

# PHYSICS

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**Q.1** **Assertion** : The stretching of a coil is determined by its shear modulus.

**Reason** : Shear modulus change only shape of a body keeping its dimensions unchanged.

**Sol.** [A]

**Q.2** **Assertion** : Steel is more elastic than rubber.

**Reason** : Under given deforming force, steel is deformed strength.

**Sol.** [A]

**Q.3** **Assertion** : Glassy solids have sharp melting point.

**Reason** : The bonds between the atoms of glassy solids get broken at the same temperature.

**Q.4** **Assertion** : Young's modulus for a perfectly plastic body is zero.

**Reason** : For a perfectly plastic body, restoring force is zero.

**Sol.** [A]

**Q.5** **Assertion** : Identical springs of steel and copper equally stretched. More work will be done on the steel spring.

**Reason** : Steel is more elastic than copper.

**Sol.** [A]

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- Q.1**
- | Column I                   | Column II   |
|----------------------------|-------------|
| (A) Stress $\times$ Strain | (P) J       |
| (B) $\frac{YA}{l}$         | (Q) N/m     |
| (C) $Yl^3$                 | (R) $J/m^3$ |
| (D) $\frac{Fl}{AY}$        | (S) m       |
- Sol.** (A)  $\rightarrow$  (R); (B)  $\rightarrow$  (Q); (C)  $\rightarrow$  (P); (D)  $\rightarrow$  (S)

- Q.2**
- | Column-I  | Column-II  |
|---|--|
| (A) Specific heat capacity S  | (P) $l_1 - l_2 = \text{constant}$<br>for $l_1\alpha_1 = l_2\alpha_2$ |
| (B) Two metals ( $l_1, \alpha_1$ ) and ( $l_2, \alpha_2$ ) are heated uniformly | (Q) Y is same  |
| (C) Thermal stress  | (R) $S = \infty$ for $\Delta T = 0$                                  |
| (D) Four wires of same material   | (S) $Y \propto \Delta t$   |

- Sol.** (A)  $\rightarrow$  (R), (B)  $\rightarrow$  (P), (C)  $\rightarrow$  (S), (D)  $\rightarrow$  (Q)
- $$S = \frac{Q}{M\Delta T}$$
- when  $\Delta T = 0$ ,  $S = \infty$
- So, (A)  $\rightarrow$  (R)
- For  $l_1 - l_2$  to remain constant, the wire/rods extension should be same. So (D  $\rightarrow$  Q)

i.e.,  $\Delta l_1 = \Delta l_2$   
 $\Rightarrow l_1\alpha_1\Delta t = l_2\alpha_2\Delta t$

or  $l_1\alpha_1 = l_2\alpha_2$

So, (B)  $\rightarrow$  (P)

$$\begin{aligned} \text{Thermal stress} &= \frac{F}{A} = Y \frac{\Delta l}{l} \\ &= Y \cdot \frac{l\alpha\Delta t}{l} = Y \alpha \Delta t \end{aligned}$$

So, (C)  $\rightarrow$  (S).

When material is same the modulus of elasticity will be the same.

- Q.3**
- | Column I                                     | Column II                           |
|--|-------------------------------------|
| (A) Thermal stress                           | (P) $\frac{1}{2Y}(\text{stress})^2$ |
| (B) Energy stored in per unit volume of wire | (Q) $\frac{Y}{2}(\text{strain})^2$  |
| (C) Young's modulus                          | (R) $Y \propto \Delta T$            |
|  | (S) $3k(1 - 200)$                   |
|  | (T) $\frac{9k\eta}{3k + \eta}$      |

**Sol.** A  $\rightarrow$  R ; B  $\rightarrow$  P,Q ; C  $\rightarrow$  S,T

- Q.4** Consider a wire of length  $\ell$ , cross-sectional area A and Young's modulus Y and match Column-I with Column-II -

- | Column-I  | Column-II                    |
|---|------------------------------|
| (A) If the wire is pulled at its ends by equal and opposite forces of magnitude F so that it undergoes an elongation x, according to Hooke's law, $F = kx$ , where k is the force constant. Force constant (k) of the wire will depend on | (P) Young's Modulus          |
| (B) Let us suspend the wire vertically from a rigid supported and attach a mass m at its lower end. If the mass is slightly pulled down and released, it executes S.H.M. The time period will depend on                                   | (Q) elongation (x)           |
| (C) If the given wire is fixed between two rigid supports and its temperature is increased thermal stress that develops in the rod will depend on   | (R) length ( $\ell$ )        |
| (D) Work done in stretching the wire to a length $\ell + x$   | (S) area of cross-section(A) |

will depend on (T) independent of  
elongation (x)

**Sol.** A  $\rightarrow$  P,R,S,T ; B  $\rightarrow$  P,R,S,T;  
C  $\rightarrow$  P,T ; D  $\rightarrow$  P,Q,R,S

**Hint :** Young's modulus is the property of material.

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**Q.1** For two different materials it is given that  $Y_1 > Y_2$  and  $B_1 < B_2$ . Here,  $Y$  is Young's modulus of elasticity and  $B$ , the Bulk modulus of elasticity.

Then we can conclude that :

- (A) 1 is more ductile
- (B) 2 is more ductile
- (C) 1 is more malleable
- (D) 2 is more malleable

**Sol.** (B, C)

The more the modulus of elasticity, the more is the resistance offered to external deforming forces.

**Q.2** A load  $w$  is suspended from a wire of length  $l$  and area of cross-section  $A$ . Change in length of the wire is say  $\Delta l$ . Change in length  $\Delta l$  can be increased to two times by increasing :

- (A)  $w$  by two times                      (B)  $l$  by two times
- (C)  $A$  by two times                      (D)  $A$  by four times

**Sol.** (A, B)  $\Delta l = \frac{fl}{AY}$

$\Delta l$  can be increased by making  $F$  or  $l$  two times.

**Q.3** Overall changes in volume and radii of a uniform cylindrical steel wire are 0.2% and 0.002 % respectively when subjected to some suitable force. If Young's modulus of elasticity of steel is  $Y = 2.0 \times 10^{11} \text{ N/m}^2$ , then -

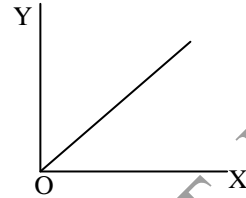
- (A) Longitudinal tensile stress acting on the wire is  $4.08 \times 10^8 \text{ N/m}^2$
- (B) Longitudinal tensile stress acting on the wire is  $3.92 \times 10^8 \text{ N/m}^2$
- (C) Longitudinal strain is 0.204 %
- (D) Longitudinal strain is 0.196 %                      **[A,C]**

**Sol.**  $\frac{dV}{V} = \frac{dl}{l} + \frac{dA}{A}$

$$\frac{0.2}{100} = \frac{dl}{l} - \frac{0.004}{100}$$

$$\frac{dl}{l} = \frac{0.204}{100}$$

**Q.4** A student plots a graph from his readings on the determination of Young's modulus of a metal but forgets to put the labels (Shown in wire figure). The quantities on X and Y-axes may be respectively-



- (A) weight hung and length increased
- (B) stress applied and length increased
- (C) stress applied and strain developed
- (D) length increased and the weight hung

**Sol.** [A,B,C,D]

**Q.5** A light rod of 2 m length is suspended from the ceiling horizontally using two vertical wires of equal length tied to its ends. One of the wire is made of steel of cross-sectional area 0.1 sq. cm and young modulus  $20 \times 10^{11} \text{ dyne/cm}^2$ , while the second is made of brass of cross-sectional area 0.2 sq. cm and Young's modulus  $10 \times 10^{11} \text{ dyne/cm}^2$ . Then -

- (A) for stress to be same, a mass can hang at 0.6 m from brass wire end
- (B) for stress to be same, a mass can hang at 0.6 m from steel end
- (C) for equal strain the mass should be hung at mid-point of the rod
- (D) for equal force the mass should be at  $\frac{1}{4}$  th from steel end

**Sol.** [A,C]

**Q.6** Which of the following is correct ?

- (A) For a small deformation of a material, the ratio  $\frac{\text{stress}}{\text{strain}}$  remains constant
- (B) For a large deformation of a material, the ratio  $\frac{\text{stress}}{\text{strain}}$  decreases
- (C) Two wires, made of different materials having the same diameter and length are connected end to end. A force is applied which stretches their combined length by 2 mm. Now, the strain is same in both the wires but stress is different
- (D) Both (B) and (C) are correct

**Q.7** A body of mass  $m$  is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is  $\ell$ . Which of the following is correct ?

- (A) Heat produced is  $\frac{1}{2} mg\ell$
- (B) Loss in gravitational potential energy of mass  $m$  is  $mg\ell$
- (C) The elastic potential energy stored in the wire is  $\frac{1}{2} mg\ell$
- (D) Only (A) and (C) are correct

**Sol.** [A,B,C]

**Q.8** Which of the following statements is correct ?

- (A) When a wire is pulled by a certain force, the elongation is inversely proportional to cross-sectional area
- (B) Energy in a stretched wire is half the product of load and extension
- (C) Bulk modulus of elasticity was first defined by Maxwell
- (D) Only (A) and (B) are correct

**Sol.** [A,B,C]

**Q.9** Which of the following is correct ?

- (A) When an iron bar is so heated that it is not permitted to expand or bend, then the gigantic force developed is independent of length
- (B) Hooke's law essentially defines elastic limit
- (C) A uniform cube is subjected to volume compression. If each side is increased by 1%, then the bulk strain is 0.03
- (D) None of these

**Sol.** [A,B,C]

**Q.10** A light rod of length 2 m is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its end. One of the wires is made of steel and is of cross-section  $0.1 \text{ cm}^2$ . The other wire is of brass of cross-section  $0.2 \text{ cm}^2$ . A weight is suspended from a certain point of the rod such that equal stresses are produced in both the wires. Which of the following is correct ?

- (A) The ratio of tensions in the steel and brass wires is 0.5
- (B) The load is suspended at a distance of  $\frac{400}{3}$  cm from the steel wire
- (C) Both (A) and (B) are correct
- (D) Neither (A) nor (B) is correct

**Sol.** [A,B,C]

**Q.11** Which of the following is correct ?

- (A) The product of bulk modulus of elasticity and compressibility is 1
- (B) A rope 1 cm in diameter breaks if the tension in it exceeds 500 N. The maximum tension that may be given to a similar rope of diameter 2 cm is 2000 N
- (C) Both (A) and (B)
- (D) Neither (A) nor (B)

**Sol.** [A,B,C]

**Q.12** Which of the following is correct ?

- (A) The shear modulus of a liquid is infinite
- (B) Bulk modulus of a perfectly rigid body is infinity
- (C) According to Hooke's law, the ratio of stress and strain remains constant
- (D) Both (A) and (B) are correct

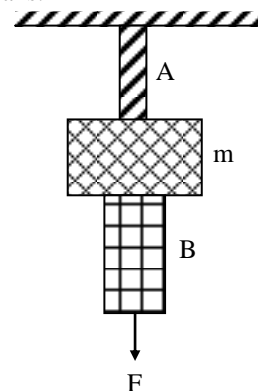
**Sol.** [A,B,C]

**Q.13** An elastic rod will change its length when it—

- (A) slides on a rough surface
- (B) rotates about an axis at one end
- (C) falls vertically under its weight
- (D) is pulled along its length by a force acting at one end

**Sol.** [B, D]

**Q.14** The wires A and B shown in the figure, are made of the same material and have radii  $r_A$  and  $r_B$  respectively. A block of mass  $m$  is connected between them; when a force  $F$  is  $mg/3$ , one of the breaks.



- (A) A will break before B if  $r_A < 2r_B$
- (B) A will break before B if  $r_A = r_B$
- (C) Either A or B will break if  $r_A = 2r_B$
- (D) The length of the wires must be known to conclude

Sol. [A,B,C]

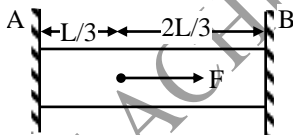
- Q.15** Two springs have identical lengths and areas of cross-section are suspended by mass M. Their Young's modulus are in the ratio 2 : 5.
- (A) They will be subjected to stresses which are in ratio 2 : 5
- (B) Their lengths will increase in the ratio 5 : 2
- (C) When stretched and released, they will oscillate with time periods in the ratio 5 : 2
- (D) Their time periods will be in the ratio  $\sqrt{5} : \sqrt{2}$

Sol. [A,B,C]

- Q.16** Two springs of same length and same area of cross - section are suspended. Their Young's modulus are in the ratio 2 : 3 –
- (A) Their lengths will increase in the ratio 3 : 2
- (B) They will be subjected to stresses which are in the ratio 2 : 3
- (C) When stretched and released, they will oscillate with time periods in the ratio 3 : 2
- (D) Their time periods will be in ratio  $\sqrt{3} : \sqrt{2}$

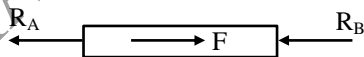
Sol. [A, D]

- Q.17** A metal rod is fixed in horizontal position and a force of magnitude F is applied as shown. If  $R_A$  = force by wall A and  $R_B$  = force by wall B, then -



- (A)  $R_A = F/2$                       (B)  $R_B = 3F/2$   
(C)  $R_A = 2F/3$                       (D)  $R_B = F/3$

Sol.[C,D]  $R_A + R_B = f$



and elongation in both parts must be same

$$\frac{R_B(2L/3)}{AY} = \frac{R_A(L/3)}{AY}$$

or  $2R_B = R_A$

- Q.18** An elastic metal rod will change its length when it
- (A) falls vertically under its weight

- (B) is pulled along its length by a force acting at one end
- (C) rotates about an axis at one end
- (D) slides on a rough surface

Sol.[B,C]  $\frac{\text{Stress}}{\text{Strain}} = \text{constant}$

without net stress no change in length.

- Q.19** Select the correct statements -
- (A) A wire under tension by two equal and opposite forces at its ends each of magnitude F the stress is  $\frac{F}{A}$ , A is the cross sectional area of wire
- (B) Stress is related to internal restoring force that comes into play due to any deformation produced by the external forces
- (C) In the option (A) stress is equal  $\frac{2F}{A}$
- (D) None of these

Sol.[A,B]

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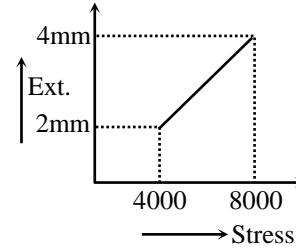
**Q.1** A steel rod with a length of 0.350 m and an aluminium rod with length of 0.250 m, both with same diameter, are placed end to end between rigid supports with no initial stress in the rods. The temperature of the rods is now raised by 60°C. The length of combined rods remains the same, but the length of individual rods change.

$$\alpha_S = 1.2 \times 10^{-5}/^\circ\text{C}; \quad \alpha_A = 2.4 \times 10^{-5}/^\circ\text{C}$$

$$Y_S = 2 \times 10^{11} \text{ N/m}^2; \quad Y_A = 0.7 \times 10^{11} \text{ N/m}^2$$

Stress developed in each rod is.....  $\times 10^7 \text{ N/m}^2$

**[0012]**



**Sol.[2]**  $\Delta l = \frac{F\ell}{AY}$

$$\frac{\Delta l}{F/A} = \frac{\ell}{Y}$$

$$Y = \frac{4000 \times 10^3}{2 \times 10^{-3}} = 2 \times 10^9 \text{ N/m}^2$$

**Q.2** A wire of length '2m' is clamped horizontally between two fixed support. A mass  $m = 5\text{kg}$  is hanged from middle of wire. The vertical and depression in wire (in cm) in equilibrium is (Young modulus of wire =  $2.4 \times 10^9 \text{ N/m}^2$ , cross-sectional area =  $1 \text{ cm}^2$ )

**Sol.[5]** At equilibrium

$$2T \sin\theta = mg$$

$$\Rightarrow 2 \cdot \left( \frac{YA}{2a} \right) \times \sin\theta \cdot \sin\theta = mg$$

$$\Rightarrow \frac{YA}{a} \times \frac{x^2}{a^2} = mg$$

$$\Rightarrow x = \left\{ \frac{a^3 mg}{YA} \right\}^{\frac{1}{3}}$$

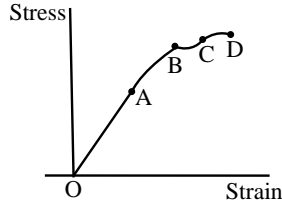
$$= \left\{ \frac{1\text{m} \times 5\text{kg} \times 10 \text{ m/s}^2}{(2.4 \times 10^9 \text{ N/m}^2) \times 10^{-4} \text{ m}^2} \right\}^{\frac{1}{3}}$$

$$= 5 \text{ cm}$$

**Q.3** In determination of young modulus of elasticity of wire, a force is applied and extension is recorded. Initial length of wire is '1m'. The curve between extension and stress is depicted then young modulus of wire will be  $K \times 10^9 \text{ N/m}^2$ , where K is

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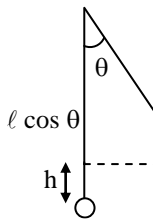
**Q.1** To determine the young's modulus by searle's method a student gets the stress v/s strain graph as shown in diagram. Which portion should give the best result –



- (A) BC    (B) CD    (C) AC    (D) OA  
[D]

**Sol.** It is proportional limit so OA is correct

**Q.2** A sphere of mass  $M$  kg is suspended by a metal wire of length  $L$  and diameter  $d$ . When in equilibrium there is a gap of  $\Delta\ell$  between the sphere and the floor. The sphere is gently pushed aside so that it makes an angle  $\theta$  with the vertical. Find  $\theta_{\max}$  so that sphere fails to rub the Floor. Young's modulus of the wire is  $Y$  -



- (A)  $\sin^{-1} \left( 1 - \frac{Y\pi d^2 \Delta\ell}{8MgL} \right)$   
 (B)  $\tan^{-1} \left( 1 - \frac{Y\pi d^2 \Delta\ell}{8MgL} \right)$   
 (C)  $\cos^{-1} \left( 1 - \frac{Y\pi d^2 \Delta\ell}{8MgL} \right)$   
 (D) none

**Sol.** [C]

$$Y = \frac{F\ell}{A\Delta\ell} = \frac{2Mg(1-\cos\theta)L}{\pi \frac{d^2}{4} \Delta\ell}$$

$$\left[ \because \frac{Mv^2}{2} = Mg\ell(1-\cos\theta) \right]$$

$$\Rightarrow \frac{Mv^2}{\ell} = 2Mg(1-\cos\theta)]$$

$$1-\cos\theta = \frac{Y\pi d^2 \Delta\ell}{8Mg\ell} \Rightarrow \cos\theta = 1 - \frac{Y\pi d^2 \Delta\ell}{8Mg\ell}$$

**Q.3** A copper wire of length 0.9 m and cross-sectional area  $1.0 \text{ mm}^2$  is stretched by a load of 1kg. Young's modulus for copper is  $1.2 \times 10^{11} \text{ N/m}^2$  and  $g = 10 \text{ m/s}^2$ . The extension in wire in mm is -  
 (A) .013    (B) .075    (C) .11    (D) .13

**Sol.** [B]  $Y = \frac{FL}{A\Delta L}$   
 $\therefore \Delta L = \frac{FL}{YA} = \frac{1 \times 10 \times 0.9}{1.2 \times 10^{11} \times 10^{-6}} = .075 \times 10^{-3} \text{ m} = .075 \text{ mm}$

**Q.4** The ratio of diameters of two wires of same material is  $n : 1$ . The length of each wire is 4 m. On applying the same load, the increase in length of thin wire will be ( $n > 1$ ) -

- (A)  $n^2$  times    (B)  $n$  times  
 (C)  $2n$  times    (D)  $(2n + 1)$  times

**Sol.** [A]

$$Y = \frac{\frac{F}{a}}{\frac{\Delta\ell}{\ell}} = \frac{F\ell}{a\Delta\ell}, \quad Y = \frac{F\ell \times 4}{\pi D^2 \times \Delta\ell}$$

$$\text{or } \Delta\ell \propto \frac{1}{D^2} \quad \text{or } \frac{\Delta\ell_2}{\Delta\ell_1} = \frac{D_1^2}{D_2^2} = \frac{n^2}{1}$$

**Q.5** In order to twist one end of a wire, 2m long and 4 mm in diameter, through  $45^\circ$ , the torque required is  $(\eta = 5 \times 10^{10} \text{ Nm}^{-2})$

- (A) 0.49 Nm    (B) 3.49 Nm  
 (C) 49 Nm    (D)  $4.9 \times 10^{10} \text{ Nm}$

**Sol.** [A]

$$\tau = \frac{22 \times 5 \times 10^{10} (2 \times 10^{-3})^4 \times 22 \times 45}{7 \times 7 \times 180 \times 2 \times 2} \text{ Nm}$$

$$= 0.49 \text{ Nm}$$

**Q.6** Given the following values for an elastic material: Young's modulus =  $7 \times 10^{10} \text{ Nm}^{-2}$  and Bulk modulus =  $11 \times 10^{10} \text{ Nm}^{-2}$ . The Poisson's ratio of the material is -



- (A) 0.12 (B) 0.24  
(C) 0.31 (D) 0.39

Sol. [D]

$$K = \frac{Y}{3(1-2\sigma)} \text{ or } 11 \times 10^{10} = \frac{7 \times 10^{10}}{3(1-2\sigma)}$$

$$\text{or } \frac{7}{33} = 1-2\sigma$$

$$\text{or } 2\sigma = 1 - \frac{7}{33}, \sigma = \frac{26}{33} = 0.39.$$

**Q.7** A wire elongates by  $\ell$  mm when a load  $W$  is hanged from it. If the wire goes over a pulley and two weights  $W$  each are hung at the two ends, the elongation of the wire will be (in mm) –

- (A) 0 (B)  $\ell/2$   
(C)  $\ell$  (D)  $2\ell$  [C]

**Q.8** A rubber ball is taken to a 100 m deep lake and its volume changes by 0.1%. The bulk modulus of rubber is nearly –

- (A)  $1 \times 10^6 \text{ N/m}^2$  (B)  $1 \times 10^8 \text{ N/m}^2$   
(C)  $1 \times 10^7 \text{ N/m}^2$  (D)  $1 \times 10^9 \text{ N/m}^2$   
[D]

**Q.9** An aluminium and steel wire of same length and cross-section are attached end to end. The compound wire is hung from a rigid support and a load is suspended from the free end.  $Y$  of steel is  $(20/7)$  times of aluminium. The ratio of increase of length of steel wire to aluminium wire is –

- (A) 20 : 3 (B) 10 : 7  
(C) 7 : 20 (D) 1 : 7 [C]

**Q.10** A gas undergoes a process in which the pressure and volume are related by  $VP^n = \text{constant}$ . The bulk modulus of the gas is –

- (A)  $nP$  (B)  $P^{1/n}$  (C)  $P/n$  (D)  $P^n$

Sol. [C]

$$VP^n = (V + \Delta V)(P + \Delta P)^n$$

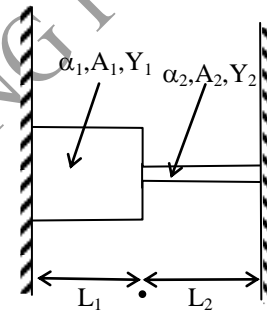
$$VP^n = VP^n \left(1 + \frac{\Delta V}{V}\right) \left(1 + n \frac{\Delta P}{P}\right)$$

$$\therefore \frac{\Delta V}{V} = -n \frac{\Delta P}{P}$$

$$K = -\frac{\Delta P}{\Delta V/V} = \frac{P}{n}$$

**Q.11** Two elastic rods are joined between fixed supports as shown in the figure. Condition for no change in the lengths of individual rods with the increase of temperature is –

( $\alpha_1, \alpha_2$  = linear expansion coefficient,  $A_1, A_2$  = Area of rods,  $Y_1, Y_2$  = young modulus)



- (A)  $\frac{A_1}{A_2} = \frac{\alpha_1 Y_1}{\alpha_2 Y_2}$  (B)  $\frac{A_1}{A_2} = \frac{L_1 \alpha_1 Y_1}{L_2 \alpha_2 Y_2}$   
(C)  $\frac{A_1}{A_2} = \frac{L_2 \alpha_2 Y_2}{L_1 \alpha_1 Y_1}$  (D)  $\frac{A_1}{A_2} = \frac{\alpha_2 Y_2}{\alpha_1 Y_1}$

Sol. [D]

Since tension in the two rods will be same, hence

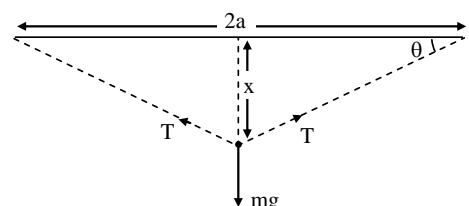
$$A_1 Y_1 \alpha_1 \Delta \theta = A_2 Y_2 \alpha_2 \Delta \theta$$

$$\Rightarrow A_1 Y_1 \alpha_1 = A_2 Y_2 \alpha_2$$

**Q.12** A wire of length '2m' is clamped horizontally between two fixed support. A mass  $m = 5 \text{ kg}$  is hanged from middle of wire. The vertical depression in wire in equilibrium is (young modulus of wire =  $2.4 \times 10^9 \text{ N/m}^2$ , cross-sectional area =  $1 \text{ cm}^2$ ) –

- (A) 4.68 cm (B) 1.52 cm  
(C) 1.12 cm (D) 0.58 cm [A]

Sol.



equation

$$2T \sin \theta = mg$$

$$\Rightarrow 2 \left( \frac{YA}{a} \right) x \sin \theta \cdot \sin \theta = mg$$

$$\Rightarrow \frac{2YA}{a} x \cdot \frac{x^2}{a^2} = mg$$

$$\Rightarrow x = \left\{ \frac{a^3 mg}{2YA} \right\}^{1/3}$$

$$= \left\{ \frac{1 \text{ m} \times 5 \text{ kg} \times 10 \text{ m/s}^2}{2 \times (2.4 \times 10^9 \text{ N/m}^2) \times 10^{-4} \text{ m}^2} \right\}^{1/3}$$

$$= 4.68 \text{ cm}$$

**Q.13** The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied -

- (A) length = 50 cm, diameter = 0.5 mm
- (B) length = 100 cm, diameter = 1 mm
- (C) length = 200 cm, diameter = 2 mm
- (D) length = 300 cm, diameter = 3 mm

[A]

**Sol.**  $Y = \frac{mg/A}{\ell/L} = \frac{mgL}{A\ell}$

$$\ell = \frac{mgL}{YA}$$

So  $\ell \propto \frac{L}{d^2}$  hence (A)

**Q.14** If the compressibility of water is  $\sigma$  per unit atmospheric pressure, then the decrease in volume (V) due to atmospheric pressure P will be -

- (A)  $\sigma P/V$
- (B)  $\sigma PV$
- (C)  $\sigma/PV$
- (D)  $\sigma V/P$

[B]

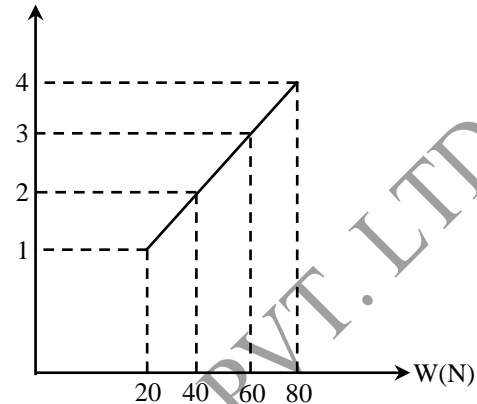
**Sol.**  $K = \frac{P}{\frac{\Delta V}{V}}$  or  $\frac{1}{K} = \frac{\Delta V/V}{P}$

or  $\sigma = \frac{\Delta V}{PV}$  or  $\Delta V = \sigma PV$ .

**Q.15** The adjacent graph shows the extension ( $\Delta l$ ) of a wire of length  $\ell$  m suspended from the top of a roof at one end and with a load W connected to

the other end. If the cross-sectional area of the wire is  $10^{-6} \text{ m}^2$ , calculate the Young's modulus of the material of the wire -

$$\Delta \ell (\times 10^{-4} \text{ m})$$



- (A)  $2 \times 10^{11} \text{ N/m}^2$
- (B)  $2 \times 10^{-11} \text{ N/m}^2$
- (C)  $3 \times 10^{12} \text{ N/m}^2$
- (D)  $2 \times 10^{13} \text{ N/m}^2$

[A]

**Sol.**  $\Delta \ell = \left( \frac{\ell}{YA} \right) \cdot W$

i.e., graph is a straight line passing through origin

(as shown in question also), the slope of which

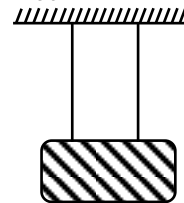
is  $\frac{\ell}{YA}$ .

$$\therefore \text{Slope} = \left( \frac{\ell}{YA} \right)$$

$$\therefore Y = \left( \frac{\ell}{A} \right) \left( \frac{1}{\text{slope}} \right)$$

$$= \left( \frac{1.0}{10^{-6}} \right) \frac{(80-20)}{(4-1) \times 10^{-4}} = 2.0 \times 10^{11} \text{ N/m}^2.$$

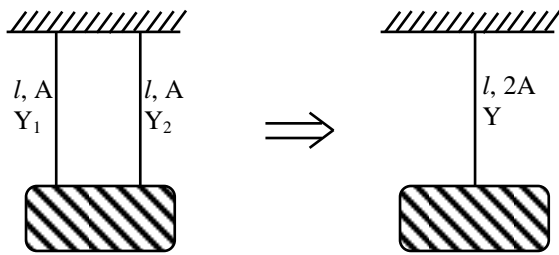
**Q.16** Two wires of equal length and cross-section are suspended as shown. Their Young's moduli are  $Y_1$  and  $Y_2$  respectively. The equivalent Young's modulus will be -



- (A)  $Y_1 + Y_2$
- (B)  $\frac{Y_1 + Y_2}{2}$
- (C)  $\frac{Y_1 Y_2}{Y_1 + Y_2}$
- (D)  $\sqrt{Y_1 Y_2}$

[B]

**Sol.**



Equivalent spring constant of a wire is given by

$$K = \frac{YA}{l}$$

$$K_{eq} = K_1 + K_2$$

$$\text{or } \frac{Y(2A)}{l} = \frac{Y_1 A}{l} + \frac{Y_2 A}{l}$$

$$\text{or } Y = \frac{Y_1 + Y_2}{2}$$

**Q.17** Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area  $A$  and wire 2 has cross-sectional area  $3A$ . if the length of wire 1 increases by  $\Delta x$  on applying force  $F$ , how much force is needed to stretch wire 2 by the same amount? [AIEEE-2009]

- (A)  $F$  (B)  $4F$   
(C)  $6F$  (D)  $9F$  [D]

**Sol.**  $\therefore \Delta x = \frac{FL}{A \cdot Y}$   
 $\therefore F = \frac{Y \cdot A \cdot \Delta x}{L} \dots\dots\dots(1)$

Volume =  $A \cdot L = A' \cdot L' = \text{constant}$   
 $\Rightarrow AL = 3A'L'$   
 $\Rightarrow L' = L/3 \dots\dots\dots(2)$

From equation (1)

$$\frac{F'}{F} = \frac{A' \cdot L}{A \cdot L'}$$

$$= 3 \times 3$$

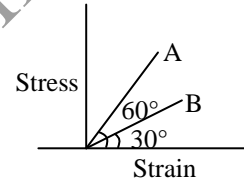
$$\Rightarrow F' = 9F$$

So option (4) is correct.

**Q.18** The rubber cord catapult has a cross-section area  $1 \text{ mm}^2$  and total unstretched length  $10 \text{ cm}$ . It is stretched to  $12 \text{ cm}$  and then released to project a stone of mass  $5 \text{ gm}$ . Taking Young's modulus  $Y$  of rubber as  $5 \times 10^8 \text{ N/m}^2$ , the velocity of projection will be -  
(A)  $20 \text{ cm/s}$  (B)  $20 \text{ m/s}$   
(C)  $2 \text{ m/s}$  (D) none of these [B]

**Sol.** P.E. =  $\frac{Y}{2} (\text{strain})^2 (AL) = \text{K.E.} = \frac{1}{2} mv^2$   
 $v = \text{strain} \sqrt{\frac{Y}{m} AL}$   
 $= \frac{2}{10} \sqrt{\frac{5 \times 10^8}{5 \times 10^{-3}} \times 10^{-6} \times 0.1}$   
 $= 20 \text{ m/s}$

**Q.19** The stress versus strain graphs for wires of two materials A and B as shown is the figure. If  $Y_A$  and  $Y_B$  are the young's modulus of the materials, then-



- (A)  $Y_B = 2Y_A$  (B)  $Y_A = Y_B$   
(C)  $Y_B = 3Y_A$  (D)  $Y_A = 3Y_B$  [D]

**Sol.**  $\frac{Y_A}{Y_B} = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3$

so  $Y_A = 3Y_B$

**Q.20** (a) Glass is more elastic than rubber  
(b) Rubber is more elastic than glass  
(c) Steel is more elastic than rubber  
(d) Rubber is more elastic than steel

For the above statements-

- (A) (a) and (b) are correct  
(B) (a) and (c) are correct  
(C) (b) and (c) are correct  
(D) (b) and (d) are correct [B]

**Q.21** Two similar balls, one of which is made of ivory while the other, of clay, are dropped from the same height, then-

- (A) the ivory ball will bounce to a greater height  
(B) the clay ball will bounce to a greater height  
(C) both the balls will bounce to the same height  
(D) the ivory ball will not at all bounce [A]

- Q.22** What is the Young's modulus of elasticity for a perfectly rigid body ?  
 (A) infinity (B) zero  
 (C) 1 (D) - 1 [A]
- Sol.** Since strain is zero therefore Y is infinite.
- Q.23** The longitudinal extension of any elastic material is very small. In order to have an appreciable change, the material must be in the form of -  
 (A) thin block of any cross section  
 (B) thick block of any cross section  
 (C) long thin wire  
 (D) short thin wire [C]
- Q.24** The modulus of elasticity of a material does not depend upon—  
 (A) shape (B) temperature  
 (C) nature of material (D) impurities mixed [A]
- Q.25** A steel wire is stretched by 1 kg. wt. If the radius of the wire is doubled, its Young's modulus will—  
 (A) remain unchanged  
 (B) become half  
 (C) become double  
 (D) become four times [A]
- Q.26** On withdrawing the applied force on some objects, the deformity caused gradually diminishes with time. This is called—  
 (A) elastic fatigue  
 (B) elastic limit  
 (C) coefficient of elasticity  
 (D) elastic after effect [A]
- Q.27** On stretching some substances, permanent elongation is caused, because—  
 (A) they are perfectly elastic  
 (B) they are perfectly plastic  
 (C) more stress acts on them  
 (D) their strain is infinite [B]
- Q.28** Out of the following whose elasticity is independent of temperature—  
 (A) steel (B) copper  
 (C) invar steel (D) glass [C]
- Q.29** A cable that can support a load W is cut into two equal parts. The maximum load that can be supported by either part is—  
 (A)  $\frac{W}{4}$  (B)  $\frac{W}{2}$   
 (C) W (D) 2 W [C]
- Q.30** On withdrawing the external applied force on bodies within the elastic limit, the body—  
 (A) regains its previous state very quickly  
 (B) regains its previous state after some time  
 (C) regain its previous state after a very long time  
 (D) does not regain its previous state [B]
- Q.31** Elasticity is the property which is caused by—  
 (A) the applied deforming forces  
 (B) gravitational force  
 (C) nuclear forces  
 (D) inter-molecular forces [D]
- Q.32** The effect of temperature on the value of Young's modulus of elasticity for various substances in general is—  
 (A) it increases with increase in temperature  
 (B) remains constant  
 (C) decrease with rise in temperature  
 (D) sometimes increases and sometimes decreases with temperature [C]
- Q.33** The number of independent elastic constants of a solid is -  
 (A) 1 (B) 2  
 (C) 3 (D) 4 [B]
- Q.34** The ratio of coefficient of isothermal and adiabatic elasticities of a gas is -  
 (A)  $\gamma$  (B)  $\gamma^2$   
 (C)  $1/\gamma$  (D)  $1/\gamma^2$  [C]
- Q.35** The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied—  
 (A) length 50 cm and diameter 0.5 mm  
 (B) length 100 cm and diameter 1 mm

- (C) length 100 cm and diameter 2 mm  
 (D) length 300 cm and diameter 3 mm [A]

**Q.36** An iron rod of length  $\ell$  and of cross-section area  $A$  is heated from  $0^\circ\text{C}$  to  $100^\circ\text{C}$ . If the rod neither expands nor bends, then the developed  $F$  is proportional to—

- (A)  $\ell$  (B)  $\ell^0$   
 (C)  $\ell^{-1}$  (D)  $A^{-1}$  [B]

**Q.37** When a wire is stretched, an amount of work is done. What is the amount of work done in stretching a wire through 0.1 mm, if its length is 2m and area of cross-section,  $10^{-6}\text{m}^2$  ( $Y = 2 \times 10^{11} \text{ N/m}^2$ )

- (A)  $5 \times 10^{-1} \text{ J}$  (B)  $5 \times 10^{-2} \text{ J}$   
 (C)  $5 \times 10^{-3} \text{ J}$  (D)  $5 \times 10^{-4} \text{ J}$  [C]

**Q.38** Two wires of the same radius and material and having lengths in the ratio 8.9 : 7.6 are stretched by the same force. The strains produced in the two cases will be in the ratio -

- (A) 1 : 1 (B) 1 : 7.6  
 (C) 8.9 : 1 (D) 1 : 3.2 [A]

**Sol.**  $Y = \frac{\text{stress}}{\text{strain}}$ ;  $\text{Strain} = \frac{\text{stress}}{Y} = \frac{F/\pi r^2}{Y}$ .

**Q.39** An iron bar of length  $\ell$  cm and cross section  $A \text{ cm}^2$  is pulled by a force of  $F$  dynes from ends so as to produce an elongation  $\ell$  cm. Which of the following statement is correct—

- (A) elongation is inversely proportional to length  
 (B) elongation is directly proportional to cross section  $A$   
 (C) elongation is inversely proportional to  $A$   
 (D) elongation is directly proportional to Young's modulus [C]

**Q.40** Bulk modulus of water is  $2 \times 10^9 \text{ Nm}^{-2}$ . The change in pressure required to increase the density of water by 0.1% is -

- (A)  $2 \times 10^9 \text{ Nm}^{-2}$  (B)  $2 \times 10^8 \text{ Nm}^{-2}$   
 (C)  $2 \times 10^6 \text{ Nm}^{-2}$  (D)  $2 \times 10^4 \text{ Nm}^{-2}$  [C]

**Sol.** The density would increase by 0.1% if the volume decrease by 0.1%,

$$K = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\Rightarrow \Delta P = K \frac{\Delta V}{V} = 2 \times 10^9 \times \frac{0.1}{100} = 2 \times 10^6 \text{ Nm}^{-2}$$

**Q.41** The ' $\sigma$ ' of a material is 0.20. If a longitudinal strain of  $4.0 \times 10^{-3}$  is caused, by what percentage will the volume change—

- (A) 0.48 % (B) 0.32 %  
 (C) 0.24 % (D) 0.50 % [C]

**Q.42** A cylinder is of length  $\ell$  and diameter  $d$ . On stretching the cylinder, an increment  $\Delta\ell$  in length and decrease  $\Delta d$  in diameter are caused. The Poisson ratio is—

$$(A) \sigma = -\frac{\Delta\ell}{\ell} \times \frac{d}{\Delta d} \quad (B) \sigma = -\frac{\ell}{d} \times \frac{\Delta d}{\Delta\ell}$$

$$(C) \sigma = -\frac{\Delta\ell}{\ell} \times \frac{\Delta d}{d} \quad (D) \sigma = -\frac{\ell}{\Delta\ell} \times \frac{d}{\Delta d}$$

[B]

**Q.43** Steel is more elastic than rubber because for a given load the strain produced in steel as compared to that produced in rubber is—

- (A) more  
 (B) less  
 (C) equal  
 (D) nothing can be said [B]

**Q.44** In a wire stretched by hanging a weight from its end, the elastic potential energy per unit volume in terms of longitudinal strain  $\sigma$  and modulus of elasticity  $Y$  is -

$$(A) \frac{Y\sigma^2}{2} \quad (B) \frac{Y\sigma}{2}$$

$$(C) \frac{2Y\sigma^2}{2} \quad (D) \frac{Y^2\sigma}{2}$$

**Sol.** [A]

$$\text{Energy density} = \frac{1}{2} \times \text{stress} \times \text{strain},$$

$$Y = \frac{\text{stress}}{\sigma} \quad \text{or} \quad \text{stress} = Y\sigma,$$

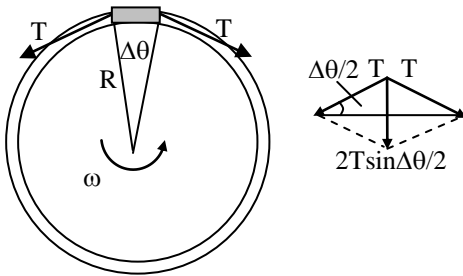
$$\therefore \text{Energy density} = \frac{1}{2} Y\sigma \times \sigma = \frac{Y\sigma^2}{2}$$

- Q.45** The formula for compressibility of a gas is—  
 (A)  $PdV/V$  (B)  $(1/P) dP/dV$   
 (C)  $V \cdot \frac{dP}{dV}$  (D)  $\frac{1}{V} \cdot \frac{dV}{dP}$  [D]
- Q.46** The potential energy of a metallic rod when it is compressed—  
 (A) increases (B) remains constant  
 (C) decreases (D) becomes infinite [C]
- Q.47** A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 100 atmospheres. The bulk modulus of its material in dynes/cm<sup>2</sup> is—  
 (A)  $10 \times 10^{12}$  (B)  $100 \times 10^{12}$   
 (C)  $1 \times 10^{12}$  (D)  $2.0 \times 10^{11}$  [C]
- Q.48** When 1 kg wt. is suspended from a wire, the increment produced is 2 mm, What will be the increment in lengths when 4 kg wt. is suspended from it—  
 (A) 4 mm (B) 8 mm  
 (C) 0.5 mm (D) 10 mm [B]
- Q.49** On increasing temperature, the elasticity of a material—  
 (A) decreases  
 (B) increases  
 (C) sometimes increases and sometimes decreases  
 (D) remains same [A]
- Q.50** Two wires, one of copper and the other of steel, are of same length and cross section. They are welded together to form a long wire. On suspending a weight at its one end, increment in length is found to be 3 cms. If Young's modulus of steel is double that of copper, the increment in steel wire will be—  
 (A) 1 cm (B) 2 cm  
 (C) 1.5 cm (D) 2.5 cm [A]

# PHYSICS

**Q.1** A thin ring of radius  $R$  is made of material of density  $\rho$  and Young's modulus  $E$ . It is spun in its own plane, about an axis through its centre, with angular velocity  $\omega$ . Determine the amount (assumed small) by which its circumference increase.

**Sol.** Let the tension in the ring be  $T$ . Its resolved component acting along the radius towards the centre of rotation is  $2T \sin (\Delta\theta/2) \approx T\Delta\theta$  and this must balance the centripetal force of  $R\Delta\theta A\rho R\omega^2$  (see figure).



It follows that the longitudinal stress in the ring,  $T/A$ , is  $RR^2\omega^2$ ; the strain  $e$  is  $E^{-1}$  times this. Finally, the increase in circumference, given by  $2\pi R\epsilon$ , is  $2\pi\rho R^3\omega^2/E$ .

**Q.2** Two identical springs, one of steel, the other of copper, are stretched by an identical amount. On which operation must more work be expended ?

**Sol.** The modulus of normal elasticity (Young's modulus) is greater for steel than for copper. Therefore if the springs are of equal dimensions and are to be stretched by the same amount, a greater force is necessary for the steel spring than for the copper one. So the first spring requires that more work should be done.

**Q.3** Two identical springs, one of steel, the other of copper, are stretched with identical forces. On which operation must more work be expended ?

**Sol.** (See previous problem). If the process of stretching is carried out with equal forces for both springs, the steel spring will be stretched less than the copper one. Therefore more work will be done this time on stretching the copper spring.

**Q.4** A load of mass  $m$  is suspended from a thread of length  $\ell$ . Find the least height to which the load must be raised so that it should break the thread in falling, assuming that the least load which would break the thread when simply suspended from it is  $M$  and that this load would stretch the thread by 1 percent of its length at the moment of breaking. Assume that Hooke's law applies to the thread right up to breaking-point.

**Sol.** A load  $m$ , falling from a height  $h$ , acquires a kinetic energy equal to the change in potential energy, i.e.  $mgh$ . This kinetic energy must be turned into energy of elastic deformation of the thread, i.e. where Hooke's law applies, it must equal  $kx^2/2$ , where  $x$  is the greatest amount of stretch in the thread (at the moment of breaking) and  $k$  is the coefficient of elasticity. In the problem it is given that  $x = 0.01 \ell$  and  $kx = Mg$ . Substituting these relationships in the equation

$$\frac{kx^2}{2} = mgh,$$

We shall get  $h = \frac{0.01M\ell}{2m}$ . **Ans.**

**Q.5** A copper ring with a radius of  $r = 100$  cm and a cross-sectional area of  $A = 4$  mm<sup>2</sup> is fitted onto a steel rod with a radius  $R = 100.125$  cm. With what force  $F$  will the ring be expanded if the modulus of elasticity of copper  $E = 12,000$  kgf/mm<sup>2</sup>? Disregard the deformation of the rod.

**Sol.**  $F = \frac{AE(R-r)}{r} = 60$  kgf.

**Q.6** What work can be performed by a steel rod with a length  $l$  and a cross-sectional area  $A$  when heated by  $t$  degrees?

**Sol.** When the rod with fastened ends is heated by  $t$  degrees, it develops an elastic force  $F$  equal, according to Hooke's law, to

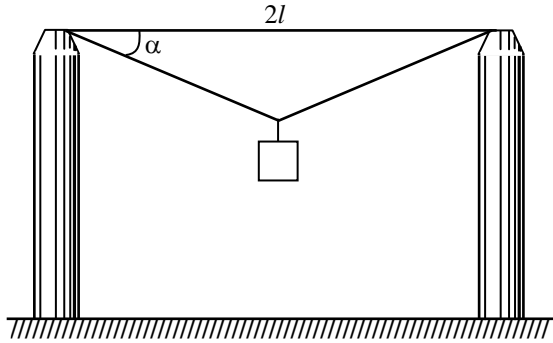
$$F = \frac{AE\Delta l}{l} = AE\alpha t$$

where  $E$  is the modulus of elasticity of steel and  $\alpha$  is its coefficient of thermal expansion.

If one of the rod ends is gradually released, the length of the rod will increase by  $\Delta l = l\alpha t$ . The force will decrease linearly from  $F$  to zero and its average magnitude will be  $F/2$ .

The sought work  $W = \frac{F}{2} \Delta l = \frac{1}{2} AE l \alpha^2 t^2$ .

**Q.7** A wire with a length of  $2l$  is stretched between two posts. A lantern with a mass  $M$  is suspended exactly from the middle of the wire. The cross-sectional area of the wire is  $A$  and its modulus of elasticity  $E$ . Determine the angle  $\alpha$  of sagging of the wire, considering it to be small (**Fig.**).



**Sol.** The tension of the wire  $T = \frac{Mg}{2\sin\alpha}$ . It follows

from Hooke's law that  $T = \frac{\Delta l}{2l} EA$ .

Since  $\Delta l = 2\left(\frac{l}{\cos\alpha} - l\right)$ , then  $T = \frac{1 - \cos\alpha}{\cos\alpha} AE$

$= \frac{Mg}{2\sin\alpha}$ . At small angles,  $\sin\alpha \cong \alpha$ , and  $\cos\alpha$

$$= 1 - 2\sin^2 \frac{\alpha}{2} \cong 1 - \frac{\alpha^2}{2}$$

Bearing this in mind, we obtain

$$\alpha = \sqrt[3]{\frac{Mg}{AE}}$$

**Q.8** A steel rod with a cross section  $A = 1 \text{ cm}^2$  is tightly fitted between two stationary absolutely rigid walls. What force  $F$  will the rod act with on the walls if it is heated by  $\Delta t = 5^\circ \text{ C}$ ?

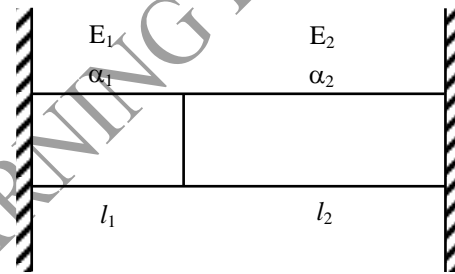
The coefficient of linear thermal expansion of steel  $\alpha = 1.1 \times 10^{-5} \text{ deg}^{-1}$  and its modulus of elasticity  $E = 20,000 \text{ kgf/mm}^2$ .

**Sol.** The rod heated by  $\Delta t$  would extend by  $\Delta l = l_0 \alpha \Delta t$  in a free state, where  $l_0$  is the initial length of the rod. To fit the heated rod between the walls, it should be compressed by  $\Delta l$ . In conformity with Hooke's law,

$$\Delta l = \frac{lF}{EA}$$

Therefore,  $F = EA\alpha\Delta t = 110 \text{ kgf}$ .

**Q.9** Two rods made of different materials are placed between massive walls (**Fig.**). The cross section of the rods is  $A$  and their respective lengths  $l_1$  and  $l_2$ . The rods are heated by  $t$  degrees.



Find the force with which the rods act on each other if their coefficients of linear thermal expansion  $\alpha_1$  and  $\alpha_2$  and moduli of elasticity of their materials  $E_1$  and  $E_2$  are known. Disregard the deformation of the walls.

**Sol.** When the rods are heated in a free state, their total length will increase by  $\Delta l = \Delta l_1 + \Delta l_2 = (\alpha_1 l_1 + \alpha_2 l_2)t$ .

Compression by the same amount  $\Delta l$  will reduce the lengths of the rods by  $\Delta l_1'$  and  $\Delta l_2'$ , where

$\Delta l_1' + \Delta l_2' = \Delta l$ . This requires the force

$$F = \frac{E_1 A}{l_1} \Delta l_1' = \frac{E_2 A \Delta l_2'}{l_2}$$

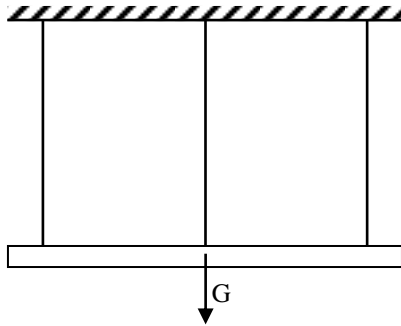
Upon solving this system of equations, we find that

$$F = \frac{\alpha_1 l_1 F \alpha_2 l_2}{\frac{l_1}{E_1} + \frac{l_2}{E_2}} A l$$

The rods will act upon each other with this force.



**Q.10** A homogeneous block with a mass  $m = 100$  kg hangs on three vertical wires of equal length arranged symmetrically (**Fig.**). Find the tension of the wires if the middle wire is of steel and the other two are of copper. All the wires have the same cross section. Consider the modulus of elasticity of steel to be double that of copper.



**Sol.** It is obvious from considerations of symmetry that the wires will elongate equally. Let us denote this elongation by  $\Delta l$ . On the basis of Hooke's law, the tension of a steel wire  $F_s = \frac{\Delta l}{l} AE_s$  and of a copper one  $F_c = \frac{\Delta l}{l} AE_c$ . It follows that the ratio between the tensions is equal to the ratio between the respective Young's moduli.

$$\frac{F_c}{F_s} = \frac{E_c}{E_s} = \frac{1}{2}$$

In equilibrium  $2F_c + F_s = mg$ .

Therefore,  $F_c = \frac{mg}{4} = 25$  kgf and  $F_s = 2F_c = 50$  kgf.

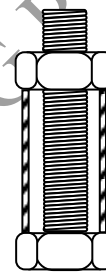
**Q.11** A reinforced-concrete column is subjected to compression by a certain load. Assuming that the modulus of elasticity of concrete  $E_c$  is one-tenth of that of iron  $E_i$ , and the cross-sectional area of the iron is one-twentieth of that of concrete, find the portion of the load acting on the concrete.

**Sol.** On the basis of Hooke's law,  
 $F_c = \frac{\Delta l}{l} A_c E_c$  and  $F_i = \frac{\Delta l}{l} A_i E_i$

It follows that  $\frac{F_c}{F_i} = 2$ .

Thus, two-thirds of the load are resisted by the concrete and one-third by the iron.

**Q.12** A steel bolt is inserted into a copper tube as shown in **Fig.** Find the forces induced in the bolt and in the tube when the nut is turned through one revolution if the length of the tube is  $l$ , the pitch of the bolt thread is  $h$  and the cross-sectional areas of the bolt and the tube are  $A_b$  and  $A_t$ , respectively.



**Sol.** The compressive force  $F$  shortens the tube by  $\frac{Fl}{A_c E_c}$  and the tensile force  $F$  extends the bolt by  $\frac{Fl}{A_s E_s}$ .

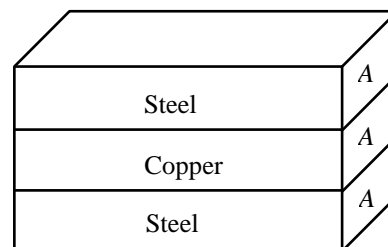
The sum  $\frac{Fl}{A_s E_s} + \frac{Fl}{A_c E_c}$  is equal to the motion to the nut along the bolt:

$$\frac{Fl}{A_s E_s} + \frac{Fl}{A_c E_c} = h$$

Hence,

$$F = \frac{h}{l} \frac{A_s E_s A_c E_c}{A_s E_s + A_c E_c}$$

**Q.13** A copper plate is soldered to two steel plates as shown in **Fig.** What tensions will arise in the plates if the temperature is increased by  $t^\circ$  C? All three plates have the same cross sections.



**Sol.** Since the coefficient of linear thermal expansion of copper  $\alpha_c$  is greater than that of steel  $\alpha_s$ , the increase in temperature will lead to compression of the copper plate and tension of the steel ones. In view of symmetry, the relative elongations of all the three plates are the same, Denoting the compressive force acting on the copper plate from the sides of the steel plates by  $F$ , we shall have for the relative elongation of the copper

$$\text{plate: } \frac{\Delta l}{l} = \alpha_{ct} - \frac{F}{AE_c}.$$

Either steel plate is subjected to the tensile force  $F/2$  from the side of the copper one. Upon equating the relative elongation of the plates, we obtain:

$$\alpha_{ct} - \frac{F}{AE_c} = \alpha_{st} + \frac{F}{2AE_s}$$

Hence

$$F = \frac{2AE_c E_s (\alpha_c - \alpha_s) t}{2E_s + E_c}$$

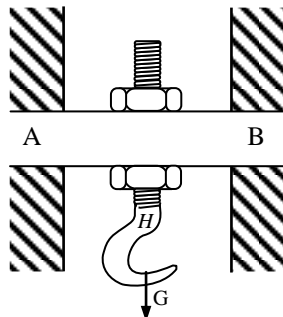
**Q.14** Find the maximum permissible linear velocity of a rotating thin lead ring if the ultimate strength of lead  $\sigma_u = 200 \text{ kgf/cm}^2$  and its density  $\rho = 11.3 \text{ g/cm}^3$ .

**Sol.** When the ring rotates, the tension  $T = \frac{mv^2}{2\pi r}$  appears in it. For a thin ring  $m = 2\pi r A \rho$ , where  $A$  is the cross section of the ring. Therefore,

$$\frac{l}{A} = \rho v^2.$$

Hence, the maximum velocity  $v = \sqrt{\frac{\sigma_u}{\rho}} \cong 41 \text{ m/s}$ .

**Q.15** An iron block AB has both ends fixed. Hook H is fastened with two nuts in a hole in the middle of the block (**Fig.**). The block is clamped by the nuts with a force  $F_0$ .



What forces will act on the upper and lower nuts from the side of the block if the hook carries a load whose weight can change from zero to  $G = 2F_0$ ? Disregard sagging of the block and the weight of the hook.

**Sol.** Initially, an elastic force  $F_0$  acts on each nut from the side of the extended bolt.

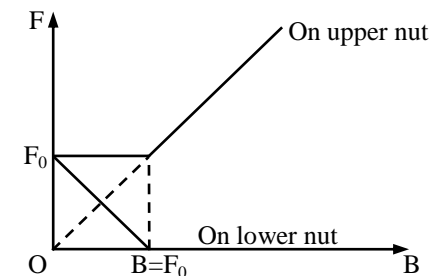
The load  $G \leq F_0$  cannot increase the length of the part of the bolt between the nuts and change its tension. For this reason the force acting on the upper nut from the side of the block will not change as long as  $G \leq F_0$ .

The lower nut is acted upon by the force  $F_0$  from the side of the top part of the bolt and by the force  $G$  from the bottom part. Since the nut is in equilibrium, the force exerted on it from the block is  $F = F_0 - G$ . Thus the action of the load  $G \leq F_0$  consists only in reducing the pressure of the lower nut on the block.

When  $G > F_0$ , the length of the bolt will increase and the force acting on the lower nut from the side of the block will disappear. The upper nut will be acted on by the force  $G$ .

The relation between the forces acting on the nuts and the weight of the load  $G$  is shown in

**Fig.**

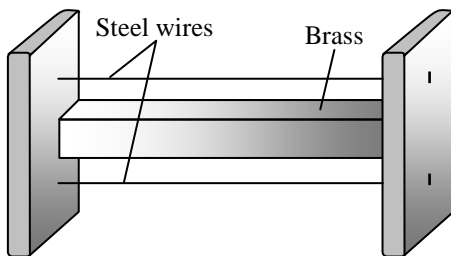


**Q.16** (a) Equation  $(F/A = -Y \alpha \Delta T)$  gives the stress required to keep the length of a rod constant as its temperature changes. Show that if the length is permitted to change by an amount  $\Delta L$  when its temperature changes by  $\Delta T$ , the stress is equal to

$$\frac{F}{A} = Y \left( \frac{\Delta L}{L_0} - \alpha \Delta T \right)$$

where  $F$  is the tension on the rod,  $L_0$  is the original length of the rod.  $A$  its cross-sectional area,  $\alpha$  its coefficient of linear expansion, and  $Y$  its Young's modulus.

(b) A heavy brass bar has projections at its ends, as in **Fig.** Two fine steel wires, fastened between the projections, are just taut (zero tension) when the whole system is at  $20^\circ\text{C}$ . What is the tensile stress in the steel wires when the temperature of the system is raised to  $140^\circ\text{C}$ ? Make any simplifying assumptions you think are justified, but state what they are.



**Sol.** (a) The change in length is due to the tension and heating

$$\frac{\Delta L}{L_0} = \frac{F}{AY} + \alpha \Delta T. \text{ Solving for } F/A, \frac{F}{A} =$$

$$Y \left( \frac{\Delta L}{L_0} - \alpha \Delta T \right).$$

(b) The brass bar is given as "heavy" and the wires are given as "fine," so it may be assumed that the stress in the bar due to the fine wires does not affect the amount by which the bar expands due to the temperature increase. This means that in the equation preceding Eq. ( $F/A = -Y \alpha \Delta T$ ),  $\Delta L$  is not zero, but is the amount  $\alpha_{\text{brass}} L_0 \Delta T$  that the brass expands, and so

$$\begin{aligned} \frac{F}{A} &= Y_{\text{steel}} (\alpha_{\text{brass}} - \alpha_{\text{steel}}) \Delta T \\ &= 20 \times 10^{10} \text{ (Pa)} (2.0 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1} - 1.2 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1}) (120^\circ\text{C}) \\ &= 1.92 \times 10^8 \text{ Pa.} \end{aligned}$$

**Q.17** A steel rod with a length of 0.350 m and an aluminum rod with a length of 0.250 m, both with the same diameter, are placed end-to-end between rigid supports with no initial stress in the rods. The temperature of the rods is now raised by  $60.0^\circ\text{C}$ . What is the stress in each rod? (Hint: The length of the combined rod remains the same, but the lengths of the individual rods do not. See Problem 9.)

**Sol.** In deriving Eq. ( $F/A = -Y \alpha \Delta T$ ), it was assumed that  $\Delta L = 0$ ; if this is not the case when there are both thermal and tensile stresses, becomes

$$\Delta L = L_0 \left( \alpha \Delta T + \frac{F}{AY} \right).$$

For the situation in this problem, there are two length changes which must sum to zero, and so Eq. ( $F/A = -Y \alpha \Delta T$ ) may be extended to two materials **a** and **b** in the form

$$L_{0a} \left( \alpha_a \Delta T + \frac{F}{AY_a} \right) + L_{0b} \left( \alpha_b \Delta T + \frac{F}{AY_b} \right) = 0.$$

Note that in the above,  $\Delta T$ ,  $F$  and  $A$  are the same for the two rods. Solving for the stress  $F/A$ .

$$\begin{aligned} \frac{F}{A} &= - \frac{\alpha_a L_{0a} + \alpha_b L_{0b}}{((L_{0a}/Y_a) + (L_{0b}/Y_b))} \Delta T \\ &= \frac{(1.2 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1})(0.350 \text{ m}) + (2.4 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1})(0.250 \text{ m})}{((0.350 \text{ m})/20 \times 10^{10} \text{ Pa}) + (0.250 \text{ m}/7 \times 10^{10} \text{ Pa})} (60.0^\circ\text{C}) \end{aligned}$$

$= -1.2 \times 10^8 \text{ Pa}$   
to two figures.

**Q.18** A liquid is enclosed in a metal cylinder that is provided with a piston of the same metal. The system is originally at a pressure of 1.00 atm ( $1.013 \times 10^5 \text{ Pa}$ ) and at a temperature of  $30.0^\circ\text{C}$ . The piston is forced down until the pressure on the liquid is increased by 50.0 atm, and then clamped in this position. Find the new

temperature at which the pressure of the liquid is again 1.00 atm. Assume that the cylinder is sufficiently strong so that its volume is not altered by changes in pressure, but only by changes in temperature. Use the result derived in Problem 17.92 (Hint: See section 11.4).

Compressibility of liquid:  $k = 8.50 \times 10^{-10} \text{ Pa}^{-1}$

Coefficient of volume expansion of liquid:  $\beta = 4.80 \times 10^{-4} \text{ K}^{-1}$

Coefficient of volume expansion of metal:  $\beta = 3.90 \times 10^{-5} \text{ K}^{-1}$ .

**Sol.** As the liquid is compressed, its volume changes by an amount  $\Delta V = -\Delta p k V_0$ . When cooled, the difference between the decrease in volume of the liquid and the decrease in volume of the metal must be this change in volume, or  $(\alpha_l - \alpha_m) V_0 \Delta T = \Delta V$ . Setting the expressions for  $\Delta V$  equal and solving for  $\Delta T$  gives

$$\begin{aligned} \Delta T &= \frac{\Delta p k}{\alpha_m - \alpha_l} = \frac{(5.065 \times 10^6 \text{ Pa})(8.50 \times 10^{-10} \text{ Pa}^{-1})}{(3.90 \times 10^{-5} \text{ K}^{-1} - 4.8 \times 10^{-4} \text{ K}^{-1})} \\ &= -9.76 \text{ C}^\circ, \end{aligned}$$

So the temperature is  $20.2^\circ\text{C}$ .

**Q.19** A vertical metal cylinder of radius 2 cm and length 2 m is fixed at the lower end and a load of 100 kg is put on it. Find (a) the stress (b) the strain and (c) the compression of the cylinder. Young's modulus of the metal =  $2 \times 10^{11} \text{ N/m}^2$ .

**Sol.** (a)  $7.96 \times 10^5 \text{ N/m}^2$  (b)  $4 \times 10^{-6}$  (c)  $8 \times 10^{-6} \text{ m}$

**Q.20** Two persons pull a rope towards themselves. Each person exerts a force of 100 N on the rope. Find the Young's modulus of the material of the rope if it extends in length by 1 cm. Original length of the rope = 2 m and the area of cross-section =  $2 \text{ cm}^2$ .

**Sol.**  $1 \times 10^8 \text{ N/m}^2$