Ellipse

Single Correct Answer Type

1. If a variable tangent of the circle $x^2 + y^2 = 1$ intersect the ellipse $x^2 + 2y^2 = 4$ at P and Q then the locus of the points of intersection of the tangents at P and Q is

A. a circle of radius 2 units

B. a parabola with fouc as (2, 3)

C. an ellipse with eccentricity $\frac{\sqrt{3}}{4}$

D. an ellipse with length of latus rectrum is 2 units

Key. D

Sol. $x^2 + y^2 = 1; x^2 + 2y^2 = 4$

Let $R(x_1, y_1)$ is pt of intersection of tangents drawn at P,Q to ellipse

$\Rightarrow PQ$ is chord of contact of $R(x_1, y_1)$

$\Rightarrow xx_1 + 2yy_1 - 4 = 0$

This touches circle $\Rightarrow r^2(\ell^2 + m^2) = n^2$

$\Rightarrow 1(x_1^2 + 4y_1^2) = 16$

$\Rightarrow x^2 + 4y^2 = 16$ is ellipse $e = \frac{\sqrt{3}}{2}; LL^1 = 2$

2. A circle S = 0 touches a circle $x^2 + y^2 - 4x + 6y - 23 = 0$ internally and the circle $x^2 + y^2 - 4x + 8y + 19 = 0$ externally. The locus of centre of the circle S = 0 is conic whose eccentricity is k then $\left[\frac{1}{k}\right]$ is where [.] denotes G.I.F

A. 7

B. 2

C. 0

D. 3

Key. A

Sol. $c_1(2, -3) r_1 = 6$

$c_2(2, -4) r_2 = 1$

Let C is the center of S = 0

$$\therefore \left.\begin{array}{l} cc_1 = r_1 - r \\ cc_2 = r_1 + r \end{array}\right\} \Rightarrow cc_1 + cc_2 = r_1 + r_2$$

$\therefore$ Locus is an ellipse whose foci are (2, -3) & (2, -4)

$$e=\frac{2ae}{2a}=\frac{c_1c_2}{r_1+r_2}=\frac{1}{7}\Rightarrow k=\frac{1}{7}$$

3. If circum centre of an equilateral triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with vertices having eccentric angles α,β,γ respectively is (x_1,y_1) then $\sum\cos\alpha\cos\beta+\sum\sin\alpha\sin\beta=$

A. $\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}+\frac{3}{2}$ B. $9x_1^2-9y_1^2+a^2b^2$ C. $\frac{9x_1^2}{2a^2}+\frac{9y_1^2}{2b^2}-\frac{3}{2}$ D. $\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}+3$

Key. C

Sol. $(x_1,y_1)=\left(\frac{a\sum\cos\alpha}{3},\frac{b\sum\sin\alpha}{3}\right)$

$$\sum\cos\alpha=\frac{3x_1}{a}.............(1)$$

$$\sum\sin\alpha=\frac{3y_1}{b}.............(2)$$

Squarding & adding

4. The ratio of the area enclosed by the locus of mid-point of PS and area of the ellipse where P is any point on the ellipse and S is the focus of the ellipse, is

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{5}$ D. $\frac{1}{4}$

Key. D

Sol. Ellipse equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, Area = πab

Let $P=(a\cos\theta,b\sin\theta)$

$S=(ae,0)$

M(h,k) mid point of PS

$$\Rightarrow h=\frac{ae+a\cos\theta}{2};k=\frac{b\sin\theta}{2}$$

$$=\frac{h-\frac{ae}{2}}{a/2}+\frac{k^2}{(b^2/4)}=1$$, locus of (h,k) is ellipse

Area $=\pi\left(\frac{a}{2}\right)\left(\frac{b}{2}\right)=\frac{1}{4}\pi ab$

5. How many tangents to the circle $x^2+y^2=3$ are there which are normal to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

A) 3 B) 2 C) 1 D) 0

Key. D

Sol. Equation of normal at $p(3\cos\theta, 2\sin\theta)$ is $3x\sec\theta-2y\operatorname{cosec}\theta=5$

$$\frac{5}{\sqrt{9\sec^2\theta+4\operatorname{cosec}^2\theta}}=\sqrt{3}$$

But Min. of $9\sec^2\theta+4\operatorname{cosec}^2\theta=25$

$\therefore$ no such $\overset{-1}{\theta}$ exists.

6. If the ellipse $\frac{x^2}{a^2-3}+\frac{y^2}{a+4}=1$ is inscribed in a square of side length $a\sqrt{2}$ then a is

A) 4 B) 2 C) 1 D) None of these

Key. D

Sol. Sides of the square will be perpendicular tangents to the ellipse so, vertices of the square will lie on director circle. So diameter of director circle is

$$2\sqrt{(a^2-3)+(a+4)}=\sqrt{2a^2+2a^2}$$

$$2\sqrt{a^2+a+1}=2a \Rightarrow a=-1$$

But for ellipse $a^2>3 \,\&\, a>-4$

So a cannot take the value $'-1'$

7. Let 'O' be the centre of ellipse for which A,B are end points of major axis and C,D are end points of minor axis, F is focus of the ellipse. If in radius of ΔOCF is $'1'$ then $|AB|\times|CD|=$

A) 65 B) 52 C) 78 D) 47

Key. A

Sol. $r=\frac{\Delta}{S}\Rightarrow\Delta=S$

$$\frac{1}{2}(ae)b=\frac{ae+b+\sqrt{a^2e^2+b^2}}{2}$$

$$ae = 6 \Rightarrow 6b = 6 + b + \sqrt{36 + b^2} \Rightarrow b = \frac{5}{2}$$

$$\Rightarrow a^2\left(1 - e^2\right) = \frac{25}{4} \Rightarrow a^2 - 36 = \frac{25}{4} \Rightarrow a = \frac{13}{2}$$

8. If the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ meet the ellipse $\frac{x^2}{1} + \frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 10b + 25$, then the value b does not satisfy

1. $(-\infty, 4)$ 2. (4, 6) 3. $(6, \infty)$ 4. [4, 6]

Key. 4

Sol. a > 1

9. The perimeter of a triangle is 20 and the points (-2, -3) and (-2, 3) are two of the vertices of it. Then the locus of third vertex is :

1. $\frac{(x-2)^2}{49} + \frac{y^2}{40} = 1$ 2. $\frac{(x+2)^2}{49} + \frac{y^2}{40} = 1$ 3. $\frac{(x+2)^2}{40} + \frac{y^2}{49} = 1$ 4. $\frac{(x-2)^2}{40} + \frac{y^2}{49} = 1$

Key. 3

Sol. PA + PB + AB = 20 where A & B are foci

10. Tangents are drawn from any point on the circle $x^2+y^2=41$ to the Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ then the angle between the two tangents is

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{3}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Key. 4

Sol. Director circle

11. The area of the parallelogram formed by the tangents at the points whose eccentric angles are $\theta, \theta + \frac{\pi}{2}, \theta + \pi, \theta + \frac{3\pi}{2}$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1. ab 2. 4ab 3. 3ab 4. 2ab

Key. 2

Sol. Put $\theta = 0^0$

12. A normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the axes in L and M. The perpendiculars to the axes through L and M intersect at P. Then the equation to the locus of P is

1. $a^2x^2-b^2y^2=\left(a^2+b^2\right)^2$
2. $a^2x^2+b^2y^2=\left(a^2+b^2\right)^2$
3. $b^2x^2-a^2y^2=\left(a^2-b^2\right)^2$
4. $a^2x^2+b^2y^2=\left(a^2-b^2\right)^2$

Key. 4

Sol. $P=\left(x_1,y_1\right),\frac{x}{x_1}+\frac{y}{y_1}=1$ Apply normal condition

13. The points of intersection of the two ellipse $x^2+2y^2-6x-12y+23=0$, $4x^2+2y^2-20x-12y+35=0$

1. Lie on a circle centered at $\left(\frac{8}{3},3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$
2. Lie on a circle centered at $\left(\frac{8}{3},-3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{3}}$
3. Lie on a circle centered at (8,9) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$
4. Are not concyclic

Key. 1

Sol. If $S_1 = 0$ and $S_2 = 0$ are the equations, Then $\lambda S_1+S_2=0$ is a second degree curve passing through the points of intersection of $S_1 = 0$ and $S_2 = 0$

$$\Rightarrow(\lambda+4)x^2+2(\lambda+1)y^2-2(3\lambda+10)x-12(\lambda+1)y+(23\lambda+35)=0$$

For it to be a circle, choose λ such that the coefficients of x^2 and y^2 are equal $\therefore \lambda=2$

This gives the equation of the circle as

$$6\left(x^2+y^2\right)-32x-36y+81=0\{u\sin g\,(1)\}$$

$$\Rightarrow x^2+y^2-\frac{16}{3}x-6y+\frac{27}{2}=0$$

Its centre is $C\left(\frac{8}{3},3\right)$ and radius is

r = $\sqrt{\frac{64}{9}+9-\frac{27}{2}}=\frac{1}{3}\sqrt{\frac{47}{2}}$

14. In a model, it is shown that an arc of a bridge is semi elliptical with major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from the horizontal; then the height of the arch, 2m from the centre of the base is (in meters)

1. $\frac{8}{3}$ 2. $\frac{\sqrt{65}}{3}$ 3. $\frac{\sqrt{56}}{3}$ 4. $\frac{9}{3}$

Key. 2

Sol. Let the equation of the semi elliptical are be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(y>0)$

Length of the major axis = 2a = 9 $\Rightarrow$ a = 9/2

So the equation of the arc becomes $\frac{4x^2}{81}+\frac{y^2}{9}=1$

If x=2, then $y^2=\frac{65}{9}\Rightarrow y=\frac{1}{3}\sqrt{65}$

15. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$ then the maximum value of ab is

1. 2 2. 4 3. 6 3. Can n't be found

Key. 2

Sol. A tangent of slope 2 is $y=2x\pm\sqrt{4a^2+b^2}$ this is normal to $x^2+y^2+4x+1=0$ then

$0=-4\pm\sqrt{4a^2+b^2}\Rightarrow 4a^2+b^2=16$ using $Am\ge GM$

$ab\le 4$

16. The distance between the polars of the foci of the Ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ w.r.to itself is

1. $\frac{25}{2}$ 2. $\frac{25}{9}$ 3. $\frac{25}{8}$ 4. $\frac{25}{3}$

Key. 1

Sol. $\frac{2a}{e}$

17. An ellipse passing through origin has its foci at (5, 12) and (24, 7). Then its eccentricity is

1. $\frac{\sqrt{386}}{38}$ 2. $\frac{\sqrt{386}}{39}$ 3. $\frac{\sqrt{386}}{47}$ 4. $\frac{\sqrt{386}}{51}$

Key. 1

Sol. Conceptual

18. If $e=\frac{\sqrt{3}}{2}$, its length of latusrectum is

1. $\frac{1}{2}$ (length of major axis)
2. $\frac{1}{3}$ (length of major axis)
3. $\frac{1}{4}$ (length of major axis)
4. Length of major axis

Key. 3

Sol. $L.L.R=\frac{2b^2}{a}$

19. Number of normals that can be drawn from the point (0, 0) to $3x^2+2y^2=30$ are

1. 2 2. 4 3. 1 4. 3

Key. 2

Sol. It is centre

20. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the axes in M and N. Then the least length of MN is

1. a + b 2. a – b 3. a^2+b^2 4. a^2-b^2

Key. 1

Sol. Standard

21. $p(\theta),\ D\left(\theta+\frac{\pi}{2}\right)$ are two points on the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Then the locus of point of intersection of the two tangents at P and D to the ellipse is

1. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{4}$ 2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$ 3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ 4. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$

Key. 3

Sol. $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ → 1eq

$\frac{x}{a}\cos\left(\frac{\pi}{2}+\theta\right)+\frac{y}{b}\sin\left(\frac{\pi}{2}+\theta\right)=1$ → 2eq

Eliminate θ from 1 and 2

22. The abscissae of the points on the ellipse $9x^2+25y^2-18x-100y-116=0$ lie between

1. 3, -5 2. -4, 6 3. 5, 7 4. 2, 5

Key. 2

Sol. $-5\geq x-1\leq 5$

23. Tangents to the ellipse $b^2x^2+a^2y^2=a^2b^2$ makes angles $\theta_1\, and\, \theta_2$ with major axis such that $\cot\theta_1+\cot\theta_2=k$. Then the locus of the point of intersection is

1. $xy=2k(y^2+b^2)$ 2. $2xy=k(y^2-b^2)$ 3. $4xy=k(y^2-b^2)$ 4. $8xy=k(y^2-b^2)$

Key. 2

Sol. Apply sum of the slopes = $\dfrac{2x_1y_1}{x_1^2-a^2}$

24. The equation $\dfrac{x^2}{10-a}+\dfrac{y^2}{4-a}=1$ represents an ellipse if

1. a < 4 2. a > 4 3. 4 <a < 10 4. a > 10

Key. 1

Sol. 10 – a > 0, 4 – a > 0

25. The locus of the feet of the perpendiculars drawn from the foci of the ellipse S=0 to any tangent to it is

1. a circle 2. an ellipse 3. a hyperbola 4. not a conic

Key. 1

Sol. Standard

26. If the major axis is “n” (n>1) times the minor axis of the ellipse, then eccentricity is

1. $\dfrac{\sqrt{n-1}}{n}$ 2. $\dfrac{\sqrt{n-1}}{n^2}$ 3. $\dfrac{\sqrt{n^2-1}}{n^2}$ 4. $\dfrac{\sqrt{n^2-1}}{n}$

Key. 4

Sol. 2a = n(2b)

$$\Rightarrow n=\frac{a}{b}$$

$$\therefore e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{1-\frac{b^2}{a^2}}=$$

$$\sqrt{1-\frac{1}{n^2}}=\frac{\sqrt{n^2-1}}{n}$$

27. If $\left(\sqrt{3}\right)bx+ay=2ab$ is tangent to the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, then eccentric angle θ is

1. $\dfrac{\pi}{4}$ 2. $\dfrac{\pi}{6}$ 3. $\dfrac{\pi}{2}$ 4. $\dfrac{\pi}{3}$

Key. 2

Sol. Equation of tangent at a point $\left(a\cos\theta, b\sin\theta\right)$ is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

But, it is the same as $\frac{x}{a}\frac{\sqrt{3}}{2}+\frac{y}{b}.\frac{1}{2}=1$

$\therefore \cos\theta=\frac{\sqrt{3}}{2}, \sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$

28. If PSQ is a focal chord of the ellipse $16x^2+25y^2=400$ such that SP = 8 then the length of SQ =

1. 2 2. $\frac{11}{3}$ 3. 16 4. 25

Key. 1

Sol. $\frac{1}{SP}+\frac{1}{SQ}=\frac{2a}{b^2}$

29. A man running round a race course notes that the sum of the distances of two flag posts from him is 8 meters. The area of the path he encloses in square meters if the distance between flag posts is 4 is

1. $15\sqrt{3}\pi$ 2. $12\sqrt{3}\pi$ 3. $18\sqrt{3}\pi$ 4. $8\sqrt{3}\pi$

Key. 4

Sol. Area = πab

30. The locus of point of intersection of the two tangents to the ellipse $b^2x^2+a^2y^2=a^2b^2$ which makes an angle 60^0 with one another is

1. $4\left(x^2+y^2-a^2-b^2\right)^2=3\left(b^2x^2+a^2y^2-a^2b^2\right)$

2. $3\left(x^2+y^2-a^2-b^2\right)^2=4\left(b^2x^2+a^2y^2-a^2b^2\right)$

3. $3\left(x^2+y^2-a^2-b^2\right)^2=2\left(b^2x^2+a^2y^2-a^2b^2\right)$

4. $3\left(x^2+y^2-a^2-b^2\right)^2=\left(b^2x^2+a^2y^2-a^2b^2\right)$

Key. 2

Sol. $Tan\theta=\frac{2ab\sqrt{S_{11}}}{x_1^2+y_1^2-a^2-b^2}$

31. If the equation of the chord joining the points $P(\theta)$ and $D\left(\theta+\frac{\pi}{2}\right)$ on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x\cos\alpha+y\sin\alpha=p$ then $a^2\cos^2\alpha+b^2\sin^2\alpha=$

1. $4p^2$ 2. p^2 3. $\frac{p^2}{2}$ 4. $2p^2$

Key. 4

Sol. $\frac{x}{a}\cos\left(\frac{\theta+\theta+\frac{\pi}{2}}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta+\theta+\frac{\pi}{2}}{2}\right)$

$=\cos\left(\frac{\theta-\theta-\frac{\pi}{2}}{2}\right)$ → 1eq

$x\cos\alpha+y\sin\alpha=P$ → 2eq

(1) = (2)

32. The locus of mid point of chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which passes through the foot of the directrix from focus is

1. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a^2}$ 2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{ae}$ 3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{a^2e}$ 4. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{ae^2}$

Key. 2

Sol. $S_1 = S_{11}$ passes through $\left(\frac{a}{e},0\right)$

33. Consider two points A and B on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, circles are drawn having segments of tangents at A and B in between tangents at the two ends of major axis of ellipse as diameter, then the length of common chord of the circles is

A) 8 B) 6 C) 10 D) $4\sqrt{2}$

Key. A

Sol. All such circles pass through foci $\therefore$ The common chord is of the length 2ae $10\times\frac{4}{5}=8$

34. If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

A) b^2 B) $2b^2$ C) $\frac{b^2}{2}$ D) $3b^2$

Key. A

Sol. $CF=\frac{ab}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}$ $PG=\frac{b}{a}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

35. The line passing through the extremity A of the major axis and extremity B of the minor axis
of the ellipse $x^2+9y^2=9,$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin 'O' is

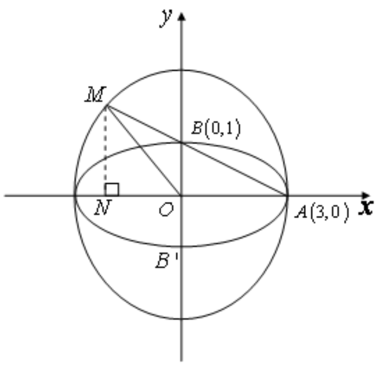
A) $\frac{31}{10}$ B) $\frac{29}{10}$ C) $\frac{21}{10}$ D) $\frac{27}{10}$

Key. D

Sol. Equation of given ellipse is $\frac{x^2}{9}+\frac{y^2}{1}=1$

Equation of auxiliary circle is $x^2+y^2=9.........(1)$

Equation of line AB is $\frac{x}{3}+\frac{y}{1}=1\Rightarrow x=3(1-y)$



Putting this in (1), we get $9(1-y)^2+y^2=9\Rightarrow 10y^2-18y=0\Rightarrow y=0,\frac{9}{5}$

Thus, y coordinate of 'M' is $\frac{9}{5}$

$$\Delta OAM=\left(\frac{1}{2}\right)(OA)(MN)=\frac{1}{2}(3)\frac{9}{5}=\frac{27}{10}$$

36. The normal at an end of a latus rectum of the ellipse $x^2/a^2+y^2/b^2=1$ passes through an end of the minor axis if

(a) $e^4+e^2=1$ (b) $e^3+e^2=1$ (c) $e^2+e=1$ (d) $e^3+e=1$

Key. A

Sol. Given ellipse equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Let $P\left(ae,\frac{b^2}{a}\right)$ be one end of latus rectum.

Slope of normal at $P\left(ae,\frac{b^2}{a}\right)=\frac{1}{e}$

Equation of normal is

$y=\frac{b^2}{a}=\frac{1}{e}(x-ae)$

It passes through $B^{'}(0,b)$ then

$b-\frac{b^2}{a}=-a$

$a^2-b^2=-ab$

$a^4e^4=a^2b^2$

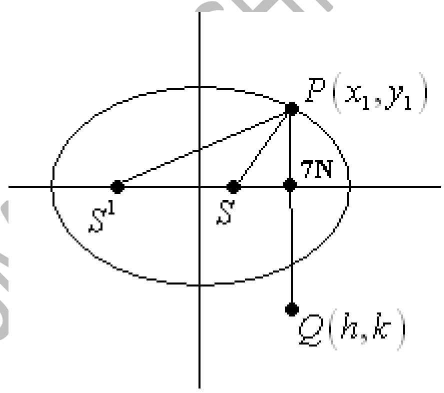
$e^4+e^2=1$

37. From any point P lying in first quadrant on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, PN is drawn perpendicular to the major axis such that N lies on major axis. Now PN is produced to the point Q such that NQ equals to PS, where S is a focus. The point Q lies on which of the following lines

(A) $2y-3x-25=0$ (B) $3x+5y+25=0$

(C) $2x-5y-25=0$ (D) $2x-5y+25=0$

Key. B



Sol.

$a^2=25$

$b^2=16$

$e=\sqrt{\frac{25-16}{25}}=\frac{3}{5}$

Let point Q be (h, k), where K < 0

Given that $|K|=a+eh(as\ x_1=h)$

$-y=a+ex$

$$-y = 5 + \frac{3}{5}x$$
$$3x + 5y + 25 = 0$$

38. A circle of radius 'r' is concentric with the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then inclination of common tangent with major axis is ________(b<r<a)

1. $\tan^{-1}\left(\frac{b}{a}\right)$ 2. $\tan^{-1}\left(\frac{rb}{a}\right)$ 3. $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$ 4. $\frac{\pi}{2}$

Key. 3

Sol. The tangent of Ellipse is $y = mx + \sqrt{a^2m^2 + b^2}$, this line touches $x^2 + y^2 = r^2$

Condition is $\left|\frac{\sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}\right| = r$

$$a^2m^2 + b^2 = r^2m^2 + r^2$$

$$m^2\left(a^2 - r^2\right) = r^2 - b^2 \Rightarrow m^2 = \frac{r^2-b^2}{a^2-r^2}$$

$$m = \sqrt{\frac{r^2-b^2}{a^2-r^2}}$$

Inclimation is $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$

39. A circle cuts the X-axis and Y-axis such that intercept on X-axis is a constant a and intercept on Y-axis is a constant b. Then eccentricity of locus of centre of circle is

1. 1 2. $\frac{1}{2}$ 3. $\sqrt{2}$ 4. $\frac{1}{\sqrt{2}}$

Key. 3

Sol. Locus of centre of circle is a rectangular hyperbola hence its eccentricity is $\sqrt{2}$

40. Consider two points A and B on the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, circles are drawn having segments of tangents at A and B in between tangents at the two ends of major axis of ellipse as diameter, then the length of common chord of the circles is

A) 8 B) 6 C) 10 D) $4\sqrt{2}$

Key. A

Sol. All such circles pass through foci $\therefore$ The common chord is of the length 2ae

$$10\times\frac{4}{5}=8$$

41. If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

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Key. A

Sol. $CF=\frac{ab}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}$ $PG=\frac{b}{a}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

42. The line passing through the extremity A of the major axis and extremity B of the minor axis
of the ellipse $x^2+9y^2=9,$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin 'O' is

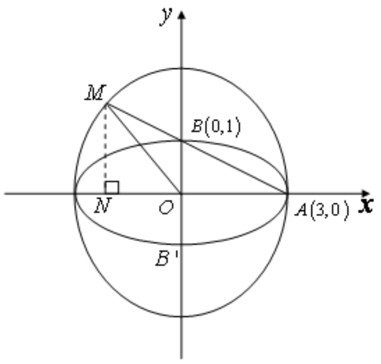
A) $\frac{31}{10}$ B) $\frac{29}{10}$ C) $\frac{21}{10}$ D) $\frac{27}{10}$

Key. D

Sol. Equation of given ellipse is $\frac{x^2}{9}+\frac{y^2}{1}=1$

Equation of auxiliary circle is $x^2+y^2=9.........(1)$

Equation of line AB is $\frac{x}{3}+\frac{y}{1}=1\Rightarrow x=3(1-y)$



Putting this in (1), we get $9(1-y)^2+y^2=9\Rightarrow 10y^2-18y=0\Rightarrow y=0,\frac{9}{5}$

Thus, y coordinate of 'M' is $\frac{9}{5}$

$$\Delta OAM=\left(\frac{1}{2}\right)(OA)(MN)=\frac{1}{2}(3)\frac{9}{5}=\frac{27}{10}$$

43. If $2x^2+y^2-24y+80=0$ then maximum value of x^2+y^2 is

A. 20 B. 40

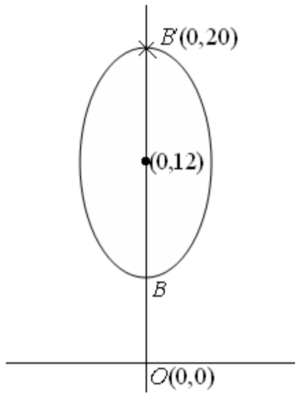
C. 200 D. 400

Key. D

Sol. Given equation is $2x^2+y^2-24y+80=0$

$$2x^2+(y-12)^2=64$$

$$\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$$



If is an ellipse with center (0, 12)

If (x, y) is any point on this distance from origin is $\sqrt{x^2+y}$

x^2+y^2 is max If $\sqrt{x^2+y^2}$ is

max

$B^1(1,\infty)$ is at max distance from 0

$\therefore \max(x^2+y^2)=400$

44. An ellipse whose foci $(2,4)(14,9)$ touches x –axis then its eccentricity is

A. $\frac{13}{\sqrt{313}}$ B. $\frac{1}{\sqrt{313}}$

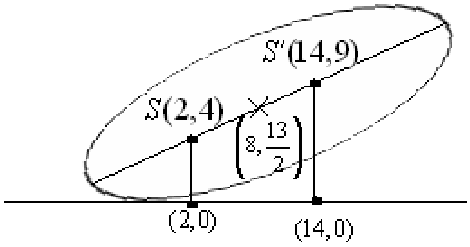
C. $\frac{2}{\sqrt{313}}$ D. $\frac{1}{\sqrt{13}}$

Key. A

Sol. Equation of aurally circle $(x-8)^2+\left(y-\frac{13}{2}\right)^2=a^2$

(2, 0) lies on it

$$36+\frac{169}{4}=a^2 \Rightarrow \frac{313}{4}=a^2$$



$$a=\frac{\sqrt{313}}{2}$$

But $SS'=2ae$

$$\sqrt{144+25}=2ae$$

$$13=2ae$$

$$e=\frac{13}{2a}=\frac{13}{\sqrt{313}}$$

45. A circle of radius 2 is concentric with the ellipse $\frac{x^2}{7}+\frac{y^2}{3}=1$ then inclination of common tangent with X-axis

A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$ D. $\frac{\pi}{6}$

Key. D

Sol. tangent is $y=mx+\sqrt{7m^2+3}$

$$\frac{x^2}{7}+\frac{y^2}{3}=1$$

$$x^2+y^2=4$$

It is also touching $x^2+y^2=4$

$$\left|\frac{\sqrt{7m^2+3}}{\sqrt{m^2+1}}\right|=2$$

$$7m^2+3=4m^2+4$$

$$m^2=\frac{1}{3}\Rightarrow m=\frac{1}{\sqrt{3}}$$

$$\therefore \tan\theta=\frac{1}{\sqrt{3}}$$

$\theta = \frac{\pi}{6}$

46. The points of intersection of two ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ be at the extremeties of conjugate diameters of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then $\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}=$

A. 1 B. 2 C. 3 D. 4

Key. B

Sol. Clearly $P(a\cos\theta, b\sin\theta)$ $Q(-a\sin\theta, b\cos\theta)$ are extremities of conjugate diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

P and Q lies m $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$

$$\frac{a^2\cos^2\theta}{\alpha^2}+\frac{b^2\sin^2\theta}{\beta^2}=1$$

$$\frac{a^2\sin^2\theta}{\alpha^2}+\frac{b^2\cos^2\theta}{\beta^2}=1$$

(+) $\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}=2$

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47. From the focus $(-5,0)$ of the ellipse $\frac{x^2}{45}+\frac{y^2}{20}=1$ a ray of light is sent which makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A) $-3/2$ B) $-7/3$ C) $-5/4$ D) $-2/11$

Key. D

Sol. Let $\theta = \cos^{-1}\left(\frac{-1}{\sqrt{5}}\right) \Rightarrow \cos\theta = \frac{-1}{\sqrt{5}} \Rightarrow \tan\theta = -2$

Foci are $(\pm 5, 0)$

Equation of line through (-5, 0) with slope –2 is $y = -29x+5) = -2x-10$

This line meets the ellipse above X-axis at $(-6, 2)$

$\therefore$ Slope = $\frac{2-0}{-6-5} = -\frac{2}{11}$.

48. If $f(x)$ is a decreasing function for all $x \in R$ and $f(x) > 0\ \forall x \in R$ then the range of *K* so that the equation $\frac{x^2}{f(K^2+2K+5)} + \frac{y^2}{f(K+11)} = 1$ represents an ellipse whose major axis is the X-axis is

A) $(-2, 3)$ B) $(-3, 2)$

C) $(-\infty, -3) \cup (2, \infty)$ D) $(-\infty, -2) \cup (3, \infty)$

Key. B

Sol. Conceptual

49. P, Q are points on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ such that PQ is a chord through the point $R(3, 0)$. If $|PR| = 2$ then length of chord PQ is

A) 8 B) 6 C) 10 D) 4

Key. C

Sol. Conceptual

50. Let $Q = (3, \sqrt{5}), R = (7, 3\sqrt{5})$. A point *P* in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A) 6 B) 12 C) 18 D) 9

Key. B

Sol. *P* lies on the ellipse for which Q, R are foci and length of major axis is 10 and eccentricity is 3/5.

51. *O* is the centre of ellipse for which A, B are end points of major axis and C, D are end points of minor axis. *F* is a focus of the ellipse. If $|OF| = 6$ and inradius of ΔOCF is 1, then $|AB| \times |CD| =$

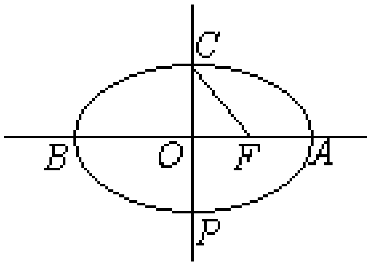
A) 65 B) 52 C) 78 D) 47

Key. A

Sol. $b^2 = \frac{25}{4} \Rightarrow a^2 - a^2e^2 = \frac{25}{4} \Rightarrow a^2 = \frac{25}{4} + 36 = \frac{169}{4} \Rightarrow a = \frac{13}{2}$

$$|OF| = ae = 6 \Rightarrow \frac{abe}{2} = 1 \times \frac{(ae + b + \sqrt{a^2e^2 + b^2})}{2}$$

$$6b = 6 + b + \sqrt{b^2 + 36} \Rightarrow (5b - 6)^2 = b^2 + 36 \Rightarrow 24b^2 = 60b \Rightarrow b = 5/2$$



52. A triangle is formed by a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the coordinate axes. The area of the triangle cannot be less than

a) $\frac{a^2 + b^2}{2}$ sq units
b) $\frac{a^2 + ab + b^2}{3}$ sq units
c) $\frac{a^2 + 2ab + b^2}{2}$ sq units
d) ab sq units

Key. D

Sol. Equation of tangent at θ is $\frac{x}{a} \cos\theta + \frac{y}{b} \sin\theta = 1$

Area with the axes is $\frac{ab}{\sin 2\theta} \geq ab$

53. Equation of circle of minimum radius which touches both the parabolas $y = x^2 + 2x + 4$ and $x = y^2 + 2y + 4$ is

a) $2x^2 + 2y^2 - 11x - 11y - 13 = 0$
b) $4x^2 + 4y^2 - 11x - 11y - 13 = 0$
c) $3x^2 + 3y^2 - 11x - 11y - 13 = 0$
d) $x^2 + y^2 - 11x - 11y - 13 = 0$

Key. B

Sol. Circle will be touching both parabolas. Circles centre will be on the common normal

54. Image of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ in the line x + y = 10 is :

a) $\frac{(x-10)^2}{16} + \frac{(y-10)^2}{25} = 1$
b) $\frac{(x-10)^2}{25} + \frac{(y-10)^2}{16} = 1$
c) $\frac{(x-5)^2}{16} + \frac{(y-5)^2}{25} = 1$
d) $\frac{(x-5)^2}{25} + \frac{(y-5)^2}{16} = 1$

Key. A

Sol. Conceptual

55. Length of common tangent to $x^2+y^2=16$ and $\frac{x^2}{25}+\frac{y^2}{7}=1$

a) $\frac{9}{4\sqrt{2}}$ b) $\frac{9}{4}$ c) $\frac{9}{2\sqrt{2}}$ d) $\frac{9}{2}$

Key. B

Sol. $y=-x+4\sqrt{2}$ is a common tangent to two curves in the 1st quadrant. Touching the curves at $P\left(2\sqrt{2},2\sqrt{2}\right)\&Q\left(\frac{25}{4\sqrt{2}},\frac{7}{4\sqrt{2}}\right)$

PQ = length of common tangent.

56. An ellipse having foci S (3, 4) & S' (6, 8) passes through the point P(0, 0). The equation of the tangent at P to the ellipse is

a) $4x+3y=0$ b) $3x+4y=0$ c) $x+y=0$ d) $x-y=0$

Key. B

Sol. Normal at a point is bisector of angle SPS'

57. The angle subtended at the origin by a common tangent of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2x}{c}=0$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2}+\frac{2x}{c}=0$, is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

Key. D

Sol. Conceptual

58. Let a hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

a) The equation of hyperbola is $\frac{x^2}{16}-\frac{y^2}{9}=1$ b) The equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

c) Focus of hyperbola is (5, 0) d) vertex of hyperbola is $\left(5\sqrt{3},0\right)$

Key. C

Sol. Conceptual

59. If the normals at 4 points having eccentric angles $\alpha,\beta,\gamma,\delta$ on an ellipse be concurrent, then $\left(\sum\cos\alpha\right)\left(\sum\sec\alpha\right)=$

a) 4 b) $(\alpha\beta\gamma\delta)^{\frac{1}{4}}$

c) $\dfrac{\alpha+\beta+\gamma+\delta}{4}$ d) None of these

Key. A

Sol. Conceptual

60. If the length of the major axis intercepted between the tangent & normal at a point $(a\cos\theta, b\sin\theta)$ on the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is equal to the length of semi-major axis, then , eccentricity of the ellipse is,

a) $\dfrac{\cos\theta}{\sqrt{1-\cos\theta}}$ b) $\dfrac{\sqrt{1-\cos\theta}}{\cos\theta}$

c) $\dfrac{\sqrt{1-\cos\theta}}{\sin\theta}$ d) $\dfrac{\sin\theta}{\sqrt{1-\sin\theta}}$

Key. B

Sol. $\dfrac{a}{\cos\theta}-\dfrac{(a^2-b^2)}{a}\cos\theta=a \Rightarrow e^2\cos^2\theta=1-\cos\theta \Rightarrow e=\dfrac{\sqrt{1-\cos\theta}}{\cos\theta}$

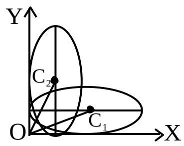
61. An ellipse with major and minor axes of lengths $10\sqrt{3}$ and 10 respectively slides along the co-ordinate axes and always remains confined in the first quadrant. The length of the arc of the locus of the centre of the ellipse is

(A) 10π (B) 5π

(C) $\dfrac{5\pi}{4}$ (D) $\dfrac{5\pi}{3}$

Key. D

Sol. The locus of the centre of the ellipse is director circle ie $x^2+y^2=100$



$C_1OC_2=\theta$

$\Rightarrow \dfrac{\pi}{2}-2\tan^{-1}\left(\dfrac{5}{5\sqrt{3}}\right)=\dfrac{\pi}{6}$

$\therefore$ arc length $= 10.\dfrac{\pi}{6}=\dfrac{5\pi}{3}$

62. Tangents drawn to the ellipse $\dfrac{x^2}{36}+\dfrac{y^2}{9}=1$ from the point P meet the co-ordinate axes at concyclic points . The locus of the point P is

(A) $x^2+y^2=7$ (B) $x^2+y^2=25$

(C) $x^2 - y^2 = 7$ (D) $x^2 - y^2 = 25$

Key. C

Sol. Let $P = (h,k)$

Equation of any tangent is $y = mx \pm \sqrt{16m^2 + 9}$

$\Rightarrow k = mh \pm \sqrt{16m^2 + 9}$

$\Rightarrow m^2\left(h^2 - 16\right) - 2mhk + \left(k^2 - 9\right) = 0$

Let m_1, m_2 are the slope of the tangents $m_1 m_2 = \dfrac{k^2 - 9}{h^2 - 16}$

For concyclic points $m_1 m_2 = 1$

$\Rightarrow h^2 - 16 = k^2 - 9$

$\Rightarrow h^2 - k^2 = 7 \quad \Rightarrow x^2 - y^2 = 7$

63. The line $2px + y\sqrt{1 - p^2} = 1(|p| < 1)$ for different value of p touches.

(A) An ellipse of eccentricity $\dfrac{2}{\sqrt{3}}$ (B) An ellipse of eccentricity $\dfrac{\sqrt{3}}{2}$

(C) Hyperbola of eccentricity 2 (D) None

Key. B

Sol. $y = \dfrac{-2p}{\sqrt{1-p^2}}x + \dfrac{1}{\sqrt{1-p^2}}$

$m = -\dfrac{2p}{\sqrt{1-p^2}} \Rightarrow p^2 = \dfrac{m^2}{4+m^2}$

$y = mx + \dfrac{1}{\sqrt{1 - \dfrac{m^2}{4+m^2}}} \Rightarrow y = mx + \sqrt{\dfrac{4+m^2}{4}}$

$\Rightarrow y = mx + \sqrt{1 + \dfrac{1}{4}m^2}$

It touches $\dfrac{x^2}{1/4} + \dfrac{y^2}{1} = 1, e = \dfrac{\sqrt{3}}{2}$

64. The normal to the curve $x^2 + 3y^2 - 4 - 0$ at the point $P\left(\pi/6\right)$ intersects the curve again at the point $Q(\theta), \theta$ being the eccentric angle at the point Q then $\theta =$ __ .

A) 0 B) $\pi/2$ C) π D) $3\pi/2$

Key. D

Sol. Given curve is $\dfrac{x^2}{4} + \dfrac{y^2}{4/3} = 1$ point $P\left(2\cos\pi/6, 2/\sqrt{3}\sin\pi/6\right) = \left(\sqrt{3}, \dfrac{1}{\sqrt{3}}\right)$.

Equation of the normal at P is $x - y = \sqrt{3} - \frac{1}{\sqrt{3}}$ it passes through

$Q(\theta) = \left(2\cos\theta, \frac{2}{\sqrt{3}}\sin\theta\right) \Rightarrow \theta = \frac{3\pi}{2}$

65. If tangents PQ and PR are drawn from a point on the circle $x^2 + y^2 = 25$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1\ (b < 4)$ so that the fourth vertex 'S' of parallelogram PQSR lies on the circum circle of triangle PQR, then the eccentricity of the ellipse is

A) $\frac{\sqrt{5}}{4}$ B) $\frac{\sqrt{7}}{3}$ C) $\frac{\sqrt{7}}{4}$ D) $\frac{\sqrt{5}}{3}$

Key. C

Sol. A cyclic parallelogram will be rectangle or square. $\therefore \angle QPR = 90^{O} \Rightarrow 'P'$ lies on director circle

$\Rightarrow b^2 = 9 \therefore e = \frac{\sqrt{7}}{4}\ \left(b^2 = a^2\left(1-e^2\right)\right)$

66. If A and B are foci of ellipse $(x-2y+3)^2 + (8x+4y+4)^2 = 20$ and P is any point on it, then PA+PB=___.

A) 2 B) 4 C) $\sqrt{2}$ D) $2\sqrt{2}$

Key. B

Sol. $\frac{\left(\frac{x-2y+3}{\sqrt{5}}\right)}{4} + \frac{\left(\frac{2x-y+1}{\sqrt{5}}\right)}{\frac{1}{4}} = 1 \Rightarrow PA + PB = 2a = 4$

67. The ratio of the area enclosed by the locus of the midpoint of PS and area of the ellipse is ___(P-be any point on the ellipse and S, its focus)

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{5}$ D) $\frac{1}{4}$

Key. D

Sol. mid point of PS is (h,k) & $h = \frac{a\cos\theta + ae}{2} \Rightarrow \cos\theta = \frac{2h - ae}{a}$; $k = \frac{b\sin\theta}{2}$

$$\frac{(2h-ae)^2}{a^2} + \frac{4k^2}{b^2} = 1 \Rightarrow \frac{\left(h - \frac{ae}{2}\right)^2}{\frac{a^2}{4}} + \frac{k^2}{\frac{b^2}{4}} = 1 \text{ its area } \Rightarrow \pi.\frac{a}{2}.\frac{b}{2} = \frac{\pi ab}{4}. \quad \therefore \text{ratio} = \frac{1}{4}$$

68. The normal to the curve $x^2 + 3y^2 - 4 = 0$ at the point $P(\pi/6)$ intersects the curve again at the point $Q(\theta)$, θ being the eccentric angle of the point Q, then θ =

(A) 0 (B) $\pi/2$

(C) π (D) $3\pi/2$

Key. D

Sol. The given curve is $\frac{x^2}{4} + \frac{y^2}{4/3} = 1$. Point P is $\left(2\cos\frac{\pi}{6}, \frac{2}{\sqrt{3}}\sin\frac{\pi}{6}\right) \equiv \left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

On differentiating the given equation, w.r.t. x, we get x + 3y $\frac{dy}{dx}=0$ $\Rightarrow$

$\left[\frac{dy}{dx}\right]_P=\left[-\frac{x}{3y}\right]_P=-1$

The equation of normal is y - $\frac{1}{\sqrt{3}}=1\left(x-\sqrt{3}\right)$ $\Rightarrow$ x - y = $\sqrt{3}-\frac{1}{\sqrt{3}}$. Normal passes through Q(θ). Hence 2cosθ - $\frac{2}{\sqrt{3}}\sin\theta=\sqrt{3}-\frac{1}{\sqrt{3}}$

$\Rightarrow 2\sqrt{3}\cos\theta - 2\sin\theta = 2 \Rightarrow \sqrt{3}\cos\theta - \sin\theta = 1 \Rightarrow \theta = \frac{3\pi}{2}$.

69. If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point $(2\alpha, \alpha)$, then a possible value of α^2 is

(A) 1 (B) 2
(C) 3 (D) 4

Key. B

Sol. The given curve is $\frac{x^2}{9}+\frac{y^2}{3}=1$, whose director circle is $x^2 + y^2 = 12$. For the required condition $(2\alpha, \alpha)$ should lie inside the circle and outside the ellipse i.e.,

$(2\alpha)^2 + 3\alpha^2 - 9 > 0$ and $(2\alpha)^2 + \alpha^2 - 12 < 0$ i.e., $\frac{9}{7}<\alpha^2<\frac{12}{5}$.

70. If the tangent at Point P to the ellipse $16x^2 + 11y^2 = 256$ is also the tangent to the circle $x^2 + y^2 - 2x = 15$, then the eccentric angle of point P is

(A) $\pm\frac{\pi}{2}$ (B) $\pm\frac{\pi}{4}$
(C) $\pm\frac{\pi}{3}$ (D) $\pm\frac{\pi}{6}$

Key. C

Sol. The equation of tangent at point $P\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse

$16x^2 + 11y^2 = 256$ is

$16x(4\cos\theta) + 11y\left(\frac{16}{\sqrt{11}}\sin\theta\right) = 256$

$4x\cos\theta + \sqrt{11}\,y\sin\theta = 16$

This touches the circle

$(x + 1)^2 + y^2 = 16$

So, $\frac{|4\cos\theta-16|}{\sqrt{16\cos^2\theta+11\sin^2\theta}} = 4$

$\Rightarrow (\cos\theta - 4)^2 = 11 + 5\cos^2\theta$

$4\cos^2\theta + 8\cos\theta - 5 = 0$

$\therefore \cos\theta = \dfrac{1}{2}$

$\therefore \theta = \pm \dfrac{\pi}{3}$

71. From the focus $(-5,0)$ of the ellipse $\dfrac{x^2}{45}+\dfrac{y^2}{20}=1$ a ray of light is sent which makes angle $\cos^{-1}\left(\dfrac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A) $-3/2$ B) $-7/3$ C) $-5/4$ D) $-2/11$

Key. D

Sol. Let $\theta=\cos^{-1}\left(\dfrac{-1}{\sqrt{5}}\right) \Rightarrow \cos\theta=\dfrac{-1}{\sqrt{5}} \Rightarrow \tan\theta=-2$

Foci are $(\pm 5,0)$

Equation of line through (-5, 0) with slope –2 is $y=-29x+5)=-2x-10$

This line meets the ellipse above X-axis at $(-6,2)$

$\therefore$ Slope = $\dfrac{2-0}{-6-5}=-\dfrac{2}{11}$.

72. If $f(x)$ is a decreasing function for all $x\in R$ and $f(x)>0\ \forall x\in R$ then the range of *K* so that the equation $\dfrac{x^2}{f(K^2+2K+5)}+\dfrac{y^2}{f(K+11)}=1$ represents an ellipse whose major axis is the X-axis is

A) $(-2,3)$ B) $(-3,2)$

C) $(-\infty,-3)\cup(2,\infty)$ D) $(-\infty,-2)\cup(3,\infty)$

Key. B

Sol. Conceptual

73. P,Q are points on the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$ such that PQ is a chord through the point $R(3,0)$. If $|PR|=2$ then length of chord PQ is

A) 8 B) 6 C) 10 D) 4

Key. C

Sol. Conceptual

74. Let $Q=(3,\sqrt{5}), R=(7,3\sqrt{5})$. A point *P* in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A) 6 B) 12 C) 18 D) 9

Key. B

Sol. *P* lies on the ellipse for which Q, R are foci and length of major axis is 10 and eccentricity is 3/5.

75. If the curve $x^2+3y^2=9$ subtends as obtuse angle at the point $(2\alpha,\alpha)$ ($(\alpha\in \text{integer})$, then a possible value of α^2 is

A) 1 B) 2 C) 3 D) 4

Key. B

Sol. The generated curve is $\frac{x^2}{9}+\frac{y^2}{3}=1$, whose director circle is $x^2+y^2=12$. For the required condition $(2\alpha,\alpha)$ should lie inside the circle and out side the ellipse i.e.

$$(2\alpha)^2+3\alpha^2-9>0\,\&\,(2\alpha)^2+\alpha^2-12<0\Rightarrow\frac{9}{7}<\alpha^2<\frac{12}{5}$$

76. Tangent at any point $'P'$ of ellipse $9x^2+16y^2-144=0$ is drawn. Eccentric angle of $'P'$ is $\theta=\frac{1}{2}\sin^{-1}\left(\frac{1}{7}\right)$. If $'N'$ is the foot of perpendicular from centre $'O'$ to this tangent then $\angle PON$ is

A) $\tan^{-1}\left(\frac{1}{12}\right)$ B) $\tan^{-1}\left(\frac{1}{24}\right)$ C) $\frac{\pi}{12}$ D) $\frac{\pi}{3}$

Key. B

Sol.

$$\tan\phi=\sin 2\theta\left(\frac{a^2-b^2}{2ab}\right)$$

$$|\tan\phi|=\frac{16-9}{2\times4\times3}\times\frac{1}{7}\Rightarrow\phi=\tan^{-1}\left(\frac{1}{24}\right)$$

77. If there are exactly two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose distance from its centre is same and is equal to $\sqrt{\frac{a^2+2b^2}{2}}$, then eccentricity of the ellipse is

A) $\frac{1}{2}$ B) $\frac{1}{\sqrt{2}}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{2\sqrt{2}}$

Key. C

Sol.

$$a=\sqrt{\frac{a^2+2b^2}{2}}$$

78. An ellipse slides between two perpendicular straight lines $x=0$ and $y=0$ then, locus of its foci is

(A) a parabola (B) an ellipse

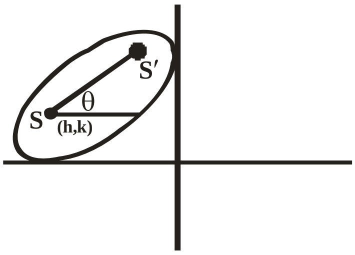
(C) a circle (D) none of these

Key. D

Sol. $(h+2ae\cos\theta)h=b^2$ ….. (1)

$(k+2ae\sin\theta)k=b^2$ …. (2)

$2ae\cos\theta=\dfrac{b^2-h^2}{h}$



$2ae\sin\theta=\dfrac{b^2-k^2}{k}$

$4a^2e^2(\cos^2\theta+\sin^2\theta)=\left(\dfrac{b^2-h^2}{h}\right)^2+\left(\dfrac{b^2-k^2}{k}\right)^2$

79. If a variable tangent to the circle $x^2+y^2=1$ intersects the ellipse $x^2+2y^2=4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

a) eccentricity is $\dfrac{\sqrt{3}}{2}$ b) eccentricity is $\dfrac{\sqrt{5}}{2}$

c) latus -rectum is of length 2 units d) foci are $\left(\pm 2\sqrt{5},0\right)$

Key: A,C

Hint: A tangent to the circle $x^2+y^2=1$ $is\, x\cos\theta+y\sin\theta=1.R\left(x_o,y_o\right)$ is the point of intersection of the tangents to the ellipse at P and Q $\Leftrightarrow x\cos\theta+y\sin\theta=1$ and $x_ox+2y_oy=4$ represent the same line

$\Leftrightarrow x_o=4\cos\theta and\ y_o=2\sin\theta$

$\Leftrightarrow \dfrac{x_0^2}{16}+\dfrac{y_0^2}{4}=1.$ Hence, locus of P is the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{4}=1$

80. From a point P, perpendicular tangents PQ and PR are drawn to ellipse $x^2 + 4y^2 = 4$. Locus of circumcentre of triangle PQR is

(A) $x^2+y^2=\dfrac{16}{5}(x^2+4y^2)^2$ (B) $x^2+y^2=\dfrac{5}{16}(x^2+4y^2)^2$

(C) $x^2 + 4y^2 = \frac{16}{5}(x^2 + y^2)^2$ (D) $x^2 + 4y^2 = \frac{5}{16}(x^2 + y^2)^2$

Key: B

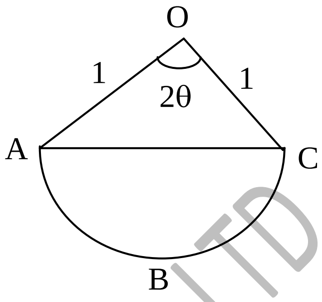
Hint

$x^2 + 4y^2 = 4$

P lies on $x^2 + y^2 = 5$

Let P($\sqrt{5}\cos\theta$, $\sqrt{5}\sin\theta$)

Comparing chord of contact with chord with middle point



$$\frac{xh}{h^2+4k^2}+\frac{y4k}{h^2+4k^2}=1$$

$$\frac{x\sqrt{5}\cos\theta}{4}+\frac{y\sqrt{5}\sin\theta}{1}=1$$

Eliminating θ

$\Rightarrow x^2 + y^2 = \frac{5}{16}(x^2 + 4y^2)^2$

81. Let PQ be a chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, which subtends an angle of $\pi/2$ radians at the centre. If L is the foot of perpendicular from (0, 0) to PQ, then

(A) locus of L is an ellipse

(B) locus of L is circle concentric with given ellipse

(C) locus of L is a hyperbola concentric with given ellipse

(D) a square concentric with given ellipse

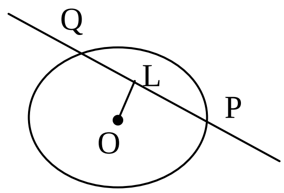
Key: B

Hint

PQ : $x\cos\alpha + y\sin\alpha - p = 0$ … (A)

Homogenising $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ with (A)

$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ = constant



82. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are at right angles then $\left(\frac{x_1x_2}{y_1y_2}\right)$ is equal to

(A) $\frac{a^2}{b^2}$ (B) $-\frac{b^2}{a^2}$ (C) $-\frac{a^4}{b^4}$ (D) $-\frac{b^4}{a^4}$

Key: C

Hint: Chord of contact from (x_1, y_1) is

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Whose slope is $-\frac{b^2}{a^2}\frac{x_1}{y_1}$

Similarly slope of another chord of contact is $-\frac{b^2}{a^2}\frac{x_2}{y_2}$

We have $\left(-\frac{b^2}{a^2}\frac{x_1}{y_1}\right)\times\left(-\frac{b^2}{a^2}\frac{x_2}{y_2}\right)=-1\Rightarrow\frac{x_1x_2}{y_1y_2}=-\frac{a^4}{b^4}$

83. If $\frac{x^2}{f(4a)}+\frac{y^2}{f(a^2-5)}=1$ represents an ellipse with major axis as y-axis and f is a decreasing function positive for all 'a' then a belongs to

A) (0,6) B) (-1,1) C) (-1,5) D) $(5,\infty)$

Key: C

Hint: $f\left(a^2-5\right)>f(4a)\Rightarrow a^2-5<4a\Rightarrow a\in(-1,5)$

84. An ellipse whose focii are $(2,4)$ and $(14,9)$ and touches X-axis then its eccentricity is

A) $\frac{\sqrt{13}}{213}$ B) $\frac{13}{\sqrt{179}}$ C) $\frac{13}{\sqrt{313}}$ D) $\frac{1}{13}$

Key: C

Hint: $2ae=13$

$b^2=36$

85. An ellipse has the point (1, -1) and (2, -1) as its foci and x + y =5 as one of its tangent then value of $a^2 + b^2$ where a, b are the length of semimajor and semiminor axis of ellipse respectively, is

a) $\frac{41}{2}$ b) 10 c) 19 d) $\frac{81}{4}$

Key: D

Hint: 2ae = SS^1 = 1

$p_1\ p_2 = b^2$, where p_1 & p_2 are the length of perpendicular from S & S^1 to the tangent

$$\frac{5}{\sqrt{2}}.\frac{4}{\sqrt{2}}=b^2\Rightarrow b^2=10\Rightarrow b^2=10=a^2-e^2a^2\Rightarrow a^2=\frac{41}{4}$$

86. If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with vertices having eccentric angles α,β,γ respectively is $\left(x_1,y_1\right)$ then $\sum\cos\alpha.\cos\beta+\sum\sin\alpha.\sin\beta$ is

(A) $\frac{9x_1^2}{2a^2}+\frac{9y_1^2}{2b^2}-\frac{3}{2}$ (B) $\frac{x_1^2}{2a^2}+\frac{y_1^2}{2b^2}-\frac{5}{2}$

(C) $\frac{x_1^2}{9a^2}+\frac{y_1^2}{9b^2}-\frac{5}{9}$ (D) $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-\frac{1}{2}$

Key: A

Hint: $A(a\cos\alpha, b\sin\alpha)$, $B(a\cos\beta, b\sin\beta)$, $C(a\cos\gamma, b\sin\gamma)$

Controid = circumcentre $=(x_1, y_1)=\left[\frac{\sum a\cos\alpha}{3}, \frac{\sum b\sin\alpha}{3}\right]$

$$\frac{3x_1}{a}=\sum\cos\alpha, \frac{3y_1}{b}=\sum\sin\alpha$$

$$\left(\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}-3\right)=2\left(\sum\cos\alpha\cos\beta+\sum\sin\alpha\sin\beta\right)$$

$$\Rightarrow \frac{9x^2}{2a^2}+\frac{9y^2}{2b^2}-\frac{3}{2}$$

87. The inclination to the major axis of the diameter of an ellipse the square of whose length is the harmonic mean between the squares of the major and minor axes is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$

c) $\frac{2\pi}{3}$ d) $\frac{\pi}{2}$

KEY : A

HINT: $4(a^2\cos^2\theta+b^2\sin^2\theta)=\frac{2(4a^2)(4b^2)}{4a^2+4b^2}$

88. An ellipse slides between two perpendicular straight lines $x=0$ and $y=0$ then, locus of its foci is

(A) a parabola (B) an ellipse

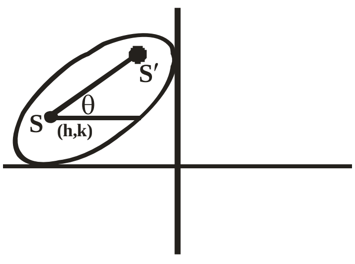
(C) a circle (D) none of these

Key. D

Sol. $(h+2ae\cos\theta)h=b^2$ ….. (1)

$(k+2ae\sin\theta)k=b^2$ …. (2)

$$2ae\cos\theta=\frac{b^2-h^2}{h}$$



$$2ae\sin\theta = \frac{b^2 - k^2}{k}$$

$$4a^2e^2(\cos^2\theta + \sin^2\theta) = \left(\frac{b^2 - h^2}{h}\right)^2 + \left(\frac{b^2 - k^2}{k}\right)^2$$

89. A circle of radius ‘r’ is concentric with the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then inclination of common tangent with major axis is _______(b<r<a)

1. $\tan^{-1}\left(\frac{b}{a}\right)$ 2. $\tan^{-1}\left(\frac{rb}{a}\right)$ 3. $\tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$ 4. $\frac{\pi}{2}$

Key. 3

Sol. The tangent of Ellipse is $y = mx + \sqrt{a^2m^2 + b^2}$, this line touches $x^2 + y^2 = r^2$

Condition is $\left|\frac{\sqrt{a^2m^2 + b^2}}{\sqrt{m^2 + 1}}\right| = r$

$$a^2m^2 + b^2 = r^2m^2 + r^2$$

$$m^2\left(a^2 - r^2\right) = r^2 - b^2 \Rightarrow m^2 = \frac{r^2 - b^2}{a^2 - r^2}$$

$$m = \sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$$

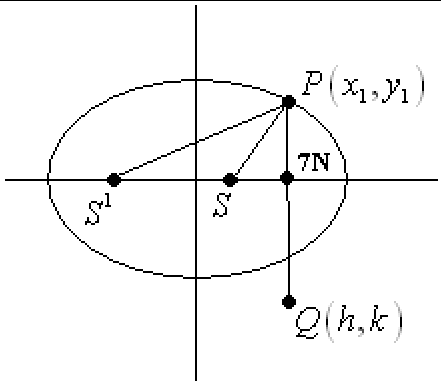
Inclimation is $\tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$

90. From any point P lying in first quadrant on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, PN is drawn perpendicular to the major axis such that N lies on major axis. Now PN is produced to the point Q such that NQ equals to PS, where S is a focus. The point Q lies on which of the following lines

(A) $2y - 3x - 25 = 0$ (B) $3x + 5y + 25 = 0$

(C) $2x - 5y - 25 = 0$ (D) $2x - 5y + 25 = 0$

Key. B



Sol.

$a^2 = 25$

$b^2 = 16$

$e = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$

Let point Q be (h, k), where K < 0

Given that $|K| = a + eh\left(as\ x_1 = h\right)$

$-y = a + ex$

$-y = 5 + \frac{3}{5}x$

$3x + 5y + 25 = 0$

91. The normal at an end of a latus rectum of the ellipse $x^2/a^2 + y^2/b^2 = 1$ passes through an end of the minor axis if

(a) $e^4 + e^2 = 1$ (b) $e^3 + e^2 = 1$ (c) $e^2 + e = 1$ (d) $e^3 + e = 1$

Key. A

Sol. Given ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let $P\left(ae, \frac{b^2}{a}\right)$ be one end of latus rectum.

Slope of normal at $P\left(ae, \frac{b^2}{a}\right) = \frac{1}{e}$

Equation of normal is

$y = \frac{b^2}{a} = \frac{1}{e}(x - ae)$

It passes through $B^{'}(0, b)$ then

$b - \frac{b^2}{a} = -a$

$a^2 - b^2 = -ab$

$a^4e^4 = a^2b^2$

$e^4 + e^2 = 1$

92. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1. $\sqrt{ab}$ 2. a/b 3. $2ab$ 4. ab

Key. 3

Sol. Let the vertices of the rectangle be $(\pm a\cos\theta, \pm b\sin\theta)$, then the Area of the rectangle is $4ab\sin\theta\cos\theta = 2ab\sin 2\theta$. The maximum value of which is $2ab$ as $\sin 2\theta \le 1$.

93. The number of values of C such that the straight line $y = 4x + c$ touches the curve $x^2/4 + y^2 = 1$ is

1. 0 2. 1 3. 2 4. Infinite

Key. 3

Sol. We know that $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$

Here $m = a^2 = 4, b^2 = 1$ so $c^2 = 4 \times 4^2 + 1 \quad \Rightarrow \quad c = \pm\sqrt{65}$

94. The normal at an end of a latus rectum of the ellipse $x^2/a^2 + y^2/b^2 = 1$ passes through an end of the minor axis if

1. $e^4 + e^2 = 1$ 2. $e^3 + e^2 = 1$
3. $e^2 + e = 1$ 4. $e^3 + e = 1$

Key. 1

Sol. Let at end of a latus rectum be $(ae, \sqrt{1-e^2})$, then the equation of the normal at this end is

$$\frac{x - ae}{ae/a^2} = \frac{y - b\sqrt{1-e^2}}{b\sqrt{1-e^2}/b^2}$$

It will pass through the end $(0, -b)$ if

$$-a^2 = \frac{-b^2(1+\sqrt{1-e^2})}{\sqrt{1-e^2}} \text{ or } \frac{b^2}{a^2} = \frac{\sqrt{1-e^2}}{1+\sqrt{1-e^2}}$$

Or $\quad (1-e^2)\left[1+\sqrt{1-e^2}\right] = \sqrt{1-e^2}$

Or $\quad \sqrt{1-e^2} + 1 - e^2 = 1$ or $e^4 + e^2 = 1$.

95. The locus of the middle points of the portions of the tangents of the ellipse $x^2/a^2 + y^2/b^2 = 1$ included between the axes is the curve.

1. $x^2/a^2 + y^2/b^2 = 4$ 2. $a^2/x^2 + b^2/y^2 = 4$

3. $a^2x^2 + b^2y^2 = 4$ 4. $b^2x^2 + a^2y^2 = 4$

Key. 2

Sol. Equation of a tangent to the ellipse can be written as $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ which meets the axes at A $(a/\cos\theta,0)$ and $B(0,b/\sin\theta)$. If (h,k) is the middle point of AB , then

$$h=a/2\cos\theta,\ k=b/2\sin\theta$$

Eliminating θ we get $(a/2h)^2 + (b/2k)^2 = 1$

$\Rightarrow$ locus of $P(h,k)$ is $a^2/x^2+b^2/y^2=4$.

96. The locus of the points of intersection of the tangents at the extremities of the chords of the ellipse $x^2+2y^2=6$ which touch the ellipse $x^2+4y^2=4$ is

1. $x^2+y^2=4$ 2. $x^2+y^2=6$ 3. $x^2+y^2=9$ 4. None of these

Key. 3

Sol. We can write $x^2+4y^2=4$ as $\frac{x^2}{4}+\frac{y^2}{1}=1$ (i)

Equation of a tangent to the ellipse (i) is

$$\frac{x}{2}\cos\theta+y\sin\theta=1 \qquad \text{(ii)}$$

Equation of the ellipse $x^2+2y^2=6$ can be written as

$$\frac{x^2}{6}+\frac{y^2}{3}=1 \qquad \text{(iii)}$$

Suppose (ii) meets the ellipse (iii) at P and Q and the tangents at P and Q to the ellipse (iii) intersect at (h,k), then (ii) is the chord of contact of (h,k) with respect to the ellipse (iii) and thus its equation is $\frac{hx}{6}+\frac{ky}{3}=1$ (iv)

Since (ii) and (iv) represent the same line

$$\frac{h/6}{(\cos\theta)/2}=\frac{k/3}{\sin\theta}=1$$

$\Rightarrow$ $h=3\cos\theta, k=3\sin\theta$

And the locus of (h,k) is $x^2+y^2=9$

97. A tangent at any point to the ellipse $4x^2+9y^2=36$ is cut by the tangent at the extremities of the major axis at T and T^1. The circle on T T^1 as diameter passes through the point.

1. $\left(0,\sqrt{5}\right)$ 2. $\left(\sqrt{5},0\right)$ 3. $(2,1)$ 4. $\left(0,-\sqrt{5}\right)$

Key. 2

Sol. Any point on the ellipse is $P(3\cos\theta, 2\sin\theta)$

Equation of the tangent at P is $\frac{x}{3}\cos\theta + \frac{y}{2}\sin\theta = 1$

Which meets the tangents $x=3$ and $x=-3$ at the extremities of the major axis at

$T\left(3, \frac{2(1-\cos\theta)}{\sin\theta}\right)$ and $T^1\left(3, \frac{2(1+\cos\theta)}{\sin\theta}\right)$

Equation of the circle on TT^1 as diameter is

$$(x-3)(x+3)+\left(y-\frac{2(1-\cos\theta)}{\sin\theta}\right)\left(y-\frac{2(1+\cos\theta)}{\sin\theta}\right)=0$$

$\Rightarrow x^2+y^2-\frac{4}{\sin\theta}y-5=0$,which passes through $(\sqrt{5},0)$

98. If $y=x$ and $3y+2x=0$ are the equations of a pair of conjugate diameters of an ellipse, then the eccentricity of the ellipse is

1. $\sqrt{2/3}$ 2. $1/\sqrt{3}$ 3. $1/\sqrt{2}$ 4. $1/\sqrt{5}$

Key. 2

Sol. Let the equation of the ellipse be $x^2/a^2 + y^2/b^2 = 1$

Slope of the given diameters are $m_1 = 1, m_2 = -2\sqrt{3}$.

$\Rightarrow \quad m_1m_2 = -2/3 = -b^2/a^2$

[using the condition of conjugacy of two diameters]

$3b^2 = 2a^2 \Rightarrow 3a^2(1-e^2) = 2a^2$,

$1-e^2 = 2/3 \Rightarrow e^2 = 1/3 \Rightarrow e = 1/\sqrt{3}$

99. On the ellipse $4x^2+9y^2=1$, the point at which the tangent is parallel to the line $8x=9y$ is

1. $(2/5, 1/5)$ 2. $(-2/5, 1/5)$ 3. $(-2/5, -1/5)$ 4. None of these

Key. 2

Sol. Let the point be $((1/2)\cos\theta, (1/3)\sin\theta)$,then the slope of the tangent is $-\frac{1/3}{1/2}\cot\theta = \frac{8}{9}$

$\Rightarrow \tan\theta = -\frac{3}{4} \Rightarrow \sin\theta = \pm\frac{3}{5}$ and $\cos\theta = \mp\frac{4}{5}$

And the required point can be $\left(-\frac{4}{5}\times\frac{1}{2},\frac{3}{5}\times\frac{1}{3}\right)=\left(-\frac{2}{5},\frac{1}{5}\right)$

100. The circle $x^2+y^2=c^2$ contains the ellipse $x^2/a^2+y^2/b^2=1\ (a>b)$ if

1. $c<a$ 2. $c<b$ 3. $c>a$ 4. $c>b$

Key. 3

Sol. Radius of the circle must be greater than the major axis of the ellipse.

101. In an ellipse, if the lines joining a focus to the extremities of the minor axis make an equilateral triangle with the minor axis, then the eccentricity of the ellipse is

1. 3/4 2. $\sqrt{3}/2$ 3. 1/2 4. 2/3

Key. 2

Sol. $a^2e^2+b^2=(2b)^2 \Rightarrow a^2e^2=3a^2(1-e^2) \Rightarrow e^2=3/4.$

102. If $(5,12)$ and $(24,7)$ are foci of an ellipse passing through origin, then the eccentricity of the ellipse is

(A) $\frac{\sqrt{386}}{24}$ (B) $\frac{\sqrt{386}}{38}$ (C) $\frac{\sqrt{386}}{25}$ (D) $\frac{1}{\sqrt{2}}$

Key. 2

Sol. Let $S(5,12)$ $S'(24,7)$ O is origin

$SO=13\quad S'O=25\quad SS'=\sqrt{386}$

$e=\frac{\sqrt{386}}{13+25}=\frac{\sqrt{386}}{38}$

103. A variable point P on the ellipse of eccentricity e is joined to foci S and S^1. The eccentricity of the locus of the in centre of triangle PSS^1 is

(A) $\sqrt{\frac{2e}{1+e}}$ (B) $\sqrt{\frac{e}{1+e}}$ (C) $\sqrt{\frac{1-e}{1+e}}$ (D) $\frac{e}{2(1+e)}$

Key. 1

Sol. Let any point be $P(a\cos\theta, b\sin\theta)$.

$SP=a[1-e\cos\theta]\quad S'P=a[1+e\cos\theta]\quad SS^1=2ae$

(h,k) be in centre of $\Delta PSS'$ upon solving we get

$\Rightarrow h=ae\cos\theta$

$k=\frac{b\sin\theta e}{1+e}$

Eliminating 'θ'

$$\frac{h^2}{a^2e^2}+\frac{k^2}{\frac{e^2b^2}{(1+e)^2}}=1$$

Locus of (h,k) $\frac{x^2}{a^2e^2}+\frac{y^2}{\frac{e^2b^2}{(1+e)^2}}=1$

$$\Rightarrow e_1^2=1-\frac{e^2b^2}{(1+e)^2e^2a^2}=\frac{2e}{1+e}$$

$$e_1=\sqrt{\frac{2e}{1+e}}$$

104. An ellipse is drawn with major and minor axis of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of circle being outside the ellipse. The radius of circle is

(A) 4 (B) 5 (C) 2 (D) 1

Key. 3

Sol. The circle must touch the end of major axis

$\because$ radius $=a-ae=a-\sqrt{a^2-b^2}=5-\sqrt{5^2-4^2}=2$

105. If the length of the major axis intercepted between the tangent & normal at a point $(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the length of semi-major axis, then , eccentricity of the ellipse is,

a) $\frac{\cos\theta}{\sqrt{1-\cos\theta}}$ b) $\frac{\sqrt{1-\cos\theta}}{\cos\theta}$ c) $\frac{\sqrt{1-\cos\theta}}{\sin\theta}$ d) $\frac{\sin\theta}{\sqrt{1-\sin\theta}}$

Key. B

Sol. $\frac{a}{\cos\theta}-\frac{(a^2-b^2)}{a}\cos\theta=a\Rightarrow e^2\cos^2\theta=1-\cos\theta\Rightarrow e=\frac{\sqrt{1-\cos\theta}}{\cos\theta}$

106. P_1, P_2 are the lengths of the perpendicular from the foci on the tangent to the ellipse and P_3, P_4 are perpendiculars from extremities of major axis and P from the centre of the ellipse on the same tangent, then $\frac{P_1P_2-P^2}{P_3P_4-P^2}$ equals (where e is the eccentricity of the ellipse)

(A) e (B) $\sqrt{e}$
(C) e^2 (D) none of these

Key. C

Sol. Let equation of tangent y = mx + c

$$P=\frac{|c|}{\sqrt{1+m^2}} \qquad P_1=\frac{|c+aem|}{\sqrt{1+m^2}}$$

$$P_2=\frac{|c-aem|}{\sqrt{1+m^2}} \qquad P_3=\frac{|c+am|}{\sqrt{1+m^2}}$$

$$P_4 = \frac{|c - am|}{\sqrt{1+m^2}}$$

So, $\dfrac{P_1P_2 - P^2}{P_3P_4 - P^2} = \dfrac{c^2 - a^2e^2m^2 - c^2}{c^2 - a^2m^2 - c^2}$

$= e^2$

107. If $\dfrac{x^2}{f(4a)} + \dfrac{y^2}{f(a^2-5)} = 1$ represents an ellipse with major axis as y-axis and f is a decreasing function positive for all 'a' then a belongs to

a) (0,5) b)(-1,1) c) (-1,5) d) $(5,\infty)$

Key. C

Sol. $f\left(a^2-5\right) > f(4a) \Rightarrow a^2 - 5 < 4a \Rightarrow a \in (-1,5)$

108. If a tangent of slope 2 of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is normal to the circle $x^2+y^2+4x+1 = 0$, then the maximum value of ab is

a)4 b)2 c) 1 d)0

Key. A

Sol. Equation of tangent with slope

$y = 2x \pm \sqrt{4a^2 + b^2}$ is a normal to the circle $\therefore 0 = -4 \pm \sqrt{4a^2+b^2} \Rightarrow 4a^2 + b^2 = 16$

Max of ab = 4

109. The eccentric angle of a point p lying in the first quadrant on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is θ. If OP makes an angle ϕ with x –axis, then $\theta - \phi$ will be maximum when $\theta =$

a) $\tan^{-1}\sqrt{\dfrac{a}{b}}$ b) $\tan^{-1}\sqrt{\dfrac{b}{a}}$ c) $\dfrac{\pi}{4}$ d) $\dfrac{\pi}{3}$

Key. A

Sol. $\tan\theta = \dfrac{b}{a}\tan\theta$

$$y = \frac{a-b}{a\cot\theta + b\tan\theta}$$ if will be maximum

If $\tan^2\theta = \dfrac{a}{b} \Rightarrow \tan\theta = \sqrt{\dfrac{a}{b}}$

110. From the focus $(-5,0)$ of the ellipse $\frac{x^2}{45}+\frac{y^2}{20}=1$ a ray of light is sent which makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A) $-3/2$ B) $-7/3$ C) $-5/4$ D) $-2/11$

Key. D

Sol. Let $\theta=\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)\Rightarrow\cos\theta=\frac{-1}{\sqrt{5}}\Rightarrow\tan\theta=-2$

Foci are $(\pm5,0)$

Equation of line through (-5, 0) with slope –2 is $y=-29x+5)=-2x-10$

This line meets the ellipse above X-axis at $(-6,2)$

$\therefore$ Slope = $\frac{2-0}{-6-5}=-\frac{2}{11}$.

111. If $f(x)$ is a decreasing function for all $x\in R$ and $f(x)>0\ \forall x\in R$ then the range of *K* so that the equation $\frac{x^2}{f(K^2+2K+5)}+\frac{y^2}{f(K+11)}=1$ represents an ellipse whose major axis is the X-axis is

A) $(-2,3)$ B) $(-3,2)$ C) $(-\infty,-3)\cup(2,\infty)$ D) $(-\infty,-2)\cup(3,\infty)$

Key. B

Sol. Conceptual

112. P,Q are points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ such that PQ is a chord through the point $R(3,0)$. If $|PR|=2$ then length of chord PQ is

A) 8 B) 6 C) 10 D) 4

Key. C

Sol. Conceptual

113. Let $Q=(3,\sqrt{5}),R=(7,3\sqrt{5})$. A point *P* in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A) 6 B) 12 C) 18 D) 9

Key. B

Sol. *P* lies on the ellipse for which Q,R are foci and length of major axis is 10 and eccentricity is 3/5.

114. *O* is the centre of ellipse for which A,B are end points of major axis and C,D are end points of minor axis. *F* is a focus of the ellipse. If $|OF|=6$ and inradius of ΔOCF is 1, then $|AB|\times|CD|=$

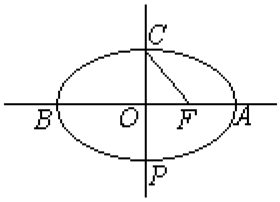
A) 65 B) 52 C) 78 D) 47

Key. A

Sol. $b^2=\frac{25}{4}\Rightarrow a^2-a^2e^2=\frac{25}{4}\Rightarrow a^2=\frac{25}{4}+36=\frac{169}{4}\Rightarrow a=\frac{13}{2}$

$|OF|=ae=6\Rightarrow \frac{abe}{2}=1\times\frac{(ae+b+\sqrt{a^2e^2+b^2})}{2}$

$6b=6+b+\sqrt{b^2+36}\Rightarrow(5b-6)^2=b^2+36\Rightarrow 24b^2=60b\Rightarrow b=5/2$



115. The angle of intersection between the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2-k^2}-\frac{y^2}{k^2-b^2}=1,(a>k>b>0)$ is

a) $\tan^{-1}\left(\frac{b}{a}\right)$ b) $\tan^{-1}\left(\frac{b}{ka}\right)$ c) $\tan^{-1}\left(\frac{a}{kb}\right)$ d) None of these

Key. D

Sol. Confocal ellipse and hyperbola cut at right angles

116. Image of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ in the line x + y = 10 is :

a) $\frac{(x-10)^2}{16}+\frac{(y-10)^2}{25}=1$ b) $\frac{(x-10)^2}{25}+\frac{(y-10)^2}{16}=1$

c) $\frac{(x-5)^2}{16}+\frac{(y-5)^2}{25}=1$ d) $\frac{(x-5)^2}{25}+\frac{(y-5)^2}{16}=1$

Key. A

Sol. Conceptual

117. Length of common tangent to $x^2+y^2=16$ and $\frac{x^2}{25}+\frac{y^2}{7}=1$

a) $\frac{9}{4\sqrt{2}}$ b) $\frac{9}{4}$ c) $\frac{9}{2\sqrt{2}}$ d) $\frac{9}{2}$

Key. B

Sol. $y=-x+4\sqrt{2}$ is a common tangent to two curves in the 1st quadrant. Touching the curves at $P\left(2\sqrt{2},2\sqrt{2}\right)\&Q\left(\frac{25}{4\sqrt{2}},\frac{7}{4\sqrt{2}}\right)$

PQ = length of common tangent.

118. An ellipse having foci S (3, 4) & S' (6, 8) passes through the point P(0, 0). The equation of the tangent at P to the ellipse is

a) $4x+3y=0$ b) $3x+4y=0$ c) $x+y=0$ d) $x-y=0$

Key. B

Sol. Normal at a point is bisector of angle SPS'

119. The angle subtended at the origin by a common tangent of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2x}{c}=0$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2}+\frac{2x}{c}=0$, is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

Key. D

Sol. Conceptual

120. If the normals at 4 points having eccentric angles $\alpha,\beta,\gamma,\delta$ on an ellipse be concurrent, then $\left(\sum\cos\alpha\right)\left(\sum\sec\alpha\right)=$

a) 4 b) $(\alpha\beta\gamma\delta)^{\frac{1}{4}}$ c) $\frac{\alpha+\beta+\gamma+\delta}{4}$ d) None of these

Key. A

Sol. Conceptual

121. If $\frac{x^2}{f(4a)}+\frac{y^2}{f(a^2-5)}=1$ represents an ellipse with major axis as y-axis and f is decreasing function, then

A) $a\in(-\infty,1)$ B) $a\in(5,\infty)$ C) $a\in(1,4)$ D) $a\in(-1,5)$

Key. D

Sol. $f(4a)<f(a^2-5)\Rightarrow 4a>a^2-5[\because f \text{ is} \downarrow \text{fn}]$

$\therefore a\in(-1,5)$