Date: 14/10/2021

Q1. Is stress a vector quantity?

Q2. The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress?

Q3. What is the Bulk modulus for a perfect rigid body?

Q4. What is the Young's modulus for a perfect rigid body?

Q5. Identical springs of steel and copper are equally stretched. On which, more work will have to be done?

Q6. A steel rod of length $2l$, cross sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the centre. If Y is the Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform.)

Q7. A steel rod ($Y = 2.0 \times 10^{11}$ Nm^{-2}; and $\alpha = 10^{-50}$ C^{-1}) of length 1 m and area of cross-section 1 cm^2 is heated from 0°C to 200°C, without being allowed to extend or bend. What is the tension produced in the rod?

Q8. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by l. Another wire of the same material of length $2L$ and radius $2r$, is pulled by a force $2f$. Find the increase in length of this wire.

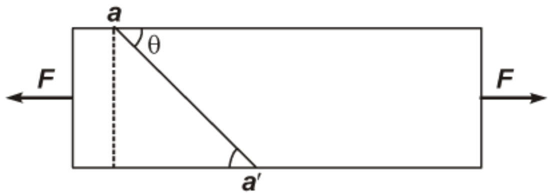
Q9. An equilateral triangle ABC is formed by two Cu rods AB and BC and one Al rod. It is heated in such a way that temperature of each rod increases by ΔT. Find change in the angle ABC. [Coeff. of linear expansion for Cu is α_1, Coeff. of linear expansion for Al is α_2]

Q10. A truck is pulling a car out of a ditch by means of a steel cable that is 9.1 m long and has a radius of 5 mm. When the car just begins to move, the tension in the cable is 800 N. How much has the cable stretched? (Young's modulus for steel is 2×10^{11} Nm^{-2}.)

Q11. Two identical solid balls, one of ivory and the other of wet-clay, are dropped from the same height on the floor. Which one will rise to a greater height after striking the floor and why?

Q12. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%. (The bulk modulus of rubber is 9.8×10^8 N m^{-2}; and the density of sea water is 10^3 kg m^{-3}.)

Q13. Consider a long steel bar under a tensile stress due to forces *F* acting at the edges along the length of the bar (see figure). Consider a plane making an angle θ with the length. What are the tensile and shearing stresses on this plane?



(a) For what angle is the tensile stress a maximum?

(b) For what angle is the shearing stress a maximum?

Q14. A stone of mass *m* is tied to an elastic string of negligible mass and spring constant *k*. The unstretched length of the string is *L* and has negligible mass. The other end of the string is fixed to a nail at a point *P*. Initially the stone is at the same level as the point P. The stone is dropped vertically from point *P*.

(a) Find the distance *y* from the top when the mass comes to rest for an instant, for the first time.

(b) What is the maximum velocity attained by the stone in this drop?

(c) What shall be the nature of the motion after the stone has reached its lowest point?

Q15. In nature, the failure of structural members usually result from large torque because of twisting or bending rather than due to tensile or compressive strains. This process of structural breakdown is called buckling and in cases of tall cylindrical structures like trees, the torque is caused by its own weight bending the structure. Thus the vertical through the centre of gravity does not fall within the base. The elastic torque caused because of this bending about the central axis of the tree is given by $\frac{Y\pi r^4}{4R}$. *Y* is the Young's modulus, *r* is the radius of the trunk and *R* is the radius of curvature of the bent surface along the height of the tree containing the centre of gravity (the neutral surface). Estimate the critical height of a tree for a given radius of the trunk.

Q16. (a) A steel wire of mass μ per unit length with a circular cross section has a radius of 0.1 cm. The wire is of length 10 m when measured lying horizontal, and hangs from a hook on the wall. A mass of 25 kg is hung from the free end of the wire. Assuming the wire to be uniform and lateral strains << longitudinal strains, find the extension in the length of the wire. The density of steel is 7860 kg m^{-3} (Young's modules $Y = 2 \times 10^{11}\ Nm^{-2}$).

(b) If the yield strength of steel is $2.5 \times 10^{8}\ Nm^{-2}$, what is the maximum weight that can be hung at the lower end of the wire?

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S1. $$\text{Stress} = \frac{\text{Magnitude of internal reaction force}}{\text{Area of cross-section}}$$

Therefore, stress is scalar quantity not a vector quantity.

S2. Young's modulus $$Y = \frac{\text{Stress}}{\text{Longitudinal strain}}$$

For same longitudinal strain, $Y \propto \text{Stress}$

$$\therefore \quad \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} = \frac{(\text{Stress})_{\text{steel}}}{(\text{Stress})_{\text{rubber}}} \quad \ldots \text{(i)}$$

But $$Y_{\text{steel}} > Y_{\text{rubber}}$$

$$\therefore \quad \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} > 1$$

Therefore, from Eq. (i),

$$\frac{(\text{Stress})_{\text{steel}}}{(\text{Stress})_{\text{rubber}}} > 1$$

$$\Rightarrow \quad (\text{Stress})_{\text{steel}} > (\text{Stress})_{\text{rubber}}.$$

S3. Bulk modulus $(K) = \dfrac{p}{\Delta V / V} = \dfrac{pV}{\Delta V}$

For perfectly rigid body, change in volume $\Delta V = 0$

$$\therefore \quad K = \frac{pV}{0} = \infty$$

Therefore, bulk modulus for a perfectly rigid body is infinity (∞).

S4. Young's modulus $(Y) = \dfrac{F}{A} \times \dfrac{l}{\Delta l}$

For a perfectly rigid body, change in length $\Delta l = 0$

$$\therefore \quad Y = \frac{F}{A} \times \frac{l}{0} = \infty$$

Therefore, Young's modulus for a perfectly rigid body is infinite (∞).

S5. Work done in stretching a wire is given by

$$W = \frac{1}{2} F \times \Delta l$$

[Where F is applied force and Δl is extension in the wire]

As springs of steel and copper are equality stretched. Therefore, for same force (F),

$$W \propto \Delta l \quad \text{... (i)}$$

$$\text{Young's modulus } (Y) = \frac{F}{A} \times \frac{l}{\Delta l} \Rightarrow \Delta l = \frac{F}{A} \times \frac{l}{Y} \quad \text{[Young's modulas = stress/strain]}$$

As both springs are identical, $\Delta l \propto \frac{1}{Y}$ ... (ii)

From Eqs (i) and (ii), we get $W \propto \frac{1}{Y}$

$$\therefore \quad \frac{W_{steel}}{W_{copper}} = \frac{Y_{copper}}{Y_{steel}} \quad [\text{As, } Y_{steel} > Y_{copper}]$$

$$\Rightarrow \quad W_{steel} < W_{copper}$$

Therefore, more work will be done for stretching copper spring.

S6. Consider an element at r of width dr. Let $T(r)$ and $T(r + dr)$ be the tensions at the two edges.

$$-T(r + dr) + T(r) = \mu\omega^2 r dr \text{ where } \mu \text{ is the mass/length}$$

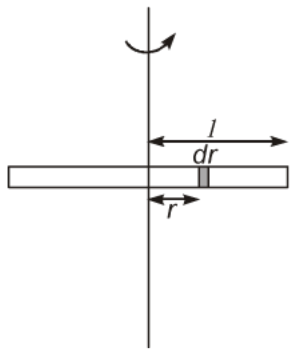
$$-\frac{dT}{dr} = \mu\omega^2 r dr$$

At $\quad r = l \quad T = 0$

$$\Rightarrow \quad C = \frac{\mu\omega^2 l^2}{2}$$

$$\therefore \quad T(r) = \frac{\mu\omega^2}{2}(l^2 - r^2)$$

Let the increase in length of the element dr be $d(\delta)$



$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{(\mu\omega^2/2)(l^2 - r^2)/A}{\frac{d(\delta)}{dr}}$$

$$\therefore \quad \frac{d(\delta)}{dr} = \frac{1}{YA}\frac{\mu\omega^2}{2}(l^2 - r^2)$$

$$\therefore \quad d(\delta) = \frac{1}{YA}\frac{\mu\omega^2}{2}(l^2 - r^2)dr$$

$$\therefore \quad \delta = \frac{1}{YA}\frac{\mu\omega^2}{2}\int_0^l (l^2 - r^2)dr$$

$$= \frac{1}{YA}\frac{\mu\omega^2}{2}\left[l^2 - \frac{l^2}{3}\right]$$

$$= \frac{1}{3YA}\mu\omega^2 l^3 = \frac{1}{3YA}\mu\omega^2 l^2$$

The total change in length is $2\delta = \frac{2}{3YA}\mu\omega^2 l^2$.

S7. Because of the increase in temperature the increase in length per unit length of the rod is

$$\frac{\Delta l}{l_0} = \alpha\Delta T = 10^{-5} \times 2 \times 10^{-2} = 2 \times 10^{-3}$$

Let the compressive tension on the rod be T and the cross sectional area be a, then

$$\frac{T/a}{\Delta l/l_0} = Y$$

$\therefore$
$$T = \frac{\Delta l}{l_0} \times a = 2 \times 10^{11} \times 2 \times 10^{-3} \times 10^{-4}$$
$$= 4 \times 10^4 \text{ N.}$$

S8. Let Y be the Young's modulus of the material. Then

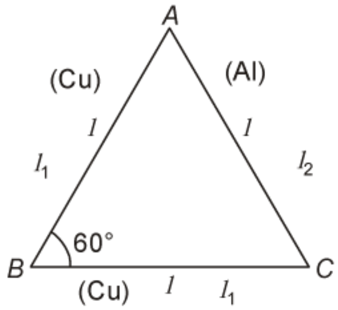
$$Y = \frac{f/\pi r^2}{l/L}$$

Let the increase in length of the second wire be l'. Then

$$\frac{\frac{2f}{4\pi r^2}}{l'/2L} = Y$$

Or,
$$l' = \frac{1}{Y} = \frac{2f}{4\pi r^2} 2L = \frac{l}{L}\frac{\pi r^2}{2f} \times \frac{2f}{4\pi r^2} 2L = l.$$

S9. Let
$$l_1 = AB, \quad l_2 = AC, \quad l_3 = BC$$



$$\cos\theta = \cos 60° = \frac{1}{2}$$

Or,
$$2 l_3 l_1 \cos\theta = l_3^2 + l_1^2 - l_2^2$$

Differentiating

$$2\,(l_3 dl_1 + l_1 dl_3)\cos\theta - 2 l_1 l_3 \sin\theta\, d\theta = 2 l_3 dl_3 + 2 l_1 dl_3 + 2 l_1\alpha_1 - 2 l_2\alpha_2$$

Now,
$$dl_1 = l_1\alpha_1\Delta t$$
$$dl_2 = l_2\alpha_1\Delta t$$
$$dl_3 = l_3\alpha_2\Delta t$$

and
$$l_1 = l_2 = l_3 = l$$

$$(l^2\alpha_1\Delta t + l^2\alpha_1\Delta t\cos\theta + l\sin\theta\, d\theta = l^2\alpha_1\Delta t + l^2\alpha_1\Delta t - l^2\alpha_2\Delta t$$
$$\sin\theta\, d\theta = 2\alpha_1\Delta t\,(1 - \cos\theta) - \alpha_2\Delta t$$

Putting $\theta = 60°$

$$d\theta\,\frac{\sqrt{3}}{2} = 2\alpha_1\Delta t \times (1/2) - \alpha_2\Delta t$$

$$= (\alpha_1 - \alpha_2)\,\Delta t$$

Or, $$d\theta = \frac{2(\alpha_1 - \alpha_2)\,\Delta t}{\sqrt{3}}.$$

S10. Let the increase in length be Δl, then

Length of steel cable $l = 9.1$

Radius $r = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$

Tension in the cable $F = 800\text{ N}$

Young's modulus for steel $Y = 2 \times 10^{11}\text{ N/m}^2$

Change in length $\Delta l = ?$

$$\text{Young's modulus }(Y) = \frac{F}{A} \times \frac{l}{\Delta l} \quad = \quad \Delta l = \frac{F}{\pi r^2} \times \frac{l}{Y}$$

$$\frac{800}{(\pi \times 25 \times 10^{-6})/\Delta l/9.1)} = 2 \times 10^{11}$$

$\therefore$ $$\Delta l = \frac{9.1 \times 800}{\pi \times 25 \times 10^{-6} \times 2 \times 10^{11}}\text{ m}$$

$$\simeq 0.5 \times 10^{-3}\text{ m}.$$

S11. As the ivory ball is more elastic than the wet-clay ball, it will tend to retain its shape instantaneously after the collision. Hence, there will be a large energy and momentum transfer compared to the wet clay ball. Thus, the ivory ball will rise higher after the collision.

S12. Given, Bulk modulus of rubber $(K) = 9.8 \times 10^8\text{ N/m}^2$

Density of sea water $(\rho) = 10^3\text{ kg/m}^3$

Percentage decrease in volume,

$$\left(\frac{\Delta V}{V} \times 100\right) = 0.1 \quad \Rightarrow \quad \frac{\Delta V}{V} = \frac{0.1}{100}$$

$\Rightarrow$ $$\frac{\Delta V}{V} = \frac{1}{1000}$$

Let the depth be h, then the pressure is

$$P = \rho gh = 10^3 \times 9.8 \times h$$

Now, $$\left|\frac{P}{\Delta V/V}\right| = B$$

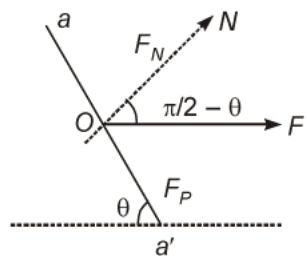
$\therefore$ $$P = B\,\frac{\Delta V}{V} = 9.8 \times 10^8 \times 0.1 \times 10^{-2}$$

$\therefore$ $$h = \frac{9.8 \times 10^8 \times 0.1 \times 10^{-2}}{9.8 \times 10^3} = 10^2\text{ m}.$$

S13. Consider the adjacent diagram.

Let the cross-sectional area of the bar be A. Consider the equilibrium of the plane aa'. A force $\boldsymbol{F}$ must be acting on this plane making an angle $\frac{\pi}{2} - \theta$ with the normal ON. Resolving $\boldsymbol{F}$ into components, along the plane (FP) and normal to the plane.

$$F_P = F\cos\theta$$
$$F_N = F\sin\theta$$



Let the area of the face aa' be A' , then

$$\frac{A}{A'} = \sin\theta$$

$$\therefore \qquad A' = \frac{A}{\sin\theta}$$

$$\text{The tensile stress} = \frac{\text{Paralleel force}}{\text{Area}} = \frac{A\sin\theta}{A'}$$

$$= \frac{F\sin\theta}{A/\sin\theta} = \frac{F}{A}\sin^2\theta$$

$$\text{Shearing stress} = \frac{\text{Paralleel force}}{\text{Area}}$$

$$= \frac{F\sin\theta}{A/\cos\theta} = \frac{F}{A}\sin\theta.\cos\theta$$

$$= \frac{F}{2A}(2\sin\theta.\cos\theta) = \frac{F}{2A}\sin 2\theta$$

(a) For tensile stress to be maximum, $\sin^2\theta = 1$

$\Rightarrow \qquad \sin\theta = 1$

$\Rightarrow \qquad \theta = \frac{\pi}{2}$

(b) For shearing stress to be maximum, $\sin 2\theta = 1$

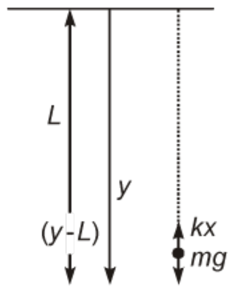
$\Rightarrow \qquad 2\theta = \frac{\pi}{2}$

$\Rightarrow \qquad \theta = \frac{\pi}{4}$

Note: We must not apply the formula for stress directly, forces must be resolved.

S14. (a) Till the stone drops through a length L it will be in free fall. After that the elasticity of the string will force it to a SHM. Let the stone come to rest instantaneously at y.

The loss in P.E. of the stone is the P.E. stored in the stretched string.



$$mgy = \frac{1}{2}k(y-L)^2$$

Or, $$mgy = \frac{1}{2}ky^2 - kyL + \frac{1}{2}kL^2$$

Or, $$\frac{1}{2}ky^2 - (kL+mg)y + \frac{1}{2}kL^2 = 0$$

$$y = \frac{(kL+mg) \pm \sqrt{(kL+mg)^2 - k^2L^2}}{k}$$

$$= \frac{(kL+mg) \pm \sqrt{2mgkL + m^2g^2}}{k}$$

Retain the positive sign and neglet the negative sign.

$\therefore$ $$y = \frac{(kL+mg) + \sqrt{2mgkL + m^2g^2}}{k}$$

(b) The maximum velocity is attained when the body passes, through the "equilibrium, position" *i.e.*, when the instantaneous acceleration is zero. That is $mg - kx = 0$ where x is the extension from L:

$\Rightarrow$ $$mg = kx$$

Let the velocity be v. Then

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = mg(L+x)$$

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = mg(L+x) - \frac{1}{2}kx^2$$

Now $$mg = kx$$

$$x = \frac{mg}{k}$$

$\therefore$ $$\frac{1}{2}mv^2 = mg\left(L + \frac{mg}{k}\right) - \frac{1}{2}k\frac{m^2g^2}{k^2}$$

$$= mgL + \frac{m^2g^2}{k} - \frac{1}{2}\frac{m^2g^2}{k}$$

$$\frac{1}{2}mv^2 = mgL + \frac{1}{2}\frac{m^2g^2}{k}$$

$\therefore$ $$v^2 = 2gL + mg^2/k$$

$$v = (2gL + mg^2/k)^{1/2}$$

(c) Consider the particle at an instantaneous position y. Then

$$\frac{md^2y}{dt^2} = mg - k(y - L)$$

$\Rightarrow$ $$\frac{d^2y}{dt^2} + \frac{k}{m}(y - L) - g = 0$$

Make a transformation of variables:

$$z = \frac{k}{m}(y - L) - g$$

Then $$\frac{d^2z}{dt^2} + \frac{k}{m}z = 0$$

$\therefore$ $$z = A\cos(\omega t + \phi) \text{ where } \omega = \sqrt{\frac{k}{m}}$$

$\Rightarrow$ $$y = \left(L + \frac{m}{k}g\right) + A'\cos(\omega t + \phi)$$

Thus the stone performs SHM with angular frequency ω about the point

$$y_0 = L + \frac{m}{k}g.$$

S15. When the tree is about to buckle

$$Wd = \frac{Y\pi r^4}{4R}$$

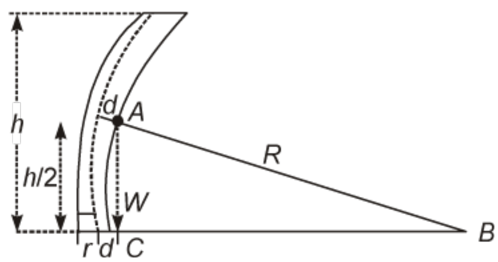
If $R >> h$, then the centre of gravity is at a height $l \simeq \frac{1}{2}h$ from the ground.

From ΔABC

$$R^2 \simeq (R - d)^2 + \left(\frac{1}{2}h\right)^2$$

If $d << R$

$$R^2 \simeq R^2 - 2Rd + \frac{1}{4}h^2$$



$\therefore$ $$d = \frac{h^2}{8R}$$

If w_0 is the weight/volume

$$\frac{Y\pi r^4}{4R} = w_0(\pi r^2 h)\frac{h^2}{8R}$$

$$\Rightarrow \quad h \simeq \left(\frac{2Y}{w_0}\right)^{1/3} r^{2/3}.$$

S16. (a) Consider an element dx at a distance x from the load ($x = 0$). If $T(x)$ and $T(x + dx)$ are tensions on the two cross sections a distance dx apart, then

$T(x + dx) - T(x) = \mu g dx$ (where μ is the mass/length) [$\mu g dx$ = mass of small element dx]

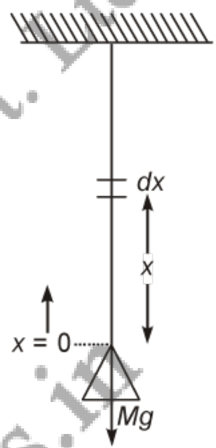
$$\frac{dT}{dx}dx = \mu g dx$$

$$\Rightarrow \quad T(x) = \mu g x + C$$

At $x = 0$, $\quad T(0) = Mg \Rightarrow C = Mg$

$$\therefore \quad T(x) = \mu g x + Mg$$

Let the length dx at x increase by dr, then



$$\frac{T(x)/A}{dr/dx} = Y$$

or $$\frac{dr}{dx} = \frac{1}{YA}T(x)$$

$$\Rightarrow \quad r = \frac{1}{YA}\int_0^L (\mu g x + Mg)\,dx$$

$$= \frac{1}{YA}\left[\frac{\mu g x^2}{2} + Mgx\right]_0^L$$

$$= \frac{1}{YA}\left[\frac{mgL}{2} + MgL\right]$$

(m is the mass of the wire)

$$A = \pi \times (10^{-3})^2 \text{m}^2, \quad Y = 200 \times 10^9 \text{ Nm}^{-2}$$

$$m = \pi \times (10^{-3}) \times 10 \times 7860 \text{ kg}$$

$$\therefore \quad r = \frac{1}{2 \times 10^{11} \times \pi \times 10^{-6}}\left[\frac{\pi \times 786 \times 10^{-7} \times 10 \times 10}{2} + 25 \times 10 \times 10\right]$$

$$= [196.5 \times 10^{-6} + 3.98 \times 10^{-3}] \sim 4 \times 10^{-3} \text{ m}$$

(b) The maximum tension would be at $x = L$.

$$T = \mu g L + Mg = (m + M)g$$

The yield force $= 250 \times 10^6 \times \pi \times (10^{-3})^2 = 250 \times \pi N$

At yield $(m + M)g = 250 \times \pi$

$$m = \pi \times (10^{-3})^2 \times 10 \times 7860 << M \qquad \therefore \quad Mg \sim 250 \times \pi$$

Hence, $$M = \frac{250 \times \pi}{10} = 25 \times \pi \sim 75\,\text{kg}.$$