Differential Equations

Single Correct Answer Type

1. The family of curves passing through (0,0) and satisfying the differential equation $\frac{y_2}{y_1}=1$

(where $y_n = d^n y/dx^n$) is

a) $y=k$ b) $y=kx$ c) $y=k(e^x+1)$ d) $y=k(e^x-1)$

Key. D

Sol. $\frac{dp}{dx}=P\left(where\ p=\frac{dy}{dx}\right)$

$\ln P = x+c \Rightarrow p=e^{x+c}$

$\frac{dy}{dx}=ke^x$

$y=ke^x+\lambda$

Satisfying $(0,0), So\ \lambda=-k$

$y=k\left(e^x-1\right)$

2. The degree of the differential equation satisfying the relation

$\sqrt{1+x^2}+\sqrt{1+y^2}=\lambda(x\sqrt{1+y^2}-y\sqrt{1+x^2})$ is

a) 1 b) 2 c) 3 d) none of these

Key. A

Sol. On Putting $x=\tan A, y=\tan B$ we get

$\sec A+\sec B=\lambda\left(\tan A\sec B-\tan B\sec A\right)$

$\cos A+\cos B=\lambda\left(\sin A-\sin B\right)$

$\tan\left(\frac{A-B}{2}\right)=\frac{1}{\lambda}$

$\tan^{-1}x-\tan^{-1}y=2\tan^{-1}\frac{1}{\lambda}$

On differentiating $\frac{1}{1+x^2}-\frac{1}{1+y^2}\frac{dy}{dx}=0$

3. S1: The differential equation of parabolas having their vertices at the origin and foci on the x-axis is an equation whose variables are separable

S2: The differential equation of the straight lines which are at a fixed distance p from the origin is an equation of degree 2

S3: The differential equation of all conics whose both axes coincide with the axes of coordinates is an equation of order 2

a) TTT b) TFT c) FFT d) TTF

Key. A

Sol. S_1 - Equation of parabola is $y^2 = \pm 4ax$

$2y\frac{dy}{dx} = \pm 4a$

D.E of parabola $\Rightarrow y^2 = 2yx\frac{dy}{dx}$

$2\frac{dy}{y} = \frac{dx}{x}$

Which is variable seperable

S_2 - Equation of line which is fixed distance. P from origin can be equation of tangent to circle $x^2 + y^2 = p^2$

Line is $y = mx + p\sqrt{1+m^2}$ $\left(m = \frac{dy}{dx}\right)$

$\left(y - x\frac{dy}{dx}\right)^2 = P\left(1 + \left(\frac{dy}{dx}\right)^2\right)$

So, degree is 2

S_3 - Equation of conic whose both axis co-incide with co-ordinate axis is $ax^2 + by^2 = 1$

As there are two constants, so order of D.E is 2

4. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

a) 1 b) 2 c) 3 d) 4

Key. A

Sol. The parametric form of the given equation is $x = t, y = t^2$. The equation of any tangent at t is $2xt = y + t^2$, Differentiating we get $2t = y_1\left(= \frac{dy}{dx}\right)$ putting this value in the above equation, we have $2x\frac{v_1}{2} = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1{}^2$

The order of this equation is 1

Hence (A) is the correct answer

5. Which of the following transformation reduces the differential equation $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$ to the form $P(x)u = Q(x)$

a) $u = \log z$ b) $u = e^z$ c) $u = (\log z)^{-1}$ d) $u = (\log z)^2$

Key. A or B or C or D

Sol. Given equ. Can be written as

$$\frac{dz}{z(\ln z)^2}+\frac{dx}{x(\ln z)}=\frac{dx}{x^2}$$

Put $u=\frac{1}{\ln z}$

$$\frac{du}{dx}=-\frac{1}{(\ln z)^2}\cdot\frac{1}{z}\cdot\frac{dz}{dx}$$

$$\therefore -\frac{du}{dx}+\frac{1}{x}.u=\frac{1}{x^2}$$

$$\frac{du}{dx}+\left(-\frac{1}{x}\right)u=\left(-\frac{1}{x^2}\right)$$

$$I.F.=e^{\int -\frac{1}{x}dx}=e^{\ln\frac{1}{x}}=\frac{1}{x}$$

$$u.\frac{1}{x}=\int -\frac{1}{x^2}.\frac{1}{x}dx$$

$$\frac{u}{x}=-\int\frac{1}{x^3}dx=+\frac{1}{4x^4}+c$$

6. If $y_1(x)$ is a solution of the differential equation $dy/dx-f(x)y=0$, then a solution of the differential equation $\frac{dy}{dx}+f(x)y=r(x)$

a) $\frac{1}{y_1(x)}\int r(x)y_1(x)dx$

b) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$

c) $\int r(x)y_1(x)dx$

d) none of these

Key. A

Sol. $\frac{dy}{dx}-f(x).y=0$

$$\frac{dy}{y}=f(x)dx$$

$$\ln y=\int f(x)dx$$

$y_1(x)=e^{\int f(x)dx}$ Then for given equation I.F = $e^{\int f(x)dx}$

Hence Solution $y.y_1(x)=\int r(x).y_1(x)dx$

$$y=\frac{1}{y_1(x)}\int r(x).y_1(x)dx$$

7. The solution of $(x+2y^3)(dy/dx)=y$ is (where c is arbitrary constant)

1) $x=y^3+cy$ 2) $x=y^3-cy$

3) $y=x^3-cy$ 4) $y=x^3+cy$

Key. 1 or 2

Sol. $\frac{dx}{dy}-\frac{1}{y}x=2y^2$

This is linear equation taking y as independent variable.

Here, I.F. $=e^{-\int\frac{1}{y}dy}=e^{-\log y}=\frac{1}{y}$

$\therefore$ solution is $x\frac{1}{y}=\int\frac{1}{y}2y^2dy+c$

$\Rightarrow\frac{x}{y}=y^2+c\Rightarrow x=y^3+cy$

8. Solution of the equation $xdy=\left(y+x\frac{f(y/x)}{f^{'}(y/x)}\right)dx$

1) $f\left(\frac{x}{y}\right)=cy$ 2) $f\left(\frac{y}{x}\right)=cx$

3) $f\left(\frac{y}{x}\right)=cxy$ 4) $f\left(\frac{y}{x}\right)=0$

Key. 2

Sol. We have, $xdy=\left(y+\frac{xf(y/x)}{f^{'}(y/x)}\right)dx$

$\Rightarrow\frac{dy}{dx}=\frac{y}{x}+\frac{f(y/x)}{f^{'}(y/x)}$ which is homogenous

Putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$,we obtain

$v+x\frac{dv}{dx}=v+\frac{f(v)}{f^{'}(v)}\Rightarrow\frac{f^{'}(v)}{f(v)}dv=\frac{dx}{x}$

Integrating, we get log $f(v)=\log x+\log c$

$\Rightarrow\log f(v)=\log cx\Rightarrow f\left(\frac{y}{x}\right)=cx$

9. The general solution of the differential equation $\frac{dy}{dx}=y\tan x-y^2\sec x$ is

1) $\tan x=(c+\sec x)y$
2) $\sec y=(c+\tan y)x$
3) $\sec x=(c+\tan x)y$
4) None of these

Key. 3

Sol. We have $\frac{dy}{dx}=y\tan x-y^2\sec x$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$

Putting $\frac{1}{y}=v \Rightarrow \frac{-1}{y^2}\frac{dy}{dx}=\frac{dv}{dx}$, we obtain

$\frac{dV}{dx}+\tan x.v=\sec x$ which is linear

$I.F=e^{\int \tan x dx}=e^{\log \sec x}=\sec x$

$\therefore$ The solution is

$v\sec x=\int \sec^2 x\,dx+c \Rightarrow \frac{1}{y}\sec x=\tan x+c$

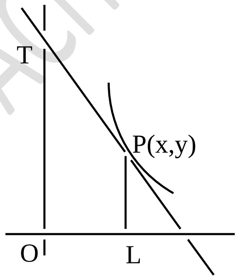
$\Rightarrow \sec x=y(c+\tan x)$

10. The curve $y=f(x)$ is such that the area of the trapezium formed by the coordinate axes, ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa. The equation of the curve can be

1) $y=cx^2\pm x$
2) $y=cx^2\pm 1$
3) $y=cx\pm x^2$
4) $y=cx^2\pm x^2\pm 1$

Key. 1

Sol. Let $P(x,y)$ be any point on the curve. Length of intercept on y-axis by any tangent at P is



$OT=y-x\frac{dy}{dx}$

$\therefore$ Area of trapezium $OLPTO=\frac{1}{2}(PL+OT)OL$

$=\frac{1}{2}\left(y+y-x\frac{dy}{dx}\right)x=\frac{1}{2}\left(2y-x\frac{dy}{dx}\right)x$

According to question

Area of trapezium $OLPTO=\frac{1}{2}x^2$

$i.e., \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x = \pm\frac{1}{2}x^2$

$\Rightarrow 2y - x\frac{dy}{dx} = \pm x \; or \; \frac{dy}{dx} - \frac{2y}{x} = \pm 1$

Which is linear differential equation and $I.F. = e^{-2\ln x} = \frac{1}{x^2}$

$\therefore$ The solution is $\frac{y}{x^2} = \int \pm\frac{1}{x^2}dx + c = \pm\frac{1}{x} + c$

$\therefore y = \pm x + cx^2 \; or \; y = cx^2 \pm x$, where c is an arbitrary constant

11. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^2y^2 = \sin x$, then

1) p > q

2) $\frac{p}{q} = \frac{1}{2}$

3) p = q

4) p < q

Key. 4

Sol. $y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x = -3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}}$

$\left(y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x\right)^3 = -9x^3\left(\frac{dy}{dx}\right)$

Here order = 2 = p

Degree = 6 = q

$\therefore p < q$

12. The solution of the differential equation $\frac{dy}{dx} - ky = 0, y(0) = 1$, approach zero when $x \to \infty$, if

1) K = 0

2) k > 0

3) k < 0

4) none of these

Key. 3

Sol. $\frac{dy}{dx} - Ky = 0, \frac{dy}{y} = Kdx$

$\ln y = Kx + c$

At $x = 0, y = 1$ $\therefore C = 0$

Now, $\ln y = Kx$

$y = e^{Kx}$

$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{Kx} = 0$

$\therefore K < 0$

13. The solution of $\frac{dv}{dt}+\frac{k}{m}v=-g$ is

1) $v=ce^{-\frac{k}{m}t}-\frac{mg}{k}$
2) $v=c-\frac{mg}{k}e^{-\frac{k}{m}t}$
3) $ve^{-\frac{k}{m}t}=c-\frac{mg}{k}$
4) $ve^{\frac{k}{m}t}=c-\frac{mg}{k}$

Key. 1

Sol. $\frac{dv}{dt}+\frac{K}{m}v=-g$

Integrating factor $(I.F.)=e^{\int\frac{k}{m}dt}=e^{\frac{K}{m}t}$

$\therefore Ve^{\frac{K}{m}t}=-\int g\,e^{K.t/m}+c$

$Ve^{\frac{K}{m}t}=\frac{-gm}{K}e^{\frac{K}{m}t}+c$

$V=C.e^{\frac{-K}{m}t}-\frac{mg}{K}$

14. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx}+f(x)y=0$, then a solution of differential equation $\frac{dy}{dx}+f(x)y=r(x)$ is

1) $\frac{1}{y(x)}\int y_1(x)dx$
2) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$
3) $\int r(x)y_1(x)dx$
4) none of these

Key. 2

Sol. i) $\frac{dy_1}{dx}+f(x)y_1=0\Rightarrow f(x)=\frac{-1}{y_1}\frac{dy_1}{dx}$

ii) $\frac{dy}{dx}-\frac{1}{y_1}\frac{dy_1}{dx}.y=r(x)$

$e^{-\int\frac{1}{y_1}\frac{dy}{dx}dx}=e^{-\int\frac{dy_1}{y_1}}=\frac{1}{y_1}$

$\frac{d}{dx}\left(\frac{y}{y_1}\right)=\frac{r(x)}{y_1}\Rightarrow\frac{y}{y_1}=\int\frac{r(x)dx+c}{y_1}$

$y=y_1\int\frac{r(x)dx}{y_1}+cy_1$

15. The solution of the differential equation $y_1y_3=3y_2^2$ is

1) $x=A_1y^2+A_2y+A_3$
2) $x=A_1y+A_2$

3) $x = A_1y^2 + A_2y$ 4)none of these

Key. 1

Sol. $y_1y_3 = 3y_2^2$

$\frac{y_3}{y_2} = 3\frac{y_2}{y_1} \Rightarrow \ln y_2 = 3\ln y_1 + \ln c$

$y_2 = cy_1^3$

$\frac{y_2}{y_1^2} = cy_1$

$-\frac{1}{y_1} = cy + c_2$

$\frac{dx}{dy} = -cy - c_2$

$x = -\frac{cy^2}{2} - c_2y + c_3$

$\therefore x = A_1y^2 + A_2y + A_3$

16. The solution of the differential

equation $\left(x^2 \sin^3 y - y^2 \cos x\right)dx + \left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy = 0$ is

1) $x^3 \sin^3 y = 3y^2 \sin x + C$ 2) $x^3 \sin^3 y + 3y^2 \sin x = C$

3) $x^2 \sin^3 y + y^3 \sin x = C$ 4) $2x^2 \sin y + y^2 \sin x = C$

Key.

Sol. $\left(x^2 \sin^2 y - y^2 \cos x\right)dx + \left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy = 0$

$\frac{dy}{dx} = \frac{y^2 \cos x - x^2 \sin^3 y}{x^3 \cos y \sin^2 - 2y \sin x}$

$\left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy$

$= \left(y^2 \cos x - x^2 \sin^3 y\right)dx = 0$

$\left(\frac{x^3}{3}d \sin^3 y - \sin dy^2\right) - \sin^3 yd\left(\frac{x^3}{3}\right) + y^2 d \sin x = 0$

$\frac{x^3}{3}d \sin^2 y + \sin^3 yd\left(\frac{x^3}{3}\right) - \left(\sin dy^2 + y^2 d \sin x\right)$

$d\left(\frac{x^3}{3}\sin^3 y\right) - d\left(y^2 \sin x\right) = 0$

$\frac{x^3}{3}\sin^3 y - y^2 \sin x = c$

17. The differential equation whose solution is $\left(x-h\right)^2 + \left(y-k\right)^2 = a^2$ is (a is a constant)

1) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$ 2) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

3) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$ 4) none of these

Key.

Sol. $(x-h)^2+(y-k)^2=a^2$ ....(1)

$2(x-h)+2(y-k)\frac{dy}{dx}=0$ ....(2)

$1+\left(\frac{dy}{dx}\right)^2+(y-k)\frac{d^2y}{dx^2}=0$ ....(3)

From (3) we have (y-k), use in (2) to get (x-h) and put(x-h) and(y-k) in(1)

18. An equation of the curve in which subnormal varies as the square of the ordinate is (k is constant of proportionality)

1) $y=Ae^{kx}$ 2) $y=e^{kx}$ 3) $y^2/2+kx=A$ 4) $y^2+kx^2=A$

Key. 1

Sol. According to the given condition $y\frac{dy}{dx}=ky^2$

$\Rightarrow \frac{dy}{y}=kdx$ (variables separable equation)

$\Rightarrow \log|y|=kx+C \Rightarrow |y|=Be^{kx} \Rightarrow y=Ae^{kx}$ where $A=\pm B$ and k is the constant of proportionality.

19. The solution of $\frac{dy}{dx}=\frac{ax+b}{cy+d}$ represents a parabola if

1) $a=0,c=0$ 2) $a=1,b=2$ 3) $a=0,c\neq 0$ 4) $a=1,c=1$

Key. 3

Sol. The given equation is with separable variables so $(cy+d)dy=(ax+b)dx$. Integrating we have $\frac{cy^2}{2}+dy+K=\frac{ax^2}{2}+bx, K$ being the constant of integration. The last equation represents a parabola if $c=0, a\neq 0$ or $a=0, c\neq 0$.

20. The equation of the curve passing through $(3,9)$ which satisfies $dy/dx=x+1/x^2$ is

1) $6xy=3x^2-6x+29$ 2) $6xy=3x^2-29x+6$

3) $6xy=3x^3+29x-6$ 4) None of these

Key. 3

Sol. The given differential equation has variable separable so integrating, we have $y = x^2/2 - 1/x + C$. This will pass through $(3,9)$ if $9 = 9/2 - 1/3 + C \Rightarrow C = 29/6$.

Hence the required equation is $6xy = 3x^3 + 29x - 6$

21. Solution of the differential equation $2y\sin x \dfrac{dy}{dx} = 2\sin x\cos x - y^2\cos x$ satisfying $y(\pi/2) = 1$ is given by

1) $y^2 = \sin x$
2) $y = \sin^2 x$
3) $y^2 = \cos x + 1$
4) $y^2\sin x = 4\cos^2 x$

Key. 1

Sol. The given equation can be written as $2y\sin x\dfrac{dy}{dx} + y^2\cos x = \sin 2x$

$\Rightarrow \dfrac{d}{dx}\left(y^2\sin x\right) = \sin 2x \Rightarrow y^2\sin x = (-1/2)\cos 2x + C.$

So $\left(y(\pi/2)\right)^2\sin(\pi/2) = (-1/2)\cos(2\pi/2) + C \Rightarrow C = 1/2.$

Hence $y^2\sin x = (1/2)(1-\cos 2x) = \sin^2 x \Rightarrow y^2 = \sin x$

22. An equation of the curve satisfying $xdy - ydx = \sqrt{x^2 - y^2}\,dx$ and $y(1) = 0$ is

1) $y = x^2\log|\sin x|$
2) $y = x\sin(\log|x|)$
3) $y^2 = x(x-1)^2$
4) $y = 2x^2(x-1)$

Key. 2

Sol. The equation can be written as $x^2\dfrac{xdy - ydx}{x^2} = x\sqrt{1-(y/x)^2}\,dx$

$\Rightarrow \dfrac{d(y/x)}{\sqrt{1-(y/x)^2}} = \dfrac{dx}{x} \Rightarrow \sin^{-1} y/x = \log|x| + \text{const}$

Since $y(1) = 0$ so const =0. Hence $y = x\sin(\log|x|)$.

23. If for the differential equation $y^1 = \dfrac{y}{x} + \phi\left(\dfrac{x}{y}\right)$ the general solution is $y = \dfrac{x}{\log|Cx|}$ then $\phi(x/y)$ is given by

1) $-x^2/y^2$
2) y^2/x^2
3) x^2/y^2
4) $-y^2/x^2$

Key. 4

Sol. Putting $v = y/x$ so that $\dfrac{xdv}{dx} + v = \dfrac{dy}{dx}$, we have

$\dfrac{xdv}{dx} + v = v + \phi(1/v) \Rightarrow \dfrac{dv}{\phi(1/v)} = \dfrac{dx}{x}$

$\Rightarrow \log|Cx| = \int\dfrac{dv}{\phi(1/v)}$ (being constant of integration.)

But $y = \dfrac{x}{\log|Cx|}$ is the general solution so

$\dfrac{x}{y} = \dfrac{1}{v} = \int\dfrac{dv}{\phi(1/v)} \Rightarrow \phi(1/v) = -1/v^2 \Rightarrow \phi(x/y) = -y^2/x^2.$

24. The solution $y(x)$ of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$ is

1) $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$
2) $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$
3) $-\frac{\cos 3x}{3} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$
4) None of these

Key. 1

Sol. Integrating the given differential equation, we have $\frac{dy}{dx} = -\frac{\cos 3x}{3} + e^x + \frac{x^3}{3} + C_1$

but $y_1(0) = 1$ so $1 = -\frac{1}{3} + 1 + C_1 \Rightarrow C_1 = \frac{1}{3}$

Again integrating, we get $y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + C_2$

but $y(0) = 0$ so $0 = 1 + C_2 \Rightarrow C_2 = -1$. Thus $y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1.$

25. The orthogonal trajectories of the family of curves $a^{n-1}y = x^n$ are given by

1) $x^n + n^2y =$ constant 2) $ny^2 + x^2 =$ constant
3) $n^2x + y^n =$ constant 4) $n^2x - y^n =$ constant

Key. 2

Sol. Differentiating, we have $a^{n-1}\frac{dy}{dx} = nx^{n-1} \Rightarrow a^{n-1} = nx^{n-1}\frac{dx}{dy}$

Putting this value in the given equation, we have $nx^{n-1}\frac{dx}{dy}y = x^n$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we have $ny = -x\frac{dx}{dy}$

$\Rightarrow nydy + xdx = 0 \Rightarrow ny^2 + x^2 =$ constant. Which is the required family of orthogonal trajectories.

26. The solution of $\left(y\left(1+x^{-1}\right)+\sin y\right)dx + \left(x + \log x + x\cos y\right)dy = 0$ is

1) $\left(1 + y^{-1}\sin y\right) + x^{-1}\log x = C$
2) $\left(y + \sin y\right) + xy\log x = C$
3) $xy + y\log x + x\sin y = C$
4) None of these

Key. 3

Sol. The given equation can be written as

$y\left(1+x^{-1}\right)dx + \left(x + \log x\right)dy + \sin y dx + x\cos y dy = 0$

$\Rightarrow d\left(y\left(x + \log x\right)\right) + d\left(x\sin y\right) = 0$

$\Rightarrow y\left(x + \log x\right) + x\sin y = C$

27. The degree of the differential equation satisfying

$\sqrt{1+x^2}+\sqrt{1+y^2}=A\left(x\sqrt{1+y^2}-y\sqrt{1+x^2}\right)$ is

1) 2 2) 3 3) 4 4) None of these

Key. 4

Sol. Put $x=\tan\theta$ and $y=\tan\phi$. Then $\sqrt{1+x^2}=\sec\theta, \sqrt{1+y^2}=\sec\phi,$ and the equation becomes $\sec\theta+\sec\phi=A(\tan\theta\sec\phi-\tan\phi\sec\theta)$

$$\Rightarrow \frac{\cos\phi+\cos\theta}{\cos\theta\cos\phi}=A\left(\frac{\sin\theta-\sin\phi}{\cos\theta\cos\phi}\right)\Rightarrow 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}=2A\sin\frac{\theta-\phi}{2}\cos\frac{\theta+\phi}{2}$$

$$\Rightarrow \cot\frac{\theta-\phi}{2}=A\Rightarrow\theta-\phi=2\cot^{-1}A$$

$$\Rightarrow \tan^{-1}x-\tan^{-1}y=2\cot^{-1}A.$$

Differentiating this, we get $\frac{1}{1+x^2}-\left(\frac{1}{1+y^2}\right)\frac{dy}{dx}=0,$ which is a differential equation of degree 1.

28. A solution of $y=2x\left(\frac{dy}{dx}\right)+x^2\left(\frac{dy}{dx}\right)^4$ is

1) $y=2c^{1/2}x^{1/4}+c$ 2) $y=2\sqrt{c}x^2+c^2$ 3) $y=2\sqrt{c}\,(x+1)$ 4) $y=2\sqrt{cx}+c^2$

Key. 4

Sol. Writing $p=\frac{dy}{dx}$ and differentiating w.r.t.x, we have

$$p=2p+2x\frac{dp}{dx}+2xp^4+4p^3x^2\frac{dp}{dx}\Rightarrow 0=p(1+2xp^3)+2x\frac{dp}{dx}(1+2p^3x)$$

$$\Rightarrow p+2x\frac{dp}{dx}=0\Rightarrow 2\frac{dp}{p}=-\frac{dx}{x}$$

$$\Rightarrow 2\log p+\log x=\text{const}\Rightarrow p^2x=c \text{ or } p=\sqrt{\frac{c}{x}}$$

Substituting this value in the given equation, we get $y=2\sqrt{cx}+c^2$

29. The solution of $y_2-7y_1+12y=0$ is

1) $y=C_1e^{3x}+C_2e^{4x}$ 2) $y=C_1xe^{3x}+C_2e^{4x}$ 3) $y=C_1e^{3x}+C_2xe^{4x}$ 4) None of these

Key. 1

Sol. The given equation can be written as $\left(\frac{d}{dx}-3\right)\left(\frac{dy}{dx}-4y\right)=0$ ...... (i)

If $\frac{dy}{dx}-4y=u$ then (I) reduces to $\frac{du}{dx}-3u=0$

$\Rightarrow \frac{du}{u} = 3dx \Rightarrow u = C_1 e^{3x}$. Therefore, we have $\frac{dy}{dx} - 4y = C_1 e^{3x}$ which is a linear equation whose $I.F.$ is e^{-4x}. So $\frac{d}{dx}\left(ye^{-4x}\right) = C_1 e^{-x}$

$\Rightarrow ye^{-4x} = -C_1 e^{-x} + C_2 \Rightarrow y = C_1 e^{3x} + C_2 e^{4x}$

30. The curves satisfying the differential equation $\left(1-x^2\right)y^1 + xy = ax$ are

1) ellipses and hyperbolas
2) ellipses and parabola
3) ellipses and straight lines
4) circles and ellipses

Key. 1

Sol. The given equation is linear in y and can be written as

$$\frac{dy}{dx} + \frac{x}{1-x^2}y = \frac{ax}{1-x^2}$$

Its integrating factor is $e^{\int \frac{x}{1-x^2}dx} = e^{-(1/2)\log\left(1-x^2\right)} = \frac{1}{\sqrt{1-x^2}}$ if $-1 < x < 1$ and if $x^2 > 1$ then

$$I.F. = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(y\frac{1}{\sqrt{1-x^2}}\right) = \frac{ax}{\left(1-x^2\right)^{3/2}} = -\frac{1}{2}a\frac{-2x}{\left(1-x^2\right)^{3/2}}$$

$$\Rightarrow y\frac{1}{\sqrt{1-x^2}} = \frac{a}{\sqrt{1-x^2}} + C \Rightarrow y = a + C\sqrt{1-x^2}$$

$$\Rightarrow (y-a)^2 = C^2\left(1-x^2\right) \Rightarrow (y-a)^2 + C^2x^2 = C^2$$

Thus if $-1 < x < 1$ the given equation represents an ellipse. If $x^2 > 1$ then the solution is of the form $-(y-a)^2 + C^2x^2 = C^2$ which represents a hyperbola.

31. Solution of the differential equation : $\frac{dy}{dx} = \frac{3x^2y^4 + 2xy}{x^2 - 2x^3y^3}$ is

(A) $x^2y^2 + \frac{x^2}{y} = c$
(B) $x^3y^2 + \frac{x^2}{y} = c$
(C) $x^3y^2 + \frac{y^2}{x} = c$
(D) $x^2y^3 + \frac{x^2}{y} = c$

Key. B

Sol.

$$x^2dy - 2x^3y^3dy = 3x^2y^4dx + 2xydx$$

$$\Rightarrow x^2dy - 2xydx = 3x^2y^4dx + 2x^3y^3dy$$

$$\Rightarrow \frac{2xydx - x^2dy}{y^2} + 3x^2y^2dx + 2x^3ydy = 0$$

$$\Rightarrow d\left(\frac{x^2}{y}\right) + d\left(x^3y^2\right) = 0$$

$$\Rightarrow \frac{x^2}{y} + x^3y^2 = C$$

32. Solution to the differential equation $\dfrac{x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \ldots\ldots..}{1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \ldots\ldots} = \dfrac{dx - dy}{dx + dy}$ is

(A) $2y\, e^{2x} = C.e^{2x} + 1$ (B) $2y\, e^{2x} = C.e^{2x} - 1$

(C) $y\, e^{2x} = C.e^{2x} + 2$ (D) $2x\, e^{2y} = C.e^{x} - 1$

Key. B

Sol. Applying C and D, we get

$\dfrac{dy}{dx} = \dfrac{e^{-x}}{e^{x}} = e^{-2x} \Rightarrow 2y = -e^{-2x} + C$

or $2y\, e^{2x} = C.e^{2x} - 1$.

33. The curve for which the x-intercept of the tangent drawn at any point P on the curve is three times the x-coordinate of the point P, is

(A) $xy = c$ (B) $xy^2 = c$ (C) $xy^3 = c$ (D) $x^2y = c$

Key. B

Sol. $x - y\dfrac{dx}{dy} = 3x \Rightarrow \dfrac{dx}{x} + 2\dfrac{dy}{y} = 0$

$\Rightarrow \ln(xy^2) = k \Rightarrow xy^2 = c$.

34. A curve is such that the mid point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets y-axis, lies on the line $y = x$. If the curve passes through (1, 0), then the curve is

(A) $2y = x^2 - x$ (B) $y = x^2 - x$ (C) $y = x - x^2$ (D) $y = 2(x - x^2)$

Key. C

Sol. The point on y-axis is $\left(0,\ y - x\dfrac{dy}{dx}\right)$.

According to given condition,

$\dfrac{x}{2} = y - \dfrac{x}{2}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = 2\dfrac{y}{x} - 1$

putting $\dfrac{y}{x} = v$ we get

$x\frac{dv}{dx}=v-1 \Rightarrow \ln\left|\frac{y}{x}-1\right|=\ln|x|+c \Rightarrow 1-\frac{y}{x}=x$ (as f (1) = 0).

35. The family of curves passing through (0,0) and satisfying the differential equation $\frac{y_2}{y_1}=1$ (where $y_n = d^n y/dx^n$) is

a) $y=k$ b) $y=kx$ c) $y=k(e^x+1)$ d) $y=k(e^x-1)$

Key. D

Sol. $\frac{dp}{dx}=P\left(\text{where } p=\frac{dy}{dx}\right)$

$\ln P = x+c \Rightarrow p=e^{x+c}$

$\frac{dy}{dx}=ke^x$

$y=ke^x+\lambda$

Satisfying $(0,0), So\, \lambda=-k$

$y=k\left(e^x-1\right)$

36. The degree of the differential equation satisfying the relation $\sqrt{1+x^2}+\sqrt{1+y^2}=\lambda(x\sqrt{1+y^2}-y\sqrt{1+x^2})$ is

a) 1 b) 2 c) 3 d) none of these

Key. A

Sol. On Putting $x=\tan A, y=\tan B$ we get

$\sec A+\sec B=\lambda\left(\tan A\sec B-\tan B\sec A\right)$

$\cos A+\cos B=\lambda\left(\sin A-\sin B\right)$

$\tan\left(\frac{A-B}{2}\right)=\frac{1}{\lambda}$

$\tan^{-1}x-\tan^{-1}y=2\tan^{-1}\frac{1}{\lambda}$

On differentiating $\frac{1}{1+x^2}-\frac{1}{1+y^2}\frac{dy}{dx}=0$

37. S1: The differential equation of parabolas having their vertices at the origin and foci on the x-axis is an equation whose variables are separable

S2: The differential equation of the straight lines which are at a fixed distance p from the origin is an equation of degree 2

S3: The differential equation of all conics whose both axes coincide with the axes of coordinates is an equation of order 2

a) TTT b) TFT c) FFT d) TTF

Key. A

Sol. S_1 - Equation of parabola is $y^2 = \pm 4ax$

$2y\frac{dy}{dx} = \pm 4a$

D.E of parabola $\Rightarrow y^2 = 2yx\frac{dy}{dx}$

$2\frac{dy}{y} = \frac{dx}{x}$

Which is variable seperable

S_2 - Equation of line which is fixed distance. P from origin can be equation of tangent to circle $x^2 + y^2 = p^2$

Line is $y = mx + p\sqrt{1+m^2}$ $\left(m = \frac{dy}{dx}\right)$

$\left(y - x\frac{dy}{dx}\right)^2 = P\left(1 + \left(\frac{dy}{dx}\right)^2\right)$

So, degree is 2

S_3 - Equation of conic whose both axis co-incide with co-ordinate axis is $ax^2 + by^2 = 1$

As there are two constants, so order of D.E is 2

38. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

a) 1 b) 2 c) 3 d) 4

Key. A

Sol. The parametric form of the given equation is $x = t, y = t^2$. The equation of any tangent at t is $2xt = y + t^2$, Differentiating we get $2t = y_1\left(= \frac{dy}{dx}\right)$ putting this value in the above equation, we have $2x\frac{y_1}{2} = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1^2$

The order of this equation is 1

Hence (A) is the correct answer

39. The solution of the differential equation $\frac{\sqrt{x}dx + \sqrt{y}dy}{\sqrt{x}dx - \sqrt{y}dy} = \sqrt{\frac{y^3}{x^3}}$ is given by

a) $\frac{3}{2}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}+c=0$

b) $\frac{2}{3}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\frac{y}{x}+c=0$

c) $\frac{2}{3}\left(\frac{y}{x}\right)+\log\left(\frac{x+y}{x}\right)+\tan^{-1}\left(\frac{y^{3/2}}{x^{3/2}}\right)+c=0$

d) None of these

Key. D

Sol. We have $\frac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\frac{y^3}{x^3}}$

$$\Rightarrow \frac{d(x^{3/2})+d(y^{3/2})}{d(x^{3/2})-d(y^{3/2)}}=\frac{y^{3/2}}{x^{3/2}}$$

$$\Rightarrow \frac{du+dv}{du-dv}=\frac{v}{u},\text{ where } u=x^{3/2} \text{ and } v=y^{3/2}$$

$$\Rightarrow udu+udv=vdu-vdv \Rightarrow udu+vdv=vdu-udv$$

$$\Rightarrow \frac{udu+vdv}{u^2+v^2}=\frac{vdu-udv}{u^2+v^2}$$

$$\Rightarrow \frac{d(u^2+v^2)}{u^2+v^2}=-2d\tan^{-1}\left(\frac{v}{u}\right)+c$$

On integrating, we get

$$\log(u^2+v^2)=-2\tan^{-1}\left(\frac{v}{u}\right)+c$$

$$\Rightarrow \frac{1}{2}\log(x^3+y^3)+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}=\frac{c}{2}$$

40. A function $y=f(x)$ has a second order derivative $f''=6(x-1)$. If its graph passes through the point(2,1) and at that point the tangent to the graph is $y=3x-5$, then the function is

a) $(x-1)^2$ b) $(x-1)^3$ c) $(x+1)^2$ d) $(x+1)^3$

Key. B

Sol. Since $f''(x)=6(x-1)$

$\Rightarrow f'(x)=3(x-1)^2+c$ (integrating) ----(i)

Also, at the point (2,1), the tangent to the graph is $y=3x-5$ and slope of the tangent = 3

$\Rightarrow f'(2)=3$

$3(2-1)^2+c=3$ [from Eq (i)

$\Rightarrow 3+c=3\Rightarrow c=0$

From Eq (i) we have

$f'(x)=3(x-1)^2$

$\Rightarrow f(x)=(x-1)^2+k$ (Integrating) ----(ii)

$\therefore 1=(2-1)^2+k\Rightarrow k=0$

Hence the equation of the function is $f(x)=(x-1)^3$.

41. The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$, where $c>0$ is a parameter, is of order and degree as follows :

a) order 1, degree 3 b) order 2, degree 3

c) order 1, degree 2 d) order 2, degree 1

Key. A

Sol. Differentiating, we get

$2yy'=2c\Rightarrow c=yy'$

$\therefore y^2=2yy'(x+\sqrt{yy'})\Rightarrow (y^2-2yy'x)^2=4(yy')^3$

$\Rightarrow$ degree= 3 and order =1

42. The normal to a curve at $P(x,y)$ meets the $x-axis$ at G. If the distance of G from the origin is twice the abscissa of P, then the curve is

a) a parabola b) a circle c) a hyperbola d) an ellipse

Key. C

Sol. The slope of the tangent = $\frac{dy}{dx}$

$\therefore$ the slope of the normal $=-\frac{dx}{dy}$

$\therefore$ the equation of the normal is $Y - y = -\dfrac{dx}{dy}(X - x)$

This meets the x-axis $(Y = 0)$, where

$$-y = \frac{-dx}{dy}(X - x) \Rightarrow X = x + y\frac{dy}{dx}$$

$\therefore$ G is $\left(x + y\dfrac{dy}{dx}, 0\right)$

$\therefore$ $OG = 2x \Rightarrow x + y\dfrac{dy}{dx} = 2x$

$\Rightarrow y\dfrac{dy}{dx} = x \Rightarrow ydy = xdx$ [variable separable integrating, we get]

$$\frac{y^2}{2} = \frac{x^2}{2} + constant$$

$\Rightarrow x^2 - y^2 = c$, which is a hyperboal

43. Which of the following transformation reduces the differential equation $\dfrac{dz}{dx} + \dfrac{z}{x}\log z = \dfrac{z}{x^2}(\log z)^2$ to the form $P(x)u = Q(x)$

a) $u = \log z$ b) $u = e^z$ c) $u = (\log z)^{-1}$ d) $u = (\log z)^2$

Key. C

Sol. Given equ. Can be written as

$$\frac{dz}{z(\ln z)^2} + \frac{dx}{x(\ln z)} = \frac{dx}{x^2}$$

Put $u = \dfrac{1}{\ln z}$

$$\frac{du}{dx} = -\frac{1}{(\ln z)^2}.\frac{1}{z}.\frac{dz}{dx}$$

$$\therefore -\frac{du}{dx} + \frac{1}{x}.u = \frac{1}{x^2}$$

$$\frac{du}{dx} + \left(-\frac{1}{x}\right)u = \left(-\frac{1}{x^2}\right)$$

$$I.F. = e^{\int -\frac{1}{x}dx} = e^{\ln\frac{1}{x}} = \frac{1}{x}$$

$$u.\frac{1}{x}=\int -\frac{1}{x^2}.\frac{1}{x}dx$$

$$\frac{u}{x}=-\int \frac{1}{x^3}dx=+\frac{1}{4x^4}+c$$

44. If $y_1(x)$ is a solution of the differential equation $dy/dx-f(x)y=0$, then a solution of the differential equation $\frac{dy}{dx}+f(x)y=r(x)$

a) $\frac{1}{y_1(x)}\int r(x)y_1(x)dx$

b) $y_1(x)\int \frac{r(x)}{y_1(x)}dx$

c) $\int r(x)y_1(x)dx$

d) none of these

Key. A

Sol. $\frac{dy}{dx}-f(x).y=0$

$$\frac{dy}{y}=f(x)dx$$

$$\ln y=\int f(x)dx$$

$y_1(x)=e^{\int f(x)dx}$ Then for given equation I.F= $e^{\int f(x)dx}$

Hence Solution $y.y_1(x)=\int r(x).y_1(x)dx$

$$y=\frac{1}{y_1(x)}\int r(x).y_1(x)dx$$

45. Solution of the differential equation : $\frac{dy}{dx}=\frac{3x^2y^4+2xy}{x^2-2x^3y^3}$ is

(A) $x^2y^2+\frac{x^2}{y}=c$

(B) $x^3y^2+\frac{x^2}{y}=c$

(C) $x^3y^2+\frac{y^2}{x}=c$

(D) $x^2y^3+\frac{x^2}{y}=c$

Key. B

Sol.

$$x^2 dy - 2x^3y^3 dy = 3x^2y^4 dx + 2xy dx$$

$$\Rightarrow x^2 dy - 2xy dx = 3x^2y^4 dx + 2x^3y^3 dy$$

$$\Rightarrow \frac{2xy dx - x^2 dy}{y^2} + 3x^2y^2 dx + 2x^3 y dy = 0$$

$$\Rightarrow d\left(\frac{x^2}{y}\right) + d\left(x^3y^2\right) = 0$$

$$\Rightarrow \frac{x^2}{y} + x^3y^2 = C$$

46. Solution to the differential equation $\dfrac{x+\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+........}{1+\dfrac{x^2}{2!}+\dfrac{x^4}{4!}+......} = \dfrac{dx-dy}{dx+dy}$ is

(A) $2y\,e^{2x} = C.e^{2x} + 1$ (B) $2y\,e^{2x} = C.e^{2x} - 1$

(C) $y\,e^{2x} = C.e^{2x} + 2$ (D) $2x\,e^{2y} = C.e^{x} - 1$

Key. B

Sol. Applying C and D, we get

$\dfrac{dy}{dx} = \dfrac{e^{-x}}{e^{x}} = e^{-2x} \Rightarrow 2y = -e^{-2x} + C$

or $2y\,e^{2x} = C.e^{2x} - 1$.

47. The curve for which the x-intercept of the tangent drawn at any point P on the curve is three times the x-coordinate of the point P, is

(A) $xy = c$ (B) $xy^2 = c$ (C) $xy^3 = c$ (D) $x^2y = c$

Key. B

Sol. $x - y\dfrac{dx}{dy} = 3x \Rightarrow \dfrac{dx}{x} + 2\dfrac{dy}{y} = 0$

$\Rightarrow \ln(xy^2) = k \Rightarrow xy^2 = c$.

48. A curve is such that the mid point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets y-axis, lies on the line y = x. If the curve passes through (1, 0), then the curve is

(A) $2y = x^2 - x$ (B) $y = x^2 - x$ (C) $y = x - x^2$ (D) $y = 2(x - x^2)$

Key. C

Sol. The point on y-axis is $\left(0,\ y - x\dfrac{dy}{dx}\right)$.

According to given condition,

$\dfrac{x}{2} = y - \dfrac{x}{2}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = 2\dfrac{y}{x} - 1$

putting $\frac{y}{x} = v$ we get

$x\frac{dv}{dx} = v - 1 \Rightarrow \ln\left|\frac{y}{x} - 1\right| = \ln|x| + c \Rightarrow 1 - \frac{y}{x}$ = x (as f (1) = 0).

49. The solution curves of the differential equation $(xdx + ydy)\sqrt{x^2 + y^2} = (xdy - ydx)\left(\sqrt{1 - x^2 - y^2}\right)$ are

a) circles of radius 1, passing through the origin

b) circles of radius $\frac{1}{2}$,passing through the origin

c) circles not passing through origin

d) solution curve is not a circle

Key: B

Hint: $x = r\cos\theta,\ x = r\cos\theta$

$\frac{dr}{\sqrt{1 - r^2}} = d\theta \Rightarrow \sin^{-1} r = \theta + \alpha$

$r = \sin(\theta + \alpha)$

$\Rightarrow x^2 + y^2 - x\sin\alpha - y\cos\alpha = 0$

$\therefore \text{radius} = \frac{1}{2}$

50. A conic C satisfies the differential equation $(1 + y^2)dx - xydy = 0$ and passes through the point (1,0). An ellipse E which is confocal with C has its eccentricity $\sqrt{2/3}$. The angle of intersection of the curves C and E is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

Key: D

Hint The conic C is $x^2 - y^2 = 1$ *and* E *is* $x^2 + 3y^2 = 3$,Angle of intersection is $\pi/2$

51. The solution of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$ is

(A) $\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$ (B) $2\tan^{-1}\left(\frac{x}{y}\right) + \ln x + C = 0$

(C) $\ln\left(y + \sqrt{x^2 + y^2}\right) + \ln y + C = 0$ (D) $\ln\left(x + \sqrt{x^2 + y^2}\right) + C = 0$

(Where C is arbitrary constant)

Key: A

Hint: $\frac{dx}{dy} + \frac{x^2}{y^2} - \frac{x}{y} + 1 = 0$

Put $x = vy$ where v is a function of 'y'

$$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\therefore v + y\frac{dv}{dy} + v^2 - v + 1 = 0 \Rightarrow -y\frac{dv}{dy} = \left(1+v^2\right) \Rightarrow \frac{dv}{v^2+1} = -\frac{dy}{y}$$

$$\tan^{-1}v + C = -\ln y \Rightarrow \tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$$

Where C is arbitrary constant

52. The solution of the differential equation $2x^3ydy + \left(1-y^2\right)\left(x^2y^2 + y^2 - 1\right)dx = 0$

[Where c is a constant]

(A) $x^2y^2 = (cx+1)\left(1-y^2\right)$
(B) $x^2y^2 = (cx+1)\left(1+y^2\right)$
(C) $x^2y^2 = (cx-1)\left(1-y^2\right)$
(D) none of these

Key : C

Hint : $\frac{2y}{\left(1-y^2\right)^2} \cdot \frac{dy}{dx} + \frac{y^2}{1-y^2}\frac{1}{x} = \frac{1}{x^3}$

Put $\frac{y^2}{1-y^2} = t \Rightarrow \frac{2y}{\left(1-y^2\right)^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow$ $\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^3}$

$\Rightarrow$ $t \cdot x = \int \frac{1}{x^2}dx + c$

$\Rightarrow$ $x^2y^2 = (cx-1)\left(1-y^2\right).$

53. The solution of differential equation $(x^5 + x + 2x^2y)\, dy + (3x^4y - y)\, dx = 0$ is
(A) $x^4y + xy^2 + y = cx$
(B) $x^4y^2 + xy + y = cx$
(C) $x^4y + x^2y^2 + xy = c$
(D) $x^4y - xy^2 - y = cx$

Key : A

Sol : Divide both sides by x^2

$$x^3dy + y.3x^2dx + \frac{xdy - ydx}{x^2} + 2ydy = 0$$

$\Rightarrow$ $d(x^3y) + d\left(\frac{y}{x}\right) + d(y^2) = 0$

$\Rightarrow$ $x^4y + xy^2 + y = cx$

$\Rightarrow$ 2. The equation to the curve which is such that portion of the axis of *x* cutoff

$\Rightarrow$ between the origin and the tangent at any point is proportional to the

⇒ ordinate of that point is (constant of proportionality is K)

⇒ a) x=y(C-K log y)

b) log x = Ky^2+C

⇒ c) x^2=y(C- K log y)

d) None of these

⇒ Key: A

⇒ Hint $x - y\dfrac{dx}{dy} = ky$

54. Let a function f(x) be such that f ′′(x) = f′(x) + e^x and f(0) = 0, f ′(0) = 1 , then ln$\left(\dfrac{(f(2))^2}{4}\right)$ equal to

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) 4

⇒ Key: D

⇒ Hint f′′(x) – f′(x) = e^x

⇒ put f′(x) = v

⇒ $\dfrac{dv}{dx} + v(-1) = e^x$

⇒ ⇒ $ve^{-x} = \int e^x.e^{-x}dx$

⇒ $ve^{-x} = x + C_1$, f ′ (0) = 1 ⇒ $C_1 = 1$

⇒ f′(x) = $xe^x + e^x$

⇒ f(x) = $xe^x + C_2$

⇒ ⇒ f(0) = 0 ⇒ $C_2 = 0$

⇒ ⇒ f(x) = xe^x ⇒ f(2) = $2e^2$

⇒ ln $\left(\dfrac{(f(2))^2}{4}\right)$ = 4.

55. The solution of $\dfrac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1, y(0) = 1;$ is given by

$2(1+x^2) - \dfrac{1}{(x+y)^2} = f(x)$ where $f(x)$ =

a) e^{-x} b) e^{-x^2} c) e^x d) e^{x^2}

Key. D

Sol. Put $x+y=Y$ then equation given becomes $\dfrac{dY}{dx} + Y = x^3Y^3$.

$$\Rightarrow \frac{1}{Y^3}\frac{dY}{dx}+\frac{x}{Y^2}=x^3 \;\; putting \;\; z=\frac{1}{Y^2} \;\; makes\,it\,\frac{dz}{dx}-2xz=-2x^3$$

$$\Rightarrow z=2+2x^2+ce^{x^2}$$

$$\Rightarrow \frac{1}{(x+y)^2}=2+2x^2+ce^{x^2} \; putting\; x=0, y=1\, gives\, c=-1$$

56. The solution of the differential equation $\dfrac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\dfrac{y^3}{x^3}}$ is given by

a) $\dfrac{3}{2}\log\left(\dfrac{y}{x}\right)+\log\left|\dfrac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\left(\dfrac{y}{x}\right)^{3/2}+c=0$

b) $\dfrac{2}{3}\log\left(\dfrac{y}{x}\right)+\log\left|\dfrac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\dfrac{y}{x}+c=0$

c) $\dfrac{2}{3}\left(\dfrac{y}{x}\right)+\log\left(\dfrac{x+y}{x}\right)+\tan^{-1}\left(\dfrac{y^{3/2}}{x^{3/2}}\right)+c=0$

d) None of these

Key. D

Sol. We have $\dfrac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\dfrac{y^3}{x^3}}$

$$\Rightarrow \frac{d(x^{3/2})+d(y^{3/2})}{d(x^{3/2})-d(y^{3/2)}}=\frac{y^{3/2}}{x^{3/2}}$$

$$\Rightarrow \frac{du+dv}{du-dv}=\frac{v}{u}, \text{ where } u=x^{3/2} \text{ and } v=y^{3/2}$$

$$\Rightarrow udu+udv=vdu-vdv \Rightarrow udu+vdv=vdu-udv$$

$$\Rightarrow \frac{udu+vdv}{u^2+v^2}=\frac{vdu-udv}{u^2+v^2}$$

$$\Rightarrow \frac{d(u^2+v^2)}{u^2+v^2}=-2d\tan^{-1}\left(\frac{v}{u}\right)+c$$

On integrating, we get

$$\log(u^2+v^2)=-2\tan^{-1}\left(\frac{v}{u}\right)+c$$

$$\Rightarrow \frac{1}{2}\log(x^3+y^3)+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}=\frac{c}{2}$$

57. A function $y = f(x)$ has a second order derivative $f'' = 6(x-1)$. If its graph passes through the point(2,1) and at that point the tangent to the graph is $y = 3x-5$, then the function is

a) $(x-1)^2$ b) $(x-1)^3$ c) $(x+1)^2$ d) $(x+1)^3$

Key. B

Sol. Since $f''(x) = 6(x-1)$

$\Rightarrow f'(x) = 3(x-1)^2 + c$ (integrating) ----(i)

Also, at the point (2,1), the tangent to the graph is $y = 3x-5$ and slope of the tangent = 3

$\Rightarrow f'(2) = 3$

$3(2-1)^2 + c = 3$ [from Eq (i)

$\Rightarrow 3 + c = 3 \Rightarrow c = 0$

From Eq (i) we have

$f'(x) = 3(x-1)^2$

$\Rightarrow f(x) = (x-1)^2 + k$ (Integrating) ----(ii)

$\therefore 1 = (2-1)^2 + k \Rightarrow k = 0$

Hence the equation of the function is $f(x) = (x-1)^3$.

58. The differential equation representing the family of curves $y^2 = 2c(x+\sqrt{c})$, where $c > 0$ is a parameter, is of order and degree as follows :

a) order 1, degree 3 b) order 2, degree 3

c) order 1, degree 2 d) order 2, degree 1

Key. A

Sol. Differentiating, we get

$2yy' = 2c \Rightarrow c = yy'$

$\therefore y^2 = 2yy'(x+\sqrt{yy'}) \Rightarrow (y^2 - 2yy'x)^2 = 4(yy')^3$

$\Rightarrow$ degree= 3 and order =1

59. The normal to a curve at $P(x, y)$ meets the $x-axis$ at G. If the distance of G from the origin is twice the abscissa of P, then the curve is

a) a parabola b) a circle c) a hyperbola d) an ellipse

Key. C or D

Sol. The slope of the tangent = $\frac{dy}{dx}$

$\therefore$ the slope of the normal $= -\frac{dx}{dy}$

$\therefore$ the equation of the normal is $Y - y = -\frac{dx}{dy}(X - x)$

This meets the x-axis $(Y = 0)$, where

$-y = \frac{-dx}{dy}(X - x) \Rightarrow X = x + y\frac{dy}{dx}$

$\therefore$ G is $\left(x + y\frac{dy}{dx}, 0\right)$

$\therefore$ $OG = 2x \Rightarrow x + y\frac{dy}{dx} = 2x$

$\Rightarrow y\frac{dy}{dx} = x \Rightarrow ydy = xdx$ [variable separable integrating, we get]

$\frac{y^2}{2} = \frac{x^2}{2} + constant$

$\Rightarrow x^2 - y^2 = c$, which is a hyperboal

60. The general solution of $x\frac{dy}{dx} + (\log x)y = x^{1-\frac{1}{2}\log x}$ is

a) $y = x^{1-\frac{1}{2}\log x} + cx^{-\frac{1}{2}\log x}$

b) $y.x^{\frac{1}{2}\log x} = x^{\frac{1}{2}\log x} + c$

c) $y = e^{\frac{(\log x)^2}{2}}(x + c)$

d) $y = e^{\frac{1}{2}(\log x)^2}(x^{1-\frac{1}{2}(\log x)} - x^{-\frac{1}{2}(\log x)}) + c$

Key. A

Sol. $\frac{dy}{dx} + \frac{\log x}{x}y = x^{-\frac{1}{2}\log x}$

$I.F = e^{\int \frac{\log x}{x}dx} = e^{\frac{(\log x)^2}{2}} = \left(e^{\log x}\right)^{\frac{\log x}{2}} = x^{\frac{1}{2}\log x}$

$G.S is x^{\frac{1}{2}\log x}.y = \int dx$

$y x^{\frac{1}{2}\log x} = x + c$

61. Solution of $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$

a) $\frac{y}{x}-\frac{1}{2\left(x^2+y^2\right)}=c$ b) $\frac{y}{x}+\frac{1}{2\left(x^2+y^2\right)}=c$

c) $\frac{x}{y}+\frac{1}{2\left(x^2+y^2\right)}=c$ d) $\frac{x}{y}-\frac{1}{2\left(x^2+y^2\right)}=c$

Key. A

Sol. $\frac{xdx+ydy}{ydx-xdy}=\frac{x^4+2x^2y^2+y^4}{x^2}$

$\frac{2xdx+2ydy}{2\left(x^2+y^2\right)^2}=-d\left(\frac{y}{x}\right)$

$\frac{xdx+ydy}{\left(x^2+y^2\right)^2}=\frac{ydx-xdy}{x^2}$

$\frac{-1}{x^2+y^2}=-2\frac{y}{x}+c$

$2\frac{y}{x}-\frac{1}{x^2+y^2}=c$

62. Solution of $\frac{y+\sin x\cos^2(xy)}{\cos^2(xy)}dx+\left(\frac{x}{\cos^2(xy)}+\sin y\right)dy=0$

a) $\tan(xy)+\cos x+\cos y=c$ b) $\tan(xy)-\cos x-\cos y=c$

c) $\tan(xy)+\cos x-\cos y=c$ c) $\tan(xy)-\cos x+\cos y=c$

Key. B

Sol. $\frac{y\,dx+x\,dy}{\cos^2(xy)}+\sin xdx+\sin ydy=0$

$\int\frac{d(xy)}{\cos^2(xy)}+\int\sin x\,dx+\int\sin y\,dy=0$

$\tan(xy)-\cos x-\cos y=c$

63. The equation of the curve which is passing through (1, 1) and whose differential equation is $\frac{dy}{dx}+\frac{y}{x}=y^3$ is

(A) $2x^2y^2-xy^2=1$ (B) $2xy^2+x^2y^2=1$

(C) $2x^2y^2+xy^2=1$ (D) $2xy^2-x^2y^2=1$

Key. D

Sol. $\frac{dy}{dx}+\frac{y}{x}=y^3 \Rightarrow \frac{1}{y^3}.\frac{dy}{dx}+\frac{1}{xy^2}=1$

Put t = $\frac{1}{y^2}$

$\Rightarrow \frac{dt}{dx}-\frac{2t}{x}=-2$

$\Rightarrow$ Solution is t. $\frac{1}{x^2}$ = $-2\int\frac{1}{x^2}dx$

$\Rightarrow 2xy^2+cx^2y^2=1$

Since this curve passes through (1, 1)

$\Rightarrow$ c = 1.

64. The solution of $\frac{dy}{dx}+x(x+y)=x^3(x+y)^3-1, y(0)=1;$ is given by

$2(1+x^2)-\frac{1}{(x+y)^2}$ = $f(x)$ where $f(x)$ =

a) e^{-x} b) e^{-x^2} c) e^{x} d) e^{x^2}

Key. D

Sol. Put $x+y=Y$ then equation given becomes $\frac{dY}{dx}+Y=x^3Y^3$.

$$\Rightarrow \frac{1}{Y^3}\frac{dY}{dx}+\frac{x}{Y^2}=x^3 \text{ putting } z=\frac{1}{Y^2} \text{ makes it } \frac{dz}{dx}-2xz=-2x^3$$

$$\Rightarrow z=2+2x^2+ce^{x^2}$$

$$\Rightarrow \frac{1}{(x+y)^2}=2+2x^2+ce^{x^2} \text{ putting } x=0, y=1 \text{ gives } c=-1.$$

65. If the solution of differential equation $\frac{dy}{dx}=\frac{ax+3}{2y+f}$ represents a circle of non zero radius then

a) $a=2; 9+4f^2>4c$ b) $a=-2; 9+4f^2<4c$

c) $a=2; 9+4f^2<4c$ d) $a=-2; 9+4f^2>4c$

Key. D

Sol. $(2y+f)dy=(ax+3)dx$ on integration

$$y^2+fy+c=\frac{ax^2}{2}+3x \Rightarrow -\frac{ax^2}{2}+y^2+fy-3x+c=0$$

$$\Rightarrow a=-2,\ 9+4f^2-4c>0.$$

66. Solution of $\left(xe^{\frac{y}{x}}-y\text{Sin}\frac{y}{x}\right)dx+x\text{Sin}\frac{y}{x}dy=0$

a) $\log x=c+\frac{1}{2}e^{\frac{-y}{x}}\left(\text{Sin}\frac{y}{x}+\text{Cos}\frac{y}{x}\right)$ b) $\log x=c+\frac{1}{2}e^{\frac{y}{x}}\left(\text{Sin}\frac{y}{x}-\text{Cos}\frac{y}{x}\right)$

c) $\log x=c+\frac{1}{2}e^{\frac{y}{x}}\left(\text{Sin}\frac{y}{x}+\text{Cos}\frac{y}{x}\right)$ d) $\log x=c+\frac{1}{2}e^{\frac{-y}{x}}\left(\text{Sin}\frac{y}{x}-\text{Cos}\frac{y}{x}\right)$

Key. A

Sol. Put y =vx $\Rightarrow \int\frac{dx}{x}+\int e^{-v}\text{Sin}v\,dv=c$

$$\Rightarrow \int e^{-v}\sin v\,dv=-\frac{e^{-v}}{2}(\text{Sin}v+\text{Cos}v)$$

$$\Rightarrow \log x = c + \frac{1}{2} e^{\frac{-y}{x}} \left(\text{Sin}\frac{y}{x} + \text{Cos}\frac{y}{x} \right)$$

67. The solution of differential equation $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$

a) $y = \tan\left(\frac{c - x - y}{2}\right)$ b) $y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$

c) $y = x\tan\left(\frac{c + x^2 + y^2}{2}\right)$ d) $y = x\tan\left(\frac{c + x^2 - y^2}{2}\right)$

Key. B

Sol. Given equation

$$\frac{1}{2}d\left(x^2 + y^2\right) + d\left(\tan^{-1}\frac{y}{x}\right) = 0 \Rightarrow y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$$

68. The differential equation of all circles in a plane must be

a) $y_3(1 + y_1^2) - 3y_1y_2^2 = 0$ b) of order 3 and degree 3

c) of order 3 and degree 2 d) $y_3(1 - y_1^2) - 3y_1y_2^2 = 0$

Key. A

Sol. Equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Differentiating two times w.r.t. x

we have f $= -\frac{1 + yy_2 + y_1^2}{y_2}$

Again differentiating

$$\frac{y_2\left[2y_1y_2 + y_1y_2 + yy_3\right] - \left[y_3(1 + y_1^2 + yy_2)\right]}{y_2^2} = 0$$

$$\Rightarrow \quad 3y_1 yy_1^2 + yy_2y_3 - y_3 - y_3y_1^2 - yy_2y_3 = 0$$

$$\Rightarrow \quad 3y_1 y_2^2 = y_3\left[1 + y_1^2\right]$$

69. If the population of a country doubles in 50 years in how many years will it become thrice the original, assume the rate of increase is proportional to the number of inhabitants

a) 75 b) $50\log_2 3$ c) $50\log_3 2$ d) 100

Key. B

Sol. P – Population, y – population after ‘t’ years

$$\frac{dy}{dt} \alpha y \Rightarrow \frac{dy}{dt} = ky \Rightarrow \int \frac{dy}{y} = \int kdt \Rightarrow \log y = kt + c$$

$$t = 0 \,\&\, y = p \Rightarrow \log p = 0 + c$$

$$t = 0 \,\&\, y = 2p \Rightarrow \log 2p = 50k + \log p$$

$$\Rightarrow \log\left(\frac{2p}{p}\right) = 50k \Rightarrow k = \frac{\log(2)}{50}$$

$$t = ? \,\&\, y = 3p \Rightarrow kt = \log y - c$$

$$\left(\frac{\log 2}{50}\right)t = \log 3p - \log p$$

$$t = \frac{\log 3}{(\log 2/50)} = 50 \log_2 3$$

70. The equation of the curve not passing through the origin and having the portion of the tangent included between the coordinate axes is bisected at the point of contact is

a) a parabola
b) an ellipse or a straight line
c) a circle or an ellipse
d) a hyperbola or a straight line

Key. D

Sol. Equation of line passing through P (x_1, y_1)

Y- y_1 = $\frac{dy}{dx}$ (X- x_1), x-int = x_1 - $\frac{y_1}{m}$, y-int = y_1 - x_1 m,

according to condition

$$\left(x - \left(x - y\frac{dx}{dy}\right)\right)^2 + y^2 = \left(y - \left(y - x\frac{dy}{dx}\right)\right)^2 + x^2$$

$$\Rightarrow \left(x\frac{dy}{dx} - y\right)\left(x\frac{dy}{dx} + y\right) = 0$$

that is y=cx or xy=c

71. Solution of the differential equation : $\frac{dy}{dx} = \frac{3x^2y^4 + 2xy}{x^2 - 2x^3y^3}$ is

(A) $x^2y^2 + \frac{x^2}{y} = c$
(B) $x^3y^2 + \frac{x^2}{y} = c$
(C) $x^3y^2 + \frac{y^2}{x} = c$
(D) $x^2y^3 + \frac{x^2}{y} = c$

Key. B

Sol.

$$x^2dy - 2x^3y^3dy = 3x^2y^4dx + 2xydx$$

$$\Rightarrow x^2dy - 2xydx = 3x^2y^4dx + 2x^3y^3dy$$

$$\Rightarrow \frac{2xydx - x^2dy}{y^2} + 3x^2y^2dx + 2x^3ydy = 0$$

$$\Rightarrow d\left(\frac{x^2}{y}\right) + d\left(x^3y^2\right) = 0$$

$$\Rightarrow \frac{x^2}{y} + x^3y^2 = C$$

72. Solution to the differential equation $\dfrac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots\ldots}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots} = \dfrac{dx - dy}{dx + dy}$ is

(A) $2y\, e^{2x} = C.e^{2x} + 1$
(B) $2y\, e^{2x} = C.e^{2x} - 1$
(C) $y\, e^{2x} = C.e^{2x} + 2$
(D) $2x\, e^{2y} = C.e^{x} - 1$

Key. B

Sol. Applying C and D, we get

$\frac{dy}{dx} = \frac{e^{-x}}{e^{x}} = e^{-2x} \Rightarrow 2y = -e^{-2x} + C$

or $2y\, e^{2x} = C.e^{2x} - 1$.

73. The curve for which the x-intercept of the tangent drawn at any point P on the curve is three times the x-coordinate of the point P, is

(A) $xy = c$ (B) $xy^2 = c$ (C) $xy^3 = c$ (D) $x^2y = c$

Key. B

Sol. $x - y\frac{dx}{dy} = 3x \Rightarrow \frac{dx}{x} + 2\frac{dy}{y} = 0$

$\Rightarrow \ln(xy^2) = k \Rightarrow xy^2 = c$.

74. A curve is such that the mid point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets y-axis, lies on the line y = x. If the curve passes through (1, 0), then the curve is

(A) $2y = x^2 - x$ (B) $y = x^2 - x$ (C) $y = x - x^2$ (D) $y = 2(x - x^2)$

Key. C

Sol. The point on y-axis is $\left(0,\ y - x\frac{dy}{dx}\right)$.

According to given condition,

$\frac{x}{2} = y - \frac{x}{2}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 2\frac{y}{x} - 1$

putting $\frac{y}{x} = v$ we get

$x\frac{dv}{dx} = v - 1 \Rightarrow \ln\left|\frac{y}{x} - 1\right| = \ln|x| + c \Rightarrow 1 - \frac{y}{x} = x$ (as f (1) = 0).

75. The solution of $\frac{dy}{dx}\sqrt{1+x+y} = x+y-1$ is

1) $2\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y} - 1\right| - \frac{4}{3}\log\left|\sqrt{1+x+y} + 2\right|\right] = x+c$

2) $2\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y}\right| - \frac{4}{3}\log\left|\sqrt{1+x+y} + 2\right|\right] = x+c$

3) $2\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y}\right| - \frac{4}{3}\log\left|\sqrt{1+x+y}\right|\right] = x+c$

4) $\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y}\right| - \frac{4}{3}\log\left|\sqrt{1+x+y}\right|\right] = x+c$

Key. 1

Sol. Putting $\sqrt{1+x+y} = v,$ we have,

$x + y - 1 = v^2 - 2 \Rightarrow 1 + \frac{dy}{dx} = 2v\frac{dv}{dx}$

Then the given equation transforms to

$\left(2v\frac{dv}{dx}-1\right)v=v^2-2\Rightarrow\frac{dv}{dx}=\frac{v^2+v-2}{2v^2}$

$\Rightarrow\int\frac{2v^2}{v^2+v-2}dv=\int dx\Rightarrow 2\int\left[1+\frac{1}{3(v-1)}-\frac{4}{3(v+2)}\right]dv=\int dx$

$\Rightarrow 2\left[v+\frac{1}{3}\log|v-1|-\frac{4}{3}\log|v+2|\right]=x+c$

Where $v=\sqrt{1+x+y}$

76. The solution of $\left(x+2y^3\right)(dy/dx)=y$ is (where c is arbitrary constant)

1) $x=y^3+cy$
2) $x=y^3-cy$
3) $y=x^3-cy$
4) $y=x^3+cy$

Key. 1 or 2

Sol. $\frac{dx}{dy}-\frac{1}{y}x=2y^2$

This is linear equation taking y as independent variable.

Here, I.F. $=e^{-\int\frac{1}{y}dy}=e^{-\log y}=\frac{1}{y}$

$\therefore$ **solution is** $x\frac{1}{y}=\int\frac{1}{y}2y^2dy+c$

$\Rightarrow\frac{x}{y}=y^2+c\Rightarrow x=y^3+cy$

77. The solution of $(1+x)\frac{dy}{dx}+1=e^{x-y}$ is

1) $e^y(x+1)=c$
2) $e^y(x+1)=e^x+c$
3) $e^y(x+1)=ce^x$
4) $(x+1)=e^x+c$

Key. 2

Sol. $(x+1)\frac{dy}{dx}+1=\frac{e^x}{e^y}\Rightarrow e^y(x+1)\frac{dy}{dx}+e^y=e^x$

$\int\frac{d}{dx}\left(e^y(x+1)\right)dx=\int e^x dx\Rightarrow e^y(x+1)=e^x+c$

78. Solution of the equation $xdy=\left(y+x\frac{f(y/x)}{f'(y/x)}\right)dx$

1) $f\left(\frac{x}{y}\right)=cy$
2) $f\left(\frac{y}{x}\right)=cx$
3) $f\left(\frac{y}{x}\right)=cxy$
4) $f\left(\frac{y}{x}\right)=0$

Key. 2

Sol. We have, $xdy=\left(y+\frac{xf(y/x)}{f^{'}(y/x)}\right)dx$

$\Rightarrow\frac{dy}{dx}=\frac{y}{x}+\frac{f(y/x)}{f^{'}(y/x)}$ which is homogenous

Putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$,we obtain

$v+x\frac{dv}{dx}=v+\frac{f(v)}{f^{'}(v)}\Rightarrow\frac{f^{'}(v)}{f(v)}dv=\frac{dx}{x}$

Integrating, we get log $f(v)=\log x+\log c$

$\Rightarrow\log f(v)=\log cx\Rightarrow f\left(\frac{y}{x}\right)=cx$

79. The general solution of the differential equation $\frac{dy}{dx}=y\tan x-y^2\sec x$ is

1) $\tan x=(c+\sec x)y$
2) $\sec y=(c+\tan y)x$
3) $\sec x=(c+\tan x)y$
4) None of these

Key. 3

Sol. We have $\frac{dy}{dx}=y\tan x-y^2\sec x$

$\Rightarrow\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$

Putting $\frac{1}{y}=v\Rightarrow\frac{-1}{y^2}\frac{dy}{dx}=\frac{dv}{dx}$, **we obtain**

$\frac{dV}{dx}+\tan x.v=\sec x$ **which is linear**

$I.F=e^{\int\tan x\,dx}=e^{\log\sec x}=\sec x$

$\therefore$ The solution is

$v\sec x=\int\sec^2 x\,dx+c\Rightarrow\frac{1}{y}\sec x=\tan x+c$

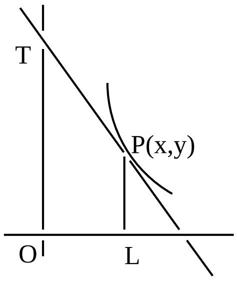
$\Rightarrow\sec x=y(c+\tan x)$

80. The curve $y=f(x)$ is such that the area of the trapezium formed by the coordinate axes, ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa. The equation of the curve can be

1) $y=cx^2\pm x$
2) $y=cx^2\pm1$
3) $y=cx\pm x^2$
4) $y=cx^2\pm x^2\pm1$

Key. 1

Sol. Let $P(x,y)$ be any point on the curve. Length of intercept on y-axis by any tangent at P is



$OT = y - x\frac{dy}{dx}$

$\therefore$ Area of trapezium $OLPTO = \frac{1}{2}(PL + OT)OL$

$= \frac{1}{2}\left(y + y - x\frac{dy}{dx}\right)x = \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x$

According to question

Area of trapezium $OLPTO = \frac{1}{2}x^2$

$i.e., \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x = \pm\frac{1}{2}x^2$

$\Rightarrow 2y - x\frac{dy}{dx} = \pm x \text{ or } \frac{dy}{dx} - \frac{2y}{x} = \pm 1$

Which is linear differential equation and $I.F. = e^{-2\ln x} = \frac{1}{x^2}$

$\therefore$ The solution is $\frac{y}{x^2} = \int \pm\frac{1}{x^2}dx + c = \pm\frac{1}{x} + c$

$\therefore y = \pm x + cx^2 \text{ or } y = cx^2 \pm x$, where c is an arbitrary constant

81. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^2y^2 = \sin x$, then

1) p > q
2) $\frac{p}{q} = \frac{1}{2}$
3) p = q
4) p < q

Key. 4

Sol. $y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x = -3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}}$

$\left(y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x\right)^3 = -9x^3\left(\frac{dy}{dx}\right)$

Here order = 2 = p
Degree = 6 = q
$\therefore p < q$

82. The solution of the differential equation $\frac{dy}{dx} - ky = 0, y(0) = 1$, approach zero when $x \to \infty$, if

1) K = 0
2) k > 0

3) k < 0 4) none of these

Key. 3

Sol. $\frac{dy}{dx} - Ky = 0,\ \frac{dy}{y} = Kdx$

$\ln y = Kx + c$

At $x = 0, y = 1$ $\therefore\ C = 0$

Now, $\ln y = Kx$

$y = e^{Kx}$

$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{Kx} = 0$

$\therefore K < 0$

83. The solution of $\frac{dv}{dt} + \frac{k}{m}v = -g$ is

1) $v = ce^{-\frac{k}{m}t} - \frac{mg}{k}$ 2) $v = c - \frac{mg}{k}e^{-\frac{k}{m}t}$

3) $ve^{-\frac{k}{m}t} = c - \frac{mg}{k}$ 4) $ve^{\frac{k}{m}t} = c - \frac{mg}{k}$

Key. 1

Sol. $\frac{dv}{dt} + \frac{K}{m}v = -g$

Integrating factor $(I.F.) = e^{\int \frac{k}{m}dt} = e^{\frac{K}{m}t}$

$\therefore Ve^{\frac{K}{m}t} = -\int g\, e^{K.t/m} + c$

$Ve^{\frac{K}{m}t} = \frac{-gm}{K}e^{\frac{K}{m}t} + c$

$V = C.e^{\frac{-K}{m}t} - \frac{mg}{K}$

84. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx} + f(x)y = 0$, then a solution of differential equation $\frac{dy}{dx} + f(x)y = r(x)$ is

1) $\frac{1}{y(x)}\int y_1(x)dx$ 2) $y_1(x)\int \frac{r(x)}{y_1(x)}dx$

3) $\int r(x)y_1(x)dx$ 4) none of these

Key. 2

Sol. i) $\frac{dy_1}{dx} + f(x)y_1 = 0 \Rightarrow f(x) = \frac{-1}{y_1}\frac{dy_1}{dx}$

ii) $\frac{dy}{dx} - \frac{1}{y_1}\frac{dy_1}{dx}.y = r(x)$

$$e^{-\int \frac{1}{y_1}\frac{dy}{dx}dx} = e^{-\int \frac{dy_1}{y_1}} = \frac{1}{y_1}$$

$$\frac{d}{dx}\left(\frac{y}{y_1}\right) = \frac{r(x)}{y_1} \Rightarrow \frac{y}{y_1} = \int \frac{r(x)dx + c}{y_1}$$

$$y = y_1 \int \frac{r(x)dx}{y_1} + cy_1$$

85. The solution of the differential

equation $\left(x^2\sin^3 y - y^2\cos x\right)dx + \left(x^3\cos y\sin^2 y - 2y\sin x\right)dy = 0$ is

1) $x^3\sin^3 y = 3y^2\sin x + C$
2) $x^3\sin^3 y + 3y^2\sin x = C$
3) $x^2\sin^3 y + y^3\sin x = C$
4) $2x^2\sin y + y^2\sin x = C$

Key.

Sol. $\left(x^2\sin^2 y - y^2\cos x\right)dx + \left(x^3\cos y\sin^2 y - 2y\sin x\right)dy = 0$

$$\frac{dy}{dx} = \frac{y^2\cos x - x^2\sin^3 y}{x^3\cos y\sin^2 - 2y\sin x}$$

$$\left(x^3\cos y\sin^2 y - 2y\sin x\right)dy$$

$$= \left(y^2\cos x - x^2\sin^3 y\right)dx = 0$$

$$\left(\frac{x^3}{3}d\sin^3 y - \sin dy^2\right) - \sin^3 yd\left(\frac{x^3}{3}\right) + y^2 d\sin x = 0$$

$$\frac{x^3}{3}d\sin^2 y + \sin^3 yd\left(\frac{x^3}{3}\right) - \left(\sin dy^2 + y^2 d\sin x\right)$$

$$d\left(\frac{x^3}{3}\sin^3 y\right) - d\left(y^2\sin x\right) = 0$$

$$\frac{x^3}{3}\sin^3 y - y^2\sin x = c$$

86. The differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$ is (a is a constant)

1) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$
2) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$
3) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$
4) none of these

Key.

Sol. $(x-h)^2+(y-k)^2=a^2$ ....(1)

$2(x-h)+2(y-k)\frac{dy}{dx}=0$ ....(2)

$1+\left(\frac{dy}{dx}\right)^2+(y-k)\frac{d^2y}{dx^2}=0$ ....(3)

From (3) we have (y-k), use in (2) to get (x-h) and put(x-h) and(y-k) in(1)

87. The solution of the differential equation $\frac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\frac{y^3}{x^3}}$ is given by

a) $\frac{3}{2}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}+c=0$

b) $\frac{2}{3}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\frac{y}{x}+c=0$

c) $\frac{2}{3}\left(\frac{y}{x}\right)+\log\left(\frac{x+y}{x}\right)+\tan^{-1}\left(\frac{y^{3/2}}{x^{3/2}}\right)+c=0$

d) None of these

Key. D

Sol. We have $\frac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\frac{y^3}{x^3}}$

$$\Rightarrow \frac{d(x^{3/2})+d(y^{3/2})}{d(x^{3/2})-d(y^{3/2)}}=\frac{y^{3/2}}{x^{3/2}}$$

$$\Rightarrow \frac{du+dv}{du-dv}=\frac{v}{u},\text{ where } u=x^{3/2} \text{ and } v=y^{3/2}$$

$$\Rightarrow udu+udv=vdu-vdv \Rightarrow udu+vdv=vdu-udv$$

$$\Rightarrow \frac{udu+vdv}{u^2+v^2}=\frac{vdu-udv}{u^2+v^2}$$

$$\Rightarrow \frac{d(u^2+v^2)}{u^2+v^2}=-2d\tan^{-1}\left(\frac{v}{u}\right)+c$$

On integrating, we get

$$\log(u^2+v^2)=-2\tan^{-1}\left(\frac{v}{u}\right)+c$$

$$\Rightarrow \frac{1}{2}\log(x^3+y^3)+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}=\frac{c}{2}$$

88. A function $y=f(x)$ has a second order derivative $f''=6(x-1)$. If its graph passes through the point(2,1) and at that point the tangent to the graph is $y=3x-5$, then the function is

a) $(x-1)^2$ b) $(x-1)^3$ c) $(x+1)^2$ d) $(x+1)^3$

Key. B

Sol. Since $f''(x)=6(x-1)$

$\Rightarrow f'(x)=3(x-1)^2+c$ (integrating) ----(i)

Also, at the point (2,1), the tangent to the graph is $y=3x-5$ and slope of the tangent = 3

$\Rightarrow f'(2)=3$

$3(2-1)^2+c=3$ [from Eq (i)

$\Rightarrow 3+c=3 \Rightarrow c=0$

From Eq (i) we have

$f'(x)=3(x-1)^2$

$\Rightarrow f(x)=(x-1)^2+k$ (Integrating) ----(ii)

$\therefore 1=(2-1)^2+k \Rightarrow k=0$

Hence the equation of the function is $f(x)=(x-1)^3$.

89. The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$, where $c>0$ is a parameter, is of order and degree as follows :

a) order 1, degree 3 b) order 2, degree 3

c) order 1, degree 2 d) order 2, degree 1

Key. A

Sol. Differentiating, we get

$2yy'=2c \Rightarrow c=yy'$

$\therefore y^2=2yy'(x+\sqrt{yy'}) \Rightarrow (y^2-2yy'x)^2=4(yy')^3$

$\Rightarrow$ degree= 3 and order =1

90. The normal to a curve at $P(x,y)$ meets the $x-axis$ at G. If the distance of G from the origin is twice the abscissa of P, then the curve is

a) a parabola b) a circle c) a hyperbola d) an ellipse

Key. C or D

Sol. The slope of the tangent = $\frac{dy}{dx}$

$\therefore$ the slope of the normal $= -\frac{dx}{dy}$

$\therefore$ the equation of the normal is $Y - y = -\frac{dx}{dy}(X - x)$

This meets the x-axis $(Y = 0)$, where

$-y = \frac{-dx}{dy}(X - x) \Rightarrow X = x + y\frac{dy}{dx}$

$\therefore$ G is $\left(x + y\frac{dy}{dx}, 0\right)$

$\therefore$ $OG = 2x \Rightarrow x + y\frac{dy}{dx} = 2x$

$\Rightarrow y\frac{dy}{dx} = x \Rightarrow ydy = xdx$ [variable separable integrating, we get]

$\frac{y^2}{2} = \frac{x^2}{2} + constant$

$\Rightarrow x^2 - y^2 = c$, which is a hyperboal

91. Which of the following transformation reduces the differential equation

$\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$ to the form $P(x)u = Q(x)$

a) $u = \log z$ b) $u = e^z$ c) $u = (\log z)^{-1}$ d) $u = (\log z)^2$

Key. A or B or C or D

Sol. Given equ. Can be written as

$$\frac{dz}{z(\ln z)^2} + \frac{dx}{x(\ln z)} = \frac{dx}{x^2}$$

Put $u = \frac{1}{\ln z}$

$$\frac{du}{dx} = -\frac{1}{(\ln z)^2}.\frac{1}{z}.\frac{dz}{dx}$$

$\therefore -\frac{du}{dx}+\frac{1}{x}.u=\frac{1}{x^2}$

$\frac{du}{dx}+\left(-\frac{1}{x}\right)u=\left(-\frac{1}{x^2}\right)$

$I.F.=e^{\int -\frac{1}{x}dx}=e^{\ln\frac{1}{x}}=\frac{1}{x}$

$u.\frac{1}{x}=\int -\frac{1}{x^2}.\frac{1}{x}dx$

$\frac{u}{x}=-\int\frac{1}{x^3}dx=+\frac{1}{4x^4}+c$

92. If $y_1(x)$ is a solution of the differential equation $dy/dx-f(x)y=0$, then a solution of the differential equation $\frac{dy}{dx}+f(x)y=r(x)$

a) $\frac{1}{y_1(x)}\int r(x)y_1(x)dx$

b) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$

c) $\int r(x)y_1(x)dx$

d) none of these

Key. A

Sol. $\frac{dy}{dx}-f(x).y=0$

$\frac{dy}{y}=f(x)dx$

$\ln y=\int f(x)dx$

$y_1(x)=e^{\int f(x)dx}$ Then for given equation I.F= $e^{\int f(x)dx}$

Hence Solution $y.y_1(x)=\int r(x).y_1(x)dx$

$y=\frac{1}{y_1(x)}\int r(x).y_1(x)dx$

93. Solution of the differential equation $(x^2\sin^3 y-y^2\cos x)dx+(x^3\cos y\sin^2 y-2y\sin x)dy=0$, is

(A) $\frac{(x\sin y)^2}{2}-y^2\sin x=c$

(B) $\frac{(x\sin y)^3}{3}-y^2\sin x=c$

(C) $x\sin y-y^2\sin x=c$

(D) $\frac{(x\sin y)^4}{4}-y^2\sin x=c$

Key. B

Sol. The given differential equation can be written as

$x^2 \sin^2 y(\sin y\, dx + x \cos y\, dy) - (y^2 \cos x\, dx + 2y \sin x\, dy) = 0$

$\Rightarrow (x \sin y)^2 d(x \sin y) - d(y^2 \sin x) = 0$

On integrating, we get

$$\int (x\sin y)^2 d(x\sin y) - \int 1.d(y^2 \sin x) = 0$$

$\Rightarrow \frac{(x\sin y)^3}{3} - y^2 \sin x + C = 0$ which is the required solution.

94. Solution of the differential equation : $\frac{dy}{dx} = \frac{3x^2y^4 + 2xy}{x^2 - 2x^3y^3}$ is

(A) $x^2y^2 + \frac{x^2}{y} = c$
(B) $x^3y^2 + \frac{x^2}{y} = c$
(C) $x^3y^2 + \frac{y^2}{x} = c$
(D) $x^2y^3 + \frac{x^2}{y} = c$

Key. B

Sol. The given differential equation can be written as

$(3x^2y^4 + 2xy)\, dx = (x^2 - 2x^3y^3)dy$

$\Rightarrow 3x^2 y^4 dx + 2x^3y^3 dy + 2xy\, dx - x^2 dy = 0$

$\Rightarrow y^2(3x^2 y^2 dx + 2x^3 y\, dy) + 2\, xy\, dx - x^2\, dy = 0$

$\Rightarrow y^2[y^2 d(x)^3 + x^3 d(y)^2] + yd\,(x)^2 - x^2 dy = 0$

$\Rightarrow y^2 d\,(x)^3 + x^3 d\,(y^2) + \frac{yd(x)^2 - x^2dy}{y^2} = 0$

$$\Rightarrow d(x^3y^2) + d\left(\frac{x^2}{y}\right) = 0$$

On integration, we obtain $x^3y^2 + \frac{x^2}{y} = C,$

as the required solution.

95. If for a curve ratio of the distance between the normal at any of its points and the origin to the distance between the same normal and the point (a, b) is equal to the constant k (k >0, k ≠1), then the curve is a

(A) circle
(B) parabola
(C) ellipse
(D) hyperbola

Key. A

Sol. Let y = f(x) be the curve and let P (x, y) be any point on the curve. The equation of the normal at P(x, y) to the given curve is

$$Y - y = -\frac{1}{\frac{dy}{dx}}(X - x)$$

The distance of (i) from the origin is

$$d_1 = \left| \frac{y + \frac{x}{\frac{dy}{dx}}}{\sqrt{1 + \frac{1}{(dy/dx)^2}}} \right| = \left| \frac{x + y\frac{dy}{dx}}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \right| \qquad d_2 = \left| \frac{(a - x) + (b - y)\frac{dy}{dx}}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \right|$$

Now $d_1 = kd_2$

On integrating, we get

$$\frac{x^2}{2}+\frac{y^2}{2}=\pm k\left[-\frac{(a-x)^2}{2}-\frac{(b-y)^2}{2}\right]+C$$

96. Solution of $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx}=\left(\frac{x+y+1}{x+y+2}\right)$, given that y = 1 when x = 1 is

(A) $\ln\left|\frac{(x-y)^2-2}{2}\right|=2(x+y)$ (B) $\ln\left|\frac{(x+y)^2-2}{2}\right|=2(x-y)$

(C) $\ln\left|\frac{(x-y)^2+2}{2}\right|=2(x+y)$ (D) $\ln\left|\frac{(x+y)^2+2}{2}\right|=2(x-y)$

Key. B

Sol. $\because$ $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx}=\left(\frac{x+y+1}{x+y+2}\right)$

Put x + y = ν

$\therefore$ $\left(\frac{\nu-1}{\nu-2}\right)\left(\frac{d\nu}{dx}-1\right)=\left(\frac{\nu+1}{\nu+2}\right)$

$\Rightarrow$ $\frac{d\nu}{dx}-1=\frac{(\nu+1)(\nu-2)}{(\nu-1)(\nu+2)}=\frac{\nu^2-v-2}{v^2+v-2}$

or $\frac{d\nu}{dx}=\frac{2\nu^2-4}{(\nu^2-\nu-2)}$

Given that y = 1, when x = 1

$\therefore$ $0+\frac{1}{2}\ln 2=c$

$(y-x)+\frac{1}{2}\left|\frac{\ln(x+y)^2-2}{2}\right|=0$

or $\ln\left|\frac{(x+y)^2-2}{2}\right|=2(x-y)$

97. Solution of the differential equation $\left\{\frac{1}{x}-\frac{y^2}{(x-y)^2}\right\}dx+\left\{\frac{x^2}{(x-y)^2}-\frac{1}{y}\right\}dy=0$ is

(A) $\ln\left|\frac{x}{y}\right|+\frac{xy}{(x-y)}=c$ (B) $\ln|xy|+\frac{xy}{(x-y)}=c$

(C) $\frac{xy}{(x-y)}=ce^{x/y}$ (D) $\frac{xy}{(x-y)}=ce^{xy}$

Key. A

Sol. The given equation can be written as

$$\left(\frac{dx}{x}-\frac{dy}{y}\right)+\frac{(x^2dy-y^2dx)}{(x-y)^2}=0$$

$$\Rightarrow \left(\frac{dx}{x}-\frac{dy}{y}\right)+\frac{\left(\frac{dy}{y^2}-\frac{dx}{x^2}\right)}{\left(\frac{1}{y}-\frac{1}{x}\right)^2}=0$$

$$\Rightarrow \quad \left(\frac{dx}{x}-\frac{dy}{y}\right)+\frac{\frac{dy}{y^2}-\frac{dx}{x^2}}{\left(\frac{1}{x}-\frac{1}{y}\right)^2}=0$$

Integrating, we get

$$\ln|x|-\ln|y|-\frac{1}{\left(\frac{1}{x}-\frac{1}{y}\right)}=c$$

or $$\ln\left|\frac{x}{y}\right|-\frac{xy}{(y-x)}=c = \ln\left|\frac{x}{y}\right|+\frac{xy}{x-y}=c$$