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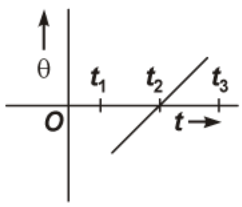
Q1. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry?

Q2. The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point?

Q3. $(n - 1)$ equal point masses each of mass m are placed at the vertices of a regular n-polygon. The vacant vertex has a position vector a with respect to the centre of the polygon. Find the position vector of centre of mass.

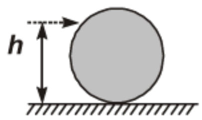
Q4. The variation of angular position θ, of a point on a rotating rigid body, with time t is shown in figure. Is the body rotating clock-wise or anti-clockwise?



Q5. The centre of gravity of a body on the earth coincides with its centre of mass for a 'small' object whereas for an 'extended' object it may not. What is the qualitative meaning of 'small' and 'extended' in this regard?

For which of the following the two coincides? A building, a pond, a lake, a mountain?

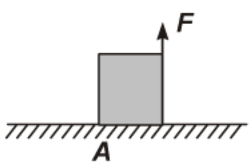
Q6. A uniform sphere of mass m and radius R is placed on a rough horizontal surface (see figure). The sphere is struck horizontally at a height h from the floor. Match the following:



(a)	$h = R/2$	(i)	Sphere rolls without slipping with a constant velocity and no loss of energy.
(b)	$h = R$	(ii)	Sphere spins clockwise, loses energy by friction.
(c)	$h = 3R/2$	(iii)	Sphere spins anti-clockwise, loses energy by friction.
(d)	$h = 7R/5$	(iv)	Sphere has only a translational motion, looses energy by friction.

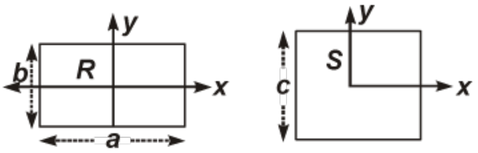
Q7. Find the centre of mass of a uniform (a) half-disc, (b) quarter-disc.

Q8. A uniform cube of mass m and side a is placed on a frictionless horizontal surface. A vertical force F is applied to the edge as shown in figure. Match the following (most appropriate choice):



(a)	$mg/4 < F < mg/2$	(i)	Cube will move up.
(b)	$F > mg/2$	(ii)	Cube will not exhibit motion.
(c)	$F > mg$	(iii)	Cube will begin to rotate and slip at A.
(d)	$F = mg/4$	(iv)	Normal reaction effectively at $a/3$ from A, no motion.

Q9. A uniform square plate S (side c) and a uniform rectangular plate R (sides b, a) have identical areas and masses (see figure).



Show that

(a) $I_{xR}/I_{xS} < 1$ (b) $I_{yR}/I_{yS} > 1$ (c) $I_{zR}/I_{zS} > 1$

Q10. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(a) Does the law of conservation of angular momentum apply to the situation? why?

(b) Find the angular speed of the two-disc system.

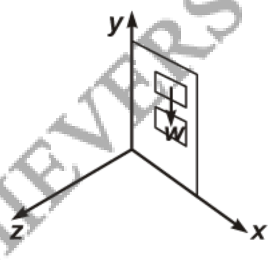
(c) Calculate the loss in kinetic energy of the system in the process.

(d) Account for this loss.

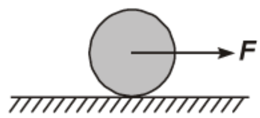
Q11. A wheel in uniform motion about an axis passing through its centre and perpendicular to its plane is considered to be in mechanical (translational plus rotational) equilibrium because no net external force or torque is required to sustain its motion. However, the particles that constitute the wheel do experience a centripetal acceleration directed towards the centre. How do you reconcile this fact with the wheel being in equilibrium?

How would you set a half-wheel into uniform motion about an axis passing through the centre of mass of the wheel and perpendicular to its plane? Will you require external forces to sustain the motion?

Q12. A door is hinged at one end and is free to rotate about a vertical axis (see figure). Does its weight cause any torque about this axis? Give reason for your answer.



Q13. A uniform disc of radius R, is resting on a table on its rim. The coefficient of friction between disc and table is μ (see figure). Now the disc is pulled with a force F as shown in the figure. What is the maximum value of F for which the disc rolls without slipping?



Q14. Two cylindrical hollow drums of radii R and $2R$, and of a common height h, are rotating with angular velocities ω (anti-clockwise) and ω (clockwise), respectively. Their axes, fixed are parallel and in a horizontal plane separated by $(3R + \delta)$. They are now brought in contact $(\delta \to 0)$.

(a) Show the frictional forces just after contact.

(b) Identify forces and torques external to the system just after contact.

(c) What would be the ratio of final angular velocities when friction ceases?

Q15. A disc of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_k.

(a) What was the velocity of its centre of mass before being brought in contact with the table?

(b) What happens to the linear velocity of a point on its rim when placed in contact with the table?

(c) What happens to the linear speed of the centre of mass when disc is placed in contact with the table?

(d) Which force is responsible for the effects in (b) and (c).

(e) What condition should be satisfied for rolling to begin?

(f) Calculate the time taken for the rolling to begin.

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S1. $I = \sum m_i r_i^2$. All the mass in a cylinder lies at distance R from the axis of symmetry but most of the mass of a solid sphere lies at a smaller distance than R and thus moment of inertia is small for sphere.

S2. No. Given $\sum_i F_i \neq 0$

The sum of torques about a certain point '0

$$\sum_i r_i \times F_i = 0$$

The sum of torques about any other point O′,

$$\sum_i (r_i - a) \times F_i = \sum_i r_i \times F_i - a \times \sum_i F_i$$

Here, the second term need not vanish.

Therefore, sum of all the torques about any arbitrary point need not be zero necessarily.

S3. Let '***b***' be the position vector of the centre of mass of a regular *n*-polygon.

$(n - 1)$ equal point masses are placed at $(n - 1)$ vertices of the regular *n*-polygon, therefore, for its centre of mass

$$f_{CM} = \frac{(n-1)mb + ma}{(n-1)m + m} = 0 \qquad (\because \text{ Centre of mass lies at centre})$$

$$\Rightarrow \qquad (n - 1)\, mb + ma = 0$$

$$\Rightarrow \qquad b = -\frac{a}{(n-1)}.$$

S4. Positive slope indicates anticlockwise rotation which is traditionally taken as positive.

S5. When the vertical height of the object is very small as compared to Earth's radius, we call the object small, otherwise it is extended.

(a) Building and pond are small objects.

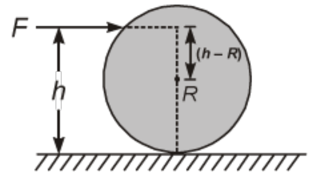
(b) A deep lake and a mountain are examples of extended objects.

S6. Consider the diagram where a sphere of m and radius R, struck horizontally at height h above the floor

The sphere will roll without slipping when $\omega = \frac{v}{r}$, where, v is linear velocity and ω is angular velocity of the sphere.

Now, angular momentum of sphere, about centre of mass.

[We are applying conservation of angular momentum just before and after struck]



$$mv(h-R) = I\omega = \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)$$

$\Rightarrow$ $$mv(h-R) = \frac{2}{5}mvR$$

$$h - R = \frac{2}{5}R \quad \Rightarrow \quad h = \frac{7}{5}R$$

Therefore, the sphere will roll without slipping with a constant velocity and hence, no loss of energy, so

(d) $\to$ (i)

Torque due to applied force, F about centre of mass

$$\tau = F(h-R) \qquad \text{(Clockwise)}$$

For t = 0, h = R, sphere will have only translational motion. It would lose energy by friction.

Hence, (b) $\to$ (iv)

The sphere will spin clockwise when $\tau > 0 \Rightarrow h > R$

Therefore, (c) $\to$ (ii)

The sphere will spin anti-clockwise when $t < 0 \Rightarrow h < R$

Therefore, (a) $\to$ (iii).

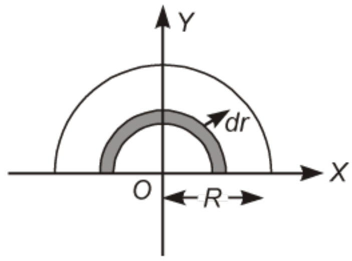
S7. Let M and R be the mass and radius of the half-disc, mass per unit area of the half-disc.

$$m = \frac{M}{\frac{1}{2}\pi R^2} = \frac{2M}{\pi R^2}$$

(a) The half-disc can be supposed to be consists of a large number of semicircular rings of mass dm and thickness dr and radii ranging from $r = 0$ to $r = R$.

Surface area of semicircular ring of radius r and of thickness $dr = \frac{1}{2}\ 2\pi r \times dr = \pi r dr$

$\therefore$ Mass of this elementary ring,



$$dm = \pi r dr \times \frac{2M}{\pi R^2}$$

$$dm = \frac{2M}{R^2}\ r dr$$

If (x, y) are coordinates of centre of mass of the element,

$$(x, y) = \left(0, \frac{2r}{\pi}\right)$$

Therefore, $x = 0$ and $y = \frac{2r}{\pi}$

Let x_{CM} and y_{CM} be the coordinates of the centre of mass of the semicircular disc.

Then, $$x_{CM} = \frac{1}{M}\int_0^R x\, dm = \frac{1}{M}\int_0^R 0\, dm = 0$$

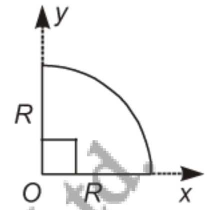
$$y_{CM} = \frac{1}{M}\int_0^R y\,dm = \frac{1}{M}\int_0^R \frac{2r}{\pi}\times\left(\frac{2M}{R^2}rdr\right)$$

$$= \frac{4}{\pi R^2}\int_0^R r^2 dr = \frac{4}{\pi R^2}\left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{4}{\pi R^2}\times\left(\frac{R^3}{3}-0\right) = \frac{4R}{3\pi}$$

$\therefore$ Centre of mass of the semicircular disc = $\left(0, \frac{4R}{3\pi}\right)$.

(b) Centre of mass of a uniform quarter-disc.



Mass per unit area of the quarter disc = $\frac{M}{\frac{\pi R^2}{4}} = \frac{4M}{\pi R^2}$.

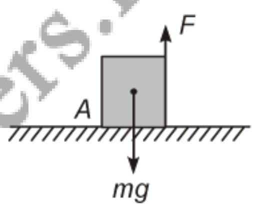
Using symmetry

For a half-disc along y-axis centre of mass will be at $x = \frac{4R}{3\pi}$.

For a half-disc x-axis centre of mass will be at $x = \frac{4R}{3\pi}$.

Hence, for the quarter-disc centre of mass = $\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$.

S8. Consider the below diagram



Momentum of the force F about point A,

$$\tau_1 = F\times a \quad \text{(anti-clockwise)}$$

Momentum of weight mg of the cube about point A.

$$\tau_2 = mg\times\frac{a}{2} \quad \text{(clockwise)}$$

Cube will not exhibit motion, if $\tau_1 = \tau_2$

[$\because$ In this case, both the torque will cancel the effect of each other]

$\therefore$ $F\times a = mg\times\frac{a}{2} \quad \Rightarrow \quad F > \frac{mg}{2}$

Cube will rotate only when, $\tau_1 > \tau_2$

Let normal reaction is acting at $\frac{a}{3}$ from point A, then

$$mg\times\frac{a}{3} = F\times a \quad \text{or} \quad F = \frac{mg}{3}$$ [For no motion]

When, $F = \frac{mg}{4}$ which is less than $\frac{mg}{3}$, $\left[F < \frac{mg}{3}\right]$

there will be no motion.

$\therefore$ (a) $\rightarrow$ (ii) (b) $\rightarrow$ (iii) (c) $\rightarrow$ (i) (d) $\rightarrow$ (iv)

S9. (a) Area of square = area of rectangle $\Rightarrow c^2 = ab$

$$\frac{I_{xR}}{I_{xS}} \times \frac{I_{yR}}{I_{yS}} = \frac{b^2}{c^2} \times \frac{a^2}{c^2} = \left(\frac{ab}{c^2}\right)^2 = 1$$

(a) and (b) $\frac{I_{yR}}{I_{yS}} > \frac{I_{xR}}{I_{xS}} \Rightarrow \frac{I_{yR}}{I_{yS}} > 1$

and $\frac{I_{xR}}{I_{xS}} < 1.$

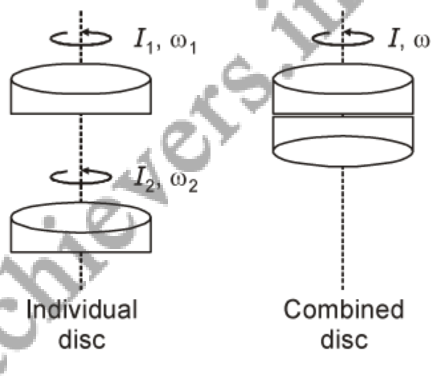
(c) $I_{zr} - I_{zS} \propto (a^2 + b^2 - 2c^2)$

$= a^2 + b^2 - 2ab > 0$

$\therefore$ $(I_{zr} - I_{zS}) > 0$

$\therefore$ $\frac{I_{zR}}{I_{zS}} > 1.$

S10. Consider the diagram below:



Let the common angular velocity of the system is ω.

(a) Yes, the law of conservation of angular momentum can be applied. Because, there is no net external torque on the system of the two discs.

External forces, gravitation and normal reaction, act through the axis of rotation, hence, produce no torque.

(b) By conservation of angular momentum.

$L_f = L_i$

$\Rightarrow$ $I\omega = I_1\omega_1 + I_2\omega_2.$

$\Rightarrow$ $\omega = \frac{I_1\omega_1 + I_2\omega_2}{I} = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}.$ $(\because I = I_1 + I_2)$

(c) $K_f = \frac{1}{2}(I_1 + I_2)\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)} = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}$

$K_i = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$

$$\Delta K = K_f - K_i = -\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

(d) The loss in kinetic energy is due to the work against the friction between the two discs.

S11. The centripetal acceleration in a wheel arise due to the internal elastic forces which in pairs cancel each other; being part of a symmetrical system.

In a half wheel the distribution of mass about its centre of mass (axis of rotation) is not symmetrical. Therefore, the direction of angular momentum does not coincide with the direction of angular velocity and hence an external torque is required to maintain rotation.

S12. No. A force can produce torque only along a direction normal to itself as $\tau = r \times f$. So, when the door is in the xy-plane, the torque produced by gravity can only be along $\pm z$-direction, never about an axis passing through y-direction.

S13. Let the acceleration of the centre of mass of disc be 'a', then

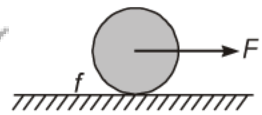
$$Ma = F - f \qquad \text{... (i)}$$

The angular acceleration of the disc is $\alpha = a/R$. (if there is no sliding).

Then, $\tau = I\alpha \qquad \left(\frac{1}{2}MR^2\right)\alpha = Rf \qquad$ ... (ii)

$$\Rightarrow \qquad Ma = 2f$$

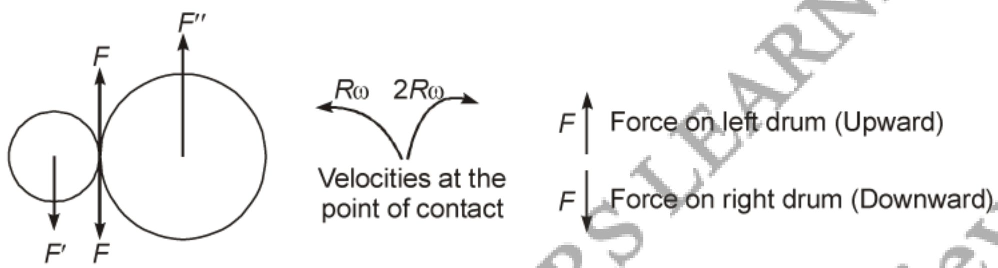
Thus, $f = F/3$. Since there is no sliding,



$$\Rightarrow \qquad f \leq \mu mg$$

$$\Rightarrow \qquad F \leq 3\mu Mg.$$

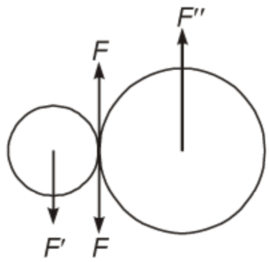
S14. (a)



(b) $F' = F = F''$ where F and F'' and external forces through support.

$F_{net} = 0$

External torque = $F \times 3R$, anticlockwise.

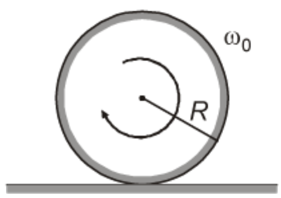


(c) Let ω_1 and ω_2 be final angular velocities (anticlockwise and clockwise respectively)

Finally there will be no friction.

Hence, $\qquad R\omega_1 = 2R\omega_2 \quad \Rightarrow \frac{\omega_1}{\omega_2} = 2.$

S15. (a) Before being brought in contact with the table the disc was in pure rotational motion hence,

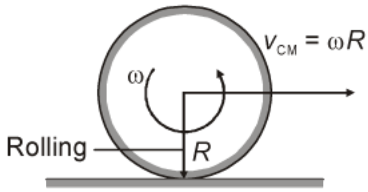


$$v_{CM} = 0.$$

(b) When the disc is placed in contact with the table due to friction centre of mass acquire some linear velocity.

(c) When the disc is placed in contact with the table with the table due to friction centre of mass acquires some linear velocity.

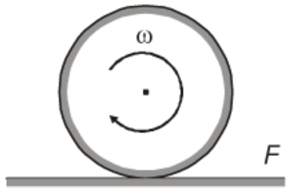
(d) Friction is responsible for the effects in (b) and (c).



(e) When rolling starts $v_{CM} = \omega R$.

where ω is angular speed of the disc when rolling just starts.

(f) Acceleration produced in centre of mass due to friction:



$$a_{cm} = \frac{F}{m} = \frac{\mu_k mg}{m} = \mu_k g \, .$$

Angular acceleration produced by the torque due to friction,

$$\alpha = \frac{\tau}{I} = \frac{\mu_k mgR}{I}$$

$\therefore$ $$v_{cm} = u_{cm} + a_{cm} t \quad \Rightarrow \quad v_{cm} = \mu_k gt$$

and $$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \omega = \omega_0 - \frac{\mu_k mgR}{I} t$$

For rolling without slipping,

$$\frac{v_{cm}}{R} = \omega_0 - \frac{\mu_k mgR}{I} t$$

$$\frac{\mu_k gt}{R} = \omega_0 - \frac{\mu_k mgR}{I} t$$

$$t = \frac{R\omega_0}{\mu_k g \left(1 + \frac{mR^2}{I}\right)}.$$