Date: 14/10/2021

Q1. Is the bulb of a thermometer made of diathermic or adiabatic wall?

Q2. Find out the increase in moment of inertia I of a uniform rod (coefficient of linear expansion α) about its perpendicular bisector when its temperature is slightly increased by ΔT.

Q3. 100 g of water is supercooled to – 10°C. At this point, due to some disturbance mechanised or otherwise some of it suddenly freezes to ice. What will be the temperature of the resultant mixture and how much mass would freeze?

$$[S_w = 1 \text{ cal/g/°C and } L^{W}_{\text{Fusion}} = 80 \text{ cal/g}]$$

Q4. A student records the initial length l, change in temperature ΔT and change in length Δl of a rod as follows:

S. No.	$l(m)$	ΔT (°C)	$\Delta l(m)$
1.	2	10	4×10^{-4}
2.	1	10	4×10^{-4}
3.	2	20	2×10^{-4}
4.	3	10	6×10^{-4}

If the first observation is correct, what can you say about observations 2, 3 and 4.

Q5. Calculate the temperature which has same numeral value on Celsius and Fahrenheit scale.

Q6. These days people use steel utensils with copper bottom. This is supposed to be good for uniform heating of food. Explain this effect using the fact that copper is the better conductor.

Q7. Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appears cooler than wooden bar if they are both colder than room temperature.

Q8. One day in the morning, Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some times, say 5-10 minutes before he could take bath. Now he had two options: (a) fill the remaining bucket completely by cold water and then attend to the work, (b) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer? Explain.

Q9. We would like to prepare a scale whose length does not change with temperature. It is proposed to prepare a unit scale of this type whose length remains, say 10 cm. We can use a bimetallic strip made of brass and iron each of different length whose length (both components) would change in such a way that difference between their lengths remain constant. If $\alpha_{\text{iron}} = 1.2 \times 10^{-5}$/K and $\alpha_{\text{brass}} = 1.8 \times 10^{-5}$/K, what should we take as length of each strip?

Q10. According to Stefan's law of radiation, a black body radiates energy σT^4 from its unit surface area every second where T is the surface temperature of the black body and $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4 is known as Stefan's constant. A nuclear weapon may be thought of as a ball of radius 0.5 m. When detonated, it reaches temperature of 10^6 K and can be treated as a black body.

(a) Estimate the power it radiates.

(b) If surrounding has water at 30°C, how much water can 10% of the energy produced evaporate in 1 s?

[S_w = 4186.0 J/kg K and $L_v = 22.6 \times 10^5$ J/kg].

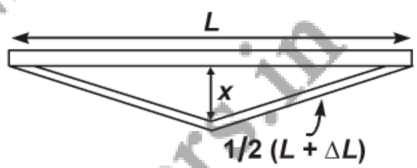
(c) If all this energy U is in the form of radiation, corresponding momentum is $p = U/c$. How much momentum per unit time does it impart on unit area at a distance of 1 km?

Q11. A thin rod having length L_0 at 0°C and coefficient of linear expansion α has its two ends maintained at temperatures θ_1 and θ_2, respectively. Find its new length.

Q12. Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and $\alpha = 1.7 \times 10^{-5}$/°C, bulk modulus for copper = 140×10^9 N/m^2.

Q13. During summers in India, one of the common practice to keep cool is to make ice balls of crushed ice, dip it in flavoured sugar syrup and sip it. For this a stick is inserted into crushed ice and is squeezed in the palm to make it into the ball. Equivalently in winter, in those areas where it snows, people make snow balls and throw around. Explain the formation of ball out of crushed ice or snow in the light of P–T diagram of water.

Q14. A rail track made of steel having length 10 m is clamped on a railway line at its two ends (see figure). On a summer day due to rise in temperature by 20°C, it is deformed as shown in figure. Find x (displacement of the centre) if $\alpha_{steel} = 1.2 \times 10^{-5}$/°C.



Q15. We would like to make a vessel whose volume does not change with temperature (take a hint from the problem above). We can use brass and iron ($\beta_{vbrass} = 6 \times 10^{-5}$/K and $\beta_{viron} = 3.55 \times 10^{-5}$/K) to create a volume of 100 cc. How do you think you can achieve this.

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S1. As diathermic walls allow exchange of heat energy between two systems and adiabatic walls do not, hence, diathermic walls are used to make the bulb of a thermometer.

S2.
$$I = \frac{1}{12} Ml^2$$
$$I' = \frac{1}{12} M(l + \Delta l)^2$$
$$= \frac{1}{12} Ml^2 + \frac{1}{12} 2Ml\Delta l + \frac{1}{12} M(\Delta l)^2 \alpha$$
$$= I + \frac{1}{12} Ml^2 2\alpha\Delta T$$
$$= I + 2I\alpha\Delta T$$
$$\therefore \quad \Delta I = 2\alpha I\Delta T.$$

S3. Given, Mass of water (m) = 100

Change in temperature $\Delta T = 0 - (-10) = 10\,°C$

Specific heat of water (S_w) = 1 cal/g/°C

Latent heat of fusion of water L^W_{Fusion} = 80 cal/g

Heat required to bring water in super cooling from – 10 °C to 0 °C
$$Q = ms_w\Delta T$$
$$= 100 \times 1 \times 10 = 1000 \text{ cal}$$

Let m gram of ice be melted
$$\therefore \quad Q = mL$$
or
$$m = \frac{Q}{L} = \frac{1000}{80} = 12.5\,\text{g}$$

As small mass of ice is melted, therefore the temperature of the mixture will remain 0 °C.

Note: To find the temperature of the mixture we must go through the two steps ($Q = ms\Delta T$) and ($Q = mL$) we should not directly apply first one.

S4. From the 1st observation $\alpha = \frac{\Delta l}{l\Delta T} \Rightarrow \alpha = \frac{4 \times 10^{-4}}{2 \times 10} = 2 \times 10^{-5}\,°C^{-1}$.

For 2nd observation
$$\Delta l = \alpha l\Delta T$$
$$= 2 \times 10^{-5} \times 1 \times 10$$
$$= 2 \times 10^{-4}\,\text{m} \neq 4 \times 10^{-4}\,\text{m} \quad \text{(Wrong)}$$

For 3rd observation $\Delta l = \alpha l \Delta T$

$= 2 \times 10^{-5} \times 2 \times 20$

$= 8 \times 10^{-4}\,\text{m} \neq 2 \times 10^{-4}\,\text{m}$ (Wrong)

For 4th observation $\Delta l = \alpha l \Delta T$

$= 2 \times 10^{-5} \times 3 \times 10$

$= 6 \times 10^{-4}\,\text{m} = 6 \times 10^{-4}\,\text{m}$ [*i.e.*, observed value (Correct)]

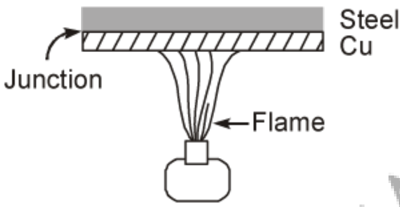
S5. Let Q be the value of temperature having same value an Celsius and Fahrenheit scale.

Now, we can write $\dfrac{°F - 32}{180} = \dfrac{°C}{100}$

$\Rightarrow$ Let $F = C = Q$

$\Rightarrow$ $\dfrac{Q - 32}{180} = \dfrac{Q}{100} = Q = -40°C$ or $-40°F$.

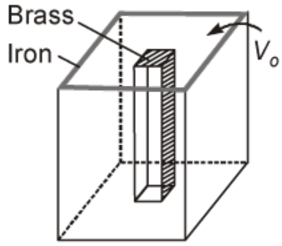
S6. Since Cu has a high conductivity compared to steel, the junction of Cu and steel gets heated quickly but steel does not conduct as quickly, thereby allowing food inside to get heated uniformly.



S7. Due to difference in conductivity, metals having high conductivity compared to wood. On touch with a finger, heat from the surrounding flows faster to the finger from metals and so one feels the heat. Similarly, when one touches a cold metal the heat from the finger flows away to the surroundings faster.

S8. The first option would have kept water warmer because according to Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature of the body and the surrounding and in the first case the temperature difference is less, so rate of loss of heat will be less.

S9. In the previous problem the difference in the length was constant.



In this problem the difference in volume is constant.

The situation is shown in the diagram

Let V_{io}, V_{bo} be the volume of iron and brass vessel at 0°C

V_i, V_b be the coefficient, of volume expansion of iron and brass

As per question, $V_{io} - V_{bo} = 100\,\text{cc} = V_i - V_b$ ... (i)

Now, $V_i = V_{io}(1 + \gamma_i \Delta\theta)$

$V_b = V_{bo}(1 + \gamma_i \Delta\theta)$

$V_i - V_b = (V_{io} - V_{bo}) + \Delta\theta\,(V_{io}\gamma_i - V_{bo}\gamma_b)$

Since, $V_i - V_b$ = constant.

So, $$V_{io}\gamma_i = V_{bo}\gamma_b$$

$$\Rightarrow \quad \frac{V_{io}}{V_{bo}} = \frac{\gamma_b}{\gamma_i} = \frac{\frac{3}{2}\beta_b}{\frac{3}{2}\beta_i} = \frac{\beta_b}{\beta_i} = \frac{6\times10^{-5}}{3.55\times10^{-5}} = \frac{6}{3.55}$$

$$\frac{V_{io}}{V_{bo}} = \frac{6}{3.55} \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$V_{io} = 244.9\,\text{cc}$$
$$V_{bo} = 144.9\,\text{cc}$$

S10. Given, $\sigma = 5.67\times10^{-8}\,\text{W/m}^2\,\text{kg}$

Radius (R) = 0.5 m, $T = 10^6\,\text{K}$

(a) Power related by Stefan's law

$$P = \sigma A T^4 = (4\pi R^2)\,T^4 \qquad \text{[Here, } \sigma = \text{stefan's constant]}$$
$$= (5.67\times10^{-4}\times4\times(3.14)\times(0.5)^2\times(10^6)\,4$$
$$= 1.78\times10^{17}\,\text{J/s} = 1.8\times10^{17}\,\text{J/s}$$

(b) Energy available per second,

$$U = 1.8\times10^{17}\,\text{J/s} = 1.8\times10^{17}\,\text{J/s}$$

Actual energy required to evaporate water = 10% of 1.8×10^{17} J/s

$= \mathbf{1.8\times10^{16}\ J/s}$.

Energy used per second to raise the temperature of m kg of water from 30 C to 100 C and then into vapour at 100C

$$= ms_w\Delta\theta + mL_v$$
$$= m\times4186\times(100-30) + m\times22.6\times10^5$$
$$= 2.93\times10^5\,m + 22.6\times10^5\,m$$
$$= 25.53\times10^5\,m\ \text{J/s}$$

As per question, $25.53\times10^5\,m = 1.8\times10^{16}$

or $$m = \frac{1.8\times10^{16}}{25.33\times10^5} = \mathbf{7.0\times10^9\ kg}.$$

(c) Momentum per unit time,

$$p = \frac{U}{c} = \frac{U}{c} = \frac{1.8\times10^{17}}{3\times10^8} \qquad \begin{bmatrix} P = \text{momentum} \\ V = \text{energy} \\ C = \text{velocity of light} \end{bmatrix}$$
$$= 6\times10^8\,\text{kg-m/s}$$

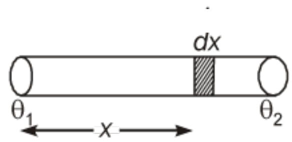
Momentum per unit time per unit

$$\text{Area } p = \frac{6 \times 10^8}{4 \times 3.14 \times (10^3)^2}$$

$\Rightarrow$ $d = \mathbf{47.7\ N/m^2}$ [$4\pi R^2$ = Surface area]

S11. **Method I:** Temperature θ at a distance x from one and (that at θ_1) is given by $\theta = \theta_1 + \frac{x}{L_0}(\theta_2 - \theta_1)$: linear temperature gradient.

New length of small element of length dx_0



$$dx = dx_0(1 + \alpha\theta)$$

$$= dx_0 + dx_0\alpha\left[\theta_1 + \frac{x}{L_0}(\theta_2 - \theta_1)\right]$$

Now, $\int dx_0 = L_0$ and $\int dx = L$: new length

Integrating

$$\therefore \quad L = L_0 + L_0\alpha\theta_1 + \frac{(\theta_2 - \theta_1)}{L_0}\alpha \int x\, dx_0$$

$$= L_0\left(1 + \frac{1}{2}\alpha(\theta_2 - \theta_1)\right) \text{ as } \int_0^{L_0} x\, dx = \frac{1}{2}L_0^2$$

Method II: If temperature of the rod varies linearly, we can assume average temperature to be $\frac{1}{2}(\theta_1 - \theta_2)$ and hence new length

$$L = L_0\left(1 + \frac{1}{2}\alpha(\theta_2 - \theta_1)\right).$$

S12. Stress = K × Strain

$$= K\frac{\Delta V}{V}$$

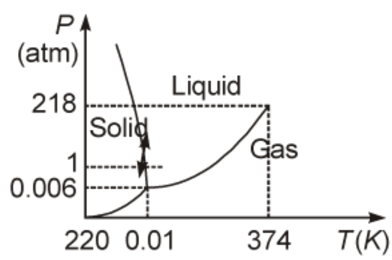
$$= K(3\alpha)\Delta t$$

$$= 140 \times 10^9 \times 3 \times 1.7 \times 10^{-5} \times 20$$

$$= 1.428 \times 10\ \text{N/m}$$

This is about 10^3 times atmospheric pressure.

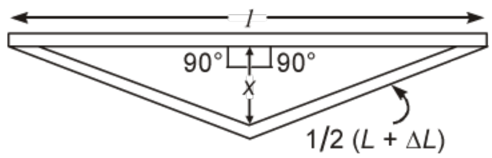
S13. Refer to the P-T diagram of water and double headed arrow. Increasing pressure at 0°C and 1 atm takes ice into liquid state and decreasing pressure in liquid state at 0°C and 1 atm takes water to ice state.



When crushed ice is squeezed, some of it melts. filling up gap between ice flakes. Upon releasing pressure, this water freezes binding all ice flakes making the ball more stable.

S14. Consider the diagram,

Applying Pythagorus theorem in right angled triangle in figure.



$$\left(\frac{L+\Delta L}{2}\right)^2 = \left(\frac{L}{2}\right)^2 + x^2$$

$$\Rightarrow \quad x = \sqrt{\left(\frac{L+\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(L+\Delta L)^2 - L^2}$$

$$= \frac{1}{2}\sqrt{(L^2 + \Delta L^2 + 2L\Delta L) - L^2}$$

$$= \frac{1}{2}\sqrt{(\Delta L^2 + 2L\Delta L)}$$

As increase in length ΔL is very small, therefore, neglecting $(\Delta L)^2$, we get

$$x = \frac{1}{2} \times \sqrt{2L\,\Delta L} \quad \text{... (i)}$$

But $\Delta L = L\alpha\Delta t$ ... (ii)

Substituting value of ΔL in Eq. (i) from Eq. (ii)

$$x = \frac{1}{2}\sqrt{2L \times L\alpha\Delta t} = \frac{1}{2}L\sqrt{2\alpha\Delta t}$$

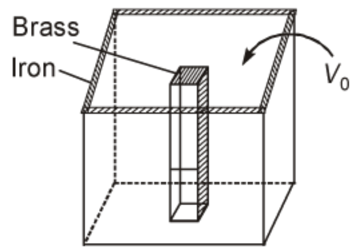
$$= \frac{10}{2} \times \sqrt{2 \times 1.2 \times 10^{-5} \times 20}$$

$$= 5 \times \sqrt{4 \times 1.2 \times 10^{-4}}$$

$$= 5 \times 2 \times 1.1 \times 10^{-2} = 0.11 \text{ m} = 11 \text{ cm}$$

Note: Here we have assumed ΔL to be very small so that it can be neglected compared to L.

S15. Iron vessel with a brass rod inside



$$\frac{V_{iron}}{V_{brass}} = \frac{6}{3.55}$$

$$V_{iron} - V_{brass} = 100 \text{ cc} = V_0$$

$$V_{brass}^{rod} = 144.9 \text{ cc} \quad V_{iron}^{inside} = 244.9 \text{ cc}$$