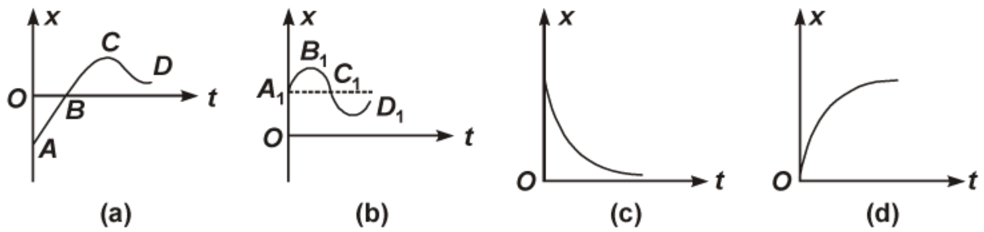
Date: 13/10/2021

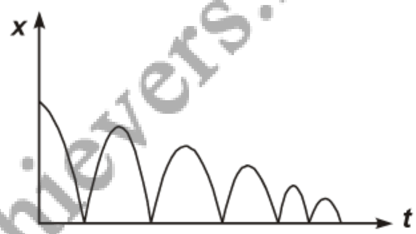
Q1. Refer to the graphs in figure, Match the following.



Graph	Characteristic
(a)	(i) has $v > 0$ and $a < 0$ throughout.
(b)	(ii) has $x > 0$ throughout and has a point with $v = 0$ and a point with $a = 0$.
(c)	(iii) has a point with zero displacement for $t > 0$.
(d)	(iv) has $v < 0$ and $a > 0$.

Q2. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has a speed of 18 m/h while the other has the speed of 27 km/h. The bird starts moving from first car towards the other and is moving with the speed of 36 km/h and when the two cars were separated by 36 km. What is the total distance covered by the bird? What is the total displacement of the bird?

Q3. A ball is dropped and its displacement vs time graph is as shown figure (displacement x is from ground and all quantities are +ve upwards).



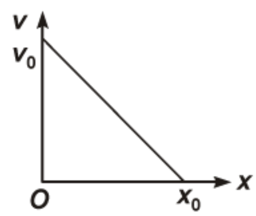
(a) Plot qualitatively velocity vs time graph.

(b) Plot qualitatively acceleration vs time graph.

Q4. Give example of a motion where $x > 0$, $v < 0$, $a > 0$ at a particular instant.

Q5. A ball is dropped from a building of height 45 m. Simultaneously another ball is thrown up with a speed 40 m/s. Calculate the relative speed of the balls as a function of time.

Q6. The velocity-displacement graph of a particle is shown in figure.



(a) Write the relation between v and x.

(b) Obtain the relation between acceleration and displacement and plot it.

Q7. An object falling through a fluid is observed to have acceleration given by $a = g - bv$ where g = gravitational acceleration and b is constant. After a long time of release, it is observed to fall with constant speed. What must be the value of constant speed?

Q8. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time. (Take acceleration in the backward direction as positive).

Q9. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is of a lower height than the first. If his speed is 9 m/s, the (horizontal) distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building? (take $g = 10\ m/s^2$)

Q10. Give examples of a one-dimensional motion where

(a) the particle moving along positive x-direction comes to rest periodically and moves *forward*.

(b) the particle moving along positive x-direction comes to rest periodically and moves *backward*.

Q11. A particle executes the motion described by $x(t) = x_0(1 - e^{-\gamma t})$; $t \geq 0$, $x_0 > 0$.

(a) Where does the particle start and with what velocity?

(b) Find maximum and minimum values of $x(t)$, $v(t)$, $a(t)$. Show that $x(t)$ and $a(t)$ increase with time and $v(t)$ decreases with time.

Q12. A motor car moving at a speed of 72 km/h can not come to a stop in less than 3.0 s while for a truck this time interval is 5.0 s. On a highway the car is behind the truck both moving at 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bump onto (collide with) the truck. Human response time is 0.5 s.

Q13. A man is standing on top of a building 100 m high. He throws two balls vertically, one at $t = 0$ and other after a time interval (less than 2 seconds). The later ball is thrown at a velocity of half the first. The vertical gap between first and second ball is +15 m at $t = 2$ s. The gap is found to remain constant. Calculate the velocity with which the balls were thrown and the exact time interval between their throw.

Q14. A monkey climbs up a slippery pole for 3 seconds and subsequently slips for 3 seconds. Its velocity at time t is given by $v(t) = 2t(3 - t)$; $0 < t < 3$ and $v(t) = -(t - 3)(6 - t)$ for $3 < t < 6$ s in m/s. It repeats this cycle till it reaches the height of 20 m.

(a) At what time is its velocity maximum?

(b) At what time is its average velocity maximum?

(c) At what time is its acceleration maximum in magnitude?

(d) How many cycles (counting fractions) are required to reach the top?

Q15. It is a common observation that rain clouds can be at about a kilometre altitude above the ground.

(a) If a rain drop falls from such a height freely under gravity, what will be its speed? Also calculate in Km/h. ($g = 10\ m/s^2$)

(b) A typical rain drop is about 4 mm diameter. Momentum is mass x speed in magnitude. Estimate its momentum when it hits ground.

(c) Estimate the time required to flatten the drop.

(d) Rate of change of momentum is force. Estimate how much force such a drop would exert on you.

(e) Estimate the order of magnitude force on umbrella. Typical lateral separation between two rain drops is 5 cm.

(Assume that umbrella is circular and has a diameter of 1 m and cloth is not pierced through !!)

Date: 13/10/2021

S1. We have to analyse slope of each curve *i.e.*, $\frac{dx}{dt}$. For peak points $\frac{dx}{dt}$ will be zero as x is maximum at peak points.

For graph (a), there is a point (B) for which displacement is zero. So, a matches with (iii).

In graph (b), x is positive (> 0) throughout and at point B_1, $v = \frac{dx}{dt} = 0$.

Since, at point of curvature changes $a = 0$, So b matches with (ii)

In graph (c), slope $v = \frac{dx}{dt}$ is negative hence, velocity will be negative.

So, matches with (iv)

In graph (d), as slope $v = \frac{dx}{dt}$ is positive hence, $v > 0$.

Hence, d matches with (i).

S2. Given, Speed of first car = 18 km/h

Speed of second car = 27 km/h

∴ Relative speed of each car w.r.t. t, each other

= 18 + 27 = 45 km/h

Distance between the cars = 36 km

$$\therefore \quad \text{Time of meeting the cars } (t) = \frac{\text{Distance between the cars}}{\text{Relative speed of cars}}$$

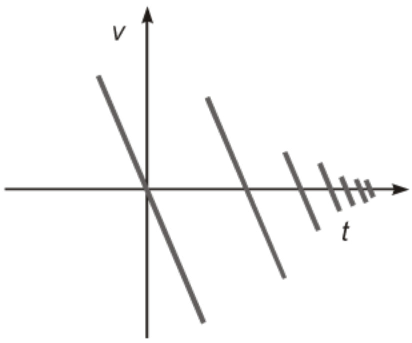
$$= \frac{36}{45} = \frac{4}{5} \text{ h} = 0.8 \text{ h}$$

Speed of the bird (v_b) = 26 km/h

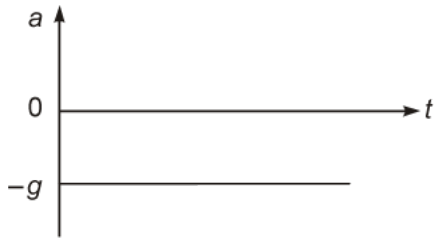
∴ Distance covered by the bird = $v_b \times t = 36 \times 0.8 = 28.8$ km.

S3. The ball is released and is falling under gravity. Acceleration is $-g$, except for the short time intervals in which the ball collides with ground, and when the impulsive force acts and produces a large acceleration.

(a) The velocity-time graph of the ball is shown in figure (a)



(a)



(b)

(b) The acceleration-time graph of the ball is shown in figure (b)

S4. Let the motion is represented by

$$x(t) = A + Be^{-\gamma t} \quad \text{... (i)}$$

Let $\quad A > B \quad \text{and} \quad \gamma > 0$

Now velocity $\quad x(t) = \dfrac{dx}{dt} = -B\gamma e^{-\gamma t}$

Acceleration $\quad a(t) = \dfrac{dx}{dt} = B\gamma^2 e^{-\gamma t}$

Suppose we are considering any instant t, then from Eq. (i) we can say that

$$x(t) > 0; \quad v(t) < 0 \quad \text{and} \quad a > 0.$$

S5. For the ball dropped from the building,

$$u_1 = 0, \quad u_2 = 40\ \text{m/s}$$

Velocity of the dropped ball after time t,

$$v_1 = u_1 + gt$$

$$v_1 = gt \qquad \text{[Downward]}$$

For the ball thrown up, $\quad u_2 = 40\ \text{m/s}$

Velocity of the ball after time t

$$v_2 = u_2 - gt$$

$$= (40 - gt) \qquad \text{[Upward]}$$

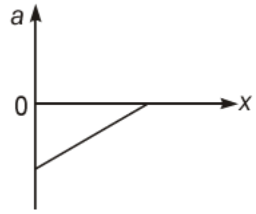
∴ Relative velocity of one ball w.r.t. another ball

$$= v_1 - v_2$$

$$= gt - [-(40 - gt)] = 40\ \text{m/s}.$$

Note: When we are applying equations for rectilinear motion we should carefully put up the signs for the physical quantities.

S6. $$v = (-v_0/x_0)\,x + v_0,$$
$$a = (v_0/x_0)^2\,x - v_0.$$



The variation of a with x is shown in the figure. It is a straight line with a positive slope and a negative intercept.

S7. When speed becomes constant acceleration

$$a = \frac{dv}{dt} = 0$$

Given acceleration $\quad a = g - bv$

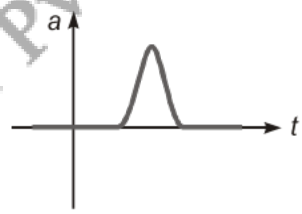
where, $\quad g$ = gravitational acceleration

Clearly, from above equation as speed increases acceleration will decrease. At a certain speed say v_0, acceleration will be zero and speed will remain constant.

Hence, $\quad a = g - bv_0 = 0$

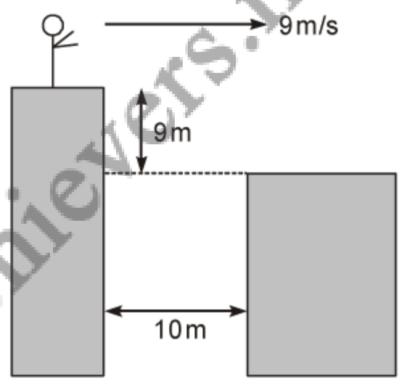
$\Rightarrow \quad v_0 = g/b.$

S8. If gravity effect is neglected then ball moving uniformly turned back with same speed when a bat hit it. Acceleration of the ball is zero just before it strikes the bat. When the ball strikes the bat, it gets accelerated due to the applied impulsive force by the bat.



The variation of acceleration with time is show in graph

S9. Suppose that the fall of 9 m will take time t. Hence



$$y - y_0 = v_{0y} - \frac{gt^2}{2}$$

Since, $v_{0y} = 0$,

$$t = \sqrt{\frac{2(y-y_0)}{g}} = \sqrt{\frac{2 \times 9\,\text{m}}{10\,\text{m/s}^2}}$$

$$= \sqrt{1.8} \approx 1/34 \text{ seconds.}$$

In this time, the distance moved horizontally is

$$x - x_0 = v_{0x}\,t = 9\,\text{m/s} \times 1.34\,\text{s} = 12.06\,\text{m}.$$

Yes-he will land.

S10. When we are writing an equation belonging to periodic nature it will involve sine or cosine function.

(a) The particle will be moving along positive x-direction only it $t > \sin t$

Hence, $\quad x(t) = 1 - \sin t$

$$\text{Velocity } v(t) = \frac{dx(k)}{dt} = -\cos t$$

$$\text{Acceleration } a(t) = \frac{dv(t)}{dt} = \sin t$$

When $t = 0; \quad x(t) = 0$

When $t = \pi; \quad x(t) = \pi > 0$

When $t = 0; \quad x(t) = 2\pi > 0$

(b) Equation can be represented by

$$x(t) = \sin t$$

$$v = \frac{d}{dt} x(t) = \cos t$$

As displacement and velocity is involving sin t and cos t hence these equations represent periodic motion.

S11. Given,

$$x(t) = x_0(1 - e^{-\gamma t})$$

$$v(t) = \frac{dx(t)}{dt} = x_0\gamma e^{-\gamma t}$$

$$a(t) = \frac{dv(t)}{dt} = -x_0\gamma^2 e^{-\gamma t}$$

(a) When $t = 0; \quad x(t) = (1 - e^{-0}) = x_0(1 - 1)$

$$x(t = 0) = x_0\gamma e^{-0} = x_0\gamma(1) = \gamma x_0$$

(b) $x(t)$ is maximum when $t = \infty$ $\quad [x(t)]_{max} = x_0$

$x(t)$ is minimum when $t = 0$ $\quad [x(t)]_{min} = 0$

$v(t)$ is maximum when $t = 0; \quad v(0) = x_0\gamma$

$v(t)$ is minimum when $t = \infty; \quad v(\infty) = 0$

$a(t)$ is maximum when $t = \infty; \quad a(\infty) = x_0\gamma$

$a(t)$ is minimum when $t = 0; \quad a(0) = -x_0\gamma^2$

Note: We should be careful about nature of variation of the curve and maximum and minimum value will be decided accordingly.

S12. Car behind the truck

$$\text{Retardation of truck} = \frac{20}{5} = 4 \text{ ms}^{-2}$$

$$\text{Retardation of car} = \frac{20}{3} \text{ ms}^{-2}$$

Let the truck be at a distance x from the car when breaks are applied

Distance of truck from A at $t > 0.5$ s is

$$x + 20t - 2t^2.$$

Distance of car from A is

$$10 + 20\,(t - 0.5) - \frac{10}{3}\,(t - 0.5)^2.$$

If the two meet $\quad x + 20t - 2t^2 = 10 + 20t - 10 - \frac{10}{3}t^2 + \frac{10}{3}t - 0.25 \times \frac{10}{3}.$

$$x = -\frac{4}{3}t^2 + \frac{10}{3}t - \frac{5}{6}.$$

To find x_{min}, $\quad \frac{dx}{dt} = -\frac{8}{3}t + \frac{10}{3} = 0.$

which gives $\quad t_{min} = \frac{10}{8} = \frac{5}{4}\text{ s}.$

Therefore, $\quad x_{min} = -\frac{4}{3}\left(\frac{5}{4}\right)^2 + \frac{10}{3} \times \frac{5}{4} - \frac{5}{6} = \frac{5}{4}.$

Therefore, $x > 1.25$ m.

Second method: This method does not require the use of calculus. If the car is behind the truck,

$$v_{car} = 20 - (20/3)(t - 0.5) \text{ for } t > 0.5\text{ s as car decelerate only after 0.5 s.}$$

$$v_{truck} = 20 - 4t$$

Find t from equating the two or from velocity vs time graph. This yields $t = 5/4$ s.

In this time truck would travel truck,

$$S_{truck} = 20\,(5/4) - (1/2)(4)(5/4)^2 = 21.875\text{ m}.$$

and car would travel,

$$S_{car} = 20\,(0.5) + 20(5/4 - 0.5) - \left(\frac{1}{2}\right)(20/3) \times \left(\frac{5}{4} - 0.5\right)^2 = 23.125\text{ m}$$

Thus, $\quad S_{car} - S_{truck} = 1.25$ m.

If the car maintains this distance initially, its speed after 1.25 s will he always less than that of truck and hence collision never occurs.

S13. Let the speeds of the two balls (1 and 2) be v_1 and v_2 where

If $\quad v_1 = 2v, \quad v_2 = v$

If $\quad y_1$ and y_2 and the distance covered by

the balls 1 and 2, respectively, before coming to rest, then

$$y_1 = \frac{v_1^2}{2g} = \frac{4v^2}{2g} \quad \text{and} \quad y_2 = \frac{v_2^2}{2g} = \frac{v^2}{2g}$$

Since, $y_1 - y_2 = 15$ m

$$\frac{4v^2}{2g} - \frac{v^2}{2g} = 15 \text{ m} \quad \text{or} \quad \frac{3v^2}{2g} = 15 \text{ m}$$

or $$v^2 = \sqrt{5 \text{ m} \times (2 \times 10)} \text{ m/s}$$

or $$v = 10 \text{ m/s}$$

Clearly, $v_1 = 20$ m/s and $v_2 = 10$ m/s

as $$y_1 = \frac{v_1^2}{2g} = \frac{(20\text{m})^2}{2 \times 10\text{m}} = 20\text{m}$$

$$y_2 = 20 - 15\text{m} = 5\text{m}$$

If t_2 is the time taken by the ball 2 toner

a distance of 5 m then from $y_2 = v\, t_2 - \frac{1}{2} g t_2^2$

$$5 = 10t_2 - 5t_2^2 \quad \text{or} \quad t_2^2 - 2t_2 + 1 = 0.$$

where $t_2 = 15$

Since, t_1 (time taken by ball 1 to cover distance of 20 m) is 2 s, time interval between the two throws

$$= t_1 - t_2 = 2\text{s} - 1\text{s} = 1\text{s}$$

Note: We should be very careful, when we are applying the equation of rectilinear motion.

S14. In this problem to calculate maximum velocity we will use $\frac{dv}{dt} = 0$, then the time corresponding to maximum velocity will be obtained.

Given velocity $v(t) = 2t(3 - t) = 6t - 2t^2$ ... (i)

Since to obtain function is maximum or minimum, we put

$$\frac{dv}{dt} = 0$$

(a) For maximum velocity $\frac{dv}{dt} = 0$

$$\Rightarrow \quad \frac{d}{dt}(6t - 2t^2) = 0$$

$$\Rightarrow \quad 6 - 4t = 0$$

$$\Rightarrow \quad t = \frac{6}{4} = \frac{3}{2} \text{ s} = 1.5 \text{ s}$$

(b) From Eq. (i), $v = 6t - 2t^2$

$$\Rightarrow \quad \frac{ds}{dt} = 6t - 2t^2$$

$$\Rightarrow \quad ds = (6t - 2t^2)\, dt$$

where, s is displacement.

$\therefore$ Distance travelled in time interval 0 to 3s.

$$s = \int_0^3 (6t - 2t^2)\,dt$$

$$= \left[\frac{6t^2}{2} - \frac{2t^3}{3}\right]_0^3 = \left[3t^2 - \frac{2}{3}t^3\right]_0^3$$

$$= 3 \times 9 - \frac{2}{3} \times 3 \times 3 \times 3$$

$$= 28 - 18 = 9\,\text{m}$$

$$\text{Average velocity} = \frac{9}{3} = 3\text{ m/s}$$

Given, $x = 6t - 2t^2$

$\Rightarrow$ $3 = 6t - 2t^2$

$\Rightarrow$ $2t^2 - 6t + 3 = 0$

$\Rightarrow$ $$t = \frac{6 \pm \sqrt{6^2 - 4 \times 2 \times 3}}{2 \times 2} = \frac{6 \pm \sqrt{36 - 24}}{4}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{3 \pm 2\sqrt{3}}{2}$$

Considering positive sigh only

$$t = \frac{3 + 2\sqrt{3}}{2} = \frac{3 + 2 \times 1.732}{2} = \frac{9}{4}\,\text{s}$$

(c) In a periodic motion when velocity is zero acceleration will be maximum putting $v = 0$ in Eq. (i)

$0 = 6t - 2t^2$

$\Rightarrow$ $0 = t(6 - 2t)$

$\Rightarrow$ $t \times 2(3 - t) = 0$

$\Rightarrow$ $t = 0$ or 3s

(d) Distance covered in 0 to 3s = 9 m

Distance covered in 3 to 6s = $\int_3^6 (18 - 9t + t^2)\,dt$

$$= \left(18t - \frac{9t^2}{2} + \frac{t^3}{3}\right)_3^6$$

$$= 18 \times 6 - \frac{9}{2} \times 6^2 + \frac{6^3}{3} - \left(18 \times 3 - \frac{9 \times 3^2}{2} + \frac{3^3}{3}\right)$$

$$= 108 - 9 \times 18 + \frac{6^3}{3} - 18 \times 3 + \frac{9}{2} \times 9 - \frac{27}{3}$$

$$= 108 - 18 \times 9 + \frac{216}{3} - 54 + 4.5 \times 9 - 9 = -4.5 \text{ m}$$

$\therefore$ Total distance travelled in one cycle = $s_1 + s_2 = 9 - 4.5 = 4.5$ m

Number of cycles covered in total distance to be covered = $\frac{20}{45} \approx 4.44 \approx 5$.

S15. Given, Height (h) = 1 km = 1000 m

$g = 10$ m/s

(a) Velocity attained by the rain drop in freely falling through a height h.

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1000} = 100\sqrt{2} \text{ m/s}$$

$$= 100\sqrt{2} \times \frac{60 \times 60}{1000} \text{ km/h}$$

$$= 360\sqrt{2} \text{ km/h} \approx 510 \text{ km/h}.$$

(b) Diameter of the drop (d) = $2r$ = 4 mm

$\therefore$ Radius of the drop (r) = 2 mm = 2×10^{-3} m

Mass of a rain drop (m) = $V \times \rho$

$$= \frac{4}{3} \pi r^3 \rho$$

$$= \frac{4}{3} \times \frac{22}{7} \times (2 \times 10^{-3})^3 \times 10^3 \quad [\because \text{ Density of water} = 10^3 \text{ kg/m}^3]$$

$$\approx 3.4 \times 10^{-5} \text{ kg}$$

Momentum of the rain drop (p) = mv

$$= 3.4 \times 10^{-5} \times 100\sqrt{2} \text{ kg}$$

$$= 4.7 \times 10^{-3} \text{ kg-m/s}$$

$$= 5 \times 10^{-3} \text{ kg-m/s}$$

(c) Time required to flatten the drop = time taken by the drop to travel the distance equal to the diameter of the drop near the ground

$$t = \frac{d}{v} = \frac{4 \times 10^{-3}}{100\sqrt{2}} = 0.028 \times 10^{-3} \text{ s}$$

$$= 2.8 \times 10^{-5} \text{ s} \approx 30 \text{ µs}$$

(d) Force exerted by a rain drop

$$F = \frac{\text{Change in momentum}}{\text{Time}} = \frac{p-0}{t}$$

$$= \frac{4.7\times10^{-3}}{2.8\times10^{-5}} \approx 168\,\text{N}$$

(e) Radius of the umbrella (R) = $\frac{1}{2}$ m

$\therefore$ Area of the umbrella (A) = $\pi R^2 = \frac{22}{7}\times\left(\frac{1}{2}\right)^2 = \frac{22}{28} = \frac{11}{14} \approx 0.8\,\text{m}^2$

Number of drops striking the umbrella simultaneously with average separation of 5 cm = 5×10^{-2} m.

$$= \frac{0.8}{(5\times10^{-2})^2} = 320$$

$\therefore$ Net force exerted on umbrella = 320 × 168 = 53760 = 54000 N

Note: In practice, the velocity of the drops decreases due to air friction.