CHAPTER 07

Ratio and Proportion

The number of times one quantity contains another quantity of the same kind is called ratio of the two unit.

Ratio

It is a tool to compare two or more numbers of same quantities.

Or

The ratio of two quantities in the same units is the fraction that one quantity is of the other.

Thus, the ratio a to b is the fraction $\dfrac{a}{b}$ written as $a:b$.

Note In the ratio $a:b$, the first term a is antecedent and second term b is consequent.

Properties of Ratios

(i) The value of a ratio remains unchanged, if each one of its terms is multiplied or divided by a same non-zero number.

(ii) $a^2:b^2$ is the duplicate ratio of $a:b$.

(iii) $a^3:b^3$ is the triplicate ratio of $a:b$.

(iv) $\sqrt{a}:\sqrt{b}$ is the sub-duplicate ratio of $a:b$.

(v) If $a:b$ and $c:d$ are two ratios, then the compounded ratio is $ac:bd$.

Proportion

The equality of two ratios is called proportion. Let a, b, c and d are four quantities, then the proportional are $a:b::c:d$.

Note In the proportion $a:b::c:d$, a and d are extreme values and b and c are mean values.

i.e., Product of means = Product of extreme

Properties of Proportion

(i) If x is the third proportional to a, b, then $a:b::b:x$.

(ii) Mean proportional between a and b is $\sqrt{ab}$.

(iii) If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$

and $\dfrac{a-b}{a+b} = \dfrac{c-d}{c+d}$ (Componendo and Dividendo)

e.g., Divide ₹ 1024 among, A, B and C so find the value of A and B.

Sol. Sum of the ratios $= 4 + 5 + 7 = 16$

∴ First part $= \left(1024 \times \dfrac{9}{16}\right) = ₹576$

and second part $= \left(1024 \times \dfrac{7}{16}\right) = ₹448$

Important Tips/Formulae

- If $\dfrac{a}{b} = 1, \dfrac{a + x}{a + x} = 1$ and $\dfrac{a - x}{a - x} = 1$
- If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = \cdots = k$

Then, $\dfrac{a + c + e + \cdots}{b + d + f + \cdots} = k$

- If $A : B$ and $B : C$ are given, then $A : B : C$ is given by
$A : B$

i.e., $A \times B : B \times B : B \times C$

e.g., If A : B = 5 : 4

and B : C = 3 : 2, then A : B : C = ?

Sol.

$$3 \quad : \quad 2$$

$$= 5 \times 3 : 3 \times 4 : 4 \times 2$$

$$= 15 : 12 : 8$$

- If $\dfrac{a}{b} = \dfrac{N_1}{D_1}, \dfrac{b}{c} = \dfrac{N_2}{D_2}, \dfrac{c}{d} = \dfrac{N_3}{D_3}$, then $a : b : c : d$ is given by

$$N_1 N_2 N_3 : D_1 N_2 N_3 : D_1 D_2 D_3$$

Solved Examples:

1. If $\frac{a}{b} = \frac{3}{4}, \frac{b}{c} = \frac{5}{6}, \frac{c}{d} = \frac{7}{8}$, then find $a : b : c : d$.

    (a) $105 : 120 : 135 : 180$
    (b) $105 : 140 : 168 : 192$
    (c) $105 : 140 : 158 : 212$
    (d) $115 : 140 : 168 : 182$

Sol. (b) $a : b : c : d$

$= N_1 N_2 N_3 : D_1 N_2 N_3 : D_1 D_2 N_3 : D_1 D_2 D_3$

$= 3 \times 5 \times 7 : 4 \times 5 \times 7 : 4 \times 6 \times 7 : 4 \times 6 \times 8 = 105 : 140 : 168 : 192$

2. What is the ratio of 80 paise and ₹5?

    (a) $2 : 25$
    (b) $4 : 35$
    (c) $4 : 25$
    (d) $6 : 35$

Sol. (c) ₹ $5 = 5 \times 100$ paise

Their ratio $= \frac{80}{500} = 4 : 25$

3. Two natural numbers are in the ratio $3 : 5$ and their product is 2160. The smaller of the numbers is

    (a) 36
    (b) 24
    (c) 18
    (d) 12

Sol. (a) Let the natural numbers are $3x$ and $5x$, then

$$3x \times 5x = 2160 \Rightarrow 15x^2 = 2160$$
$$\Rightarrow \quad x^2 = 144 \Rightarrow x = 12$$

Hence, smaller $= 3x = 3 \times 12 = 36$

4. What must be added to each term of the ratio $7 : 11$ so as to make it equal to $3 : 4$?

    (a) 8
    (b) 7.5
    (c) 6.5
    (d) 5

Sol. (d) Let the required number is x, then $\frac{7+x}{11+x} = \frac{3}{4} \Rightarrow 28 + 4x = 33 + 3x \Rightarrow 4x - 3x = 33 - 28 \Rightarrow x = 5$

5. If $x:y = 3:1$, then find the ratio of $x^3 - y^3 : x^3 + y^3$.

    (a) $12:11$
    (b) $13:21$
    (c) $13:14$
    (d) $21:42$

Sol. (c) Let $x = 3k; y = k$

Then, $x^3 - y^3 = 27k^3 - k^3 = 26k^3$

and $x^3 + y^3 = 27k^3 + k^3 = 28k^3$

Their ratio $= \dfrac{26k^3}{28k^3} = 13:14$

6. If two numbers are in the ratio of $5:8$ and if 9 be added to each, the ratio becomes $8:11$. Find the lower number.

    (a) 10
    (b) 13
    (c) 12
    (d) 15

Sol. (d) Let the numbers are $5x$ and $8x$.

By given condition, $\dfrac{5x+9}{8x+9} = \dfrac{8}{11}$

$\Rightarrow \qquad 64x + 72 = 55x + 99$
$\Rightarrow \quad 9x \qquad\qquad = 27$
$\Rightarrow \qquad\qquad x = 3$

Hence, lower number $= 5 \times 3 = 15$

Practice Questions

1. The least ratio among $10:18, 7:21, 12:16, 8:20$ is

    (a) $12:16$
    (b) $8:20$
    (c) $10:18$
    (d) $7:21$

2. If x be the mean proportion between $(x - 2)$ and $(x - 3)$, then the value of x is

    (a) 6
    (b) 5
    (c) 6/5
    (d) 5/6

3. The fourth proportional of the numbers 12,16,18 is

(a) 28
(b) 30
(c) 20
(d) 24

4. The ratio of land and water in the whole world is $1:2$. If this ratio in the Northern hemisphere be $2:3$, then the ratio of land and water in the Southern hemisphere is

(a) $4:7$
(b) $4:11$
(c) $3:4$
(d) $4:3$

5. A certain number is divided into two parts such that 5 times the first part added to 11 times, the second part makes 7 times the whole. The ratio of the first part to the second part is

(a) $2:1$
(b) $5:11$
(c) $1:2$
(d) $2:3$

6. What sum of money is to be divided among 3 persons in the ratio $3:4:7$, so that the second person receives ₹ 12 only?

(a) ₹ 21
(b) ₹ 32
(c) ₹ 9
(d) ₹ 42

7. In a certain examination, the number of those who passed was 4 times the number of those who failed. If there had been 35 fewer candidates and 9 more had failed, the ratio of passed and failed candidates would have been $2:1$, then the total number of candidates was

(a) 135
(b) 155
(c) 145
(d) 150

8. The monthly income of H and W is in the ratio $4:3$ and the expenditure is in the ratio $3:2$. If each of them saves ₹ 600 per month, the income of W, in rupees is

(a) 1200
(b) 2400
(c) 1800
(d) 9000

9. If $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$, then $(a + b):(b + c) = ?$

(a) $3:4$
(b) $4:5$
(c) $5:9$
(d) $20:27$

10. Ratio between the monthly incomes of A and B is $9:8$ and the ratio between their expenditures is $8:7$. If they save ₹ 500 each, find A 's monthly income.

(a) ₹ 3500
(c) ₹ 4500
(b) ₹ 4000
(d) ₹ 5000

11. If $x:y = 3:4$, then the value of $\frac{5x-2y}{7x+2y}$.

(a) $\frac{7}{25}$
(b) $\frac{7}{23}$
(c) $\frac{7}{29}$
(d) $\frac{7}{17}$

12. If $\frac{1}{5}:\frac{1}{x} = \frac{1}{x}:\frac{1}{1.25}$, then the value of x is

(a) 1.5
(b) 2
(c) 2.5
(d) 3.5

13. If $a:b = c:d$, then $\frac{ma+nc}{mb+nd}$ is equal to

(a) $m:n$
(b) $na:mb$
(c) $a:b$
(d) $md:nc$

14. A bag contains an equal number of ₹1,50 paise and 25 paise coins respectively. If the total value is ₹ 35 , how many coins of each type are there?

(a) 20
(b) 16
(c) 18
(d) 25

15. If $a:b = c:d$, then the value of $\frac{a^2+b^2}{c^2+d^2}$ is

(a) $\frac{1}{2}$

(b) $\frac{a+b}{c+d}$

(c) $\frac{a-b}{c-d}$

(d) $\frac{b^2}{d^2}$

16. A dog pursues a cat and takes 5 leaps for every 6 leaps of the cat, but 4 leaps of the dog are equal to 5 leaps of the cat. Compare the speeds of the dog and the cat.

(a) $15:22$

(b) $9:25$

(c) $25:21$

(d) $25:24$

17. What should be subtracted from each of the numbers 54,71,75 and 99, so that the remainders are in continued proportion?

(a) 9

(c) 4

(b) 7

(d) 3

18. Divide 170 into three parts such that the first part is 10 more than the second and its ratio with third part is $2:5$.

(a) $22:35:113$

(b) $35:55:80$

(c) $40:30:100$

(d) $35:65:70$

19. Ratio of boys to the girls in a class is $5:4$. Which of the following cannot be the number of students in the class?

(a) 45

(b) 72

(c) 108

(d) 98

20. If there are ₹ 495 in a bag in denominations of ₹ 1, 50 paisa and 25 paisa coins, which are in the ratio $1:8:16$. How many 50 paisa coins are there in the bag?

(a) 50

(b) 220

(c) 440

(d) None of these

21. The monthly salary of A, B, C are in the ratio of $2:3:5$. If C 's monthly salary is ₹ 1200 more than that of A, then B 's annual salary is

(a) ₹ 14400
(b) ₹ 24000
(c) ₹ 1200
(d) ₹ 2000

22. The ratio of 40 m and 2 km is

(a) $1:50$
(b) $2:35$
(c) $1:40$
(d) $3:25$

23. If $p:q = 3:4$ and $q:r = 8:9$, then $p:r$ is

(a) $1:3$
(b) $3:2$
(c) $2:3$
(d) $1:2$

24. If $a+b:b+c:c+a = 6:7:8$ and $a+b+c = 14$, then the value of c is

(a) 6
(b) 7
(c) 8
(d) 14

25. If $\frac{1}{3}$ of $A = 75\%$ of $B = 0.6$ of C, then $A:B:C$ is

(a) $4:5:9$
(b) $5:9:4$
(c) $9:5:4$
(d) $9:4:5$

26. A certain sum of money is divided between P and Q in the ratio of $3\frac{1}{2}:5\frac{1}{2}$. If P gets ₹ 180 less than Q, then the square of Q is

(a) ₹ 315
(b) ₹ 495
(c) ₹ 630
(d) ₹ 810

27. Four numbers in the ratio $1:3:4:7$ add upto give a sum of 105. Find the value of the biggest number.

(a) 42
(b) 35
(c) 49
(d) 63

28. What number must be taken from each term of the fraction 27/35 that it may become 2: 3?

    (a) 9
    (b) 10
    (c) 7
    (d) 11

29. The present ratio of ages of A and B is 4: 5.18yr ago, this ratio was 11: 16. Find the sum of total of their present ages.

    (a) 90yr
    (b) 105 yr
    (c) 110yr
    (d) 80yr

30. If two numbers are in the ratio of 5: 8 and if 9 added to each, the ratio becomes 8: 11. Find the sum of the numbers.

    (a) 39
    (b) 49
    (c) 64
    (d) 19

ANSWERS

1. (c)	2. (c)	3. (d)	4. (b)	5. (a)	6. (b)	7. (b)	8. (c)	9. (d)	10. (c)
11.(c)	12.(c)	13.(c)	14.(a)	15.(d)	16.(d)	17.(d)	18. (c)	19.(d)	20. (c)
21.(a)	22.(a)	23.(c)	24.(a)	25.(d)	26.(b)	27.(c)	28.(d)	29.(a)	30.(a)

Hints & Solutions

1. $\frac{10}{18} = \frac{2}{9} = 0.22, \frac{7}{21} = \frac{1}{3} = 0.33,$

$\frac{12}{16} = \frac{3}{4} = 0.75, \frac{8}{20} = \frac{2}{5} = 0.4$

Thus, 10:18 is the least among the given ratios.

2. Since, x is mean proportion of $(x - 2)$ and $(x - 3)$.

    $\therefore\ x^2 = (x - 2)(x - 3)$
    $\Rightarrow x^2 = x^2 - 5x + 6$
    $\Rightarrow 5x = 6$
    $\Rightarrow x = \frac{6}{5}$

3. Let x be the fourth proportion of $12, 16$ and 18, then

$$x = \frac{16 \times 18}{12} \Rightarrow x = 24$$

4. Let the whole world having 30 equal parts the fraction of the land to the world $= \frac{1}{3} \times 30 = 10$ parts

and the fraction of the water to the world $= \frac{2}{3} \times 30 = 20$ parts

∵ Northern hemisphere carries 15 parts of a whole, hence the fraction of land at Northern hemisphere

$$= 15 \times \frac{2}{5} = 6 \text{ parts}$$

and the fraction of water at Northern hemisphere $= 15 \times \frac{3}{5} = 9$ parts

∴ Ratio of land and water in

Southern hemisphere $= (10 - 6) : (20 - 9) = 4 : 11$

5. Let first and second part of the number be x and y, respectively.
Then,

$$
\begin{aligned}
5x + 11y &= 7(x + y) \\
\Rightarrow \quad 11y - 7y &= 7x - 5x \\
\Rightarrow \quad 4y &= 2x \\
\therefore \quad x : y &= 2 : 1
\end{aligned}
$$

6. Let required money be ₹x.
Then,

$$\frac{4}{14} x = 12$$

$$\Rightarrow x = \frac{12 \times 14}{4} = ₹42$$

7. Let number of failed and passed candidates be x and $4x$, respectively. Therefore, total number of candidates was $5x$ According to the question, if total number of students had been $5x - 35$,

Then,

$$\frac{4x - 35 - 9}{x + 9} = \frac{2}{1}$$
$$\Rightarrow 4x - 44 = 2(x + 9)$$
$$\Rightarrow 4x - 2x = 18 + 44$$
$$\Rightarrow 2x = 62$$
$$\therefore x = 31$$

Thus, total number of candidates was 31×5 i.e., 155.

8. Let the monthly income of H and W be ₹$4x$, ₹ $3x$ respectively and the expenditure be ₹ $3y$, ₹ $2y$ respectively.

By given condition,

$$4x - 3y = 600$$
$$3x - 2y = 600$$

From Eqs. (i) and (ii),

$x = 600$

∴ Monthly income of $W = 600 \times 3$

9. $\dfrac{a}{b} = \dfrac{2}{3}, \dfrac{b}{c} = \dfrac{4}{5}$

$= ₹1800$

$\Rightarrow a:b:c = 8:12:15$

Let $a = 8k, b = 12k$ and $c = 15k$

$\therefore \dfrac{a+b}{b+c} = \dfrac{8k+12k}{12k+15k} = \dfrac{20}{27}$

10. Let A's monthly income $= ₹9x$ and B's monthly income $= ₹8x$ According to question,

Ratio between their expenditures $= 8:7$

$$\frac{9x - 500}{8x - 500} = \frac{8}{7}$$
$$6x - 3500 = 64x - 4000$$
$$x = 500$$

∴ A's monthly income $= 9 \times 500 = ₹4500$

11. ∵ $\dfrac{x}{y} = \dfrac{3}{4} \Rightarrow y = \dfrac{4x}{3}$

$\therefore \dfrac{5x - 2y}{7x + 2y} = \dfrac{5x - 2\left(\frac{4x}{3}\right)}{7x + 2\left(\frac{4x}{3}\right)}$

$= \dfrac{5x - \frac{8x}{3}}{7x + \frac{8x}{3}} = \dfrac{15x - 8x}{21x + 8x} = \dfrac{7x}{29x} = \dfrac{7}{29}$

12. $\frac{1}{5} \times \frac{1}{1.25} = \frac{1}{x^2}$

$\Rightarrow x^2 = 5 \times 1.25$

$x = \sqrt{6.25} = 2.5$

13. Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk, c = dk$ $\therefore \frac{ma+nc}{mb+nd} = \frac{mbk+ndk}{mb+nd} = k = \frac{a}{b}$

14. Number of coins of each type Total value

Sum of value of each type of coin $= \frac{35}{1+0.5+0.25} = 20$

15. $\frac{a}{b} = \frac{c}{d} = k; a = bk, c = dk$

$\left(\frac{a^2 + b^2}{c^2 + d^2}\right) = \frac{b^2(k^2 + 1)}{d^2(k^2 + 1)} = \frac{b^2}{d^2}$

16. 5 leaps of dog $= \frac{5 \times 5}{4}$ leaps of cat

$\therefore$ Speed of dog : Speed of cat

$= \frac{25}{4} : 6 = 25 : 24$

17. Let number x be subtracted from each of the numbers, then

$\frac{54 - x}{71 - x} = \frac{75 - x}{99 - x}$

For $x = 3$, this relation is correct.

18. Let first and third part be $2x$ and $5x$, then second part be $2x - 10$.

$(2x) + (2x - 10) + 5x = 170$

$\Rightarrow x = 20$

Therefore, three parts are 40, 30 and 100.

19. The total number of students should be divisible by $(5 + 4) = 9$

20. Ratio of number of coins $= 1:8:16$

Ratio of value of each type of coins $= 1:4:4$

$\therefore$ Value of 50 paisa coins $= \frac{4}{9} \times 495 = 220$

Therefore, number of coins of 50 paisa $= 220 \times 2 = 440$

21. Let the monthly salary of

A, B and C be $2x, 3x$ and $5x$.

$5x = 2x + 1200$

$\Rightarrow x = ₹400$

So, annual salary of B

$= 3 \times 400 \times 12$
$= ₹14400$

22. 24 m $= 2 \times 1000 = 2000$ m

So that, ratio of 40 m and 2000 m

$= 40:2000 = 1:50$

23. $p:q = 3:4$ and $q:r = 8:9$

i.e., $\frac{p}{q} = \frac{3}{4}$ and $\frac{q}{r} = \frac{8}{9}$

On multiplying these two

$\frac{p}{r} = \frac{p}{q} \times \frac{q}{r} = \frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$

$\therefore p:r = 2:3$

24. Given $a + b : b + c : c + a = 6 : 7 : 8$

Let $a + b = 6k, b + c = 7k$ and $c + a = 8k$

$\therefore 2(a + b + c) = 6k + 7k + 8k$

$\Rightarrow a + b + c = \dfrac{21}{2}k$

$\Rightarrow 14 = \dfrac{21}{2}k$

$\Rightarrow k = \dfrac{28}{21} = \dfrac{4}{3}$

Therefore, $c = (a + b + c) - (a + b)$

$\qquad = \dfrac{21}{2}k - 6k = \dfrac{9k}{2} = \dfrac{9}{2} \times \dfrac{4}{3}$

$\qquad\qquad \therefore c = 6$

25. $\dfrac{1}{3}$ of $A = 75\%$ of $B = 0.6$ of C

$\Rightarrow \dfrac{1}{3} \times A = \dfrac{75 \times B}{100} = 0.6 \times C$

$\Rightarrow \dfrac{A}{3} = \dfrac{3B}{4} = \dfrac{3C}{5}$

Now, $A = \dfrac{9}{4}B$ and $C = \dfrac{5}{4}B$

Then, ratio of

$A : B : C = \dfrac{9}{4}B : B : \dfrac{5}{4}B$

Required ratio $= 9 : 4 : 5$

26. The ratio of P 's share and Q 's share $= 3\dfrac{1}{2} : 5\dfrac{1}{2} = \dfrac{7}{2} : \dfrac{11}{2} = 7 : 11$

By given condition,

$11x - 7x = 180 \Rightarrow x = 45$

$\therefore$ The share of $Q = 11x = 11 \times 45 = ₹495$

27. Let the numbers are $x, 3x, 4x$ and $7x$

By given condition,

$x + 3x + 4x + 7x = 105$

$\Rightarrow 15x \qquad\qquad\qquad\qquad = 105$

$\Rightarrow x \qquad\qquad\qquad\qquad\qquad = 7$

$\therefore$ Biggest number $\qquad = 7 \times 7 = 49$

28. Let x be taken, then $\frac{27-x}{35-x} = \frac{2}{3}$

$\Rightarrow 70 - 2x = 81 - 3x \Rightarrow x = 11$

29. Let the present ages of A and B are $4x$ and $5x$, respectively.

By given condition, $\frac{4x-18}{5x-18} = \frac{11}{16}$

$\Rightarrow 64x - 18 \times 16 = 55x - 18 \times 11$

$\Rightarrow 9x = 18(16 - 11)$

$\Rightarrow x = 10$

$\therefore$ The sum of their present ages $= 40 + 50 = 90$yr

30. Let the numbers are $5x$ and $8x$.

By given condition,

$\frac{5x + 9}{8x + 9} = \frac{8}{11}$
$\Rightarrow 64x + 72 = 55x + 99$
$\Rightarrow 9x = 27$
$\Rightarrow x = 3$

Hence, the numbers are $5 \times 3, 8 \times 3$ i.e., 15 and 24.

The sum of the numbers $= 15 + 24 = 39$