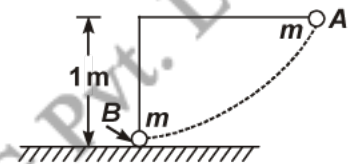


- Q14.** A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy?
- Q15.** In an elastic collision of two billiard balls, which of the following quantities remain conserved during the short time of collision of the balls (*i.e.*, when they are in contact).
- (a) Kinetic energy. (b) Total linear momentum?
- Give reason for your answer in each case.

- Q16.** A ball of mass m , moving with a speed $2v_0$, collides in elastically ($e > 0$) with an identical ball at rest. Show that
- (a) For head-on collision, both the balls move forward.
 (b) For a general collision, the angle between the two velocities of scattered balls is less than 90° .

- Q17.** The bob A of a pendulum released from horizontal to the vertical hits another bob B of the same mass at rest on a table as shown in figure. If the length of the pendulum is 1 m , calculate
- (a) the height to which bob A will rise after collision.
 (b) the speed with which bob B starts moving.



Neglect the size of the bobs and assume the collision to be elastic.

- Q18.** Consider a one-dimensional motion of a particle with total energy E . There are four regions A , B , C and D in which the relation between potential energy V , kinetic energy (K) and total energy E is as given below:

Region A : $V > E$ Region B : $V < E$ Region C : $K > E$ Region D : $V > K$

State with reason in each case whether a particle can be found in the given region or not.

- Q19.** A balloon filled with helium rises against gravity increasing its potential energy. The speed of the balloon also increases as it rises. How do you reconcile this with the law of conservation of mechanical energy? You can neglect viscous drag of air and assume that density of air is constant.
- Q20.** A rocket accelerates straight up by ejecting gas downwards. In a small time interval Δt , it ejects a gas of mass Δm at a relative speed u . Calculate K.E. of the entire system at $t + \Delta t$ and t and show that the device that ejects gas does work $= (1/2) \Delta m u^2$ in this time interval (neglect gravity).

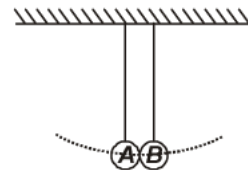
- Q21.** A curved surface is shown in figure. The portion BCD is free of friction. There are three spherical balls of identical radii and masses. Balls are released from rest one by one from A which is at a slightly greater height than C .

With the surface AB , ball 1 has large enough friction to cause rolling down without slipping; ball 2 has a small friction and ball 3 has a negligible friction.

- (a) For which balls is total mechanical energy conserved?
 (b) Which ball (s) can reach D ?
 (c) For balls which do not reach D , which of the balls can reach back A ?



Q22. Two pendulums with identical bobs and lengths are suspended from a common support such that in rest position the two bobs are in contact (see figure). One of the bobs is released after being displaced by 10° so that it collides elastically head-on with the other bob.



- (a) Describe the motion of two bobs.
 (b) Draw a graph showing variation in energy of either pendulum with time, for $0 \leq t \leq 2T$, where T is the period of each pendulum.

Q23. Two identical steel cubes (masses 50 g, side 1 cm) collide head-on face to face with a speed of 10 cm/s each. Find the maximum compression of each. Young's modulus for steel = $Y = 2 \times 10^{11} \text{ N/m}^2$.

Q24. An adult weighing 600 N raises the centre of gravity of his body by 0.25 m while taking each step of 1 m length in jogging. If he jogs for 6 km, calculate the energy utilised by him in jogging assuming that there is no energy loss due to friction of ground and air. Assuming that the body of the adult is capable of converting 10% of energy intake in the form of food, calculate the energy equivalents of food that would be required to compensate energy utilised for jogging.

Q25. On complete combustion a litre of petrol gives off heat equivalent to $3 \times 10^7 \text{ J}$. In a test drive a car weighing 1200 kg, including the mass of driver, runs 15 km per litre while moving with a uniform speed on a straight track. Assuming that friction offered by the road surface and air to be uniform, calculate the force of friction acting on the car during the test drive, if the efficiency of the car engine were 0.5.

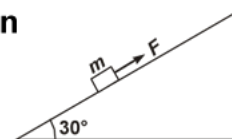
Q26. A raindrop of mass 1.00 g falling from a height of 1 km hits the ground with a speed of 50 m s^{-1} . Calculate

- (a) the loss of P.E. of the drop. (b) the gain in K.E. of the drop.
 (c) Is the gain in K.E. equal to loss of P.E.? If not why.

Take $g = 10 \text{ m s}^{-2}$

Q27. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 30° by a force of 10 N parallel to the inclined surface (see figure). The coefficient of friction between block and the incline is 0.1. If the block is pushed up by 10 m along the incline, calculate

- (a) work done against gravity (b) work done against force of friction
 (c) increase in potential energy (d) increase in kinetic energy
 (e) work done by applied force.



- S1.** Given, Mass (m) = 100 kg
 Height (h) = 10 m
 Time duration (t) = 20 s

Power = Rate of work done

$$= \frac{\text{Change of P.E.}}{\text{Time}} = \frac{mgh}{t}$$

$$= \frac{100 \times 9.8 \times 10}{20}$$

$$= 5 \times 98 = 490 \text{ W.}$$

- S2.** Work done = change in K.E.

Both bodies had same K.E. and hence same amount of work is needed to be done. Since force applied is same, they would come to rest within the same distance.

- S3.** When a charged particle moves in a uniform normal magnetic field, the path of the particle is circular, as given field is uniform hence, radius of the circular path is also constant.

As the force is central and movement is tangential work done by the force is zero. As speed is also constant we can say that $\Delta K = 0$.

- S4.** No, because resistive force of air also acts on the body which is a non-conservative force. So the gain in K.E. would be smaller than the loss in P.E.

- S5.** No, work done over each closed path is necessarily zero only if all the forces acting on the system are conservative.

- S6.** Force of gravity acts on the car vertically downward while car is moving along horizontal road, i.e., angle between them is 90° .

Work done by the car against gravity

$$W = Fs \cos 90^\circ = 0, \quad [\because \cos 90^\circ = 0]$$

- S7.** (a) Force is applied on the body to lift it in upward direction and displacement of the body is also in upward direction, therefore, angle between the applied force and displacement is $\theta = 0^\circ$.

\therefore Work done by the applied force

$$W = Fs \cos \theta = Fs \cos 0^\circ = Fs \quad [\because \cos 0^\circ = 1]$$

i.e., $W = \text{Positive.}$

- (b) The gravitational force acts in downward direction and displacement in upward direction, therefore, angle between them is $\theta = 180^\circ$.

∴ Work done by the gravitational force

$$W = Fs \cos 180^\circ = -Fs \quad [\because \cos 180^\circ = -1]$$

S8. When the elevator is descending, then electric power is required to prevent it from falling freely under gravity.

Also, as the weight inside the elevator increases its speed of descending increases, therefore, there should be a limit on the number of passengers in the elevator to prevent the elevator from descending with large velocity.

S9. Given, Mass of the system (m) = 50,000 kg

Speed of the system (v) = 36 km/h

$$= \frac{36 \times 1000}{60 \times 60} = 10 \text{ m/s}$$

Compression of the spring (x) = 10 m

$$\text{K.E. of the system} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 50000 \times (10)^2$$

$$= 25000 \times 100 = 2.5 \times 10^6 \text{ J}$$

Since, 90% of K.E. of the system is lost due to friction, therefore, energy transferred to shock absorber, is given by

$$\Delta E = \frac{1}{2} kx^2 = 10\% \text{ of total K.E. of the system.}$$

$$= \frac{10}{100} \times 2.5 \times 10^6 \quad \text{or} \quad k = \frac{2 \times 2.5 \times 10^6}{10 \times (1)^2}$$

$$= 50 \times 10^5 \text{ N/m}$$

S10. Given, average work done by a human heart per beat = 0.5 J

Total work done during 72 beats

$$P = \frac{\Delta E}{\Delta t} = \frac{0.5 \times 72}{60} = 0.6 \text{ watts.}$$

S11. Given, Average mass of rain drop (m) = 3.0×10^{-5} kg

Average terminal velocity = (V) = 9 m/s

Height (h) = 100 cm = 1 m

Density of water (ρ) = 103 kg/m³

Area of the surface (A) = 1 m²

Volume of the water due to rain (V) = Area \times height

$$= A \times h = 1 \times 1 = 1 \text{ m}^3$$

$$\begin{aligned} \text{Mass of the water due to rain } (M) &= \text{Volume} \times \text{density} \\ &= V \times \rho = 1 \times 10^3 = 10^3 \text{ kg} \end{aligned}$$

$$\begin{aligned} \therefore \text{Energy transferred to the surface} &= \frac{1}{2} MV^2 \\ &= \frac{1}{2} \times 10^3 \times (9)^2 \\ &= 40.5 \times 10^3 \text{ J} = 4.05 \times 10^4 \text{ J.} \end{aligned}$$

S12. K.E. versus x graph

We know that Total M.E. = K.E. + P.E.

$$\Rightarrow E_0 = \text{K.E.} + V(x)$$

$$\Rightarrow \text{K.E.} = E_0 - V(x)$$

$$\text{at } A_1 \ x = 0, \ V(x) = E_0$$

$$\Rightarrow \text{K.E.} = E_0 - E_0 = 0$$

$$\text{at } B_1 \ V(x) < E_0$$

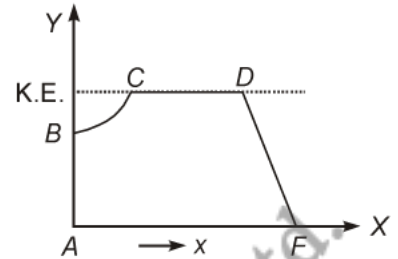
$$\Rightarrow \text{K.E.} > 0$$

$$\text{at } C \text{ and } D_1 \ V(x) = 0$$

$$\Rightarrow \text{K.E. is maximum at } F_1 \ V(x) = E_0$$

$$\text{Hence, K.E.} = 0$$

The variation is shown in adjacent diagram.



[Positive]

Velocity versus x graph

$$\text{As } \text{K.E.} = \frac{1}{2} mv^2$$

$$\therefore \text{At } A \text{ and } F, \text{ where K.E.} = 0, \ v = 0$$

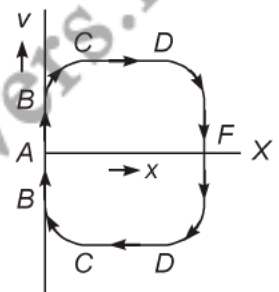
At C and D, K.E. is maximum. Therefore v is \pm max.

At B, K.E. is positive but not maximum.

Therefore, v is \pm some value

[< max.]

The variation is shown in the diagram.



S13. When bob is whirled into a vertical circle, the required centripetal force is obtained from the tension in the string. When string is cut, tension in string becomes zero and centripetal force is not provided, hence, bob start to move in a straight line path along the direction of its velocity.

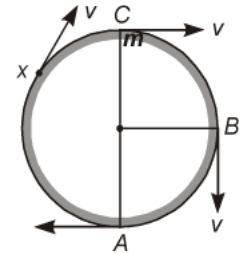
(a) At point B, the velocity of B is vertically downward, therefore, when string is cut at B, bob moves vertically down ward.

(b) At point C, the velocity is along the horizontal towards right, therefore, when string is cut at C, bob moves horizontally towards right.

Also, the bob moves under gravity simultaneously with horizontal uniform speed. So, it traversed on a parabolic path with vertex at C.

- (c) At point X, the velocity of the bob is along the tangent drawn at point X, therefore when string is cut at point C, bob moves along the tangent at that point X.

Also, the bob move under gravity simultaneously with horizontal uniform speed. So, it traversed on a parabolic path with vertex higher than C.

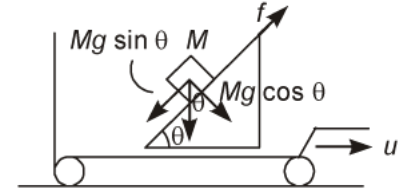


- S14.** Consider the adjacent diagram. As the block M is at rest.

Hence,

$$f = \text{frictional force} = Mg \sin \theta$$

The force of friction acting between the block and incline opposes the tendency of sliding of the block. Since, block is not in motion, therefore, no work is done by the force of friction. Hence, no dissipation of energy takes place.



- S15.** (b) Total linear momentum.

While balls are in contact, there may be deformation which means elastic potential energy which came from part of K.E. Momentum is always conserved.

- S16.** (a) For head on collision:

$$\text{Conservation of momentum} \Rightarrow 2mv_0 = mv_1 + mv_2$$

$$\text{Or} \quad 2v_0 = v_1 + v_2$$

$$\text{and} \quad e = \frac{v_2 - v_1}{2v_0}$$

$$\Rightarrow v_2 = v_1 + 2v_0 e \quad [\text{Here } e = \text{coefficient of restitution}]$$

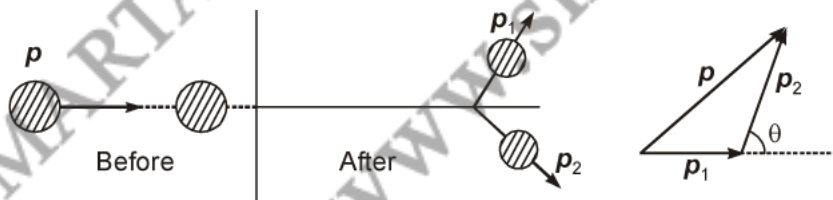
$$\therefore 2v_1 = 2v_0 - 2ev_0$$

$$\therefore v_1 = v_0(1 - e)$$

Since $e < 1 \Rightarrow v_1$ has the same sign as v_0 , therefore the ball moves on after collision.

- (b) Conservation of momentum $\Rightarrow \mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$

$$\text{But K.E. is lost} \Rightarrow \frac{p^2}{2m} > \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$$



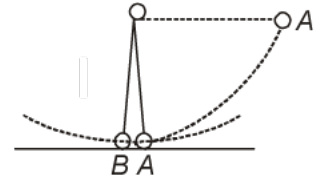
$$\therefore p^2 > p_1^2 + p_2^2.$$

Thus \mathbf{p} , \mathbf{p}_1 and \mathbf{p}_2 are related as shown in the figure.

θ is acute (less than 90°) ($p^2 = p_1^2 + p_2^2$ would give $\theta = 90^\circ$)

- S17.** When ball A reaches bottom point its velocity is horizontal, hence, we can apply conservation of linear momentum in the horizontal direction.

- (a) Two balls have same mass and the collision between them is elastic, therefore, ball A transfers its entire linear momentum to ball B. Hence, ball A will come to at rest after collision and does not rise at all.



- (b) Speed with which bob B starts moving
 = Speed with which bob A hits bob B

$$= \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1}$$

$$= \sqrt{19.6} = 4.42 \text{ m/s}$$

Note: When the bob A is at the bottommost point, its velocity is horizontal and tension is the external force on the bob but still momentum can be considered to be conserved in horizontal direction, because the tension has no effect in horizontal direction at the bottommost point.

S18. We know that

$$\text{Total energy } E = \text{P.E.} + \text{K.E.}$$

$$\Rightarrow E = V + K \quad \dots (i)$$

For Region A : Given, $V > E$, From Eq. (i)

$$K = E - V$$

$$\text{as } V > E \Rightarrow E - V < 0.$$

Hence, $K < 0$, this is not possible.

For Region B : Given, $V < E \Rightarrow E - V > 0$

This is possible because total energy can be greater than P.E. (V).

For Region C : Given, $K > E \Rightarrow K - E > 0$

$$\text{From Eq. (i), P.E.} = V = E - K < 0$$

Which is possible, because P.E. can be negative.

For Region D : Give, $V > K$

This is possible because for a system P.E. (V) may be greater than K.E. (K)

S19. Let m , V , ρ_{He} denote respectively the mass, volume and density of helium balloon and ρ_{air} be density of air

Volume V of balloon displaces volume V of air.

$$\text{So, } V(\rho_{air} - \rho_{He})g = ma \quad \dots (i)$$

Integrating with respect to t ,

$$V(\rho_{air} - \rho_{He})g = mv \quad [\text{Here net force is upthrust which gives } ma]$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}m \frac{V^2}{m^2} (\rho_{air} - \rho_{He})^2 g^2 t^2$$

$$= \frac{1}{2m} V^2 (\rho_{air} - \rho_{He})^2 g^2 t^2 \quad \dots (ii)$$

If the balloon rises to a height h , from $s = ut + \frac{1}{2} at^2$.

we get
$$h = \frac{1}{2} at^2 = \frac{1}{2} \frac{V^2(\rho_{\text{air}} - \rho_{\text{He}})}{m} gt^2 \quad \dots \text{(iii)}$$

From Eqs. (iii) and (ii),

$$\begin{aligned} \frac{1}{2} mv^2 &= [V(\rho_a - \rho_{\text{He}})g] \left[\frac{1}{2m} V(\rho_a - \rho_{\text{He}}) gt^2 \right] \\ &= V(\rho_a - \rho_{\text{He}}) gh \end{aligned}$$

Rearranging the terms,

$$\Rightarrow \frac{1}{2} mv^2 + V_{\rho_{\text{He}}} gh = V_{\rho_{\text{air}}} hg$$

$$\Rightarrow \text{K.E.}_{\text{balloon}} + \text{P.E.}_{\text{balloon}} = \text{change in P.E. of air.}$$

So, as the balloon goes up, an equal volume of air comes down, increase in P.E. and K.E. of the balloon is at the cost of P.E. of air [which comes down].

S20. Let M be the mass of rocket at any time t and v_1 the velocity of rocket at the same time t .

Let $\Delta m =$ mass of gas ejected in time interval Δt .

Relative speed of gas ejected = u .

Consider at time $t + \Delta t$

$$\begin{aligned} (\text{K.E.})_t + \Delta t &= \text{K.E. of rocket} + \text{K.E. of gas} \\ &= \frac{1}{2} (M - \Delta m) (v + \Delta v)^2 + \frac{1}{2} \Delta m (v - u)^2 \\ &\quad \text{rocket} \qquad \qquad \qquad \text{gas} \\ &= \frac{1}{2} Mv^2 - Mv\Delta v - \Delta mvu + \frac{1}{2} \Delta mu^2 \end{aligned}$$

$$(\text{K.E.})_t = \text{K.E. of the rocket at time } t = \frac{1}{2} Mv^2$$

$$\begin{aligned} \Delta K &= (\text{K.E.})_t + \Delta t - (\text{K.E.})_t \\ &= (M\Delta v - \Delta mu)v + \frac{1}{2} \Delta mu^2 \end{aligned}$$

Since, action-reaction forces are equal.

Hence,
$$M \frac{dv}{dt} = \frac{dm}{dt} |u|$$

$$\Rightarrow M\Delta v = \Delta mu$$

$$\Delta v = \frac{1}{2} \Delta mu^2$$

Now, by work-energy theorem,

$$\begin{aligned} \Delta K &= \Delta W \\ \Delta W &= \frac{1}{2} \Delta mu^2 \end{aligned}$$

- S21.** (a) As ball 1 is rolling down without slipping there is no dissipation of energy hence, total mechanical energy is conserved.
Ball 3 is having negligible friction hence, there is no loss of energy.
- (b) Ball 1 acquires rotational energy, ball 2 loses energy by friction. They cannot cross at C. Ball 3 can cross over.
- (c) Ball 1, 2 turn back before reaching C. Because of loss of energy, ball 2 cannot reach back to A. Ball 1 has a rotational motion in "wrong" sense when it reaches B. It cannot roll back to A, because of kinetic friction.

- S22.** (a) Consider the adjacent diagram in which the bob B is displaced through an angle θ and released.

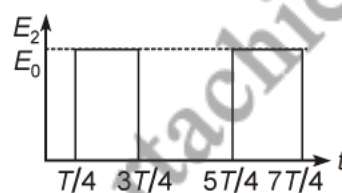
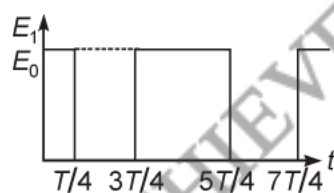
At $t = 0$, suppose bob B is displaced by $\theta = 10^\circ$ to the right. It is given potential energy $E_1 = E$. Energy of A, $E_2 = 0$.

When B is released, it strikes A at $t = T/4$. In the head-on elastic collision between B and A comes to rest and A gets velocity of B. Therefore, $E_1 = 0$ and $E_2 = E$. At $t = 2T/4$, B reaches its extreme right position when K.E. of A is converted into P.E. = $E_2 = E$. Energy of B, $E_1 = 0$.

At $t = 3T/4$, A reaches its mean position, when its P.E. is converted into K.E. = $E_2 = E$. It collides elastically with B and transfers whole of its energy to B. Thus, $E_2 = 0$ and $E_1 = E$. The entire process is repeated.

- (b) The values of energies of B and A at different time intervals are tabulated here. The plot of energy with time $0 \leq t \leq 2T$ is shown separately for B and A in the figure below.

Time (t)	0	$T/4$	$2T/4$	$3T/4$	$4T/4$	$5T/4$	$6T/4$	$7T/4$	$8T/4$
Energy of A (E_1)	E	0	0	E	E	0	0	E	E
Energy of B (E_2)	0	E	E	0	0	E	E	0	0



- S23.** Hooke's law: Stress \propto Strain \Rightarrow Stress = Y strain

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

where A is the surface area and L is length of the side of the cube. If k is spring or compression constant, then $F = k\Delta L$

$$\therefore k = Y \frac{A}{L} = YL$$

$$\text{Initial K.E.} = 2 \times \frac{1}{2} mv^2 = 5 \times 10^{-4} \text{ J}$$

$$\text{Final P.E.} = 2 \times \frac{1}{2} k(\Delta L)^2$$

$$\therefore \Delta L = \sqrt{\frac{\text{K.E.}}{k}} = \sqrt{\frac{\text{K.E.}}{YL}} = \sqrt{\frac{5 \times 10^{-4}}{2 \times 10^{11} \times 0.1}} = 1.58 \times 10^{-7} \text{ m.}$$

S24. Given, Weight of the adult (w) = $mg = 600 \text{ N}$

Height of each step (h) = 0.25 m

Length of each step = 1 m

Total distance travelled = $6 \text{ km} = 6000 \text{ m}$

$$\therefore \text{Total number of steps} = \frac{6000}{1} = 6000.$$

Total energy utilised in jogging = $n \times mgh$

$$= 6000 \times 600 \times 0.25 \text{ J} = 9 \times 10^5 \text{ J}$$

Since, 10% of intake energy is utilised in jogging

$$\therefore \text{Total intake energy} = 10 \times 9 \times 10^5 \text{ J} = 9 \times 10^6 \text{ J.}$$

S25. Energy is given by the petrol in the form of heat of combination.

Thus, by question,

Energy given by 1 litre of petrol = $3 \times 10^7 \text{ J}$

Efficiency of the car engine = 0.5

\therefore Energy used by the car = $0.5 \times 3 \times 10^7 \text{ J}$

$$E = 1.5 \times 10^7 \text{ J}$$

Total distance travelled (s) = $15 \text{ km} = 15 \times 10^3 \text{ m}$

If f is the force of friction then,

$$E = f \times s \quad [\because \text{Energy is utilised in working against friction}]$$

$$1.5 \times 10^7 = f \times 15 \times 10^3$$

$$\Rightarrow f = \frac{1.5 \times 10^7}{15 \times 10^3}$$

$$f = 1000 \text{ N.}$$

S26. Given, Mass of the rain drop (m) = 1 g

$$= 1 \times 10^{-3} \text{ kg}$$

Height of falling (h) = $1 \text{ km} = 10^3 \text{ m}$

$$g = 10 \text{ m/s}^2$$

(a) Loss of P.E. = $mgh = 1 \times 10^{-3} \times 10 \times 10^3 = 10 \text{ J}$

(b) Gain in K.E. = $\frac{1}{2} mv^2 = \frac{1}{2} \times 10^{-3} \times 2500 = 1.25 \text{ J}$

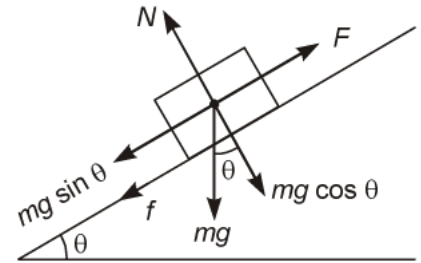
(c) No, because a part of P.E. is used up in doing work against the viscous drag of air.

S27. Consider the adjacent diagram the block is pushed up by applying a force F .

Normal reaction (N) and frictional force (f) is shown.

Given, Mass = $m = 1 \text{ kg}$, $\theta = 30^\circ$

$F = 10 \text{ N}$, $m = 0.1$ and $s =$ distance moved by the block along the inclined plane = 10 m



(a) Work done against gravity = Increase in P.E. of the block

$$= mg \times \text{Vertical distance travelled}$$

$$= mg \times s (\sin \theta) = (mgs) \sin \theta$$

$$= 1 \times 10 \times 10 \times \sin 30^\circ = 50 \text{ J}$$

$$[\because g \times 10 \text{ m/s}^2]$$

(b) Work done against friction

$$W_f = f \times s = \mu mg \cos \theta \times s$$

$$= 0.1 \times 1 \times 10 \cos 30^\circ \times 10$$

$$= 10 \times 0.866 = 8.66 \text{ J}$$

(c) Increase in P.E. = $mgh = mg (s \sin \theta)$

$$= 1 \times 10 \times 10 \times \sin 30^\circ$$

$$= 100 \times \frac{1}{2} = 50 \text{ J}$$

(d) By work-energy theorem, we know that work done by all the forces = change in K.E.

$$(W) = \Delta K$$

$$\Delta k = W_g + W_f + W_f$$

$$\Rightarrow = -mgh - fs + FS$$

$$= -50 - 8.66 + 10 \times 10$$

$$= 50 - 8.66 = 41.34 \text{ J}$$

(e) Work done by applied force,

$$F = FS$$

$$= (10)(10) = 100 \text{ J.}$$