CHAPTER 15

Volume and Surface Area

In this chapter, we have to deal with the problem of finding the volume and surface area of solid figures. As we know that, solid figures have length, breadth and height (thickness). Hence, this chapter can be featured as 3D and mensuration too.

Volume

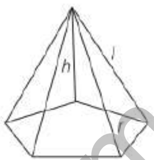
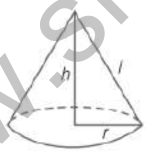
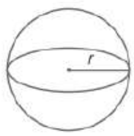
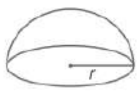
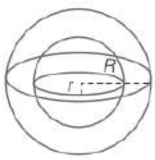
It is the space occupied within the boundary of a 3D figure.

Unit cu cm, cu m (i.e., cube units) etc.

Surface Area

It is the total area that can be measured on the entire surface. This can only be measured, if the object is a 3D object surface area is measured in square unit.

Important Formulae Related to Solid Figures

Name	Figure	Lateral/Curved Surface Area	Total Surface Area	Volume	Nomenclature
Right pyramid		(Perimeter of the base) × (Slant height)	Area of the base + Lateral surface area	$\frac{1}{3}$ (Area of the base) × Height	
Right circular cone		πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$	h = Height, r = Radius, l = Slant height
Sphere		—	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r = Radius
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r = Radius
Spherical shell		—	$4\pi(R^2 - r^2)$	$\frac{4}{3}\pi(R^3 - r^3)$	R = Outer radius, r = Inner radius

Solved Examples:

1. If the surface of a cube is 216 sq cm, its volume will be

(a) 108 sq cm
(b) 36 sq cm
(c) 216 sq cm
(d) 216 sq cm

Sol. (d) Surface of a cube $= 6 \times (\text{ Side })^2$

$\therefore\ 216 = 6 \times x^2$

Where, x is the side of cube.

$$x^2 = \frac{216}{6} \qquad = 36$$

$$\therefore\ x = \sqrt{36} \qquad = 6 \text{ cm}$$

$\therefore$ Volume of the cube $= 6 \times 6 \times 6 = 216$cucm

2. If the area of the floor of a rectangular room be 15sqm and the height of the room be 4 m, then how much air is in the room?

(a) 11 cm
(b) $\frac{15}{4}$ cm
(c) 60 cm
(d) 19 cm

Sol. (c) Volume $=$ Area $\times\, h = 15 \times 4 = 60$cm

3. One rectangular tank is 2.1 m long and 2 m broad. If the capacity of the tank is 21 hecto liter, then the height is

(a) 0.9 m
(b) 0.5 m
(c) 0.7 m
(d) 10 m

Sol. (b) Capacity $= 21$ hectolitre $= 2100$ L $= 2.1$cum

$\therefore$ Capacity $= l \times b \times h \Rightarrow 2.1 = 2.1 \times 2 \times h$

$$\therefore\ h = \frac{2.1}{2.1 \times 2} = 0.5 \text{ m}$$

4. If each edge of a cube is decreased by 40%. Find the per cent decrease in the surface area of the cube.

(a) 64
(b) 84
(c) 80
(d) 74

Sol. (a) According to the formula

Percentage decrease in surface area

$$= \left[2(-40) + \frac{(-40)^2}{100}\right] = \left[-80 + \frac{(1600)}{100}\right]$$
$$= (-80 + 16) = -64\%$$

Negative sign shows that decrease takes place here.

Practice Questions

1. The base of right prism is an equilateral triangle with a side of 7 m and its height is 24 m. Find its volume.

(a) 509 m^3
(c) 529 m^3
(b) 1018 m^3
(d) 519 m^3

2. To raise the height of a low land 48 m long and 31.5 m broad to 6.5dm, a ditch 27 m long and 18.2 m broad was dug in a side plot, the depth of the ditch will be

(a) 5 m
(c) 1 m
(b) 7 m
(d) 2 m

3. Weight of a solid metal sphere of radius 4 cm is 4 kg. The weight of a hollow sphere made with same metal, whose outer diameter is 16 cm and inner diameter is 12 cm, is

(a) 20.5 kg
(b) 15.5 kg
(c) 16.5 kg
(d) 18.5 kg

4. The radii of a sphere and a right circular cylinder are equal and their curved surface areas are also equal. The ratio of their volumes is

(a) $3:4$
(b) $2:3$
(c) $3:2$
(d) $4:3$

5. A sphere and a cylinder have equal volume and equal radius. The ratio of the curved surface area of the cylinder to that of the sphere is

(a) $4:3$
(b) $2:3$
(c) $3:2$
(d) $3:4$

6. A sphere exactly fits inside a hollow cylinder closed at both ends. The ratio of the volume of the empty space in the cylinder to the volume of the sphere is

(a) $2:1$
(c) $2:3$
(b) $1:2$
(d) $3:2$

7. If the length of longest rod that can be placed within the cuboid is $5\sqrt{5}$ m long and the sum of lenght breadth and height is 19 m long, then find the whole surface area of that cuboid.

(a) $236\ \mathrm{m}^2$
(c) $125\ \mathrm{m}^2$
(b) $256\ \mathrm{m}^2$
(d) $361\ \mathrm{m}^2$

8. The number of spherical bullets that can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being of 4 cm diameter, is $\left(\text{take } \pi = \frac{22}{7}\right)$

(a) 2541
(b) 2451
(c) 2514
(d) 2415

9. The length of longest pole that can be placed in a room of 12 m long, 8 m broad and 9 m high is

(a) 12 m
(b) 17 m
(c) 19 m
(d) 21 m

10. Three cubes of iron of edges 9 cm, 12 cm and 15 cm respectively are melted to form a large single cube. The edge of the new cube is

(a) 10 cm
(b) 14 cm
(c) 18 cm
(d) 16 cm

11. Three cubes of iron of edges 9 cm, 12 cm and 15 cm respectively are melted to form a large single cube. The edge of the new cube is

(a) 10 cm
(b) 14 cm
(c) 18 cm
(d) 16 cm

12. If the radius of base and height of a cone are increased by 10%, then the volume of the cone is increased by

(a) 30%
(b) 33.1%
(c) 40%
(d) 42%

13. A sphere of radius 2 cm is put into water contained in a cylinder of radius 4 cm. If the sphere is completely immersed in the water, the water level in the cylinder rises by

(a) 2 cm
(b) $\frac{1}{3}$ cm
(c) $\frac{1}{2}$ cm
(d) $\frac{2}{3}$ cm

14. A sphere and a right circular cylinder have the same radius r. If their volumes are equal, the height of the cylinder is

(a) $\frac{4}{3}r$

(b) $\frac{3}{4}r$
(c) $\frac{2}{3}r$
(d) $\frac{3}{2}r$

15. If the ratio of surface areas of two sphere is $9:16$, then the ratio of their volume is

(a) $3:4$
(b) $9:16$
(c) $27:64$
(d) $81:256$

16. The radius of the base and height of a cone are 3 cm and 5 cm respectively whereas the radius of the base and height of a cylinder are 2 cm and 4 cm respectively. The ratio of the volume of the cone to that of the cylinder is

(a) $15:8$
(b) $45:16$
(c) $15:16$
(d) $1:3$

17. A rectangular block 6 cm × 42 cm × 45 cm is cut up into exact number of equal cubes. The least possible number of cubes will be

(a) 30
(b) 210
(c) 330
(d) 420

18. Two cylindrical buckets have their diameters in the ratio $3:1$ and their heights are as $1:3$. Their volumes are in the ratio

(a) $1:2$
(b) $2:3$
(c) $3:1$
(d) $3:4$

19. A sphere is cut into two hemispheres. One of them is used as a bowl. It takes 8 bowlfuls of this to fill a conical vessel of height 12 cm and radius 6 cm. The radius of the sphere is

(a) 2 cm
(b) 3 cm
(c) 4 cm
(d) 6 cm

20. A cone of height 7 m and of base radius 3 m is carved from a rectangular block of wood of dimensions $10 \text{ m} \times 5 \text{ m} \times 4 \text{ m}$. The percentage of volume of the block left out is

(a) 67%
(b) 66%
(c) 34%
(d) 33%

21. Three cubes of metal whose edges are in the ratio $3:4:5$ are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. The edges of the three cubes (in cm) are

(a) 9,12,15
(b) 15,20,25
(c) 6,8,10
(d) 8,10,12

22. A solid cylinder of diameter 14 mm and length 25 mm has a volume 3850 mm^3. If the length were doubled and the diameter halved, the new volume would be

(a) 1172 mm^3
(c) 3850 mm^3
(b) 1925 mm^3
(d) 7700 mm^3

23. The surface areas of a cylinder, a cone and a hemisphere of same radii are equal. The ratio between height of the cylinder and cone is

(a) $2\sqrt{3}:1$
(b) $1:2\sqrt{3}$
(c) $2:\sqrt{3}$
(d) $\sqrt{3}:2$

24. If the side of two cubes are in the ratio $3:1$, the ratio of their total surface areas is

(a) $3:1$
(b) $8:1$
(c) $9:1$
(d) $12:1$

ANSWERS

1.	(a)	**2.**	(d)	**3.**	(d)	**4.**	(b)	**5.**	(c)	**6.**	(b)	**7.**	(b)	**8.**	(a)	**9.**	(a)	**10.**	(b)
11.	(c)	**12.**	(b)	**13.**	(d)	**14.**	(a)	**15.**	(c)	**16.**	(c)	**17.**	(d)	**18.**	(c)	**19.**	(b)	**20.**	(a)
21.	(c)	**22.**	(b)	**23.**	(b)	**24.**	(c)												

Hints & Solutions

1. Area of base of right prism

$$= \frac{\sqrt{3}}{4}(7)^2 = \frac{49\sqrt{3}}{4}\ \text{m}^2$$

Volume of right prism

= Area of base × Height

$$= \frac{49\sqrt{3}}{4} \times 24$$

$$= 6 \times 49 \times \sqrt{3}$$

$$= 294 \times 1.732$$

$$= 509.222 \approx 509\ \text{m}^3$$

2. Let the depth of the ditch be h.

According to the question,

$$h = \frac{48 \times 31.5 \times \frac{6.5}{10}}{27 \times 18.2}$$

$$= \frac{9828}{4914} = 2\ \text{m}$$

3. Volume of solid sphere of radius 4 cm $= \frac{4}{3}\pi(4)^3$

Volume of hollow sphere

$$= \frac{4}{3}\pi[(8)^3 - (6)^3]$$

$$\because \text{ Weight of } \frac{4}{3}\pi(4)^3 \text{ cm}^3 = 4 \text{ kg}$$

$$\therefore \text{ Weight of } \frac{4}{3}\pi[(8)^3 - (6)^3]\text{cm}^3$$

$$= \frac{4}{\frac{4}{3}\pi(4)^3} \cdot \frac{4}{3}\pi[(8)^3 - (6)^3]$$

$$= \frac{4(512 - 216)}{4^3} = 18.5 \text{ kg}$$

4. Given, $4\pi r^2 = 2\pi rh$

$\Rightarrow h = 2r$

Now, required ratio $= \frac{4}{3}\pi r^3 : \pi r^2 h$

$$= 4r : 3h$$
$$= 4r : 6r$$
$$= 2 : 3$$

$(\because h = 2r)$

5. According to question,

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$$h = \frac{4}{3}r$$

$\therefore$ Required ratio $= \dfrac{\text{Curved surface of sphere}}{\text{Curved surface of cylinder}}$

$$= \frac{4\pi r^2}{2\pi rh} = \frac{4\pi r^2}{2\pi r\left(\frac{4}{3}r\right)} = \frac{3}{2}$$

6. Let radius of sphere $= r$ and side of cube $= a$

According to question,

Surface area of sphere

$$= \text{Surface area of cube}$$

$$4\pi r^2 = 6a^2$$

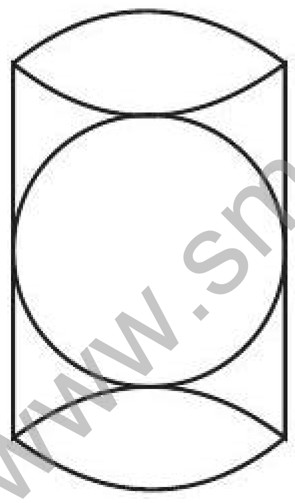
$$\frac{r}{a} = \frac{\sqrt{3}}{\sqrt{2\pi}}$$

$\therefore$ Ratio of volumes $= \dfrac{\frac{4}{3}\pi r^3}{a^3}$

$$= \frac{4}{3}\pi\left(\frac{r}{a}\right)^3 = \frac{4}{3}\pi\left(\frac{\sqrt{3}}{\sqrt{2\pi}}\right)^3$$

$$= \frac{4\pi \times 3\sqrt{3}}{3 \times 2\sqrt{2} \times \pi\sqrt{\pi}} = \frac{\sqrt{6}}{\sqrt{\pi}}$$

7. Radius of the sphere = Radius of the base of the cylinder and height of the cylinder = Diameter of the sphere $= 2r$ Volume of sphere $= \frac{4}{3}\pi r^3$



and volume of the cylinder $= \pi r^2(2r) = 2\pi r^3$

$\therefore$ Volume of the empty space

$$= 2\pi r^3 - \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\pi r^3$$

∴ Required ratio $= \frac{2\pi r^3}{3} : \frac{4\pi r^3}{3} = 1:2$

8. Given, $l + b + h = 19$

and $\sqrt{l^2 + b^2 + h^2} = 5\sqrt{5}$

On squaring Eq. (i), we get

$$(l + b + h)^2 = (19)^2$$
$$l^2 + b^2 + h^2 + 2(lb + bh + hl) = 361$$

Now, on squaring Eq. (ii),

$$\left(\sqrt{l^2 + b^2 + h^2}\right)^2 = (5\sqrt{5})^2$$
$$l^2 + b^2 + h^2 = 125$$

Now putting the obtaining value in Eq. (iii), we get

$$125 + 2(lb + bh + hl) = 361$$

$$2(lb + bh + hl) = 361 - 125$$

Whole surface area of cuboid $= 236$ m^2

9. Total number of spherical bullets

$$= \frac{\text{Volume of solid cube}}{\text{Volume of 1 bullet}}$$
$$= \frac{44 \times 44 \times 44}{\frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2} = 2541$$

10. Length of the longest pole

$$= \sqrt{12^2 + 8^2 + 9^2}$$
$$= \sqrt{144 + 64 + 81}$$
$$= \sqrt{289} = 17 \text{ m}$$

11. Volume of the new cube = $(9)^3 + (12)^3 + (15)^3$

$= 729 + 1728 + 3375 = 5832$ cm^3

∴ One side of the new cube $= 18$ cm

12. Let the original radius and height be r and h, respectively.

$\therefore$ Volume of the original cone $= \frac{1}{3}\pi r^2 h$

and increased volume

$$= \frac{1}{3}\pi\left(\frac{110r}{100}\right)^2 \times \left(\frac{110h}{100}\right)$$
$$= \frac{1}{3}\pi \times 1.331r^2 h$$

$\therefore$ Percentage of increase

$$= \frac{\frac{1}{3}\pi r^2 h(1.331 - 1)}{\frac{1}{3}\pi r^2 h} \times 100$$
$$= 33.1\%$$

13. Volume of the sphere $= \frac{4}{3}\pi(2)^3 = \frac{32\pi}{3}$ cm^3

Let the water raised h m when a sphere is immersed in it.

Volume of water (cylinder)

= Volume of sphere

$$\pi r^2 h = \frac{32\pi}{3}$$

$$(4)^2 h = \frac{32}{3} \quad (\because r = 4 \text{ cm})$$

$$\Rightarrow h = \frac{2}{3} \text{ cm}$$

14. Volume of the sphere $= \frac{4}{3}\pi r^3$ and volume of the cylinder

$$= \pi r^2 h$$
$$\because \quad \pi r^2 h = \frac{4}{3}\pi r^3$$
$$\therefore \quad h = \frac{4}{3} r$$

15. Let r_1 and r_2 be the radii of spheres.

$$\therefore \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{9}{16}$$
$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{9}{16} \Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$$
$$\therefore \frac{4}{3}\pi r_1^3 : \frac{4}{3}\pi r_2^3 = r_1^3 : r_2^3$$
$$= 3^3 : 4^3$$
$$= 27 : 64$$

16. Volume of the cone $= \frac{1}{3}\pi(3)^2 \times 5 = 15\pi \text{cm}^3$

and volume of the cylinder

$$= \pi(2)^2 \times 4$$
$$= 16\pi \text{cm}^3$$

$\therefore$ Required ratio $= 15 : 16$

17. For the least number of cubes, the edge of the cube will be largest.

$\therefore$ HCF of 6,42 and $45 = 3$

$\therefore$ Volume of 1 cube $= 3 \times 3 \times 3 = 27 \text{ cm}^3$

$\therefore$ Required number of cubes $= \frac{6 \times 42 \times 45}{27} = 420$

18. Let the diameter and height of one bucket $3x$ m and h m.

$\therefore$ The diameter and height of the second bucket will x m and $3h$ m.

$$\therefore V_1 : V_2 = \pi\left(\frac{3x}{2}\right)^2 \times h : \pi\left(\frac{x}{2}\right)^2 \cdot 3h$$
$$= \frac{9\pi x^2 h}{4} : \frac{3\pi x^2 h}{4} = 3 : 1$$

19. Volume of the cone

$$= \frac{1}{3}\pi \times 36 \times 12$$
$$= 144\pi \text{cm}^3$$

If the radius of the sphere be r cm, then

$$\frac{2}{3}\pi r^3 \times 8 = 144\pi$$
$$\therefore\ r = 3 \text{ cm}$$

20. Volume of the block left $= (10 \times 5 \times 4) - \left(\frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7\right)$

$$= 200 - 66 = 134 \text{ m}^3$$

$\therefore$ Required percentage $= \frac{134 \times 100}{200} = 67\%$

21. Let the edges of the three cubes be $3x$ cm, $4x$ cm and $5x$ cm.

$\therefore$ Volume of the resulting cube

$$= (3x)^3 + (4x)^3 + (5x)^3$$
$$= 216x^3$$

$\therefore$ Edge of the resulting cube $= 6x$ cm

$\therefore$ Diagonal the resulting cube

$$= 6x\sqrt{3} \text{ cm}$$
$$6x\sqrt{3} = 12\sqrt{3}$$
$$\therefore\ x = \frac{12\sqrt{3}}{6\sqrt{3}} = 2 \text{ cm}$$

$\therefore$ The edges of three cubes are 6 cm, 8 cm and 10 cm.

22. $\because$ Diameter = 14 mm

$\therefore$ Diameter of new cylinder = 7 mm

$\therefore$ Length of new cylinder

$$\pi\left(\frac{7}{2}\right)^2 \times 50 = x$$

$$\therefore\ x = 1925\ \text{mm}^3$$

23. Let the radius of each solid be r and the heights of the cylinder and the cone be h_1 and h_2 respectively.

$$2\pi r^2 + 2\pi r h_1 = 3\pi r^2$$

$$\therefore\ h_1 = \frac{r}{2}$$

and also $3\pi r^2 = \pi r\sqrt{h_2^2 + r^2} + \pi r^2$

$$\Rightarrow\ \pi r\sqrt{h_2^2 + r^2} = 2\pi r^2$$

$$\Rightarrow\ h_2^2 + r^2 = 4r^2$$

$$\Rightarrow\ h_2^2 = 3r^2$$

$$\therefore\ h_2 = \sqrt{3}r$$

$$\therefore\ \frac{h_1}{h_2} = \frac{\frac{r}{2}}{\sqrt{3}r} = 1:2\sqrt{3}$$

24. The ratio of their total surface areas $= \frac{(3)^2}{(1)^2} = 9:1$