Date: 14/10/2021

Q1. A vessel filled with water is kept on a weighing pan and the scale adjusted to zero. A block of mass M and density ρ is suspended by a massless spring of spring constant k. This block is submerged inside into the water in the vessel. What is the reading of the scale?

Q2. A cubical block of density ρ is floating on the surface of water. Out of its height L, fraction x is submerged in water. The vessel is in an elevator accelerating upward with acceleration a. What is the fraction immersed?

Q3. Is surface tension a vector?

Q4. Is viscosity a vector?

Q5. Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged if the density of ice is $\rho_i = 0.917\ \text{g cm}^{-3}$?

Q6. Two mercury droplets of radii 0.1 cm. and 0.2 cm. collapse into one single drop. What amount of energy is released? The surface tension of mercury $T = 435.5 \times 10^{-3}\ \text{N m}^{-1}$.

Q7. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R, break into N small droplets each of radius r. Estimate the drop in temperature.

Q8. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$ m. The surface tension of sap is $T = 7.28 \times 10^{-2}\ \text{Nm}^{-1}$ and the angle of contact is 0°. Does surface tension alone account for the supply of water to the top of all trees?

Q9. (a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure dp over a differential height dh?

(b) Considering the pressure p to be proportional to the density, find the pressure p at a height h if the pressure on the surface of the Earth is p_0.

(c) If $p_0 = 1.03 \times 10^5\ \text{N m}^{-2}$, $\rho_0 = 1.29\ \text{kg m}^{-3}$ and $g = 9.8\ \text{m s}^{-2}$, at what height will the pressure drop to (1/10) the value at the surface of the Earth?

(d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

Q10. The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating the free surface will be titled by an angle θ. If the acceleration is $a\ \text{m s}^{-2}$, what will be the slope of the free surface?.

Q11. The surface tension and vapour pressure of water at 20°C is $7.28 \times 10^{-2}\ \text{Nm}^{-1}$ and 2.33×10^3 Pa, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20°C?

Q12. A hot air balloon is a sphere of radius 8 m. The air inside is at a temperature of 60°C. How large a mass can the balloon lift when the outside temperature is 20°C? (Assume air is an ideal gas, $R = 8.314\ \text{J mole}^{-1}\text{K}^{-1}$, 1 atm. = 1.013×10^5 Pa; the membrane tension is $5\ \text{N m}^{-1}$.)

Q13. **Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decreases with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporisation for water L_v = 540 k cal kg^{-1}, the mechanical equivalent of heat J = 4.2 J cal^{-1}, density of water $\rho_w = 10^3$ kg l^{-1}, Avagadro's No $N_A = 6.0 \times 10^{26}$ k $mole^{-1}$ and the molecular weight of water M_A = 18 kg for 1 k mole.**

(a) estimate the energy required for one molecule of water to evaporate.

(b) show that the inter-molecular distance for water is $d = \left[\frac{M_A}{N_A} \times \frac{1}{\rho_w}\right]^{1/3}$ and find its value.

(c) 1 g of water in the vapor state at 1 atm occupies 1601 cm^3. Estimate the intermolecular distance at boiling point, in the vapour state.

(d) During vaporisation a molecule overcomes a force *F*, assumed constant, to go from an inter-molecular distance *d* to *d′*. Estimate the value of *F*.

(e) Calculate *F*/*d*, which is a measure of the surface tension.

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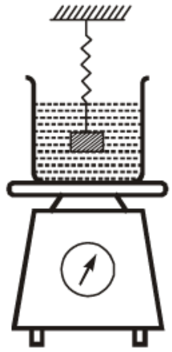
S1. Let x be the compression on the spring. As the block is in equilibrium

$$Mg - (kx + \rho_w Vg) = 0$$

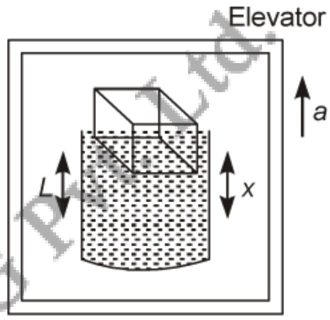
where ρ_w is the density of water and V is the volume of the block. The reading in the pan is the force applied by the water on the pan *i.e.*,

$$m_{\text{vessel}} + m_{\text{water}} + \rho_w Vg.$$

Since the scale has been adjusted to zero without the block, the new reading is $\rho_w Vg$.



S2. Consider the diagram.



Let the density of water be ρ_w and a cubical block of ice of side L be floating in water with x of its height (L) submerged in water.

Volume of the block (V) = L^3

Mass of the block (m) = $V\rho = L^3\rho$

Weight of the block = $mg = L^3\rho g$

Case I: Volume of the water displaced by the submerged part of the block xL^3

$\therefore$ Weight of the water displaced by the block

In floating condition $xL^2\rho_w g$

Weight of the block = Weight of the water displaced by the block

$$L^3\rho g = xL^2\rho_w g$$

or
$$\frac{x}{L} = \frac{\rho}{\rho_w} = x$$

Case II: When elevator s accelerating upward with an acceleration a, then effective acceleration

$= (g + a)$ [$\because$ Pseudo force is downward)

Then, Weight of the block = $m(g + a)$

$= L^3\rho(g + a)$

Let x_1 fraction be submerged in water when elevator is accelerating upwards.

Now, in the floating condition,

Weight of the block = Weight of the displaced water

$$L^3\rho(g + a) = (x_1 L^2)\rho_w(g + a)$$

or
$$\frac{x_1}{L} = \frac{\rho}{\rho_w} = x$$

From case I and II, we see that, the fraction of the block submerged in water is independent of the acceleration of the elevator.

S3. No, surface tension is a scalar quantity.

$$\text{Surface tension} = \frac{\text{Work done}}{\text{Surface area}},$$

where work done and surface area both are scalar quantities.

S4. No.

S5. Let the volume of the iceberg be V. The weight of the iceberg is $\rho_i Vg$. If x is the fraction submerged, then the volume of water displaced is xV. The buoyant force is $\rho_w xVg$ where ρ_w is the density of water.

$$\rho_i Vg = \rho_w xVg$$

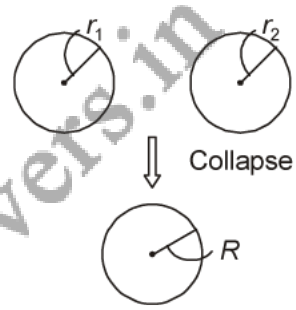
$$\therefore \quad x = \frac{\rho_i}{\rho_w} = 0.917$$

S6. Consider the diagram,

Radii of mercury droplets $r_1 = 0.1 \text{ cm} = 1 \times 10^{-3}$ m

$r_2 = 0.2 \text{ cm} = 2 \times 10^{-3}$ m

Surface tension (T) = 435.5×10^{-3} N/m



Let the radius of the big drop formed by collapsing be R.

$\therefore$ Volume of big drop = Volume of small droplets

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$$

or

$$R^3 = r_1^3 + r_2^3$$
$$= (0.1)^3 + (0.2)^3$$
$$= 0.001 + 0.008$$
$$= 0.009$$

or

$$R = 0.21 \text{ cm} = 2.1 \times 10^{-3} \text{ m}$$

$\therefore$ Change in surface area $\Delta A = 4\pi R^2 - (4\pi r_1^2 + 4\pi r_2^2)$

$$= 4\pi [R^2 - (r_1^2 + r_2^2)]$$

$\therefore$ Energy released = $T \cdot \Delta A$ (where T is surface tension of mercury)

$$= T \times 4\pi [R^2 - (r_1^2 + r_2^2)]$$
$$= 435.5 \times 10^{-3} \times 4 \times 3.14 [(2.1 \times 10^{-3})^2 - (1 \times 10^{-6} + 4 \times 10^{-6})]$$
$$= 435.4 \times 4 \times 3.14 [4.41 - 5] \times 10^{-6} \times 10^{-3}$$
$$= -32.23 \times 10^{-7}$$ (Negative sigh shows absorption)

Therefore, 3.22×10^{-6} J energy will be absorbed.

S7. When a big drop of radius R, breaks into N droplets each of radius r, the volume remains constant.

$\therefore$ Volume of big drop = $N \times$ Volume of each small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

or $$R^3 = Nr^3$$

or $$N = \frac{R^3}{r^3}$$

Now, Change in surface area $= 4\pi R^2 - N4\pi r^2$

$$= 4\pi (R^2 - Nr^2)$$

Energy released $= T \times \Delta A = S \times 4\pi (R^2 - Nr^2)$ [T = Surface tension]

Due to releasing of this energy, the temperature is lowered.

If ρ is the density and s is specific heat of liquid and its temperature is lowered by $\Delta\theta$, then energy released $= ms\Delta\theta$ [s = specific heat $\Delta\theta$ = change in temperature]

$$T \times 4\pi (R^2 - Nr^2) = \left(\frac{4}{3} \times R^3 \times \rho\right) s\Delta\theta \qquad \left[\therefore\ m = v\rho = \frac{4}{3}\pi R^3 \rho\right]$$

$$\Rightarrow \quad \Delta\theta = \frac{T \times 4\pi (R^2 - Nr^2)}{\frac{4}{3}\pi R^3 \rho \times s}$$

$$= \frac{3T}{\rho s}\left[\frac{R^2}{R^3} - \frac{Nr^2}{R^3}\right]$$

$$= \frac{3T}{\rho s}\left[\frac{1}{R} - \frac{(R^3/r^3) \times r^2}{R^3}\right]$$

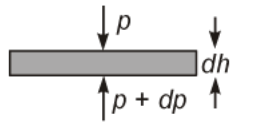
$$= \frac{3T}{\rho s}\left[\frac{1}{R} - \frac{1}{r}\right].$$

S8. The height to which the sap will rise is

$$h = \frac{2T\cos 0^\circ}{\rho g r} = \frac{2(7.2 \times 10^{-2})}{10^3 \times 9.8 \times 2.5 \times 10^{-5}} = 0.6\ \text{m}$$

This is the maximum height to which the sap can rise due to surface tension. Since many trees have heights much more than this, capillary action alone cannot account for the rise of water in all trees.

S9. (a) Consider a horizontal parcel of air with cross section A and height dh. Let the pressure on the top surface and bottom surface be p and $p + dp$. If the parcel is in equilibrium, then the net upward force must be balanced by the weight.



i.e., $(p + dp)A - pA = -PgAdh$

$$\Rightarrow \quad dp = -\rho\, gdh.$$

(b) Let the density of air on the Earth's surface be ρ_0, then

$$\frac{p}{p_0} = \frac{\rho}{\rho_0}$$

$$\Rightarrow \qquad \rho = \frac{\rho_0}{p_0}\, p$$

$$\therefore \qquad dp = -\frac{\rho_0 g}{p_0}\, p\,dh$$

$$\Rightarrow \qquad \frac{dp}{p} = -\frac{\rho_0 g}{p_0}\, dh$$

$$\Rightarrow \qquad \int_{p_0}^{p} \frac{dp}{p} = -\frac{\rho_0 g}{p_0} \int_0^{h_0} dh$$

$$\Rightarrow \qquad \ln\frac{p}{p_0} = -\frac{\rho_0 g}{p_0}\, h$$

$$\Rightarrow \qquad p = p_0 \exp\left(-\frac{\rho_0 g}{p_0}\, h\right)$$

(c)

$$\ln\frac{1}{10} = -\frac{\rho_0 g}{p_0}\, h_0$$

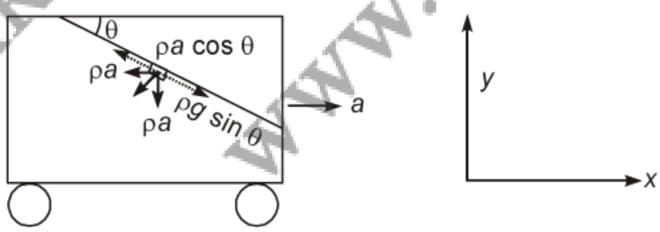
$$\therefore \qquad h_0 = -\frac{p_0}{\rho_0 g} \ln\frac{1}{10}$$

$$= \frac{p_0}{\rho_0 g} \times 2.303$$

$$= \frac{1.013 \times 10^5}{1.29 \times 9.8} \times 2.303 = 0.16 \times 10^5\ \text{m} = 16 \times 10^3\ \text{m}$$

(d) The assumption $p \propto \rho$ is valid only for the isothermal case which is only valid for small distances.

S10. If the tanker accelerates in the positive x direction, then the water will bulge at the back of the tanker. The free surface will be such that the tangential force on any fluid partical is zero.



Consider a partical at the surface, of unit volume. The forces on the fluid are

$$-\rho g \hat{y} \quad \text{and} \quad -\rho a \hat{x}$$

The component of the weight along the surface is $\rho g \sin\theta$. The component of the acceleration force along the surface is $\rho a \cos\theta$

$\therefore \qquad \rho g \sin\theta = \rho a \cos\theta$

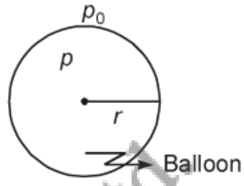
Hence, $\qquad \tan\theta = a/g$

S11. The drop will evaporate if the water pressure is more than the vapour pressure. The membrane pressure (water)

$$p = \frac{2T}{r} = 2.33 \times 10^3 \text{ Pa}$$

$$\therefore \qquad r = \frac{2T}{p} = \frac{2(7.28\times10^{-2})}{2.33\times10^3} = 6.25\times10^{-5}\text{ m}$$

S12. Let the pressure inside the balloon be Pi and the outside pressure be P_0



$$P_i - P_0 = \frac{2\gamma}{r}$$

Considering the air to be an ideal gas

$P_iV = n_iRT_i$ where V is the volume of the air inside the balloon, n_i is the number of moles inside and T_i is the temperature inside, and $P_0V = n_0R\,T_0$ where V is the volume of the air displaced and n_0 is the number of moles displaced and T_0 is the temperature outside.

$n_{\text{i}} = \dfrac{P_iV}{RT_i} = \dfrac{M_i}{M_A}$ where M_i is the mass of air inside and M_A is the molar mass of air and

$n_{\text{i}} = \dfrac{P_0V}{RT_0} = \dfrac{M_0}{M_A}$ where M_0 is the mass of air outside that has been displaced. If W is the load it can raise, then

$$W + M_ig = M_0g$$

$$\Rightarrow \qquad W = M_0g - M_ig$$

Air is 21% O_2 and 79% N_2

$\therefore$ Molar mass of air $M_A = 0.21\times32 + 0.79\times28 = 28.84$ g.

$$\Rightarrow \qquad W = \frac{M_AV}{R}\left(\frac{P_0}{T_0} - \frac{P_i}{T_i}\right)g$$

$$= \frac{0.02884\times\frac{4}{3}\pi\times8^3\times9.8}{8.314}\left(\frac{1.013\times10^5}{293} - \frac{1.013\times10^5}{333} - \frac{2\times5}{8\times313}\right)\text{N}$$

$$= \frac{0.02884\times\frac{4}{3}\pi\times8^3\times9.8}{8.314}\times1.013\times10^5\left(\frac{1}{293} - \frac{1}{333}\right)\times9.8\text{ N}$$

$$= 3044.2\text{ N.}$$

S13. (a) 1 kg of water requires L_v k cal

$\therefore$ M_A kg of water requires M_AL_v k cal

Since there are N_A molecules in M_A kg of water the energy required for 1 molecule to evaporate is

$$u = \frac{M_A L_v}{N_A}\text{ J} \qquad [N_A = \text{Avogadro's number}]$$

$$= \frac{18 \times \overset{90}{\cancel{540}} \times 4.2 \times 10^3}{\cancel{6} \times 10^{26}}\text{ J}$$

$$= 90 \times 18 \times 4.2 \times 10^{-23}\text{ J}$$

$$= 6.8 \times 10^{-20}\text{ J}.$$

(b) Consider the water molecules to be points at a distance d from each other.

N_A molecules occupy $\frac{M_A}{\rho_w}\, l$

Thus, the volume around one molecule is $\frac{M_A}{N_A \rho_w}\, l$

The volume around one molecule is $d^3 = (M_A/N_A\, \rho_w)$

$$\therefore \qquad d = \left(\frac{M_A}{N_A \rho_w}\right)^{1/3} = \left(\frac{18}{6 \times 10^{26} \times 10^3}\right)^{1/3}$$

$$= (30 \times 10^{-30})^{1/3}\text{ m} \simeq 3.1 \times 10^{-10}\text{ m}$$

(c) 1 kg of vapour occupies $1601 \times 10^{-3}\text{ m}^3$.

$\therefore$ 18 kg of vapour occupies $18 \times 1601 \times 10^{-3}\text{ m}^3$

$\Rightarrow$ 6×10^{26} molecules occupies $18 \times 1601 \times 10^{-3}\text{ m}^3$

$\therefore$ 1 molecule occupies $\frac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}}\text{ m}^3$

If d' is the inter molecular distance, then

$$d'^3 = (3 \times 1601 \times 10^{-29})\text{ m}^3$$

$$\therefore \qquad d' = (30 \times 1601)^{1/3} \times 10^{-10}\text{ m}$$

$$= 36.3 \times 10^{-10}\text{ m}.$$

(d) $$F(d' - d) = u$$

$$\Rightarrow \qquad F = \frac{u}{d' - d} = \frac{6.8 \times 10^{-20}}{(36.3 \times 3.1) \times 10^{-10}} = 0.2048 \times 10^{-10}\text{ N}.$$

(e) $$F/d = \frac{0.2048 \times 10^{-10}}{3.1 \times 10^{-10}} = 0.066\text{ Nm}^{-1} = 6.6 \times 10^{-2}\text{ Nm}^{-1}.$$