CHAPTER 03

Square Root and Cube Root

"Square and Cubes are the basic operations with equal importance as the binary operations. To cater such problem you must have the conceptual clarity regarding the same".

Square

The square of any number is the number multiplied by itself.

e.g., $2^2, 3^2, 4^2, \ldots, n^2$

$$2^2 = 2 \times 2 = 4$$
$$4^2 = 4 \times 4 = 16$$
$$n^2 = n \times n$$

In $2^2, 3^2, 4^2, \ldots, n^2$, figure $2,3,4, \ldots, n$ are called the base and at the top of a number tells you to square it. The above statements can be expressed by saying that the square of 2 is 4 or two squared is 4 and so on.

Properties of Square

- It cannot be a negative number.
- It cannot have odd number of zeros at its end.
- It cannot end with 2,3,7 or 8 .

Perfect Square

A natural number is called a perfect square or a square number. If it is the square of some natural number. $n = m^2$, for some natural number m, then n is said to be a perfect square.

e.g., 4 is a perfect square of 2.9 is a perfect square of 3 . Note

(i) Squares of even numbers are always even.

(ii) Squares of odd numbers are a/ways odd.

Square Root

The square root of a number is that factor of the number which, when multiplied by itself, will give that number.

The above statement can be expressed by if a is the square root of ' b ', then ' b ' is the square of ' a '.

The square root of a number is indicated by the sign $\sqrt{\ }$ or $\sqrt[2]{\ }$.
The square root of 49 is written as $\sqrt{49}$.

Thus, $\sqrt{49} = \sqrt{7 \times 7} = 7$

Square root of a number can be learnt with the help of given below.

$\sqrt{81} = 9$ $\sqrt{100} = 10$ $\sqrt{121} = 11$
$\sqrt{144} = 12$ $\sqrt{169} = 13$ $\sqrt{196} = 14$
$\sqrt{225} = 15$ $\sqrt{256} = 16$ $\sqrt{289} = 17$
$\sqrt{324} = 18$ $\sqrt{361} = 19$ $\sqrt{400} = 20$
$\sqrt{441} = 21$ $\sqrt{484} = 22$ $\sqrt{529} = 23$
$\sqrt{576} = 24$ $\sqrt{525} = 25$ $\sqrt{676} = 26$
$\sqrt{729} = 27$ $\sqrt{784} = 28$ $\sqrt{841} = 29$

$\sqrt{900} = 30$ and so on ....

Unit Digit in Square			
Number	Unit Digit	Number	Unit Digit
$(\ldots 1)^2$	... 1	$(\ldots 6)^2$	... 6
$(\ldots 2)^2$	... 4	$(\ldots 7)^2$	... 9
$(\ldots 3)^2$	... 9	$(\ldots 8)^2$	... 4
$(\ldots 4)^2$	... 6	$(\ldots 9)^2$	... 1
$(\ldots 5)^2$	... 5	$(\ldots 0)^2$	... 0

How to Calculate the Square Root?

There are two methods to calculate the square root.

Prime Factorisation Method

This method has the following steps.

Step I Express the given number as the product of prime factors

Step II Keep these factors in pairs.

Step III Take the product of these prime factors taking one out of every pair of the same primes. This product gives us the square root of the given number.
e.g. Find the square root of 1089 .

Sol. Prime factors of $1089 = 11 \times 11 \times 3 \times 3$

11	1089
11	99
3	9
3	3
	1

$$\Rightarrow \sqrt{1089} = \sqrt{11 \times 11 \times 3 \times 3}$$

Now, taking one number from each pair and multiplying them, we get

$\sqrt{1089} = 11 \times 3 = 33$

e.g. Find the square root of 1024.

Sol. Prime factors of 1024

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow \sqrt{1024} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

Now, taking one number from each pair and multiplying them, we get $\sqrt{1024} = 2 \times 2 \times 2 \times 2 \times 2 = 32$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2

Division Method

The steps of this method can be easily understood with the help of following examples

e.g. Find the square root of 18769.

Sol. Step I In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one digit (if any), is called a period.

Step II Now, on subtracting, we get 0 (zero) as remainder.

Step III Bring down the next period, i.e., 87. Now the trial divisor is $1 \times 2 = 2$ and trial dividend is 87.

So, we take 23 as divisor and put 3 as quotient. The remainder is 18 now.

	137
1	1 87 69 1
23	87 69
267	1869 1869
	×

Step IV Bring down the next period, which is 69. Now, trial divisor is $13 \times 2 = 26$ and trial dividend is 1869. So, we take 267 as dividend and 7 as quotient. The remainder is 0 now.

Step V The process (processes like III and IV) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero). Hence, the required square root $= 137$

e.g. What is the square root of 151321?

Sol.

	389
3	15 13 21 9
68	613 544
769	6921 6921

x

$\therefore$ Required square root = 389

Note Division method should be applied, when the given number is so large that it is very difficult to find its square root by the prime factorisation method.

Square of Two Digits

Let AB is 2 Digit number, then

Step I B^2

Step II $2(A \times B)$

Step III $A^2 = A^2 + 2AB + B^2$

e.g. $(\mathbf{16})^2$

Sol. Here, $1 = A$ and $6 = B$. Then,

$$\begin{array}{ll} \text{Step I} & (6)^2 = 36 \\ \text{Step II} & 2(1 \times 6) = 12 \\ \text{Step III} & (1)^2 = 1 = 1 + 12 + 36 = 2 + {}_15 + {}_36 = 256 \end{array}$$

e.g. $(\mathbf{23})^2$

Sol. Here, $2 = A$ and $3 = B$. Then,

$$\begin{array}{lrl} \text{Step I} & (3)^2 & = 9 \\ \text{Step II} & 2(2 \times 3) & = 12 \\ \text{Step III} & (2)^2 & = 4 = 4 + 12 + 9 = 5 + {}_12 + 9 = 529 \end{array}$$

e.g. $(\mathbf{26})^2$

Sol. Here, $2 = A$ and $6 = B$. Then,

$$\begin{array}{lc} \text{Step I} & (6)^2 = 36 \\ \text{Step II} & 2(2 \times 6) = 24 \\ \text{Step III} & (2)^2 = 4 = 4 + 24 + 36 = 6 + {}_27 + {}_36 = 676 \end{array}$$

e.g. $(\mathbf{47})^2$

Sol. Here, $4 = A$ and $7 = B$. Then,

$$\begin{array}{lrl} \text{Step I} & (7)^2 & = 49 \\ \text{Step II} & 2(4 \times 7) & = 56 \\ \text{Step III} & (4)^2 & = 16 = 16 + 56 + 49 \\ & & = 22 + {}_60 + {}_49 = 2209 \end{array}$$

Square of a Number Ending in 5

Step I Multiply the number formed after deleting 5 at the unit's place with the number one higher than it.

Step II Annex 25 on the right side of the product and you will get the square of the given number.

e.g. $(\mathbf{15})^{\mathbf{2}}$

Sol. Step I $1 \times 2 = 2$
Step II $= 25 \Rightarrow (15)^2 = 225$
e.g. $(35)^2$

Square of Decimal Numbers

Step I Find the square of the number ignoring the decimal point.

Step II Put the decimal point in such a way that the number of decimal places in the square is twice of that in the original number.

Step III The square of a decimal number will lie between the square of its integral part and the square of the number one higher than the integral part.

e.g. $(\mathbf{3.6})^{\mathbf{2}} = (36)^2 = 1296 = 12.96$

e.g.$(\mathbf{4.5})^{\mathbf{2}} = (45)^2 = 2025 = 20.25$

e.g.$(\mathbf{10.5})^{\mathbf{2}} = (105)^2 = 11025 = 110.25$

Square of a Number Ending in 25

Step I Multiply the number formed after leaving 25 by a number 5 suffixed to it.

Step II Annex 625 to the right side of the product.

e.g. $(\mathbf{125})^{\mathbf{2}} = 1 \times 15 = 15$ hence, $(125)^2 = 15625$

e.g. $(\mathbf{225})^{\mathbf{2}} = 2 \times 25 = 50$ hence, $(225)^2 = 50625$

e.g. $(\mathbf{325})^{\mathbf{2}} = 3 \times 35 = 105$ hence, $(325)^2 = 105625$

How to Calculate the Square Root of Decimal Numbers?

If in a given decimal number, the number of digits after decimal are not even, then we put a 0 (zero) at the extreme right. Now, we mark off the periods and try to calculate the square root applying the division method.

1	$147 \cdot 1369$ 1
22	47 44
241	313 241
242 3	7269 7269

e.g. Find the square root of 147.1369.

Sol. 12.13

∴ Required square root $= 12.13$

How to Calculate the Square Root of a Fraction?

To find square root of a fraction, we have to find the square roots of numerators and denominators, separately.

e.g. $\sqrt{\dfrac{2704}{81}}$ is equal to

Sol. $\sqrt{\dfrac{2704}{81}} = \dfrac{\sqrt{2704}}{\sqrt{81}} = \dfrac{52}{9}$

5	2704
102	204
	204

Note Sometimes, numerator and denominator are not a complete square. In these types of cases, it is better to convert the given fraction into decimal fraction to find the square root.

Cube

The cube of any number is the number multiplied by itself and by itself again.

e.g. $\mathbf{2^3, 5^3, \ldots, n^3}$

A small figure 3 at the top of given number tells you to cube it. 2^3 is usually read as 'two cubed' or the 'cube of 2 ' and so on.

The cube of 5 is $5 \times 5 \times 5 = 125$

Perfect Cube

A natural number is said to be a perfect cube, if it is the cube of some natural number. A natural number n is a perfect cube, if $n = m^3$, where m is a natural number.

Numbers such as 1,8,27,64, ... are called perfect cube.

Note Cube of a number is the triple product obtained on multiplying the number by itself.

e.g. $9^3 = 9 \times 9 \times 9 = 729$

$11^3 = 11 \times 11 \times 11 = 1331$

$12^3 = 12 \times 12 \times 12 = 1728$ and so on....

Cube of a number can be learnt through this below

$0^3 = 0$	$1^3 = 1$	$2^3 = 8$
$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$6^3 = 216$	$7^3 = 343$	$8^3 = 512$
$9^3 = 729$	$10^3 = 1000$	$11^3 = 1331$
$12^3 = 1728$	$13^3 = 2197$	$14^3 = 2744$
$15^3 = 3375$	$16^3 = 4096$	$17^3 = 4913$
$18^3 = 5832$	$19^3 = 6859$	$20^3 = 8000$
$21^3 = 92621$	$22^3 = 10648$	$23^3 = 12167$
$24^3 = 13824$	$25^3 = 15625$	

Cube Root

The cube root of a number multiplied by itself and by itself again gives the number.

A natural number m is the cube root of a number n if $n = m^3$.

The above statement can be expressed by if m is the cube root of n, then n is the cube of m.e.g., 125 is the cube of 5 and therefore 5 is the cube root of 125.

How to Calculate the Cube Root?

Prime Factorisation Method

This method has following steps

Step I Express the given number as the product of prime factors.

Step II Keep these factors in a group of three.

Step III Take the product of these prime factors picking one out of every group (group of three) of the same primes. This product gives us the cube root of given number.

e.g., Find the cube root of 9261.

Sol. Prime factors of $9261 = (3 \times 3 \times 3) \times (7 \times 7 \times 7)$

$$\sqrt[3]{9261} = \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$$

Now, taking one number from each group of these,

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

we get $\sqrt[3]{9261} = 3 \times 7 = 21$

Important Tips/Formulae

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- The value of $\sqrt{x \pm \sqrt{x \pm \sqrt{x \pm \cdots \infty}}}$

Suppose ' a ' and ' b ' are consecutive factors of x where $b > a$. If their sign is (+) in the expression, the answer is b i.e., bigger factor and if their sign is (-), the answer is a i.e., the smaller factor.

e.g., $\sqrt{20-\sqrt{20-\sqrt{20-\sqrt{20 \ldots \infty}}}}$ is equal to

Sol. $20 = 4 \times 5$. Since, the sign is $(-)$, then the required answer is the smaller factor i.e., 4.

- $\sqrt{x \cdot \sqrt{x \cdot \sqrt{x \cdot \sqrt{x \ldots \infty}}}}$ If the root goes upto ∞ in multiplication, the answer is x itself.

Solved Examples:

1. Find the value of $\frac{\sqrt{288}}{\sqrt{128}}$.

(a) $\frac{2}{3}$
(b) $\frac{3}{2}$
(c) $\frac{1}{2}$
(d) 2

Sol. (b) $\frac{\sqrt{288}}{\sqrt{128}} = \sqrt{\frac{288}{128}} = \sqrt{\frac{144}{64}} = \frac{12}{8} = \frac{3}{2}$

2. If $\sqrt{1369} = 37$; find the value of

$\sqrt{13.69} + \sqrt{0.1369} + \sqrt{0.001369} + \sqrt{0.00001369}$

(a) 4.1207
(b) 4.1109
(c) 4.1107
(d) 3.8506

Sol. (c) We have,

Given expression $= \sqrt{\frac{1369}{100}} + \sqrt{\frac{1369}{10000}}$

$$+ \sqrt{\frac{1369}{1000000}} + \sqrt{\frac{1369}{100000000}}$$
$$= \frac{37}{10} + \frac{37}{100} + \frac{37}{1000} + \frac{37}{10000}$$
$$= 3.7 + 0.37 + 0.037 + 0.0037 = 4.1107$$

3. Find the smallest number which should be added to 8958 so that the sum is a perfect square.

(a) 65
(b) 63
(c) 69
(d) 67

Sol. (d)

9	8958	94
$\frac{9}{184}$	$\underline{81}$	858
$\frac{4}{188}$	$\frac{736}{122}$	

∴ Required number $= (95)^2 - 8958 = 9025 - 8958 = 67$

4. Find $\sqrt[3]{1 - \frac{91}{216}}$.

(a) $\frac{4}{5}$
(b) $\frac{5}{6}$
(c) $\frac{7}{8}$
(d) $\frac{3}{4}$

Sol. (b) $\sqrt[3]{\frac{216-91}{216}} = \sqrt[3]{\frac{125}{216}}$

$= \sqrt[3]{\frac{5 \times 5 \times 5}{6 \times 6 \times 6}} = \frac{5}{6}$

5. Evaluate $\sqrt[3]{0.064} + \sqrt{1.21}$.

(a) 1.8
(b) 1.3
(c) 1.5
(d) 2.2

Sol. (c) $\sqrt[3]{0.064} + \sqrt{1.21} = \sqrt[3]{\frac{64}{1000}} + \sqrt{\frac{121}{100}}$

$= \frac{\sqrt[3]{64}}{\sqrt[3]{1000}} + \frac{\sqrt{121}}{\sqrt{100}}$

$= \frac{4}{10} + \frac{11}{10} = 0.4 + 1.1 = 1.5$

6. Find the value of n, if $\sqrt{3^n} = 729$

(a) 11
(b) 10
(c) 12
(d) 13

Sol. (c) $\sqrt{3^n} = 729$

$\sqrt{3^n} = 3^6$

$\therefore\ 3^n = (3^6)^2 = 3^{12}$

$\Rightarrow\ n = 12$

Practice Questions

1. Evaluate $\sqrt{129 + \sqrt{216 + \sqrt{68 + \sqrt{169}}}}$

(a) 13
(b) 15
(c) 9
(d) 12

2. The simplified value of $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{12} \times \frac{\sqrt{256}}{8}$ is

(a) 12
(b) 8
(c) 16
(d) 32

3. The value of $\sqrt[6]{0.000729}$ is

(a) 0.027
(b) 0.3
(c) 0.03
(d) 0.09

4. $1499 \times 1499 = ?$

(a) 19501
(b) 1900501
(c) 2247001
(d) 2204701

5. $\sqrt{14161} = ?$

(a) 129
(b) 119
(c) 121
(d) None of these

6. $\sqrt{0.04} = ?$

(a) 0.002
(b) 0.02
(c) 0.2
(d) None of these

7. $\sqrt{\frac{0.441}{0.625}} = ?$

(a) 0.048
(b) 0.084
(c) 0.48
(d) 0.84

8. The value of $\sqrt{\frac{1.21 \times 0.9}{1.1 \times 0.11}}$ is

(a) 2
(b) 3
(c) 9
(d) 11

9. If $\sqrt{4^n} = 1024$, then the value of n is

(a) 5
(b) 8
(c) 10
(d) 12

10. The value of $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$ is

(a) 14
(b) 16
(c) 16.6
(d) 18.2

11. If $\sqrt{1+\frac{x}{144}}=\frac{13}{12}$, then x is equal to

(a) 1
(b) 12
(c) 13
(d) 25

12. Three-fifth of the square of a certain number is 126.15. What is the number?

(a) 14.5
(b) 75.69
(c) 145
(d) 210.25

13. Given that $\sqrt{1225}=35$, then the value of $\sqrt{12.25}+\sqrt{0.1225}+\sqrt{0.001225}$ is

(a) 0.3885
(b) 388.5
(c) 38.85
(d) 3.885

14. If $\sqrt{5}=2.24$ and $\sqrt{6}=2.45$, then the value of $\sqrt{\frac{2}{3}}+\sqrt{\frac{5}{6}}$ is

(a) 1.37
(b) 1.57
(c) 1.73
(d) 1.75

15. If $\frac{\sqrt{1296}}{x}=\frac{x}{2.25}$, then find the value of x.

(a) 10
(b) 9
(c) 8
(d) 6

16. If $a=0.1039$, then the value of $\sqrt{4a^2-4a+1}+3a$ is

(a) 0.1039
(b) 0.2078
(c) 1.1039
(d) 2.1039

17. The largest number of five digits when it is a perfect square is

(a) 99967
(b) 99764
(c) 99856
(d) 99999

18. The least number to be added to 269 to make it a perfect square is

(a) 31
(b) 16
(c) 7
(d) 20

19. Find the least number which when subtracted from 1850 makes it a perfect square.

(a) 5
(b) 7
(c) 1
(d) 11

20. The value of $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}}$ is

(a) $6\frac{2}{3}$
(b) $3\frac{1}{2}$
(c) 6
(d) 3

21. Find the value of $\sqrt{5\cdot\sqrt{5\cdot\sqrt{5\cdot\sqrt{5\cdot\ldots\cdot\infty}}}}$.

(a) 125
(b) 25
(c) $\sqrt{5}$
(d) 5

22. The least perfect square number divisible by each one of 3,4,5,6,8 is

(a) 1200
(b) 1500
(c) 3600
(d) 700

23. $\frac{7.2}{\sqrt[3]{0.729}} = \frac{(?)^3}{(2)^3}$, then find the value of (?)

(a) 5
(b) 4
(c) 3
(d) 6

24. What least number should be subtracted from 6860 so that 19 be the cube root of the result from this subtraction?

(a) 3
(b) 2
(c) 4
(d) 1

25. If $p = 999$, then $\sqrt[3]{p(p^2 + 3p + 3) + 1} =$?

(a) 1000
(b) 999
(c) 1002
(d) 998

26. By what least number, 3600 be divided to get a perfect cube?

(a) 9
(b) 50
(c) 300
(d) 450

27. By what least number, 675 be multiplied to obtain a number which is a perfect cube?

(a) 5
(b) 6
(c) 7
(d) 8

28. The largest four-digit number which is a perfect cube is

(a) 9999
(b) 9261
(c) 8000
(d) None of these

29. The value of $\sqrt{2^4} + \sqrt[3]{64} + \sqrt[4]{2^8}$ is

(a) 12
(b) 16
(c) 18
(d) 24

30. The value of $\sqrt[3]{(216)^{-3} \div (343)^{-2}}$ is

(a) $\frac{36}{156}$
(b) $\frac{17}{54}$
(c) $\frac{49}{216}$
(d) $\frac{49}{412}$

ANSWERS

1. (d)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (b)
11. (d)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (c)	**20.** (d)
21. (d)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (a)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (c)

Hints & Solutions

1. $\sqrt{129 + \sqrt{216 + \sqrt{68 + \sqrt{169}}}}$

$= \sqrt{129 + \sqrt{216 + \sqrt{68 + 13}}}$

$= \sqrt{129 + \sqrt{216 + \sqrt{81}}}$

$= \sqrt{129 + \sqrt{216 + 9}}$

$= \sqrt{129 + \sqrt{225}}$

$= \sqrt{129 + 15} = \sqrt{144} = 12$

2. $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{12} \times \frac{\sqrt{256}}{8}$

$= \frac{112}{14} \times \frac{24}{12} \times \frac{16}{8}$

$= 8 \times 2 \times 2 = 32$

3. $\sqrt[6]{0.000729} = \sqrt[6]{(0.3)^6} = 0.3$

4. $? = 1499 \times 1499$

$\Rightarrow\ ? = 2247001$

5. $? = \sqrt{14161}$

$\Rightarrow\ ? = 119$

6. $? = \sqrt{0.04}$

$\Rightarrow\ ? = 0.2$

7. $? = \sqrt{\frac{0.441}{0.625}}$

$\Rightarrow\ ? = \sqrt{\frac{441}{625}}$

$\Rightarrow\ ? = \frac{21}{25}$

$\Rightarrow\ ? = 0.84$

8. $\frac{\sqrt{1.21\times0.9}}{\sqrt{1.1\times0.11}} = \sqrt{\frac{121\times9}{11\times11}} = \sqrt{9} = 3$

9. $\sqrt{4^n} = 1024 \Rightarrow (4)^{\frac{n}{2}} = 1024$

$\Rightarrow (4)^{\frac{n}{2}} = 4^5$

$\Rightarrow \frac{n}{2} = 5$

$\Rightarrow\ n = 10$

10. $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$

$= \sqrt{248 + \sqrt{52 + 12}}$

$= \sqrt{248 + \sqrt{64}}$

$= \sqrt{248 + 8} = \sqrt{256} = 16$

11. $\sqrt{1+\frac{x}{144}} = \frac{13}{12}$

$\Rightarrow\ 1+\frac{x}{144} = \frac{169}{144}$

$\Rightarrow\ \frac{x}{144} = \frac{169}{144} - 1$

$= \frac{25}{144}$

$\Rightarrow\ x = \frac{25}{144} \times 144 = 25$

Alternate Method

$\Rightarrow\ x = 13 + 12 = 25$

12. $\frac{3}{5}x^2 = 126.15$

$x^2 = \frac{126.15 \times 5}{3} = 42.05 \times 5$

$x = \sqrt{210.25}$

$x = 14.5$

13. $\sqrt{12.25} + \sqrt{0.1225} + \sqrt{0.001225}$

$= \sqrt{\frac{1225}{100}} + \sqrt{\frac{1225}{10000}} + \sqrt{\frac{1225}{1000000}}$

$= \frac{35}{10} + \frac{35}{100} + \frac{35}{1000}$

$= 3.5 + 0.35 + 0.035 = 3.885$

14. $\sqrt{\frac{2}{3}} + \sqrt{\frac{5}{6}} = \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{6}}$

$= \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{3}\times\sqrt{2}} + \frac{\sqrt{5}}{\sqrt{6}}$

$= \frac{2+\sqrt{5}}{\sqrt{6}} = \frac{2+2.24}{2.45}$

$= \frac{4.24}{2.45} = 1.73$

15. $\frac{\sqrt{1296}}{x} = \frac{x}{2.25}$

$\Rightarrow x^2 = 2.25 \times \sqrt{1296}$

$\Rightarrow x^2 = 2.25$

$\times \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3} \Rightarrow x^2 = 2.25 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow x^2 = 2.25 \times 36$

$\Rightarrow x = \sqrt{2.25 \times 36}$

$\Rightarrow x = 1.5 \times 6$

$\Rightarrow x = 9$

16. $\sqrt{4a^2 - 4a + 1} + 3a$

$= \sqrt{(1)^2 - 2 \times 2a + (2a)^2} + 3a$

$= \sqrt{(1 - 2a)^2} + 3a$

$= 1 - 2a + 3a$

$= 1 + a$

$= 1 + 0.1039$

$= 1.1039$

17. Largest number of 5 digits is 99999.

3	$9\ \overline{99}\ \overline{99}$	316
3	9	
61	99	
1	61	
626	3899	
	3756	
	143	

∴ Required number

$= 99999 - 143$
$= 99856$

18. As we know that $16^2 = 256$ and $17^2 = 289$

Hence, 289 - 269 (Given number) = 20

So, 20 is the required number to be added to make given number a perfect square.

19. Let's denote the unknown least number as x.

According to the given information:

$1850 - x =$ perfect square

To find the least number x, we want to minimize the perfect square.

The perfect squares less than 1850 are:

$44^2 = 1936$

$43^2 = = 1849$

So, the least number x is $1850 - 1849 = 1$

20. $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots\infty}}}}$

6 has two factors 2 and 3.

Since, here is (+ ve) sign, so that bigger factor is the answer i.e., 3 is the answer.

21. Clearly, 5 is answer

Since, root goes upto ∞

$\sqrt{5\cdot\sqrt{5\cdot\sqrt{5\cdot\sqrt{5\ldots\infty}}}} = 5$ (itself)

22. By Hit and Trial

As we know that 3600 is the only given option which is a perfect square too and again it is divisible by all the given numbers 3,4,5,6,8

Hence, 3600 is the required number.

23. $\frac{7.2}{\sqrt[3]{0.729}} = \frac{(?)^3}{(2)^3}$

$\Rightarrow \frac{7.2}{\sqrt[3]{\frac{729}{1000}}} = \frac{(?)^3}{8}$

$\Rightarrow (?)^3 = \frac{7.2 \times 8}{\sqrt[3]{\frac{9 \times 9 \times 9}{10 \times 10 \times 10}}}$

$\Rightarrow (?)^3 = \frac{7.2 \times 8}{9/10}$

$\Rightarrow (?)^3 = \frac{7.2 \times 8 \times 10}{9}$

$\Rightarrow (?)^3 = 64$

$\Rightarrow ? = \sqrt[3]{64} = 4$

24. $\because (19)^3 = 6859$

$\therefore$ Required number

$= 6860 - 6859 = 1$

24. Let's denote the unknown least number as x.

According to the given information: $6860 - x = 19^3$

The cube root of 19 is 2 (because $2^3 = 8$ and $3^3 = 27$, and 19 falls between 8 and 27).

So, we want to find x such that:

$6860 - x = 2^3$

$6860 - x = 8$

Solving for x:

$x = 6860 - 8 = 6852$

25. $24\sqrt[3]{p(p^2+3p+3)+1}$

$= \{p^3+3p^2+3p+1\}^{\frac{1}{3}}$
$= [(p+1)^3]^{\frac{1}{3}}$
$= p+1 = 999+1 = 1000$

26. Prime factorization of 3600 is

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$\therefore\ 3600 = 2^3 \times 2 \times 3^2 \times 5^2$

To get perfect cube, it must be divided by $2 \times 3^2 \times 5^2 = 450$

27. Prime factorization of 675 is

5	675
5	135
3	27
3	9
3	3
	1

$\therefore\ 675 = 5^2 \times 3^3$

To get perfect cube, it must be multiplied by 5.

28. By given option, 9261 is clearly a perfect cube.

29. $\sqrt{2^4} + \sqrt[3]{64} + \sqrt[4]{2^8}$

$= 2^2 + 4 + \sqrt[4]{2^4 \times 2^4}$

$= 4 + 4 + 4$

$= 12$

30. $\sqrt[3]{(216)^{-3} \div (343)^{-2}} = \sqrt[3]{\frac{(216)^{-3}}{(343)^{-2}}}$

$= \sqrt[3]{\frac{(343)^2}{(216)^3}}$

$= \sqrt[3]{\frac{(7^3)^2}{(6^3)^3}}$

$= \sqrt[3]{\frac{7^3 \times 7^3}{6^3 \times 6^3 \times 6^3}}$

$= \frac{7 \times 7}{6 \times 6 \times 6}$

$= \frac{49}{216}$