Date: 14/10/2021

Q1. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature *T*. Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)

Q2. The molecules of a given mass of a gas have root mean square speeds of 100 m s”1 at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure?

Q3. Two molecules of a gas have speeds of $9 \times 10^6 \, ms^{-1}$ and $1 \times 10^6 \, ms^{-1}$, respectively. What is the root mean square speed of these molecules.

Q4. The volume of a given mass of a gas at 27°C, 1 atm is 100 cc. What will be its volume at 327°C?

Q5. Calculate the number of atoms in 39.4 g gold. Molar mass of gold is 197 g $mole^{-1}$.

Q6. Explain why

(a) there is no atmosphere on moon. (b) there is fall in temperature with altitude.

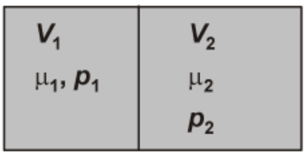
Q7. When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case?

Q8. An insulated container containing monoatomic gas of molar mass *m* is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.

Q9. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP.

Q10. A gas mixture consists of molecules of types *A*, *B* and *C* with masses $m_A > m_B > m_C$. Rank the three types of molecules in decreasing order of (a) average K.E., (b) r.m.s. speeds.

Q11. The container shown in figure has two chambers, separated by a partition, of volumes V_1 = 2.0 litre and V_2 = 3.0 litre. The chambers contain μ_1 = 4.0 and μ_2 = 5.0 moles of a gas at pressures p_1 = 1.00 atm and p_2 = 2.00 atm. Calculate the pressure after the partition is removed and the mixture attains equilibrium.



Q12. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1Å and 2Å. The gases may be considered under identical conditions of temperature, pressure and volume.

Q13. We have 0.5 g of hydrogen gas in a cubic chamber of size 3 cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law, in the final state?

(Hydrogen molecules can be consider as spheres of radius 1 Å).

Q14. A balloon has 5.0 g mole of helium at 7°C. Calculate

(a) the number of atoms of helium in the balloon,

(b) the total internal energy of the system.

Q15. Ten small planes are flying at a speed of 150 km/h in total darkness in an air space that is $20 \times 20 \times 1.5$ km^3 in volume. You are in one of the planes, flying at random within this space with no way of knowing where the other planes are. On the average about how long a time will elapse between near collision with your plane. Assume for this rough computation that a safety region around the plane can be approximated by a sphere of radius 10 m.

Q16. Consider a rectangular block of wood moving with a velocity v_0 in a gas at temperature T and mass density ρ. Assume the velocity is along x-axis and the area of cross-section of the block perpendicular to v_0 is A. Show that the drag force on the block is $4\rho A v_0 \sqrt{\frac{kF}{m}}$, where m is the mass of the gas molecule.

Q17. A box of 1.00 m^3 is filled with nitrogen at 1.50 atm at 300 K. The box has a hole of an area 0.010 mm^2. How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm.

Q18. Consider an ideal gas with following distribution of speeds.

Speed (m/s)	200	400	600	800	1000
% of molecules	10	20	40	20	10

(a) Calculate $v_{\text{r.m.s.}}$ and hence T. ($m = 3.0 \times 10^{-26}$ kg)

(b) If all the molecules with speed 1000 m/s escape from the system, calculate new $V_{\text{r.m.s.}}$ and hence T.

Date: 14/10/2021

S1. O_2 has 5 degrees of freedom. Therefore, energy per mole = $\frac{5}{2}RT$

$\therefore$ For 2 moles of O_2, energy = $5RT$

Neon has 3 degrees of freedom $\therefore$ Energy per mole = $\frac{3}{2}RT$

$\therefore$ For 4 mole of neon, energy = $4 \times \frac{3}{2}RT = 6RT$

$\therefore$ Total energy = $11RT$.

S2.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{P_2T_1}{P_1T_2} = \frac{2\times 300}{400} = \frac{3}{2}$$

$$P_1 = \frac{1}{3}\frac{M}{V_1}c_1^{-2}; \qquad P_2 = \frac{1}{3}\frac{M}{V_2}c_2^{-2}$$

$\therefore$

$$c_2^2 = c_1^2 \times \frac{V_2}{V_1} \times \frac{P_2}{P_1}$$

$$= (100)^2 \times \frac{2}{3} \times 2$$

$$c_2 = \frac{200}{\sqrt{3}}\,\text{m s}^{-1}.$$

S3.

$$v_{\text{r.m.s.}} = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

$$= \sqrt{\frac{(9\times 10^6)^2 + (1\times 10^6)^2}{2}}$$

$$= \sqrt{\frac{(91+1)\times 10^{12}}{2}} = \sqrt{41}\times 10^6\,\text{m s}^{-1}.$$

S4. We have to convert the given temperatures in kelvin.

If pressure of a given mass of the gas is kept constant, then

$$V \propto T$$

$\Rightarrow$ $\frac{V}{T}$ = Constant $\left[\begin{array}{l} V = \text{Volume of gas} \\ T = \text{Temperature of gas} \end{array}\right]$

$$\Rightarrow \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow \qquad V_2 = V_1\left(\frac{T_2}{T_1}\right)$$

$$T_1 = 273 + 27 = 300\,\text{K}$$
$$T_2 = 273 + 327 = 600\,\text{K}$$

But $V_1 = 100\,\text{cc}$

$$V_2 = V_1\left(\frac{600}{300}\right)$$

$$V_2 = 2V_1$$

$$\therefore \qquad V_2 = 2 \times 100 = 200\,\text{cc}.$$

Note: To apply ideal gas equation, we must convert the given temperature in kelvin.

S5. $\therefore$ Molar mass of gold is 197 g mole^{-1}, the number of atoms $= 6.0 \times 10^{23}$.

$$\therefore \qquad \text{No. of atoms in } 39.4\,\text{g} = \frac{6.0 \times 10^{23} \times 39.4}{197} = 1.2 \times 10^{23}.$$

S6. (a) The Moon has small gravitational force and hence the escape velocity is small. As the Moon is in the proximity of the Earth as seen from the Sun, the Moon has the same amount of heat per unit area as that of the Earth. The air molecules have large range of speeds. Even though the r.m.s. speed of the air molecules is smaller than the escape velocity on the Moon, a significant number of molecules have speed greater than escape velocity and they escape. Now rest of the molecules arrange the speed distribution for the equilibrium temperature. Again a significant number of molecules escape as their speeds exceed escape speed. Hence, over a long time the Moon has lost most of its atmosphere.

At 300 K $\qquad V_{\text{r.m.s.}} = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3 \times 1.38 \times 10^{-23} \times 300}{7.3 \times 10^{-26}}} = 1.7\ \text{km/s}$

[k = Boltzman constant, T = Temperature]

V_{esc} for Moon = 4.6 km/s.

(b) As the molecules move higher their potential energy increases and hence kinetic energy decreases and hence temperature reduces.

At greater height more volume is available and gas expands and hence some cooling takes place.

S7. When air is pumped, more molecules are pumped in. Boyle's law is stated for situation where number of molecules remain constant.

S8. According to kinetic interpretation of temperature, absolute temperature of a given sample of a gas is proportional to the total translational kinetic energy of its molecules.

Hence, any change in absolute temperature of a gas will contribute to corresponding change in translational K.E. and *vice versa*.

Assuming n = Number of noices.

Given, m = Molar mass of the gas.

When, the container stops, its total K.E. is transferred to gas molecules in the form of translational K.E., thereby increasing the absolute temperature.

If ΔT = change in absolute temperature

Then, K.E. of molecules due to velocity v_0,

$$\text{K.E.} = \frac{1}{2}(mv)v_0^2 \quad \text{... (i)}$$

$$\text{Increase in translational K.E.} = n\frac{3}{2}R(\Delta T) \quad \text{... (ii)}$$

According to kinetic theory Eqs. (i) and (ii) are equal

$$\Rightarrow \quad \frac{1}{2}(mv)v_0^2 = n\frac{3}{2}R(\Delta T)$$

$$(mv)v_0^2 = n3R(\Delta T)$$

$$\Rightarrow \quad \Delta T = \frac{(mn)v_0^2}{3nR}$$

$$\Rightarrow \quad \Delta T = \frac{mv_0^2}{3R}$$

S9. Volume occupied by 1gram mole of gas at NTP = 22400 cc

$\therefore$ Number of molecules in 1 cc of hydrogen

$$= \frac{6.023\times10^{23}}{22400} = 2.688\times10^{19}$$

As each diatomic molecule has 5 degrees of freedom, hydrogen being diatomic also has 5 degrees of freedom

$\therefore$ Total no of degrees of freedom $= 5\times2.688\times10^{19}$

$$= 1.344\times10^{20}.$$

S10. (a) The average K.E. will be the same as conditions of temperature and pressure are the same

Now as, $$v_{\text{r.m.s.}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}} \quad [\because\ PV = nRT]$$

$$= \sqrt{\frac{3RT}{mN}} = \sqrt{\frac{3kT}{m}}$$

where, M = Molar mass of the gas
m = Mass of each molecular of the gas
R = Gas constant

Clearly, $v_{\text{r.m.s.}} = \sqrt{\dfrac{1}{m}}$

(b) As k = Boltzmann constant
T = Absolute temperature (same for all)

But, $m_A > m_B > m_C$

$(v_{\text{r.m.s.}})_A < (v_{\text{r.m.s.}})_B < (v_{\text{r.m.s.}})_C$

∴ or, $(v_{\text{r.m.s.}})_C > (v_{\text{r.m.s.}})_B > (v_{\text{r.m.s.}})_A$.

S11. $V_1 = 2.0$ litre $\quad V_2 = 3.0$ litre
$\mu_1 = 4.0$ moles $\quad \mu_2 = 5.0$ moles
$P_1 = 1.00$ atm $\quad P_2 = 2.00$ atm

$P_1V_1 = \mu_1 RT_1 \quad P_2V_2 = \mu_2 RT_2$
$\mu = \mu_2 + \mu_2 \quad V = V_1V_2$

For 1 mole $\quad PV = \dfrac{2}{3}E$

For μ_1 moles $\quad P_1V_1 = \dfrac{2}{3}\mu_1 E_1$

For μ_2 moles $\quad P_2V_2 = \dfrac{2}{3}\mu_2 E_2$

Total energy is $\quad (\mu_1 E_1 + \mu_2 E_2) = \dfrac{2}{3}(P_1V_1 + P_2V_2)$

$$PV = \frac{2}{3}E_{\text{total}} = \frac{2}{3}\mu E_{\text{permole}}$$

$$P(V_1 + V_2) = \frac{2}{3} \times \frac{2}{3}(P_1V_1 + P_2V_2)$$

$$= \left(\frac{1.00 \times 2.0 \times 2.00 \times 3.0}{2.0 + 3.0}\right) \text{atm}$$

$$= \frac{8.0}{5.0} = \mathbf{1.60\ atm}.$$

Comment: This form of ideal gas law represented by Equation marked* becomes very useful for adiabatic changes.

S12. Mean free path of a molecule is given by

$$l = \frac{1}{\sqrt{2}d^2 n}$$

where, n = number of molecules/volume

d = diameter of the molecule

Now, we can write $\quad l \propto \frac{1}{d^2}$

Given, $\quad d_1 = 1\text{Å} \qquad d_2 = 2\text{Å}$

As $\quad l_1 \propto \frac{1}{d_1^2} \quad$ and $\quad l_2 \propto \frac{1}{d_2^2}$

$\Rightarrow$ So, $\quad \frac{l_1}{l_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$

Hence, $\quad l_1 : l_2 =$ **4 : 1**.

S13. We have $0.25 \times 6 \times 10^{23}$ molecules, each of volume $10^{-30}\,\text{m}^3$.

Total volume of all Molecules = $2.5 \times 10^{-7}\,\text{m}^3$

Supposing Ideal gas law is valid.

Final volume = $\frac{V_{\text{in}}}{100} = \frac{(3)^3 \times 10^{-6}}{100} \approx 2.7 \times 6.02 \times 10^{23}$.

which is about the molecular volume. Hence, intermolecular forces cannot be neglected. Therefore the ideal gas situation does not hold.

S14. (a)

$$\mu = 5.0$$

$$T = 280\text{K}$$

$$\text{No of atoms} = \mu N_A = 5.0 \times 6.02 \times 10^{23}$$

$$= 30 \times 10^{23}$$

(b) Average kinetic energy per molecule = $\frac{2}{3}kT$

$\therefore$ Total internal energy = $\frac{2}{3}kT \times N$

$$= \frac{2}{3} \times 30 \times 10^{23} \times 1.38 \times 10^{-23} \times 280$$

$$= 1.74 \times 10^4\,\text{J}.$$

S15. Time $t = \frac{\lambda}{v}$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}, \quad d = \text{diameter and } n = \text{number density}$$

$$n = \frac{N}{V} = \frac{10}{20 \times 20 \times 1.5} = 0.0167 \text{ km}^{-3}$$

$$t = \frac{1}{\sqrt{2}\pi d^2 (N/V) \times v}$$

$$= \frac{1}{1.414 \times 3.14 \times (20)^2 \times 0.0167 \times 10^{-3} \times 150}$$

$$= 225 \text{ h.}$$

S16. n = no. of molecules per unit volume

$v_{\text{r.m.s.}}$ = r.m.s. speed of gas molecules

When block is moving with speed v_0, relative speed of molecules w.r.t. front face = $v + v_0$.

Coming head on, momentum transferred to block per collision = $2m\,(v + v_0)$, where m = mass of molecule.

No. of collision in time $\Delta t = \frac{1}{2}(v - v_0)\, n\, \Delta t\, A$, where A = area of cross section of block and factor of 1/2 appears due to particles moving towards block.

$\therefore$ Momentum transferred in time $\Delta t = m\,(v + v_0)^2 n\Delta t A$ from front surface.

Similarly momentum transferred in time $\Delta t = m\,(v + v_0)^2 n\Delta t A$ from back surface.

$\therefore$ Net force (drag force) = $mnA\,[(v + v_0) - (v - v_0)^2]$ from front

$= mnA\,(4vv_0) = (4mnAv)\,v_0$

$= (4\rho\, Av)\,v_0$

We also have, $\frac{1}{2}mv^2 = \frac{1}{2}kT$ (v - is the velocity along x-axis)

Therefore, $v = \sqrt{\frac{kT}{m}}$.

Thus, drag $= 4\rho\sqrt{\frac{kT}{m}}\,Av_0$.

S17. v_{1x} = speed of molecule inside the box along x direction

n_1 = number of molecules per unit volume

In time Δt, particles moving along the wall will collide if they are within $(v_{1x}\,\Delta t)$ distance. Let a = area of the wall. No. of particles colliding in time $\Delta t = \frac{1}{2}n_i\,(v_{1x}\,\Delta t)\,a$ (factor of 1/2 due to motion towards wall).

In general, gas is in equilibrium as the wall is very large as compared to hole.

$$\therefore \qquad v_{1x}^2 + v_{1y}^2 + v_{1z}^2 = v_{\text{r.m.s.}}^2$$

$$\therefore \qquad v_{1x}^2 = \frac{V_{\text{r.m.s}}^2}{3}$$

$$\frac{1}{2} m v_{\text{r.m.s}}^2 = \frac{3}{2} kT \quad \Rightarrow \quad v_{\text{r.m.s.}}^2 = \frac{3kT}{m}$$

$$\therefore \qquad v_{1x}^2 = \frac{kT}{m}$$

$\therefore$ No. of particles colliding in time $\Delta t = \frac{1}{2} n_i \sqrt{\frac{kT}{m}} \Delta t\, a$. If particles collide along hole, they move out. Similarly outer particles colliding along hole will move in.

$\therefore$ Net particle flow in time $\Delta t = \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}} \Delta t\, a$ as temperature is same in and out.

$$pV = \Delta RT \Rightarrow \mu = \frac{PV}{RT}$$

$$n = \frac{\mu N_A}{V} = \frac{PN_A}{RT}$$

After some time τ pressure changes to p_1' inside

$$\therefore \qquad n_1' = \frac{P_1' N_A}{RT}$$

$$n_1 V - n_1' V = \text{no. of particle gone out } \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}}\, \tau a$$

$$\therefore \qquad \frac{P_1 N_A}{RT} V - \frac{P_1' N_A}{RT} V = \frac{1}{2}(P_1 - P_2)\frac{N_A}{RT}\sqrt{\frac{kT}{m}}\, \tau a$$

$$\therefore \qquad \tau = 2\left(\frac{P_1 - P_1'}{P_1 - P_2}\right)\frac{V}{a}\sqrt{\frac{m}{kT}}$$

$$= 2\left(\frac{1.5 - 1.4}{1.5 - 1.0}\right)\frac{5 \times 1.00}{0.01 \times 10^{-6}}\sqrt{\frac{46.7 \times 10^{-27}}{1.38 \times 10^{-23} \times 300}}$$

$$= 1.38 \times 10^5 \text{ s.}$$

S18. (This problem is designed to give an idea about cooling by evaporation)

(a) $$v_{\text{r.m.s.}}^2 = \frac{\sum_i n_i v_i^2}{\sum n_i}$$

$$= \frac{10 \times (200)^2 + 20 \times (400)^2 + 40 \times (600)^2 + 20 \times (800)^2 + 10 \times (1000)^2}{100}$$

$$= \frac{10 \times 100^2 \times (1 \times 4 \times 2 \times 16 + 4 \times 36 + 2 \times 64 + 1 \times 100)}{100}$$

$$= 1000 \times (4 + 32 + 144 + 128 + 100) = 408 \times 1000\,\text{m}^2/\text{s}^2$$

$$\therefore \qquad v_{\text{r.m.s.}} = 639\,\text{m/s}$$

$$\frac{1}{2}\, m v^2_{\text{r.m.s.}} = \frac{2}{3}\, kT$$

[*k* = Boltzman constant]

$$\therefore \qquad T = \frac{1}{3}\frac{m v^2_{\text{r.m.s.}}}{\text{k}} = \frac{1}{3} \times \frac{3.0 \times 10^{-26} \times 4.08 \times 10^5}{1.38 \times 10^{-23}}$$

$$= 2.96 \times 10^2\ \text{K} = 296\text{K}.$$

(b)

$$v^2_{\text{r.m.s.}} = \frac{10 \times (200)^2 + 20 \times (400)^2 + 40 \times (600)^2 + 20 \times (800)^2}{90}$$

$$= \frac{10 \times 100^2 \times (1 \times 4 + 2 \times 16 + 4 \times 36 + 2 \times 64)}{90}$$

$$= 10000 \times \frac{308}{9} = 342 \times 1000\,\text{m}^2/\text{s}^2$$

$$v_{\text{r.m.s.}} = 584\,\text{m/s}$$

$$T = \frac{1}{3}\frac{mV^2}{k} = 248\,\text{K}\,.$$