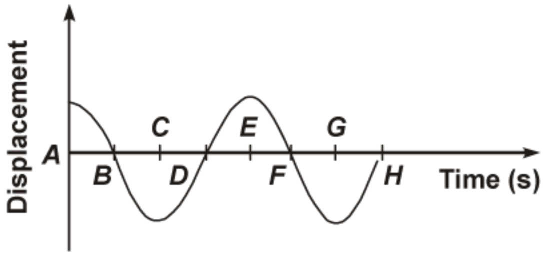
Date: 14/10/2021

Q1. Displacement versus time curve for a particle executing S.H.M. is shown in figure. Identify the points marked at which

(a) velocity of the oscillator is zero,

(b) speed of the oscillator is maximum.



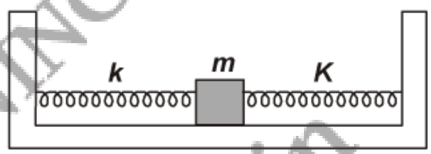
Q2. When will the motion of a simple pendulum be simple harmonic?

Q3. What is the ratio between the distance travelled by the oscillator in one time period and amplitude?

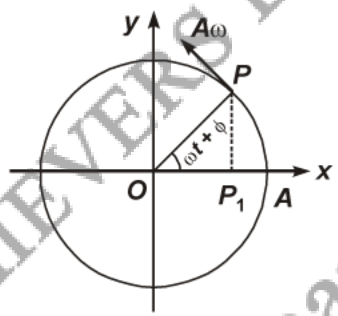
Q4. What are the two basic characteristics of a simple harmonic motion?

Q5. What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic oscillator?

Q6. Two identical springs of spring constant K are attached to a block of mass m and to fixed supports as shown in figure. When the mass is displaced from equilibrium position by a distance x towards right, find the restoring force.



Q7. In figure, what will be the sign of the velocity of the point P', which is the projection of the velocity of the reference particle P. P is moving in a circle of radius R in anticlockwise direction.



Q8. Show that for a particle executing S.H.M, velocity and displacement have a phase difference of $\pi/2$.

Q9. Draw a graph to show the variation of P.E., K.E. and total energy of a simple harmonic oscillator with displacement.

Q10. Find the displacement of a simple harmonic oscillator at which its P.E. is half of the maximum energy of the oscillator.

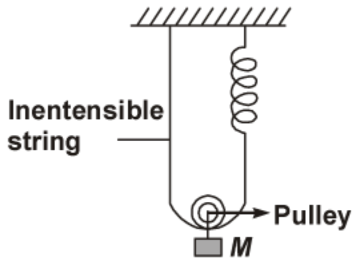
Q11. A body of mass m is situated in a potential field $U(x) = U_0 (1 - \cos \alpha x)$ when U_0 and α are constants. Find the time period of small oscillations.

Q12. The length of a second's pendulum on the surface of Earth is 1 m. What will be the length of a second's pendulum on the moon?

Q13. Consider a pair of identical pendulums, which oscillate with equal amplitude independently such that when one pendulum is at its extreme position making an angle of 2° to the right with the vertical, the other pendulum makes an angle of 1° to the left of the vertical. What is the phase difference between the pendulums?

Q14. A mass of 2 kg is attached to the spring of spring constant 50 Nm^{-1}. The block is pulled to a distance of 5cm from its equilibrium position at $x = 0$ on a horizontal frictionless surface from rest at $t = 0$. Write the expression for its displacement at anytime t.

Q15. Find the time period of mass M when displaced from its equilibrium position and then released for the system shown in



Q16. A person normally weighing 50 kg stands on a massless platform which oscillates up and down harmonically at a frequency of 2.0 s^{-1} and an amplitude 5.0 cm. A weighing machine on the platform gives the persons weight against time.

(a) Will there be any change in weight of the body, during the oscillation?

(b) If answer to part (a) is yes, what will be the maximum and minimum reading in the machine and at which position?

Q17. One end of a V-tube containing mercury is connected to a suction pump and the other end to atmosphere. The two arms of the tube are inclined to horizontal at an angle of 45° each. A small pressure difference is created between two columns when the suction pump is removed. Will the column of mercury in V-tube execute simple harmonic motion? Neglect capillary and viscous forces. Find the time period of oscillation.

Q18. A tunnel is dug through the centre of the Earth. Show that a body of mass 'm' when dropped from rest from one end of the tunnel will execute simple harmonic motion.

Q19. Show that the motion of a particle represented by $y = \sin \omega t - \cos \omega t$ is simple harmonic with a period of $2\pi/\omega$.

Q20. A body of mass m is attached to one end of a massless spring which is suspended vertically from a fixed point. The mass is held in hand so that the spring is neither stretched nor compressed. Suddenly the support of the hand is removed. The lowest position attained by the mass during oscillation is 4 cm below the point, where it was held in hand.

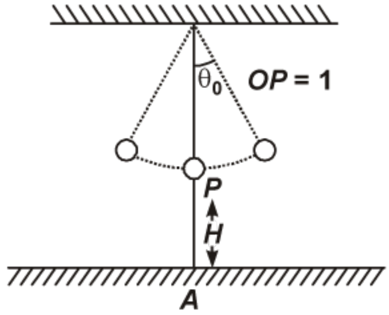
(a) What is the amplitude of oscillation? (b) Find the frequency of oscillation?

Q21. A cylindrical log of wood of height h and area of cross-section A floats in water. It is pressed and then released. Show that the log would execute S.H.M. with a time period.

$$T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

where m is mass of the body and ρ is density of the liquid.

Q22. A simple pendulum of time period 1*s* and length *l* is hung from a fixed support at *O*, such that the bob is at a distance *H* vertically above *A* on the ground (see figure).

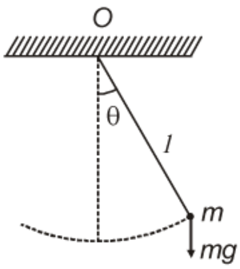


The amplitude is θ_0. The string snaps at $\theta = \theta_0 /2$. Find the time taken by the bob to hit the ground. Also find distance from A where bob hits the ground. Assume θ_0 to be small so that $\sin \theta_0 \simeq \theta_0$ and $\cos \theta_0 \simeq 1$.

Date: 14/10/2021

S1. (a) (A), (C), (E), (G) (b) (B), (D), (F), (H)

S2. Consider the diagram of a simple pendulum.



The bob is displaced through an angle θ shown.

The restoring torque about the fixed point *O* is

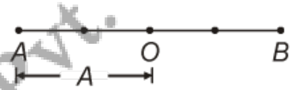
$$\tau = -mg \sin\theta$$

If θ is small angle in radians, then $\sin\theta \approx \theta$

$\Rightarrow$ $$\tau = -mg\,\theta \quad \Rightarrow \quad \tau \propto (-\theta)$$

Hence, motion of a simple pendulum is SHM for small angle of oscillations.

S3. The diagram represents the motion of a particle excuting S.H.M. between *A* and *B*.



Total distance travelled while it goes from *A* to *B* and returns to *A* is

$$= AO + OB + BO + OA \qquad [\because\ OA = A]$$
$$= A + A + A + A = 4A$$

Amplitude = $OA = A$.

Hence, ratio of distance and amplitude = $\dfrac{4A}{A} = 4$.

S4. The two basic characteristics of a simple harmonic motion

(a) Acceleration is directly proportional to displacement.

(b) Acceleration is directed opposite to displacement.

S5. Let $x = A\sin\omega t$ is the displacement function of S.H.M.

Velocity, $$v = \frac{dx}{dt} = A\omega\cos\omega t$$
$$v = A\omega\,|\cos\omega t|_{max}$$
$$= A\omega \times 1 = \omega A \qquad [\because\ |\cos\omega t|_{max} = 1] \quad \ldots \text{(i)}$$

Acceleration, $$a = \frac{dv}{dt} = -\omega A \cdot \omega \sin\omega t$$
$$= -\omega^2 A \sin\omega t$$

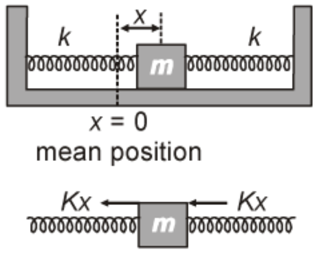
$$|a_{max}| = |(-\omega^2 A)(+1)| \qquad [\because\ (\sin\omega t)_{max} = 1]$$
$$|a_{max}| = \omega^2 A \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{v_{max}}{a_{max}} = \frac{\omega A}{\omega^2 A} = \frac{1}{\omega}$$

$$\Rightarrow \qquad \frac{a_{max}}{v_{max}} = \omega.$$

S6. Consider the diagram in which the block is displaced right through x.

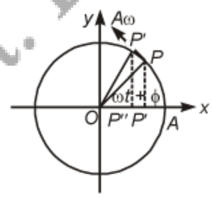


The right spring gets compressed by x developing a restoring force kx towards left on the block. The left spring is stretched by an amount x developing a restoring force kx towards left on the block as given in the free body diagram of the block.

Hence, Total force (restoring) = $(kx + kx)$ [$\because$ Both force are in same direction]

$= 2kx$ towards left.

S7. As the particle on reference circle moves in anti-clockwise direction. The projection will move from P to O towards left.



Hence, in the position shown the velocity is directed from $P' \rightarrow P''$ *i.e.*, from right to left, hence sign is negative.

S8. Let us assume the displacement function of S.H.M.

$$x = A \cos \omega t$$

where, a = Amplitude of motion

$$\text{Velocity } v = \frac{dx}{dt}$$

or $$\frac{dx}{dt} = a(-\sin \omega t)\,\omega = -\omega \sin \omega t\, a$$

or $$v = -\omega a \sin \omega t$$

$$= \omega a \cos\left(\frac{\pi}{2} + \omega t\right) \qquad \left[\because \sin \omega t = \cos\left(\frac{\pi}{2} + \omega t\right)\right]$$

Now, phase of displacement = ωt

$$\text{Phase of velocity} = \frac{\pi}{2} + \omega t$$

$\therefore$ Difference in phase of velocity to that of phase of displacement

$$= \frac{\pi}{2} + \omega t - \omega t = \frac{\pi}{2}.$$

S9. Potential energy (P.E.) of a simple harmonic oscillator is

$$= \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \qquad \text{... (i)}$$

where, k = Force constant = $m\omega^2$

When, P.E. is plotted against displacement x, we will obtain a parabola.

When, $x = 0$, P.E. = 0

When, $x = \pm A$, P.E. = maximum

$$= \frac{1}{2}m\omega^2x^2$$

KE. of a simple harmonic oscillator $= \frac{1}{2}mv^2$ $\left[\because \; v = \omega\sqrt{A^2 - x^2}\right]$

$$= \frac{1}{2}m\left[\omega\sqrt{A^2 - x^2}\right]^2$$

$$= \frac{1}{2}m\omega^2(A^2 - x^2) \qquad \text{... (ii)}$$

This is also parabola, if plot K.E. against displacement x.

i.e., K.E. = 0 at $x = \pm A$

and K.E. $= \frac{1}{2}m\omega^2A^2$ at $x = 0$

Now, total energy of the simple harmonic oscillator = P.E. + K.E. [Using Eqs. (i) and (ii)]

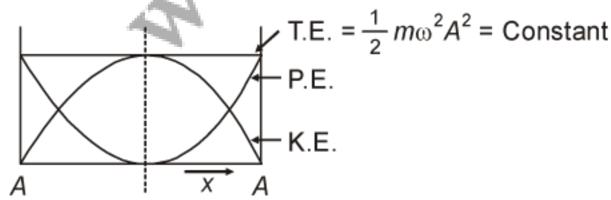
$$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2$$

$$\text{T.E} = \frac{1}{2}m\omega^2A^2$$

Which is constant and does not depend on x.

Plotting under the above guidelines K.E., P.E. and T.E. versus displacement x-graph as follows:



S10. Let us assume that the required displacement be x.

$\therefore$ Potential energy of the simple harmonic oscillator = $\frac{1}{2}kx^2$

where k = Force constant = $m\omega^2$

$$\therefore \quad \text{P.E.} = \frac{1}{2}m\omega^2x^2 \quad \text{... (i)}$$

Maximum energy of the oscillator

$$\text{T.E.} = \frac{1}{2}m\omega^2A^2 \quad [\because x_{max} = A] \quad \text{... (ii)}$$

where, A = Amplitude of motion

Given, $$\text{P.E.} = \frac{1}{2}\text{T.E.}$$

$$\Rightarrow \quad \frac{1}{2}m\omega^2x^2 = \frac{1}{2}\left[\frac{1}{2}m\omega^2A^2\right]$$

$$\Rightarrow \quad x^2 = \frac{A^2}{2}$$

or $$x = \sqrt{\frac{A^2}{2}} = \pm\frac{A}{\sqrt{2}}$$

Sign ± indicates either side of mean position.

S11. Given, potential energy associated with the field

$$U(x) = U_0(1 - \cos\alpha x) \quad \text{... (i)}$$

Now, $$\text{Force } (F) = -\frac{dU(x)}{dx} \quad \left[\because \text{ for conservative force } f\text{, we can write } f = \frac{-du}{dx}\right]$$

We have assumed the field to be conservative

$$F = -\frac{d}{dx}(U_0 - U_0\cos\alpha x) = -U_0\alpha\sin\alpha x$$

$$F = -U_0\alpha^2x \quad [\because \text{ for small oscillations } \alpha x \text{ is small, } \sin\alpha x = \alpha x]$$

$$\Rightarrow \quad F \propto (-x)$$

As, U_0, a being S.H.M. for small oscillations.

Standard equation for S.H.M. $F = -m\omega^2x$ ... (iii)

Comparing Eqs. (ii) and (iii), we get

$$m\omega^2 = U_0\alpha^2$$

$$\omega^2 = \frac{U_0\alpha^2}{m} \quad \text{or} \quad \omega = \sqrt{\frac{U_0\alpha^2}{m}}$$

$\therefore$ Time period $(T) = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{U_0\alpha^2}}$.

S12. A second's pendulum means a simple pendulum having time period $T = 2$ s.

For a simple pendulum, $T = 2\pi\sqrt{\dfrac{l}{g}}$

where, l = Length o the pendulum and

g = Acceleration due to gravity on surface of the Earth

$$T_e = 2\pi\sqrt{\frac{l_e}{g_e}} \qquad \text{... (i)}$$

On the surface of the Moon, $T_m = 2\pi\sqrt{\dfrac{l_m}{g_m}}$ ... (ii)

$$\therefore \quad \frac{T_e}{T_m} = \frac{2\pi}{2\pi}\sqrt{\frac{l_e}{g_e}} \times \sqrt{\frac{g_m}{l_m}}$$

$T_e = T_m$ to maintain the second's pendulum time period

$$\therefore \quad 1 = \sqrt{\frac{l_e}{l_m} \times \frac{g_m}{g_e}} \qquad \text{... (iii)}$$

But the acceleration due to gravity at Moon is 1/6 of the acceleration due to gravity at Earth, *i.e.*,

$$g_m = \frac{g_e}{6}$$

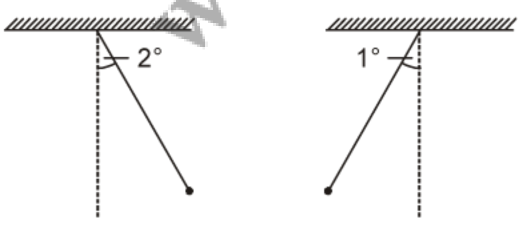
Squaring Eq. (iii) and putting this value,

$$1 = \frac{l_e}{l_m} \times \frac{g_e/6}{g_e} = \frac{l_e}{l_m} \times \frac{1}{6}$$

$$\Rightarrow \quad \frac{l_e}{6l_m} = 1 \quad = \quad 6l_m = l_e$$

$$\Rightarrow \quad l_m = \frac{1}{6}l_e = \frac{1}{6} \times 1 = \frac{1}{6}\,m .$$

S13. Consider the situations shown in the diagram (i) and (ii)



(i) (ii)

Assuming the two pendulums follow the following functions of their angular displacements

$$\theta_1 = \theta_0 \sin(\omega t + \phi_1) \quad \text{... (i)}$$

and

$$\theta_2 = \theta_0 \sin(\omega t + \phi_2) \quad \text{... (ii)}$$

As it is given that amplitude and time period being equal but phases being different.
Now, for first pendulum at any time t

$$\theta_1 = +\theta_0 \quad \text{[Right extreme]}$$

From Eq. (i), we get

$$\Rightarrow \quad \theta_0 = \theta_0 \sin(\omega t + \phi_1) \quad \text{or} \quad 1 = \sin(\omega t + \phi_1)$$

$$\Rightarrow \quad \frac{\pi}{2} = \sin(\omega t + \phi_1)$$

or

$$(\omega t + \phi_1) = \frac{\pi}{2} \quad \text{... (iii)}$$

Similarly, at the same instant t for pendulum second, we have

$$\theta_2 = -\frac{\theta_0}{2}$$

where, $\theta_0 = 2$ is the angular amplitude of first pendulum. For the second pendulum, the angular displacement is one degree, therefore $\theta_2 = \frac{\theta_0}{2}$ and negative sign is taken to show for being left to mean position.

From Eq. (ii), then

$$-\frac{\theta_0}{2} = \theta_0 \sin(\omega t + \phi_2)$$

$$\Rightarrow \quad \sin(\omega t + \phi_2) = -\frac{1}{2} \quad \Rightarrow \quad (\omega t + \phi_2) = -\frac{\pi}{6} \text{ or } \frac{7\pi}{6}$$

or

$$(\omega t + \phi_2) = -\frac{\pi}{6} \text{ or } \frac{7\pi}{6} \quad \text{... (iv)}$$

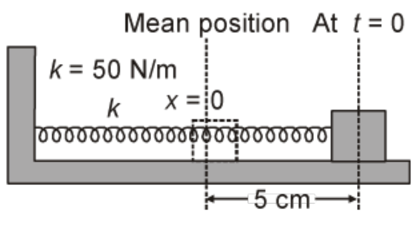
From Eqs. (iv) and (iii), the difference in phases

$$(\omega t + \phi_2) - (\omega t + \phi_1) = \frac{7\pi}{6} - \frac{\pi}{2} = \frac{7\pi - 3\pi}{6} = \frac{4\pi}{6}$$

or

$$(\phi_2 - \phi_1) = \frac{4\pi}{6} = \frac{2\pi}{3} = 120°.$$

S14. Consider the diagram of the spring block system. It is S.H.M. with amplitude of 5 cm about the mean position shown.



Given, Spring constant $k = 50$ N/m

$$m = \text{Mass attached} = 2 \text{ kg}$$

$$\therefore \quad \text{Angular frequency } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}} = \sqrt{25} = 5 \text{ rad/s}$$

Assuming the displacement function

$$y(t) = A \sin(\omega t + \phi)$$

where, ϕ = initial phase

But given at $t = 0$, $y(t) = +A$

$$y(0) = +A = A \sin(\omega \times 0 + \phi)$$

or $$\sin\phi = 1 \quad \Rightarrow \quad \phi = \frac{\pi}{2}$$

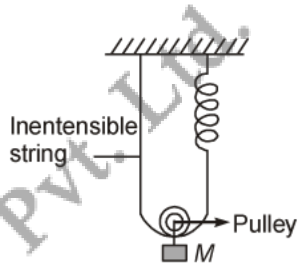
$\therefore$ The desired equation is $y(t) = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos\omega t$

Putting $A = 5\,\text{cm}$, $\omega = 5$ rad/s

We get, $y(t) = 5 \sin 5t$

where, t is in second and y is in centimetre.

S15. For the calculation purpose, in this situation we will neglect gravity because it is constant throughout and will not effect the net restoring force.



Let in the equilibrium position, the spring has extended by an amount x_0,

Now, if the mass is given a further displacement downwards by an amount x. The string and spring both should increase in length by x.

But, string is inextensible, hence the spring alone will contribute the total extension $x + x = 2x$, to lower the mass down by x from initial equilibrium mean position x_0. So net extension in the spring $(= 2x + x_0)$

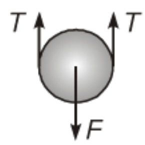
Now, force on the mass before bulling (in the x_0, extension case)

$$F = 2T$$

But, $T = kx_0$ [where k is spring constant]

$\therefore$ $F = 2kx_0$ ... (i)

When the mass is lowered down further by x,



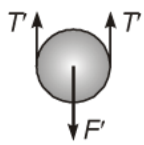
$$F' = 2T$$

But new spring length $= (2x + x_0)$

$\therefore$ $F' = 2k(2x + x_0)$ ... (ii)

Restoring force on the system

$$F_{\text{restoring}} = -[F' - F]$$



Using Eqs. (i) and (ii), we get

$$F_{\text{restoring}} = 2k(2x + x_0) - 2kx_0]$$
$$= -[2 \times 2kx + 2kx_0 - 2kx_0]$$
$$= -4kx$$

or $Ma = -4kx$

where, a = Acceleration (As, $F = ma$)

$$a = -\left(\frac{4k}{M}\right)x$$

k, M being constant

$\therefore \qquad a = -x$

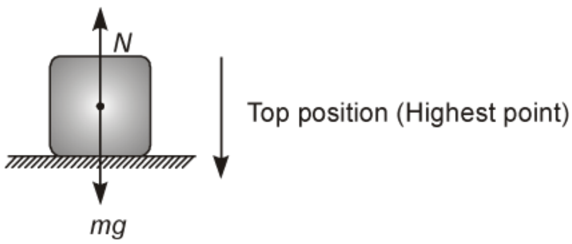
Hence, motion is S.H.M.

Comparing the above acceleration expression with standard S.H.M. equation $a = -\omega^2 x$, we get

$$\omega^2 = \frac{4k}{M} \Rightarrow \omega = \sqrt{\frac{4k}{M}}$$

$$\therefore \qquad \text{Time period } T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4k}{M}}} = 2\pi\sqrt{\frac{M}{4k}}\ .$$

S16. In this case acceleration is variable. In accelerated motion, weight of body depends on the magnitude and direction of acceleration for upward or downward motion.



(a) Hence, the weight of the body changes during oscillations.

(b) Considering the situation in two extreme positions, as their acceleration is maximum in magnitude.

We, have $\qquad mg - N = ma$

$\because$ At the highest point, the platform is accelerating downward.

$\Rightarrow \qquad N = mg - ma$ [Weight measred by the machine is reaction]

But $\qquad a = \omega^2 A$ [In magnitude]

$\therefore \qquad N = mg - m\omega^2 A$

where, $\qquad A$ = Amplitude of motion.

Give, $\qquad m = 50\,\text{kg}$, Frequency $\nu = 2\,\text{s}^{-1}$

$\therefore \qquad \omega = 2\pi\nu = 4\pi$ rad/s

$A = 5\,\text{cm} = 5 \times 10^{-2}\,\text{m}$

$$\therefore \qquad N = 50 \times 9.8 - 50 \times (4)^2 \times 5 \times 10^{-2}$$

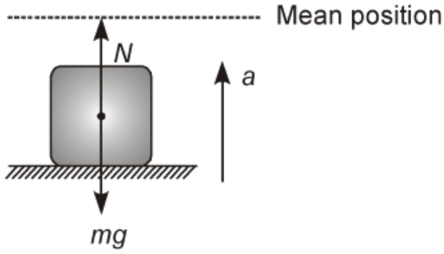
$$= 50[9.8 - 16\pi^2 \times 5 \times 10^{-2}]$$

$$= 50[9.8 - 7.89]$$

$$= 50 \times 1.91$$

$$= 95.5\,\text{N}$$

When the platform is at the lowest position of its oscillation,



It is accelerating towards mean position that is vertically upwards.

Writing equation of motion

$$N - mg = ma = m\omega^2 A$$

or $$N = mg + m\omega^2 A$$

$$= m[g + \omega^2 A]$$

Putting the date $$N = 50[9.8 + (4\pi)^2 \times 5 \times 10^{-2}]$$

$$= 50[9.8 + (12.56)^2 \times 5 \times 10^{-2}]$$

$$= 50[9.8 + 7.88]$$

$$= 50 \times 17.884 \text{ N}$$

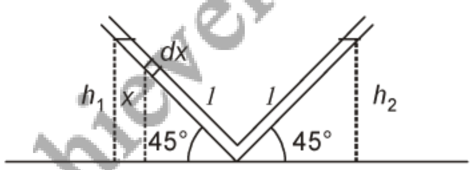
Now, the machine reads the normal reaction. It is clear that

Maximum weight = 884 N [At lowest point]

Minimum weight = 95.5 N. [At top point]

S17. Consider the liquid in the length dx. It's mass is $A\rho dx$ at a height x.

$$\text{P.E.} = A\rho dx\, gx$$



The P.E. of the left column = $\int_0^{h_1} A\rho g x\, dx$

$$= A\rho g \left.\frac{x^2}{2}\right|_0^{h_1}$$

$$= A\rho g \frac{h_2^2}{2} = \frac{A\rho g l^2 \sin^2 45^\circ}{2}$$

Similarly, P.E. of the right column = $A\rho g \dfrac{h_2^2}{2} = \dfrac{A\rho g l^2 \sin^2 45^\circ}{2}$

$$h_1 = h_2 = l \sin 45^\circ$$

where l is the length of the liquid in one arm of the tube.

$$\text{Total P.E.} = A\rho g h^2 = A\rho g l^2 \sin^2 45^\circ = \frac{A\rho g l^2}{2}.$$

If the change in liquid level along the tube in left side in *y*, then length of the liquid in left side is $l - y$ and in the right side is $l + y$.

$$\text{Total P.E.} = A\rho g\,(l - y)^2 \sin^2 45° + A\rho g\,(l + y)^2 \sin^2 45°$$

$$\text{Change in PE} = (\text{P.E.})_f - (\text{P.E.})_i$$

$$= \frac{A\rho g}{2}[(l-y)^2 + (l-y)^2 - l^2]$$

$$= \frac{A\rho g}{2}[(\cancel{l}^2 + y^2 - 2\cancel{ly} - l^2 + y^2 + 2\cancel{ly} - \cancel{l}^2]$$

$$= \frac{A\rho g}{2}[y^2 + l^2]$$

$$\text{Change in K.E.} = \frac{1}{2}A\rho 2l y^2$$

$$\text{Change in total energy} = 0$$

$$\Delta(\text{P.E}) + \Delta(\text{K.E}) = 0$$

$$A\rho g\,[l^2 + y^2] + A\rho l\dot{y}^2 = 0$$

Differentiating both sides w.r.t. time,

$$A\rho g\left[0 + 2y\frac{dy}{dt}\right] + 2A\rho l\dot{y}\ddot{y} = 0$$

$$2A\rho g y + 2A\rho l\dot{y} = 0$$

$$l\ddot{y} + gy = 0$$

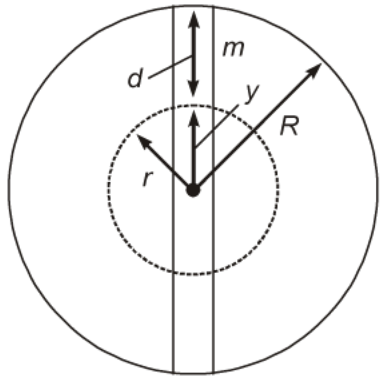
$$\ddot{y} + \frac{g}{l}y = 0$$

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

S18. Consider the situation shown in the diagram.



The gravitational force on the particle at a distance r from the centre of the Earth arises entirely from that portion of matter of the Earth in shells internal to the position of the particle. The external shells exert no force on the particle.

More clearly,

Let g' be the acceleration at P.

So, $$g' = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right)$$

From figure, $R - d = y$

$\Rightarrow$ $$g' = g\,\frac{y}{R}$$

Force on body at p $$F = -mg' = \frac{-mg}{R}\,y \quad \text{... (i)}$$

$\Rightarrow$ $F \propto -y$ [where, y is distance from the centre]

So, motion is S.H.M.,

For time period, we can write Eq. (i)

As $$ma = -\frac{Mg}{R}\,y \quad \Rightarrow \quad a = -\frac{g}{R}\,y$$

Comparing with $a = -\omega^2 y$

$$\omega^2 = \frac{g}{R}$$

$\Rightarrow$ $$\left(\frac{2\pi}{T}\right) = \frac{g}{R} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{R}{g}}$$

S19. We have to convert the given combination of two harmonic functions to a single harmonic (sine or cosine) function.

Given, displacement function $y = \sin\omega t - \cos\omega t$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cdot\sin\omega t - \frac{1}{\sqrt{2}}\cdot\cos\omega t\right)$$

$$= \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)\cdot\sin\omega t - \sin\left(\frac{\pi}{4}\right)\cdot\cos\omega t\right)$$

$$= \sqrt{2}\left[\sin\left(\omega t - \frac{\pi}{4}\right)\right] = \sqrt{2}\left[\sin\left(\omega t - \frac{\pi}{4}\right)\right]$$

Comparing with standard equation

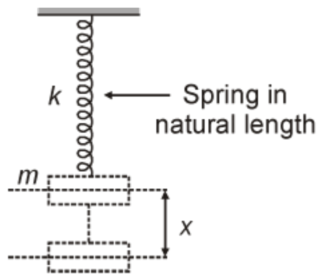
$$y = a\sin(\omega t + \phi),$$

we get, $$\omega = \frac{2\pi}{T} \quad \Rightarrow \quad T = \frac{2\pi}{\omega}$$

Clearly, the function represents S.H.M. with a period $T = \frac{2\pi}{\omega}$.

S20. (a) When the support of the hand is removed, the body oscillates about a mean position.

Suppose x is the maximum extension in the spring when it reaches the lowest point in oscillation.



Loss in P.E. of the block = mgx ... (i)

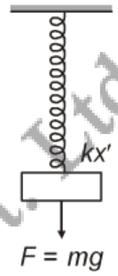
where, m = Mass of the block

Gain in elastic potential energy of the spring

$$= \frac{1}{2}kx^2 \quad \text{... (ii)}$$

As the two are equal, conserving the mechanical energy,

we get, $$mgx = \frac{1}{2}kx^2 \quad \text{or} \quad x = \frac{2mg}{k} \quad \text{... (iii)}$$



Now, the mean position of oscillation will be, when the block is balanced by the spring.

It x' is the extension in that case, then

$$F = +kx'$$

But $$F = mg$$

⇒ $$mg = +kx'$$

or $$x = \frac{mg}{k} \quad \text{... (iv)}$$

Dividing Eq. (iii) by Eq. (iv).

$$\frac{x}{x'} = \frac{2mg}{k} \Big/ \frac{mg}{k} = 2$$

⇒ $$x = 2x'$$

But given x = 4 cm (maximum extension from the unstretched position)

∴ $$2x' = 4$$

∴ $$x' = \frac{4}{2} = 2 \text{ cm}$$

But the displacement of mass from the mean position to the position when spring attains its natural length is equal to amplitude of the oscillation.

∴ $$A = x' = 2 \text{ cm}$$

where, A = Amplitude of the motion.

(b) Time period of the oscillating system depends on mass spring constant given by]

$$T = 2\pi\sqrt{\frac{m}{k}}$$

It does not depend on the amplitude.

But from Eq. (iii), $\frac{2mg}{k} = x$ [Maximum extension]

$$\frac{2mg}{k} = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$$

$$\therefore \quad \frac{m}{k} = \frac{4 \times 10^{-2}}{2g} = \frac{2 \times 10^{-2}}{g}$$

$$\therefore \quad \frac{k}{m} = \frac{g}{2 \times 10^{-2}}$$

and $\nu = \text{Frequency} = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

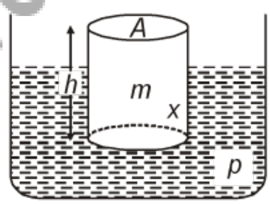
$$\therefore \quad \nu = \frac{1}{2 \times 3.14}\sqrt{\frac{g}{2 \times 10^{-2}}}$$

$$= \frac{1}{2 \times 3.14}\sqrt{\frac{4.9}{10^{-2}}} = \frac{1}{6.28} \times \sqrt{4.9 \times 100}$$

$$= \frac{10}{6.28} \times 221 = 351 \text{ Hz.}$$

Note: The block during oscillation cannot go above the position it was released, as it is given that there is no velocity in the upward direction either by the system or external agent.

S21. Consider the diagram.



Let the log be pressed and let the vertical displacement at the equilibrium position be x_0.

At equilibrium mg = Buoyant force $(\rho A x_0)\, g$ $[\because \; m = V\rho = (Ax_0)\, \rho]$

When it is displaced by a further displacement x, the buoyant force is $A\,(x_0 + x)\,\rho g$

$\therefore$ Net restoring force = Buoyant force – Weight

$= A\,(x_0 + x)\,\rho g - mg$

$= (A\rho g)\, x$ $[\because \; mg = \rho A x_0 g]$

As displacement x is downward and restoring force is upward,

we can write $F_{\text{restoring}} = -(A\rho g)\, x$

$= -kx$

where, $k = \text{Constant} = A\rho g$

So, the motion is S.H.M. $[\because \; F \propto -x]$

Now, Acceleration $a = \dfrac{F_{\text{restoring}}}{m} = -\dfrac{k}{m}x$

Comparing with $a = -\omega^2 x$

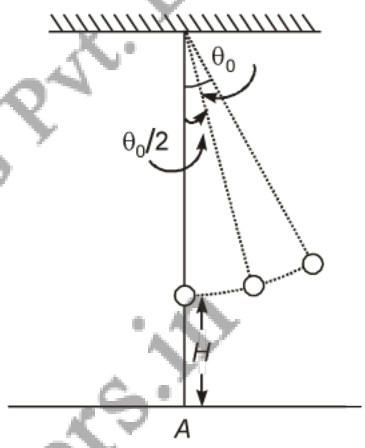
$\Rightarrow \quad \omega^2 = \dfrac{k}{m} \quad \Rightarrow \quad \omega = \sqrt{\dfrac{k}{m}}$

$\Rightarrow \quad \dfrac{2\pi}{T} = \sqrt{\dfrac{k}{m}} \quad \Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k}}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{A\rho g}}$.

S22. Assume that $t = 0$ when $\theta = \theta_0$. Then,

$$\theta = \theta_0 \cos \omega t$$



Given a seconds pendulum $\omega = 2\pi$

At time t_1, let $\theta = \theta_0/2$

$\therefore \quad \cos 2\pi t_1 = 1/2 \quad \Rightarrow \quad t_1 = \dfrac{1}{6}$

$$\dot{\theta} = -\theta_0 2\pi \sin 2\pi t \quad \left[\dot{\theta} = \frac{d\theta}{dt}\right]$$

At $t_1 = 1/6$

$$0 = -\theta_0 2\pi \sin\frac{2\pi}{6} = -\sqrt{3}\pi\theta_0$$

Thus the linear velocity is $u = -\sqrt{3}\pi\theta_0 l$ perpendicular to the string.

The vertical component is

$$u_y = -\sqrt{3}\pi\theta_0 l \sin\theta_0$$

and the horizontal component is

$$u_x = -\sqrt{3}\pi\theta_0 l \cos\theta_0$$

At the time it snaps, the vertical height is

$$H' = H + l(1 - \cos(\theta/2))$$

Let the time required for fall be t, then

$H' = u_y t + (1/2)gt^2$ (notice g is also in the negative direction)

Or, $\dfrac{1}{2}gt^2 + \sqrt{3}\pi\theta_0 l \sin\theta_0 t - H' = 0$

$$\therefore \qquad t' = \frac{-\sqrt{3}\pi\theta_0 l \sin\theta_0 \pm \sqrt{3\pi^2\theta_0^2 e^2 \sin^2\theta_0 + 2gH'}}{g}$$

$$\simeq \frac{-\sqrt{3}\pi l\theta_0^2 \pm \sqrt{3\pi^2\theta_0^4 l^2 + 2gH'}}{g}$$

Neglecting terms of order θ_0^2 and heigher,

$$t \simeq \sqrt{\frac{2H'}{g}}.$$

Now $\qquad H' \simeq H + l(1 - 1) = H \quad \therefore \; t \simeq \sqrt{\frac{2H}{g}}.$

The distance travelled in the x direction is $u_x t$ to the left of where it snapped.

$$X = \sqrt{3}\pi\theta_0 l \cos\theta_0 \sqrt{\frac{2H}{g}}$$

To order of θ_0,

$$X = \sqrt{3}\pi\theta_0 l \sqrt{\frac{2H}{g}} = \sqrt{\frac{6H}{g}}\,\theta_0 l.$$

At the time of snapping, the bob was $l \sin\theta \simeq l\theta_0$ distance from A.

Thus, the distance from A is

$$l\theta_0 - \sqrt{\frac{6H}{g}}\, l\theta_0 = l\theta_0\left(1 - \sqrt{6H/g}\right).$$