PHYSICS

The following questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true.

Q. 1 **Assertion :** In transverse wave motion in a string both potential energy & kinetic energy transported through string have maximum value at the equilibrium position of particle.
Reason : Particle velocity is maximum at the equilibrium position of particle **[A]**

Sol. Rate of kinetic energy & potential energy transport
is given as –

$$\frac{dv}{dt} = \frac{dk}{dt} = \frac{1}{2}\mu v\omega^2 A^2 \cos 2(kx - \omega t)$$

Q.2 **Assertion :** In a small segment of string carrying sinusoidal wave, total energy is conserved.
Reason : Every small part moves in SHM and in SHM total energy is conserved. **[D]**

Sol. Every small segment is acted upon by forces from both sides of it hence energy in not conserved, rather it is transmitted by the element.

Q.3 **Assertion :** In a standing wave formed in a stretched wire the energy of each element of wire remains constant.
Reason : The net energy transfer in a standing wave is zero. **[D]**

Sol. y = 2A sin k x cos wt

Power = Energy /sec = $\frac{T\partial y}{\partial x}.\frac{\partial y}{\partial t}$

Power = T 2Ak cos k x cos ωt.
2A ωsin kx sin ωt

= $4A^2$ kT ω cos kx cos ωt sin kx sin ω t
energy is variable with time.

Q.4 **Assertion** : Phase difference $= \frac{2\pi}{\lambda} \times$ path difference.
Reason : Phase difference corresponding to a path difference of λ is π. **[C]**

Q.5 **Assertion** : Acceleration of a progressive wave is constant.
Reason : A progressive wave travels through a medium with constant velocity. **[D]**

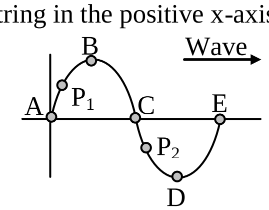
Q.6 **Assertion** : An observer standing on the bank of sea finds that 54 waves are reaching the bank per minute. If the wavelength of waves is 10 m, then their speed will be 9 m/s.
Reason : This follows from v = nλ. **[A]**

Q.7 **Assertion** : Waves of length 25 cm are propagating on a string. The phase difference between 2 points separated by 10 cm is 0.8 π radian.
Reason : $\phi = \frac{2\lambda}{\lambda} x = \frac{2\pi}{25} \times 10 = 0.8\,\pi$ radian **[A]**

Q.8 **Assertion** : The equation of a stationary wave is
$y = 20 \sin \frac{\pi x}{4} \cos \omega t$
The distance between two consecutive antinodes will be 4 m.
Reason : The data is insufficient. **[C]**

Q.9 **Statement I :** A wave can represented by function y = f(kx ± ωt).
Statement II : Because it satisfy the differential equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\left(\frac{\partial^2 y}{\partial t^2}\right)$ where $v = \frac{\omega}{K}$.
[A]

Q.10 **Statement I :** The transverse wave is travelling along a string in the positive x-axis is shown.



Points (A & P_1) moving downward and point (C & P_2) moving upward.
Statement II : In a wave propagating in positive x direction, the points with +ve slope move downward and vice versa. **[A]**

Q.11 **Statement I :** Sonometer is used to determine frequency of unknown T.F.
Statement II : In sonometer riders used as indicator. **[B]**

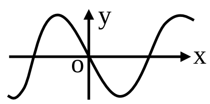
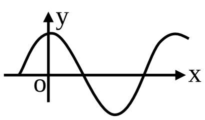
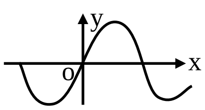
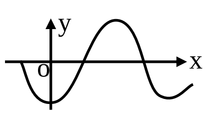
Q.12 **Statement I :** For same length but different gas COP fundamental frequency are different.
Statement II : For same length but different gas COP fundamental wavelength remains same.
[A]

Q.13 **Statement I :** Mechanical transverse waves cannot be generated in gaseous medium.
Statement II : Mechanical transverse waves can be produced only in such medium which have shearing property. **[A]**

Q.14 Consider the following statements :
Statement-I: The velocity of sound in the air increases due to presence of moisture in it.
Statement-II: The presence of moisture in air lowers the density of air.

Sol.[C]

Q.1 For four sine waves, moving on a string along positive X direction, displacement-distance curves (y–x curves) are shown at time t = 0. In the right column, expressions for y as function of distance X and time t for sinusoidal waves are given. All terms in the equations have general meaning. Correctly match y–x curves with corresponding equations.

Column I	Column II
(A) 	(P) $y = A\cos(\omega t - kx)$
(B) 	(Q) $y = -A\cos(kx - \omega t)$
(C) 	(R) $y = A\sin(\omega t - kx)$
(D) 	(S) $y = A\sin(kx - \omega t)$

Sol. Use x = o; t = o for y and particle velocity $\frac{dy}{dt}$

Like for (A), y = 0 at x = o and t = o.

$\frac{dy}{dt}$ > o i.e.

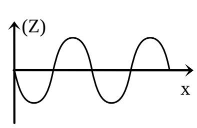
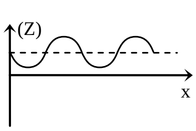
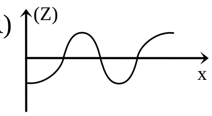
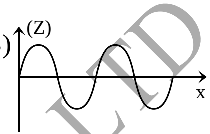
positive therefore it matches with (R)

(A → R) **(B → P)**
(C → S) **(D → Q)**

Q.2 The displacement equation of a standing wave in air is given by

$y = A\cos kx \cos \omega t$

Match the physical quantities (Z) in the column-I, to the correct plots in the column-II

Column I	Column II
(A) Displacement y of the particles at t = T/2	(P) 
(B) Velocity of the particles at t = T/4	(Q) 
(C) Change in pressure of the medium at t = 0	(R) 
(D) Density of the medium at t = T/2	(S) 

Sol. **A → R ; B → R ; C → S ; D → Q**

(A) $y = A\cos kx \cos\frac{2\pi}{T}\times T/2 = -A\cos kx$ (at t = T/2)

Hence plot is (R)

(B) $v_p = \frac{\partial y}{\partial t} = -A\,\omega\cos kx \sin\omega t = -A\omega\cos kx \sin \pi/2 = -A\omega\cos kx$ (at t = T/4)

Hence plot is (R)

(C) $\Delta P = -B\frac{\partial y}{\partial x} = +BkA\sin kx \cos\omega t$

$= +BkA\sin kx$ (at t = 0)

Hence plot is (S)

(D) Pressure at any point $\boxed{P = P_0 \pm \Delta P}$

$\Delta P = (BAk)\sin kx$

Density $\boxed{\rho = \frac{PM}{RT}}$

$\rho = \frac{M}{RT}[P_0 \pm \Delta P]$

$\boxed{\rho = \rho_0 \pm \Delta\rho}$

$\rho = \rho_0 \pm \Delta\rho$

$= \rho_0 \pm \frac{M}{RT}\Delta P$

$= \rho_0 \pm \frac{M}{RT}[-(BAk)\sin kx]$ at (t = T/2)

$\rho = \rho_0 \pm \rho'_0\,(-\sin kx)$ where $\rho'_0 = BAk$

Hence plot is (Q)

Q.3 Match the statements in column-I with the statements in column-II:

Column-I	**Column-II**
(A) A tight string is fixed at both ends and sustaining standing wave	(P) At the middle, antinode is formed in odd harmonic
(B) A tight string is fixed at one end and free at on end and free at the other end	(Q) At the middle,node is formed in even harmonic
(C) Standing wave is formed in an open organ pipe. End correction is not negligible	(R) At the middle, neither node nor antinode is formed
(D) Standing wave is formed in a closed organ pipe. End correction is not negligible	(S) Phase difference between SHM's of any two particles will be either π or zero.

Sol. **A $\rightarrow$ P,Q,S ; B $\rightarrow$ R,S ; C $\rightarrow$ S ; D $\rightarrow$ R,S**

(A) Number of loops (of length $\lambda/2$) will be even or odd and node or antinode will respectively be formed at the middle.

Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

(B) and (D) Number of loops will bot be integral. Hence neither a node nor and antinode will be formed in the middle.

Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

Q.4 For an harmonic wave, $y = (2.0 \text{ cm}) \sin(3t - 4x)$ where y and x are in cm and t is in seconds. Match the entries of column I with the entries of column II.

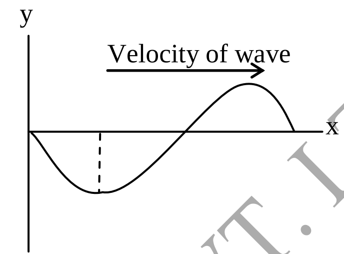
Column-I	**Column-II**
(A) The particle at x = 0 at t = 0	(P) is moving downwards
(B) The particle at $x = \frac{\pi}{8}$ cm at t = 0	(Q) is moving upwards
(C) The particle at x = 0 at $t = \frac{\pi}{3}$ sec.	(R) is accelerated up wards
(D) The particle at $x = \frac{\pi}{8}$ cm at $t = \frac{\pi}{3}$ sec.	(S) is accelerated downwards

Sol. **A $\rightarrow$ Q, B $\rightarrow$ R, C $\rightarrow$ P, D $\rightarrow$ S**

At t = 0

The shape of the string would be as shown in figure v_p at x = 0, would be upwards from,

$v_p = -v_x = \frac{\delta y}{\delta x}$



Acceleration of a particle is given by:

$a = -\omega^2 y$

At x = 0, y = 0, so a = 0

At, $x = \frac{\pi}{8}, \frac{\delta y}{\delta x} = 0$

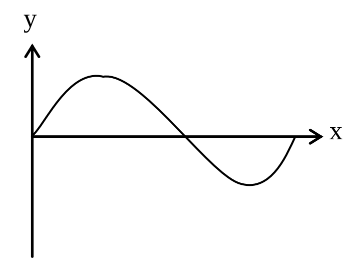
So, $v_p = 0$, and $a = \omega^2 A$

i.e., accelerated upwards

At $t = \frac{\pi}{3}$ s

$y = 2 \sin(\pi - 4x)$

$= 2 \sin 4x$



$v_{p|x=0} = -$ve, ie., moving downwards

$a_{p|x=0} = 0$

$v_{p|x=\frac{\pi}{8}} = 0$

and $a_{p|x=\frac{\pi}{8}} = -\omega^2 A$

Q.5 Match the standing waves formed in column – II due to plane progressive waves in Column – I and also with conditions in column - I

	Column-I	**Column-II**
(A)	Incident wave is $y = A \sin(kx - \omega t)$	(P) $y = 2A \cos kx \sin \omega t$
(B)	Incident wave is $y = A \cos(kx - \omega t)$	(Q) $y = 2A \sin kx \cos \omega t$
(C)	x = 0 is rigid support	(R) $y = 2A \sin kx \cos \omega t$
(D)	x = 0 is flexible support	(S) $y = 2A \cos kx \cos \omega t$

Sol. **(A) → P, R** **(B) → Q, S**

(C) → Q, R **(D) → P, S**

$y_1 = A \sin(kx - \omega t)$

$y_2 = A \sin(kx + \omega t)$

$y = y_1 + y_2 = 2A \sin kx \cos \omega t$

$y_1 = A \sin(kx - \omega t)$

$y_2 = -A \sin(kx + \omega t)$

$y = y_1 + y_2 = 2A \cos kx \sin \omega t$

$y_1 = A \cos(kx - \omega t)$

$y_2 = A \cos(kx + \omega t)$

$y = y_1 + y_2 = 2A \cos kx \cos \omega t$

$y_1 = A \cos(kx - \omega t)$

$y_2 = -A \cos(kx + \omega t)$

$y = y_1 + y_2 = 2A \sin kx \sin \omega t$

x = 0, rigid support i.e., nodes is

At nodes y = 0. This is satisfied by equations –

$y = 2A \sin kx \cos \omega t$

and $y = 2A \sin kx \sin \omega t$

x = 0, flexible support i.e., antinodes

At antinodes y is maximum. This is satisfied by equations –

$y = 2A \cos kx \sin \omega t$

and $y = 2A \cos kx \cos \omega t$.

Q.6 The equation of a travelling wave is given by (all quantities are in SI units)

$y = (0.02) \sin 2\pi (10t - 5x)$

Column-I	**Column-II**
(A) Speed of wave	(P) 10
(B) Angular frequency of wave	(Q) 0.4 π
(C) Wavelength of wave	(R) 2
(D) Maximum particle speed	(S) 0.2

Sol. **A → R; B → P; C → Q; D → S**

$y = A \sin(\omega t - kx)$

$\therefore\ \omega = 10$ rad/s $\quad k = 5\ m^{-1}$

as $k = 5 \Rightarrow \frac{2\pi}{\lambda} = 5 \Rightarrow \lambda = 0.4\pi$

$v = \frac{\omega}{k} = 2$ m/s

$(v_{Pa})_{max} = Akv = 0.02 \times 5 \times 2 = 0.2$ m/s

Q.7 In the equation, $y = A \sin 2\pi (ax + bt + \pi/4)$ match the following :

Column-I	**Column-II**
(A) Frequency of wave	(P) a
(B) Wavelength of wave	(Q) b
(C) Phase difference between two points $\frac{1}{4a}$ distance apart	(R) π
(D) Phase difference of a point after a time interval of $\frac{1}{8b}$.	(S) π/2
	(T) none

Ans. **A → Q ; B → T ; C → S ; D → T**

Q.8 From a single source, two wave trains are sent in two different strings. String-2 is 4 times heavy than string-1. The two wave equations are : (area of cross-section and tension of both strings is same).

$y_1 = A \sin(\omega_1 t - k_1 x)$ and $y_2 = 2A \sin(\omega_2 t - k_2 x)$

Suppose u = energy density, P = power transmitted and I = intensity of the wave, then match the following :

Column-I	**Column-II**
(A) u_1/u_2 is equal to	(P) 1/8
(B) P_1/P_2 is equal to	(Q) 1/16
(C) I_1/I_2 is equal to	(R) 1/4

Ans. **A → Q ; B → P ; C → P**

Q.9 A string is suspended from the ceiling. A wave train is produced at the bottom at regular interval. As the wave moves upwards :

Column-I	**Column-II**
(A) mass per unit length of the string	(P) increases
(B) tension in the string	(Q) decreases
(C) wave speed	(R) remains same
(D) wavelength	

Ans. **A → R ; B → P ; C → P ; D → P**

Q.10 Following is given the equation of a travelling wave (all in SI units)

$y = (0.02) \sin 2\pi (10t - 5x)$

Match the following :

Column-I	**Column-II**
(A) Speed of wave	(P) 10
(B) Frequency of wave	(Q) 0.4π
(C) Wavelength of wave	(R) 2
(D) Maximum particle speed	(S) 0.2

Ans. $\mathbf{A \rightarrow R ; B \rightarrow P ; C \rightarrow S ; D \rightarrow Q}$

Q.11 The equation of a stationary wave (all quantities are in SI units)

$$y = (0.06) \sin (2\pi x) \cos (5\pi t)$$

Column-I	**Column-II**
(A) Amplitude of constituent wave	(P) 0.06
(B) Position of node is at x_1 equal to	(Q) 0.5
(C) Position of anti node is at x_2 equal to	(R) 0.25
(D) Amplitude at $x = \frac{3}{4}$ m is	(S) 0.03

Sol. $\mathbf{A \rightarrow S;\ B \rightarrow Q;\ C \rightarrow R;\ D \rightarrow P}$

Sol. $y = 2A \sin kx \cos \omega t$

$\therefore 2A = 0.06 \Rightarrow A = 0.03$ m

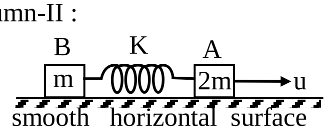
$\rightarrow$ At nodes $y = 0 \Rightarrow \sin 2\pi x = 0 \Rightarrow x = 0.5$ m

$\rightarrow$ At antinodes y = maximum $\Rightarrow \sin 2\pi x = 1$

$\Rightarrow x = 0.25$ m

$\rightarrow |y_{max}| = \left|2A \sin 2\pi \times \frac{3}{4}\right| = 0.06$ m

Q.12 Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column-I, certain statements are given and corresponding results are given in column-II, Match the statements in column-I to the corresponding results in column-II :



Column I	**Column II**
(A) The velocity of block A	(P) Can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is minimum at maximum extension of spring
	(T) constant

Sol. **A** $\rightarrow$ P **; B** $\rightarrow$ Q **; C** $\rightarrow$ P,R **; D** $\rightarrow$ Q,S

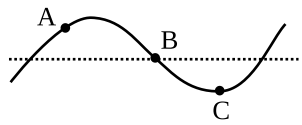
(A) If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. The K.E. of block B = $\frac{1}{2}$ m $(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of a never be zero.

(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also K.E of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.

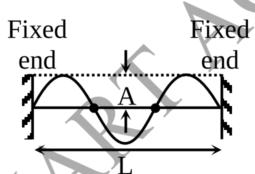
Q.13 The figure shows a string at a certain moment as a transverse wave passes through it. Three particles A, B and C of the string are also shown. Match the physical quantities in the left column with the description in the column on the right.



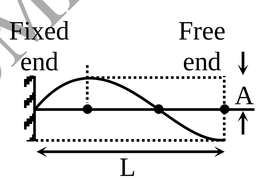
Column-I	Column-II
(A) Velocity of A	(P) Downwards, if the wave is travelling towards right
(B) Acceleration of A	(Q) Downwards, if the wave is travelling towards left
(C) Velocity of B	(R) Downwards, no matter which way the wave is travelling
(D) Velocity of C	(S) Zero

Q.14 The diagrams in Column I show transverse sinusoidal standing/travelling waveforms on stretched strings. In each case, the string is oscillating in a particular mode and its shape and other characteristics are shown at time t = 0. The maximum amplitude (in all the cases) is A, the velocity of the waveform on the string is e, the mass per unit length of the string is μ and the frequency of vibration is f(angular frequency = ω)
The kinetic energy of the string (of length L) is represented by the functions in Column II. Match the correct entries in Column II.

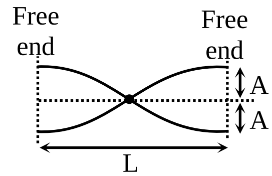
Column-I **Column-II**

(A) 

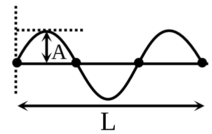
(P) $\frac{9\pi^2}{4}\frac{\mu c^2}{L}A^2$

(B) 

(Q) $\frac{9\pi^2}{4}\frac{\mu c^2}{L}A^2\sin^2\omega t$

(C) 

(R) $\frac{9\pi^2}{16}\frac{\mu c^2}{L}A^2\sin^2\omega t$

(D) 

Travelling wave

(S) $\frac{\pi^2}{4}\frac{\mu c^2}{L}A^2\sin^2\omega t$

Q.15 In the wave equation $y = A\sin 2\pi\left(ax + bt + \frac{\pi}{4}\right)$ match the following.

Column-I	Column-II
(A) Frequency of wave	(P) a
(B) Wavelength of wave	(Q) b
(C) Phase difference between two points $\frac{1}{4a}$ distance apart	(R) π
(D) Phase difference between two points $\frac{1}{8a}$ distance apart	(S) π/2
	(T) None

Sol. $A \rightarrow Q$; $B \rightarrow T$; $C \rightarrow S$; $D \rightarrow T$

PHYSICS

Q.1 In a standing wave on a string

(A) In one lime period all the particles are simultaneously at rest twice

(B) All the particles must be at their positive extremes simultaneously once in one time period

(C) All the particles are never at rest simultaneously.

(D) All the particles may be at their positive extremes simultaneously once in a time period.

[A, D]

Sol. $y = 2A \sin kx \sin\omega t$

$V_y = dy/dt = 2A\omega \sin kx. \cos \omega t$

$V_y = 0 \Rightarrow t = T/4, 3T/4 \left(T = \dfrac{2\pi}{\omega}\right)$

(2 times in one time period)

Q.2 A string is stretched along the x-axis. There is a transverse wave along the string.

$$y\ (x\ ,\ t) = 4\sin\left(\frac{\pi x}{3}\right) \cos(10\pi t)\ [\text{cm}]$$

Where x is in cms and t is in seconds then : -

(A) The amplitude of the component waves is 4 cm

(B) Velocity of component waves is 30cm/sec

(C) Wavelength of waves is 6 cm

(D) Distance between two adjacent junctions is 3 cm

Sol. **[B,C,D]**

$y = 2\ A \sin k \times \cos wt$

$2\ A = 4 \Rightarrow A = 2$ cm

$k = 2\pi/\lambda = \pi/3$

$\lambda = 6$ cm

distance between adjacent nodes $\lambda/2 = 3$ cm

$w = kv = \dfrac{2\pi}{\lambda}.v$

$v = \dfrac{w\lambda}{2\pi} = \dfrac{10\pi \times 6}{2\pi}$

$V = 30$ cm/s

Q.3 Which of the following waves represents standing wave (a, b and A are positive constant)

(A) A sin ax cos bt

(B) 2A sin (ax – bt) cos bt

(C) 2A sin (ax – bt) sin ax

(D) 2A cos (ax + t) cos (bx + t) **[A,C]**

Sol. In standing wave amplitude should be function of x only.

Q.4 Which of the following is valid wave equation traveling on string -

(A) $Ae^{-b(x + vt)}$ (B) $A \sec(kx - \omega t)$

(C) $\dfrac{1}{1+\{x(1+t/x)\}^2}$ (D) $A \sin(x^2 - vt^2)$

[A,C]

Sol. Expression representing wave equation of string must be (i) Bound function (ii) A function of $(Ax \pm Bt)$, where 'A' and 'B' are constant.

Q.5 The equation $y = 4 + 2 \sin(6t - 3x)$ represents a wave motion with –

(A) amplitude 6 units

(B) amplitude 2 units

(C) wave speed 2 units

(D) wave speed 1/2 units

Sol.(B, C) The shape of the equation is as follows:

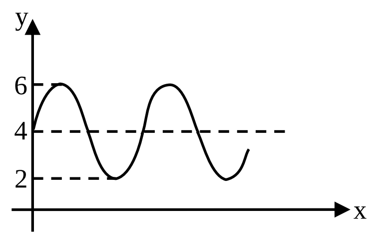


Fig.

From this we can see that amplitude of wave is 2 units.

$$\text{Wave speed } v = \frac{\text{coefficient of } t}{\text{coefficient of } x}$$

$$= \frac{6}{3} = 2 \text{ units}$$

Q.6 The tension in a stretched string fixed at both ends is changed by 2%, the fundamental frequency is found to get changed by 15 Hz. Select the correct statement(s) –

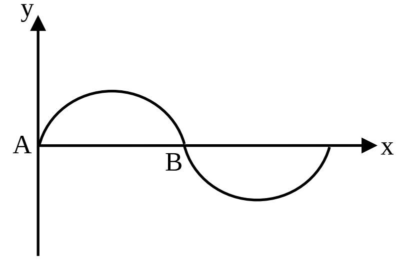


Fig.

(A) Wavelength of the string of fundamental frequency does not change

(B) Velocity of propagation of wave changes by 2%

(C) Velocity of propagation of wave changes by 1%

(D) Original frequency is 1500 Hz

[A, C, D]

Sol. Wavelength depends on length which is fixed. Therefore wavelength does not change.

Further $v = \sqrt{T/m}$ or $v \propto T^{1/2}$

$\therefore$ % change in v $= \frac{1}{2}$ (% change in T)

$= \frac{1}{2}(2) = 1\%$

i.e., Speed and hence frequency will change by 1%.

Change in frequency is 15 Hz which is 1% of 1500 Hz.

Therefore, original frequency should be 1500 Hz.

Q.7 For a certain stretched string, three consecutive resonance frequencies are observed as 105, 175, 245 Hz respectively. Then select the correct alternative(s) –

(A) The string is fixed at both ends

(B) The string is fixed at one end only

(C) The fundamental frequency is 35 Hz

(D) The fundamental frequency is 52.5 Hz

[B, C]

Sol. As $f_1 : f_2 : f_3 = 3 : 5 : 7$,

string is fixed at one end. Its fundamental frequency is $f_0 = \frac{f_1}{3} = \frac{105}{3} = 35$ Hz.

Q.8 A transverse sinusoidal wave of amplitude **a**, wavelength **λ** and frequency **f** is traveling on a stretched string. The maximum speed of any point on the string is **v/10**, where v is the speed of propagation of the wave. If **a** = 10^{-3} m and **v** = 10 m/s, then λ and f are given by–

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-3}$ m

(C) $f = \frac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

[A,C]

Sol. Maximum speed of any point on the string = aω = a(2πf)

$\therefore = \frac{v}{10} = \frac{10}{10} = 1$ (Given : v = 10 m/s)

$\therefore 2\pi a f = 1$

$\therefore f = \frac{1}{2\pi a}$

$a = 10^{-3}$ m (Given)

$\therefore f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}$ Hz

Speed of wave, v = fλ

$\therefore (10 \text{ m/s}) = \left(\frac{10^3}{2\pi} s^{-1}\right)\lambda$

$\therefore \lambda = 2\pi \times 10^{-2}$ m

Q.9 As a wave propagates –

(A) The wave intensity remains constant for a plane wave

(B) The wave intensity decreases as the inverse of the distance from the source for a spherical wave

(C) The wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) The wave intensity decreases as the inverse of the distance for a line source

[A,C,D]

Sol. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.

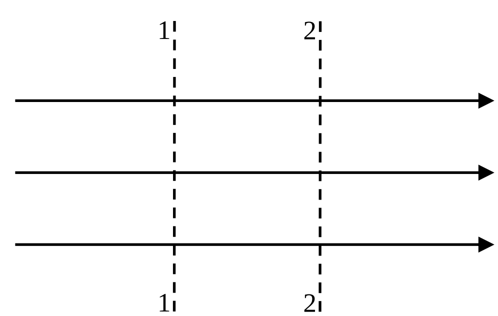


Fig. (a)

But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

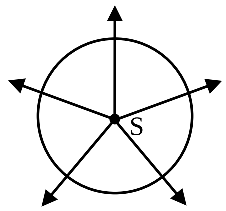


Fig.(b)

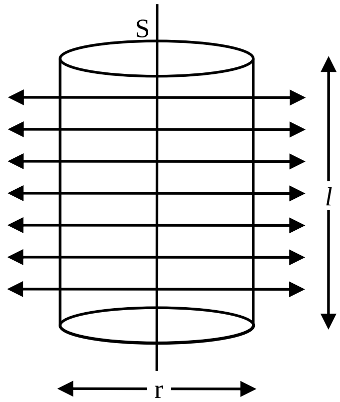


Fig.(c)

$I = \dfrac{P}{4\pi r^2}$ or $I \propto \dfrac{1}{r^2}$

For a line source $I \propto \dfrac{1}{r}$

because $I = \dfrac{P}{\pi r l}$

Q.10 A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero moves along the negative x-direction with a velocity of 300 m/s. A and B are two points 6 m apart on the line of propagation of the wave. At any instant the phase difference between A and B is ϕ. The maximum difference in the displacements at A and B is Δ –

(A) $\phi = \pi$ (B) $\phi = 0$

(C) $\Delta = 0$ (D) $\Delta = 5 \times 10^{-5}$ m

[A,D]

Sol. $\lambda = \dfrac{300\text{ m/s}}{25\text{ Hz}} = 12$ m.

Separation between A and B = 6 m = $\lambda/2$.

Q.11 A string of length L is stretched along the x-axis and is rigidly clamped at its two ends. It undergoes transverse vibration. If n is an integer, which of the following relations may represent the shape of the string at any time t ?

(A) $y = A \sin\left(\dfrac{n\pi x}{L}\right) \cos \omega t$

(B) $y = A \sin\left(\dfrac{n\pi x}{L}\right) \sin \omega t$

(C) $y = A \cos\left(\dfrac{n\pi x}{L}\right) \cos \omega t$

(D) $y = A \cos\left(\dfrac{n\pi x}{L}\right) \sin \omega t$ **[A,B]**

Sol. y = 0 at x = 0. This can be satisfied by the term $\sin\left(\dfrac{n\pi x}{L}\right)$.

Q.12 In a stationary wave system, all the particles –

(A) of the medium vibrate in the same phase

(B) in the region between two antinodes vibrate in the same phase

(C) in the region between two nodes vibrate in the same phase

(D) on either side of a node vibrate in opposite phase **[C,D]**

Q.13 P, Q and R are three particles of a medium which lie on the x-axis. A sine wave of wavelength λ is traveling through the medium in the x-direction. P and Q always have the same speed, while P and R always have the same velocity. The minimum distance between –

(A) P and Q is $\lambda/2$ (B) P and Q is λ

(C) P and R is $\lambda/2$ (D) P and R is λ

[A,D]

Q.14 The equation of a stationary wave in a string is y = (4 mm) sin [(3.14 m^{-1}) x] cos ωt.

Select the correct alternative(s) –

(A) The amplitude of component waves is 2 mm

(B) The amplitude of component waves is 4 mm

(C) The smallest possible length of string is 0.5 m

(D) The smallest possible length of string is 1.0 m

[A, D]

Sol. Comparing the given equation with standard equation of stationary wave

$y = 2a \sin (kx) \cos (\omega t)$

we have a = 2 mm

$k = \dfrac{2\pi}{\lambda} = 3.14$

or $\dfrac{\lambda}{2} = 1.0$ m

$\therefore$ The smallest possible length is $\lambda/2$ or 1.0 m.

Q.15 The equation of a wave traveling on a string is given by $y = 8 \sin [(5\ m^{-1})\ x - (4\ s^{-1})t]$. Then –

(A) velocity of wave is 0.8 m/s

(B) the displacement of a particle of the string at t = 0 and $x = \frac{\pi}{30}$ m from the mean position is 4 m (C) the displacement of a particle from the mean position at t = 0, $x\ \frac{\pi}{30}$ m is 8 m

(D) velocity of the wave is 8 m/s **[A, B]**

Sol. $v = \frac{\omega}{k} = \frac{4}{5} = 0.8$ m/s, y at t = 0 and $x = \frac{\pi}{30}$

$y = 8 \sin 5 \times \frac{\pi}{30} = 8 \sin \frac{\pi}{6} = 4$m.

Q.16 Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1 and T_2 the higher and the lower initial tension in the strings, then it higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension –

(A) T_2 was decreased

(B) T_2 was increased

(C) T_1 was decreased

(D) T_1 was increased **[B, C]**

Sol. $T_1 > T_2$

$\therefore v_1 > v_2$

or $f_1 > f_2$ and $f_1 - f_2 = 6$ Hz

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore (D) option is wrong.

If T_1 is decreased, f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore (C) option is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. So, (B) is also correct but (A) is wrong.

Q.17 A vibrating string produces 2 beats per second when sounded with a tuning fork of frequency 256 Hz. Slightly increasing the tension in the string produces 3 beats per second. The initial frequency of the string may have been –

(A) 253 Hz (B) 254 Hz

(C) 258 Hz (D) 259 Hz **[B, C]**

Q.18 A light ring slipped in a horizontal string and one end of string is tied with a rigid support. Other end string is vibrated up and down with frequency 'ν'. If the ring moves along string irregularly, value of 'ν' cannot be : (Tension in string = 10 N, Length of string = 1 m, Mass of string = 25 gm)

(A) 4 Hz (B) 5 Hz

(C) 15 Hz (D) 20 Hz **[B,C]**

Sol. It is clear that standing wave are not getting formed on the string as it continuous move irregularly. Hence 'ν' cannot be a natural frequency of string.

Q.19 Equation of standing wave travelling on a string stretched along x-axis is given by

$y = A \sin (kx + \frac{\pi}{2}) \cos \omega t$

(A) Nodes occurs at x = 0

(B) Anti node occurs at x = 0

(C) Anti node occurs at $x = \frac{\pi}{2k}$

(D) Amplitude of interfering wave is $\frac{A}{2}$

[B,C,D]

Sol. Amplitude of standing wave is equal to

$|A \sin (kx + \frac{\pi}{2})|$

$\therefore \quad |A \sin (kx + \frac{\pi}{2})| = 0$

$\Rightarrow \quad x = \frac{\pi}{2}$

$|A \sin (kx + \frac{\pi}{2})| = A$

$\Rightarrow \quad x = \frac{\pi}{2k}$

Q.20 Transverse mechanical waves can travel in–

(A) Iron rod (B) Hydrogen gas

(C) Water (D) Stretched strings

[A,D]

Q.1 A 50 cm string fixed at both ends produces resonant frequency 384 Hz and 288 Hz with out there being any other resonant frequency between two. Wave speed for the string is ......cm/sec.

Sol. **9600 cm/sec.**

$f_n - f_{n-1} = \dfrac{v}{2\ell} = 384 \times 288$

$\dfrac{v}{2 \times 0.5} = 96$

v = 96 m/sec

[v = 9600 cm/sec]

Q.2 The waves is a string (all in SI units) are $y_1 = 0.6 \sin (10\, t - 20\, x)$ and $y_2 = 0.4 \sin (10\, t + 20x)$, mass per unit length of string is 10^{-2} kg/m.
Net energy transfer per unit time through any section of the string is –

Sol. **[0.0500 J/s]**

$\rho s = \mu$ = mass per unit length = 10^{-2} kg

$v = \dfrac{\omega}{k} = \dfrac{10}{20} = 0.5$ m/s

$P_1 = \dfrac{1}{2} \times 10^{-2} \times (10)^2 \times (0.6)^2 (0.5) = 0.09$ J/s

Q.3 A string of length ℓ is fixed at both ends and is vibrating in second harmonic. The amplitude at antinodes is 2mm and the amplitude of a particle at a distance $\dfrac{\ell}{8}$ from fixed end is $\dfrac{a}{\sqrt{2}}$ mm then the value of a is…………mm.

Sol. **[0002]**

y = 2a sin kx cos ωt

$\therefore$ A = 2a sin kx

as from second harmonic

$\lambda = \ell$

$\therefore k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{\ell}$

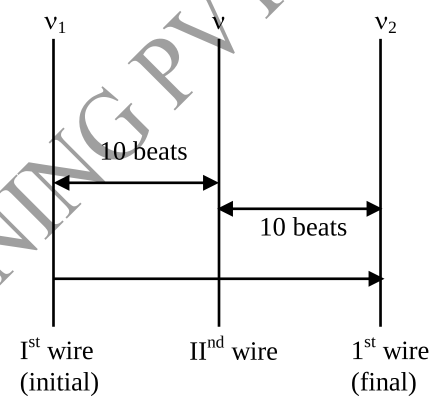
$\therefore A = 2a \sin kx = 2 \sin \left(\dfrac{2\pi}{\lambda} \times \dfrac{\ell}{8}\right)$

$= 2 \sin \dfrac{\pi}{4} = \dfrac{2}{\sqrt{2}}$ mm

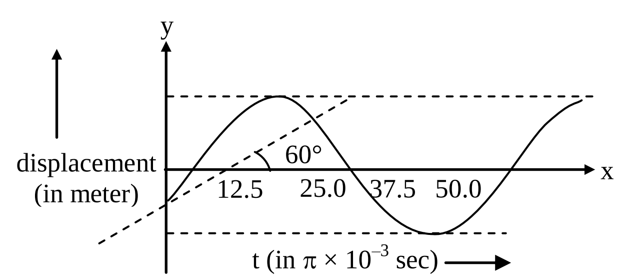
Q.4 Two wires are vibrating together to produce 10 beats/sec. Frequency of one wire is 200 Hz. When tension in this wire is increased, beat frequency remains unchanged. Frequency of other wire (in Hertz) is. **[210 Hz]**

Sol. Frequency of first wire is less than that of second wire as upon increasing tension beats frequency remains unchanged.

$\therefore \nu = \nu_1 + 10 = 210$ Hz.



Q.5 Figure shows, displacement of a particle on a string transmitting wave along x-axis as a function of time. At t = 0 particle is at half its maximum displacement. Amplitude of wave (in cm) is. **[0005]**



Sol. Phase of particle at t = 0 = – π/6

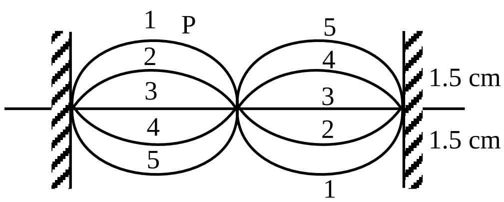
$\therefore$ Velocity of particle = $v_{max} \cos (-\pi/6)$

$= \dfrac{\sqrt{3}}{2} A\omega = \sqrt{3}$ (By figure)

$\therefore$ A = 5 cm

$\because \omega = \dfrac{2\pi}{50 \times \pi \times 10^{-3}}$ sec^{-1}

Q.6 A vibrating string 50.0 cm long is under a tension of 1.00 N. The results from five successive stroboscopic pictures are shown is observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between. Speed of the traveling waves on the string is (x.y) m/s where x and y are single non zero digit number. Find x.



Sol.[5] x = 5

$$\frac{1}{2}\,T = \frac{4}{5000}\ \text{min}$$

$T = 1.6 \times 10^{-3}$ min

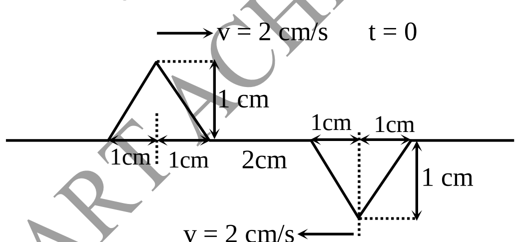
$= 9.6 \times 10^{-2}$ s

$$f = \frac{1}{9.6} \times 10^{-2}\ \text{s} = 10.4\ \text{Hz}$$

$\lambda = L = 0.5$ m

$v = f\lambda = 5.2$ m/s

Q.7 Two triangular wave pulses are traveling towards each other on a stretched string as shown in figure.



Speed of each pulse is 2 cm/s. Find maximum displacement of particle of string at t = 1s. The leading edges of the pulses are 2.00 cm apart at t = 0. **[0]**

Sol. At t = 1s, both pulses superimpose and cancel out.

Q.8 A wave pulse described by the function

$$\gamma(x, t) = \frac{A^3}{A^2 + (x - vt)^2}$$

propagates down the string, where A = 1.00 cm, and v = 20.0 m/s. At the point x = 4.00 cm, at the t = n × 10^{-3}s the displacement is maximum. Find n.

Sol. **[2]** Displacement will be maximum when x – vt = 0

$$t = x/v = \frac{4\text{cm}}{20\text{m/s}} = 2 \times 10^{-3}\ \text{s}$$

$\therefore$ n = 2

Q.9 The equation of a transverse wave is given by

$$\Psi = 10^{-2} \sin \pi[30t - \sqrt{3} - y]$$

where x, y and Ψ are in metre, and t in second. If phase difference between two points A ($2\sqrt{3}$ m, 2m) and B($3\sqrt{3}$ m, 3m) be nπ. Find value of n.

Sol. **[4]**

$$\Psi = a \sin\left[\omega t - \frac{2\pi}{\lambda}(x\cos\alpha + y\cos\beta)\right]$$

represents a wave travelling along a line in x-y plane through origin making an angle α with x-axis and β with the y-axis.

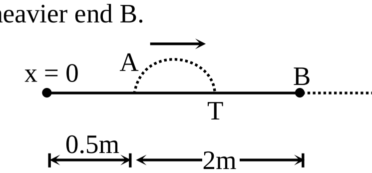
$$\Delta\phi = \frac{2\pi}{\lambda}\,[(x_2 - x_1)\cos\alpha + (y_2 - y_1)\cos\beta]$$

comparing with the given equation, we get

α = 30º, β – 60º, λ = 1m, ω = 30/s

Also, Δϕ = 4π

Q.10 A transverse wave pulse starts propagating in the +x direction along a non-uniform wire of length (2m) under tension 9N. The mass per unit length of wire varies as m = $4x^2$ kg, where x is measured from the lighter end of the wire as shown. Find the time (in seconds) taken by the pulse to reach to the heavier end of the wire, it is starts travelling from lighter end A towards heavier end B.



Sol. **[2]**

$$v = \frac{dx}{dt} = \sqrt{\frac{9N}{4x^2}}$$

integrate from $\frac{L}{4}$ to $\frac{5L}{4}$

$$t = \frac{3\times(2)^2}{4}\sqrt{\frac{4}{9}} = 2 \text{ sec.}$$

Q.11 $y = f(x, t) = \dfrac{10}{(2x+10t)^2+2}$ represents a moving pulse where, x and y are in meters and t is in seconds. Find the numerical ratio of maximum displacement of moving pulse and the wave speed.

Sol. **[1]**

$y_{max} = \frac{10}{2} = 5$, putting $(2x + 10t)^2 = 0$

comparing with f(x + vt), v = – 5m/sec

ratio $= \frac{5}{5} = 1$

Q.12 The equation (wave function) of a transverse wave travelling on a stretched string is $Y = (4\text{mm})\ e^{-(2x+5t)^2}$ where x is in cm and t in sec. If tension in the string be 20N, determine the rate of energy flow along + x axis through the point x = 0 at a time t = 0.

Sol. **[0]**

$$\left(\frac{\partial Y}{\partial x}\right)_{\substack{x=0\\t=0}} = 0$$

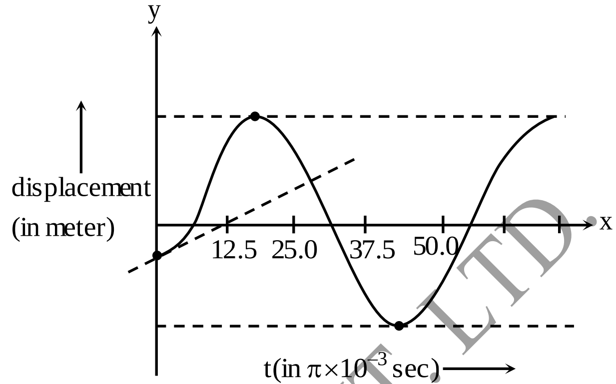
$$P = -T\left(\frac{\partial Y}{\partial x}\right)\left(\frac{\partial Y}{\partial t}\right) = 0$$

at x = 0, t = 0

Q.13 Two boats are floating on a pond in same direction and with the same speed v. Each boat sends through the water, a signal to the other. The frequencies ν_0 of the generated signals are the same. Find the ratio of frequencies received by the boats.

Sol. **[1]**

Q.14 Figure shows displacement of a particle on a string transmitting wave along x-axis as a function of time. At t = 0 particle is at half its maximum, displacement. Amplitude of wave (in cm) is -



Sol. **[5]**

Phase of particle at t = 0 = – π/6

$\therefore$ velocity of particle $= V_{max}\cos\left(\frac{-\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}A\omega \qquad = \sqrt{3}$ (By figure)

$\therefore$ A = 5 cm $\because$ $\omega = \frac{2\pi}{50\times\pi\times10^{-3}}$ se

Q.15 A standing wave exist in a string of length 150 cm which is fixed at both ends with rigid supports the displacement amplitude of a point at a distance 10 cm from one end is $5\sqrt{3}$ mm. The distance between the two nearest point within the same loop and having displacement amplitude $5\sqrt{3}$ mm is 10 cm. Max. displacement amplitude of the particles in the string (in mm) divided by two is -

Sol. **[5]**

y = A sin kx cos ωt

$5\sqrt{3}$ = A sin 10k = A sin 20k

$\Rightarrow k = \frac{\pi}{30}$

$$\therefore A = \frac{5\sqrt{3}}{\sin(\pi/30\times10)} = 10 \text{ mm}$$

PHYSICS

Q.1 The speed of a wave in a string is 20 m/s and frequency is 50 Hz. The phase difference between two points on the string 10 cm apart will be -

(A) $\pi/2$ (B) π (C) $3\pi/2$ (D) 2π

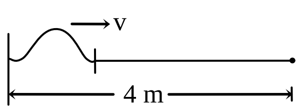
Sol.[A] $\Delta\phi = \frac{2\pi}{\lambda}\ \Delta x = \frac{2\pi}{v}.\ n.\ \Delta x$

$= \frac{2\pi}{20} \times 50 \times \frac{10}{100} = \frac{\pi}{2}$

Q.2 A wire is 4 m long and has a mass 0.2 kg. The wire is kept horizontally. A transverse pulse is generated by plucking one end of the taut (tight) wire. The pulse makes four trips back and forth along the cord in 0.8 sec. The tension is the cord will be -

(A) 80 N (B) 160 N (C) 240 N (D) 320 N

Sol.[A] 4 trips means 32 m



$t = \frac{d}{v} \quad \Rightarrow v = \frac{d}{t} = \frac{32}{0.8} = 40$ m/s

$v = \sqrt{\frac{T}{\mu}}$

$\Rightarrow T = \mu v^2$

$T = \frac{0.2}{4} \times (40)^2 = \frac{2\times16\times10}{4}$

$T = 80$ N

Q.3 A taut string for which $\mu = 5 \times 10^{-2}$ kg/m is under tension of 80 N. How much is the average rate of transport of potential energy if the frequency is 60Hz and amplitude is 6 cm (Given $4\pi^2 = 39.5$)

(A) 52W (B) 256 W

(C) 512 W (D) 215 W

Sol. [B] Average power $\cong P_{av} = \frac{1}{2}\mu v\omega^2 A^2$

Average rate of transport potential energy

$= \left(\frac{dU}{dt}\right) = \frac{P_{av}}{2} = \frac{1}{4}\ \mu v\omega^2 A^2$

$=$ 256 W

Q.4 In case of standing waves -

(A) At nodes particles displacement is time dependent

(B) At antinodes displacement of particle may or may not be zero

(C) Wave does not travel but energy is transmitted

(D) Components waves traveling in same direction having same amplitude and same frequency are superimposed **[B]**

Q.5 A 1 cm log string vibrates with fundamental frequency of 256 Hz. If the length is reduced to $\frac{1}{4}$ cm keeping the tension unaltered, the new fundamental frequency will be :

(A) 64 Hz (B) 256 Hz

(C) 512 Hz (D) 1024 Hz **[D]**

Q.6 A stretched string of length ℓ, fixed at both ends can sustain stationary waves of wavelength λ, given by :

(A) $\lambda = \frac{n^2}{2\ell}$ (B) $\lambda = \frac{\ell^2}{2n}$

(C) $\lambda = \frac{2\ell}{n}$ (D) $\lambda = 2\ \ell\ n$ **[C]**

Q.7 If the tension of sonometer's wire increases four times then the fundamental frequency of the wire will increase by :

(A) 2 times (B) 4 times

(C) 1/2 times (D) None of these **[A]**

Q.8 A string of 7 m length has a mass of 0.035 kg. If tension in the string is 60.5 N, then speed of a wave on the string is :

(A) 77 m/s (B) 102 m/s

(C) 110 m/s (D) 165 m/s **[C]**

Q.9 The fundamental frequency of a sonometre wire is n. If its radius is doubled and its tension becomes half, the material of the wire remains same, the new fundamental frequency will be :

(A) n (B) $\frac{n}{\sqrt{2}}$ (C) $\frac{n}{2}$ (D) $\frac{n}{2\sqrt{2}}$ **[D]**

Q.10 The fundamental frequency of a string stretched with a weight of 4 kg is 256 Hz. The weight required to produce its octave is :

(A) 4 kg wt (B) 8 kg wt

(C) 12 kg wt (D) 16 kg wt **[D]**

Q.11 The frequency of transverse vibrations in a stretched string is 200 Hz. If the tension is increased four times and the length is reduced to non-fourth the original value, the frequency of vibration will be :

(A) 25 Hz (B) 200 Hz

(C) 400 Hz (D) 1600 Hz **[D]**

Q.12 To increase the frequency from 100 Hz to 400 Hz the tension in the string has to be changed by :

(A) 4 times (B) 16 times

(C) 20 times (D) None of these **[B]**

Q.13 The tension of a stretched string is increased by 69 %. In order to keep its frequency of vibration constant, its length must be increased by :

(A) 20 % (B) 30 % (C) $\sqrt{69}$ % (D) 69 %

Sol. **[B]**

Q.14 A string vibrates according to the equation $y = 5 \sin\left(\frac{2\pi x}{3}\right) \cos 20\pi t$, where x and y are in cm and t in sec. The distance between two adjacent nodes is

(A) 3 cm (B) 4.5 cm

(C) 6 cm (D) 1.5 cm **[D]**

Q.15 For the stationary wave $y = 4\sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$, the distance between a node and the next antinode is :

(A) 7.5 (B) 15 (C) 22.5 (D) 30 **[A]**

Q.16 The frequency of a stretched uniform wire under tension is in resonance with the fundamental frequency of a closed tube. If the tension in the wire is increased by 8 N, it is in resonance with the first overtone of the closed tube. The initial tension in the wire is :

(A) 1 N (B) 4 N

(C) 8 N (D) 16 N **[A]**

Q.17 The displacement of a particle in string stretched in X direction is represented by y. Among the following expressions for y, those describing wave motions are :

(A) $\cos kx \sin \omega t$ (B) $k^2x^2 - \omega^2t^2$

(C) $\cos(kx - \omega t)$ (D) $\cos (k^2x^2 - \omega^2t^2)$

[A]

Q.18 Wave in string is represented by :
$y(x, t) = 2 \sin^2 (k_1x - \omega_1 t) \cos^2(k_2x - \omega_2 t)$.
Numbers of component waves are -

(A) two (B) three

(C) four (D) six **[C]**

Sol. $y (x, t) = 2 \sin^2 (k_1x - \omega_1 t) \cos^2 (k_2x - \omega_2 t)$

$$= \frac{1}{2} \{(1 - \cos (2k_1x - 2\omega_1 t)\} \{1 + \cos (2k_2x - 2\omega_2 t)\}$$

$$= \frac{1}{2} [1 - \cos (2k_1x - 2\omega_1 t) - \cos (2k_2x - 2\omega_2 t)$$

$$+ \frac{1}{2} \{\cos (2k_1x + 2k_2x - 2\omega_1 t - 2\omega_2 t)\}$$

$$+ \frac{1}{2} \{\cos (2k_1x - 2k_2x - 2\omega_1 t + 2\omega_2 t)]$$

$$= \frac{1}{4} [1 - 2 \cos (2k_1x - 2\omega_1 t) - 2 \cos (2k_2x - 2\omega_2 t) + \cos \{2(k_1 + k_2) x - 2 (\omega_1 + \omega_2) t\} + \cos \{2 (k_1 - k_2) x - 2 (\omega_1 - \omega_2)t\}]$$

Q.19 A string is stretched along x-axis. Shape of string transfering wave at t = t_1 sec is given by $y = e^{-2x}$. If wave velocity is 'v_0' in negative x-direction, then equation of waves is -

(A) $e^{-2(x - vt)}$ (B) $e^{-2x-v(t-t_0)}$

(C) $e^{-2\{x-v(t-t_0)\}}$ (D) $e^{-2\{x+v(t-t_0)\}}$

[D]

Sol. If shape of string at t = t_0 is given by y = f(x), the equation of wave traveling in negative x-direction will be $y(x, t) = f\{x + v(t - t_0)\}$.

Q.20 The amplitude of wave disturbance propagating in the positive x-axis is given by y = $\frac{1}{x^2-2x+1}$ at t = 2 sec and y = $\frac{1}{x^2+2x+5}$ at t = 6 sec, where x and y are in meters. Velocity of the pulse is -

(A) 1 m/s in positive x-direction

(B) + 2 m/s in negative x-direction

(C) 0.5 m/s in negative x-direction

(D) 1 m/s in negative x-direction **[C]**

Sol. At t = 2 sec,

$$y = \frac{1}{x^2-2x+1}$$

At t = 6 sec,

$$y = \frac{1}{x^2+2x+5}$$

$$\Rightarrow y = \frac{1}{(x+2)^2-2x+1}$$

$\therefore$ Wave velocity = $\frac{2}{4}$ = $\frac{1}{2}$ m/s in negative x-direction.

Q.21 In meldey experiment, 16 kg mass is hanged with wire and tuning fork is vibrated with its prong parallel to wire. Wire vibrates with 2 loops. When an unknown mass is hanged with wire and tuning fork is vibrated with its prong perpendicular to wire, wire vibrates in 8 loops. Unknown mass is -

(A) 1 kg (B) 2 kg

(C) 6 kg (D) 4 kg **[D]**

Sol. $\nu = \frac{n\sqrt{F/\mu}}{2\ell}$

As ℓ & μ are not changing

$\nu \propto n\sqrt{F}$

$$\frac{\nu_2}{\nu_1} = \frac{n_2\sqrt{F_2}}{n_1\sqrt{F_1}}$$

$$\Rightarrow \frac{\nu_2}{\nu_1} = \frac{n_2}{n_1}\sqrt{\frac{m_2}{m_1}}$$

$$\therefore m_2 = \left(\frac{\nu_2}{\nu_1}\times\frac{n_1}{n_2}\right)^2 . m_1$$

$\Rightarrow m_2 = 4$ kg

Q.22 Two strings of copper are stretched to the same tension. If their cross-section area are in the ratio 1 : 4, then the respective wave velocities will be -

(A) 4 : 1 (B) 2 : 1

(C) 1 : 2 (D) 1 : 4 **[B]**

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$

where, ρ = density

A = Area of cross-section

$\mu = \rho A$ = mass of unit length

Q.23 The equation of stationary wave is $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$. The distance between a node and its next antinode is -

(A) 7.5 units (B) 1.5 units

(C) 22.5 units (D) 30 units **[A]**

Sol. $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ ….(1)

$y = 2a \sin kx \cos \omega t$ ….(2)

from comparison 1 & 2

$$k = \frac{\pi}{15} = \frac{2\pi}{\lambda}$$

$\lambda = 30$

Distance b/w nearest node and Antinode = $\lambda/4$

$= \frac{30}{4} = 7.5$ unit

Q.24 The equation of a wave disturbance is given as y = 0.02 $\sin\left(\frac{\pi}{2}+50\pi t\right)\cos(10\pi x)$, where **x** and **y** are in metres and **t** is in seconds. Choose the correct statement(s) –

(A) The wavelength of wave is 0.2 m

(B) Displacement node occurs at x = 0.15 m

(C) Displacement antinode occurs at x = 0.3 m

(D) All of the above **[D]**

Sol. From the given expression for y :

amplitude A = 0.02 m

angular frequency $\omega = 50\pi$ rad/s and wave number $k = 10\pi\ m^{-1}$

Now wave speed $v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s

Therefore, option (D) is wrong.

Displacement node occurs at

$10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

or $x = \frac{1}{20}, \frac{3}{20}$

or x = 0.05 m and 0.15 m

Displacement antinode occurs at

$10\pi x = 0, \pi, 2\pi, 3\pi$ etc.

or x = 0, 0.1 m, 0.2 m and 0.3 m

Wavelength λ = 2 (distance between two consecutive nodes or antinodes)

= 2(0.1) = 0.2 m

Q.25 In a stationary wave system, all the particles of the medium –

(A) have zero displacement simultaneously at some instant

(B) have maximum displacement simultaneously at some instant

(C) are at rest simultaneously at some instant

(D) All of the above **[D]**

Q.26 For a sine wave passing through a medium, let **y** be the displacement of a particle, **v** be its velocity and **a** be its acceleration –

(A) y and a are always in opposite phase

(B) Phase difference between y and v is $\pi/2$

(C) Phase difference between v and a is $\pi/2$

(D) All of the above **[D]**

Q.27 A wire under tension vibrates with a frequency of 450 per second. What would be the fundamental frequency if the wire were half as long, twice as thick and under one-fourth tension ?

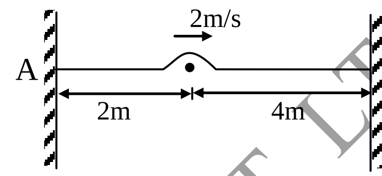
(A) 225 Hz (B) 190 Hz

(C) 247 Hz (D) 174 Hz **[A]**

Sol. $n = \frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2 \rho}} = 450$

$n' = \frac{1}{2\left(\frac{\ell}{2}\right)}\sqrt{\frac{T/4}{\pi 4 r^2 \rho}} = \frac{n}{2} = \frac{450}{2} = 225$ Hz

Q.28 A string is tied at two rigid support. A pulse is generated on the string as shown in figure. Minimum time after which string will regain its shape : (Neglect the time during reflection)



(A) 2 sec (B) 4 sec

(C) 6 sec (D) None of these **[C]**

Sol. At 'B' shape of pulse gets inverted. The pulse will become erect after reflection at 'A'.

Q.29 Equation of wave traveling on string is given by $y = A \sin(kx - \omega t)$. It is reflected at $x = \pi/k$. Equation of reflected wave will be –

(A) $A \sin(kx + \omega t + \pi)$

(B) $A \sin(kx + \omega t - 2\pi)$

(C) $A \sin(-kx - \omega t + \pi)$

(D) $A \sin(kx + \omega t + 2\pi)$ **[B]**

Sol. Let equation of reflected wave be

$y = A \sin(-kx - \omega t + \phi)$

Phase difference between incident wave and reflected wave must be 'π' at $x = \pi/k$.

$\Rightarrow -2k\left(\frac{\pi}{k}\right) + \phi = \pi$

$\Rightarrow \phi = 3\pi$

$\therefore$ Equation of reflected wave

$= A \sin(kx + \omega t - 2\pi)$

Q.30 A mechanical wave moving in a gas -

(A) must be longitudinal

(B) may be longitudinal

(C) must be transverse

(D) may be transverse **[A]**

Q.31 Which of the following waves can not travel through vacuum ?

(A) Light waves (B) X-rays

(C) Heat waves (D) Sound waves **[D]**

Q.32 A mechanical wave propagates in a medium along the X axis. The particles of the medium-

(A) must move on the X axis

(B) must move on the Y axis

(C) may move on the X axis

(D) may move on the Y axis **[D]**

Q.33 The property of medium necessary for wave propagation is its -

(A) Inertia (B) Elasticity

(C) Low resistance (D) All of above **[D]**

Q.34 Elastic waves in solid are-

(A) Transverse

(B) Longitudinal

(C) Either transverse or Longitudinal

(D) Neither transverse nor longitudinal **[C]**

Q.35 The displacement of a particle of a string carrying a travelling wave is given by

y = (3 cm) sin 6.28 (0.50 x – 50 t)

where x is in centimeter and t is in second. The velocity of the wave is-

(A) 100 m/s (B) 50 cm/s

(C) 100 cm/s (D) 10 m/s **[C]**

Q.36 A transverse wave is described by equation

$$y = a \sin\ 2\pi\left(vt - \frac{x}{\lambda}\right)$$

The maximum velocity of the particle will be four times the velocity of the wave provided-

(A) $\lambda = \pi a$ (B) $\lambda = \pi\frac{a}{2}$

(C) $\lambda = 2\pi a$ (D) $\lambda = \pi\frac{a}{4}$ **[B]**

Q.37 The equation of a wave travelling in the (+) x-direction -

(A) $y = a\sin 2\pi\left(\frac{vt}{\lambda} - x\right)$

(B) $y = a\sin\frac{2\pi}{\lambda}(vt + x)$

(C) $y = a\sin\frac{2\pi}{\lambda}(vt - x)$

(D) All of the above **[C]**

Q.38 The equation of a progressive wave is

$$y = 0.4\sin\left(120\pi t - \frac{4\pi}{5}x\right)$$

Where distance is in meters and time is in seconds. Calculate frequency and wavelength.

(A) 60 Hz, 2.5 m (B) 30 Hz, 3 m

(C) 90 Hz, 2.5 m (D) 60 Hz, 5 m **[A]**

Q.39 The relation between frequency ν, wavelength λ and velocity of propagation v of a wave is-

(A) $v\lambda = \upsilon$ (B) $\frac{\lambda\upsilon}{v} = 1$

(C) $\frac{v\upsilon}{\lambda} = 1$ (D) $\frac{\upsilon}{v} + \frac{\lambda}{v} = 1$ **[B]**

Q.40 The velocity of a wave is 330 m/s and its frequency is 330 Hz. Then its wavelength is-

(A) 100 cm. (B) 10 cm.

(C) 1 cm. (D) 330 cm. **[A]**

Q.41 If the frequency of wave is 100 Hz than the particles of the medium cross the mean position in one second-

(A) 100 times (B) 200 times

(C) 400 times (D) 50 times **[B]**

Q.42 A string is stretched by a force of 40 Newton. The mass of 10m length of this string is 0.01 kg. The speed of transverse waves in this string will be -

(A) 400 m/s (B) 40 m/s

(C) 200 m/s (D) 80 m/s **[C]**

Q.43 Two strings A and B, made of same material are stretched by same tension. The radius of string A is double of the radius of B. A transverse wave travels on A with speed v_A and on B with speed v_B. The ratio $\frac{v_A}{v_B}$ is –

(A) $\frac{1}{2}$ (B) 2

(C) $\frac{1}{4}$ (D) 4 **[A]**

Q.44 A string of length 2x is streched by 0.1x and the velocity of transverse wave along it is v. When it is streched by 0.4x, the velocity of the wave is-

(A) $\sqrt{\frac{5}{6}}.v$ (B) $\sqrt{\frac{11}{7}}.v$

(C) $\sqrt{\frac{32}{7}}.v$ (D) $\sqrt{\frac{27}{6}}.v$ **[C]**

Q.45 A long string having a cross-sectional area 0.80 mm^2 and density 12.5 g/cm^3 is subjected to a tension of 64 N along the X-axis. One end of the string is attached to a vibrator moving in transverse direction. At t = 0, the source is at maximum displacement y = 1 cm. Find the speed of wave travelling on the string.

(A) 40 m/s (B) 80 m/s
(C) 20 m/s (D) 100 m/s **[B]**

Q.46 A transverse wave described by
$y = (0.02m) \sin [(1.0\ m^{-1})\ x + (30s^{-1})\ t]$
propagates on a stretched string having a linear mass density of 1.2×10^{-4} kg/m. Find the tension in the string.

(A) 0.108 N (B) 1 N
(C) .02 N (D) 2 N **[A]**

Q.47 Two waves are represented by
$y_1 = a_1 \cos (\omega t - kx)$ and
$y_2 = a_2 \sin(\omega t - kx + \pi/3)$
Then the phase difference between them is-

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$
(C) $\frac{5\pi}{6}$ (D) $\frac{\pi}{6}$ **[D]**

Q.48 Standing waves are produced by superposition of two waves
$y_1 = 0.05 \sin (3\pi t - 2x)$ and
$y_2 = 0.05 \sin (3\pi t + 2x)$
Where x and y are measured in meter and t in second. Find the amplitude of particle at x = 0.5m
[cos 57.3 = 0.54]

(A) 0.54 m (B) 5.4 m
(C) 54 m (D) 0.054 m **[D]**

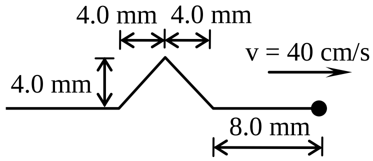
Q.49 The principle on which a stethoscope works is-

(A) Reflection (B) Interference
(C) Refraction (D) Diffraction **[A]**

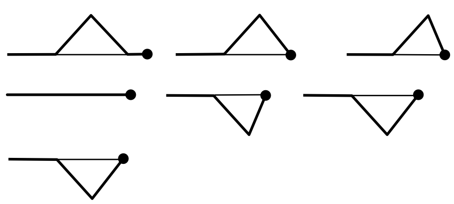
Q.50 A standing wave is produced on a string clamped at one end and free at the other. The length of the string -

(A) Must be an integral multiple of $\frac{\lambda}{4}$
(B) Must be an integral multiple of $\frac{\lambda}{2}$
(C) Must be an integral multiple of λ
(D) May be an integral multiple of $\frac{\lambda}{2}$ **[A]**

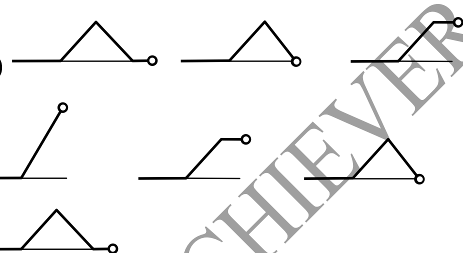
Q.1 A wave pulse on a string has the dimensions shown in Fig. at t = 0. The wave speed is 40 cm/s. **(a)** If point O is a fixed end, draw the total wave on the string at t = 15 cm, 20 ms, 25 ms, 30 ms, 35 cm, 40 ms and 45 ms. **(b)** Repeat part (a) for the case in which point O is a free end [**Reflection.**].



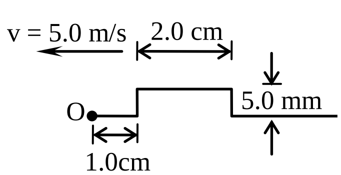
Sol. **(a)** The wave form for the given times, respectively, is shown.



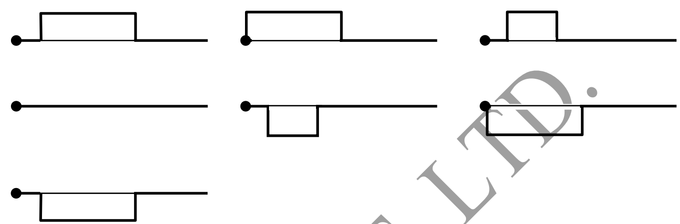
(b)



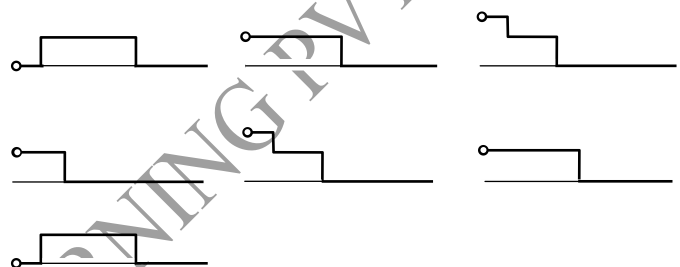
Q.2 A wave pulse on a string has the dimensions shown in Fig. at t = 0. The wave speed is 5.0 m/s. **(a)** If point O is a fixed end, draw the total wave on the string at t = 1.0 ms, 2.0 ms, 3.0 ms, 4.0 ms, 5.0 ms, and 7.0 ms. **(b)** Repeat part (a) for the case in which point O is a free end [**Reflection**].



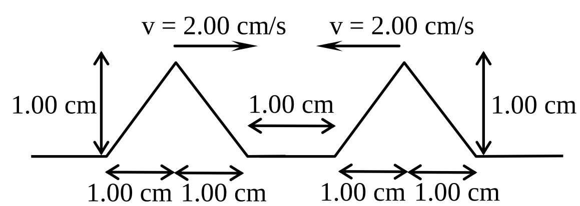
Sol. **(a)** The wave form for the given times, respectively, is shown.



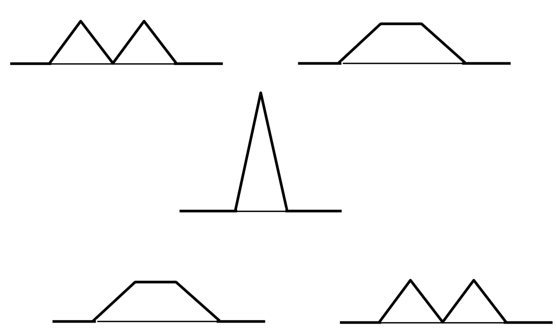
(b



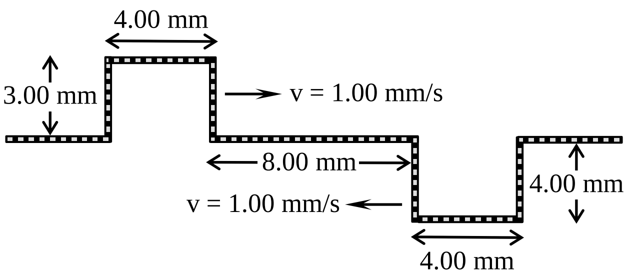
Q.3 Two triangular wave pulses are traveling toward each other on a stretched string as shown in Fig. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at t = 0. Sketch the shape of the string at t = 0.250 s, t = 0.500 s, t = 0.750 s, t = 1.000 s, and t = 1.250 s [**Interference of Triangular Pulses**].



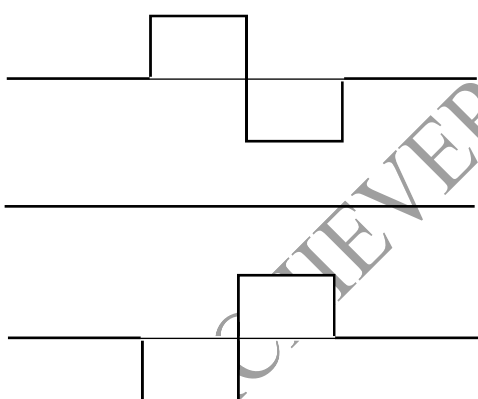
Sol.



Q.4 Figure shows two rectangular wave pulses on a stretched string traveling toward each other. Each pulse is traveling with a speed of 1.00 mm/s and has the height and width shown in the figure. If the leading edges of the pulses are 8.00 mm apart at t = 0, sketch the shape of the string at t = 4.00 s, t = 6.00 s, and t = 10.0 s [**Interference of Rectangular Pulses**].



Sol.

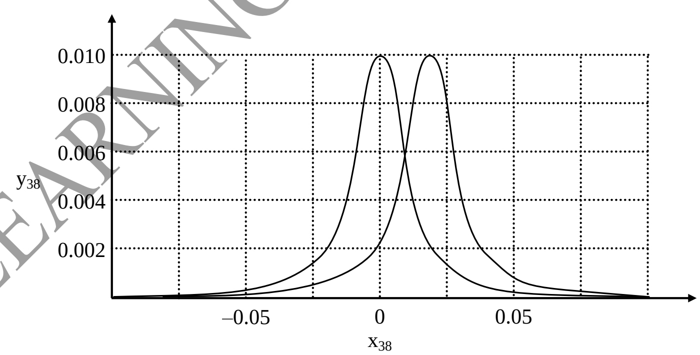


Q.5 The sinusoidal waves can be produced on a string by continually oscillating one end of the string. If instead the end of the string is given a single shake, a wave pulse propagates down the string. A particular wave pulse is described by the function

$$y(x, t) = \frac{A^3}{A^2 + (x - vt)^2}$$

where A = 1.00 cm and v = 20.0 m/s. **(a)** Sketch the pulse as a function of x at t = 0. How far along the string does the pulse extend? **(b)** Sketch the pulse as a function of x at t = 0.001 s. **(c)** At the point x = 4.5 cm, at what time **t** is the displacement maximum? At which two times is the displacement at x = 4.50 cm equal to half its maximum value? **(d)** Show that y(x, t) satisfies the wave equation Eq. $\left(\frac{d^2y(x,t)}{dx^2} = \frac{1}{v^2}\frac{d^2y(x,t)}{dt^2}\right)$ [**Motion of a Wave Pulse**].

Sol. **(a), (b)**



(c) The displacement is a maximum when the term in parentheses in the denominator is zero; the denominator is the sum of two squares and is minimized when x = vt, and the maximum displacement is A. At x = 4.50 cm, the displacement is a maximum at
$t = (4.50 \times 10^{-2}\text{ m})/(20.0\text{ m/s}) = 2.25 \times 10^{-3}$s.
The displacement will be half of the maximum when $(x - vt)^2 = A^2$, or $t = (x \pm A)/v = 1.75 \times 10^{-3}$ s and 2.75×10^{-3}s.

(d) Of the many ways to obtain the result, the method presented saves some algebra and minor calculus, relying on the chain rule for partial derivatives. Specifically, let **u** = u(x, t) = x – vt, so that if f(x, t) = g(u), $\frac{\partial f}{\partial x} = \frac{dg}{du}\frac{\partial u}{\partial t} = \frac{dg}{du}$ and $\frac{\partial f}{\partial t} = \frac{dg}{du}\frac{\partial u}{\partial t} = -\frac{dg}{du}v$.

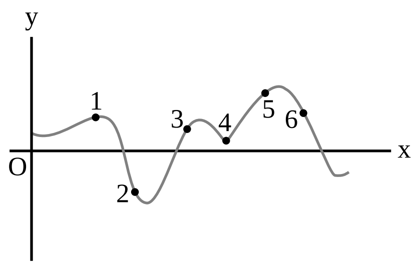
(In this form it may be seen that any function of this form satisfies the wave equation; In this case, $y(x, t) = A^3(A^2 + u^2)^{-1}$, and so

$$\frac{\partial y}{\partial x} = \frac{-2A^3u}{(A^2+u^2)^2}, \quad \frac{\partial^2 y}{\partial x^2} = -\frac{2A^3(A^2-3u^2)}{(A^2+u^2)^3}$$

$$\frac{\partial y}{\partial t} = v\frac{2A^3u}{(A^2+u^2)^2}, \quad \frac{\partial^2 y}{\partial t^2} = -v^2\frac{2A^3(A^2-3u^2)}{(A^2+u^2)^3},$$

and so the given form for y(x, t) is a solution to the wave equation with speed v.

Q.6 The shape of a wave on a string at a specific instant is shown in Fig.



The wave is propagating to the right, in the +x-direction. **(a)** Determine the direction of the transverse velocity of each of the six labeled points on the string. If the velocity is zero, state it as such Explain your reasoning. **(b)** Determine the direction of the transverse acceleration of each of the six labeled points on the string. Explain your reasoning. **(c)** How would your answers be affected if the wave was propagating to the left, in the — x-direction? [**A Non-Sinusoidal Wave**]

Sol. **(a)** and **(b)** (1): The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will begin to move downward, and $\mathbf{a_y} < 0$. (2): As the wave moves in the +x-direction, the particle will move upward so $\mathbf{v_y} > 0$. The portion of the curve to the left of the point is steeper, so $\mathbf{a_y} > 0$. (3): The point is moving down, and will increase its speed as the wave moves; $\mathbf{v_y} < 0$, $\mathbf{a_y} < 0$. (4): The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will move away from the x-axis, and $\mathbf{a_y} > 0$. (5): The point is moving downward, and will increase its speed as the wave moves; $\mathbf{v_y} < 0$, $\mathbf{a_y} < 0$. (6): The particle is moving upward, but the curve that represents the wave appears to have no curvature, so $\mathbf{v_y} > 0$ and $\mathbf{a_y} = 0$.

(c) The accelerations, which are related to the curvatures, will not change. The transverse velocities will all change sign.

Q.7 A stretched string lies along the x-axis. The string is displaced along both the y-and z-direction, so that the transverse displacement of the string is given by

$y(x, t) = A\cos(kx - \omega t)$ $\quad$ $z(x, t) = A\sin(kx - \omega t)$

(a) Draw a graph of z versus y for a particle on the string at x = 0. This shows the trajectory of the particle as seen by an observer on the +x-axis looking back toward x = 0. Indicate the position of the particle at t = 0, $t = \pi/2\omega$, $t = \pi/\omega$, and $t = 3\pi/2\omega$. **(b)** Find the velocity vector of a particle at an arbitrary position x on the string. Show that this represents the tangential velocity of a particle moving in a circle of radius A with angular velocity ω, and show that the speed of the particle is constant (i.e. the particle is in uniform circular motion). **(c)** Find the acceleration vector of the particle in part (b). Show that the acceleration is always directed toward the centre of the circle and that its magnitude is $a = \omega^2 A$. Explain these results in terms of uniform circular motion. Suppose that the displacement of the string was instead given by

$y(x, t) = A\cos(kx - \omega t)$ $\quad$ $z(x, t) = -A\sin(kx - \omega t)$

Describe how the motion of a particle at x would be different from the motion described in part (a) [**Two-Dimensional Waves**].

Sol. **(a)** $y^2(x, y) + z^2(x, y) = A^2$

The trajectory is a circle of radius A.

At t = 0, y(0, 0) = A, z(0, 0) = 0.

At $t = \pi/2\omega$, $y(0, \pi/2\omega) = 0$, $z(0, \pi/2\omega) = -A$.

At $t = \pi/\omega$, $y(0, \pi/\omega) = -A$, $z(0, \pi/2\omega) = 0$.

At $t = 3\pi/2\omega$, $y(0, 3\pi/2\omega) = 0$, $z(0, 3\pi/2\omega) = +A$.

(b) $v_y = dy/dt = +A\omega \sin(kx - \omega t)$, $v_z = dz/dt = -A\omega \cos(kx - \omega t)$

$v = \sqrt{v_y^2 + v_z^2} = A\omega$, so the speed is constant.

$\vec{r} = y\hat{j} + z\hat{k}$

$\vec{r}\,.\,\vec{v} = yv_y + zv_z = A^2\omega \sin(kx - \omega t)\cos(kx - \omega t) - A^2\omega \cos(kx - \omega t) \sin(kx - \omega t)$

$\vec{r}\,.\,\vec{v} = 0$, so $\vec{v}$ is tangent to the circular path.

(b) $a_y = dv_y/dt = -A\omega^2 \cos(kx - \omega t)$ $a_z = dv_z/dt = -A\omega^2 \sin(kx - \omega t)$

$\vec{r}\,.\,\vec{a} = ya_y + za_z = A^2\omega^2[\cos^2(kx - \omega t) + \sin^2(kx - \omega t)] = -A^2\omega^2$

$r = A$, $a = A\omega^2$, so $\vec{r}\,.\,\vec{a} = -ra$

$\vec{r}\,.\,\vec{a} = ra \cos\phi$ so $\phi = 180°$ and $\vec{a}$ is opposite in direction to $\vec{r}$; $\vec{a}$ is radially inward.

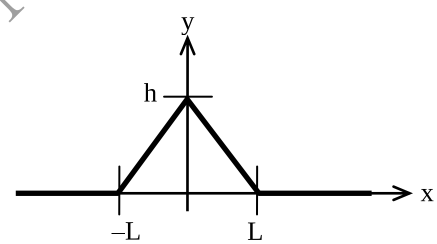
$y^2 + z^2 = A^2$, so the path is again circular, but the particle rotates in the opposite sense compared to part (a).

Q.8 A triangular wave pulse on a taut string travels in the positive x-direction with speed v. The tension in the string is F and the linear mass density of the string is **μ**. At t = 0, the shape of the pulse is given by

$$y(x, 0) = \begin{cases} 0 & \text{if } x < -L \\ h(L+x)/L & \text{for } -L < x < 0 \\ h(L-x)/L & \text{for } 0 < x < L \\ 0 & \text{for } x > L \end{cases}$$

(a) Draw the pulse at t = 0. **(b)** Determine the wave function y(x, t) at all times t. **(c)** Find the instantaneous power in the wave. Show that the power is zero except for $-L < (x - vt) < L$ and that in this interval the power is constant. Find the value of this constant power [**Energy in a Triangular Pulse**].

Sol. **(a)**



(b) The wave moves in the +x direction with speed v, so in the expression for y(x, 0) replace x with x – vt:

$$y(x, t) = \begin{cases} 0 & \text{for } (x - vt) < -L \\ h(L + x - vt)/L & \text{for } -L < (x - vt) < 0 \\ h(L - x + vt)/L & \text{for } 0 < (x - vt) < L \\ 0 & \text{for } (x - vt) > L \end{cases}$$

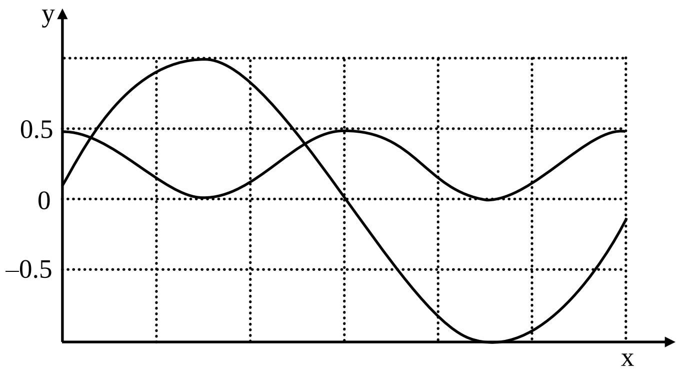
(c) $P(x, t) = F\dfrac{\partial y(x,t)}{\partial x}\dfrac{\partial y(x,t)}{\partial t} =$

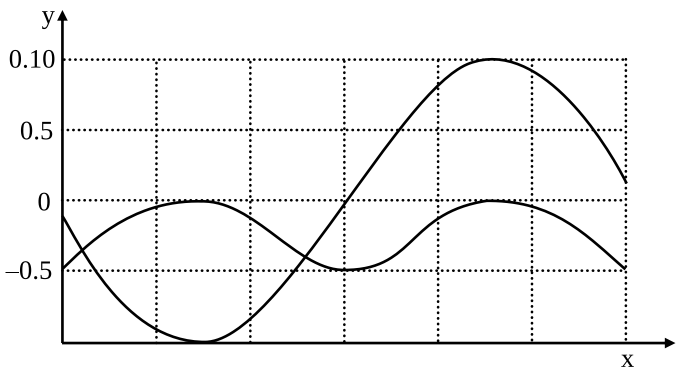
$$\begin{cases} -F(0)(0) = 0 & \text{for } (x - vt) < -L \\ -F(h/L)(-hv/L) = Fv(h/L)^2 & \text{for } -L < (x - vt) < 0 \\ -F(-h/L)(hv/L) = Fv(h/L)^2 & \text{for } 0 < (x - vt) < L \\ -F(0)(0) = 0 & \text{for } (x - vt) > L \end{cases}$$

Thus the instantaneous power is zero except for $-L < (x - vt) < L$, where it has the constant value $\mathbf{Fv(h/L)^2}$.

Q.9 **(a)** Graph y(x, t) given as a function of x for a given time t (say, t = 0). On the same axes, make a graph of the instantaneous power P(x, t). **(b)** Explain the connection between the slope of the graph of y (x, t) versus x and the value of P(x, t). In particular, explain what is happening at points where P = 0, where there is no instantaneous energy transfer. **(c)** The quantity P(x, t) always has the same sign. What does this imply about the direction of energy flow? **(d)** Consider a wave moving in the –x-direction, for which $y(x, t) = A \cos(kx + \omega t)$. Calculate P(x, t) for this wave, and make a graph of y(x, t) and P(x, t) as functions of x for a given time t (say, t = 0). What differences arise from reversing the direction of the wave? [**Instantaneous Power in a Wave**]

Sol. **(a), (d)**





(b) The power is a maximum where the displacement is zero, and the power is a minimum of zero when the magnitude of the displacement is a maximum.

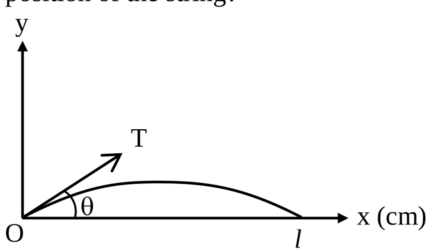
(c) The direction of the energy flow is always in the same direction.

(d) In this case, $\frac{\partial y}{\partial x} = -kA\sin(kx + \omega t)$, and so Eq.(15.22) becomes

$P(x, t) = -Fk\omega A^2 \sin^2(kx + \omega t)$.

The power is now negative (energy flows in the –x-direction), but the qualitative relations of part (b) are unchanged.

Q.10 A guitar string is 80 cm long and has a fundamental frequency of 400 Hz. In its fundamental mode the maximum displacement is 2 cm at the middle. If the tension in the string is 10^6 dynes, what is the maximum of that component of the force on the end support which is perpendicular to the equilibrium position of the string?



Sol. Use Cartesian coordinates with the x-axis along the equilibrium position of the string and the origin at one of its fixed ends. Then the two fixed ends are at x = 0 and x = l = 80 cm, as shown in Fig. At x = 0, the y-component of the force on the support is

$$F_y = T\sin\theta \simeq T\theta \simeq T\frac{\partial y}{\partial x},$$

where T is the tension in the string. The guitar string has a sinusoidal form

$$y = y_0 \sin\left[\omega\left(t - \frac{x}{v}\right)\right]$$

with $\omega = \frac{2\pi v}{\lambda} = \frac{2\pi v}{2l} = \frac{\pi v}{80}$, $y_0 = 2$ cm. Thus

$$y = 2\sin\left(\omega t - \frac{\pi x}{80}\right) \text{cm}.$$

Hence at x = 0,

$$F_y = -\frac{2\pi T}{80}\cos(\omega t)$$

and

$$F_{y\ max} = \frac{\pi T}{40} = 7.85 \times 10^4 \text{ dynes}.$$

Q.11 A vibrating string 50.0 cm long is under a tension of 1.00 N. The results from five successive stroboscopic pictures are shown in Fig. The strobe rate is set at 5000 flashes per minute and observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between.

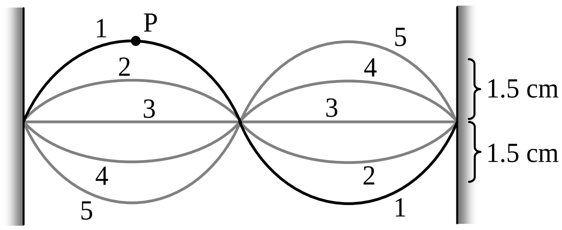
a) Find the period, frequency, and wavelength for the traveling waves on this string.

b) In what normal mode (harmonic) is the string vibrating?

c) What is the speed of the traveling waves on the string?

d) How fast is point P moving when the string is in: (i) position 1? (ii) position 3?

e) What is the mass of this string?



Sol. **a)** The string vibrates through ½ cycle in $4 \times \frac{1}{5000}$ min, so

$\frac{1}{2}T = \frac{4}{5000}$ min $\rightarrow$ T = 1.6×10^{-3} min = 9.6×10^{-2}s

f = 1/T = $1/9.6 \times 10^{-2}$s = 10.4 Hz

λ = L = 50.0 cm = 0.50 m

b) Second harmonic.

c) v = fλ = (10.4 Hz)(0.50 m) = 5.2 m/s

d) (i) Maximum displacement, so v = 0 (ii) $v_y = \frac{\partial y}{\partial t} = \frac{\partial}{\partial t}$ (1.5 cm sin kx sin ωt)

Speed = $|v_y|$ = ω(1.5 cm)sin kx sin ωt

at maximum speed, sin kx = sin ωt = 1

$|v_y|$ = w(1.5 cm) = 2πf(1.5 cm) = 2π(10.4 Hz)(1.5 cm)

= 98cm/s = 0.98m/s

e) $v = \sqrt{F/\mu} \rightarrow \mu = F/v^2$

$M = \mu L = \frac{F}{v^2}L = \frac{(1.00N)(0.500m)}{(5.2m/s)^2} = 1.85 \times 10^{-2}$ kg

= 18.5 g

Q.12 A guitar string of length L is plucked in such a way that the total wave produced is the sum of the fundamental and the second harmonic. That is, the standing wave is given by

$y(x, t) = y_1(x, t) + y_2(x, t)$

where

$y_1(x, t) = C \sin \omega_1 t \sin k_1 x$

$y_2(x, t) = C \sin \omega_2 t \sin k_2 x$

with $\omega_1 = vk_1$ and $\omega_2 = vk_2$.

a) At what values of x are the nodes of y_1?

b) At what values of x are the nodes of y_2?

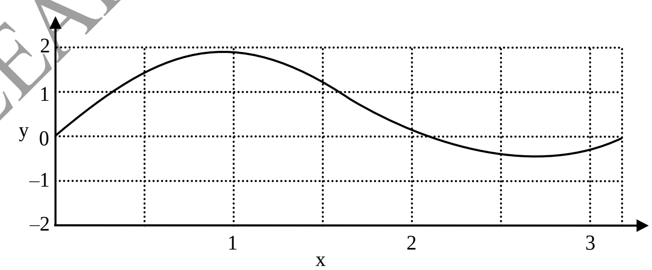
c) Graph the total wave at t = 0, $t = \frac{1}{8}f_1$, $t = \frac{1}{4}f_1$, $t = \frac{3}{8}f_1$, and $t = \frac{1}{2}f_1$.

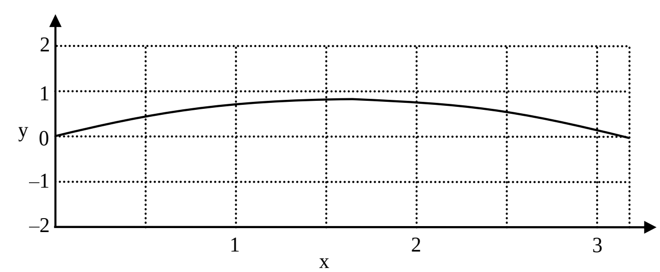
d) Does the sum of the two standing waves y_1 and y_2 produce a standing wave? Explain.

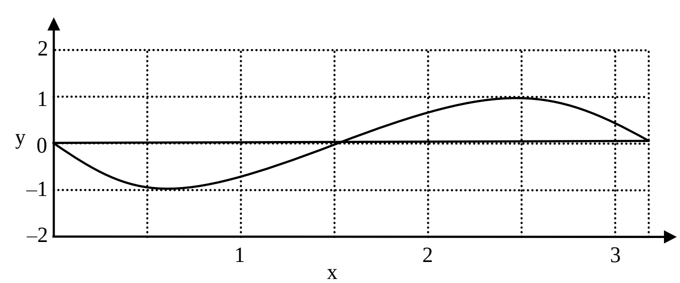
Sol. **a)** The fundamental has nodes only at the ends, x = 0 and x = L.

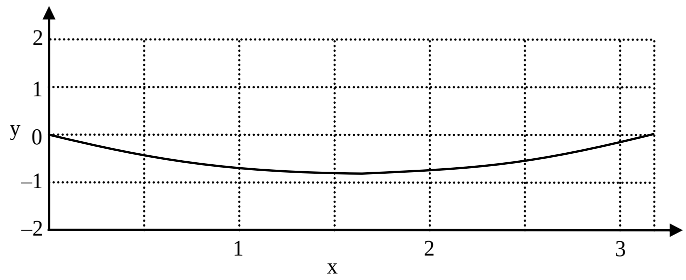
b) For the second harmonic, the wavelength is the length of the string, and the nodes are at x = 0, x = L/2 and x = L.

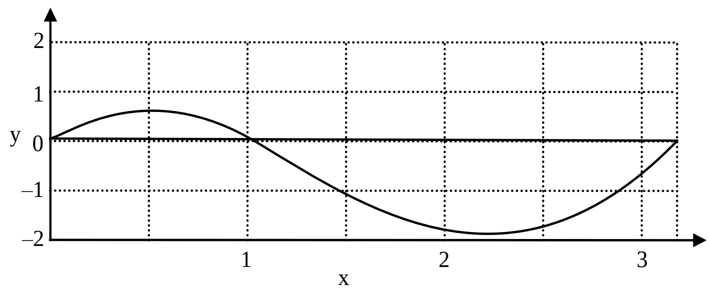
c)











d) No; no part of the string except for x = L/2, oscillates with a single frequency.

Q.13 A massive aluminum sculpture is hung from a steel wire. The fundamental frequency for transverse standing waves on the wire is 200 Hz. The sculpture is then immersed in water so that 1/3of its volume is submerged.

a) What is the new fundamental frequency?

b) Why is it a good approximation to treat the wire as being fixed at both ends?

Sol. **a)** The new tension F'. in the wire is

$$F' = F - B = w - \frac{(1/3w)\rho_{water}}{\rho_{A1}} = w\left(1 - \frac{1}{3}\frac{\rho_{water}}{\rho_{A1}}\right)$$

$$= w\left(1 - \frac{(1.00\times10^3 kg/m^3)}{3(2.7\times10^3 kg/m^3)}\right) = (0.8765)\ w = (0.87645)\ F.$$

The frequency will be proportional to the square root of the tension, and so

$f' = (200\ Hz)\sqrt{0.8765} = 187$ Hz.

b) The water does not offer much resistance to the transverse waves in the wire, and hence the node will be located at the point where the wire attaches to the sculpture and not at the surface of the water.

Q.14 A cellist tunes the C-string of her instrument to a fundamental frequency of 65.4 Hz. The vibrating portion of the string is 0.600 m long and has a mass of 14.4 g.

a) With what tension must she stretch it?

b) What percent increase in tension is needed to increase the frequency from 65.4 Hz to 73.4 Hz, corresponding to a rise in pitch from C to D?

Sol. **a)** $F = 4L^2f_1^2\ \mu = 4mLf_1^2 = 4(14.4 \times 10^{-3}$ kg)(0.600 m)(65.4 Hz)^2 = 148 N.

b) The tension must increase by a factor of $\left(\frac{73.4}{65.4}\right)^2$, and the percent increase in $(73.4/65.4)^2 - 1 = 26.0\%$.

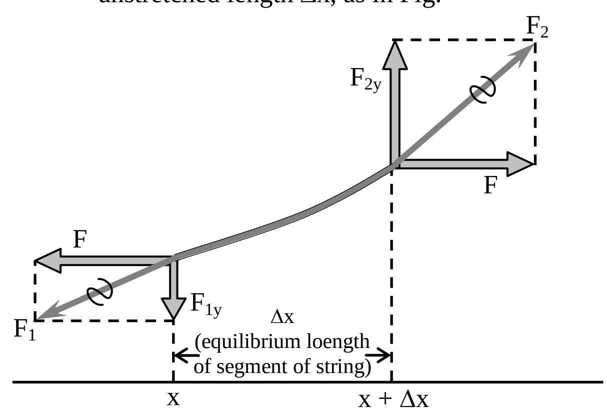
Q.15 **a)** Show that for a wave on a string, the kinetic energy per unit length of string is

$$u_k(x, t) = \frac{1}{2}\mu v_y^2(x, t) = \frac{1}{2}\mu\left(\frac{\partial y(x,t)}{\partial t}\right)^2$$

where μ is the mass per unit length.

b) Calculate $u_k(x, t)$ for a sinusoidal wave given by Eq. [y(x, t) = A cos(kx – ωt)].

c) There is also elastic potential energy in the string, associated with the work required to deform and stretch the string. Consider a short segment of string at position x that has unstretched length Δx, as in Fig.



Ignoring the (small) curvature of the segment, its slope is ∂y(x, t)/∂x. Assume that the displacement of the string from equilibrium is small, so that ∂y/∂x has a magnitude much less than unity. Show that the stretched length of the segment is approximately

$$\Delta x\left[1 + \frac{1}{2}\left(\frac{\partial y(x,t)}{\partial x}\right)^2\right]$$

(Hint: Use the relationship $\sqrt{1+u} \approx 1 + \frac{1}{2}u$, valid for |u| << 1.)

d) The potential energy stored in the segment equals the work done by the string tension F (which acts along the string) to stretch the segment from its unstretched length Δx to the length calculated in part (c). Calculate this work and show that the potential energy per unit length of string is

$$u_p(x, t) = \frac{1}{2} F\left(\frac{\partial y(x,t)}{\partial x}\right)^2$$

e) Calculate $u_p(x, t)$ for a sinusoidal wave given by Eq. [$y(x, t) = A\cos(kx - \omega t)$].

f) Show that $u_k(x, t) = u_p(x, t)$ for (x, t) for all x and t.

g) Show $y(x, t)$, $u_k(x, t)$, and $u_p(x, t)$ as functions of x for t = 0 in one graph with all three functions on the same axes. Explain why u_k and u_p are maximum where y is zero, and vice versa.

h) Show that the instantaneous power in the wave, given by Eq. [$p(x, t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$], is equal to the total energy per unit length multiplied by the wave speed v. Explain why this result is reasonable.

Sol. **a)** $u_k = \dfrac{\Delta K}{\Delta x} = \dfrac{(1/2)\Delta m v_y^2}{\Delta m/\mu} = \dfrac{1}{2}\mu\left(\dfrac{\partial y}{\partial t}\right)^2$.

b) $\dfrac{\partial y}{\partial t} = \omega A \sin(kx - \omega t)$ and so

$$u_k = \frac{1}{2}\mu\omega^2 A^2 \sin^2(kx - \omega t).$$

c) The piece has width Δx and height $\Delta x \dfrac{\partial y}{\partial x}$, and so the length of the piece is

$$\left((\Delta x)^2 + \left(\Delta x \frac{\partial y}{\partial x}\right)^2\right)^{1/2} = \Delta x\left(1 + \left(\frac{\partial y}{\partial x}\right)^2\right)^{1/2}$$

$$\approx \Delta x\left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right].$$

where the relation given in the hint has been used.

d) $u_p = F\dfrac{\Delta x\left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right] - \Delta x}{\Delta x} = \dfrac{1}{2} F\left(\dfrac{\partial y}{\partial x}\right)^2$.

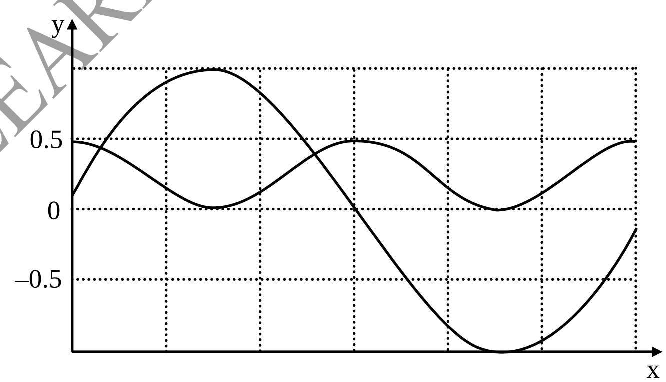
e) $\dfrac{\partial y}{\partial x} = -kA\sin(kx - \omega t)$, and so

$$u_p = \frac{1}{2} F k^2 A^2 \sin^2(kx - \omega t)$$

and

f) Comparison with the result of part (c) with $k^2 = \omega^2/v^2 = \omega^2\,\mu/F$, shows that for a sinusoidal wave $u_k = uv_p$.

g) In this graph, u_k and u_p coincide, as shown in part (f).



At y = 0, the string is stretched the most, and is moving the fastest, so u_k and u_p are maximized. At the extremes of y, the string is unstretched and is not moving, so $\mathbf{u_k}$ and $\mathbf{u_p}$ are both at their minimum of zero.

h) $u_k + u_p = Fk^2A^2\cos^2(\omega t - kx) = Fk(\omega/v)A^2\cos^2(\omega t - kx) = \dfrac{P}{v}$.

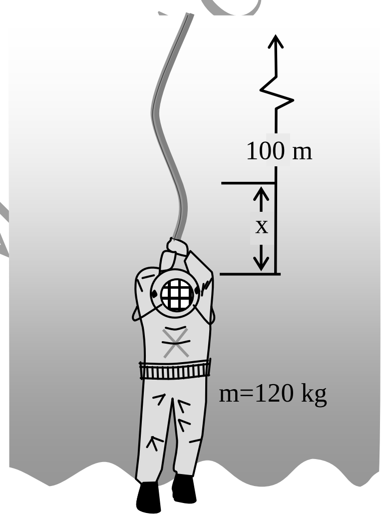
The energy density travels with the wave, and the rate at which the energy is transported is the product of the density per unit length and the speed.

Q.16 A deep-sea diver is suspended beneath the surface of Loch Ness by a 100-m long cable that is attached to a boat on the surface (Fig.). The diver and his suit have a total mass of 120 kg and a volume of 0.0800 m^3. The cable has a diameter of 2.00 cm and a linear mass density of μ = 1.10 kg/m. The diver thinks he sees something moving in the murky depths and jerks the end of the cable back and forth to send transverse waves up the cable as a signal to his companions in the boat.

a) What is the tension in the cable at its lower end, where it is attached to the diver? Do not forget to include the buoyant force that the water (density 1000 kg/m^3) exerts on him.

b) Calculate the tension in the cable a distance x above the diver. The buoyant force on the cable must be included in your calculation.

c) The speed of transverse waves on the cable is given by $v = \sqrt{F/\mu}$. The speed therefore varies along the cable, since the tension is not constant. (This expression neglects the damping force that the water exerts on the moving cable.) Integrate to find the time required for the first signal to reach the surface.



Sol. **a)** The tension is the difference between the diver's weight and the buoyant force,

$F = (m - \rho_{water}V)g = (120 \text{ kg} - 1000 \text{ kg/m}^3)(0.0800 \text{ m}^3)(9.80 \text{ m/s}^2)) = 392 \text{ N}.$

b) The increase in tension will be the weight of the cable between the diver and the point at x, minus the buoyant force. This increase in tension is then

$(\mu x - \rho(Ax))\, g = (1.10 \text{ kg/m} - (1000 \text{ kg/m}^3)\pi(1.00 \times 10^{-2}\text{m})^2)(9.80 \text{ m/s}^2)\ x = (7.70 \text{ N/m})x$

The tension as a function of x is then $F(x) = (392 \text{ N}) + (7.70 \text{ N/m})x$.

c) Denote the tension as $F(x) = F_0 + ax$, where $F_0 = 392$ N and $a = 7.70$ N/m. Then, the speed of transverse waves as a function of x is $v = \frac{dx}{dt} = \sqrt{(F_0 + ax)/\mu}$ and the time t needed for a wave to reach the surface is found from

$$t = \int dt = \int \frac{dx}{dx/dt} = \int \frac{\sqrt{\mu}}{\sqrt{F_0 + ax}}\, dx.$$

Let the length of the cable be L, so

$$t = \sqrt{\mu}\int_0^L \frac{dx}{\sqrt{F_0 + ax}} = \sqrt{\mu}\,\frac{2}{a}\sqrt{F_0 + ax}\Big|_0^L$$

$$= \frac{2\sqrt{\mu}}{a}\left(\sqrt{F_0 + aL} - \sqrt{F_0}\right)$$

$$= \frac{2\sqrt{1.10 \text{ kg/m}}}{7.70 \text{ N/m}}$$

$$(\sqrt{392\text{N} + (7.70\text{N/m})(100\text{m})} - \sqrt{392\text{N}})$$

= 3.98 s.

Q.17 A uniform rope with length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. You can ignore the force of gravity on the rope. Find the time required for a transverse wave to travel from one end of the rope to the other.

Sol. The tension in the rope will vary with radius r. The tension at a distance r from the center must supply the force to keep the mass of the rope that is further out than r accelerating inward. The mass of this piece in $m\dfrac{L-r}{L}$, and its center of mass moves in a circle of radius $\dfrac{L+r}{2}$, and so

$$T(r) = \left[m\frac{L-r}{L}\right] w^2\left[\frac{L+r}{L}\right] = \frac{m\omega^2}{2L}(L^2 - r^2).$$

An equivalent method is to consider the net force on a piece of the rope with length dr and mass dm = dr m/L. The tension must vary in such a way that

$T(r) - T(r + dr) = -\omega^2 r\ dm$, or $\dfrac{dT}{dr} = -(m\omega^2/L)r dr$. This is integrated to obtained

$T(r) - (m\omega^2/2L)r^2 + C$, where C is a constant of integration. The tension must vanish at r = L, from which $C = (m\omega^2 L/2)$ and the previous result is obtained.

The speed of propagation as a function of distance is

$$v(r) = \frac{dr}{dt} = \sqrt{\frac{T(r)}{\mu}} = \sqrt{\frac{TL}{m}} = \frac{\omega}{\sqrt{2}}\sqrt{L^2 - r^2},$$

where $\dfrac{dr}{dt} > 0$ has been chosen for a wave traveling from the center to the edge. Separating variables and integrating, the time t is

$$t = \int dt = \frac{\sqrt{2}}{\omega}\int_0^L \frac{dr}{\sqrt{L^2 - r^2}}.$$

The integral is done explicitly by letting $r = L\sin\theta$, $dr = L\cos\theta\, d\theta$, $\sqrt{L^2 - r^2} = L\cos\theta$, so that

$$\int \frac{dr}{\sqrt{L^2 - r^2}} = \theta = \arcsin\frac{r}{L}, \text{ and}$$

$$t = \frac{\sqrt{2}}{\omega}\arcsin(1) = \frac{\pi}{\omega\sqrt{2}}.$$

Q.18 From Eq. $[x = 0, \dfrac{\lambda}{2}, \dfrac{2\lambda}{2}, \dfrac{3\lambda}{2}, \ldots]$, the instantaneous rate at which a wave transmits energy along a string (instantaneous power) is

$$P(x, t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

where F is the tension.

a) Evaluate P(x, t) for a standing wave of the form given by Eq.$[y(x, t) = (A\omega \sin kx)\sin \omega t]$.

b) Show that for all values of x, the average power P_{av} carried by the standing wave is zero. (Equation $[p_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2]$ does not apply here. Can you see why?)

a) For a standing wave given by Eq. $[y(x, t) = (A\omega \sin kx)\sin \omega t]$, graph P(x, t) and the displacement y(x, t) as functions of x for t = 0, $t = \pi/4\omega$, $t = \pi/2\omega$, and $t = 3\pi/4\omega$. (Positive P(x, t) means energy is flowing in the +x-direction; negative P(x, t) means the flow is in the –x-direction.)

b) The kinetic energy per unit length of string is greatest where the string has the greatest transverse speed, and the potential energy per unit length of string is greatest where the string has the steepest slope (because there the string is stretched the most).

Using these ideas, discuss the flow of energy along the string.

Sol. **a)** $\dfrac{\partial y}{\partial x} = kA_{SW}\cos kx \sin \omega t$, $\dfrac{\partial y}{\partial t} = -\omega A_{SW}\,\omega \sin kx \cos\omega t$, and so the instantaneous power is

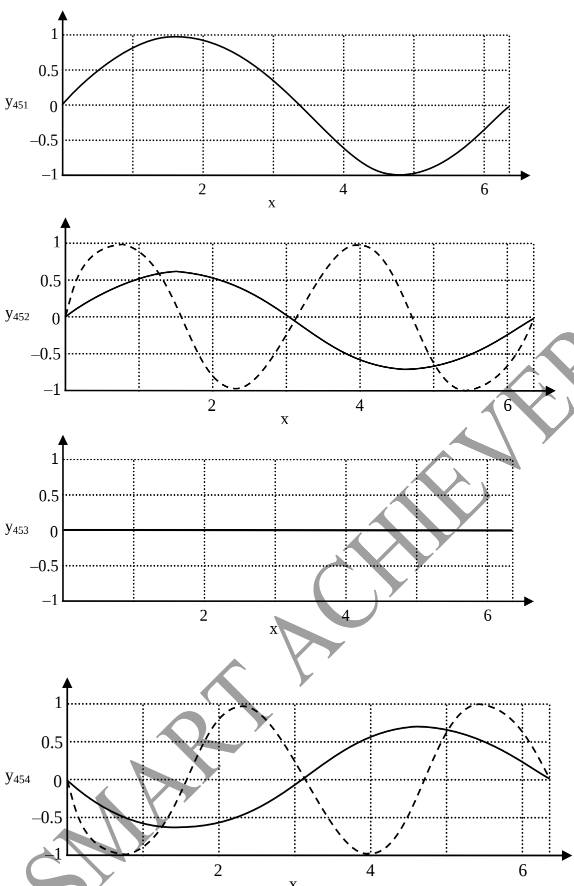
$$P = FA^2_{SW}\,\omega k(\sin kx \cos kx)(\sin \omega t \cos \omega t)$$

$$= \frac{1}{4}FA^2_{SW}\,\omega k \sin(2kx)\sin(2\omega t).$$

b) The average value of P is proportional to the average value of sin ($2\omega t$), and the average of the since function is zero; $P_{av} = 0$.

c) The waveform is the solid line, and the power is the dashes line. At time $t = \pi/2\omega$, $y = 0$ and $P = 0$ and the graphs coincide.

d) When the standing wave is at its maximum displacement at all points, all of the energy is potential, and is concentrated at the places where the slope is steepest (the nodes). When the standing wave has zero displacement, all of the energy is kinetic, concentrated where the particles are moving the fastest (the antinodes). Thus, the energy must be transferred from the nodes to the antinodes, and back again, twice in each cycle. Note that |P| is greatest midway between adjacent nodes and antinodes, and that P vanishes at the nodes and antinodes.



Q.19 A string is stretched between two rigid supports 100 cm apart. In the frequency range between 100 and 350 cps only the following frequencies can be excited: 160, 240, 320 cps. What is the wavelength of each of these modes of vibration?

Sol. As the two ends of the string are fixed, we have $n\lambda = 2L$, where L is the length of the string and n an integer. Let the wavelengths corresponding to frequencies 160, 240, 320 Hz be λ_0, λ_1, λ_2 respectively. Then

$n\lambda_0 = (n + 1)\lambda_1 = (n + 2)\lambda_2 = 200$,

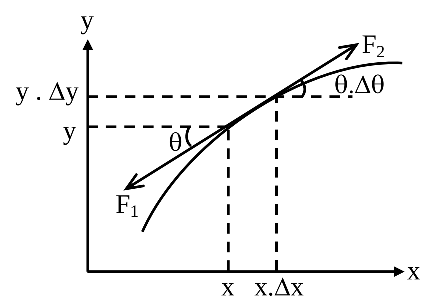
$160\lambda_0 = 240\lambda_1 = 320\lambda_2$.

Hence n = 2, and

$\lambda_0 = 100$ cm, $\lambda_1 = 67$ cm, $\lambda_2 = 50$ cm.

Q.20 **a)** Give the equation which relates the fundamental frequency of a string to the physical and geometrical properties of the string.

b) Derive your result from Newton's equations by analyzing what happens to a small section of the string.



Sol. **a)** Let ω be the fundamental frequency of a string of length l, linear density ρ and tension F. The equation relating F, l and ρ is

$$\omega = \frac{\pi}{l}\sqrt{\frac{F}{\rho}}\,.$$

b) Consider a small length Δl of a string along the x direction undergoing small oscillations and let F_1, F_2 be the tensions at its two ends, as shown in Fig. For small oscillations, $\theta \approx 0$ and $\Delta\theta$ is a second-order small quality. Furthermore as there is no x motion, we can take the x-component of the net force on Δl to be zero. Thus

$f_z = F_2 \cos(\theta + \Delta\theta) - F_1 \cos\theta$

$\approx (F_2 - F_1)\cos\theta - F_2\Delta\theta \sin\theta$

$\approx F_2 - F_1 = 0,$

or $F_2 \approx F_1$. Then

$$f_y = F \sin(\theta + \Delta\theta) - F \sin\theta \approx F \frac{d\sin\theta}{d\theta}\Delta\theta$$

$$= F\cos\theta \frac{d\theta}{dx}\Delta x \approx F\frac{d\theta}{dx}\Delta x.$$

For small θ,

$$\theta \approx \frac{dy}{dx},\ \frac{d\theta}{dx} = \frac{d^2y}{dx^2},$$

and the above becomes

$$\rho\Delta l \frac{\partial^2 y}{\partial t^2} = F\frac{\partial^2 y}{\partial x^2}\Delta x$$

by Newton's second law. As $\Delta l \approx \Delta x$, this gives

$$\frac{\partial^2 y}{\partial x^2} - \frac{\rho}{F}\frac{\partial^2 y}{\partial t^2} = 0,$$

which is the equation for a wave with velocity of propagation

$$v = \sqrt{\frac{F}{\rho}}.$$

For the fundamental mode in a string of length l with the two ends fixed, the wavelength λ is given by $l = \lambda/2$. Hence the fundamental angular frequency is

$$\omega = \frac{2\pi v}{\lambda} = \frac{\pi v}{l} = \frac{\pi}{l}\sqrt{\frac{F}{\rho}}.$$