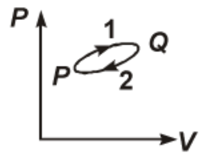
Date: 14/10/2021

Q1. Can a system be heated and its temperature remains constant?

Q2. If a refrigerator's door is kept open, will the room become cool or hot? Explain.

Q3. Is it possible to increase the temperature of a gas without adding heat to it? Explain.

Q4. A system goes from *P* to *Q* by two different paths in the *P-V* diagram as shown in figure. Heat given to the system in path 1 is 1000 J. The work done by the system along path 1 is more than path 2 by 100 J. What is the heat exchanged by the system in path 2?



Q5. Air pressure in a car tyre increases during driving. Explain.

Q6. Consider a Carnot's cycle operating between T_1 = 500 K and T_2 = 300 K producing 1 kJ of mechanical work per cycle. Find the heat transferred to the engine by the reservoirs.

Q7. Consider a cycle tyre being filled with air by a pump. Let *V* be the volume of the tyre (fixed) and at each stroke of the pump $\Delta V (\ll V)$ of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from P_1 to P_2?

Q8. A person of mass 60 kg wants to lose 5 kg by going up and down a 10 m high stairs. Assume he burns twice as much fat while going up than coming down. If 1 kg of fat is burnt on expending 7000 kilo calories, how many times must he go up and down to reduce his weight by 5 kg?

Q9. The initial state of a certain gas is (P_i, V_i, T_i). It undergoes expansion till its volume becomes V_f. Consider the following two cases:

(a) the expansion takes place at constant temperature.

(b) the expansion takes place at constant pressure.

Plot the *P-V* diagram for each case. In which of the two cases, is the work done by the gas more?

Q10. If the co-efficient of performance of a refrigerator is 5 and operates at the room temperature (27°C), find the temperature inside the refrigerator.

Q11. In a refrigerator one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor is of 1 kW power, and heat is transferred from – 3°C to 27°C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.

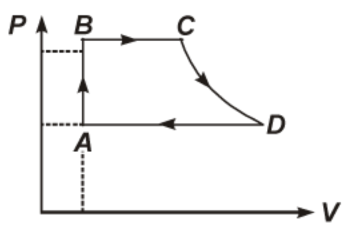
Q12. A cycle followed by an engine (made of one mole of an ideal gas in a cylinder with a piston) is shown in figure. Find heat exchanged by the engine, with the surroundings for each section of the cycle. ($C_v = (3/2) R$)

AB : constant volume

BC : constant pressure

CD : adiabatic

DA : constant pressure



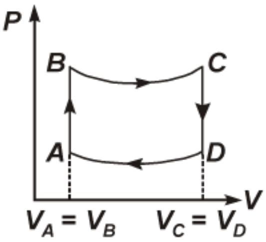
Q13. A cycle followed by an engine (made of one mole of perfect gas in a cylinder with a piston) is shown in figure.

A to B : volume constant

B to C : adiabatic

C to D : volume constant

D to A : adiabatic



$$V_C = V_D = 2V_A = 2V_B$$

(a) In which part of the cycle heat is supplied to the engine from outside?

(b) In which part of the cycle heat is being given to the surrounding by the engine?

(c) What is the work done by the engine in one cycle? Write your answer in term of P_A, P_B, V_A.

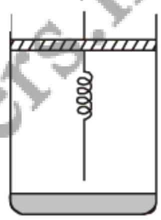
(d) What is the efficiency of the engine?

[$\gamma = 5/3$ for the gas], ($C_v = \frac{3}{2}R$ for one mole)

Q14. Consider that an ideal gas (n moles) is expanding in a process given by $P = f(V)$, which passes through a point (V_0, P_0). Show that the gas is absorbing heat at (V_0, P_0) if the slope of the curve $P = f(V)$ is larger than the slope of the adiabat passing through (V_0, P_0).

Q15. Consider one mole of perfect gas in a cylinder of unit cross section with a piston attached (see figure). A spring (spring constant k) is attached (unstretched length L) to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas causing an increase of volume from V_0 to V_1.

Atomospheric pressure = P_a



(a) What is the initial pressure of the system?

(b) What is the final pressure of the system?

(c) Using the first law of thermodynamics, write down a relation between Q, P_a, V, V_0 and k.

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S1. If the system does work against the surroundings so that it compensates for the heat supplied, the temperature can remain constant.

S2. Here heat removed is less than the heat supplied and hence the room, including the refrigerator (which is not insulated from the room) becomes hotter.

S3. Yes. When the gas undergoes adiabatic compression, its temperature increases.

$$dQ = dU + dW$$

As $dQ = 0$ (adiabatic process)

So, $dU = -dW$

In compression, work is done on the system So, $dW = -$ve

$\Rightarrow$ $dU = +$ve

So internal energy of the gas increases, *i.e.*, its temperature increases.

S4. **For path 1:** Heat given $Q_1 = +1000$ J

Work done $= W_1$ (let)

For path 2: Work done $(W_2) = (W_1 - 100)$ J

Heat given $Q_1 = +1000$ J

As change in internal energy between two states for different path is same

$$\therefore \quad \Delta U = Q_1 - W_1 = Q_2 - W_2$$

$$1000 - W_1 = Q_2 - (W_1 - 100)$$

$$\Rightarrow \quad Q_2 = 1000 - 100 = 900 \text{ J}$$

S5. During driving, temperature of the gas increases while its volume remains constant.

So according to Charle's law, at constant V, $P \alpha T$.

Therefore, pressure of gas increases.

S6. Given, Temperature of the source $T_1 = 500$ K

Temperature of the sink $T_2 = 300$ K

Work done per cycle $W = 1$ k J $= 1000$ J

Heat transferred to the engine per cycle $Q_1 = ?$

Efficiency of a Carnot engine $(\eta) = 1$

and $$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = \frac{200}{500} = \frac{2}{5}$$

$$\Rightarrow \quad Q_1 = \frac{W}{\eta} = \frac{1000}{(2/5)} = 2500 \text{ J.}$$

S7.

$$P(V+\Delta V)^{\gamma} = (P+\Delta p)V^{\gamma}$$

$$P\left[1+\gamma\frac{\Delta V}{V}\right] = P\left[1+\frac{\Delta p}{P}\right]$$

$$\gamma\frac{\Delta V}{V} = \frac{\Delta p}{P}; \quad \frac{dV}{dp} = \frac{V}{\gamma p}$$

$$\text{W.D.} = \int_{p_1}^{p_2} p\,dV = \int_{p_1}^{p_2} p\,\frac{\Delta p}{\gamma p}\,dp = \frac{(P_2 - P_1)}{\gamma}V\ .$$

S8. Given, Height of the stairs = h = 10 m

Energy produced by burning 1 kg of fat = 7000 k cal

$\therefore$ Energy produced by burning 5 kg of fat = 5×7000 = 35000 k cal

$= 35 \times 10^5$ cal

Energy utillised in going up and down one time

$$= mgh + \frac{1}{2}mgh = \frac{3}{2}mgh$$

$$= \frac{3}{2} \times 60 \times 10 \times 10$$

$$= 9000\ \text{J} = \frac{9000}{4.2} = \frac{3000}{1.4}\ \text{cal}$$

$\therefore$ Number of times, the person has to go up and down the stairs

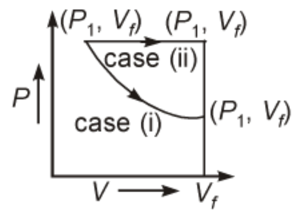
$$= \frac{35 \times 10^6}{(3000/1.4)} = \frac{35 \times 1.4 \times 10^6}{3000}$$

$= 16.3 \times 10^3$ times.

S9. Consider the diagram p-V, where variation is shown for each process.

Process 1 is isobaric and process 2 is isothermal.

Since, work done = area under the p-V curve. Here, area under the p-V curve 1 is more. So, work done is more when the gas expands in isobaric process.



S10. Given, Coefficient of performance (β) = 5

T_1 = (27 + 273) K = 300 K, T_2 = ?

$$\text{Coefficient of performance } (\beta) = \frac{T_2}{T_1 - T_2}$$

$$6 = \frac{T_2}{300 - T_2} \quad \Rightarrow \quad 1500 - 5T_2 = T_2$$

$\Rightarrow$ $6T_2 = 1500 \quad \Rightarrow \quad T_2 = 250$ K

$\Rightarrow$ $T_2 = (250 - 273)\,°\text{C} = -23\,°\text{C}.$

S11. Given, temperature of the source is 27°C

$\Rightarrow \quad T_1 = (27 + 273)\text{ K} = 300\text{ K}$

Temperature of sink $\quad T_2 = (-3 + 273)\text{ K} = 270\text{ K}$

Efficiency of a perfect heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{270}{300} = \frac{1}{10}$$

Efficiency of refrigerator is 50% of a perfect engine

$$\therefore \quad \eta' = 0.5 \times \eta = \frac{1}{2}\eta = \frac{1}{20}.$$

$\therefore$ Coefficient of performance of the refrigerator

$$\beta = \frac{Q_2}{W} = \frac{1-\eta'}{\eta'} \qquad [Q = \text{Heat taken, } W = \text{Work done on gas}]$$

$$= \frac{1-(1/20)}{(1/20)} = \frac{19/20}{1/20} = 19$$

$$\Rightarrow \quad Q_2 = \beta W = 19\,W \qquad \left(\because \ \beta = \frac{Q_2}{W}\right)$$

$$= 19 \times (1\text{ kW}) = 19\text{ kW} = 19\text{ k J/s}$$

Therefore, heat is taken out of the refrigerator at a rate of 19 kJ per second.

S12. (a) For process AB.

Volume is constant, hence work done $dW = 0$

Now, by first law of thermodynamics,

$$dQ = dU + dW = dU + 0 = dU$$

$$= n\,C_v\,dT = n\,C_v(T_B - T_A)$$

$$= \frac{3}{2}\,R(T_B - T_A) \qquad (\because \ n = 1)$$

$$= \frac{3}{2}\,(RT_B - RT_A) = \frac{3}{2}\,(p_B V_B - p_A V_A)$$

$$\text{Heat exchanged} = \frac{3}{2}\,(p_B V_B - p_A V_A)$$

(b) For process BC, $\quad p = \text{Constant}$

$$dQ = dU + dW = \frac{3}{2}\,R(T_C - T_B) = p_B(V_C - V_B)$$

$$= \frac{3}{2}\ (p_C V_C - p_B V_B) + p_B (V_C - V_B) = \frac{5}{2}\ p_B (V_C - V_B)$$

$$\text{Heat exchanged} = \frac{5}{2}\ p_B (V_C - V_B) \qquad (\because\ p_B = p_C \text{ and } p_B = V_A)$$

(c) For process CD, Because CD is adiabatic, dQ = Heat exchanged = 0.

(d) DA involves compression of gas from V_D to V_A at constant pressure p_A.

$\therefore$ Heat exchanged can be calculated by similar way as BC_1.

Hence, $$dQ = \frac{5}{2}\ p_A (V_A - V_D).$$

S13. (a) For the process AB.

$$dV = 0 \quad \Rightarrow \quad dW = 0 \qquad (\because \text{ volume is constant})$$

$$dQ = dU + dW = dU$$

$\Rightarrow$ $$dQ = dU = \text{Change in internal energy.}$$

Hence, in this process heat supplied is ultilised to increase, internal energy of the system. Since, $p = \left(\frac{nR}{V}\right)T$, in isochoric process, $T \propto P$. So, temperature increases with increases of pressure in process AB which in turn increases internal energy of the system *i.e.*, $dU > 0$. This imply that $dQ > 0$. So, heat is supplied to the system in process AB.

(b) For the process CD, volume is constant but pressure decreases.

Hence, temperature also decreases and is given to surroundings.

(c) $W_{AB} = \int_A^B pdV = 0; \quad W_{CD} = 0.$

Similarly, $$W_{BC} = \left[\int_B^C pdV = k\int_B^C \frac{dV}{V^r} = k\ \frac{V^{-r+1}}{-R+1}\right]_{V_B}^{V_C}$$

$$= \frac{1}{1-\gamma}(P_C V_C - P_B V_B)$$

Similarly, $$W_{DA} = \frac{1}{1-\gamma}(P_A V_A - P_D V_D)$$

Now, $$P_C = P_B \left(\frac{V_B}{V_C}\right)^{\gamma} = 2^{-\gamma} P_B$$

Similarly, $$P_D = P_A 2^{-\gamma}$$

$$\text{Total work done} = W_{BC} + W_{DA}$$

$$= \frac{1}{1-\gamma}[P_B V_B (2^{-\gamma+1} - 1) - P_A V_A (2^{-\gamma+1} - 1)]$$

$$= \frac{1}{1-\gamma}(2^{1-\gamma}-1)(P_B - P_A)V_A$$

$$= \frac{3}{2}\left(1-\left(\frac{1}{2}\right)^{2/3}\right)(P_B - P_A)V_A$$

(d) Heat supplied during process A, B

$$dQ_{AB} = dU_{AB}$$

$$Q_{AB} = \frac{3}{2}nR(T_B - T_A) = \frac{3}{2}(P_B - P_A)V_A$$

$$\text{Efficiency} = \frac{\text{Net work done}}{\text{Heat supplies}}\left[1-\left(\frac{1}{2}\right)^{2/3}\right].$$

S14. Slope of $P = f(V)$, curve at (V_0, P_0)

$$= f(V_0)$$

Slope in adiabatic process at $(V_0, P_0) = k(-\gamma)\,V_0^{-1-\gamma} = -\gamma P_0/V_0$

Now heat absorbed in the process $P = f(V)$

$$dQ = dV + dW$$

$$= nC_v\,dT + P\,dV$$

Since, $T = (1/nR)\,PV = (1/nR)\,V f(V)$

$$dT = (1/nR)\,[f(V) + V f'(V)]\,dV$$

Thus $$\frac{dQ}{dV}_{V=V_0} = \frac{CV}{R}[f(V_0) + V_0 f'(V_0)] + f(V_0)$$

$$= \left[\frac{1}{\gamma-1}+1\right]f(V_0) + \frac{V_0 f'(V_0)}{\gamma-1}$$

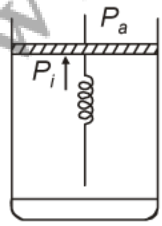
$$= \frac{\gamma}{\gamma-1}P_0 + \frac{V_0}{\gamma-1}f'(V_0).$$

Heat is absorbed when $dQ/dV > 0$ when gas expands, that is when

$$\gamma P_0 + V_0 f'(V_0) > 0$$

$$f'(V_0) > -\gamma P_0/V_0.$$

S15. (a) Initially the piston is in equilibrium hence, $P_i = P_a$



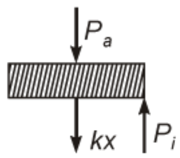
(b) On supplying heat, the gas expands from V_0 to V_1

$\therefore$ Increase in volume of the gas $= V_1 - V_0$

As the piston is of unit cross-sectional area hence, extension in the spring

$$x = \frac{V_1 - V_0}{\text{Area}} = V_1 - V_0 \qquad \text{[Area = 1]}$$

$\therefore$ Force exerted by the spring on the piston



$$= F = kx = k(V_1 - V_0)$$

Hence, Final pressure $= P_i = P_a + kx$

$$= P_a + k \times (V_1 - V_0).$$

(c) From first law of thermodynamics $dQ = dU + dW$

If T is final temperature of the gas, then increase in internal energy

$$dU = C_V(T - T_0) \qquad [C_V = \text{Molar specific heat at constant volume}]$$

We can write, $$T = \frac{P_f V_1}{R} = \left[\frac{P_a + k(V_1 - V_o)}{R}\right]\frac{V_1}{R}$$

Work done by the gas = pdV + increase in P.E. of the spring

$$= P_a(V_1 - V_0) + \frac{1}{2}kx^2$$

Now, we can write $dQ = dU + dW$

$$= C_V(T - T_0) = P_a(V - V_0) + \frac{1}{2}kx^2$$

$$= C_V(T - T_0) + P_a(V - V_0) + \frac{1}{2}(V_1 - V_0)^2$$

This is the required relation.