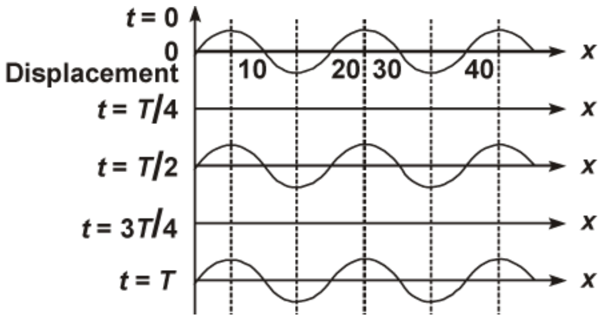
Date: 13/10/2021

Q1. The wave pattern on a stretched string is shown in figure. Interpret what kind of wave this and find its wavelength.



Q2. A tuning fork A, marked 512 Hz, produces 5 beats per second, where sounded with another unmarked tuning fork *B*. If *B* is loaded with wax the number of beats is again 5 per second. What is the frequency of the tuning fork *B* when not loaded?

Q3. A sonometer wire is vibrating in resonance with a tuning fork. Keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be is resonance with the wire?

Q4. The displacement of an elastic wave is given by the function

$$y = 3 \sin \omega t + 4 \cos \omega t.$$

where y is in cm and t is in second. Calculate the resultant amplitude.

Q5. An organ pipe of length L open at both ends is found to vibrate in its first harmonic when sounded with a tuning fork of 480 Hz. What should be the length of a pipe closed at one end, so that it also vibrates in its first harmonic with the same tuning fork?

Q6. A train standing at the outer signal of a railway station blows a whistle of frequency 400 Hz still air. The train begins to move with a speed of 10 m s^{-1} towards the platform. What is the frequency of the sound for an observer standing on the platform? (sound velocity in air = 330 m s^{-1})

Q7. A steel wire has a length of 12 m and a mass of 2.10 kg. What will be the speed of a transverse wave on this wire when a tension of 2.06×10^4 N is applied?

Q8. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a source of 1237.5 Hz? (sound velocity in air = 330 m s^{-1})

Q9. When two waves of almost equal frequencies n_1 and n_2 reach at a point simultaneously, what is the time interval between successive maxima?

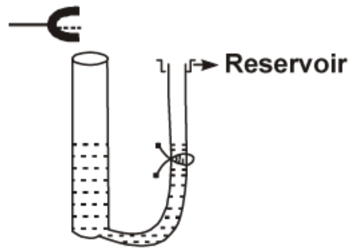
Q10. At what temperatures (in °C) will the speed of sound in air be 3 times its value at O°C?

Q11. A sitar wire is replaced by another wire of same length and material but of three times the earlier radius. If the tension in the wire remains the same, by what factor will the frequency change?

Q12. Show that when a string fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio 1 : 2 : 3 : 4.

Q13. If c is r.m.s. speed of molecules in a gas and v is the speed of sound waves in the gas, show that c/v is constant and independent of temperature for all diatomic gases.

Q14. A tuning fork vibrating with a frequency of 512 Hz is kept close to the open end of a tube filled with water (see figure). The water level in the tube is gradually lowered. When the water level is 17 cm below the open end, maximum intensity of sound is heard. If the room temperature is 20°C, calculate



(a) speed of sound in air at room temperature.

(b) speed of sound in air at 0°C.

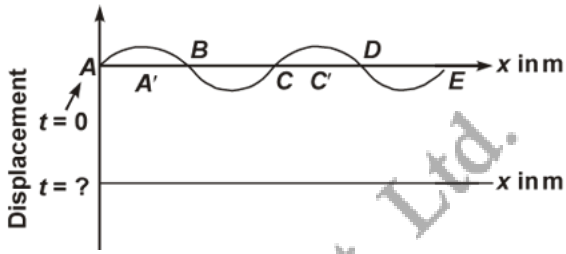
(c) if the water in the tube is replaced with mercury, will there be any difference in your observations?

Q15. The pattern of standing waves formed on a stretched string at two instants of time are shown in figure. The velocity of two waves superimposing to form stationary waves is 360 m s^{-1} and their frequencies are 256 Hz.

(a) Calculate the time at which the second curve is plotted.

(b) Mark nodes and antinodes on the curve.

(c) Calculate the distance between A' and C'.



Q16. Given below are some functions of x and t to represent the displacement of an elastic wave.

(a) $y = 5 \cos (4x) \sin (20t)$ (b) $y = 4 \sin (5x - t/2) + 3 \cos (5x - t/2)$

(c) $y = 100 \cos (100 \pi t + 0.5x)$ (d) $y = 10 \cos [(252 - 250) \pi t] \cos [(252 + 250) \pi t]$

State which of these represent

(a) a travelling wave along $-x$ direction (b) a stationary wave

(c) a travelling wave along $+x$ direction. (d) beats

Given reasons for your answers.

Q17. The Earth has a radius of 6400 km. The inner core of 1000 km radius is solid. Outside it, there is a region from 1000 km to a radius of 3500 km which is in molten state. Then again from 3500 km to 6400 km the Earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that the P wave has a speed of 8 km s^{-1} in solid parts and of 5 km s^{-1} in liquid parts of the Earth. An Earthquake occurs at some place close to the surface of the Earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the Earth if wave travels along diameter?

Q18. For the harmonic travelling wave $y = 2 \cos 2\pi (10t - 0.0080x + 3.5)$ where x and y are in cm and t is second. What is the phase difference between the oscillatory motion at two points separated by a distance of

(a) 4 m (b) 0.5 m (c) $\frac{\lambda}{2}$ (d) $\frac{3\lambda}{4}$ (at a given instant of time)

(e) What is the phase difference between the oscillation of a particle located at $x = 100$cm, at $t = T$s and $t = 5$s?

Q19. In the given progressive wave $y = 5 \sin (100 \pi t - 0.4 \pi x)$ where y and x are in m, t is in s. What is the

(a) amplitude (b) wave length (c) frequency

(d) wave velocity (e) particle velocity amplitude.

Date: 13/10/2021

S1. We have to observe the displacement and position of different points, then accordingly nature of two wave is decided.

Points on positions $x = 10, 20, 30, 40$ never move, always at mean position with respect to time. These are forming nodes which characterise a stationary wave. Since, stationnery wave does not move with time.

$\because$ Distance between two successive nodes $= \dfrac{\lambda}{2}$

$$\therefore \quad \lambda = 2 \times \text{(node to node distance)}$$

$$= 2 \times (20 - 10)$$

$$= 2 \times 10 = \mathbf{20\ cm}.$$

Stationary waves = 20 cm

S2. Frequency of tuning fork A,

$$\nu_A = 512 \text{ Hz}$$

Probable frequency of tuning fork B,

$$\nu_B = \nu_A \pm 5 = 512 \pm 5 = 517 \text{ or } 507 \text{ Hz}$$

When B is loaded, its frequency reduces.

If it is 517 Hz, it might reduced to 507 Hz given again a beat frequency of 5 Hz.

If it is 507 Hz, reduction will always increase the beat frequency, hence $\nu_B = 517$ Hz.

Note: For production of beats frequencies of the two tuning forks must be nearly equal *i.e.*, slight difference in frequencies.

S3. Wire of twice the length vibrates in its second harmonic. Thus if the tuning fork resonates at L, it will resonate at $2L$. This can be explained as below:

The sonometer frequency is given by

$$\nu = \frac{n}{2L}\sqrt{\frac{T}{m}} \qquad [n = \text{number of loops}]$$

Now, as it vibrates with length L, we assume $\nu = \nu_1$

$$n = n_1$$

$$\therefore \quad \nu_1 = \frac{n_1}{2L}\sqrt{\frac{T}{m}} \qquad \ldots \text{(i)}$$

When length is doubled, then

$$\nu_2 = \frac{n_2}{2 \times 2L}\sqrt{\frac{T}{m}} \quad \ldots \text{(ii)}$$

Dividing E. (i) by Eq. (ii), we get

$$\frac{\nu_1}{\nu_2} = \frac{n_1}{n_2} \times 2$$

To keep the resonance

$$\frac{\nu_1}{\nu_2} = 1 = \frac{n_1}{n_2} \times 2$$

$\Rightarrow$ $n_2 = 2n_1$

Hence, when the wire is doubled the number of loops also get doubled to produce the resonance. That is it resonates in second harmonic.

S4. Given displacement of an elastic wave $y = 3 \sin \omega t + 4 \cos \omega t$

Assume, $3 = a \cos \phi$ ... (i)

$4 = a \sin \phi$ ... (ii)

In dividing Eq. (ii) by Eq. (i)

$$\tan \phi = \frac{4}{3} \quad \Rightarrow \quad \phi = \tan^{-1}(4/3)$$

Also, $a^2 \cos^2 \phi + a^2 \sin^2 \phi = 3^2 + 4^2$

$\Rightarrow$ $a^2 (\cos^2 \phi + \sin^2 \phi) = 25$

$a^2 \cdot 1 = 25 \quad \Rightarrow \quad a = 5$

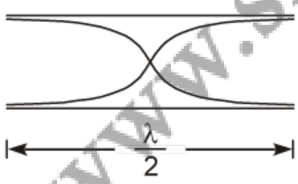
Hence, $Y = 5 \cos \phi \sin \omega t + 5 \sin \phi \cos \omega t$

$= 5 [\cos \phi \sin \omega t + \sin \phi \cos \omega t] = 5 \sin (\omega t + \phi)$

where $\phi = \tan^{-1}(4/3)$

Hence, amplitude = 5 cm.

S5. Consider the situation shown in the diagram.



Open organ pipe

As the organ pipe is open at both ends, hence for first harmonic

$$l = \frac{\lambda}{2}$$

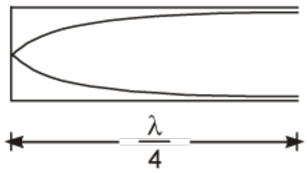
$\Rightarrow$ $\lambda = 2l \quad \Rightarrow \quad \frac{c}{\nu} = 2l \quad \Rightarrow \quad \nu = \frac{c}{2l}$

where c is speed of the sound wave in air.

For pipe closed at one end

$$\nu' = \frac{c}{\nu}$$

c for first harmonic



Closed organ pipe

Hence, $\nu = \nu'$ [for resonance with same tuning fork]

$\Rightarrow$ $$\frac{c}{2L} = \frac{c}{4L'}$$ [$\because$ speed remains constant]

$\Rightarrow$ $$\frac{L'}{L} = \frac{2}{4} = \frac{1}{2} \quad \Rightarrow \quad L' = \frac{L}{2}.$$

S6. As the source (train) is moving towards the observer (platform) hence apparent frequency observed is more than the natural frequency.

Frequency of whistle $\nu = 400$ Hz

Speed of train $v_t = 10$ m/s

Velocity of sound in air $v = 330$ m/s

Apparent frequency when source is moving,

$$\nu_{app} = \left(\frac{v}{v - v_t}\right)\nu$$

$$= \left(\frac{330}{330 - 10}\right) 400$$

$\Rightarrow$ $$\nu_{app} = \frac{330}{320} \times 400 = \mathbf{412.5\ Hz}.$$

S7. Given, length of the wire $l = 12$ m

Mass of wire $m = 2.10$ kg

Tension $T = 2.06 \times 10^4$ N

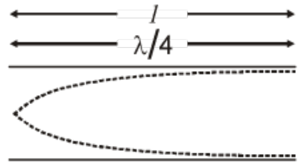
Speed of transverse wave $v = \sqrt{\frac{T}{\mu}}$ [where μ = mass per unit length]

$$= \sqrt{\frac{2.06 \times 10^4}{\left(\frac{2.10}{12}\right)}} = \sqrt{\frac{2.06 \times 12 \times 10^4}{2.10}} = 343 \text{ m/s}.$$

S8. Length of pipe

$$l = 20\text{ cm} = 20 \times 10^{-2}\text{ m}$$

$$\nu_{\text{funda}} = \frac{v}{4L} = \frac{330}{4 \times 20 \times 10^{-2}} \qquad \text{[For closed pipe]}$$



$$\nu_{\text{funda}} = \frac{330 \times 100}{80} = 412.5\text{ Hz}$$

$$\frac{\nu_{\text{Given}}}{\nu_{\text{Funda}}} = \frac{1237.5}{412.5} = 3$$

Hence, 3rd harmonic node of the pipe is resonantly by the source of given frequency.

S9. Let $n_1 > n_2$

Beat frequency $v_b = n_1 - n_2$

$\therefore$ Time period of beats = $T_b = \dfrac{1}{v_b} = \dfrac{1}{n_1 - n_2}$.

S10. We know that speed of sound in air $v \propto \sqrt{T}$

$$\therefore \quad \frac{v_T}{v_0} = \sqrt{\frac{T_T}{T_0}} = \sqrt{\frac{T_T}{273}} \qquad \text{[where } T \text{ is in kelvin]}$$

But $\dfrac{v_T}{v_0} = \dfrac{3}{1}$ [Q speed becomes three times]

$$\therefore \quad \frac{3}{1} = \sqrt{\frac{T_T}{T_0}} \Rightarrow \frac{T_T}{273} = 9$$

$$\therefore \quad T_T = 273 \times 9 = 2457\text{ K}$$

$$= 2457 - 273 = 2184\,°\text{C}.$$

S11. Frequency of vibration produced by a stretched wire

$$\nu = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

Mass per unit length $\mu = \dfrac{\pi r^2 l \rho}{l} = \pi r^2 \rho$ $\quad [\because\ M = vp = Al\rho = \pi r^2 l\rho]$

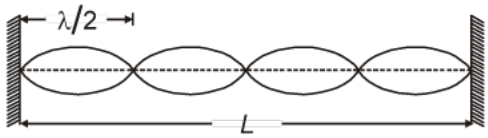
$$\therefore \quad \nu = \frac{n}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow \nu \propto \sqrt{\frac{1}{r^2}}$$

$$\nu \propto \frac{1}{r}$$

Hence, when radius is tripled, v will be $\frac{1}{3}$rd of previous value.

1/3. Since frequency $\alpha\sqrt{\frac{1}{m}}\ m = \pi r^2 \rho$.

S12. Let, there are n number of loops in the string



Length corresponding each loop is $\frac{\lambda}{2}$

Now, we can write

$$L = \frac{n\lambda}{2} \quad \Rightarrow \quad \lambda = \frac{2L}{n} \qquad \text{[For } n \text{ loops]}$$

$$\Rightarrow \qquad \frac{v}{\nu} = \frac{2L}{n} \quad \Rightarrow \quad [\because\ v = \nu\lambda]$$

$$\Rightarrow \qquad \nu = \frac{n}{2L} v = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad [\because \text{ velocity of transverse waves } = \sqrt{T/\mu}\,]$$

$$\Rightarrow \qquad \nu \propto n \qquad [\because \text{ length and speed are constants}]$$

So, $\nu_1 : \nu_2 : \nu_3 : \nu_4 = n_1 : n_2 : n_3 : n_4$
$= 1 : 2 : 3 : 4.$

S13. We know that r.m.s. speed of molecules of a gas

$$c = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3RT}{M}} \qquad \text{... (i)}$$

where M = molar mass of the gas

Speed of sound wave in gas $\quad v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \qquad$ ... (ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{c}{v} = \sqrt{\frac{3RT}{M} \times \frac{M}{\gamma RT}} \Rightarrow \frac{c}{v} = \sqrt{\frac{3}{\gamma}}$$

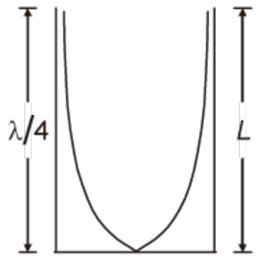
where γ = adiabatic constant for diatomic gas

$$\gamma = \frac{7}{5} \qquad \left[\text{Since } \gamma = \frac{C_p}{C_v}\right]$$

Hence, $\qquad \frac{c}{v}$ = Constant.

S14. Consider the diagram frequency of tuning fork ν = 512 Hz.

For observation of first maxima of intensity



(a) $L = \dfrac{\lambda}{4} \Rightarrow \lambda = 4L$ [for closed pipe]

$$v = \nu\lambda = 512 \times 4 \times 17 \times 10^{-2}$$
$$= 348.16 \text{ m/s}$$

(b) We know that $v \propto \sqrt{T}$

where temperature (T) is in kelvin,

$$\frac{v_{20}}{v_0} = \sqrt{\frac{273+20}{273+0}} = \sqrt{\frac{293}{273}}$$

$$\frac{v_{20}}{v_0} = \sqrt{1.073} = 1.03$$

$$v_0 = \frac{v_{20}}{1.03} = \frac{38.16}{1.03} = 338 \text{ m/s.}$$

(c) Resonance will be observed at 17 cm length of air column, only intensity of sound heard may be greater due to more complete reflection of the sound waves at the mercury surface because mercury is more denser than water..

S15. Given, Frequency of the wave ν = 256 Hz

Time period $T = \dfrac{1}{\nu} = \dfrac{1}{256}$ s $= 3.9 \times 10^{-3}$ s.

(a) Time taken to pass through mean position is

$$t = \frac{T}{4} = \frac{1}{40} = \frac{3.9 \times 10^{-3}}{4} \text{ s}$$
$$= \mathbf{9.8 \times 10^{-4}\ s}.$$

(b) Nodes are A, B, C, D, E (*i.e.*, zero displacement).

Antinodes - A', C' (*i.e.*, maximum displacement).

(c) It is clear from the diagram A' and C' are consecutive antinodes, hence separation = wavelength (λ)

$$= \frac{v}{\nu} = \frac{360}{256} = 1.41 \text{ m.} \qquad [\because\ v = \nu\lambda]$$

S16. (a) The equation $y = 100 \cos(100\pi t + 0.5x)$ is representing a travelling wave along negative x-direction.

(b) The equation $y = 5\cos(4x)\sin(20t)$ represents a stationary wave, because it contains sin, cos terms *i.e.*, combination of two progressive waves.

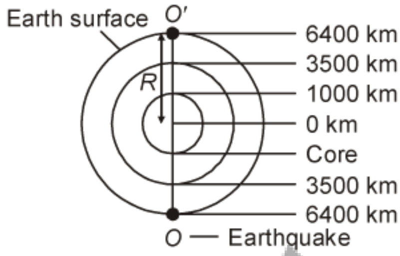
(c) As the equation $y = 4 \sin (5x - t/2) + 3 \cos (5x - t/2)$ involves negative sign with x, have if represents a travelling wave along x-direction.

(d) As the equation $y = 10 \cos [252 - 250) \pi t] \cdot \cos [252 + 250 \pi t]$ involving sum and difference of two near by frequencies 252 and 250 have this equation represents beats formation.

Note: We must not confuse with sign connected with x and direction of propagation of wave, it is just reversed, positive sign with x shown propagration of the wave in negative x-direction and vice-versa.

S17. Speed of wave in solid = 8 km/s

Speed of wave in liquid = 5 km/s



$$\text{Required time} = \left[\frac{1000}{8} + \frac{2500}{5} + \frac{2900}{8}\right] \times 2$$

[∵ Diameter = Radius × 2]

$$= \left[\frac{1000}{8} + \frac{2500}{5} + \frac{2900}{8}\right] \times 2 \qquad \left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$= [125 + 500 + 362.5] \times 2 = 1975$$

As we are considering at diametrically opposite point, hence there is a multiplication of 2.

S18. Given, wave functions are $y = 2 \cos 2\pi (10 - 0.0080x + 3.5)$

$$= 2 \cos (20 \pi t - 0.016 \pi x + 7\pi)$$

Now, standard equation of a travelling wave can be written as

$$y = a \cos (\omega t - kx + \phi)$$

On comparing with above equation, we get

$$a = 2 \text{ cm}; \quad \omega = 20\pi \text{ rad/s}; \quad k = 0.016\pi$$

Path difference = 4 cm

(a) Phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times \text{Path difference}$

$\therefore$ $$\Delta\phi = 0.016\pi \times 4 \times 100 \qquad \left(\because \frac{2\pi}{\lambda} = k\right)$$

$$= 6.4\pi \text{ rad}$$

(b) $$\Delta\phi = \frac{2\pi}{\lambda} \times (0.5 \times 100) \qquad [\because \text{Path difference} = 0.5 \text{ m}]$$

$$= 0.016\pi \times 0.5 \times 100$$

$$= 0.8\pi \text{ rad}$$

(c) $$\Delta\phi = \frac{2\pi}{\lambda} \times \left(\frac{\lambda}{2}\right) = \pi \text{ rad} \qquad [\because \text{ Path difference} = \lambda/2]$$

(d) $$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2} \text{ rad}$$

(e) $$T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} \text{ s}$$

$\therefore$ At $x = 100$ cm,

$$t = T$$

$$\phi_1 = 20\pi T - 0.016\pi(100) + 7\pi$$

$$= 20\pi\left(\frac{1}{10}\right) - 1.6\pi + 7\pi = 2\pi - 1.6\pi + 7\pi \qquad \text{... (i)}$$

Again, at $x = 100$ cm, $t = 5$ s

$$\phi_2 = 20\pi(5) - 0.016\pi(100) + 7\pi$$
$$= 100\pi - (0.016 \times 100) + 7\pi$$
$$= 100\pi - 1.6\pi + 7\pi \qquad \text{... (ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$\Delta\phi$ = Phase difference = $\phi_2 - \phi_1$

$$= (100\pi - 1.6\pi + 7\pi) - (2\pi - 1.6\pi + 7\pi)$$
$$= 100\pi - 2\pi = \mathbf{98\,\pi\ rad}.$$

S19. Standard equation of a progressive wave is given by

$$y = a \sin(\omega t - kx + \phi)$$

This is travelling along positive x-direction.

Given, equations $y = 5 \sin(100\pi t - 0.4\pi x)$

Comparing with the standard equation

(a) Amplitude = 5 m

(b) $$k = \frac{2\pi}{\lambda} = 0.4\pi$$

$\therefore$ $$\text{Wavelength } \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.4\pi} = \frac{20}{4} = 5 \text{ m}$$

(c) $\omega = 100\pi$

$$\omega = 2\pi\nu = 100\pi$$

$\therefore$ $$\text{Frequency } \nu = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

(d) Wave velocity $v = \frac{\omega}{k}$, where k is wave number and $k = \frac{2\pi}{\lambda}$.

$$= \frac{100\pi}{0.4\pi} = \frac{1000}{4}$$

$$= 250 \text{ m/s}$$

(e) $$y = 5 \sin (10\pi t - 0.4\pi x) \quad \text{... (i)}$$

$$\frac{dy}{dt} = \text{Particle velocity}$$

From Eq. (i), $$\frac{dy}{dt} = 5(100\pi) \cos [100\pi t - 0.4\pi x]$$

For particle velocity amplitude $\left(\frac{dy}{dt}\right)_{max}$

Which will be for $\{\cos [100\pi t - 0.4\pi x]\}_{max} = 1$

∴ Particle velocity amplitude

$$= \left(\frac{dy}{dt}\right)_{max} = 5(100\pi) \times 1$$

$$= 500\pi \text{ m/s.}$$