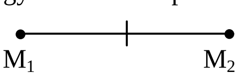
(A) $\sqrt{\frac{2G(M_1+M_2)}{d}}$ (B) $\sqrt{\frac{G(M_1+M_2)}{2d}}$

(C) $2\sqrt{\frac{G(M_1-M_2)}{2d}}$ (D) $2\sqrt{\frac{G(M_1+M_2)}{d}}$

Sol. **[D]**

Total potential energy at mid point is

$\left[-\frac{GM_1m}{d/2} - \frac{GM_2m}{d/2}\right]$



or $-\frac{2G}{d}(M_1 + M_2)m$

If v is required escape velocity, the

$\frac{1}{2}mv^2 = \frac{2G}{d}(M_1 + M_2)m$

$v = 2\sqrt{\frac{G(M_1+M_2)}{d}}$

Q.49 A body of mass m and radius r falls on earth from a great height. If M is mass and R is the radius of earth while $r = \frac{R}{100}$ then the acceleration of the body when it hits the earth is : (acceleration due to gravity at earth surface is g)

(A) g (B) 0.98 g

(C) $\frac{g}{0.98}$ (D) 9.8 g **[B]**

Sol. $a = \frac{GM}{x^2} = \frac{GM}{(R+r)^2} = \frac{GM}{\left(R+\frac{R}{100}\right)^2}$

$= \frac{GM}{\left(\frac{101}{100}\right)^2 R^2} = 0.98\text{ g}$

Q.50 The increase in gravitational potential energy of an object of mass m raised from the surface of earth to a height equal to radius R of earth is :

(A) mgR (B) $\frac{mgR}{2}$

(C) $\frac{mgR}{3}$ (D) $\frac{mgR}{4}$ **[B]**

Sol. Increase in potential energy

$= -\frac{GmM}{2R} - \left(-\frac{GmM}{R}\right)$

$= \frac{GmM}{2R} = \frac{GM}{R^2} \times \frac{mR}{2}$

$= g \times \frac{1}{2} mR = \frac{1}{2} mgR$

$U = \frac{-GMm}{R} + \frac{GMm}{4R} = \frac{-3GMm}{4R}$

Q.1 A point mass of 0.5 kg moving with a constant speed of $5ms^{-1}$ on a elliptical track experiences an outward force of 10N when at either endpoint of the major axis and a similar force of 125N at each end of minor axis. How long are the axes of the ellipse ?

Sol. We can prove that the radii of curvature of the ellipse at the endpoints of its axes are $\mathbf{b^2/a}$ and $\mathbf{a^2/b}$, where 2a and 2b are the lengths of the major and minor axes, respectively. This geometrical result can be deduced using calculus or by considering one of a number of physical situations; what follows is one possibility.

Consider a planet orbiting the Sun in an ellipse. Newton's second motion applied at the endpoint of the major axis, a distance **r** from the Sun gives

$$G\frac{M}{r^2} = \frac{v^2}{R},$$

Where R is the radius of curvature at the endpoint and M is the mass of the Sun. According to Kepler's third law the period of the orbit is $2\pi\sqrt{a^3/GM}$ and the radius vector sweeps out area at a constant rate. The area of the ellipse is π**ab**, and so equating two expressions for that constant the planet is at the endpoint of the major axis, we obtain

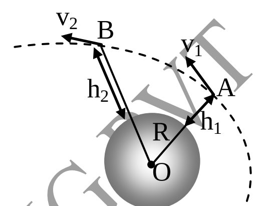
$$\frac{vr}{2} = \frac{ab}{2}\sqrt{\frac{GM}{a^3}}.$$

Comparing the above two equations we conclude that $R = b^2/a$. For this argument we utilized the fact that the foci of the ellipse are on the major axes; we cannot therefore apply the same proof at the endpoints of the minor axis. However, in respect of their corresponding radii of curvature the two axes are symmetrical.

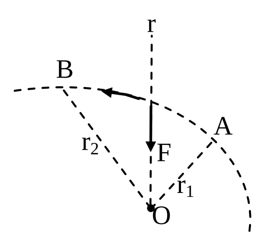
The uniformly moving point mass of the problem obeys the equation of motion $F = mv^2/R$, where R is the appropriate radius of curvature. Using the data given we obtain ; $b^2/a = 1.25$ m; $a^2/b = 10$ m and hence; $2a = 10$ m,

$2b = 5$m.

Q.2 A satellite of mass **m** is put into an elliptical orbit around the earth. At point A, its distance from the earth is $h_1 = 500$ km and it has a velocity $v_1 = 30000$ km/h. Determine the velocity $\mathbf{v_2}$ of the satellite as it reaches point B, a distance $h_2 = 1200$ km from the earth.



Sol. The satellite is moving outside of the earth's atmosphere so that the only force acting on it is the gravitational attraction of the earth. With the mass and radius of the earth expressed by $\mathbf{m_e}$ and **R**, respectively, the gravitational law of Eq. gives $F = Gmm_e/r^2 = gR^2m/r^2$ when the substitution. $Gm_e = gR^2$ is made for the surface values $F = mg$ and $r = R$. The work done by F is due only to the radial component of motion along the line of action of F and is negative for increasing **r**.



$$U_{1-2} = -\int_{r_1}^{r_2} F\,dr = -mgR^2\int_{r_1}^{r_2}\frac{dr}{r^2} = mgR^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

The work-energy equation $U_{1-2} = \Delta T$ gives

$$mgR^2\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = \frac{1}{2}m(v_2^2 - v_1^2)\ v_2^2 = v_1^2 + 2gR^2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

Substituting the numerical values gives

$$v_2^2 = \left(\frac{30000}{3.6}\right)^2 + 2(9.81)\ [(6371)\ (10^3)]^2 \left(\frac{10^{-3}}{6371+1200}-\frac{10^{-3}}{6371+500}\right)$$

$= 69.44(10^6) - 10.72(10^6) = 58.73\ (10^6)\ (m/s)^2$

$v_2 = 7663$ m/s or $v_2 = 7663(3.6) =$ **27590 km/h**

Ans.

Helpful Hints :

(i) Note that the result is independent of the mass of the satellite.

(ii) Consult Table D/2, appendix D, to find the radius R of the earth.

Q.3 An orbiting space station is observed to remain always vertically above the same point on the earth. Where on earth is the observer ? Describe the orbit of the space station as completely as possible.

Sol. The observer must be on the equator of the earth. The orbit of the space station is a large circle in the equatorial plane with center at the center of the earth. The radius of the orbit can be figured out using the orbiting period of 24 hours as follows. Let the radius of the orbit be **R** and that of the earth be $\mathbf{R_0}$.

We have $\frac{mv^2}{R} = \frac{GMm}{R^2}$,

Where **v** is the speed of the space station, **G** is the universal constant of gravitation, **m** and **M** are the masses of the space station and the earth respectively, giving

$v^2 = \frac{GM}{R}$,

As $mg = \frac{GMm}{R_0^2}$,

We have $GM = R_0^2 g$.

Hence $v^2 = \frac{R_0^2 g}{R}$.

For circular motion with constant speed **v**, the orbiting period is

$T = \frac{2\pi R}{v}$.

Hence $\frac{4\pi^2 R^2}{T^2} = \frac{R_0^2 g}{R}$

And $R = \left(\frac{R_0^2 T^2 g}{4\pi^2}\right)^{\frac{1}{3}} = 4.2 \times 10^4$ km.

Q.4 Consider a rotating spherical planet. The velocity of a point on its equator is **V**. The effect rotation of the planet is to make **g** at the equator **1/2** of **g** at the pole. What is the escape velocity for a polar particle on the planet expressed as a multiple of **V** ?

Sol. Let **g** and **g′** be the gravitational accelerations at the pole and at the equator respectively and consider a body of mass **m** on the surface of the planet, which has a mass M. at the pole,

$mg = \frac{GMm}{R^2}$,

giving $GM = gR^2$.

At the equator, we have

$$\frac{mV^2}{R} = \frac{GMm}{R^2} - mg' = mg - \frac{mg}{2} = \frac{mg}{2}.$$

Hence $g = 2V^2/R$.

If we define gravitational potential energy with respect to a point at infinity from the planet, the body will have potential energy

$$-\int_{\infty}^{R} -\frac{GMm}{r^2}dr = -\frac{GMm}{R}.$$

Note that the negative sign in front of the gravitational force takes account of its attractiveness. The body at the pole then has total energy

$$E = \frac{1}{2}mV^2 - \frac{GMm}{R}.$$

For it to escape from the planet, its total energy must be at least equal to the minimum energy of a body at infinity, i.e. zero. Hence the escape velocity **v** is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0$$

$$\text{or } v^2 = \frac{2GM}{R} = 2gR = 4V^2,$$

i.e. **v = 2V**.

Q.5 The sun is about 25,000 light years from the center of the galaxy and travels approximately in a circle with a period of 170,000,000 years. The earth is 8 light minutes from the sun. From these data alone, find the a approximate gravitational mass of the galaxy in units of the sun's mass. You may assume that the gravitational force on the sun may be approximated by assuming that all the mass of the galaxy is at its center.

Sol. For the motion of the earth around the sun,

$$\frac{mv^2}{r} = \frac{Gmm_s}{r^2},$$

Where **r** is the distance from the earth to the sun, **v** is the velocity of the earth, **m** and **m_s** are the masses of the earth and the sun respectively.

For the motion of the sun around the center of the galaxy,

$$\frac{m_sV^2}{R} = \frac{Gm_sM}{R^2},$$

Where **R** is the distance from the sun to the center of the galaxy, **V** is the velocity of the sun and **M** is the mass of the galaxy.

$$\text{Hence } M = \frac{RV^2}{G} = \frac{R}{r}\left(\frac{V}{v}\right)^2 m_s.$$

Using $V = 2\pi R/T$, $v = 2\pi r/t$, where T and t are the periods of revolution of the sun and the earth respectively, we have

$$M = \left(\frac{R}{r}\right)^3\left(\frac{t}{T}\right)^2 m_s.$$

With the data given, we obtain

$M = 1.53 \times 10^{11} m_s$.

Q.6 Calculate the ratio of the mean densities of the earth and the sun from the following approximate data.

θ = angular diameter of the sun seen from the earth = $\frac{1}{2}^{\circ}$.

ℓ = length of 1° of latitude on the earth's surface = 100 km.

T = one year = 3×10^7s.

g = 10 ms^{-2}.

Sol. Let **r** be the distance between the sun and the earth, **M_e** and **M_s** be the masses and **R_e** and **R_s** be the radii of the earth and the sun respectively, and G be the gravitational constant. We then have

$$\frac{GM_eM_s}{r^2} = M_e r\omega^2,$$

$$\frac{2R_s}{r} = \frac{1}{2}\frac{2\pi}{360} = \frac{\pi}{360}\text{ rad},$$

$$\text{i.e. } r = \frac{720R_s}{\pi}.$$

$$\text{The above gives } \frac{GM_s}{(720R_s/\pi)^3} = \omega^2,$$

$$\text{or } \frac{GM_s}{R_s^3} = \left(\frac{720}{\pi}\right)^3\left(\frac{2\pi}{3\times10^7}\right)^2.$$

For a mass **m** on the earth's surface,

$$\frac{GmM_e}{R_e^2} = mg,$$

Giving $\frac{GM_e}{R_e^3} = \frac{g}{R_e} = \frac{g}{\left(\frac{360\times100}{2\pi}\right)}$

$$= \frac{g\pi}{18\times10^3}.$$

Hence $\frac{\rho_e}{\rho_s} = \frac{g\pi}{18\times10^3}\left(\frac{720}{\pi}\right)^{-3}\left(\frac{2\pi}{3\times10^7}\right)^{-2}$

$= 3.31.$

Q.7 A satellite in stationary orbit above a point on the equator is intended to send energy to ground stations by a coherent microwave beam of wavelength one meter from a one-km mirror.

(a) What is the height of such stationary orbit ?

(b) Estimate the required size of a ground receptor station.

Sol. **(a)** The revolving angular velocity ω of the synchronous satellite is equal to the spin angular velocity of the earth and is given by

$$m(R+h)\omega^2 = \frac{GMm}{(R+h)^2}.$$

Hence the height of the stationary orbit is

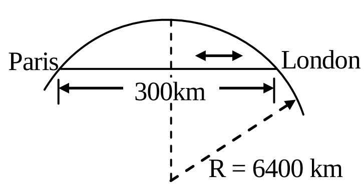
$$h = \left(\frac{GM}{\omega^2}\right)^{\frac{1}{3}} - R = 3.59\times10^4 \text{ km},$$

using $G = 6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}$, $M = 5.98\times10^{24}$ kg, $R = 6.37\times10^4$ km.

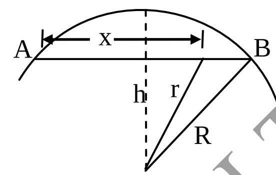
(b) Due to diffraction, the linear size of the required receptor is about

$$\frac{\lambda h}{D} = 1\times\left(\frac{3.59\times10^4}{1}\right) = 3.59\times10^4 \text{ m}.$$

Q.8 Paris and London are connected by a straight subway tunnel **(see Fig.)**. A train travels between the two cities powered only by the gravitational force of the earth. Calculate the maximum speed of the train and the time taken to travel from London to Paris. The distance between the two cities is 300 km and the radius of the earth is 6400 km. Neglect friction.



Sol. Define **x**, **h**, **r** as in **Fig.** and assume the earth to be a stationary homogeneous sphere of radius **R**. Taking the surface of the earth as reference level, the gravitational potential energy of the train at **x** is



$$V = \int_R^r \frac{GmM}{R^3} r\,dr = \frac{GmM}{2R^3}(r^2 - R^2),$$

Where **m**, **M** are the masses of the train and the earth respectively. Conservation of mechanical energy gives, as the train starts from rest at the earth's surface,

$$\frac{mv^2}{2} + \frac{GmM(r^2 - R^2)}{2R^3} = 0,$$

or $v^2 = \frac{g(R^2 - r^2)}{R}$,

where **g = GM/R²** is the acceleration of gravity at the earth's surface. As

$r^2 = h^2 + (150 - x)^2 = (R^2 - 150^2) + (150 - x)^2 = R^2 - 300x + x^2$,

$$v^2 = \frac{gx(300 - x)}{R}.$$

v is maximum when x = 150 km

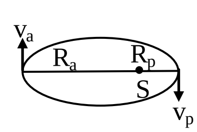
$$v_{max} = \sqrt{\frac{9.8\times150\times150\times1000}{6400}} = 185.6 \text{ m/s}.$$

The time from London to Paris is

$$T = \int_0^{300}\frac{dx}{v} = \int_0^{300}\sqrt{\frac{R}{g}}\frac{dx}{\sqrt{x(300-x)}}$$

$$= \int_0^1 \sqrt{\frac{R}{g}}\frac{dt}{\sqrt{t(1-t)}} = \pi\sqrt{\frac{R}{g}} = 42.3 \text{ min}.$$

Q.9 A comet in an orbit about the sun has a velocity 10 km/sec at aphelion and 80 km/sec at perihelion **(Figure)**. If the earth's velocity in a circular orbit is 30 km/sec and the radius of its orbit is 1.5×10^8 km, find the aphelion distance R_a for the comet.



Sol. Let **v** be the velocity of the earth **R** the radius of the earth's orbit, **m** and $\mathbf{m_s}$ the masses of the earth and the sun respectively. Then

$$\frac{mv^2}{R} = \frac{Gmm_s}{R^2},$$

or $Gm_s = Rv^2$.

By the conservation of the mechanical energy and of the angular momentum of the comet, we have

$$\frac{-Gm_c m_s}{R_a} + \frac{m_c v_a^2}{2} = \frac{-Gm_c m_s}{R_p} + \frac{m_c v_p^2}{2},$$

Where $\mathbf{m_c}$ is the mass of the comet, and $\mathbf{v_a}$ and $\mathbf{v_p}$ are the velocities of the comet at aphelion and at perihelion respectively. The above equations give

$$R_a = \frac{2Gm_s}{v_a(v_a + v_p)} = \frac{2Rv^2}{v_a(v_a + v_p)}$$

$= \mathbf{3 \times 10^8}$ **km.**

Q.10 A comet moves towards the sun with initial velocity $\mathbf{v_0}$. The mass of the sun is **M** and its radius is **R**. Find the total cross section σ for striking the sun. Take the sun to be at rest and ignore all other bodies.

Sol. Let the impact parameter of the comet be **b**. At the closest approach to the sun (closest distance **r** from the sun's center), we have from the conservation of mechanical energy and angular momentum

$$\frac{mV_0^2}{2} = \frac{mV^2}{2} - \frac{GMm}{r},$$

$mbV_0 = mrV$,

where **m** is the mass of the comet and **V** its velocity at closest approach. From these, we find

$$b = r\sqrt{1 + \frac{2GM}{V_0^2 r}}.$$

If **r** < **R**, the comet will strike the sun. Hence the total cross section for striking the sun is

$$\sigma = \pi\,[b(R)]2 = \pi R^2\left(1 + \frac{2GM}{V_0^2 R}\right).$$

Q.11 A meteorite of mass 1.6×10^3 kg moves about the earth in a circular orbit at an altitude of 4.2×10^6m above the surface. It suddenly makes a head-on collision with another meteorite that is much lighter, and loses 2.0% of its kinetic energy without changing its direction of motion or its total mass.

(a) What physics principles apply to the motion of the heavy meteorite after its collision ?

(b) Describe the shape of the meteorite's orbit after the collision.

(c) Find the meteorite's distance of closest approach to the earth after the collision.

Sol. **(a)** The laws of conservation of mechanical energy and conservation of angular momentum apply to the motion of the heavy meteorite after its collision.

(b) For the initial circular motion, E < 0, so after the collision we still have E < 0. After it loses 2.0% of its kinetic energy, the heavy meteorite will move in a elliptic orbit.

(c) From $\frac{mv^2}{r} = \frac{GmM}{r^2}$,

We obtain the meteorite's kinetic energy before collision :

$$\frac{1}{2}mv^2 = \frac{GmM}{2r} = \frac{mgR^2}{2r}$$

$$= \frac{m \times 9.8 \times 10^3 \times 6400^2}{2(6400 + 4200)} = 1.89 \times 10^7 \text{m Joules.}$$

Where **m** is the mass of the meteorite in kg. The potential energy of the meteorite before collision is

$$-\frac{GmM}{r} = -mv^2 = -3.78 \times 10^7 \text{m Joules.}$$

During the collision, the heavy meteorite's potential energy remains constant, while its kinetic energy is suddenly reduced to

$1.89 \times 10^7\text{m} \times 98\% = 1.85 \times 10^7$m Joules.

Hence the total mechanical energy of the meteorite after the collision is

$E = (1.85 - 3.78) \times 10^7 m = -1.93 \times 10^7 m$ Joules.

From $E = \dfrac{-GmM}{2a} = \dfrac{-mR^2 g}{2a}$,

We obtain the major axis of the ellipse as

$$2a = \frac{R^2 g}{1.93 \times 10^7} = \frac{(6400 \times 10^3)^2 \times 9.8}{1.93 \times 10^7}$$

$= 2.08 \times 10^7 m = 2.08 \times 10^4$ km.

As after the collision, the velocity of the heavy meteorite is still perpendicular to the radius vector from the center of the earth, the meteorite is at the apogee of the elliptic orbit. Then the distance of the apogee from the center of the earth is 6400 + 4200 = 10600 km and the distance of the perigee from the center of the earth is

r_{min} = 20800 – 10600 = 10200 km.

Thus the meteorite's distance of closest approach to the earth after the collision is 10200 – 6400 = 3800 km.

From the above calculations, we see that it is unnecessary to know the mass of the meteorite. Whatever the mass of the meteorite, the answer is the same as the conditions remain unchanged.

Q.12 Given that an earth satellite near the earth's surface takes about 90 min per revolution and that a moon satellite (of our moon, i.e., a spaceship orbiting our moon) takes also about 90 min per revolution, what interesting statement can you derive about the moon's composition ?

Sol. From the equation $mr\omega^2 = GmM/r^2$ for a body **m** to orbit around a fixed body M under gravitation, we find

$r^3\omega^2 = GM$.

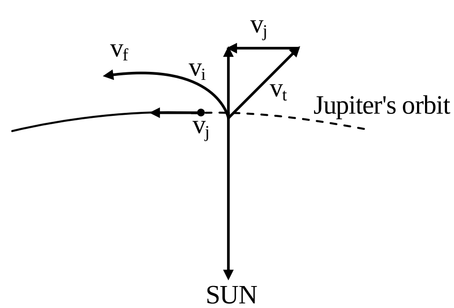
Then if $\mathbf{M_e}$, $\mathbf{M_m}$ are the masses and $\mathbf{r_e}$, $\mathbf{r_m}$ are the radii of the earth and moon respectively, and the periods of revolution of the earth and moon satellites are the same, we have

$$\frac{r_m^3}{r_e^3} = \frac{M_m}{M_e},$$

or $\dfrac{M_e}{V_e} = \dfrac{M_m}{V_m}$,

where V_e and V_m are the volumes of the earth and moon respectively. It follows that the earth and moon have the same density.

Q.13 A satellite is launched from the earth on a radial trajectory away from the sun with just sufficient velocity to escape from the sun's gravitational field. It is timed so that it will intercept Jupiter's orbit a distance **b** behind Jupiter, interact with Jupiter's gravitational field and the deflected by 90º, i.e., its velocity after the collision is tangential to Jupiter's orbit **(figure)**. How much energy did the satellite gain in the collision ? Ignore the sun's gravitational field during the collision and assume that the duration of the collision is small compared with Juipter's period.



Sol. Let **r** represent the distance from zuipter to the sun, $\mathbf{v_i}$ the velocity of the satellite with respect to the sun at the time it intercepts Juipter's orbit a distance **b** behind it and before any interaction with it, and **m** and $\mathbf{M_s}$ the masses of the satellite and the sun respectively. As the satellite just escapes the sun's gravitational field, we have

$$\frac{mv_i^2}{2} = \frac{GmM_s}{r},$$

Giving $v_i = \sqrt{\frac{2GM_s}{r}} = \sqrt{\frac{2\times 4.01\times 10^{14}\times 3.33\times 10^5}{7.78\times 10^{11}}}$

$= 1.85 \times 10^4$ m/s = 18.5 km/s,

Where we have used $M_s = 3.33 \times 10^5 M_e$ (M_e is the earth's mass), $GM_e = gR^2$ (R is the radius of the earth). $= 4.01 \times 10^{14}$ m^3/s^2, $r = 7.78 \times 10^{11}$m.

The velocity v_J of Juipter with respect to the sun is given by

$$\frac{v_J^2}{r} = \frac{GM_s}{r^2},$$

i.e. $v_J = \sqrt{\frac{GM}{r}} = \frac{v_i}{\sqrt{2}}$ = 13.1 km/s.

When the satellite just enters the gravitational field of Jupiter, its velocity in the Jupiter frame is

$v_r = v_i - v_J$,

or $v_r = \sqrt{18.5^2 + 13.1^2}$ = 22.67 km/s.

If **b** does not change during the encounter, conservation of the angular momentum of the satellite in the Jupiter frame shows that this is also the speed of the satellite in the Jupiter frame when it leaves the gravitational field of Jupiter. After the encounter, the satellite leaves the gravitational field of Jupiter with a velocity in the sun's frame tangential to Jupiter's orbit. Thus the speed of the satellite with respect to the sun is

$v_f = v_r + v_J = 22.67 + 13.1 = 35.77$ km/s.

The energy gained by unit mass of the satellite in the collision is therefore

$$\frac{35.77^2 - 18.5^2}{2} = 468.6 \times 10^6 \text{ J/kg}.$$

Q.14 Mariner 4 was designed to travel from earth to Mars in an elliptical orbit with its perihelion at earth and its aphelion at Mars. Assume that the orbits of earth and Mars are circular with radii $\mathbf{R_E}$ and $\mathbf{R_M}$ respectively. Neglect the gravitational effects of the planets on Mariner 4.

(a) With what velocity, relative to earth, does Mariner 4 have to leave earth, and in what direction ?

(b) How long will it take to reach Mars ?

(c) With what velocity, relative to Mars, will it reach the orbit of Mars ?

(The time at which Mariner 4 leaves earth must be properly chosen if it is to arrive at Mars. Assume this is done.)

Sol. As the gravitational force on Mariner 4, which is a central force, is conservative, we have

$$= \frac{m\dot{r}^2}{2} - \frac{GmM}{r} + \frac{mh^2}{2r^2},$$

Where **m** and **M** are the mass of Mariner 4 and the sun respectively, G is the gravitational constant, and $\mathbf{h = r^2}\dot{\theta}$ is a constant. At the perihelion and aphelion of the elliptical orbit, $\dot{r} = 0$, $r = R_E$ and $r = R_M$ respectively.

Then $E = \frac{-GmM}{R_M} + \frac{mh^2}{2R_M^2} = \frac{-GmM}{R_E} + \frac{mh^2}{2R_E^2}$,

Giving $h = \sqrt{\frac{2GMR_M R_E}{R_M + R_E}}$.

At the perihelion we obtain its velocity relative to the sun as

$$v = \frac{h}{R_E} = \sqrt{\frac{2GMR_M}{R_E(R_M + R_E)}}.$$

Suppose Mariner 4 is launched in a direction parallel to the earth's revolution around the sun. The velocity relative to the earth with which Mariner 4 is to leave the earth is then

$$v_r = v - v_E = \sqrt{\frac{2GMR_M}{R_E(R_M + R_E)}} - \sqrt{\frac{GM}{R_E}},$$

Where v_E is the velocity of revolution of the earth. Similarly at the aphelion the velocity, relative to Mars, which Mariner 4 must have is

$$v'_r = v' - v_M = \sqrt{\frac{2GMR_E}{R_M(R_M + R_E)}} - \sqrt{\frac{GM}{R_E}}.$$

Applying Kepler's third law we have for the period T of revolution of Mariner 4 around the sun

$$T^2 = T_E^2 \left(\frac{R_E + R_M}{2}\right)^3 R_E^{-3},$$

Where T_E = period of revolution of the earth = 1 year. Hence the time taken for Mariner 4 to reach Mars in year is

$$t = \frac{T}{2} = \frac{1}{2}\left(\frac{R_E + R_M}{2R_E}\right)^{\frac{3}{2}}.$$

Q.15 Estimate how big an asteroid you could escape by jumping.

Sol. Generally speaking, before jumping, one always bends one's knees to lower the center of gravity of the body by about 50 cm and then jumps up. You can usually reach a height 60 cm above our normal height. In the process, the work done is **(0.5 + 0.6)mg,** where m is the mass of your body and **g** is the acceleration of gravity.

It is reasonable to suppose that when one jumps on an asteroid of mass M and radius R one would consume the same energy as on the earth. Then to escape from the asteriod by jumping we require that

$$1.1mg = \frac{GMm}{R}.$$

If we assume that the density of the asteroid is the same as that of the earth, we obtain

$$\frac{M}{M_E} = \frac{R^3}{R_E^3},$$

Where M_E and R_E are respectively the mass and radius of the earth. As $g = GM_E/R^2_E$, we find

$$R = \frac{GM}{1.1g} = \frac{R^3}{1.1R_E},$$

or $R = \sqrt{1.1R_E} = \sqrt{1.1 \times 6400 \times 10^3} = 2.7 \times 10^3$m.

Q.16 You know that the acceleration due to gravity on the surface of the earth is 9.8 m/sec^2, and that the length of a great circle around the earth is 4×10^7m. You are given that the ratios of moon/earth diameters and masses are $\frac{D_m}{D_e} = 0.27$ and $\frac{M_m}{M_e} = 0.0123$ respectively.

(a) Compute the minimum velocity required to escape from the moon's gravitational field when starting from its surface.

(b) Compare this speed with thermal velocities of oxygen molecules at the moon's temperature which reaches 100ºC.

Sol. **(a)** Let the velocity required to escape from the moon's gravitational field be v_{min}, then

$$\frac{mv_{min}^2}{2} = \frac{GM_m m}{r_m},$$

Giving $v_{min} = \sqrt{\frac{2GM_m}{r_m}} = \sqrt{\left(\frac{0.0123}{0.27}\right)\left(\frac{GM_e}{r_e^2}\right)2r_e}$

$$= \sqrt{\left(\frac{0.0123}{0.27}\right).g.D_e} = 2.38 \times 10^3 \text{ m/s}$$

Using $g = GM_e/r_e^2$, $D_m/D_e = 0.27$ and $M_m/M_e = 0.0123$.

(b) The average kinetic energy of the translational motion of oxygen molecules at a temperature of 100ºC is 3kT/2 :

$$\frac{1}{2}mv^2 = \frac{3}{2}kT.$$

Hence $v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times373}{32\times1.67\times10^{-27}}}$

= 538 m/s.

v, which is the root-mean-square speed of an oxygen molecules at the highest moon temperature, is smaller than v_{min}, the speed required to escape from the moon.

Q.17 Derive formulas and calculate the values of (a) the gravitational acceleration at the surface of the moon, and (b) the escape velocity from the moon.

Sol. **(a)** Let M and R be the mass and radius of the moon respectively. Then by the law of universal gravitation and the definition of gravitational acceleration at the surface of the moon we have

$$\frac{GMm}{R^2} = mg,$$

Where m is the mass of a body on the surface of the moon. The relation gives the gravitational acceleration at the surface of the moon as

$$g = \frac{GM}{R^2} = \frac{6.67\times10^{-11}\times7.35\times10^{22}}{(1.74\times10^6)^2} = 1.62$$

m/s^2.

(b) The potential energy of a projectile of mass m at infinite distance from the moon $\rho \to \infty$ is

$$-\frac{GmM}{\rho} = -\frac{mgR^2}{\rho} \to 0.$$

Its kinetic energy, a positive quantity, is at least zero. Hence for the projectile to reach infinity from the surface of the moon, its total mechanical energy must be at least zero, by the conservation of energy.

At the surface of the moon, the projectile has total energy

$$E = \frac{1}{2}mv_0^2 - mgR.$$

If v_0 is the escape velocity, we require E = 0, or

$v_0 = \sqrt{2gR} = \sqrt{2\times1.62\times1.74\times10^6} = 2.37 \times 10^3$ m/s.

Q.18 Consider the earth-moon system and for simplicity assume that any interaction with other objects can be ignored. The moon, which moves around the earth more slowly than the earth rotates, creates tides on the earth. A similar situation exists on Mars, but with the difference that one of its moons revolves about Mars faster than the planet rotates. Show that one consequence of tidal friction is that in one system the moon-planet distance is increasing, and in the other it is decreasing. In which one is it decreasing ?

Sol. For the earth-moon system, the frictional force caused by the tides slows down the rotational speed of the earth. However, the total angular momentum of the earth-moon system is conserved because the interaction between this system and other objects can be ignored. The decrease in the earth's rotational angular momentum will lead to an increase in the angular momentum of the moon about the earth (to be exact, about the center of mass of the system). The angular momentum of the moon is $J = mR^2\omega$.

$$mR\omega^2 = \frac{GMm}{R^2},$$

we have $J = mR^2\sqrt{\frac{GM}{R^3}} = m\sqrt{GMR}$.

Here we consider the center of the earth to be approximately fixed, so that R is the earth-moon distance. Then as J increases, R will increase also. Thus for the earth-moon system, the effect of tides is to increase the distance between the moon and the earth.

For the Mars-moon system, the moon revolves about Mars faster than the latter rotates, so the frictional force caused by tides will speed up the rotation of Mars, whose rotational angular momentum consequently increases. As the total angular momentum is conserved, the angular momentum of the moon will decrease. The argument above then shows that the distance between Mars and its moon will decrease.

Q.19 A parachutist jumps at an altitude of 3000 meters. Before the parachute opens she reaches a terminal speed of 30 m/sec.

(a) Assuming that air resistance is proportional to speed, about how long does it take her to reach this speed?

(b) How far has she traveled in reaching this speed ?

After her parachute opens, her speed is slowed to 3 m/sec. As she hits the ground, she flexes her knees to absorb the shock.

(c) How far must she bend her knees in order to experience a deceleration no greater than 10g ? Assume that her knees are like a spring with a resisting force proportional to displacement.

(d) Is the assumption that air resistance is proportional to speed a reasonable one? Show that this is or is not the case using qualitative arguments.

Sol. **(a)** Choose the downward direction as the positive direction of the x-axis. Integrating the differential equation of motion

$$\frac{dv}{dt} = g - \alpha v,$$

Where α is a constant, we obtain

$$v = \frac{g}{\alpha}(1 - e^{-\alpha t}).$$

This solution shows that v approaches its maximum, the terminal speed g/α, when $t \to \infty$.

(b) Integrating the above equation, we obtain

$$x = \frac{gt}{\alpha} + \frac{ge^{-\alpha t}}{\alpha^2}.$$

Thus $x \to \infty$ as $t \to \infty$. This means that when the parachutist reaches the terminal speed she has covered an infinite distance.

(c) As her speed is only 3 m/s, we may neglect any air resistance after she hits the ground with this speed. Conservation of mechanical energy gives

$$\frac{k\xi^2}{2} = mg\xi + \frac{mv^2}{2},$$

where ξ is the distance of knee bending and v is the speed with which she hits the ground, considering the knee as a spring of constant k. Taking the deceleration –10g as the maximum allowed, we have

$mg - k\xi = -10mg$,

i.e. $\xi = 11mg/k$.

The energy equation then gives

$$\xi = \frac{v^2}{9g} = \frac{3^2}{9 \times 9.8} = 0.102 \text{ m}.$$

(d) We have seen that if the air resistance is proportional to speed, the time taken to reach the terminal speed is ∞ and the distance traveled is also ∞. However, the actual traveling distance is no more than 3000 m and the traveling time is finite before she reaches the terminal speed of 30 m/s. Hence the assumption that air resistance is proportional to speed is not a reasonable one.

Q.20 A defective satellite of mass 950 kg is being towed by a spaceship in empty space. The two vessels are connected by a uniform 50 m rope whose mass per unit length is 1 kg/m. The spaceship is accelerating in a straight line with acceleration 5 m/sec^2.

(a) What is the force exerted by the spaceship on the rope ?

(b) Calculate the tension along the rope.

(c) Due to exhaustion, the crew of the spaceship falls asleep and a short circuit in one of the booster control circuits results in the acceleration changing to a deceleration of 1 m/sec^2. Describe in detail the consequences of this mishap.

Sol. (a) $F = (m_{rope} + m_{satellite}) . a$

$= (950 + 50) \times 5 = 5 \times 10^3$N.

(b) Choose the point where the rope is attached to the satellite as the origin and the x-axis along the rope towards the spaceship. The tension along the rope is then

$F(x) = (m_{satellite} + m_{rope}(x)).a$

$= [950 + 1 \times (50 - x)] \times 5$

$= (5 \times 10^3 - 5x)$N.

(c) After the mishap, the spaceship moves with an initial velocity v_0 and a deceleration of $1 m/s^2$, while the satellite moves with a constant speed v_0. After the mishap, the two vessels will collide at a time t given by

$$v_0 t = 50 + v_0 t - \frac{a}{2} t^2,$$

$$\text{or } t = \sqrt{\frac{100}{1}} = 10s.$$