Functions

Single Correct Answer Type

1. Types of Functions

1. $f:\mathbf{R}\to\mathbf{R}, f(x)=x|x|$ is

1) one-one but not onto
2) onto but not one-one
3) Both one-one and onto
4) neither one-one nor onto

Key. 3

Sol. Give that $f(x)=\begin{cases} x^2 \ if\ x>0 \\ 0\ if\ \ x=0 \\ -x^2\ if\ x<0 \end{cases}$

2. Let $f:\left[0,\sqrt{3}\right]\to\left[0,\frac{\pi}{3}+\log_e 2\right]$ defined by $f(x)=\log_e\sqrt{x^2+1}+\tan^{-1}x$ then $f(x)$ is

A) one – one and onto
B) one – one but not onto
C) onto but not one – one
D) neither one – one nor onto

Key. A

Sol. $f'(x)=\frac{x+1}{x^2+1}\Rightarrow f(x)\ is\ increasing\ in\ \left[0,\sqrt{3}\right]$

3. If $f:N\to N$ is defined by $f(n)=n-(-1)^n$, then

(A) *f* is one-one but not onto
(B) *f* is both one-one and onto
(C) *f* is neither one-one nor onto
(D) *f* is onto but not one-one

Key. B

Sol. This function *f* maps

$1\to 2,\ 2\to 1$

$3\to 4,\ 4\to 3$

$5\to 6,\ 6\to 5$

i.e., $2m-1\to 2m$ and $2m\to 2m-1$

So *f* is one-one and onto.

4. Given A={x,y,z}, B={u,v,w}, the function $f:A\to B$ defined by $f(x)=u, f(y)=v, f(z)=w$ is

1) Surjective
2) bijective
3) injective
4) all of the above

Key. 4

Sol. Conceptual

2. Domain & Range

6. The domain of $\sqrt{\sin(\cos x)}$

1) $\left[2n\pi, 2n\pi+\frac{\pi}{2}\right], n\in I$
2) $\left[2n\pi+\frac{\pi}{2}, 2n\pi+\pi\right], n\in I$
3) $\left[2n\pi+\pi, 2n\pi+\frac{3\pi}{2}\right], n\in I$
4) $\left[2n\pi-\frac{\pi}{2}, 2n\pi+\frac{\pi}{2}\right], n\in I$

Key. 4

Sol. F(x) is defined when $\sin(\cos x) \geq 0$

$$\cos x \geq \sin^{-1} 0 \Rightarrow \cos x \geq 0$$

X lies on I and IV quadrant

$$2n\pi - \frac{\pi}{2} \leq x \leq 2n\pi + \frac{\pi}{2},\ n \in I$$

7. The domain of the function $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$ is

1) $[-2, 2]$ 2) $[-2, -1]$ 3) $[1, 2]$ 4) $[-2, -1] \cup [1, 2]$

Key. 4

Sol.

$$f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right) \in \mathbf{R} \Leftrightarrow -1 \leq \log_2\left(\frac{x^2}{2}\right) \leq 1 \Leftrightarrow \frac{1}{2} \leq \frac{x^2}{2} \leq 2 \Leftrightarrow 1 \leq x^2 \leq 4 \Leftrightarrow x \in [-2, -1] \cup [1, 2]$$

8. The domain of definition of the function, f(x) given by the equation $2^x + 2^y = 2$ is

(A) $0 < x \leq 1$ (B) $0 \leq x \leq 1$ (C) $-\infty < x \leq 0$ (D) $-\infty < x < 1$

Key. D

Sol. It is given that $2^x + 2^y = 2 \forall x, y \in R$

Therefore, $2^x = 2 - 2^y < 2 \Rightarrow 0 < 2^x < 2$

Taking log for both side with base 2.

$\Rightarrow \log_2 0 < \log_2 2^x < \log_2 2$

Hence domain is $-\infty < x < 1$.

9. The domain of the function $f(x) = \frac{1}{x} + \sin^{-1} x + \frac{1}{\sqrt{x-2}}$ is

1) $[-1, 1] \setminus \{0\}$ 2) $[-1, 1]$ 3) $(-1, 0)$ 4) ϕ

Key. 4

Sol. $x \neq 0,\ -1 \leq x \leq 1,\ x - 2 > 0$

10. If $f : \mathbf{R} \to \mathbf{R}$ is defined by $f(x) = \frac{\sin[x]\pi + \tan[x]\pi}{1 + [x]^2}$, then the range of f= (where [x] denotes integral part of x)

1) $[-1, 1]$ 2) $\{-1, 1\}$ 3) $\{1\}$ 4) $\{0\}$

Key. 4

Sol. $[x] = n \in Z \Leftrightarrow \sin[x]\pi = \tan[x]\pi = 0$

11. The range of $f(x) = \frac{3}{5 + 4\sin 3x}$ is

1) $\left[\frac{1}{3}, 3\right]$ 2) $\left[\frac{1}{3}, 1\right]$

3) $[1,3]$ 4) $\left(-\infty,\frac{1}{3}\right)\cup(3,\infty)$

Key. 1

Sol. $-1\le \sin 3x \le 1$

12. Let $f:R\to[0.\frac{\pi}{2})$ be defined by $f(x)=Tan^{-1}\left(x^2+x+a\right)$. Then the set of values of a for which f is onto is

1) $[0,\infty)$ 2) $[\frac{1}{4},\infty)$ 3) $[\frac{1}{4},(-\infty,\frac{1}{4}]$ 4) $\left\{\frac{1}{4}\right\}$

Key. 4

Sol. $x^2+x+a=0$ has a real solution

$\Rightarrow 1-4a\ge 0$

13. The range of $x^2+4y^2+9z^2-6yz-3xz-2xy$ is

1) $\varnothing$ 2) R 3) $[0,\infty)$ 4) $(-\infty,0)$

Key. 3

Sol. $x^2+4y^2+9z^2-6yz-3xz-2xy=(x)^2+(2y)^2+(3z)^2-(2y)(3z)-(x)(3z)-(x)(2y)\ge 0$

$\therefore$ Range = $[0,\infty)$.

14. The range of $\frac{x^2-x+1}{x^2+x+1}$ is

1) $\left[\frac{1}{3},3\right]$ 2) $\left[\frac{1}{3},1\right]$

3) $[1,3]$ 4) $(-\infty,\frac{1}{3}]\cup[3,\infty)$

Key. 1

Sol. Let $y=\frac{x^2-x+1}{x^2+2x+7}\Rightarrow yx^2+yx+y=x^2-x+1\Rightarrow(y-1)x^2+(y+1)x+(y-1)=0$

$x\in R\Rightarrow$ Discriminant $\ge 0\Rightarrow(y+1)^2-4(y-1)^2\ge 0\Rightarrow -3y^2+10y-3\ge 0$

$\Rightarrow 3y^2-10y+3\le 0\Rightarrow(3y-1)(y-3)\le 0\Rightarrow\frac{1}{3}\le y\le 3$

Range = $\left[\frac{1}{3},3\right]$

15. The range of $|x-2|+|x-5|$ is

1) $[2,\infty)$ 2) $[3,\infty)$ 3) $[4,\infty)$ 4) $[5,\infty)$

Key. 2

Sol. $f(x)=|x-2|+|x-5|$ and domain f=R

For x<2, $f(x)=2-x+5-x=7-2x>3$;

For 2<x<5, $f(x)=x-2+5-x=3$;

For x>5, $f(x)=x-2+x-5=2x-7>3$;

Range $f=[3,\infty)$

16. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

1) $\{1,2,3\}$ 2) $\{1,2,3,4,5\}$ 3) $\{1,2,3,4\}$ 4) $\{1,2,3,4,5,6\}$

Key. 1

Sol. f(x) is defined $\Rightarrow x-3 \ge 0, x-3 \le 7-x \Rightarrow x \ge 3, 2x \le 10 \Rightarrow 3 \le x \le 5 \Rightarrow x = 3 \, or \, 4 \, or \, 5$

Range = $\{f(3), f(4), f(5)\} = \{{}^4P_0, {}^3P_1, {}^2P_2\} = \{1,3,2\}$

17. The range of $\sin^{-1}x - \cos^{-1}x$ is

1) $\left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$ 2) $\left[\frac{-5\pi}{2}, \frac{\pi}{3}\right]$ 3) $\left[\frac{-3\pi}{2}, \pi\right]$ 4) $\left[0, \frac{\pi}{2}\right]$

Key. 1

Sol. $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \frac{\pi}{2} - 2\cos^{-1}x$

$0 \le \cos^{-1}x \le \pi \Rightarrow 0 \le 2\cos^{-1}x \le 2\pi \Rightarrow -2\pi \le -2\cos^{-1}x \le 0 \Rightarrow \frac{-3\pi}{2} \le \frac{\pi}{2} - 2\cos^{-1}x \le \frac{\pi}{2}$

$\therefore$ Range = $\left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$

18. The range of the function $f(x) = \frac{2+x}{2-x}, x \ne 2$ is

1) R 2) R-{-1} 3) R-{1} 4) R-{2}

Key. 2

Sol. $y = \frac{2+x}{2-x} \Rightarrow 2y - yx = 2 + x \Rightarrow x(y+1) = 2y - 2 \Rightarrow x = \frac{2y-2}{y+1} \Rightarrow f^{-1}(x) = \frac{2x-2}{x+1}$

$\therefore$ Range = f = Domain $f^{-1} = R - \{-1\}$

19. The domain of $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$ is

1) (1, 2) 2) $(-1,0) \cup (1,2)$

3) $(-1,0) \cup (2,\infty)$ 4) $(-1,0) \cup (1,2) \cup (2,\infty)$

Key. 4

Sol. $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$ is defined $\Rightarrow 4 - x^2 \ne 0, x^3 - x > 0 \Rightarrow x \ne \pm 2, (x+1)x(x-1) > 0$

$\therefore$ Domain = $(-1,0) \cup (1,2) \cup (2,\infty)$

20. The domain of $\frac{\sqrt{2+x} + \sqrt{2-x}}{x}$ is

1) [-2, 2] 2) (-2, 2) 3) $[-2,0) \cup (0,2]$ 4) $R - \{0\}$

Key. 3

Sol. $\frac{\sqrt{2+x} + \sqrt{2-x}}{x}$ is defined $\Rightarrow 2 + x \ge 0, x - x \ge 0, x \ne 0 \Rightarrow x \ge -2, x \le 2, x \ne 0$

$\therefore$ Domain = $[-2,0)\cup(0,2]$

21. The domain of the function $f(x)=\left[9^{x}+27^{\frac{2}{3}(x-2)}-219-3^{2(x-1)}\right]^{1/4}$

A) $[-3,3]$ B) $[3,\infty)$ C) $\left[\frac{5}{2},\infty\right)$ D) $[0,1]$

Key. C

Sol. We must have

$9^{x}+27^{\frac{2}{3}(x-2)}-219-3^{2(x-1)}\ge 0$

$\left(3^{x}\right)^{2}+3^{2(x-2)}-219-3^{2x-2}\ge 0$

$\left(3^{2x}\right)+\frac{3^{2x}}{81}-219-\frac{3^{2x}}{9}\ge 0$

$\left(1+\frac{1}{81}-\frac{1}{9}\right)3^{2x}\ge 219$

$\frac{73}{81}\times 3^{2x}\ge 219$

$3^{2x}\ge 3\times 81=3^{5}$

$2x\ge 5$

$x\ge\frac{5}{2}$

$\Rightarrow$ Domain is $\left[\frac{5}{2},\infty\right)$.

22. The domain of the function $f(x)=\sqrt{10-\sqrt{x^{4}-21x^{2}}}$ is

(A) $[5,\infty)$ (B) $\left[-\sqrt{21},\sqrt{21}\right]$

(C) $\left[-5,-\sqrt{21}\right]\cup\left[\sqrt{21},5\right]\cup\{0\}$ (D) $(-\infty,-5]$

Key. C

Sol. We must have $x^{4}-21x^{2}\ge 0$ and $10-\sqrt{x^{4}-21x^{2}}\ge 0$

$\Rightarrow x^{2}\left(x^{2}-21\right)\ge 0$ ----(1)

and $100\ge x^{4}-21x^{2}$ ------- (2)

(1) gives $x=0$ or $x\le-\sqrt{21}$ or $x\ge\sqrt{21}$

(2) $\Rightarrow x^{4}-21x^{2}-100\le 0$

$\Rightarrow\left(x^{2}-25\right)\left(x^{2}+4\right)\le 0$

$\Rightarrow x^{2}-25\le 0$ (as $x^{2}+4>0$ always)

$\Rightarrow -5\le x\le 5$

Domain is given by $\left[-5,-\sqrt{21}\right]\cup\left[\sqrt{21},5\right]$ and $x=0$.

23. $f(x)=\frac{x^{2}-3x+4}{x^{2}+3x+4}$ then range of $f(x)$ is

1) $\left[0,\frac{1}{7}\right]$ 2) $\left(-\infty,\frac{1}{7}\right)U(7,\infty)$ 3) ¡ 4) $\left[\frac{1}{7},7\right]$

Key. 4

Sol. $y=\frac{x^2-3x+4}{x^2+3x+4}$

$yx^2+3xy+4y=x^2-3x+4$

$x^2(y-1)+3x(y+1)+4(y-1)=0$

Dis $1\geq 0\Rightarrow 9(y+1)^2-4x4(y-1)^2\geq 0$

$(3(y+1)-4(y-1)(3(y+1)4-4(y-1)\geq 0$

$(-y+7)(7y-1)\geq 0\backslash$

$(y-7)\left(y-\frac{1}{7}\right)\leq 0$

$\frac{1}{7}\leq y\leq 7$

24. If $2f(\sin x)+f(\cos x)=x\forall x\in$ ¡ then range of $f(x)$ is

1) $\left[\frac{-\pi}{3},\frac{\pi}{3}\right]$ 2) $\left[\frac{-2\pi}{3},\frac{\pi}{3}\right]$ 3) $\left[\frac{-2\pi}{3},\frac{\pi}{6}\right]$ 4) $\left[\frac{-\pi}{6},\frac{\pi}{6}\right]$

Key. 2

Sol. Put $x=\sin^{-1}x$

$2f(x)+f\left(\sqrt{1-x^2}\right)=\sin^{-1}x\ \rightarrow(1)$

$x=\cos^{-1}x$

$\Rightarrow 2f\left(\sqrt{1-x^2}\right)+f(x)=\cos^{-1}x\ \rightarrow(2)$

(1) x (2) $\Rightarrow 4f(x)+2f\left(\sqrt{1-x^2}\right)=2\sin^{-1}x$

$f(x)+2f\left(\sqrt{1-x^2}\right)=\cos^{-1}x$

$3f(x)=2\sin^{-1}x-\cos^{-1}x$

$f(x)=\frac{2}{3}\sin^{-1}x-\frac{1}{3}\left(\frac{\pi}{3}-\sin^{-1}x\right)$

$=\sin^{-1}x-\frac{\pi}{6}$

$f_{max}=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3},\ f_{min}=-\frac{\pi}{2}-\frac{\pi}{6}=\frac{-4\pi}{6}=\frac{-2\pi}{3}$

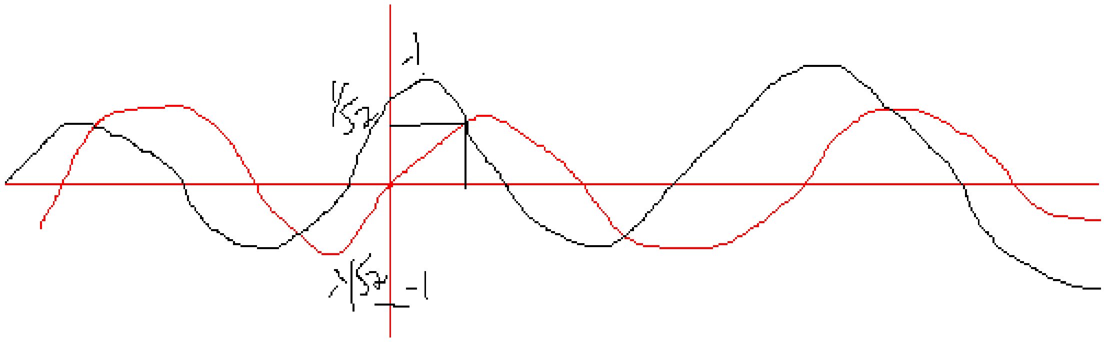
$=\left[\frac{-2\pi}{3},\frac{\pi}{3}\right]$

25. $f(x)=Max\{\sin x,\cos x\}\ \forall x\in$ ¡ then Range of $f(x)$ is.

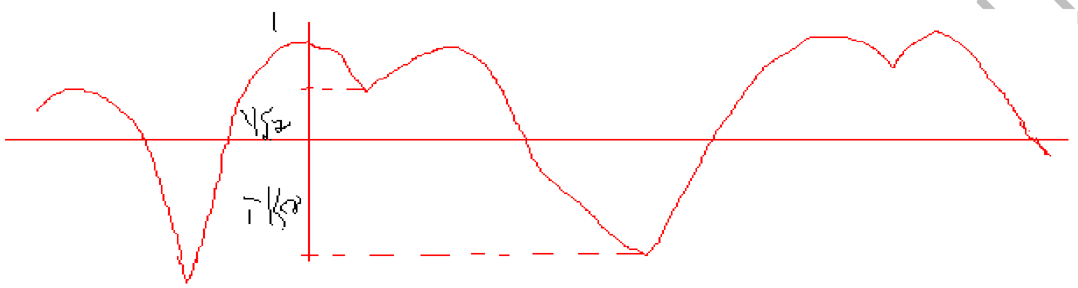
1) $\left[\frac{-1}{\sqrt{2}},1\right]$ 2) $\left[\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ 3) $[-1,1]$ 4) ϕ

Key. 1

Sol.



$f(x) = \max\{\sin x, \cos x\}$



Required range = $\left[-\frac{1}{\sqrt{2}}, 1\right]$

26. The range of $f(x) = \tan^{-1}(x^2 + x + a) \forall x \in \mathbb{R}$ is a subset of $[0, \frac{\pi}{2})$ then range of a is

1) $\mathbb{R}$ 2) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ 3) $\left[-\sqrt{3}, -1\right]$ 4) $[\frac{1}{4}, \infty)$

Key. 4

Sol. $\tan^{-1}(x^2 + x + a) \geq 0 \Rightarrow x^2 + x + a \geq 0$

$\Rightarrow disc \leq 0 \Rightarrow 1 - 4a \leq 0 \Rightarrow a \geq \frac{1}{4}$

$\Rightarrow a \in [\frac{1}{4}, \infty)$

27. The domain of the function $f(x) = \frac{1}{x - [x]}$.

(A) N (B) $(0, \infty)$ (C) $R - \{0, \pm 1, \pm 2, \pm 3, ...\}$ (D) $R - N$

Key. C

Sol. Observe that when x is an integer x = [x]. Hence, f(x) is not defined when x is an integer. Domain is R excluding $0, \pm 1, \pm 2$, …..

28. Domain of the function f(x) = $\log_2\left(\log_4\left(\log_2\left(\log_3(x^2 + 4x - 23)\right)\right)\right)$ is

(A) (–8, 4) (B) $(-\infty, -8) \cup (4, \infty)$

(C) $(-4, 8)$ (D) $(-\infty, -4) \cup (8, \infty)$

Key. B

Sol. The given function is defined when

$\log_2 \log_3(x^2+4x-23) > 1$

i.e., when $\log_3(x^2+4x-23) > 2$

i.e., when $x^2+4x-23 > 3^2$

i.e., when $x^2+4x-32 > 0$

i.e., when $x < -8$ or $x > 4$

29. Domain of the function $f(x) = \sqrt{5|x| - x^2 - 6}$ is

(A) $(-\infty, 2) \cup (3, \infty)$ (B) $[-3, -2] \cup [2, 3]$ (C) $(-\infty, -2) \cup (2, 3)$ (D) $R - \{-3, -2, 2, 3\}$

Key. B

Sol. $5|x| - x^2 - 6 \ge 0 \quad \Rightarrow \quad x^2 - 5|x| + 6 \le 0$

when $x < 0$, $x^2 + 5x + 6 \le 0$, $-3 \le x \le -2$

when $x > 0$, $x^2 - 5x + 6 \le 0$, $2 \le x \le 3$

x = 0 will not satisfy the condition.

Domain is $[-3, -2] \cup [2, 3]$.

30. Range of the function $y = \dfrac{2^x - 2^{-x}}{2^x + 2^{-x}}$ is

(A) R (B) (–1, 1) (C) [–1, 1] (D) (0, 1)

Key. B

Sol. $2^x + 2^{-x}$ is always > 0 i.e., domain is R

$y = \dfrac{2^x - 2^{-x}}{2^x + 2^{-x}} = \dfrac{2^{2x}-1}{2^{2x}+1}$

$\Rightarrow \dfrac{1+y}{1-y} = \dfrac{2.2^{2x}}{2}$ (Componendo Dividendo)

$= 2^{2x} > 0$

$\Rightarrow \dfrac{1+y}{1-y} > 0$ i.e., $\dfrac{(1+y)^2}{1-y^2} > 0$

$\Rightarrow 1 - y^2 > 0 \quad \Rightarrow \quad -1 < y < 1$

31. The range of the function $f(x) = \dfrac{x+3}{|x+3|}$, $x \ne -3$ is

(A) {3, –3} (B) R – {–3} (C) all positive integers (D) {–1, 1}

Key. D

Sol. f(x) = 1 when x + 3 > 0

f(x) = –1 when x + 3 < 0

Range = {–1, 1}

32. The range of the function $f(x) = \cos^2 \frac{x}{4} + \sin\frac{x}{4},\ x \in R$ is

(A) $\left[0, \frac{5}{4}\right]$ (B) $\left[1, \frac{5}{4}\right]$ (C) $\left(-1, \frac{5}{4}\right)$ (D) $\left[-1, \frac{5}{4}\right]$

Key. D

Sol. $f(x) = 1 - \sin^2\frac{x}{4} + \sin\frac{x}{4} = -\left\{\sin^2\frac{x}{4} - \sin\frac{x}{4}\right\} + 1 = -\left\{\left(\sin\frac{x}{4} - \frac{1}{2}\right)^2 - \frac{1}{4}\right\} + 1$

$= \frac{5}{4} - \left(\sin\frac{x}{4} - \frac{1}{2}\right)^2$

Maximum f(x) $= \frac{5}{4}$

Minimum f(x) $= \frac{5}{4} - \left(-1 - \frac{1}{2}\right)^2 = \frac{5}{4} - \frac{9}{4} = -1$

Range of f(x) $= \left[-1, \frac{5}{4}\right]$

33. The domain of the function $f(x) = \log_e(x^2 + x + 1) + \sin\sqrt{x-1}$ is

(A) (–2, 1) (B) $(-2, \infty)$ (C) $[1, \infty)$ (D) None of these

Key. C

Sol. We must have $x - 1 \geq 0$.

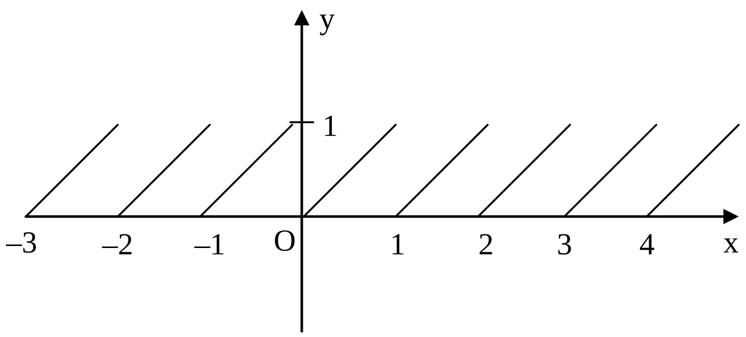
Note that $(x^2 + x + 1)$ is always positive combining, the domain is $[1, \infty)$.

34. Let $f(x) = \frac{x - [x]}{1 + x - [x]},\ x \in R$, where [ ] denotes the greatest integer function. Then, the range of f is

(A) (0, 1) (B) $\left[0, \frac{1}{2}\right)$ (C) [0, 1] (D) $\left[0, \frac{1}{2}\right]$

Key. B

Sol. The graph of y = x – [x] is as shown below



When x is an integer, x – [x] = 0

Hence, f(x) = 0 when x is an integer

$x \to [x]$ as x tends to an integer.

As $x \to 1, \frac{x}{1+x} \to \frac{1}{2}$

Hence, the range of f(x) is $\left[0, \frac{1}{2}\right)$.

35. Let $f(x) = [x]\cos\left(\frac{\pi}{[x+2]}\right)$ where, [ ] denotes the greatest integer function. Then, the domain of f is

(A) $x \in R$, x not an integer (B) $(-\infty, -2) \cup [-1, \infty)$

(C) $x \in R, x \neq -2$ (D) $(-\infty, -1]$

Key. B

Sol. $[x+2] \neq 0$

$[x] + 2 \neq 0$

$[x] \neq -2$

x should not belong to [–2, –1)

Domain of f is $(-\infty, -2) \cup [-1, \infty)$.

36. Range of $f(x) = \frac{\tan\left(\pi[x^2 - x]\right)}{1+\sin(\cos x)}$ is (where [x] denotes the greatest integer function)

(A) $(-\infty, \infty) \sim [0, \tan 1]$ (B) $(-\infty, \infty) \sim [\tan 2, 0)$

(C) $[\tan 2, \tan 1]$ (D) {0}

Key. D

Sol. $f(x) = \frac{\tan\left(\pi[x^2 - x]\right)}{1+\sin(\cos x)}$ = {0} because of $[x^2 - x]$ is integer.

37. Range of the function $f(x) = x^2 + \frac{1}{x^2+1}$, is

(A) $[1, \infty)$ (B) $[2, \infty)$ (C) $\left[\frac{3}{2}, \infty\right)$ (D) $(-\infty, \infty)$

Key. A

Sol. $f(x) = x^2 + 1 + \frac{1}{x^2+1} - 1$

$x^2 + 1 + \frac{1}{x^2+1} \geq 2$ [$\because$ AM $\geq$ GM]

$x^2 + \frac{1}{x^2+1} \geq 1$

$\therefore f(x) \in [1, \infty)$

38. If $f(x) = \ln\left(\frac{x^2+e}{x^2+1}\right)$, then range of f(x) is

(A) (0, 1) (B) (0, 1] (C) [0, 1) (D) {0, 1}

Key. B

Sol. $f(x) = \ln\left(\frac{x^2+e}{x^2+1}\right) = \ln\left(\frac{x^2+1-1+e}{x^2+1}\right) = \ln\left(1+\frac{e-1}{x^2+1}\right)$

Clearly range is (0, 1]

Hence (B) is correct answer.

39. The inverse of $f(x) = \left(5-(x-8)^5\right)^{\frac{1}{3}}$ is

(A) $5-(x-8)^5$ (B) $8+(5-x^3)^{1/5}$

(C) $8-(5-x^3)^{1/5}$ (D) $\left(5-(x-8)^{1/5}\right)^3$

Key. B

Sol. Let y = f(x) = $\left(5-(x-8)^5\right)^{1/3}$, then

$y^3 = 5-(x-8)^5 \quad \Rightarrow \quad (x-8)^5 = 5-y^3$

$\Rightarrow \quad x = 8 + (5-y^3)^{1/5}$

Let, z = $g(x) = 8+(5-x^3)^{1/5}$

Now, $f\left(g(x)\right) = \left[5-(x-8)^5\right]^{1/3}$

$= \left(5-\left[(5-x^3)^{1/5}\right]^5\right)^{1/3} = (5-5+x^3)^{1/3} = x$

Similarly, we can show that $g\left(f(x)\right) = x$.

Hence, g(x) = 8 + $\left(5-x^3\right)^{1/5}$ is the inverse of f(x).

40. The range of the function $f(x) = |x-1|+|x-2|+|x+1|+|x+2|$ where, $x \in [-2, 2]$ is

(A) [6, 8] (B) [2, 4] (C) [0, 4] (D) {1, 2}

Key. A

Sol. $f(x) = |x-1|+|x-2|+|x+1|+|x+2|$

when $x \in [-2, -1]$

f(x) $= -(x-1)-(x-2)-(x+1)+x+2 = -2x + 4$

when $x \in [-1, 1]$, $f(x) = -(x-1)-(x-2)+x+1+x+2$

$= -x+1-x+2+x+1+x+2 = 6$

when $x \in [1, 2]$, $f(x) = (x-1)-(x-2)+x+1+x+2 = 2x + 4$

Plotting the graph of the function, range of f(x) = [6, 8]

41. Range of the function $f(x)=\dfrac{x^2+x+2}{x^2+x+1};\ x\in R$ is

(A) $(1, \infty)$ (B) $\left(1, \dfrac{11}{7}\right]$ (C) $\left(1, \dfrac{7}{3}\right]$ (D) $\left[1, \dfrac{7}{5}\right]$

Key. C

Sol. We have, f(x) $= \dfrac{x^2+x+2}{x^2+x+1}=\dfrac{(x^2+x+1)+1}{x^2+x+1}=1+\dfrac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$

We can see here that as $x\to\infty$, f(x) $\to$ 1 which is the min value of f(x). Also f(x) is max when $\left(x+\dfrac{1}{2}\right)^2+\dfrac{3}{4}$ is min which is so when x = $-\dfrac{1}{2}$ and then $\dfrac{3}{4}$.

$\therefore$ $f_{max} = 1+\dfrac{1}{3/4}=\dfrac{7}{3}$

$\therefore$ $R_j = \left(1, \dfrac{7}{3}\right]$

42. Domain of the function $f(x)=\sqrt{\log_e \dfrac{1}{|\sin x-1|}}$ is

(A) $n\pi+(-1)^n\alpha$ where n is any integer and $\alpha\in\left[0, \dfrac{\pi}{2}\right)$

(B) $n\pi+(-1)^n\dfrac{\pi}{2}$, n = 1, 2, 3, ... (C) $2n\pi-\alpha$ where $\alpha\in\left(0, \dfrac{\pi}{2}\right)$, n any integer

(D) $\dfrac{(2n+1)\pi}{2}$, n any integer

Key. A

Sol. $|\sin x-1| \neq 0$ …(i)

$\sin x \neq 1$

$|\sin x - 1| \leq 1$

$-1 \leq \sin x - 1 \leq 1$

$0 \leq \sin x \leq 2$ …(ii)

From (i) and (ii), $\sin x \in [0, 1)$

$\sin x \in [0, 1)$

$\sin x = 0 \to x = n\pi$

$\sin x = 1 \to x = n\pi + (-1)^n \dfrac{\pi}{2}$

$\Rightarrow$ Domain of f(x) is

x = $n\pi$ (n any integer)

$\sin x \le 1$

$x \in \left[0, \frac{\pi}{2}\right)$

General solution is

. $x = n\pi + (-1)^n \alpha$

where, $\alpha \in \left[0, \frac{\pi}{2}\right)$.

43. If the range of $f(x) = 2+\sqrt[3]{x}, -3 \le x < -1$ is $\left[0, \sqrt[3]{n}\right]$ where $n \in N$ then n=

$= x^{\frac{2}{3}} \quad , -1 \le x \le 2$

(A) 1 (B) 2

(C) 4 (D) 6

Key. C

Sol. The given function has local maximum at x= -1 ,minimum at x=0 and F(0) = 0,F(-1) =1,

F(-3) = $2-\sqrt[3]{3}$ $f(2) = 2^{\frac{2}{3}} = \sqrt[3]{4}$

$\therefore$ range of f(x) =[0, $\sqrt[3]{4}$]

44. If $2f(\sin x) + f(\cos x) = x \forall x \in ¡$ then range of $f(x)$ is

1) $\left[\frac{-\pi}{3}, \frac{\pi}{3}\right]$ 2) $\left[\frac{-2\pi}{3}, \frac{\pi}{3}\right]$ 3) $\left[\frac{-2\pi}{3}, \frac{\pi}{6}\right]$ 4) $\left[\frac{-\pi}{6}, \frac{\pi}{6}\right]$

Key. 2

Sol. Put $x = \sin^{-1} x$

$2f(x) + f\left(\sqrt{1-x^2}\right) = \sin^{-1} x \rightarrow (1)$

$x = \cos^{-1} x$

$\Rightarrow 2f\left(\sqrt{1-x^2}\right) + f(x) = \cos^{-1} x \rightarrow (2)$

(1) x (2) $\Rightarrow 4f(x) + 2f\left(\sqrt{1-x^2}\right) = 2\sin^{-1} x$

$$\frac{f(x) + 2f\left(\sqrt{1-x^2}\right) = \cos^{-1} x}{3f(x) = 2\sin^{-1} x - \cos^{-1} x}$$

$f(x) = \frac{2}{3}\sin^{-1} x - \frac{1}{3}\left(\frac{\pi}{3} - \sin^{-1} x\right)$

$= \sin^{-1} x - \frac{\pi}{6}$

$f_{\max} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}, \; f_{\min} = -\frac{\pi}{2} - \frac{\pi}{6} = \frac{-4\pi}{6} = \frac{-2\pi}{3}$

$= \left[\frac{-2\pi}{3}, \frac{\pi}{3}\right]$

45. Range of $f\left(x\right)=\tan^{-1}\left[\frac{2}{\pi}\left(2\tan^{-1}x-\sin^{-1}x+\cot^{-1}x-\cos^{-1}x\right)\right]$ contains

(A) Only one integer (B) More than 2 integers
(C) Only two integers (D) No integer

Key. A

Sol. $y=\tan^{-1}\left(\frac{2}{\pi}\tan^{-1}x\right),\ -1\le x\le 1$

$-\frac{\pi}{4}\le \tan^{-1}x\le\frac{\pi}{4}$

$-\frac{1}{2}\le\frac{2}{\pi}\tan^{-1}x\le\frac{1}{2}$

$-\tan^{-1}\frac{1}{2}\le\tan^{-1}\left(\frac{2}{\pi}\tan^{-1}x\right)\le\tan^{-1}\left(\frac{1}{2}\right)$

y = 0, is only integer hence one integer

46. If $f\left(x\right)=\sqrt{\cos\left(\sin x\right)}+\sqrt{\sin\left(\cos x\right)}$, then the range of $f(x)$ is

(A) $\left[\sqrt{\cos 1},\ \sqrt{\sin 1}\ \right]$ (B) $\left[\sqrt{\cos 1}, 1+\sqrt{\sin 1}\ \right]$

(C) $\left[\ 1-\sqrt{\cos 1},\ \sqrt{\sin 1}\ \right]$ (D) $\left[\sqrt{\cos 1},1\right]$

Key. B

Sol. Period of f(x) is 2π, but f(x) is not defined for x ∈ (π/2, 3π/2). Hence it suffices to consider x ∈ [–π/2, π/2]. Further since f(x) is even, we consider x ∈ [0, π/2].

Now $\sqrt{\cos\left(\sin x\right)}$ and $\sqrt{\sin\left(\cos x\right)}$ are decreasing functions for x ∈ [π,π/2].

$\Rightarrow\ R_f=\left[f\left(\pi/2\right),\ f\left(0\right)\right]=\left[\sqrt{\cos 1},\ 1+\sqrt{\sin 1}\ \right]$

47. The range of $f\left(x\right)=-x^3+x^2-x+\cos^{-1}x,$ is

A) $\left[-1,3+\pi\right]$ B) $\left[0,\pi-1\right]$ C) $\left[-1,2+\pi\right]$ D) $\left[-1,\pi\right]$

Key. A

Sol. $f\left(x\right)=-x^3+x^2-x+\cos^{-1}x$

Domain $=\left[-1,1\right]$

$f'\left(x\right)=-3x^2+2x-1-\frac{1}{\sqrt{1-x^2}}<0$

'f' is a decreasing function

∴ Min of $f\left(x\right)$ is $f\left(1\right)=-1+1-1+0=-1$

Max of f(x) is $f\left(-1\right)=1+1+1+\pi=3+\pi$

Range $=[-1,3+\pi]$

48. The domain of the function

$f(x)=\log_e\{\operatorname{sgn}(9-x^2)\}+\sqrt{[x]^3-4[x]}$ where [.] = G.I.F

A) $[-2,1)\cup[2,3)$ B) $[-4,1)\cup[2,3)$

C) $[4,1)\cup[2,3)$ D) $[2,1)\cup[2,3)$

Key. A

Sol. Given $f(x)=\log_e\{\operatorname{sgn}(9-x^2)\}+\sqrt{[x]^3-4[x]}=y_1+y_2(say)$

Now, y_1 is defined if $\operatorname{sgn}(9-x^2)>0$

But sgn x = 1(i.e. > 0) if x > 0

$\therefore \operatorname{sgn}(9-x^2)>0 \Rightarrow 9-x^2>0 \Rightarrow x^2-9<0 \Rightarrow (x-3)(x+3)<0 \Rightarrow -3<x<3$ ...(A)

Again, y_2 is defined if $[x]^3-4[x]\ge 0 \Rightarrow [x]\{[x]^2-4\}\ge 0 \Rightarrow [x]([x]-2)([x]+2)\ge 0$.

Following the wavy curve method, we find

Thus $[x]\ge 2$ or $[x]$ lies between – 2 and 0, i.e. [x] = - 2, - 1 or 0

Now, $[x]\ge 2 \Rightarrow x\ge 2$ ...(B)

[x] = - 2 $\Rightarrow$ - 2 $\le$ x < 1

[x] = - 1 $\Rightarrow$ - 1 $\le$ x < 0

[x] = 0 $\Rightarrow$ 0 $\le$ x < 1.

Hence [x] = - 2, - 1, 0 $\Rightarrow -2\le x<1$

$\therefore (B)\cup(C)=(x\ge 2)$ or $(-2\le x<1)$ ...(C)

Hence $D_f=(A)\cup(C)=[-2,1)\cup[2,3)$.

49. The Range of the function

$f(x)=\log_{10}\left\{\sin^{-1}\left(\sqrt{x-5}\right)+\frac{3\pi}{2}\right\}$ is

A) $\left[\log\frac{3\pi}{2},\log 2\pi\right]$ B) $\left[\log\frac{3\pi}{2},\log 3\pi\right]$

C) $\left[\log\frac{3\pi}{2},\log \pi\right]$ D) $\left[\log\frac{3\pi}{4},\log 2\pi\right]$

Key. A

Sol. Let $f(x)=\log_{10}\left\{\sin^{-1}\left(\sqrt{x-5}\right)+\frac{3\pi}{2}\right\}$.

The function is defined if (i) $x-5\ge 0$ (ii) $-1\le\sqrt{x-5}\le 1$ and

(iii) $\sin^{-1}\left(\sqrt{x-5}\right)+\frac{3\pi}{2}>0$.

Now (i) $\Rightarrow x\ge 5$

(ii) $\Rightarrow 0\le x-5\le 1 \Rightarrow 6\le x\le 6$.

(iii) is satisfied by virtue of (ii).

Hence, considering (i) and (ii), we find that the domain of the function viz. $D_f = [5,6]$.

Let $y_1 = \sin^{-1}\left(\sqrt{x-5}\right)$ and $y_2 = \sin^{-1}\left(\sqrt{x-5}\right) + \frac{3\pi}{2}$ so that $y = \log_{10}(y_2)$ where $y_2 = y_1 + \frac{3\pi}{2}$

Now, for y_1 since $x \in [5,6], y_1 \geq 0$ so that $0 \leq y_1 \leq \frac{\pi}{2}\left(Q -\frac{\pi}{2} \leq \sin^{-1}(z) \leq \frac{\pi}{2}\right)$

Consequently $0 + \frac{3\pi}{2} \leq y_1 + \frac{3\pi}{2} \leq \frac{\pi}{2} + \frac{3\pi}{2} \Rightarrow \frac{3\pi}{2} \leq y_2 \leq 2\pi$

$\Rightarrow \quad \log\left(\frac{3\pi}{2}\right) \leq \log(y_2) \leq \log(2\pi)$, since u = log z is an increasing function

$\Rightarrow \quad \log\left(\frac{3\pi}{2}\right) \leq \log(y_2) \leq \log(2\pi)$.

Hence the range of f(x) is $\left[\log\frac{3\pi}{2}, \log 2\pi\right]$.

50. The domain of $f(x) = \sqrt{x-2-2\sqrt{x-3}} - \sqrt{x-2+2\sqrt{x-3}}$, is

A) $[3,5]$ B) $(3,5)$ C) $[5,\infty)$ D) $[3,\infty)$

Key. D

Sol. $x-3 \geq 0 \Rightarrow x \geq 3$

$x-2-2\sqrt{x-3} \geq 0$ For $x \geq 3$

$\Rightarrow x-2 \geq 2\sqrt{x-3}$ and $x-2+2\sqrt{x-3} \geq 0$

$\Rightarrow x^2-8x+16 \geq 0 \Rightarrow (x-4)^2 \geq 0 \ \forall \ x \in R$

Domain $= [3,\infty)$

51. Minimum value of function $f(x) = x^3(x^3+1)(x^3+2)(x^3+3) : x \in R$, is

(A) –2 (B) –1 (C) 1 (D) none

Key. B

Sol. Let $t = x^3(x^3+3); t = (x^3+\frac{3}{2})^2 - \frac{9}{4} \in [-\frac{9}{4}, \infty)$

$f(x) = g(t) = t(t+2) = (t+1)^2 - 1$ is least when t = –1

and $-1 \in [-\frac{9}{4}, \infty) \therefore$ min $f(x) = -1$

52. The domain of the function $f(x) = \sqrt{[x]^2 - 6[x] + 8}$ where [.] = G. I. F

A) (- 4, 4) B) $(-\infty,3) \cup [4,\infty)$ C) (3, 4) D) $(3,4) \cup (5,\infty)$ Key. B

Sol. (i) The function is defined if sin x $-\frac{1}{2} \geq 0$

$\Rightarrow \sin x \geq \frac{1}{2} \Rightarrow x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] \Rightarrow x \in \left[2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right]$

(ii) The function is defined if $-1 \le \frac{1}{|x-1|} - 2 \le 1;\ x \ne 1$

$\Rightarrow \quad 1 \le \frac{1}{|x-1|} \le 3 \Rightarrow \frac{1}{|x-1|} \ge 1$ ...(1)

And $\frac{1}{|x-1|} \le 3$ ...(2)

(1) $\Rightarrow \quad |x-1| \le 1 \Rightarrow -1 \le x-1 \le 1 \Rightarrow 0 \le x \le 2$ ...(A)

(2) $\Rightarrow \quad |x-1| \ge \frac{1}{3} \Rightarrow -\frac{1}{3} \le x-1 \le \frac{1}{3} \Rightarrow \frac{2}{3} \le x \le \frac{4}{3}$ ...(B)

Combining (A) and (B), we find that $x \in \left[0, \frac{2}{3}\right] \cup \left[\frac{4}{3}, 2\right]$ with is the domain of the given function.

53. The domain of the function of $f(x) = \log_{[x]}\left\{\text{sgn}\left(x^2\right)\right\}$

(where [.] G.I.F) is

A) $[2, \infty)$ B) (- 2, 2) C) $(-\infty, 2)$ D) None

Key. A

Sol. (i) f(x) is defined if (i) (4 - |x|) > 0 (iii) $\left[x^2\right] > 0$ but $\left[x^2\right] \ne 1$

Now, (i) $\Rightarrow |x| < 4 \Rightarrow -4 < x < 4$ ...(A)

From (iii), $\left[x^2\right] > 0 \Rightarrow \left[x^2\right] = 1, 2, 3, ...$

But $\left[x^2\right] \ne 1$.

$\therefore \quad \left[x^2\right] = 2, 3, 4, ...\ i.e. \left[x^2\right] \ge 2$

$\Rightarrow \quad x^2 \ge 2; \because \left[f(x)\right] \ge n \Rightarrow f(x) \ge n$.

$\Rightarrow \quad x \le -\sqrt{2}\ \ or\ \ x \ge \sqrt{2}$

Combining (A) and (B), we find that $-4 < x \le -\sqrt{2}\ \ or\ \ \sqrt{2} \le x < 4$. ...(B)

Hence the domain of the given function is $\left(-4, -\sqrt{2}\right] \cup \left[\sqrt{2}, 4\right)$.

(ii) The function is defined if (*i) sgn $\left(x^2\right) > 0$ and (ii) [x] > 0 but [x] $\ne$ 1.

We know that sgn $\left(x^2\right) = \begin{cases} 1\ if\ \ x^2 > 0 \\ 0\ if\ \ x = 0 \\ -1\ if\ \ x^2 < 0 \end{cases}$

(i) Since sgn $\left(x^2\right)$ is non-negative, we have $x^2 > 0 \Rightarrow x \in R - \{0\}$. ...(A)

(ii) $\Rightarrow$ [x] = 2, 3, 4, ... $\therefore x \in [2, \infty)$ ...(B)

Hence, $D_f = A \cap B = [2, \infty)$.

54. The domain of the function

$f(x) = \log_{10}\left\{1 - \log_{10}\left(x^2 - 5x + 10\right)\right\}$ is

A) $(0,\infty)$ B) (0, 5) C) $(-\infty,0)$ D) None

Key. B

Sol. (a) The function f(x) is defined if (i) $x^2-5x+10>0$, (ii) $1-\log_{10}\left(x^2-5x+10\right)>0$

Now, (ii) $\Rightarrow \log_{10}\left(x^2-5x+10\right)<1 \Rightarrow x^2-5x+10<10$

$\Rightarrow x^2-5x<0 \Rightarrow x(x-5)<0 \Rightarrow 0<x<5$ ...(A)

Again, $x^2-5x+10>0$ for all x, ...(B)

Since the discriminant of the corresponding equation $x^2-5x+10=0$ is negative, so that the roots of the equation are imaginary.

Combining (A) and (B), we find that the domain of f(x) is (0, 5).

(b) The function g(x) is defined if (i) $(x-4)^2>0$, (ii) $\log_4(x-4)^2>0$

(iii) $\log_3\left\{\log_4(x-4)^2\right\}>0$

(i) is true for all x. ...(A)

(ii) is true if $(x-4)^2>1 \Rightarrow x^2-8x+15>0 \Rightarrow (x-3)(x-5)>0 \Rightarrow x<3 \text{ or } x>5$ ...(B)

(iii) is true if $\log_4(x-4)^2>1 \Rightarrow (x-4)^2>4 \Rightarrow x^2-8x+12>0$

$\Rightarrow$ (x – 2)(x – 6) > 0 $\Rightarrow$ x < 2 or x > 6 ...(C)

Hence combining (A), (B) and (C), we find that the domain of g(x) is $(-\infty,2)\cup(6,\infty)$.

55. The domain of the function

$f(x)=\log_e\left\{\text{sgn}\left(9-x^2\right)\right\}+\sqrt{[x]^3-4[x]}$ where [x] = G.I.F

A) $[-2,1)\cup[2,3]$ B) $[-4,1)\cup[2,3]$ C) $[4,1)\cup[2,3]$ D) $[2,1)\cup[2,3]$

Key. A

Sol. Given $f(x)=\log_e\left\{\text{sgn}\left(9-x^2\right)\right\}+\sqrt{[x]^3-4[x]}=y_1+y_2(say)$

Now, y_1 is defined if $\text{sgn}\left(9-x^2\right)>0$

But sgn x = 1(i.e. > 0) if x > 0

$\therefore \text{sgn}\left(9-x^2\right)>0 \Rightarrow 9-x^2>0 \Rightarrow x^2-9<0 \Rightarrow (x-3)(x+3)<0 \Rightarrow -3<x<3$ ...(A)

Again, y_2 is defined if $[x]^3-4[x]\geq 0 \Rightarrow [x]\left\{[x]^2-4\geq 0 \Rightarrow [x]\left([x]-2\right)\geq 0\right.$.

Following the wavy curve method, we find

Thus $[x]\geq 2$ or $[x]$ lies between – 2 and 0, i.e. [x] = - 2, - 1 or 0

Now, $[x]\geq 2 \Rightarrow x\geq 2$ ...(B)

[x] = - 2 $\Rightarrow$ - 2 $\leq$ x < 1

[x] = - 1 $\Rightarrow$ - 1 $\leq$ x < 0

[x] = 0 $\Rightarrow$ 0 $\leq$ x < 1.

Hence [x] = - 2, - 1, 0 $\Rightarrow -2\leq x<1$

$\therefore (B)\cup(C)=(x\geq 2) \text{ or } (-2\leq x<1)$ ...(C)

Hence $D_f=(A)\cup(C)=[-2,1)\cup[2,3)$.

56. The range of the function $f(x)=3x^4-4x^3-12x^2+1$ is

A) $[31,\infty)$ B) $[-31,\infty)$ C) $[3,\infty)$ D) $[-3,\infty)$

Key. B

Sol. Given that $y=f(x)=3x^4-4x^3-12x^2+1$.

It cuts the y-axis at the point (x = 0, x = 1).

Differentiating, we get $\frac{dy}{dx}=12x^3-12x^2-24x$

i.e. $\frac{dy}{dx}=12x(x^2-x-2)=12x(x-2)(x+1)$.

Now, $\frac{dy}{dx}=0\Rightarrow x(x-2)(x+1)=0\Rightarrow x=0,2,-1$

Also, $\frac{dy}{dx}>0\Rightarrow x(x-2)(x+1)>0$.

Using wavy-curve method, we have

Thus $\frac{dy}{dx}>0$ when x > 2 or $x\in(-1,0)$.

Similarly, $\frac{dy}{dx}<0$ when 0 < x < 2 or x < - 1.

Hence the graph of the curve will be as follows:

At x = 2, f(x) = 3 × 16 – 4 × 8 – 12 × 4 + 1 = 48 – 32 – 48 + 1 = - 31.

At x = - 1, f(x) = 3. 1 + 4. 1 – 12.1 + 1 = - 4.

$\therefore$ The least value of the function is – 31.

Hence the range of the function is $[-31,\infty)$.

57. The range of the function $f(x)=\sqrt{e^{\cos^{-x}\left(\log_4^{x^2}\right)}}$ is

A) $\left[1,\sqrt{e^{\pi}}\right]$ B) $\left[4,\sqrt{e^{\pi}}\right]$ C) $\left[2,\sqrt{e^{\pi}}\right]$ D) $\left[3,\sqrt{e^{\pi}}\right]$

Key. A

Sol. (iii) Given that $y(x)=\sqrt{e^{\cos^{-1}\left(\log_4 x^2\right)}}$.

The function is defined if (i) $x^2>0$ which is true for all x (ii) $-1\le\log_4 x^2\le 1$.

Now, (ii) $\Rightarrow 4^{-1}\le x^2\le 4\Rightarrow\frac{1}{4}\le x^2\le 4\Rightarrow x\in\left[\frac{1}{2},2\right] or x\in\left[-2,-\frac{1}{2}\right]$.

Hence the domain of the function is $\left[-2,-\frac{1}{2}\right]\cup\left[\frac{1}{2},2\right]$.

To find out the range, let $y_1=\log_4 x^2$ so that $y=\sqrt{e^{\cos^{-1}(y_1)}}$.

Again, let $y_2=\cos^{-1}(y_1)$.

$\therefore y=\sqrt{e^{y_2}}$ where $y_2=\cos^{-1}(y_1)$ and $y_1=\log_4 x^2$.

Now, for $x=\frac{1}{2}\left(or-\frac{1}{2}\right)y_1=\log_4\left(\frac{1}{4}\right)=\log_4\left(4^{-1}\right)=-1$

And for x = 2 (or – 2), $y_1 = \log_4(4) = 1$.

Hence y_1 lies between -1 and 1 i.e. $-1 \le y_1 \le 1 \Rightarrow \cos^{-1}(-1) \ge \cos^{-1}(y_1) \ge \cos^{-1}(1)$

$\Rightarrow \pi \ge y_2 \ge 0 \Rightarrow 0 \le y_2 < \pi$.

Again $0 \le y_2 \le \pi \Rightarrow e^{y_2} \le e^{\pi} \Rightarrow 1 \le e^{y_2} \le e^{\pi} \Rightarrow 1 \le \sqrt{e^{y_2}} \le \sqrt{e^{\pi}} \Rightarrow 1 \le y \le \sqrt{e^{\pi}}$.

Hence the range of the function is $\left[1, \sqrt{e^{\pi}}\right]$.

58. The Range of the function

$f(x) = \log_{10}\left\{\sin^{-1}\left(\sqrt{x-5}\right) + \frac{3\pi}{2}\right\}$ is

A) $\left[\log\frac{3\pi}{2}, \log 2\pi\right]$ B) $\left[\log\frac{3\pi}{2}, \log 3\pi\right]$ C) $\left[\log\frac{3\pi}{2}, \log \pi\right]$ D) $\left[\log\frac{3\pi}{4}, \log 2\pi\right]$ Key. A

Sol. Let $f(x) = \log_{10}\left\{\sin^{-1}\left(\sqrt{x-5}\right) + \frac{3\pi}{2}\right\}$.

The function is defined if (i) $x - 5 \le 0$ (ii) $-1 \le \sqrt{x-5} \le 1$ and (iii) $\sin^{-1}\left(\sqrt{x-5}\right) + \frac{3\pi}{2} > 0$.

Now (i) $\Rightarrow x \ge 5$

(ii) $\Rightarrow 0 \le x - 5 \le 1 \Rightarrow 6 \le x \le 6$.

(iii) is satisfied by virtue of (ii).

Hence, considering (i) and (ii), we find that the domain of the function viz. $D_f = [5, 6]$.

Let $y_1 = \sin^{-1}\left(\sqrt{x-5}\right)$ and $y_2 = \sin^{-1}\left(\sqrt{x-5}\right) + \frac{3\pi}{2}$ so that $y = \log_{10}(y_2)$ where $y_2 = y_1 + \frac{3\pi}{2}$

Now, for y_1 since $x \in [5, 6], y_1 \ge 0$ so that $0 \le y_1 \le \frac{\pi}{2}\left(Q - \frac{\pi}{2} \le \sin^{-1}(z) \le \frac{\pi}{2}\right)$

Consequently $0 + \frac{3\pi}{2} \le y_1 + \frac{3\pi}{2} \le \frac{\pi}{2} + \frac{3\pi}{2} \Rightarrow \frac{3\pi}{2} \le y_2 \le 2\pi$

$\Rightarrow \quad \log\left(\frac{3\pi}{2}\right) \le \log(y_2) \le \log(2\pi)$, since u = log z is an increasing function

$\Rightarrow \quad \log\left(\frac{3\pi}{2}\right) \le \log(y_2) \le \log(2\pi)$.

Hence the range of f(x) is $\left[\log\frac{3\pi}{2}, \log 2\pi\right]$.

59. The range of the function $f(x) = \cos^{-1}\sqrt{\log_{[x]}^{\frac{|x|}{x}}}$ is where [.] = G.I.F

A) $\left[\frac{\pi}{2}\right]$ B) {0} C) $\{\pi\}$ D) $\{2\pi\}$

Key.

Sol. The function is defined if (i) [x] > 0 and [x] $\ne$ 1 (ii) $\frac{|x|}{x} > 0$

(iii) $\log_{[x]}\frac{|x|}{x} \ge 0$ (iv) $0 \le \sqrt{\log_{[x]}\frac{|x|}{x}} \le 1$.

Now, (i) [x] = 2, 3, … i.e. $[x] \ge 2 \Rightarrow x \ge 2$ i.e. the domain of the function is $[2, \infty)$.

For this value of x ($\ge$ 2) (ii) is true,

(iii) is also true and $\sqrt{\log_{[x]}\frac{|x|}{x}} = \sqrt{\log_{[x]}1} = 0$.

Hence $f(x) = \cos^{-1}(0) = \frac{\pi}{2}$.

Hence the range of the function is $\left\{\frac{\pi}{2}\right\}$.

60. The range of the function

$f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$ where [.] = G. I. F

A) $\{\pi\}$ B) $\left\{\frac{\pi}{2}\right\}$ C) $\{2\pi\}$ D) $\{0\}$

Key. A

Sol. Let $y_1 = \sin^{-1}\left[x^2 + \frac{1}{2}\right]$ *and* $y_2 = \cos^{-1}\left[x^2 - \frac{1}{2}\right]$. Then $y = y_1 + y_2$.

Now, $y_1 = \sin^{-1}\left[x^2 + \frac{1}{2}\right]$ is defined

if $-1 \le \left[x^2 + \frac{1}{2}\right] \le 1 \Rightarrow -1 \le x^2 + \frac{1}{2} < 2 \Rightarrow -\frac{3}{2} \le x^2 < \frac{3}{2}$ ...(1)

Again $y_2 = \cos^{-1}\left[x^2 - \frac{1}{2}\right]$ is defined

If $-1 \le \left[x^2 - \frac{1}{2}\right] \le 1 \Rightarrow -1 \le x^2 - \frac{1}{2} < 2 \Rightarrow -\frac{1}{2} \le x^2 < \frac{5}{2}$ ...(2)

Taking the intersection of (1) and (2), we find that

$-\frac{1}{2} \le x^2 < \frac{3}{2} \Rightarrow 0 \le x^2 < \frac{3}{2}$, since x^2 cannot be negative.

Now, for x^2 so that $\frac{1}{2} \le x^2 + \frac{1}{2} \le 1$ and $-\frac{1}{2} \le x^2 - \frac{1}{2} \le 0$, we have

$y = \sin^{-1}(0) + \cos^{-1}(-1) = 0 + \pi - \cos^{-1}(1) = 0 + \pi - 0 = \pi$.

Similarly for $\frac{1}{2} \le x^2 < \frac{3}{2}$, we have $y = \sin^{-1}(1) + \cos^{-1}(0) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$.

Hence the range of the given function is $[\pi]$.

61. Consider the real-valued function satisfying 2f(sin x) + f(cos x) = x. Find the domain and range of f(x).

Sol. Given 2f(sin x) + f(cos x) = x ...(1)

Replacing x by $\frac{\pi}{2} - x$, we have 2f(cos x) + f(sin x) $= \frac{\pi}{2} - x$ ...(2)

(1) + (2) $\Rightarrow 3f(\sin x) + 3f(\cos x) = \frac{\pi}{2} \Rightarrow f(\sin x) + f(\cos x) = \frac{\pi}{6}$ ...(3)

(1) – (3) $\Rightarrow f(\sin x) = x - \frac{\pi}{6} \Rightarrow f(x) = \sin^{-1} x - \frac{\pi}{6}$.

Hence $D_f = [-1,1]$ and $R_f = \left[-\frac{\pi}{2} - \frac{\pi}{6}, \frac{\pi}{2} - \frac{\pi}{6}\right] = \left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$.]

62. If $f(x) = x^2 + x + \frac{3}{4}$ and $g(x) = x^2 + ax + 1$ be two real functions, then the range of a for which $g(f(x)) = 0$ has no real solution is ______

A) $(-\infty, -2)$ B) $(-2, 2)$ C) $(-2, \infty)$ D) $(2, \infty)$

Key. C

Sol. $f(x) = x^2 + x + \frac{3}{4} = (x + \frac{1}{2})^2 + \frac{1}{2} \geq \frac{1}{2}$

$g(f(x)) = (f(x))^2 + af(x) + 1$, for $g(f(x)) = 0$ $a = -\left(f(x) + \frac{1}{f(x)}\right) \leq -2$

$\therefore$ If $a > -2$, $g(f(x)) = 0$ has no solutions

63. The number of integers in the domain of real function $f(x) = \log_{10} \sin(x-3) - \sqrt{16 - x^2}$ is

A) 4 B) 8 C) 9 D) infinite

Key. A

Sol. The domain of the given function is $(3-2\pi, 3-\pi) \cup (3,4]$. The integers in the domain are $\{-3, -2, -1, 4\}$

64. if $f(x)$ is a polynomial function such that $|f(x)| \leq 1 \forall x \in R$ and $g(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}$, then the range of $g(x)$ is

A) [0, 1] B) $\left[0, \frac{e^2+1}{e^2-1}\right]$

C) $\left[0, \frac{e^2-1}{e^2+1}\right]$ D) $\left[\frac{1-e^2}{1+e^2}, 0\right]$

Key. D

Sol. For $0 \leq f(x) < 1$ $g(x) = 0$

For $-1 < f(x) < 0$

$g(x) = \frac{e^{2f(x)} - 1}{e^{2f(x)} + 1} \Rightarrow g(x) \in \left[\frac{1-e^2}{1+e^2}, 0\right)$

$\therefore$ range of $g(x) = \left[\frac{1-e^2}{1+e^2}, 0\right]$

65. The domain of definition of the function, f(x) given by the equation $2^x + 2^y = 2$ is

(A) $0 < x \le 1$ (B) $0 \le x \le 1$ (C) $-\infty < x \le 0$ (D) $-\infty < x < 1$

Key. D

Sol. It is given that $2^x + 2^y = 2 \forall x,\ y \in R$

Therefore, $2^x = 2 - 2^y < 2 \Rightarrow 0 < 2^x < 2$

Taking log for both side with base 2.

$\Rightarrow \log_2 0 < \log_2 2^x < \log_2 2$

Hence domain is $-\infty < x < 1$

66. The range of the function f(x) = $\log_{\{x\}}$ [x], where [x] and {x} respectively denote the integral and fractional parts of x, is

(A) (-∞, 0] (B) (-∞, 0)

(C) {ln k; k∈N} (D) (-∞, -1)

Key. A

Sol. For x∈ (1, 2) , f(x) = 0

for x∈ (2, 3) , f(x) = $\log_{\{x\}}2 = \frac{1}{\log_2\{x\}}$ ∈ (-∞, 0)

67. If $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ then range of the function $f(x)$ is _______

(1) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (2) $[0, \pi]$ (3) $(0, \pi)$ (4) R

Key. 1

Sol. Domain [-1, 1] and Range [f(-1), f(1)]

68. Let R be set of real numbers, the function $f : R^- \to R,\ f(x) = \log_2 \log_2 \left|x + \sqrt{1+x^2}\right|$, then range of f(x) is

(1) ϕ (2) R (3) R+ (4) R–

Key. 1

Sol. for $x < 0$

$0 < x + \sqrt{1+x^2} < 1$

so $\log_2\left(x + \sqrt{x + x^2}\right) < 0$

69. If $e^x + e^{f(x)} = e$, then for f(x)

(1) domain $= (-\infty, 1)$, range $= (-\infty, 1)$

(2) domain $= (-\infty, -1]$, range $= (-\infty, 1]$

(3) domain $= (-\infty, 0]$, range $= (-\infty, 1]$

(4) domain $= (-\infty, 0]$, range $= (-\infty, 3]$

Key. 1

Sol. $e^{f(x)} = e - e^x \Rightarrow f(x) = \log_e(e - e^x)$

$e - e^x > 0 \Rightarrow e^1 > e^x \Rightarrow x < 1$

$D_f = (-\infty, 1)$

Let $y = f(x) = \log_e(e - e^x) \Rightarrow e^y = e - e^x$

$\Rightarrow \quad e^x = e - e^y \Rightarrow x = \log_e(e - e^y)$

$\Rightarrow \quad e - e^y > 0 \Rightarrow e^1 > e^y$

$\therefore \quad y < 1$

$R_f = (-\infty, 1)$

70. The domain of $f(x) = \sin\left(\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$ is

(1) $(0,5)$ (2) $(1,5)$ (3) $(-2,1)$ (4) $(2,3)$

Key. 3

Sol. $4 - x^2 > 0$ and $1 - x > 0$

$\therefore -2 < x < 2$ and $x < 1$

71. The range of the function $Y = [x^2] - [x]^2, x \in [0,2]$ where [.] denotes the integral part, is

(1) $\{0\}$ (2) $\{0,1\}$ (3) $\{1,2\}$ (4) $\{0,1,2\}$

Key. 4

Sol. We have, $y = [x^2] - [x]^2, \quad x \in [0,2]$

i.e., $y = [x^2], \qquad 0 \le x < 1$

$y = [x^2] - 1, \quad 1 \le x < 2$

$= [x^2] - 1, \quad x = 2$

$= 0 \qquad x = 2$

i.e., y = 0, $\quad 0 \le x < 1$

$= 1 - 1 = 0 \quad 1 \le x < \sqrt{2}$

$= 2 - 1 = 1, \quad \sqrt{2} \le x < \sqrt{3}$

$= 3 - 1 = 2, \quad \sqrt{3} \le x < 2$

$= 0 \qquad x = 2$

Hence, the range is {0, 1, 2 }

72. Let $f(x) = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$ then

(1) Greatest value of $f(x)$ is $\frac{5\pi^2}{8}$ (2) Greatest value of $f(x)$ is $\frac{7\pi^2}{4}$

(3) Least value of $f\left(x\right)$ is $\frac{\pi^2}{8}$ (4) Least value of $f\left(x\right)$ is $\frac{\pi^2}{12}$

Key. 3

Sol. $f\left(x\right)=2\left(\sin^{-1}x\right)^2-\pi\sin^{-1}x+\frac{\pi^2}{4}$

$$=2\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{8}$$

$$\Rightarrow f\left(x\right)\in\left[\frac{\pi^2}{8},\frac{5\pi^2}{4}\right]$$

73. If [a , b] be the range of $\frac{1}{\pi^2}\left((\cos^{-1}x)^2+(\sin^{-1}x)^2\right)$ then b – a =

A. 1 B. $\frac{9}{8}$ C. $\frac{3}{4}$ D. $\frac{5}{4}$

Key. B

Sol. $(\cos^{-1}x)^2+(\sin^{-1}x)^2=\frac{1}{2}\left\{(\cos^{-1}x+\sin^{-1}x)^2+(\cos^{-1}x-\sin^{-1}x)^2\right\}$

$$=\frac{1}{2}\left\{\left(\frac{\pi}{2}\right)^2+\left(\frac{\pi}{2}-2\sin^{-1}x\right)^2\right\}\geq\frac{\pi^2}{8}$$

$$a=\frac{1}{\pi^2}\left(\frac{\pi^2}{8}\right)=\frac{1}{8}$$

$$b=\frac{1}{2\pi^2}\left\{\frac{\pi^2}{4}+\left(\frac{\pi}{2}+\pi\right)^2\right\}, at\ x=\frac{-\pi}{2}$$

$$=\frac{5}{4}$$

$$\therefore b-a=\frac{9}{8}$$

74. The domain of the function $\sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is

A. (0,5) B. (1,4) C. [0,5] D. [1,4]

Key. D

Sol. $\frac{5x-x^2}{4}\geq1\Rightarrow x^2-5x+4\leq0\Rightarrow x\in[1,4]$

75. The greatest and least values of $(\sin^{-1}x)^3+(\cos^{-1}x)^3$ are

A. $\frac{\pi^3}{32}, \frac{7\pi^3}{32}$ B. $\frac{7\pi^3}{8}, \frac{\pi^3}{32}$ C. $\frac{-\pi^3}{8}, \frac{7\pi^3}{8}$ D. $\frac{\pi^3}{8}, \frac{\pi^3}{32}$

KEY. B

SOL. $(\sin^{-1} x)^3 + (\cos^{-1} x)^3 = \left(\frac{\pi}{2}\right)^3 - 3\sin^{-1} x \cos^{-1} x \left(\frac{\pi}{2}\right)$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)$$

$$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left\{\sin^{-1} x - \frac{\pi}{4}\right\}^2$$

$$\min = \frac{\pi^3}{32}, \max = \frac{7\pi^3}{8}$$

3.Odd & Even Functions

76. Let $f(x) = e^x + \sin x$ be defined on the interval [-4, 0], the odd extension of $f(x)$ in the interval [-4, 4]

1) $e^{-x} + \sin x,\ x \in (0,4)$ 2) $-e^{-x} + \sin x,\ x \in (0,4)$
3) $e^{-x} - \sin x,\ x \in (0,4)$ 4) $-e^{-x} - \sin x,\ x \in (0,4)$

Key. 2

Sol. $f(x) = -f(-x)$

77. The function $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x+21\pi}{\pi}\right] - 41}$ is (where [.] = G.I.F)

A) An odd function. B) An even function
C) Neither even nor odd function D) None of these

Key. A

Sol. The denominator is $= 2\left[\dfrac{x+21\pi}{\pi}\right] - 41 = 2\left[\dfrac{x}{\pi} + 21\right] - 41$

.

$$\therefore f(x) = \frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi}\right] + \frac{1}{2}}$$

$$\Rightarrow f(-x) = \frac{-x\{\sin(-x) + \tan(-x)\}}{\left[-\frac{x}{\pi}\right] + \frac{1}{2}} = \frac{x(\sin x + \tan x)}{-1 - \left[\frac{x}{\pi}\right] + \frac{1}{2}} \ (if\ \ x \neq n\pi)$$

$$= -\frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} = -f(x).\left(if \quad x = n\pi \quad then \quad f(x) = 0\right)$$

78. If $f:[-20,20] \to R$ defined by $f(x) = \left[\frac{\pi^2}{a}\right]\sin x + \cos x$ is an even function, then the set of values of 'a' is [.] G.I.F

A) $a \in (500, \infty)$
B) $a \in (400, \infty)$
C) $a \in (-400, 0)$
D) $a \in (-300, 900)$

Key. B

Sol. If $f:[-20,20] \to R$ defined by $f(x) = \left[\frac{x^2}{a}\right]\sin x + \cos x$ is an even function , find the set of values of a; [.] = G. I. F.

[Hint: If f(x) is an even function, then $f(x) = f(-x) \Rightarrow f(x) - f(-x) = 0 \Rightarrow \left[\frac{x^2}{a}\right]\sin x = 0$

$\Rightarrow \left[\frac{x^2}{a}\right] = a \Rightarrow 0 \le \frac{x^2}{a} < 1 \Rightarrow a > x^2 \Rightarrow a > 400$ (since $-20 \le x \le 20$) $\therefore a \in (400, \infty)$.]

79. If $f:[-4,4] - \{-\pi, o, \pi\} \to R$, such that $f(x) = \cot(\sin x) + \left[\frac{x^2}{|a|}\right] + \frac{\sin 2x}{x^2}$,([.] denotes greatest integer function) is an odd function, then the complete set of values of 'a' , is

A) $[-\infty, -4] \cup [4, \infty)$
B) $(-\infty, -16) \cup (16, \infty)$
C) $[-16, -16]$
D) $(-\infty, -16] \cup [16, \infty)$

Key. B

Sol. For $f(x)$ to be odd, $\left[\frac{x^2}{|a|}\right]$ should not depend on value of x.

Since $x \in [-4,4] \Rightarrow 0 \le x^2 \le 16$

$\Rightarrow \left[\frac{x^2}{|a|}\right] = 0$ if $|a| > 16 \Rightarrow a \in (-\infty, -16) \cup (16, \infty)$

4. Periodic Functions

80. If $f: R \to R$ is a function satisfying the property $f(x+1) + f(x+3) = 2$ for all $x \in R$ than f is _____

(1) Periodic with period 3
(2) Periodic with period 4
(3) non periodic
(4) Periodic with period 5

Key. 2

Sol. $f(x+1) = f(x+5)$

81. A real valued function f satisfies $f(10+x) = f(10-x)$ and $f(20-x) = -f(20+x)$, for all $x \in R$ which of the following statements is true?

(1) f is an even function (2) f is an odd function

(3) f is a constant function (4) f is a non-periodic function

Key. 2

Sol. Change x to 10 – x to obtain

$f(20-x)=f(x)$

We have $f(20-x)=-f(20+x)$

$\Rightarrow f(x)=-f(20+x)$

Now change x to 20 + x

$f(20+x)=-f(40+x)$

$-f(x)=-f(40+x)$

$f(x)=f(40+x)$, so f is periodic

Again $f(-x)=-f(20-x)=-f(x)$

Thus f is odd

82. The period of the function $f(x)=\frac{1}{2}\left(\frac{|\sin x|}{\cos x}+\frac{\sin x}{|\cos x|}\right)$ is

(A) π (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

Key. B

Sol. Since $|\sin x|$ and $\cos x$ are periodic function with period π and 2π respectively.

Therefore, $\frac{|\sin x|}{\cos x}$ is periodic with period 2π.

Similarly, $\frac{|\cos x|}{\sin x}$ is periodic with period 2π.

So, period of f(x) is L.C.M. of $\{2\pi,\ 2\pi\}=2\pi$.

83. Let $f:R\to R-\{3\}$ be a function such that for some $p>0$, $f(x+p)=\frac{f(x)-5}{f(x)-3}$ for all $x\in R$.

Then, period of f is

(A) $2p$ (B) $3p$ (C) $4p$ (D) $5p$

Key. C

Sol. 3 does not belong to the range of *f* implies 2 also cannot belong to range of *f* because, if $f(x)=2$ for some $x\in R$. Then $f(x+p)=\frac{2-5}{2-3}=3$ which is not in the range of *f*. Hence 2 and 3 are not in the range of *f*. If $f(x+2p)=f(x)$, this implies

$$f(x)=f(x+p+p)$$
$$=\frac{f(x+p)-5}{f(x+p)-3}$$

$$=\frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}$$

$$=\frac{-4f(x)+10}{-2f(x)+4}=\frac{2f(x)-5}{f(x)-2}$$

so that $\left[f(x)-2\right]^2=-1$ which is absurd. Therefore, $2p$ is not a period. Again

$$f(x+3p)=\frac{2f(x+p)-5}{f(x+p)-2}$$

$$=\frac{3f(x)-5}{f(x)-1}\neq f(x).$$

Now $f(x+4p)=f(x+3p+p)$

$$=\frac{f(x+3p)-5}{f(x+3p)-3}$$

$$=\frac{\frac{3f(x)-5}{f(x)-1}-5}{\frac{3f(x)-5}{f(x)-1}-3}$$

$$=\frac{-2f(x)}{-2}=f(x).$$

Therefore $4p$ is a period.

84. Period of the function f(x) = [x] + [2x] + [3x] + [4x] + … + [nx] – $\frac{n(n+1)x}{2}$, where $n \in N$ and [ ] denotes the greatest integer function, is

(A) 1 (B) n (C) $\frac{1}{n}$ (D) 2n

Key. A

Sol. $f(x)=[x]+[2x]+.....+[nx]-(x+2x+.....+nx) \quad =[x]-x+[2x]-2x+....+[nx]-(nx)$

$=-[\{x\}+\{2x\}+.....+\{nx\}]$

Period of $\{rx\}$ = $\frac{1}{r}$

$\therefore$ Period of f(x) = $\text{LCM}\left(1,\frac{1}{2},\frac{1}{3},......,\frac{1}{n}\right)=1$

85. Which of the following is non-periodic

(A) $\frac{\tan x}{\tan x}$ (B) $\sin\sqrt{x}$ (C) $\cos|x|$ (D) $\frac{\sin x}{\sin x}$

Key. B

Sol. $f(x)=\sin\sqrt{x}$ is non-periodic because f(T) = f(0) = f(–T) is not satisfied.

86. If $f(2+x)=a+\left[1-(f(x)-a)^4\right]^{1/4}$ for all $x\in R$, then f(x) is periodic with period

(A) 1 (B)2 (C) 4 (D) 8

Key. C

Sol. $f(2+x)-a = \{1-[f(x)-a]^4\}^{1/4}$

$\Rightarrow$ $[f(2+x)-a]^4 = 1-[f(x)-a]^4$

$[f(2+x)-a]^4+[f(x)-a]^4=1$ …(i)

(i) is true for all x

Replace x by (x + 2) in (i)

$[f(x+4)-a]^4+[f(x+2)-a]^4=1$ …(ii)

(i) and (ii) gives, $f(x)-a]^4=[f(x+4)-a]^4$

$\Rightarrow$ $f(x+4)-a=f(x)-a$

$\Rightarrow$ $f(x+4)=f(x)$

87. Period of f(x) = sgn([x] + [–x]) is equal to
(where [.] denotes greatest integer function)

(A) 1 (B) 2
(C) 3 (D) does not exist

Key. A

Sol. Let f(x) = sgn([x] + [–x])

$$=\begin{cases}0; & x\in I\\ -1; & x\notin I\end{cases}$$

Hence f(x) is periodic with period 1.

88. The period of the function

$$f(x)=\exp\left[x-[x]+\sqrt{x-[x]}+(x-[x])^2\right.$$
$$\left.+|\sin\pi x|+|\cos\pi x|+|\tan\pi x|\right]$$

A) 1 B) 2 C) 3 D) 4

Key. A

Sol. The period of x – [x] is 1.

The period of $\sqrt{x-[x]}$ is 1.

The period of $(x-[x])^2$ is 1.

The period of $|\sin\pi x|=\frac{\pi}{\pi}=1$.

The period of $|\cos\pi x|=1$.

The period of $|\tan\pi x|=1$.

Thus each of the above functions is a periodic function with period 1. Therefore their L.C.M. is 1. Hence the function f(x) is periodic with fundamental period = 1.

89. The period of the function $f(x)=\sin 3x\cos[3x]-\cos 3x\sin[3x]$, where $[\cdot]$ denotes the greatest integer function is

(1) 6 (2) 3 (3) 1/3 (4) 1/6

Key. 3

Sol. f(x) = sin 3{x}, where {.} is a fractional part function.

90. If $f(2+x)=a+\left[1-(f(x)-a)^4\right]^{1/4}$ for all $x\in R$, then f(x) is periodic with period

(A) 1 (B) 2 (C) 4 (D) 8

Key. C

Sol. $f(2+x)-a = \{1-[f(x)-a]^4\}^{1/4}$

$\Rightarrow$ $[f(2+x)-a]^4 = 1-[f(x)-a]^4$

$[f(2+x)-a]^4+[f(x)-a]^4=1$ …(i)

(i) is true for all x

Replace x by (x + 2) in (i)

$[f(x+4)-a]^4+[f(x+2)-a]^4=1$ …(ii)

(i) and (ii) gives, $f(x)-a]^4=[f(x+4)-a]^4$

$\Rightarrow$ $f(x+4)-a=f(x)-a$

$\Rightarrow$ $f(x+4)=f(x)$

91. The period of the function $f(x)=(-1)^{[x]}$ where [.] = G.I.F

A) 2 B) 1 C) 3 D) 4

Key. A

Sol. Given: $f(x)=(-1)^{[x]}$.

First of all, we sketch the graph of f(x) with the help of piecewise defined functions as follows:

$$f(x)=(-1)^{[x]}=\begin{cases}1; & -2x<-1\\ -1; & -1x<0\\ 1; & 0\le x<1\\ -1: & 1\le x<2\\ 1; & 2\le x<3.\end{cases}$$

The graph of f(x) is given by

From the above graph of f(x), we see that the function f(x) repeats its value after the least interval of 2. Therefore the function f(x) is periodic with period 2.

92. If $2f(x)+3.f\left(\frac{1}{x}\right)=x^2-1$ then f(x) is _______

(1) Periodic function (2) an even function

(3) an odd function (4) one one function on domain R

Key. 2

Sol. replace x by $\frac{1}{x}$

5. Inverse, Composition Functions

93. If the functions of *f* and *g* are defined by $f(x)=3-x, g(x)=2+3x$ *for* $x\in R$ respectively, then $g^{-1}(f^{-1}(2))=$

A. 1 B. $-\frac{1}{3}$ C. $-\frac{4}{3}$ D. $\frac{1}{4}$

Key. B

Sol. $f^{-1}(x)=3-x, g^{-1}(x)=\frac{x-2}{3}$

$g^{-1}[f^{-1}(2)]=g^{-1}(1)=-\frac{1}{3}$

94. Which among the functions is inverse of itself?

(A) $y=a^{2\log x}$ (B) $y=5^{x^2+2}$ (C) $y=\frac{1+x^2}{1-x^2}$ (D) $y=\frac{1-x}{1+x}$

Key. D

Sol. Out of 4 choices, if f(x) = $\frac{1-x}{1+x}$.

f[f(x)] = $\frac{1-\frac{(1-x)}{(1+x)}}{1+\frac{(1-x)}{(1+x)}}$ = x

$\therefore$ $\frac{1-x}{1+x}$ is the inverse of itself.

95. The inverse of $f(x)=\left(5-(x-8)^5\right)^{\frac{1}{3}}$ is

(A) $5-(x-8)^5$ (B) $8+(5-x^3)^{1/5}$

(C) $8-(5-x^3)^{1/5}$ (D) $\left(5-(x-8)^{1/5}\right)^3$

Key. B

Sol. Let y = f(x) = $\left(5-(x-8)^5\right)^{1/3}$, then

y^3 = $5-(x-8)^5$ $\Rightarrow$ $(x-8)^5=5-y^3$

$\Rightarrow$ x = 8 + $(5-y^3)^{1/5}$

Let, z = $g(x)=8+(5-x^3)^{1/5}$

Now, $f\left(g(x)\right)$ = $\left[5-(x-8)^5\right]^{1/3}$

= $\left(5-\left[(5-x^3)^{1/5}\right]^5\right)^{1/3}$ = $(5-5+x^3)^{1/3}=x$

Similarly, we can show that $g(f(x)) = x$.

Hence, g(x) = 8 + $(5-x^3)^{1/5}$ is the inverse of f(x).

96. If $f(x) = x - x^2 + x^3 - x^4 + ...\infty$ when $|x| < 1$ then $f^{-1}(x) =$

1) $\frac{x}{1-x}$ 2) $\frac{x}{1+x}$ 3) $\frac{1}{1-x}$ 4) $\frac{1}{1+x}$

Key. 1

Sol. $f(x) = x - x^2 + x^3 - x^4 + ... = \frac{x}{1+x}$

$$f^{-1}(x) = t \Rightarrow f(t) = \frac{t}{1+t} \Rightarrow x + xt = t \Rightarrow x = t(1-x) \Rightarrow t = \frac{x}{1-x} \Rightarrow f^{-1}(x) = \frac{x}{1-x}$$

97. If $f:(0,\infty) \to R$ defined by $f(x) = \log_{10}^{x}$ then $f^{-1}(x) =$

1) $\log_x^{10}$ 2) x^{10} 3) 10^x 4) None

Key. 3

Sol. $f^{-1}(x) = y \Rightarrow x = f(y) \Rightarrow x = \log_{10}^{y} \Rightarrow y = 10^x \Rightarrow f^{-1}(x) = 10^x$

98. If $f(x) = (1-x^n)^{1/n}, 0 < x < 1, n$ being an odd positive integer and $h(x) = f(f(x)),$ then $h'(1/2)$

A. 2^n B. 2 C. $n.2^{n-1}$ D. 1

Key. D

Sol. $h(x) = (1 - f(x)^n)^{\frac{1}{n}} = (1-(1-x^n)^{\frac{1}{n}} = x \quad \therefore h^1(\frac{1}{2}) = 1$

99. If $f(x) = x - \frac{1}{x}$ then number of solutions of $f(f(f(x)) = 1.$

1) 1 2) 4 3) 6 4) 2

Key. 2

Sol. $f(x) = x - \frac{1}{x}, \quad \Rightarrow f(f(x)) = \frac{x^4 - 3x^2 + 1}{x(x^2-1)}$

$\Rightarrow f(f(f(x))) = 1 \Rightarrow f(f(x)) = f^{-1}(1) = \frac{1+\sqrt{5}}{2}$ $\to$ 2 values exist

Or $f^{-1}(1) = \frac{1-\sqrt{5}}{2}$ $\to$ 2 values exist

100. Which among the functions is inverse of itself?

(A) $y = a^{2\log x}$ (B) $y = 5^{x^2+2}$ (C) $y = \frac{1+x^2}{1-x^2}$ (D) $y = \frac{1-x}{1+x}$

Key. D

Sol. Out of 4 choices, if f(x) = $\frac{1-x}{1+x}$.

$$f[f(x)] = \frac{1-\frac{(1-x)}{(1+x)}}{1+\frac{(1-x)}{(1+x)}} = x$$

$\therefore$ $\frac{1-x}{1+x}$ is the inverse of itself.

101. If $f(x) = x(x-1)$ is a function from $\left[\frac{1}{2},\infty\right)$ to $\left[-\frac{1}{4},\infty\right)$, then $\{x \in R / f^{-1}(x) = f(x)\}$ is

(A) null set (B) {1}

(C) {0, 2} (D) a set containing 3 elements

Key. C

Sol. $\{x \in R / f^{-1}(x) = f(x)\} = \{x \in R / f\,f(x) = x\}$

$f\left(f(x)\right) = f\left(x(x-1)\right) = [x(x-1)][x(x-1)-1] = x(x-1)[x^2 - x - 1]$

$f(f(x)) = x \Rightarrow x(x-1)(x^2-x-1) = x$

$\Rightarrow x(x^3 - 2x^2) = 0 \Rightarrow x = 0, 2$

102. Let $f(x) = 3x^2 - 7x + c$, where 'c' is a variable co-efficient and $x > \frac{7}{6}$. The value of 'c' such that $f(x)$ touches $f^{-1}(x)$ is……

(A) 6 (B) 7 (C) $\frac{16}{3}$ (D) $\frac{4}{3}$

Key. C

Sol. $f(x)$ and $f^{-1}(x)$ can only intersect on the line $y = x$

$\therefore y = x$ must be tangent

Solving $3x^2 - 7x + c = x$ p

$\Rightarrow 3x^2 - 8x + c = 0$

The above equation has real and equal roots

$\Rightarrow 64 - 12c = 0$

$c = \frac{16}{3}$

103. Let $f : \left[\frac{-\pi}{3}, \frac{2\pi}{3}\right] \to [0,4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$ then $f^{-1}(x)$ is given by

(1) $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{x+2}{2}\right) + \frac{\pi}{6}$

(3) $\frac{2\pi}{3} - \cos^{-1}\left(\frac{x-2}{2}\right)$ (4) Does not exist

Key. 3

Sol. $f(x)=2\sin\left(x-\frac{\pi}{6}\right)+2$

Since f is one – one onto

f is invertible

Now $fof^{-1}(x)=x$

$\Rightarrow 2\sin\left(f^{-1}(x)-\frac{\pi}{6}\right)+2=x$

$f^{-1}(x)=\sin^{-1}\left(\frac{x}{2}-1\right)+\frac{\pi}{6}$ $\left(\because\left|\frac{x}{2}-1\right|\le 1\forall x\in[0,4]\right)$

Also using $\sin^{-1}\alpha+\cos^{-1}\alpha=\frac{\pi}{2}$

$f^{-1}(x)=\frac{\pi}{2}-\cos^{-1}\frac{x-2}{2}+\frac{\pi}{6}=\frac{2\pi}{3}-\cos^{-1}\left(\frac{x-2}{2}\right)$

104. The value of the parameter α, for which the function $f(x)=1+\alpha x,\ \alpha\neq 0$ is the inverse of itself, is

(A) –2 (B) –1 (C) 1 (D) 2

Key. B

Sol. $y = 1+\alpha x \Rightarrow x=\frac{y-1}{\alpha}$

$f^{-1}(x)=\frac{x-1}{\alpha}=f(x)=1+\alpha x$

$\Rightarrow \frac{x-1}{\alpha}=1+\alpha x \quad \Rightarrow x-1=\alpha+\alpha^2 x$

Equating the coefficient of x

$\alpha^2=1$ and $\alpha=-1$

$\alpha=\pm 1$

$\alpha=-1$

6. Functional Equations

105. If f is real function satisfying the relation $f(x+y)=f(x).f(y)$ for all $x,y\in R$ and f(1) = 2 and $a\in N$, for which $\sum_{K=1}^{n} f(a+k)=16(2^n-1)$ then a = _______

(1) 2 (2) 4 (3) 3 (4) 8

Key. 3

Sol. $f(a)=a^n$; $\sum f(a+K)=\sum f(a)f(K)$

$=2^a\sum_{K=1}^{n}2^K=2^a(2^n-1)$

$\therefore a=3$

106. A real valued function f(x) satisfies the functional equation

$f(x-y)=f(x).f(y)-f(a-x)f(a+y)$ for some given constant a and f(0) = 1 then f(2a-x) =

(1) $f(x)$ (2) $-f(x)$ (3) $f(-x)$ (4) $f(a)+f(a-x)$

Key. 2

Sol. Put $x=y=0 \Rightarrow f(a)=0$

$$f(a-x)=f(a-(x-a))=f(a).f(x-a)-f(a-a)$$

107. If $f:R\to R$ is a function satisfying $f(x+y)=f(xy)$ for all $x,y\in R$ and $f\left(\frac{3}{4}\right)=\left(\frac{3}{4}\right)$, then $f\left(\frac{9}{16}\right)=$

1) $\frac{3}{4}$ 2) $\frac{9}{16}$ 3) $\frac{\sqrt{3}}{2}$ 4) 0

Key. 1

Sol. Let f(0)=k, then f(x) = f(x+0) = f(0) = k, f is a constant function. But $f\left(\frac{3}{4}\right)=\left(\frac{3}{4}\right)$

$\therefore f(x)=\left(\frac{3}{4}\right)$ for all x and hence $f\left(\frac{9}{16}\right)=\left(\frac{3}{4}\right)$

108. If for nonzero x, $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$, then $f(x^2)=$

1) $\frac{3+2x^4-x^2}{5x^2}$ 2) $\frac{3-2x^4+x^2}{5x^2}$ 3) $\frac{3-2x^4-x^2}{5x^2}$ 4) $\frac{3+2x^4+x^2}{5x^2}$

Key. 3

Sol. $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1\Rightarrow 4f(x^2)+6f\left(\frac{1}{x^2}\right)=2x^2-2$ ....(1)

$2f\left(\frac{1}{x^2}\right)+3f(x^2)=\frac{1}{x^2}-1\Rightarrow 9f(x^2)+6f\left(\frac{1}{x^2}\right)=\frac{3}{x^2}-3$ ....(2)

$(2)-(1)\Rightarrow 5f(x^2)=\frac{3}{x^2}-2x^2-1\Rightarrow f(x^2)=\frac{3-2x^4-x^2}{5x^2}$

109. If $f:\mathbf{R}\to\mathbf{R}$ satisfies $f(x+y)=f(x)+f(y)$ for all $x,y\in\mathbf{R}$ and f (1) =7, then $\sum_{r=1}^{n} f(r)$ is

1) $\frac{7(n+1)}{2}$ 2) $\frac{7n(n+1)}{2}$ 3) $\frac{7n}{2}$ 4) $7n(n+1)$

Key. 2

Sol. $f(1)=7, f(2)=f(1+1)=f(1)+f(1)=2f(1), f(n)=nf(1)$

110. If f is a real valued function satisfying $f(x)+f(x+6)=f(x+3)+f(x+9)$, then $f(x)=$

1) $f(x+3)$ 2) $f(x+6)$ 3) $f(x+9)$ 4) $f(x+12)$

Key. 4

Sol. Replace x with x+3

111. If $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)\forall x\in R-\{0\}$, where f(x) be a polynomial function and f(5) = 126, then f(3) =

(A) 28 (B)26 (C) 27 (D) 25

Key. A

Sol. $f(x) = 1 \pm x^n$ or $f(5) = 1 \pm 5^n$

or, $126 = 1 \pm 5^n$ or $\pm 5^n = 125 \Rightarrow \pm 5^n = 5^3$

$n = 3$

$f(3) = 1 + 3^3 = 28$

112. If g(x) is a polynomial satisfying g(x) g(y) = g(x) + g(y) + g(xy) –2 for all real x and y and g(2) = 5, then g(3) is equal to

(A) 10 (B) 24

(C) 21 (D) 15

Key. A

Sol. Putting x = 1, y = 2, then

g(1) g(2) = g(1) + g(2) + g(2) – 2

$\Rightarrow$ 5g(1) = 8 + g(1)

$\therefore$ g(1) = 2

Also, replacing y by $\frac{1}{x}$ in the given relation, then

$$g(x)g\left(\frac{1}{x}\right)=g(x)+g\left(\frac{1}{x}\right)+g(1)-2$$

or $g(x)g\left(\frac{1}{x}\right)=g(x)+g\left(\frac{1}{x}\right)$

$\Rightarrow$ $g(x)=1\pm x^n$

$\Rightarrow$ $\pm 2^n = 2^2$

Taking +ve sign

$2^n = 2^2$

$\therefore$ n = 2

$\Rightarrow$ $g(x) = 1 + x^2$

$\therefore$ $g(3) = 1 + 3^2 = 10$

113. Let $f\left(\frac{x+y}{2}\right)=\frac{1}{2}(f(x)+f(y))$ for real x and y. If $f'(0)$ exists and equals to -1 and f(0)=1 then the value of f(2) is

a) 1 b) -1 c) $\frac{1}{2}$ d) 2

Key. B

Sol.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{f(2x)+f(2h)}{2}-f(x)}{h}$$

$f'(x)=-1$; $f(2x)=2f(x)\text{-}1$

$\Rightarrow f(x)=1-x$

114. A function f : R → R satisfies the equation f(x) f(y) – f(xy) = x + y ∀ x, y ∈ R and f(1) > 0, then

(A) $f(x)\ f^{-1}(x)=x^2-4$ (B) $f(x)\ f^{-1}(x)=x^2-6$

(C) $f(x)\ f^{-1}(x)=x^2-1$ (D) none of these

Key. C

Sol. Taking x = y = 1, we get

$f(1)f(1)-f(1)=2$

$\Rightarrow f^2(1)-f(1)-2=0 \Rightarrow (f(1)-2)(f(1)+1)=0$

$\Rightarrow f(1)=2$ (as f(1) > 0)

Taking y = 1, we get

f(x). f(1) – f(x) = x + 1

$\Rightarrow$ f(x) = x + 1 $\Rightarrow f^{-1}(x)=x-1$

$\therefore\ f(x).f^{-1}(x)=x^2-1$

∴ (C) is the correct answer.

115. A function f satisfies the equation $3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30,\ (x\neq 1)$

then the value of $\frac{f(11)}{f(7)}$ is

A) 7 B) 11 C) -7 D) -11

Key. B

Sol. At $x=11$

$3f(11)+2f(7)=140$.(1)

but $x=7$ to get

$3f(7)+2f(11)=100$ ..(2)

$\frac{(1)}{(2)}\Rightarrow\frac{3f(1)+2f(7)}{3f(7)+2f(11)}=\frac{7}{5}$

Using componendo and dividendo

$\frac{5(f(11)+f(7))}{(f(11)-f(7))}=\frac{6}{1} \Rightarrow \frac{(f(11)+f(7))}{(f(11)-f(7))}=\frac{6}{5}$

$\frac{f(11)}{f(7)}=11$

116. Let f be a real-valued function with domain R. If for some positive constant a, the equation

$$f(x + a) = 1 + (1 - 3f(x) + 3(f(x))^2 - (f(x))^3\}^{1/3}$$

holds good for all $x \in R$, prove that f(x) is a periodic function with period 2a.

Sol. Given $f(x+a)=1+\left\{1-3f(x)+3(f(x))^2-(f(x))^3\right\}^{1/3}$

$\Rightarrow \quad f(x+a)-1=\left\{(1-f(x))^3\right\}^{1/3} \Rightarrow \{f(x+a)-1\}^3=\{1-f(x)\}^3$

$\Rightarrow$ f(x + a) – 1 = 1 – f(x) f(x + a) + f(x) = 2 ...(1)

Replacing x by x – a, the equation (1) becomes f(x) + f(x – a) = 2 ...(2)

Subtracting (2) from (1), we get f(x + a) – f(x – a) = 0.

Finally replacing x by x + a, we get f(x + 2a) – f(x) = 0

$\Rightarrow$ f(x + 2a) = f(x) and hence f is periodic with period 2a.

117. Let $f: R \to R$ be continuous and periodic with period T > 0. then

(A) $f(x_0+T/2)=f(x_0)$ for some $x_0 \in [k, k+T/2], K \in R$

(B) $f(x_0+T/2)=f(x_0)$ for some $x_0 \in (k, k+T/4), K \in R$

(C) $f(x_0+T/2)=f(x_0)$ for some $x_0 \in (k, k+T/3), K \in R$

(D) $f(x_0+T/2)=f(x_0)$ for some $x_0 \in (k, k+T/6), K \in R$

Key. A

Sol. Let $g(x)=f(x+T/2)-f(x)$

then $g(k)=f(k+T/2)-f(k)$ ….. (1)

and $g(k+T/2)=f(k+T)-f(k+T/2)$

$=f(k)-f(k+T/2)$

$=-g(k)$

Hence by intermediate value property there exist an $x_0 \in [k, k+T/2]$ for which g(x) = 0

118. A function f : R $\to$ R satisfies the equation f(x) f(y) – f(xy) = x + y $\forall$ x, y $\in$ R and f(1) > 0, then

(A) $f(x)\, f^{-1}(x)=x^2-4$ (B) $f(x)\, f^{-1}(x)=x^2-6$

(C) $f(x)\, f^{-1}(x)=x^2-1$ (D) none of these

Key: C

Hint: Taking x = y = 1, we get

$f(1)f(1)-f(1)=2$

$\Rightarrow f^2(1)-f(1)-2=0 \Rightarrow (f(1)-2)(f(1)+1)=0$

$\Rightarrow \ f(1) = 2$ (as f(1) > 0)

Taking y = 1, we get

f(x). f(1) – f(x) = x + 1

$\Rightarrow$ f(x) = x + 1 $\Rightarrow \ f^{-1}(x) = x - 1$

$\therefore \ f(x).f^{-1}(x) = x^2 - 1$

$\therefore$ (C) is the correct answer.

119. Let f be a function such that $f(x + f(y)) = f(x) + y;\ \forall x, y \in R$ then $f(2013) =$ ___

(1) 0 (2) 1 (3) 2013 (4) 4026

Key. 3

Sol. Put $y = x \Rightarrow f(x + f(x)) = f(x) + x$

$\Rightarrow f(t) = t$ (Identity function)

120. If f(x) is a polynomial function satisfying the condition f(x).f(1/x) = f(x) + f(1/x), x$\in$R-{0} and f(2) = 9 then

(1) 2 f(4) = 3 f(6) (2) 7 f(1) = f(3) 3) 9 f(3) = 2 f(5) (4) f(10) = f(11)

Key. 3

Sol. f(x) = 1 + x^n put x = 2, we get n = 3

$\therefore$ f(x) = 1 + x^3

$\therefore$ 2 f(4) = 130 $\neq$ 3 f(6)

14 f(1) = 28 = 3 f(3)

9 f(3) = 252 = 2 f(5)

f(10) $\neq$ f(11)

121. If $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \forall x \in R - \{0\}$, where f(x) be a polynomial function and f(5) = 126, then f(3) =

(A) 28 (B)26 (C) 27 (D) 25

Key. A

Sol. f(x) = 1 $\pm$ x^n or f(5) = 1 $\pm$ 5^n

or, 126 = 1 $\pm$ 5^n or $\pm$ 5^n = 125 $\Rightarrow$ $\pm$ $5^n = 5^3$

n = 3

f(3) = 1 + 3^3 = 28

7. Different Types of Functions

122. Let $f(x) = a_1 \tan x + a_2 \tan\left(\frac{x}{2}\right) + a_3 \tan\left(\frac{x}{3}\right) + - - -$

$+ a_n.\tan\left(\frac{x}{n}\right)$ $where\, a_1, a_2, a_3 - - - a_n$ are real numbers and

$n \in Z^{+}, \left|f(x)\right| \le \left|\tan x\right|$ for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ then $\left|a_1 + \frac{a_2}{2} + \frac{a_3}{3} + ---- + \frac{a_n}{n}\right|$ is

(A) 1 (B) ≤ 1 (C) > 1 (D) $\ge \frac{1}{2}$

Key. B

Sol. Clearly $f^1(0)$ is required $\left|f^1(0)\right| = \left|\underset{n\to 0}{\text{lt}} \frac{f(h)}{h}\right|$

$= \underset{h\to 0}{\text{lt}} \frac{|f(h)|}{|h|} \le \underset{h\to 0}{\text{lt}} \left|\frac{\tan h}{h}\right| = 1$

123. If $[x]$ denotes the integral part of x. for real x. then the value of

$\left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{200}\right] + \left[\frac{1}{4} + \frac{1}{100}\right] + \left[\frac{1}{4} + \frac{3}{200}\right] + \_\_\_\_\_\_ + \left[\frac{1}{4} + \frac{199}{200}\right]$

1) 50 2) 100 3) 25 4) 75

Key. 1

Sol. $\left[200.\frac{1}{4}\right] = [50] = 50$

124. If $g = \{(1,1),(2,3),(3,5),(4,7)\}$ is described by the formula $g(x) = \alpha x + \beta$, then $(\alpha, \beta) =$

1) (2, 1) 2) (2,-1) 3) (-2, 1) 4) (-2,-1)

Key. 2

Sol. $g(1) = \alpha + \beta = 1$

$g(2) = 2\alpha + \beta = 3$

125. If $f(x) = \cos\left[\pi^2\right]x + \cos\left[-\pi^2\right]x$, then (where $[\alpha]$ is integral part of α)

1) $f\left(\frac{\pi}{2}\right) = -1$ 2) $f(\pi) = 1$ 3) $f(-\pi) = 1$ 4) $f\left(\frac{\pi}{4}\right) = 2$

Key. 1

Sol. $f(x) = \cos 9x + \cos 10x, \because 9 < \pi^2 < 10$

126. Set A has 3 elements and set B has 4 elements. The number of injections that can be defined from A to B is

1) 144 2) 12 3) 24 4) 64

Key. 3

Sol. ${}^{n(B)}P_{n(A)} = {}^4P_3 = 4.3.2 = 24$

127. $f : \mathbf{N} \to \mathbf{Z}$ is defined by $f(n) = \begin{cases} 2, if\ n = 3k, k \in Z \\ 10 - n, if\ n = 3k+1, k \in \mathbf{Z}. Then \{n \mid f(n) > 2\} = \\ 0, if\ n = 3k+2, k \in \mathbf{Z} \end{cases}$

1) $\{3,6,3\}$ 2) $\{1,4,7\}$ 3) $\{4,7\}$ 4) $\{7\}$

Key. 2

Sol. $\{n \backslash (f(n) > 2)\} = \{n \backslash 10 - n > 2, n = 3k+1\}$

$$= \{n \backslash n < 8, n = 3k+1\}$$

128. Let $f_1(x) = \begin{cases} x, 0 \le x \le 1 \\ 1, \quad x > 1 \\ 0, \text{ otherwise} \end{cases}$ and $f_2(x) = f_1(-x)$ for all x

$f_3(x) = -f_2(x)$ for all x

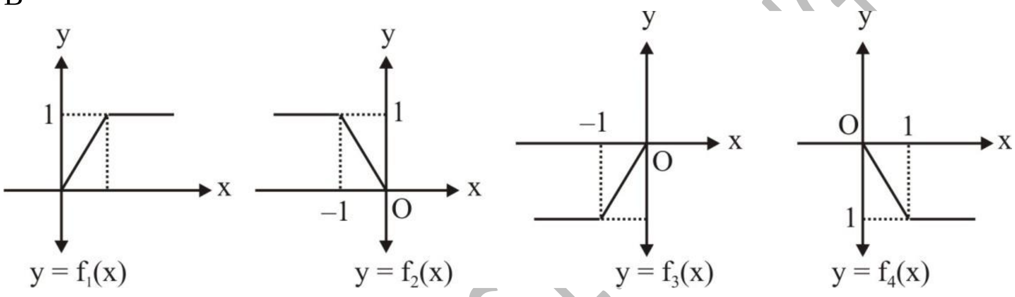
$f_4(x) = f_3(-x)$ for all x

Which of the following is necessarily true?

(A) $f_4(x) = f_1(x)$ for all x
(B) $f_1(x) = -f_3(-x)$ for all x
(C) $f_2(-x) = f_4(x)$ for all x
(D) $f_1(x) = f_3(x) = 0$ for all x

Key. B

Sol.



129. If $\log_{(2x+3)}(6x^2+23x+21) = 4 - \log_{(3x+7)}(4x^2+12x+9)$, then the value of $-4x$ is

A) 0
B) 1
C) 2
D) $-\frac{1}{4}$

Key. B

Sol. First note that $2x+3>0$ and $2x+3\neq 1$, that is, $x>-3/2$ and $x\neq -1$. Also, $3x+7>0$ and $3x+7\neq 1$, that is, $x>-7/3$ and $x\neq -2$. Suppose $x>-3/2,\ x\neq -1$. Then the given equation can be written as

$$\frac{\log[(2x+3)(3x+7)]}{\log(2x+3)} = 4 - \frac{2\log(2x+3)}{\log(3x+7)}$$

$$1 + \frac{\log(3x+7)}{\log(2x+3)} = 4 - \frac{2\log(2x+3)}{\log(3x+7)}$$

Put $\frac{\log(3x+7)}{\log(2x+3)} = y$

Then $1 + y = 4 - \frac{2}{y}$

Therefore $y = 3 - \frac{2}{y}$

$$y^2 - 3y + 2 = 0$$

$$(y-1)(y-2) = 0$$

This gives $y = 1$ or 2

Case 1: suppose that $y = 1$. Then

$$\log(3x+7) = \log(2x+3)$$
$$3x+7 = 2x+3$$
$$x = -4$$

This is rejected because $x > -3/2$.

Case 2: Suppose that $y = 2$. Then

$$\log(3x+7) = 2\log(2x+3) = \log(2x+3)^2$$

Therefore $3x+7 = 4x^2+12x+9$

$$4x^2+9x+2 = 0$$
$$(4x+1)(x+2) = 0$$
$$x = -1/4 \text{ or } -2$$

Here $x = -1/4$ (since $x > -3/2$) so $-4x = 1$

130. If f and g are two functions defined on N, such that $f(n) = \begin{cases} 2n-1 \text{ if } n \text{ is even} \\ 2n+2 \text{ if } n \text{ is odd} \end{cases}$ and $g(n) = f(n) + f(n+1)$. Then range of g is

A) $\{m \in N / m$ = multiple of 4$\}$

B) { set of even natural numbers}

C) $\{m \in N / m = 4k+3, k$ is a natural number$\}$

D) $\{m \in N / m =$ multiple of 3 or multiple of 4$\}$

Key. C

Sol. $g(n) = f(n) + f(n+1)$

If n is even, n+1 is odd.

$\therefore g(n) = 2n-1+2(n+1)+2 = 4n+3$

If n is odd, n+1 is even.

$\therefore g(n) = 2n+2+2(n+1)-1 = 4n+3$.

131. The number of solution of $y = \frac{1}{3}\left[\sin x + \left[\sin x + [\sin x]\right]\right]$ and $\left[y + [y]\right] = 2\cos x$ where [.] denotes the greatest integer function is

a) 4 b) 0 c) 2 d) 7

Key. B

Sol. $y = [\sin x]$ and $2\cos x = 2[y]$ is impossible for every $x \in R$.

132. Let W be the set of whole numbers and $f : W \to W$ be defined by

$$f(x) = \begin{cases} \left(x - 10\left[\frac{x}{10}\right]\right)10^{[\log_{10} x]} + f\left(\left[\frac{x}{10}\right]\right) & if \quad x > 0 \\ 0 \quad if \quad x = 0 \end{cases}$$

where $[y]$ denotes the largest integer $\le y$. Then $f(7752) =$

(A) 7527 (B) 5727 (C) 7257 (D) 2577

Key. D

Sol. This function simply writes the digits of the given number in the reverse order.

133. $f(x)=\sin[x]+[\sin x], 0<x<\frac{\pi}{2}$, where [ ] represents the greatest integer function, can also be represented as

(A) $\begin{cases} 0 & , 0<x<1 \\ 1+\sin 1 & , 1\le x<\frac{\pi}{2} \end{cases}$

(B) $\begin{cases} \frac{1}{\sqrt{2}} & , 0<x<\frac{\pi}{4} \\ 1+\frac{1}{2}+\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2} & , \frac{\pi}{4}\le x<\frac{\pi}{2} \end{cases}$

(C) $\begin{cases} 0 & , 0<x<1 \\ \sin 1 & , 1\le x<\frac{\pi}{2} \end{cases}$

(D) $\begin{cases} 0 & , 0<x<\frac{\pi}{4} \\ 1 & , \frac{\pi}{4}<x<1 \\ \sin 1 & , 1\le x<\frac{\pi}{2} \end{cases}$

Key. C

Sol. $0<x<\frac{\pi}{2}$

$\therefore [x]=\begin{cases} 0 \;\; if \;\; 0<x<1 \\ 1 \;\; if \;\; 1\le x<\frac{\pi}{2} \end{cases}$

$\Rightarrow \sin[x]=\begin{cases} \sin 0=0 \;\; if \;\; 0<x<1 \\ \sin 1 \;\; if \;\; 1\le x<\frac{\pi}{2} \end{cases}$

We have $0<\sin x<1$ when $0<x<\frac{\pi}{2}$.

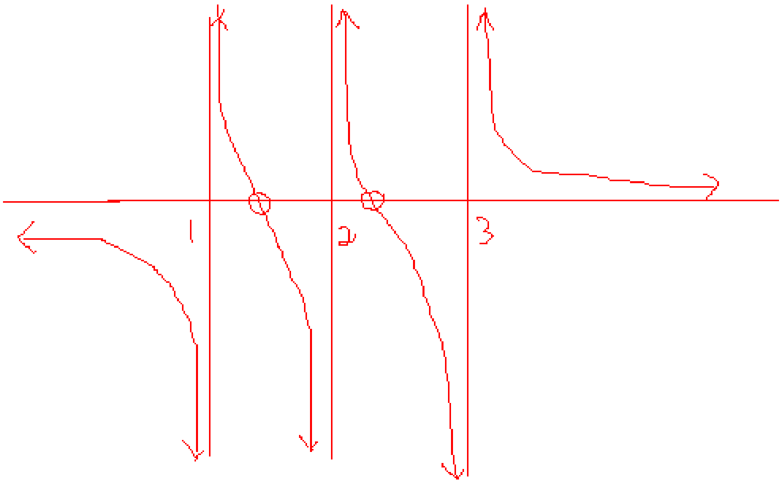
$\therefore [\sin x]=0 \;\; for \;\; 0<x<\frac{\pi}{2}$

$\therefore \sin[x]+[\sin x]=\begin{cases} 0 \;\; if \;\; 0<x<1 \\ \sin 1 \;\; if \;\; 1\le x<\frac{\pi}{2} \end{cases}$

134. $f(x)=\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}$ then number of points where $f(x)=0$

1) 1 2) 2 3) 3 4) 4

Key. 2



Sol.

135. $f(x)=x^2+\lambda x+\mu\cos x,\ \lambda\in\mathbb{Z},\ \mu\in\mathbb{R}$. The number of ordered pairs $(\lambda,\ \mu)$ for which $f(x)=0$ and $f(f(x))=0$ have same set of real roots.

1) 4 2) 6 3) 8 4) 10

Key. 1

Sol. $f(x)=x^2+\lambda x+u\cos x$

Let α be the root of $f(x)=0\Rightarrow f(\alpha)=0$

$\Rightarrow f(f(\alpha))=f(0)=0$ $(\because \alpha$ is root of $f(f(x)=0$ also)

Now $f(0)=\mu=0$

$f(x)=x^2+\lambda x=0\Rightarrow x=0,x=-\lambda$

$f(f(x))=f(x^2+\lambda x)=(x^2+\lambda x)^2+\lambda(x^2+\lambda x)$

$=(x^2+\lambda x)\{x^2+\lambda x+\lambda\}=0$

Will have same root $x=0,\ x=-\lambda$ If

$x^2+\lambda x+\lambda=0$ have no real roots

$\Rightarrow \lambda^2-4\lambda<0$

$\Rightarrow 0<\lambda<4\Rightarrow \lambda=1,2,3$

But $\lambda=0$ is also satisfy

$(0,0),(0,1),(,2)(0,3)$ are 4 or diff. (λ,μ) does exist.

136. $f(x)=x^5+x^2+1$ has roots x_1,x_2,x_3,x_4,x_5 *and* $g(x)=x^2-2$ then $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5)-30g(x_1x_2x_3x_4x_5)=$ ____

1) 2 2) 5 3) 7 4) 11

Key. 3

Sol. Put $g(x)=y=x^2-2\Rightarrow x=\sqrt{y+2}\Rightarrow f(\sqrt{y+2})=0$

$\Rightarrow y^5+20y^4+40y^3+79y^2+74y+23=0$

Roots are $g(x_1),g(x_2),g(x_3),g(x_4),g(x_5)$

$g(x_1).g(x_2).g(x_3).g(x_4).g(x_5)=-23$

And $x_1x_2x_3x_4x_5=-1$

$g(x_1x_2x_3x_4x_5)=g(-1)=-1$

$\therefore g(x_1).g(x_2).g(x_3).g(x_4).g(x_5)-30g(x_1x_2x_3x_4x_5)$

$= -23 + 30 = 7$

137. $f(x) = \cos^{-1}\left(\dfrac{2[|\sin x| + |\cos x|]}{\sin^2 x + 2\sin x + \frac{11}{4}}\right)$

([ ] denotes greatest integer function}. Then domain of $f(x)$ is the internal $[0, 2\pi]$ is.

1) $\left[0, \dfrac{7\pi}{6}\right] U \left[\dfrac{11\pi}{6}, 2\pi\right]$ 2) $[0, 2\pi]$

3) $\left[\dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right]$ 4) $\left[\dfrac{3\pi}{2}, \dfrac{11\pi}{6}\right]$

Key. 1

Sol. $| \le |\sin x| + |\cos x| \le \sqrt{2}$

$[|\sin x| + |\cos x|] = 1 \quad \forall x \in$ ¡

Now $\sin^2 x + 2\sin x + \dfrac{11}{4} = (\sin x + 1)^2 + \dfrac{7}{4}$

For f to be well defined $(\sin x + 1)^2 + \dfrac{7}{4} \ge 2$

$(\sin x + 1)^2 \ge \dfrac{1}{4}$

$\Rightarrow \sin x + 1 \ge \dfrac{1}{2}, \quad \sin x + 1 \le -\dfrac{1}{2}$

$\sin x \ge -\dfrac{1}{2}, \quad \sin x \le -\dfrac{3}{2}$ (This is impossible

$\Rightarrow x \in \left[0, \dfrac{7\pi}{6}\right] U \left[\dfrac{11\pi}{6}, 2\pi\right]$ Hence (A) is correct

138. If $f(x)$ is a polynomial of degree 4 with leading coefficient one satisfying $f(1) = 1$, $f(2) = 2$, $f(3) = 3$ then $\left[\dfrac{f(-1) + f(5)}{f(0) + f(4)}\right] =$ ([.] denotes GIF)

1) 0 2) 5 3) 1 4) -1

Key. 2

Sol. $f(x) - x = (x-1)(x-2)(x-3)(x-\alpha)$

$f(-1) = 24(1+\alpha) - 1$

$f(0) = 6\alpha$

$f(4) = 6(4-\alpha) + 4$

$f(5) = 24(5-\alpha) + 5$

$\left[\dfrac{f(-1) + f(5)}{f(0) + f(4)}\right] = \left[\dfrac{148}{28}\right] = 5$

139. A function 'f' defined as $f(\alpha) = (-1)^{\alpha_1} + (-1)^{\alpha_2} + (-1)^{\alpha_3} + ... + (-1)^{\alpha_k}$ where $\alpha \in$ ¥ , and α_1, α_2, α_k are all divisors of α including 1 and itself such that $\alpha_1, \alpha_2 ...... \alpha_k = \alpha$ and $\alpha_1, \alpha_2 ...... \alpha_k \in$ ¥

If $f(\alpha)=4$ and $\alpha<60$ then number of possible values of α .

1) 3 2) 6 3) 10 4) 4

Key. 1

Sol. $4=(-1)^2+(-1)^2+(-1)^2+(-1)^2=\qquad \alpha=16<60$

$4=(-1)^4+(-1)^2+(-1)^2+(-1)^2=\qquad \alpha=32<60$

$4=(-1)^6+(-1)^2+(-1)^2+(-1)^2=\qquad \alpha=48<60$

140. $f(x+1)=(-1)^{x+1}x-1f(x)$ for $x\in$ ¥ and $f(1)=f(1986)$. Then sum of digits of $(f(1)+f(2)+........+f(1985))$ is

1) 4 2) 3 3) 7 4) 11

Key. 3

Sol. $\sum_{x=1}^{1985} f(x+1)=\sum_{x=1}^{1985}(-1)^{x+1}x-2\sum_{x=1}^{1985} f(x)$

Since $f(1)=f(1986)$

$3\sum_{x=1}^{1985} f(x)=1-2+3-4+5.........+1985$

$=(1+3+.......+1985)-2(1+2+3+......992)$

$=\frac{993}{2}(1986)-2\left(\frac{992\text{x}993}{2}\right)$

$=(993)^2-993\,\text{x}\,992$

= 993

$\therefore \sum_{x=1}^{1985} f(x)=\frac{993}{3}=331$

Sum of digits = 3+3+1= 7

141. $f(x)=ax^2-C^{satisfy}-4\le f(1)\le -1$ and $-1\le f(2)\le 5$ then which of the following is true

1) $-7\le f(3)\le 26$ 2) $-4\le f(3)\le 15$

3) $-1\le f(3)\le 20$ 4) $\frac{-28}{3}\le f(3)\le \frac{35}{3}$

Key. 1

Sol. $f(x)=ax^2-c$

$-4\le f(1)\le -1 \Rightarrow -4\le a-c\le -1;$

$1\le c-a\le 4\rightarrow(1)$

$-1\le f(2)\le 5\Rightarrow -1\le 4a-c\le 5\rightarrow(2)$

$(1)+(2)\Rightarrow 0\le 3a\le a$

$0\le a\le 3\rightarrow(3)$

From (1)

$-16\le 4a-4c\le -4$

$\Rightarrow 4\le 4c-4a\le 16$

From (2)

$-1\le 4a-c\le 5$

$3 \le 3c \le 21 \Rightarrow 1 \le c \le 7 \rightarrow (4)$

Now $f(3) = 9a - c$ is max of a is max and c is min

$f(3)_{\max} = 9(3) - 1 = 26$

$f(3)_{\min} = 9(0) - 7 = -7$

$\therefore -7 \le f(3) \le 26$

142. Let $f(x) = 5 + \sum_{r=1}^{2010} a_{2r-1} x^{2r-1}$ and $f(-1) = 4$ then $f(1) =$

1) 2 2) 6 3) 5 4) 4

Key. 2

Sol. $f(x) = 5 + a_1 x + a_3 x^3 + a_5 x^5 + ... + a_{4019} x^{4019}$

$f(-1) = 5 - a_1 - a_3 - a_5 .... - a_{4019} = 4$

$f(1) = 5 + a_1 + a_3 + a_5 .... + a_{4019} = \lambda \ say$

$10 = 4 + \lambda \Rightarrow \lambda = 6$

143. Let $f(x) = ax + b$ where a and b are rational numbers (where $b \ne 0$). Such that $f(1) \le f(2)$, $f(3) \ge f(4)$ then value of $\left(\dfrac{\sum_{r=1}^{2n-1} f\left(\sqrt{2}r\right)}{f(\sqrt{3})} \right)$ $\left(where\ n \in ¥ \right)$ is

1) n 2) 1 3) 0 4) n^2

Key. 4

Sol. For fixed values of a and b $f(x) = ax + b$ is a straight line

But given $f(1) \le f(2)$ and $f(3) \ge f(4)$

$\therefore f(1) = f(2) = f(3) = f(4) = \lambda$

$\Rightarrow f(x)$ should be constant function $\Rightarrow a = 0$

$\Rightarrow f(x) = b \Rightarrow f(\sqrt{2}r) = b$ and $f\left(\sqrt{3}\right) = b$

Given expression is $\dfrac{n^2 b}{b} = n^2$

144. A linear function that map the set {–2, 2} onto the set {0, 4} is

(A) f(x) = (x – 2) (B) f(x) = (2 – x) (C) f(x) = (2 + x) (D) (B) and (C)

Key. D

Sol. Let the linear function be

f(x) = ax + b

Let f(–2) = 0 and f(2) = 4 $\Rightarrow$ f(x) = x + 2

Let f(–2) = 4 and f(0) = 0 $\Rightarrow$ f(x) = –x + 2

The two linear function as are

f(x) = (x + 2) and f(x) = (2 – x)

145. Suppose f(x) = $(x + 2)^2$ for $x \ge -2$. If g(x) is the function whose graph is the reflection of the graph of f(x) in the line y = x, then g(x) equals

(A) $-\sqrt{x}-2,\ x\ge 0$ (B) $\sqrt{x}-2,\ x\ge 0$ (C) $\frac{1}{(x+2)^2},\ x>2$ (D) $\sqrt{x+2},\ x>-2$

Key. B

Sol. $y = (x + 2)^2$

Equation of the reflection curve in y = x is obtained by interchanging x and y in $y = (x + 2)^2$

$\Rightarrow$ reflection curve is

$x = (y + 2)^2$

$y + 2 = \sqrt{x}$

$y = \sqrt{x}-2,\ x\ge 0$

Since x is always ≥ 2.

146. If $\log_4\left(\log_3\left(\log_2 x\right)\right)=1$, then x is

(A) 2^{3^4} (B) 9 (C) 24 (D) 4^{3^2}

Key. A

Sol. $\log_4\left[\log_3 \log_2 x\right] = 1 \Rightarrow \log_3 \log_2 x = 4$

$\Rightarrow \log_2 x = 3^4 \quad \Rightarrow x = 2^{3^4}$

147. The value of the parameter α, for which the function $f(x) = 1+\alpha x,\ \alpha \ne 0$ is the inverse of itself, is

(A) –2 (B) –1 (C) 1 (D) 2

Key. B

Sol. $y = 1+\alpha x \Rightarrow x = \frac{y-1}{\alpha}$

$f^{-1}(x) = \frac{x-1}{\alpha} = f(x) = 1+\alpha x$

$\Rightarrow \frac{x-1}{\alpha} = 1+\alpha x \quad \Rightarrow x - 1 = \alpha+\alpha^2 x$

Equating the coefficient of x

$\alpha^2 = 1$ and $\alpha = -1$

$\alpha = \pm 1$

$\alpha = -1$

148. If $f(x) = x^2+2bx+2c^2$ and $g(x) = -x^2-2cx+b^2$ such that min f(x) > max g(x), then the relation between b and c, is

(A) no real value of b and c (B) $0<c<b\sqrt{2}$

(C) $|c| < |b|\sqrt{2}$ (D) $|c| > |b|\sqrt{2}$

Key. D

Sol. We have, $f(x) = x^2 + 2bx + 2c^2$; $g(x) = -x^2 - 2cx + b^2$

$\Rightarrow$ $f(x) = (x+b)^2 + 2c^2 - b^2$

and, $g(x) = -(x + c)^2 + b^2 + c^2$

$\Rightarrow$ $f_{min} = 2c^2 - b^2$ and $g_{max} = b^2 + c^2$

for, $f_{min} > g_{max}$ $\Rightarrow$ $2c^2 - b^2 > b^2 + c^2$

$\Rightarrow$ $c^2 > 2b^2$ $\Rightarrow$ $|c| > |b|\sqrt{2}$

149. Let $A_1, A_2, A_3, \ldots, A_{40}$ are 40 sets each with 7 elements and $B_1, B_2, \ldots, B_n$ are n sets each with 7 elements. If $\bigcup_{i=1}^{40} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly ten of A_i's and exactly 9 of B_j's, then n equals

(A) 42 (B) 35

(C) 28 (D) 36

Key. D

Sol. $n(S) \times 10 = 40 \times 7$

$n(S) = 28$

$28 \times 9 = n \times 7$

$n = 36$

150. The number of functions f from the set A={0,1,2} in to the set B={0,1,2,3,4,5,6,7} such that $f(i) \le f(j)$ for i<j and $i, j \in A$ is

a) ${}^8C_3$ b) ${}^8C_3 + 2({}^8C_2)$

c) ${}^{10}C_3$ d) ${}^{10}C_4$

Key. C

Sol.

$$0 < 1 < 2$$
$$\Rightarrow f(0) \le f(1) \le f(2)$$
$$f(0)<f(1)<f(2) \Rightarrow {}^8C_3$$
$$f(0)<f(1)=f(2) \Rightarrow {}^8C_2$$
$$f(0)=f(1)<f(2) \Rightarrow {}^8C_2$$
$$f(0)=f(1)=f(2) = {}^8C_1$$

151. Find the value of $\sum_{r=1}^{n} \sum_{s=1}^{n} \delta_{rs}\, 2^r 3^s$ where $\begin{cases} \delta_{rs} = 0,\ if\ \ r \ne s \\ \delta_{rs} = 1,\ if\ \ r = s \end{cases}$

a) $\frac{6}{5}\left(6^n - 1\right)$ b) $6^n - 1$ c) $\frac{1}{5}\left(6^n - 1\right)$ d) none

Key. A

Sol. $\sum_{r=1}^{n} 2^r \left\{ \sum_{s=1}^{n} \delta_{rs}\, 3^s \right\}$

$$= \sum_{r=1}^{n} 2^r \left\{ \delta_{r1} 3^1 + \delta_{2r} 3^2 + \delta_{r3} 3^3 + ... + \delta_{rn} 3^n \right\}$$

$$= 2^1 3^1 + 2^2 3^2 + 2^3 3^3 + ... + 2^n 3^n = 6 + 6^2 + ... + 6^n = \frac{6}{5}\left(6^n - 1\right)$$

152. Consider $\int_0^x \left(t^2 - 8t + 13\right) dt = x \sin\left(\frac{a}{x}\right)$ and $\left(a, x \in R - \{0\}\right)$ x takes the values for which the equation has a solution, then the number of values of a $\in [0,100]$ is ___

a) 1 b) 2 c) 3 d) 4

Key. C

Sol. $\left(\frac{t^3}{3} - \frac{8t^2}{2} + 13t\right)_0^x = x \sin\left(\frac{a}{x}\right)$

$$x\left\{\frac{x^2}{3} - 4x + 13 - \sin\left(\frac{a}{x}\right)\right\} = 0$$

Here $x \neq 0 \Rightarrow \frac{1}{3}\left(x^2 - 12x + 39\right) = \sin\left(\frac{a}{x}\right)$

$$\Rightarrow \frac{1}{3}(x-6)^2 + 1 = \sin\left(\frac{a}{x}\right)$$

$$\Rightarrow \frac{a}{6} = \frac{\pi}{2} \text{ or } \frac{5\pi}{2} \text{ or } \frac{9\pi}{2}$$

$\therefore \quad a = 3\pi, 15\pi, 27\pi$ (3 values)

153. Let f be a function defined on the set of non-negative integers and taking values in the same set. Given that

i) $x - f(x) = 19\left[\frac{x}{19}\right] - 90\left[\frac{f(x)}{90}\right]$ $\forall$ non-negative integers x. [x] denotes greatest integer functions.

ii) $1900 \leq f(1990) \leq 2000$. Then possible values of $f(1990)$ can take.

a) 2004, 2094 b) 1804, 1994 c) 1904, 1994 d) 1894

Key. C

Sol. Since $1900 \leq f(1990) \leq 2000$

$$\Rightarrow \left[\frac{1900}{90}\right] \leq \left[\frac{f(1990)}{90}\right] \leq \left[\frac{2000}{90}\right] \Rightarrow 21 \leq \left[\frac{f(1990)}{90}\right] \leq 22$$

Case - I

If $\left[\frac{f(1990)}{90}\right] = 21,\ x - f(x) = 19\left[\frac{x}{19}\right] - 90\left[\frac{f(x)}{90}\right]$

Substitute $x = 1990$

$$1990 - f(1990) = 19\left[\frac{1990}{19}\right] - 90\left[\frac{f(1990)}{90}\right]$$

$$1990 - f(1990) = 19\times104 - 90\times21 \Rightarrow f(1990) = 1904$$

Case – II

If $\left[\frac{f(1990)}{90}\right] = 22$

$$\Rightarrow 1990 - f(1990) = 19\times104 - 90\times22 \Rightarrow f(1990) = 1994$$

154. If $g:[-1,1]\to \mathbf{R}$ is a function and the area of the equilateral triangle with two of its vertices at (0,0) and (x,g(x)) is $\frac{\sqrt{3}}{4}$, then g(x)=

1) $+\sqrt{x^2-1}$ 2) $+\sqrt{1-x^2}$ 3) $+\sqrt{1+x^2}$ 4) $+x$

Key. 2

Sol. If a = length of a side = $\sqrt{(x-0)^2+(g(x)-0)^2} = \sqrt{x^2+g^2(x)}$

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \Rightarrow x^2+g^2(x)=1 \Rightarrow g(x) = +\sqrt{1-x^2}$

155. Let $S=\sum_{r=1}^{117}\frac{1}{2[\sqrt{r}]+1}$ where [.] denotes the greatest integer function. The value of S is

(A) $\frac{69}{7}$ (B) $\frac{206}{21}$ (C) $\frac{76}{7}$ (D) $\frac{227}{21}$

Key. A

Sol. $[\sqrt{117}]=10$; If $r\in[n^2,(n+1)^2):n\in ¥$ then $[\sqrt{r}]=n$

The interval $[n^2,(n+1)^2)$ has 2n + 1 integers

$$S=\frac{1}{2.1+1}.3+\frac{1}{2.2+1}.5+....+\frac{1}{2.9+1}.19+\frac{1}{2.10+1}.18$$

$$=9+\frac{18}{21}=\frac{69}{7}.$$

156. If $x+[y]+\{z\}=1.1$ [.] is G.I.F and {.} is fractional part

$[x]+\{y\}+z=2.2$

$\{x\}+y+[z]=3.3$ then

(A) $x+y+z=3.3$ (B) $y-2x=1$

(C) $2(z+1)=5y$ (D) $\{x\}+\{y\}+\{z\}=0.3$

Key. A, B, C, D

Sol. $x+[y]+\{z\}=1.1$ (1)

$[x]+\{y\}+z=2.2$ (2)

$\{x\}+y+[z]=3.3$ (3)

(1) + (2) + (3)

$\Rightarrow$ $2(x+y+z)=6.6$

$\Rightarrow \quad x+y+z=3.3 \qquad (4)$

$(4)-(1)$

$\{y\}+[z]=2.2 \quad \Rightarrow \{y\}=0.2 \,\&\, [z]=2$

$(4)-(2)$

$\{x\}+[y]=1.1 \quad \Rightarrow \{x\}=0.1,\ [y]=1$

$(4)-(3)$

$[x]+\{z\}=0 \quad \Rightarrow [x]=0 \ \&\ \{z\}=0$

$$\begin{cases} x=0.1 \\ y=1.2 \\ z=2 \end{cases}$$

157. If $0<x<1000$ and $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31}{30}x,$ where [x] is the greatest integer less than or equal to x, the number of possible values of x is

(A) 34 (B) 33

(C) 32 (D) none of these

Key : B

Sol : $\because$ LHS is an integer

$\therefore$ RHS is must be an integer for which x is multiple of 30.

$\therefore\ x=30,60,90,120,.....,990$

$\Rightarrow$ Number of possible values of x is 33.

158. If $f(x)=[x^2]-[x]^2$, [ ] denotes greatest integer function and $x\in[0,n],\ n\in N$, then the number of elements in the range of $f(x)$ is

A) 1 B) $n-1$ C) n D) $2n-1$

Key. D

Sol. If $x\in(n-1,n)$ then $[x]=n-1 \Rightarrow [x]^2=(n-1)^2$

and $(n-1)^2 \le [n^2] \le n^2-1$

$0\le[x^2]-[x]^2 \le n^2-1-(n-1)^2$

$0\le f(x)\le 2n-2$

Since $f(x)$ has to be integer, range of $f(x)=\{0,1,2,3,.....2n-2\}$

$\therefore$ The number of elements in range of f is $(2n-1)$

159. If $\frac{5^m+3}{40}-\left[\frac{5^m+3}{40}\right]=\lambda$ (m$\in$N, m $\ge$ 3) and [ ] denote the G.I.F., then λ can take

(A) two values (B) one value

(C) infinite values (D) four values

Key: A

Hint: $\frac{5^m+3}{40}=\frac{1}{10}$ $(5 + 5^2 + 5^3 + ... + 5^{m-1} + 2)$ $\Rightarrow \lambda = \frac{1}{5}, \frac{7}{10}$

160. The sum of all positive integral values of $'a'$, $a \in [1, 500]$ for which the equation $[x]^3 + x - a = 0$ has solution is ([.] denote G.I.F)

(A) 462 (B) 512 (C) 784 (D) 812

Key: D

Hint: a is integer then x must be integer, i.e., [x] = x

$$a = x^3 + x$$

$$1 \le a \le 500 \Rightarrow 1 \le x \le 7, \ x \in I$$

$$\sum a_i = \sum_{x=1}^{7}\left(x^3 + x\right) = \left(\frac{7.8}{2}\right)^2 + \left(\frac{7.8}{2}\right) = 812$$

161. $f : [0,1] \to R$ is a differentiable function such that $f(0) = 0$ and $|f(x)| \le k|f(x)|$ for all x $\in [0,1], (k > 0)$, then which of the following is/are always true ?

(A) $f(x) = 0, \forall x \in R$ (B) $f(x) = 0, \forall x \in [0,1]$

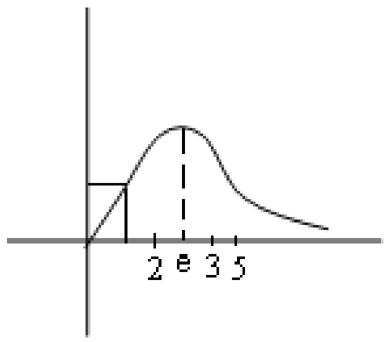
(C) $f(x) \ne 0, \forall x \in [0,1]$ (D) $f(1) = k$

Key: B

Hint: $(f(x))^2 - k^2 (f(x))^2 \le 0$

$\Rightarrow (f'(x) - kf(x))(f'(x) + kf(x)) \le 0$

$\Rightarrow \left(f(x)e^{-kx}\right)' \left(f(x)e^{kx}\right)' \le 0$



$\Rightarrow$ Exactly one of the functions $g_1(x) = f(x)e^{-kx}$ or $g_2(x) = f(x)e^{kx}$ is non decreasing.

But $f(0) = 0 \Rightarrow$ both function g_1 and g_2 have a value zero at x = 0

$\forall x \in [0,1], g_1(0) = 0$ and g_1 increasing $\Rightarrow g_1(x) \ge 0 \Rightarrow f(x) \ge 0$

$g_2(0) = 0$ and g_2 decreasing $\Rightarrow g_2(x) \le 0 \Rightarrow f(x) \le 0$

$\Rightarrow f(x) = 0 \forall x \in [0,1]$

162. Let f be a one one function with domain = {x, y, z} and range = {1, 2, 3}. It is given that exactly one of the following statements is true and the remaining two are false : $f(x) = 1, \ f(y) \ne 1, \ f(z) \ne 2$, then $f^{-1}(1) =$ ____

(1) x (2) y (3) z (4) 1

Key. 2

Sol. $f(x) = 1 \ (F) \Rightarrow f(x) = 2 \ or \ 3$

$f(y) \neq 1 \ (F) \Rightarrow f(y) = 1$

$f(z) \neq 2 \ (T) \Rightarrow f(z) = 1 \ or \ 3$

163. If $f(x) = 1 + x; \quad x \geq 0$
$= 1 - x; \quad x < 0$

Which of the following are true ?

(1) Range of f(x) is $[2, \infty)$

(2) $f(f(x))$ is not a one one function

(3) Graph of $y = f(f(x))$ is symmetric about y axis.

(4) All the above

Key. 4

Sol. $f(x) = 1 + |x|; \ x \in R$

$f(f(x)) = f(1 + |x|) = 1 + 1 + |x| = 2 + |x| \quad \forall x \in R$

164. If $[x]$ is the greatest integral function, then $\sum_{k=1}^{4020} \left[\frac{1}{2} + \frac{k-1}{4020} \right]$ is equal to

(1) 2010 (2) 2009 (3) 2011 (4) 2005

Key. 1

Sol. For $k = 1, 2, 3......$ upto 2010, the value of $\left[\frac{1}{2} + \frac{k-1}{4020} \right]$ is equal to zero

For $k = 2011$, 2010, ----- 4020, the value of $\left[\frac{1}{2} + \frac{k-1}{4020} \right]$ = 1

$\therefore$ The sum value is 2010.

165. Let $f(x) = [x]$ and $g(x) = x + [x]$. Then the number of solutions of the equality ($[\cdot]$ is G.I.F)

$4(x - f(x)) = g(x)$ is

(1) 2 (2) 3 (3) 4 (4) 0

Key. 1

Sol. The given equation is

$4(x - [x]) = x + [x] = 2[x] + \{x\}$

$4\{x\} = 2[x] + \{x\}$

$\therefore 0 \leq \frac{2[x]}{3} < 1 \Rightarrow x = 0, 5/3$

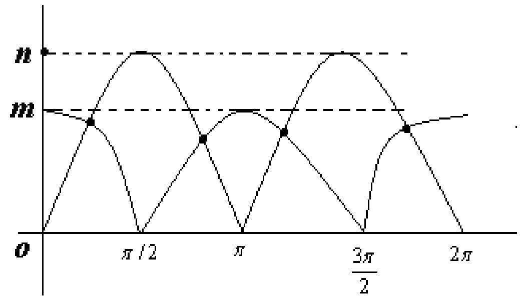
166. If m, n (n > m) are positive integers, then number of solutions of the equation

$n \ |\sin x| = m \ |\cos x| \ in \ [0, 2\pi]$ is

(1) 2 (2) 4 (3) m (4) n

Key. 2

Sol. No.of solutions = 4



167. Let $f(x)=ln\left(\frac{1-x}{1+x}\right)$. The set of values of $'\alpha'$ for which $f(\alpha)+f(\alpha^2)=f\left(\frac{\alpha}{\alpha^2-\alpha+1}\right)$ is satisfied are

A) $(-\infty,-1)\cup(1,\infty)$ B) $(-1,1)$ C) $(0,1)$ D) $(1,2)$

Key. B

Sol. $$f(\alpha)+f(\alpha)^2=ln\left[\left(\frac{1-\alpha}{1+\alpha}\right)\left(\frac{1-\alpha^2}{1+\alpha^2}\right)\right]=ln\left[\frac{(1-\alpha)^2}{1+\alpha^2}\right]$$

$$f\left(\frac{\alpha}{\alpha^2-\alpha+1}\right)=ln\left[\frac{1-\frac{\alpha}{\alpha^2-\alpha+1}}{1+\frac{\alpha}{\alpha^2-\alpha+1}}\right]=ln\left[\frac{(1-\alpha)^2}{1+\alpha^2}\right]$$

$\therefore\ f(\alpha)+f(\alpha^2)=f\left(\frac{\alpha}{\alpha^2-\alpha+1}\right)$ for all values of α for which the functions are defined, therefore

(i) $\frac{1-\alpha}{1+\alpha}>0\Rightarrow -1<\alpha<1....(1)$

(ii) $\frac{1-\alpha^2}{1+\alpha^2}>0\Rightarrow 1-\alpha^2>0\Rightarrow -1<\alpha<1....(2)$

From (1) and (2), we have $-1<\alpha<1$

$\therefore$ The set of values of $\alpha=(-1,1)$.

168. If $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10,10)$ and $f(x)=kf\left(\frac{200x}{100+x^2}\right)$, then k =

A. 2 B. 10 C. $\frac{1}{2}$ D. $\frac{1}{10}$

Key. C

Sol. $f(x)=\log_e\left(\frac{10+x}{10-x}\right)$

$$f\left(\frac{200x}{100+x^2}\right)=\log_e\left(\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right)=2\log\left(\frac{10+x}{10-x}\right)=2f(x)$$

169. The number of solutions of $\sin\{x\}=\cos\{x\}$ (where {.} denotes fractional part) in [0, 2π] is equal to

A. 5 B. 6 C. 7 D. 8

KEY. B

SOL. Draw Graph

170. A linear function that map the set {–2, 2} onto the set {0, 4} is

(A) f(x) = (x – 2) (B) f(x) = (2 – x) (C) f(x) = (2 + x) (D) (B) and (C)

Key. D

Sol. Let the linear function be

f(x) = ax + b

Let f(–2) = 0 and f(2) = 4 $\Rightarrow$ f(x) = x + 2

Let f(–2) = 4 and f(0) = 0 $\Rightarrow$ f(x) = –x + 2

The two linear function as are

f(x) = (x + 2) and f(x) = (2 – x)

171. Suppose $f(x) = (x + 2)^2$ for $x \ge -2$. If g(x) is the function whose graph is the reflection of the graph of f(x) in the line y = x, then g(x) equals

(A) $-\sqrt{x}-2,\ x\ge 0$ (B) $\sqrt{x}-2,\ x\ge 0$ (C) $\frac{1}{(x+2)^2},\ x>2$ (D) $\sqrt{x+2},\ x>-2$

Key. B

Sol. $y = (x + 2)^2$

Equation of the reflection curve in y = x is obtained by interchanging x and y in $y = (x + 2)^2$

$\Rightarrow$ reflection curve is

$x = (y + 2)^2$

$y + 2 = \sqrt{x}$

$y = \sqrt{x}-2,\ x\ge 0$

Since x is always ≥ 2.

172. If f and g are two functions defined on N, such that $f(n)=\begin{cases}2n-1 \text{ if } n \text{ is even}\\ 2n+2 \text{ if } n \text{ is odd}\end{cases}$ and

$g(n)=f(n)+f(n+1)$. Then range of g is

A) $\{m\in N/m=$ multiple of 4}

B) { set of even natural numbers}

C) $\{m\in N/m=4k+3, k$ is a natural number}

D) $\{m\in N/m=$ multiple of 3 or multiple of 4}

Key. C

Sol. $g(n)=f(n)+f(n+1)$

If n is even, n+1 is odd.

$\therefore g(n)=2n-1+2(n+1)+2=4n+3$

If n is odd, n+1 is even.

$\therefore g(n) = 2n + 2 + 2(n+1) - 1 = 4n + 3.$

173. A function 'f' defined as $f(\alpha) = (-1)^{\alpha_1} + (-1)^{\alpha_2} + (-1)^{\alpha_3} + ...+ (-1)^{\alpha_k}$ where $\alpha \in ¥$, and α_1, α_2, α_k are all divisors of α including 1 and itself such that $\alpha_1, \alpha_2......\alpha_k = \alpha$ and $\alpha_1, \alpha_2......\alpha_k \in ¥$ If $f(\alpha) = 4$ and $\alpha < 60$ then number of possible values of α .

1) 3 2) 6 3) 10 4) 4

Key. 1

Sol. $4 = (-1)^2 + (-1)^2 + (-1)^2 + (-1)^2 = \quad \alpha = 16 < 60$

$4 = (-1)^4 + (-1)^2 + (-1)^2 + (-1)^2 = \quad \alpha = 32 < 60$

$4 = (-1)^6 + (-1)^2 + (-1)^2 + (-1)^2 = \quad \alpha = 48 < 60$

174. $f(x+1) = (-1)^{x+1} x - 1 f(x)$ for $x \in ¥$ and $f(1) = f(1986)$. Then sum of digits of $(f(1) + f(2) + ...+ f(1985))$ is

1) 4 2) 3 3) 7 4) 11

Key. 3

Sol. $\sum_{x=1}^{1985} f(x+1) = \sum_{x=1}^{1985} (-1)^{x+1} x - 2\sum_{x=1}^{1985} f(x)$

Since $f(1) = f(1986)$

$3\sum_{x=1}^{1985} f(x) = 1 - 2 + 3 - 4 + 5.........+1985$

$= (1 + 3 + ...+1985) - 2(1 + 2 + 3 + ...992)$

$= \frac{993}{2}(1986) - 2\left(\frac{992x993}{2}\right)$

$= (993)^2 - 993\,x\,992$

= 993

$\therefore \sum_{x=1}^{1985} f(x) = \frac{993}{3} = 331$

Sum of digits = 3+3+1= 7

175. $f(x) = ax^2 - C^{satisfy} - 4 \le f(1) \le -1$ and $-1 \le f(2) \le 5$ then which of the following is true

1) $-7 \le f(3) \le 26$ 2) $-4 \le f(3) \le 15$

3) $-1 \le f(3) \le 20$ 4) $\frac{-28}{3} \le f(3) \le \frac{35}{3}$

Key. 1

Sol. $f(x) = ax^2 - c$

$-4 \le f(1) \le -1 \Rightarrow -4 \le a - c \le -1;$

$1 \le c - a \le 4 \to (1)$

$-1 \le f(2) \le 5 \Rightarrow -1 \le 4a - c \le 5 \to (2)$

$(1) + (2) \Rightarrow 0 \le 3a \le a$

$0 \le a \le 3 \to (3)$

From (1)

$-16 \le 4a - 4c \le -4$
$\Rightarrow 4 \le 4c - 4a \le 16$

From (2)
$-1 \le 4a - c \le 5$
$3 \le 3c \le 21 \Rightarrow 1 \le c \le 7 \rightarrow (4)$

Now $f(3) = 9a - c$ is max of a is max and c is min
$f(3)_{\max} = 9(3) - 1 = 26$

$f(3)_{\min} = 9(0) - 7 = -7$
$\therefore -7 \le f(3) \le 26$

176. Let $f(x) = 5 + \sum_{r=1}^{2010} a_{2r-1} x^{2r-1}$ and $f(-1) = 4$ then $f(1) =$

1) 2 2) 6 3) 5 4) 4

Key. 2

Sol. $f(x) = 5 + a_1 x + a_3 x^3 + a_5 x^5 + ... + a_{4019} x^{4019}$
$f(-1) = 5 - a_1 - a_3 - a_5 .... - a_{4019} = 4$
$f(1) = 5 + a_1 + a_3 + a_5 .... + a_{4019} = \lambda$ *say*
$10 = 4 + \lambda \Rightarrow \lambda = 6$

177. Let $f(x) = ax + b$ where a and b are rational numbers (where $b \ne 0$). Such that $f(1) \le f(2)$, $f(3) \ge f(4)$ then value of $\left(\frac{\sum_{r=1}^{2n-1} f\left(\sqrt{2r}\right)}{f(\sqrt{3})} \right)$ $\left(where\ n \in ¥\right)$ is

1) n 2) 1 3) 0 4) n^2

Key. 4

Sol. For fixed values of a and b $f(x) = ax + b$ is a straight line
But given $f(1) \le f(2)$ and $f(3) \ge f(4)$
$\therefore f(1) = f(2) = f(3) = f(4) = \lambda$
$\Rightarrow f(x)$ should be constant function $\Rightarrow a = 0$
$\Rightarrow f(x) = b \Rightarrow f(\sqrt{2r}) = b$ and $f\left(\sqrt{3}\right) = b$

Given expression is $\frac{n^2 b}{b} = n^2$

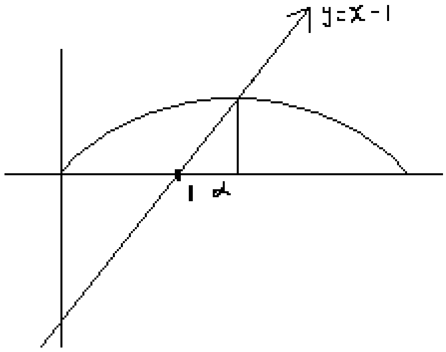
178. If α is a root of the equation $\sin x + 1 = x$ then $\lim_{x \to \alpha} \left[\frac{\min\left(\sin x, \{x\}\right)}{x - 1} \right]$ is

Where [.] $\rightarrow$ denotes greatest integer function
{x}$\rightarrow$ fractional part of x.

1) 1 2) 0 3) does not exist 4) -1

Key. 3

Sol. LHL :



$$\lim_{x\to\alpha-}\left[\frac{\min(\sin x,\ x-[x]}{(x-1)}\right]$$

When $1<x<\alpha$

$\{x\}=x-1<\sin x$

$\min\{\sin x,\ x-1\}=x-1$

Required limit = $\lim_{x\to\infty-}\left[\frac{x-1}{x-1}\right]=1$

RHL :

$\lim_{x\to\alpha+}\left[\frac{\sin x}{x-1}\right]=0$

Hence $LHL\neq RHL$

Limit does not exist

$x\to\alpha+$

$\sin x<x-1$

$\frac{\sin x}{x-1}<1$

$\left[\frac{\sin x}{x-1}\right]=0$

179. $f(x)=x^5+x^2+1$ has roots x_1,x_2,x_3,x_4,x_5 *and* $g(x)=x^2-2$ then $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5)-30g(x_1x_2x_3x_4x_5)=$ ____

1) 2 2) 5 3) 7 4) 11

Key. 3

Sol. Put $g(x)=y=x^2-2\Rightarrow x=\sqrt{y+2}\Rightarrow f(\sqrt{y+2})=0$

$\Rightarrow y^5+20y^4+40y^3+79y^2+74y+23=0$

Roots are $g(x_1),g(x_2),g(x_3),g(x_4),g(x_5)$

$g(x_1).g(x_2).g(x_3).g(x_4).g(x_5)=-23$

And $x_1x_2x_3x_4x_5=-1$

$g(x_1x_2x_3x_4x_5)=g(-1)=-1$

$\therefore g(x_1).g(x_2).g(x_3).g(x_4).g(x_5)-30g(x_1x_2x_3x_4x_5)$

$=-23+30=7$

180. $f(x)=x^2+\lambda x+\mu\cos x$, $\lambda\in$ ¢, $\mu\in$ ¡ . The number of ordered pairs $(\lambda,\ \mu)$ for which $f(x)=0$ and $f(f(x))=0$ have same set of real roots.

1) 4 2) 6 3) 8 4) 10

Key. 1

Sol. $f(x)=x^2+\lambda x+u\cos x$

Let α be the root of $f(x)=0\Rightarrow f(\alpha)=0$

$\Rightarrow f(f(\alpha))=f(0)=0\quad (\because \alpha \text{ is root of } f(f(x)=0 \text{ also})$

Now $f(0)=\mu=0$

$$f(x)=x^2+\lambda x=0 \Rightarrow x=0, x=-\lambda$$

$$f(f(x))=f(x^2+\lambda x)=(x^2+\lambda x)^2+\lambda(x^2+\lambda x)$$

$$=(x^2+\lambda x)\left\{x^2+\lambda x+\lambda\right\}=0$$

Will have same root $x=0,\ x=-\lambda$ If

$x^2+\lambda x+\lambda=0$ have no real roots

$\Rightarrow \lambda^2-4\lambda<0$

$\Rightarrow 0<\lambda<4 \Rightarrow \lambda=1,2,3$

But $\lambda=0$ is also satisfy

$(0,0),(0,1),(,2)(0,3)$ are 4 or diff. (λ,μ) does exist.

181. A polynomial of 6th degree $f(x)$ satisfies $f(x)= f(2- x)"\ \ x\hat{I}\ R$.

If $f(x)= 0$ has

four distinct and two equal roots then sum of roots of $f(x)= 0$ is

a) 4 b) 5 c) 6 d) 7

Key. C

Sol. Let α be a root of f(x) = 0 \ $f(a)= f(2- a)$

f(x) has 4 distinct and two equal roots $\therefore$ Sum of roots = 6

182. The number of the functions f from the set X = {1, 2, 3} to the Y = {1, 2, 3, 4, 5, 6, 7} such that f(i) ≤ f(j) for i < j and i, j $\in$ X is

(A) $^6C_3$ (B) $^7C_3$ (C) $^8C_3$ (D) $^9C_3$

Key: D

Hint $^7C_3 + 2 \times {}^7C_2 + {}^7C_1 = {}^9C_3$.

183. $f(x)=\sin[x]+[\sin x], 0<x<\frac{\pi}{2}$, where [ ] represents the greatest integer function, can also be represented as

(A) $\begin{cases} 0 & , 0<x<1 \\ 1+\sin 1 & , 1\le x<\frac{\pi}{2} \end{cases}$

(B) $\begin{cases} \frac{1}{\sqrt{2}} & , 0<x<\frac{\pi}{4} \\ 1+\frac{1}{2}+\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2} & , \frac{\pi}{4}\le x<\frac{\pi}{2} \end{cases}$

(C) $\begin{cases} 0 & , 0<x<1 \\ \sin 1 & , 1\le x<\frac{\pi}{2} \end{cases}$

(D) $\begin{cases} 0 & , 0<x<\frac{\pi}{4} \\ 1 & , \frac{\pi}{4}<x<1 \\ \sin 1 & , 1\le x<\frac{\pi}{2} \end{cases}$

Key. C

Sol. $0 < x < \frac{\pi}{2}$

$\therefore [x] = \begin{cases} 0 \ \ if \ \ 0 < x < 1 \\ 1 \ \ if \ \ 1 \le x < \frac{\pi}{2} \end{cases}$

$\Rightarrow \sin[x] = \begin{cases} \sin 0 = 0 \ \ if \ \ 0 < x < 1 \\ \sin 1 \ \ \ \ if \ \ 1 \le x < \frac{\pi}{2} \end{cases}$

We have $0 < \sin x < 1$ when $0 < x < \frac{\pi}{2}$.

$\therefore [\sin x] = 0 \ \ for \ \ 0 < x < \frac{\pi}{2}$

$\therefore \sin[x] + [\sin x] = \begin{cases} 0 \ \ \ \ if \ \ 0 < x < 1 \\ \sin 1 \ \ if \ \ 1 \le x < \frac{\pi}{2} \end{cases}$

184. Domain of function $f(x) = \ln\left|\frac{2b^2 + x^2}{b^3 - x^3} - \frac{2x}{bx + b^2 + x^2} - \frac{1}{b - x}\right|$ is

(A) R
(B) R^+
(C) $R - \left\{\frac{b}{2}\right\}$
(D) $R - \left\{b, \frac{b}{2}\right\}$

Key. D

Sol. $\left|\frac{2b^2 + x^2}{b^3 - x^3} - \frac{2x}{bx + b^2 + x^2} - \frac{1}{b - x}\right| > 0$

$\Rightarrow \quad \frac{2b^2 + x^2}{b^3 - x^3} - \frac{2x}{bx + b^2 + x^2} - \frac{1}{b - x} \ne 0$

$\Rightarrow \quad \frac{2x^2 - 3bx + b^2}{b^3 - x^3} \ne 0 \qquad x \ne b$

$\Rightarrow \quad 2x^2 - 3bx + b^2 \ne 0 \qquad \Rightarrow \quad x \ne b, \frac{b}{2}$

185. Which of the following is a function ([.] denotes the greatest integer function, {.} denotes the fractional part function)?

(A) $\frac{1}{\log[1-|x|]}$
(B) $\frac{x!}{\{x\}}$
(C) $x\,!\,\{x\}$
(D) $\frac{\log(x-1)}{\sqrt{1-x^2}}$

Key. C

Sol.For a, b & d Domain is Null set.

$\therefore$ they are not functions.

186. If $f(x)=x^{101}-2x^{11}+2x+1$ and 'g' be the inverse of 'f', then $g'(1)$ is equal to

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) $-\frac{1}{2}<g(1)<\frac{1}{2}$ (D) none of these

Key. B

Sol. $g[f(x)]=x$

$\Rightarrow g'[f(x)].f'(x)=1$

$\Rightarrow$ on putting x = 0

$\Rightarrow g'[f(0)].f'(0)=1$

$\Rightarrow g'(1).2=1$

$\Rightarrow g'(1)=\frac{1}{2}$

187. Let g (x) be a polynomial of degree 8 satisfying $g(r)=\frac{1}{r},\ r=1,2,3,.....,9,$ and

$$f(x)=\begin{cases}\dfrac{\left(\frac{x}{1}-1\right)\left(\frac{x}{2}-1\right)\left(\frac{x}{3}-1\right)........\left(\frac{x}{9}-1\right)}{x}+\frac{1}{x}, & x\neq 0\\ 1+\frac{1}{2}+\frac{1}{3}+.........+\frac{1}{9}, & x=0\end{cases}$$ Then $\frac{f(-1)}{g(10)}=$

a) 50 b)45 c)5 d)50

Key. B

Sol. Notes that $f(x)=g(x)$

$\Rightarrow \frac{f(-1)}{g(10)}=\frac{f(-1)}{f(10)}$

$\Rightarrow \frac{9}{\frac{1}{5}}=45$

188. The range of function f(x) defined by $f(x)=\frac{x^2+1}{\log_e(x^2+1)},\ x\in R-\{0\}$ *is*

a) $(-0,1)$ b) $(0,2)$ c) $[e,\infty)$ d) $(-\infty,\infty)$

Key. C

Sol. Let $t=x^2+1$, then $(t>1)$ and $g(t)=\frac{t}{\log t}\ \Rightarrow g'(t)=\frac{\log t-1}{\log^2 t}$.

Thus g(t) decreases for $t\in[1,e]$ and

increases for $t\in[e,\infty), g(e)=\frac{e}{\log e}=e$.

We observe that $\lim_{t\to 1^+} g(t)\to\infty$ and $\lim_{t\to\infty} g(t)\to\infty$

Thus range of g is $[e,\infty)$. Hence range of f is $[e,\infty)$.

189. Let f(x) be a polynomial one – one function such that

$f(x)f(y)+2=f(x)+f(y)+f(xy), \forall x,y \in R-\{0\} \quad f(1)\neq 1, f'(1)=3$.

Let $g(x)=\frac{x}{4}(f(x)+3)-\int_0^x f(x)dx,$ then

a) $g(x)=0$ has exactly one root for $x\in(0,1)$

b) $g(x)=0$ has exactly two roots for $x\in(0,1)$

c) $g(x)\neq 0 \ \ \forall x\in R-\{0\}$

d) $g(x)=0 \ \ \forall \ x\in R-\{0\}$

Key. D

Sol. Put x = y = 1 $\Rightarrow f(1)=2$ again put $y=\frac{1}{x} \Rightarrow f(x)+f\left(\frac{1}{x}\right)=f(x)f\left(\frac{1}{x}\right)$

$\Rightarrow f(x)=x^3+1 \Rightarrow g(x)=0 \quad \forall x\in R-\{0\}$.

190. Let 'm' be the least value of the function f(x) = |x. lnx| , x∈ [e, ∞), then the number of values of x for which $e^{|x^2-4x+5|}$ = m is true is

(A) 2 (B) 4

(C) 1 (D) zero

Key. D

Sol. f(x) = ⇒ |x ln x|

Graph of f(x):

Obviously least value

Ocurs at x = e

∴ m = |e ln e| = e.

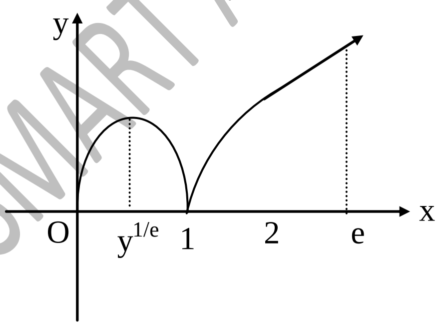
∴ $e^{|x^2-4x+5|}=e^1$

⇒ $x^2-4x+4=0$ and $x^2-4x+6=0$

⇒ x = 2 and no solution

But x = 2 ∉ [e, ∞)

⇒ No value of x is possible.



191. If f(x) = 2x + |x| , g(x) = $\frac{1}{3}$ (2x – |x|) and h(x) = f(g(x)) then $\sin^{-1}$ (h (h(h(h .... h.h(x))))) is n times

(A) $\sin^{-1}$ (sinx) (B) x
(C) $\sin^{-1}$ x (D) $\sin^{-1}$ (|x| + 2x)

Key. C

Sol. Since f[g(x)] = x, $\forall x \in R \Rightarrow$ h(x) = x
$\Rightarrow \sin^{-1}$ [h (h(h... h(x)))] = $\sin^{-1}$x

192. The range of the function defined as f(x) = $\cos^{-1}(-\{-x\})$ is (where {x} is fractional part of x)

(A) $\left[\frac{\pi}{2}, \pi\right)$ (B) $(0, \pi)$
(B) $\left[0, \frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{2}, \pi\right]$

Key. A

Sol. $0 \le \{-x\} < 1 \ \forall \ x \in R \Rightarrow -1 < -\{-x\} \le 0$, so range of f(x) is $\left[\frac{\pi}{2}, \pi\right)$

193. If f(x) . f$\left(\frac{1}{x}\right)$ = f(x) + f$\left(\frac{1}{x}\right)$, then f(x) can be

(A) $1 \pm x^n$ (B) $\frac{2}{1+k\ \ln|x|}$, where k is a fixed real number
(C) $\frac{\pi}{2\tan^{-1}|x|}$ (D) All of these

Key. D

Sol. Consider f(x) = $1 \pm x^n \Rightarrow$ (f(x) – 1) (f(1/x) – 1) = ($\pm x^n$) ($\pm \frac{1}{x^n}$) = 1

$\Rightarrow$ f(x) . $f\left(\frac{1}{x}\right)$ = f(x) + $f\left(\frac{1}{x}\right)$

Consider f(x) = $\frac{2}{1+k\ln|x|}$

f(x). $f\left(\frac{1}{x}\right)$ = $\frac{2}{1+k\ln|x|} \times \frac{1}{1-k\ln|x|}$ = $\frac{4}{1-k^2\ln^2|x|}$

$\Rightarrow$ f(x) $f\left(\frac{1}{x}\right) = f(x) f\left(\frac{1}{x}\right)$

Consider f(x) = $\frac{\pi}{2\tan^{-1}|x|}$

f(x) . f$\left(\frac{1}{x}\right)$ = $\frac{\pi}{2\tan^{-1}|x|} . \frac{\pi}{2\tan^{-1}\left|\frac{1}{x}\right|}$ = $\frac{\pi^2}{4\tan^{-1}|x|.\cot^{-1}|x|}$

f(x) + $f\left(\frac{1}{x}\right) = \frac{\pi}{2\tan^{-1}|x|} + \frac{\pi}{2\tan^{-1}\frac{1}{|x|}}$ = $\frac{\pi}{2}\left(\frac{1}{\tan^{-1}|x|} + \frac{1}{\cot^{-1}|x|}\right)$

= $\frac{\pi}{2} \frac{\cot^{-1}|x| + \tan^{-1}|x|}{\cot^{-1}|x|.\tan^{-1}|x|} = \frac{\pi^2}{4\tan^{-1}|x|\cot^{-1}|x|}$

$\Rightarrow$ f(x) f(1/x) = f(x) + f(1/x)

194. A function f: R$\rightarrow$R is defined by $f\left(x\right)=x^4-10x^3+9x^2-x+1$ then f is

A) A bijection B) one-one but not onto

C) Onto but not one-one D) Neither one-one nor onto

Key. D

Sol. Conceptual

195. If $f:R\rightarrow R$ and $f(x)=\dfrac{x^2+4x+7}{x^2+x+1}$ then $f(x)$ is

(A) one-one function (B) bijective function

(C) many one function (D) Identity function

Key. C

Sol. $f(x)$ is monotonic function

196. If $f(x)=x^3+x^2 \quad 0\le x\le 2$

$=x+2 \quad 2<x\le 4$ and $g(x)$ is even extension of $f(x)$ then

(A) $g(x)=-x+2 \quad -4\le x<-2$
$=-x^3+x^2 \quad -2\le x\le 0$

(B) $g(x)=x-2 \quad -4\le x<-2$
$=x^3-x^2 \quad -2\le x\le 0$

(C) $g(x)=-x+2 \quad -4\le x<-2$
$=x^3-x^2 \quad -2\le x\le 0$

(D) $f(x)=x-2 \quad -4\le x<-2$
$=-x^3+x^2 \quad -2\le x\le 0$

Key. A

Sol. Conceptual

197. $f(x)=\dfrac{\cos x}{\left[\dfrac{2x}{\pi}\right]+\dfrac{1}{2}}$ (where x is not integral multiple of π and [.] denotes the greatest integer function) is

(A) an odd function (B) an even function (C) neither odd nor even (D) none of these

Key. A

Sol. $\left[\dfrac{-2x}{\pi}\right]+\dfrac{1}{2}=-\left(\left[\dfrac{2x}{\pi}\right]+\dfrac{1}{2}\right)$

198. If $f(x)+2f(1-x)=x^2+2 \ \forall x\in R$ then $f(x)$ is given as

(A) $\dfrac{(x-2)^2}{3}$ (B) x^2-2 (C) 1 (D) none of these

Key. A

Sol. Replace x with $(1-x)$ in the given expression

199. If $f(x)=\dfrac{x-[x]}{1+x-[x]}$ $x\in R$ (where [.] denotes the greatest integer function) then $f(R)$ can not contain

(A) 1 (B) $\dfrac{3}{4}$ (C) $\dfrac{1}{4}$ (D) $-\dfrac{1}{2}$

Key. A,B,D

Sol. Find the range of $f(x)$

200. Equation of the locus of points equidistant from two points $(f(-1), f(0), f(1))$ and $(f'(1), f'(-2), f'(2))$ where 'f' is a differentiable function satisfying the equation $f(x-f(y)) = f(f(y)) + x\,f(y) + f(x) - 1, \forall x, y \in R$

(a) $6x-4y+10z+15=0$ (b) $3x-2y+5z+15=0$

(c) $6x+4y+10z-15=0$ (d) $3x+2y-5z-15=0$

Key. A

Sol.

$$f(x-f(y)) = f(f(y)) + xf(y) + f(x) - 1 \rightarrow \quad I$$
$$put\ x = f(y) = 0$$
$$\Rightarrow f(0) = f(0) + 0 + f(0) - 1$$
$$\Rightarrow f(0) = 1$$
$$put\ x = f(y) = k\ in \quad I$$
$$f(0) = f(k) + k(k) + f(k) - 1$$
$$1 = k^2 + 2f(k) - 1$$
$$\Rightarrow 2f(k) = 2 - k^2$$
$$\Rightarrow f(k) = 1 - \frac{k^2}{2}$$
$$\Rightarrow f(x) = 1 - \frac{x^2}{2}$$
$$\Rightarrow f^1(x) = -x$$

$$A\left(\frac{1}{2}, 1, \frac{1}{2}\right), B(-1, 2, -2)$$

Let $p(x, y, z)$ be the point on the locus

$$\Rightarrow PA^2 = PB^2$$
$$\left(x-\frac{1}{2}\right)^2 + (y-1)^2 + \left(z-\frac{1}{2}\right)^2 = (x+1)^2 + (y-2)^2 + (z+2)^2$$
$$\Rightarrow 6x-4y+10z+15=0$$

201. The function defined by

$$f(x) = \begin{cases} x|x| & x \le -1 \\ [1+x]+[1-x] & -1 < x < 1 \\ -x|x| & x \ge 1 \end{cases}$$ is

a) an odd function b) an even function c) neither even nor odd d) even as well as odd

Key. B

Sol. Draw graph.

202. The range of the function $\dfrac{2+x^2}{5+4x^2+x^4}$

a) $(0,1)$ b) $\left(0, \frac{3}{4}\right)$ c) $\left(0, \frac{2}{3}\right)$ d) None of these

Key. D

Sol. $f(x)=\dfrac{1}{(x^2+x)+\dfrac{1}{(x^2+2)}}, x^2+2=t,\ t\ge 2$

$\max f(x)=\dfrac{1}{2+\dfrac{1}{2}}=\dfrac{2}{5}$

$\min f(x)\to 0 \Rightarrow \text{range}\in\left(0,\dfrac{2}{5}\right]$

203. The function f(x) is defined on [0, 1] as following $f(x)=\begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational}\end{cases}$ then for all $x\in[0,1]$ f f(x) is equal to

a) 0 b) 1 + x c) x d) 1

Key. C

Sol. $f(f(x))=f(x)=x$, if x is rational.

$=1-f(x)=1-(1-x)$ if x is irrational

204. If $f(x)=\log_{100x}\left(\dfrac{2(\log_{10}x)+2}{-x}\right), g(x)=\{x\}$ where {} denotes the fractional part of x, then range of g(x) for existence of fog(x) is

a) $(0,100)\cup(100,200)$ b) $\left(0,\dfrac{1}{100}\right)\cup\left(\dfrac{1}{100},1\right)$

c) $\left(0,\dfrac{1}{100}\right)\cup\left(\dfrac{1}{100},\dfrac{1}{10}\right)$ d) None of these

Key. C

Sol. Range of g(x) $\subseteq$ domain of f(x)

Domain of f(x) is $\left(0,\dfrac{1}{100}\right)\cup\left(\dfrac{1}{100},\dfrac{1}{10}\right)$

205. If $f(x)=\dfrac{9^x}{9^x+3}$ for all $x\in R$ then the sum $f\left(\dfrac{1}{1996}\right)+f\left(\dfrac{2}{1996}\right)+\dots\dots\dots+f\left(\dfrac{1995}{1996}\right)$ is

a) 997.5 b) 997 c) 996.5 d) 996

Key. A

Sol. $f(x)=\dfrac{9x}{9^x+3}\Rightarrow f(1-x)=\dfrac{3}{3+3^x}\ \therefore f(x)+f(x)=1$

G.E. $\left[f\left(\dfrac{1}{1996}\right)+f\left(\dfrac{1995`}{1996}\right)\right]+\left[f\left(\dfrac{2}{1996}\right)+f\left(\dfrac{1994}{1996}\right)\right]+$

$$........+\left[f\left(\frac{997}{1996}\right)+f\left(\frac{999}{1996}\right)\right]+f\left(\frac{998}{1996}\right)$$

$$=\left[1+1+.............\text{upto 99 times}\right]+\frac{1}{2}=997.5$$

206. The function $f:R\to R$ defined by $f(x)=\dfrac{x^2+4x+30}{x^2-8x+18}$

a) injective but not surjective
b) Surjection but not injective
c) Both injective and surjective
d) Neither injective nor surjective

Key. D

Sol. $x^2-8x+18$ is not zero for any real number because $x^2-8x+18$ can be written as $(x-4)^2+2$ and numerator $x^2+4x+30$ is also +ve because $(x+2)^2+26\equiv x^2+4x+30$ since f take values which are only + ve for real 'x'. Range f is a sub set of $(0,\infty)$ $\therefore$ range $f\neq R\Rightarrow f$ is not on to also

$$f(0)=\frac{5}{3}\Rightarrow\frac{x^2+4x+30}{x^2-8x+18}=\frac{5}{3}\Rightarrow 2x^2=52x\Rightarrow x=26 \text{ if } x\neq 0$$

$\therefore f(10)=\dfrac{5}{3}=26$ $\qquad\therefore f$ is not injective.

207. If $f\left(x+\dfrac{y}{8},\ x-\dfrac{y}{8}\right)=xy$, then f(m, n) + f(n, m) = 0

A) only when m = n
B) only when $m\neq n$
C) only when m = - n
D) for all m & n

Key. D

Sol. Let $x+\dfrac{y}{8}=m$ $\qquad x-\dfrac{y}{8}=n$

$x=\dfrac{m+n}{2},\ y=4(m-n)$

F(m, n) = $2\left(m^2-n^2\right)$

Similarly f(n, m) = $2\left(n^2-m^2\right)$

= f(m, n) + f(n, m) = 0 $\forall\ m, n$

208. If $y=\sqrt{\log_{\sin x}\left(\dfrac{|x|}{x}\right)}$ then the possible set of values of x and y are

A) $x\in[2n\pi, 2n\pi+\pi],\ y\in\{0,1\}$

B) $x\in(0,\infty), y\in\{1\}$

C) $x\in\bigcup_{n\in W}\left(2n\pi, 2n\pi+\dfrac{\pi}{2}\right)\cup\left(2n\pi+\dfrac{\pi}{2},(2n+1)\pi\right)$ and $y\in\{0\}$

D) $x\in\bigcup_{n\in W}\left(2n\pi,(2n+1)\pi\right)$ and $y\in\{0,1\}$

Key. C

Sol. $\log_{\sin x}\dfrac{|x|}{x}$ $\qquad\Rightarrow\qquad \sin x\in(0,1)$ and $x\in(0,\infty)$

$\therefore \quad x \in \bigcup_{n \in W}\left(2n\pi, 2n\pi+\frac{\pi}{2}\right) \cup \left(2n\pi+\frac{\pi}{2},(2n+1)\pi\right)$ and $y \in \{0\}$

209. Let S be the set of all triangles and R^+ be the set of positive real numbers. Then the function, $f: S \to R^+, F(\Delta) =$ area of Δ, where $\Delta \in S$ is

A) injective but not surjective
B) surjective but not injective
C) injective as well as surjective
D) neither injective nor surjective

Key. B

Sol. Two triangle may have equal areas

$\therefore$ f is not one-one

Since each positive real number can represent area of a triangle

$\therefore$ f is onto

210. $f(x) = |x-1|, f: R^+ \to R$ and $g(x) = e^x, g: [-1, \infty) \to R$ if the function fog(x) is defined, then its domain and range respectively are

A) $(0,\infty) \& [0,\infty)$
B) $[-1,\infty) \& [0,\infty)$
C) $[-1,\infty) \& \left[1-\frac{1}{e},\infty\right)$
D) $[-1,\infty) \& \left[\frac{1}{e},-1,\infty\right)$

Key. B

Sol. $f(x) = |x-1| = \begin{cases} 1-x & 0<x<1 \\ x-1 & x \ge 1 \end{cases}$

$g(x) = e^x \qquad x \ge -1$

$(fog)(x) = \begin{cases} 1-g(x) & 0<g(x)<1 \quad \text{ie} \quad -1 \le x < 0 \\ g(x)-1 & g(x) \ge 1 \quad \text{ie} \quad 0 \le x \end{cases}$

$= \begin{cases} 1-e^x & -1 \le x < 0 \\ e^x - 1 & x \ge 0 \end{cases}$

$\therefore$ domain $= [-1, \infty)$

Fog is decreasing in [- 1, 0] and increasing in $(0, \infty)$

$fog(-1) = 1 - \frac{1}{e}$ and fog (0) = 0

lie $x \to \infty$ fog (x) = ∞ $\therefore$ range $[0,\infty)$

211. The range of the function $f(x) = \log_{\sqrt{2}}(2-\log_2)\left(2-\log_2\left(16\sin^2 x+1\right)\right)$ is

A) $(-\infty,1)$
B) $(-\infty,2)$
C) $(-\infty,1]$
D) $(-\infty,2]$

Key. D

Sol. $f(x) = \log_{\sqrt{2}}\left(2-\log_2\left(16\sin^2 x+1\right)\right)$

$1 \le 16\sin^2 x + 1 \le 17$

$\therefore \quad 0 \le \log_2\left(16\sin^2 x+1\right) \le \log_2 17$

$\therefore \quad 2-\log_2 17 \le 2-\log_2\left(16\sin^2 x+1\right) \le 2$

Now consider

$0 < 2-\log_2\left(16\sin^2 x+1\right) \le 2$

$-\infty < \log_{\sqrt{2}}\left[2-\log_2\left(16\sin^2 x+1\right)\right] \le \log_{\sqrt{2}} 2 = 2$

the range is $(-\infty, 2]$

212. If $f(x).f(y) = f(x) + f(y) + f(xy) - 2 \ \forall x, y \in R$ and iff(x) is not a constant function, then the value of f(1) is equal to

A) 1 B) 2 C) 0 D) – 1

Key. B

Sol. Put x = y = 1, $\left(f(1)^2 = 3f(1) - 2\right)$ $\Rightarrow$ $f(1) = 1$ or 2

Let f(1) = 1, then put y = 1

f(x). f(1) = f(x) + f(1) + f(x) – 2

$\Rightarrow$ f(x) = 1 constant function

$\therefore$ $f(1) \neq 1$, hence f(1) = 2

213. Let $f(x) = \tan x$, $g(f(x)) = f\left(x - \frac{\pi}{4}\right)$, where f(x) and g(x) are real valued functions. For all possible value of x, f(g(x)) =

A) $\tan\left(\frac{x-1}{x+1}\right)$ B) tan(x – 1) – tan (x + 1)

C) $\frac{f(x)+1}{f(x)-1}$ D) $\frac{x - \pi/4}{x + \pi/4}$

Key. A

Sol. $g(f(x)) = \tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$ $\Rightarrow$ $g(x) = \frac{x-1}{x+1}$

$f(g(x)) = \tan\left(\frac{x-1}{x+1}\right)$.

214. Let h(x) = |kx + 5|, domain of f(x) is [- 5, 7], domain of f(h(x)) is [- 6, 1] and range of h(x) is the same as the domain of f(x), then value of k is

A) $\frac{1}{3}$ B) $\frac{4}{5}$ C) 1 D) none of these

Key. D

Sol. $-5 \le |kx + 5| \le 7$ $\Rightarrow$ $-12 \le kx \le 2$ where $-6 \le x \le 1$

$\Rightarrow$ $-6 \le \frac{k}{2}x \le 1$ where $-6 \le x \le 1$

$\therefore$ k = 2 { $\because$ range of h(x) = domain of f(x)}

215. The function $(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is

A) an odd function B) an even function

C) neither an odd nor an even function D) a periodic function

Key. B

Sol. $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1 = \frac{2x + xe^x - x}{2(e^x - 1)} + 1 = \frac{x + xe^x}{2(e^x - 1)} + 1$

$f(-x) = \frac{-x - xe^{-x}}{2(e^{-x} - 1)} + 1 = \frac{x + xe^x}{2(e^x - 1)} + 1$

$\therefore$ f(-x) = f(x) for all x

$\therefore$ f(x) is an even function.

216. Let f: {x, y, z} $\rightarrow$ {1, 2, 3} be a one-one mapping such that only one of the following three statements is true and remaining two are false: $f(x) \neq 2, f(y) = 2, f(z) \neq 1$, then

A) f(x) > f(y) > f(z) B) f(x) < f(y) < f(z) C) f(y) < f(x) < f(z) D) f(y) < f(z) < f(x)

Key. C

Sol. **Case – I** f(x) $\neq$ 2 is true, f(y) = 2 and f(z) $\neq$ 1 are false, then

f(x) = 1 or 3, f(y) = 1 or 3 and f(z) = 1

$\Rightarrow$ f is not one-one

Case – II f(x) $\neq$ 2 is false, f(y) = 2 is true, f(z) $\neq$ 1 is false, then

f(x) = 2, f(y) = 2, f(z) = 1

$\Rightarrow$ not possible

Case – III f(x) $\neq$ 2 is false, f(y) = 2 is false, f(z) $\neq$ 1 is true, then

f(x) = 2, f(y) = 1 or 3 f(z) = 2 or 3

$\Rightarrow$ f(x) = 2, f(z) = 3, f(y) = 1

217. The image of the interval [- 1, 3] under the mapping specified by the function $f(x) = 4x^3 - 12x$ IS

A) [f(+ 1), f(- 1)] B) [f(-1), f(3)] C) [- 8, 16] D) [- 8, 72]

Key. D

Sol. f(x) = 4x $(x^2 - 3)$

$f'(x) = 12x^2 - 12 = 0$

or $x = \pm 1$

f(x) $\in$ [f(1), max (f(- 1), f(3))] = [- 8, 72]

218. If $f(x) = 2\sin^2\theta + 4\cos(x+\theta)\sin x.\sin\theta + \cos(2x+2\theta)$ then value of $f^2(x) + f^2\left(\frac{\pi}{4} - x\right)$

A) 0 B) 1 C) – 1 D) x^2

Key. B

Sol. $f(x) = 2\sin^2\theta + 4\cos(x+\theta)\sin x.\sin\theta + \cos(2x+2\theta)$

$= 2\sin^2\theta + \cos(2x+2\theta) + 2\cos(x+\theta)\cos(x-\theta) - 2\cos^2(x+\theta)$

$= 2\sin^2\theta + 2\cos^2(x+\theta) - 1 + 2\cos^2 x - 2\sin^2\theta - 2\cos^2(x+\theta)$

= cos 2x

$\therefore f^2(x) + f^2\left(\frac{\pi}{4} - x\right) = \cos^2 2x + \sin^2 2x = 1$

219. Let $G(x) = \left(\frac{1}{a^x - 1} + \frac{1}{2}\right)$ F9x), where ‘a’ is a positive real number not equal to 1 and F(x) is an odd function, Which of the following statements is true?

A) G(x) is an odd function

B) G(x) is an even function

C) G(x) is neither even nor odd function

D) Whether G(x) is an odd or even function depends on the value of ‘a’

Key. B

Sol. $G(x) = \left(\frac{1}{a^x - 1} + \frac{1}{2}\right)F(x)$

$G(-x) = \left(\frac{1}{a^{-x} - 1} + \frac{1}{2}\right)F(-x) = -\left(\frac{a^x}{1 - a^x} + \frac{1}{2}\right)F(x) = \left(\frac{a^x}{a^x - 1} - \frac{1}{2}\right)F(x)$

$= \left(\frac{a^x - 1 + 1}{a^x - 1} - \frac{1}{2}\right)F(x) = \left(1 + \frac{1}{a^x - 1} - \frac{1}{2}\right)F(x) = \left(\frac{1}{a^x - 1} + \frac{1}{2}\right)F(x) = G(x)$

$\therefore$ G(x) is an even function.