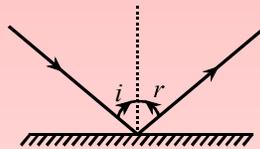# RAY OPTICS

Reflection of Light

❖ **Laws of reflection**

(i) The incident ray, the reflected ray and normal to interface, all lines in the same plane.

(ii) The angle of reflection r is equal to the angle of incident, i.e., $i = r$

❖ **Reflection from plane surfaces:**

(a) The image formed is same size, virtual, erect and as far behind the mirror as the object in front of the mirror.

(b) If an object moves towards (or away from) a plane mirror at a speed v, the image will also approach (or recede) at the same speed v, *i.e.*, the speed of image relative to the object will be $v-(-v) = 2v$.

(c) If keeping the incident ray fixed, the plane mirror is rotated by an angle θ about an axis in the plane of the mirror, the reflected ray is rotated through an angle 2θ.
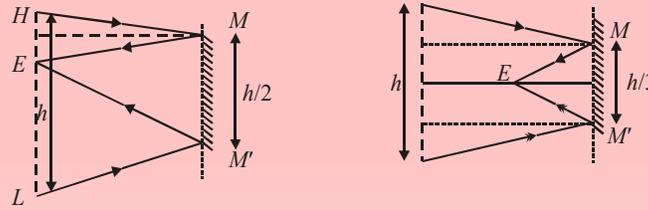

$i = r$ ...(i)

$(i - \theta) = (r + \theta - x)$ ...(ii)

$\boxed{x = 2\theta}$
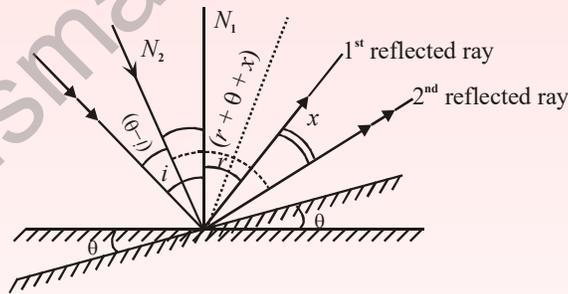
(d) To see his full image in a plane mirror, a person requires a mirror of at least half of his height.

To see a complete wall behind himself a person requires a mirror of at least 1/3 the height of wall and he must be in the middle of wall and mirror.

Example-1: **What is the minimum length of a plane mirror required for a person to see his or her full image? Is there any restriction on the position of the top edge of the mirror?**

Solution: The man can view his entire image if the light rays from the top of his head and from his feet reach his eye.

Let AB be the mirror PQ represents the man of height h and R is the position of his eyes. Light rays from P gets reflected at A and reach his eyes. Light from Q gets reflected at B and reaches his eyes. AM and BN are normal to the mirror AB.

Now, $AB = MN = MR + RN$

$= \dfrac{1}{2} (PR + RQ)$ ($\because \Delta APM \cong \Delta ARM; \Delta BQN \cong \Delta BRN$)

$= \dfrac{PQ}{2} = \dfrac{h}{2}$. Hence the length of the mirror $= \dfrac{h}{2}$

It is clear from the ray diagram that the top edge of the plane mirror (A) must be at a horizontal level half-way between the eyes (R) and the top of his head (P).
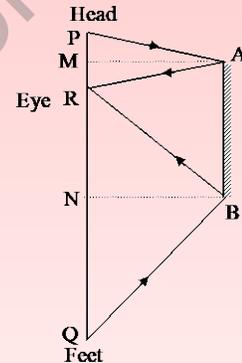
(e) If there are two plane mirrors inclined to each other at an angle θ, the number of images of a point object formed will be n where n is given as:

(i) If $\dfrac{360}{\theta} =$ even (say m)

$n = (m-1)$ (for all positions of object)

(ii) If $\dfrac{360}{\theta} =$ odd (say m)

(a) $n = m$ (if the object is not the bisector angle of mirrors)

(b) $n = (m-1)$ (if the object is on the bisector angle of mirrors)

(iii) If $\dfrac{360}{\theta}$ is a fraction, the number of images will be equal to its integral part.

Example-2: **Rays of light are incident on a plane mirror at 45°. At what angle with the first should a second mirror be placed such that the rays emerge from the second mirror parallel to the first mirror?**
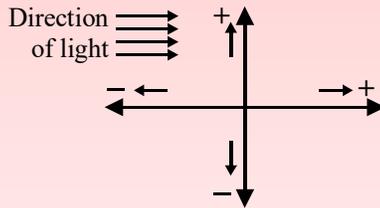
Solution: In triangle BOC, we have $2\theta + 45° = 180°$

or $\theta = \dfrac{135°}{2} = \mathbf{67.5°}$
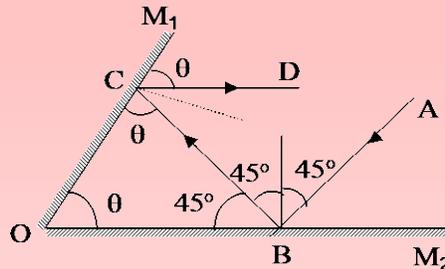
❖ **Reflection from curved surface:**

(a) Sign Convention:

(i) Along the principal axis distance in the direction of light movement are taken as +ve.

(ii) focal length f of concave mirror is –ve.
focal length f of convex mirror is +ve.

(b) Rules for image formation:

(i) A ray parallel to principal axis, after reflection from mirror, passes (concave) or appears to come (convex) from focus.

(ii) A ray passing through (concave) or directed towards (convex) the focus, after reflection from the mirror becomes parallel to the principal axis.

(iii) A ray passing through (concave) or directed towards (convex) Centre of curvature, after reflection from the mirror, retraces it's path.

(iv) For pole principal axis acts as normal $\angle i = \angle r$.

(c) Formulae:

(i) $\boxed{f = \dfrac{R}{2}}$ (–ve for concave and +ve for convex)

(ii) $\boxed{P = -\dfrac{1}{f\,(\text{in m})} = -\dfrac{100}{f\,(\text{in cm})}}$

(iii) $m = \dfrac{\text{height of image}}{\text{height of object}} = -\left[\dfrac{v}{u}\right]$ (lateral magnification)

If a 2 – dimensional object is placed with its plane perpendicular to principal axis its magnification is called **superficial magnification** (areal magnification) and will be

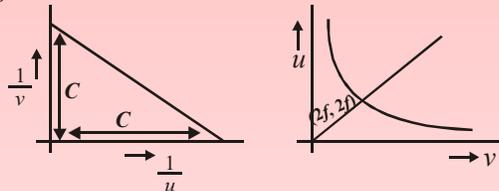$m_s = m^2 = \dfrac{\text{area of image}}{\text{area of object}}$

(iv) $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ (Known quantities are to be substituted with proper sign)

If $R \to \infty$ *i.e.*, becomes plane mirror $f \to \infty$

$\dfrac{1}{v} + \dfrac{1}{u} = 0$ or $\boxed{v = -u}$

(v) If in case of spherical mirrors, object distance x_1 and image distance x_2 are measured from focus instead of pole, then $\boxed{x_1 x_2 = f^2}$.

(vi)

Example-3: **An object is placed in front of a concave mirror at a distance of 7.5 cm from it. If the real image is formed at a distance of 30 cm from the mirror, find the focal length of the mirror. What would be the focal length if the image is virtual?**

Solution: **Case I:** When the image is real.

We have $u = -7.5$ cm; $v = -30$ cm; $f = ?$

We know $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ or, $f = \dfrac{uv}{u+v} = \dfrac{(-7.5) \times (-30)}{-7.5 - 30} = \mathbf{-6\ cm}$

The negative sign shows that the spherical mirror is concave.

Case II: When the image is virtual

In this case, $u = -7.5$ cm, $v = +30$ cm

We know $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ or, $f = \dfrac{uv}{u+v} = \dfrac{(-75)(30)}{-7.5 + 30} = \mathbf{-10\ cm}$

Example-4: **An object 0.2 cm high is placed 15 cm from a concave mirror of focal length 5 cm. Find the position and size of the image.**

Solution: We have $u = -15$ cm; $v = ?$ $f = -5$ cm

We know that $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ or, $v = \dfrac{uf}{u-f} = \dfrac{(-15)(-5)}{-15+5} = -7.5$ cm

The image is formed at a distance of 7.5 cm in front of mirror.

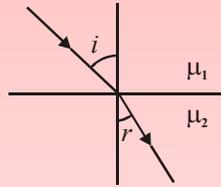Now, $m = \dfrac{I}{O} = -\dfrac{v}{u}$ or, $\dfrac{I}{O} = -\dfrac{(-7.5)}{(-15)}$ or, $I = \dfrac{1}{2}(-0.2) = -0.1$ cm

The negative sign indicates that image is inverted.

❖ **Laws of refraction:**

(a) Frequency (hence color) and phase do not change but wavelength and velocity will change.

(b) Incident ray, refracted ray and normal lie in the same plane.

(c) $\mu \sin i = $ constant. $\mu_1 \sin i_1 = \mu_2 \sin i_2$

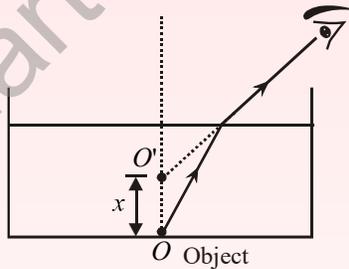if $i_1 = i$ and $i_2 = r$ then $\boxed{\mu_1 \sin i = \mu_2 \sin r}$.

Case I: If $i = 0$ *i.e.*, light is normally incident then it will go undedicated. $\therefore r = 0$ (no refraction).

Case II: If $\mu_1 = \mu_2 = \mu$ then $i = r \neq 0$

(a) Refractive index $\boxed{\mu = \dfrac{c}{v}} = \sqrt{\dfrac{\in \mu}{\in_0 \mu_0}} = \sqrt{\in_r \mu_r}$. *i.e.*, $\mu \propto \dfrac{1}{v}$ or lesser the velocity in the medium, greater is the μ.

(b) Relative refractive index $\boxed{_1\mu_3 = {}_1\mu_2 \times {}_2\mu_3}$ $\boxed{_1\mu_3 = \dfrac{1}{_3\mu_1}}$.

$$\boxed{\frac{\mu_1}{\mu_2} = \frac{d_{actual}}{d_{appearent}}}$$

Case I: If $\mu_1 > \mu_2$ (object is in denser medium) then $\boxed{d_{ac} > d_{ap}}$

The shift in the position of object is $\boxed{x = d_{ac} - d_{ap} = t - \dfrac{t}{\mu} = t\left(1 - \dfrac{1}{\mu}\right)}$

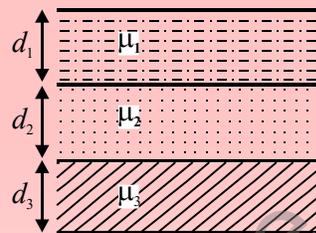**Case II:** If $\mu_2 > \mu_1$ (object is in rarer medium)

$$\boxed{d_{ac} < d_{ap}} \quad x = d_{ap} - d_{ac} = \mu t - t \quad \boxed{x = t(\mu - 1)}$$

If there are number of liquids of different depths, one over the another then

$$d_{ac} = d_1 + d_2 + d_3 + \ldots$$

$$d_{ap} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + \ldots$$

then $\boxed{\mu = \dfrac{d_{ac}}{d_{ap}} = \dfrac{d_1 + d_2 + \ldots}{\dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + \ldots}}$

If $d_1 = d_2$ (for two liquid medium) then $\boxed{\mu = \dfrac{2\mu_1\mu_2}{\mu_1 + \mu_2}}$ = harmonic mean

❖ **Total internal reflection.** The phenomena of reflection of light ray at the interface blocked by rare radium when the angle of incident is greater than critical angle is called **total internal reflection.**

In passing through a medium of thickness t and refractive index μ, a ray incident on a small angle θ is displaced parallel to itself by 'y' called **lateral displacement**.

$$y = x\sin\theta = \left(\frac{\mu - 1}{\mu}\right)t\theta \quad \boxed{y = \left(1 - \frac{1}{\mu}\right)t\theta}.$$

The value of incident angle for which $r = 90°$, is called critical angle (θ_C).

$$\mu_1 \sin i = \mu_2 \sin r \; ; \quad \mu_D \sin\theta_C = \mu_R \sin 90$$

$$\boxed{\sin\theta_C = \frac{\mu_R}{\mu_D}} \text{ if } \frac{\mu_D}{\mu_R} = \mu \text{ then } \boxed{\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)}.$$

Example-5: **A ray of light is incident at an angle of 60° on one face of a rectangular glass slab of thickness 0.1 m and refractive index 1.5. Calculate the lateral shift produced.**

Solution: Here $i = 60°$; $\mu = 1.5$ and $t = 0.1$ m

Now, $\mu = \dfrac{\sin i}{\sin r}$ or, $\sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 60°}{1.5} = \dfrac{.866}{1.5} = .5773 \quad \Rightarrow r = 35°6'$

Now, lateral shift $d = \dfrac{t}{\cos r}\sin(i - r) = \dfrac{0.1}{\cos 35°6'}\sin(60° - 35°6')$

$$= \mathbf{0.0512 \text{ m}}$$

Example-6: **A fish rising vertically to the surface of water in a lake uniformly at the rate of 3 m/s observes a king fisher bird diving vertically towards water at the rate 9 m/s vertically above it. If the refractive index of water is 4/3, find the actual velocity of the dive of the bird.**
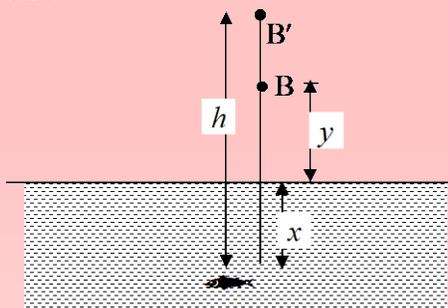

Solution: If at any instant, the fish is at a depth x below water surface while the bird at a height y above the surface, then the apparent height of the bird from the surface as seen by the fish will be given by

$$\mu = \frac{\text{Apparent height}}{\text{Real height}} \quad \text{or,} \quad \text{Apparent height} = \mu y$$

So, the total apparent distance of the bird as seen by the fish in water will be

$$h = x + \mu y \quad \text{or,} \quad \frac{dh}{dt} = \frac{dx}{dt} + \mu\frac{dy}{dt} \quad \text{or,} \quad 9 = 3 + \mu\left(\frac{dy}{dt}\right) \quad \text{or,} \quad \frac{dy}{dt} = \frac{6}{(4/3)} = \textbf{4.5 m/s}$$

Example-7: **A ray of light from a denser medium strikes a rarer medium at an angle of incidence i. If the reflected and refracted rays are mutually perpendicular to each other, what is the value of the critical angle?**
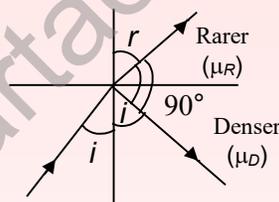
Solution: From Snell's law, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D} \quad \text{or,} \quad \mu = \frac{\mu_D}{\mu_R} = \frac{\sin r}{\sin i} \qquad \text{... (i)}$$

According to the given problem $i + r + 90° = 180°$ or, $r = 90° - i$

Substituting the above value of r in equation (i), we get

$$\mu = \frac{\sin (90 - i)}{\sin i} \qquad \text{or,} \qquad \mu = \cot i \qquad \text{... (ii)}$$

By definition $C = \sin^{-1}\left(\frac{1}{\mu}\right)$

or, $C = \sin^{-1}\left(\frac{1}{\cot i}\right)$ (using equation (ii)

or, $C = \textbf{sin}^{-1}\,(\textbf{tan}\,\textit{i}\,)$

❖ **Refraction from the curved surface:**

The image distance v of an object placed at distance u from the pole of a surface of curvature of radius R is related by an equation.

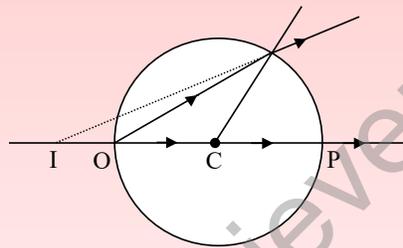$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$$

The magnification of the object is

$$\boxed{m = \frac{I}{0} = \frac{\mu_1}{\mu_2}\left(\frac{v}{u}\right)}$$

Note: [μ_2 is the refractive index of that medium from where object is to be seen]

All known terms are put with proper sign.

Example-8: **If a mark of size 0.2 cm on the surface of a glass sphere of diameter 10 cm and μ = 1.5 is viewed through the diametrically opposite point, where will the image be seen and of what size?**

Solution: As the mark is on one surface, refraction will take place on the other surface (which is curved). Further refraction is taking place from glass to air.

So, $\mu_1 = 1.5$; $\mu_2 = 1$; $R = -5$ cm; $u = -10$ cm; $v = ?$

Using the formula

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$, we have

$\dfrac{1}{v} - \dfrac{1.5}{(-10)} = \dfrac{1-1.5}{-5}$ or, $v = -20$ cm
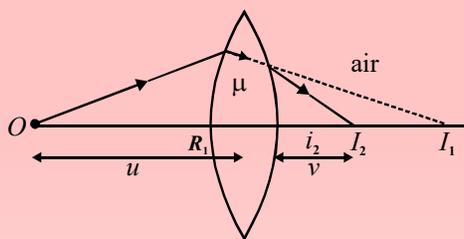
Hence, the image is at a distance of 20 cm from P towards O.

In case of refraction at a curved surface, we have

$$m = \frac{I}{O} = \frac{\mu_1}{\mu_2}\frac{v}{u} = \frac{(1.5)\times(-20)}{1\times(-10)} = +3$$

So, the image is virtual, erect and of size I = $m \times$ O = 3 × 0.2 = **0.6 cm**

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Power $p = \dfrac{1}{f(\text{in m.})} = \dfrac{100}{f(\text{in cm})}$ (diopter)

Example-9: Calculate the focal length of a concave lens in water ($\mu_w = 4/3$) if the surface has radii equal to 40 cm and 30 cm. $\mu_g = 1.5$

Solution: $R_1 = 30$ cm; $R_2 = + 40$ cm

we have $\dfrac{1}{f} = \left(\dfrac{\mu}{\mu_w} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right] = \left[\dfrac{1.5}{4/3} - 1\right]\left[\dfrac{1}{-30} - \dfrac{1}{40}\right]$

$\Rightarrow f = -\dfrac{960}{7} = -137.1$ cm

Example-10: A plane convex lens has a focal length 12 cm and is made up of glass with refractive index 1.5. Find the radius of curvature of its curved side.

Solution: Let x = magnitude of the radius of curvature

Now, $R_1 = + x$: $R_2 = \infty$, $f = + 12$ cm

We have $\dfrac{1}{f} = \left(\dfrac{\mu}{\mu_0} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

$= \left(\dfrac{1.5}{1} - 1\right)\left[\dfrac{1}{x} - \dfrac{1}{\infty}\right]$ or, $\dfrac{1}{+12} = (.5) \times \dfrac{1}{x}$

$\Rightarrow x = 6$ cm

If two lenses are placed at a separation (x) then $\boxed{\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{x}{f_1 f_2}}$

Example-11: Two plano-concave lens of glass of refractive index 1.5 have radii of curvature 20 cm and 30 cm. They are placed in contact with curved surfaces towards each-other and the space between them is filled with a liquid of refractive index 4/3. Find the focal length of the system.

Solution: As shown in figure, the system is equivalent to combination of three lenses in contact,

i.e., $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$

By lens maker's formula

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{\infty} - \frac{1}{20}\right] = \frac{1}{40} \text{ cm}$$
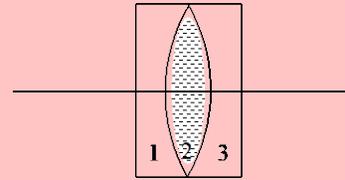
$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left[\frac{1}{20} - \frac{1}{-30}\right] = \frac{5}{180} \text{ cm}$$

$$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{-30} - \frac{1}{\infty}\right] = -\frac{1}{60} \text{ cm}$$

$$\therefore \quad \frac{1}{F} = -\frac{1}{40} + \frac{5}{180} - \frac{1}{60}$$

$$F = -\,\mathbf{72 \text{ cm}}$$

Thus, the system will behave as a concave lens of focal length 72 cm.

❖ **Lenses with silvered surface**

Power $= P_L + P_m + P_L$

$$P_L = \frac{1}{f_L} \text{ where } \frac{1}{f_L} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P_m = -\frac{1}{f_m} \text{ where } f_m = -\frac{R_2}{2}$$

(i) If $R_2 = \infty$ (plane) is silvered then $P_L = \frac{1}{f_L} = \frac{(\mu - 1)}{R_1}$

$$R_1 = R_1, \ P = 2P_L + P_m = \frac{2(\mu - 1)}{R_1} + \frac{1}{f_m} = \frac{2(\mu + 1)}{R_1} + 0 \quad (f_m = \infty) \ \therefore \ \frac{1}{F} = \frac{2}{R}(\mu + 1)$$

(ii) If R_2 is silvered $\dfrac{1}{f_L} = (\mu - 1)\left(\dfrac{1}{\infty} - \dfrac{1}{-R_2}\right) = \dfrac{(\mu - 1)}{R_2}; f_m = \left(-\dfrac{R_2}{2}\right)$

$$R_1 = \infty$$

$$P_L = \frac{(\mu - 1)}{R_2}, P_m = -\frac{1}{f_m} = \frac{2}{R_2}$$

$$\therefore \ \frac{1}{F} = \frac{2(\mu - 1)}{R_2} + \frac{2}{R_2} = \frac{2\mu}{R_2}$$

Example-12: **The plane surface of a plano-convex lens of focal length 60 cm is silvered. A point object is placed at a distance 20 cm from the lens. Find the position and nature of the final image formed.**

Solution: Let f be the focal length of the equivalent spherical mirror.

We have $\dfrac{1}{F} = \dfrac{1}{f_l} + \dfrac{1}{f_m} + \dfrac{1}{f_l}$ or, $\dfrac{1}{F} = \dfrac{2}{f_l} + \dfrac{1}{f_m}$

Here, $f_l = +60$ cm $\qquad f_m = \infty$

$$\therefore \ \frac{1}{F} = \frac{2}{60} + \frac{1}{\infty} = \frac{1}{30} \quad \text{or,} \quad F = +30 \text{ cm}$$

The problem is reduced to a simple case where a point object is placed in front of a concave

mirror. Now, using the mirror formula $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$, we have $\dfrac{1}{-20} + \dfrac{1}{v} = \dfrac{1}{-30}$

$\Rightarrow v = 60$ cm, the image is erect and virtual.
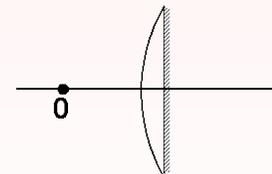
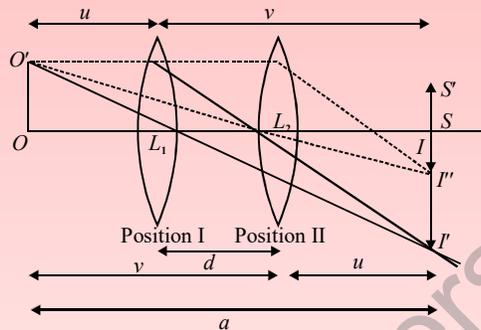➤ If the distance between the two object pins is greater than the four times of the focal length of the lens, real and inverted image of one pin is formed on the other side at the two positions of the convex lens. If the separation between the pins is *a*, difference between the two positions is *d* then

$$f = \frac{a^2 - d^2}{4a} \qquad f = \frac{ma}{(1+m)^2}$$

Where *m* is the magnification,

Proof:
$$d = v - u \qquad \text{…(i)}$$
$$a = v + u \qquad \text{…(ii)}$$

on solving

$$u = \frac{a-d}{2}, v = \frac{a+d}{2} \quad ; \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \ f = \frac{a^2 - d^2}{4a}$$

Size of object: $OO' = \sqrt{II' \times II''}$ or size of object $= \sqrt{I_1 I_2}$

If $m_1 = \dfrac{v}{u}$ and $m_2 = \dfrac{u}{v}$ and $d = v - u$ then $f = \dfrac{d}{m_1 - m_2}$.

Example-13: **An object is kept at a distance of 100 cm from a screen. A convex lens placed between them produces a real & magnified image on the screen. If the lens is shifted 30 cm towards the screen, real image is again obtained. Find the focal length of the lens. Also calculate the size of the object if the image sizes are 16 mm and 9 mm respectively.**

Solution: Here, $a = 100$ cm; $d = 30$ cm $\quad \therefore f = \dfrac{a^2 - d^2}{4a} = \dfrac{(100)^2 - (30)^2}{4(100)} = \dfrac{91}{4}$ cm

$I_1 = 16$ mm; $I_2 = 9$ mm $\quad \therefore$ Object size $= \sqrt{I_1 I_2} = \sqrt{16 \times 9} = $ **12 mm**

➢ A transparent medium bounded by any number of surfaces in such a way that the surface in which light is incident and the surface from which emerge are plane and non-parallel.

❖ **Refraction through a prism:**

In ΔPLM $\delta = \angle PLM + \angle PML \Rightarrow (i_1 - r_1) + (i_2 - r_2)$

$\delta = (i_1 + i_2) - (r_1 + r_2)$

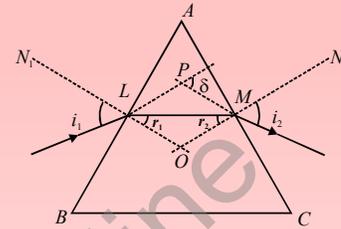In ΔOLM $\angle O + r_1 + r_2 = 180$

$\angle O = 180 - (r_1 + r_2)$

$\angle A + \angle O = 180$ [In quadrilateral ALOM, $\angle L + \angle M = 180$]

$\angle O + r_1 + r_2 = \angle A + \angle O$

$\boxed{r_1 + r_2 = A}$, $\boxed{\delta = (i_1 + i_2) - A}$

$\delta = (\mu r_1 + \mu r_2) - A$

$\boxed{\delta_m = (\mu - 1)A}$ (For small refracting angle only)

❖ **Prism's formula:** $\boxed{\mu = \dfrac{\sin\left(\dfrac{A + \delta m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}}$

Example-14: **The material of an equilateral prism has refractive index 1.5. Find the angle of minimum deviation, the angle of incidence, when the angle of deviation is minimum, and also the angle of refraction at first face.**

Solution: Given, $\mu = 1.5$, $A = 60°$

$\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$ or, $\sin\dfrac{(60 + \delta_m)}{2} = 1.5 \times 0.5 = 0.75$

$\dfrac{60 + \delta_m}{2} = \sin^{-1}(0.75) = 48°36'$ ∴ $60 + \delta_m = 97°12'$ $\Rightarrow \delta_m = 37°12'$

When the deviation is minimum $i_1 + i_2 = 2i = A + \delta_m = 60° + 37°12' = 97°12'$

$i_1 = \dfrac{97°12'}{2} = 48°36'$ ∴ $r_1 = \dfrac{A}{2} = $ **30°**

Example-15: **A prism with refracting angle 60° has its minimum deviation as 37° when placed in air. Find its minimum deviation angle when immersed completely in water. ($\mu_{water} = 1.33$).**

Solution:
$$\mu_g = \frac{\sin\left(\dfrac{60+37}{2}\right)}{\sin\left(\dfrac{60}{2}\right)} = 1.498, \qquad {}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{1.498}{1.33} = 1.126$$

$$1.126 = \frac{\sin\left(30° + \dfrac{\delta_m}{2}\right)}{\sin 30°} \Rightarrow \sin\left(30 + \frac{\delta_m}{2}\right) = 0.563 \Rightarrow 30 + \frac{\delta_m}{2} = 34.26°$$
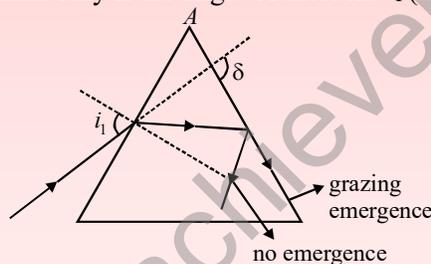
$$\Rightarrow \frac{\delta_m}{2} = 4.26° \Rightarrow \delta_m = \textbf{8.52°}$$

❖ **Condition of no emergence**

if $\boxed{\begin{array}{l} A > 2\theta_C \\[4pt] \mu > \operatorname{cosec}\left(\dfrac{A}{2}\right) \end{array}}$ the ray will not come out.

or if

❖ **Condition of grazing emergence:**
Light will come out of a prism only if the angle of incident i_1 (min) is

grazing
emergence

no emergence

$$\boxed{i_1(\min) = \sin^{-1}\left[\sqrt{(\mu^2 - 1)}\sin A - \cos A\right]} \quad (\delta = i_1 + 90 - A)$$

Example-16: **The refractive index of the material of a prism of refracting angle 45° is 1.6 for a certain monochromatic ray. What should be the minimum angle of incidence of this ray on the prism so that no total internal reflection takes place as the ray comes out of the prism?**

Solution: Given $A = 45°$, $\mu = 1.6$

$$\therefore \quad \sin C = \frac{1}{\mu} = \frac{1}{1.6} \quad C = 38.68°$$
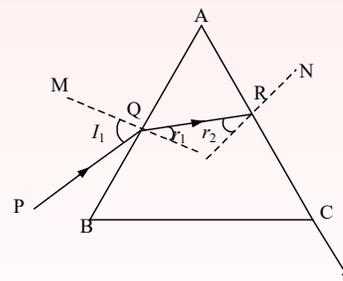
For total internal reflection not to take place at the face AC, the angle of incidence in that face $r_2 \le C$

In the limiting case $r_2 = C$

Now, $r_1 = A - r_2 = 45 - 38.68° = 6.32°$

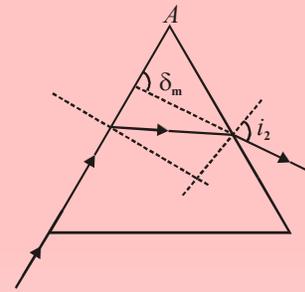$$\mu_a \sin i_1 = \mu_g \sin r_1$$

$$\sin i_1 = 1.6 \sin(6.32°) = \textbf{10.14°}$$

❖ **Condition for maximum deviation:**

$$\boxed{\delta_{max.} = (i_2 + 90 - A)} \quad \text{and} \quad \boxed{i_2 = \sin^{-1}[\mu \sin(A - \theta_c)]}$$

❖ **Condition for min. deviation:**

$$\boxed{\delta_m = (\mu - 1)A} \quad \boxed{\mu = \dfrac{\sin\dfrac{A + \delta_m}{2}}{\sin\dfrac{A}{2}}}$$

Where $i_1 = i_1 = i = \dfrac{A + \delta_m}{2}$ and $r_1 = r_2 = r = \dfrac{A}{2}$ also $\sin i = \mu \sin \dfrac{A}{2}$

Dispersion of light

➤ When white light passes through a prism, it splits up into constituent colours. μ for different colours, is different.

(a) $\boxed{\mu_v > \mu_R}$ as $\boxed{v_v < v_R}$ $\mu \propto \dfrac{1}{v}$.

(b) $\mu = A + \dfrac{B}{\lambda^2} + \dfrac{C}{\lambda^4} + - - -$ (Cauchy's formula)

$\therefore$ $\lambda \propto \dfrac{1}{\mu}$ or $\boxed{\lambda_v < \lambda_R}$

(c) As $\delta_m = (\mu - 1)A$ $\therefore$ $\boxed{\delta_v > \delta_R}$
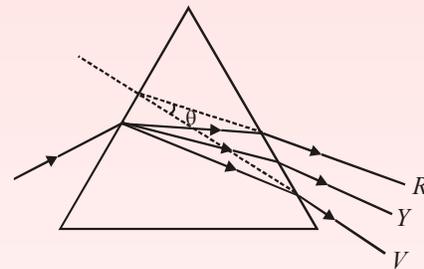
Angular dispersion $\boxed{\theta = \delta_v - \delta_R = (\mu_v - \mu_R)A}$. Dispersive power $\boxed{\omega = \dfrac{\delta_v - \delta_R}{\delta_y} = \dfrac{(\mu_v - \mu_R)}{(\mu_y - 1)}}$

Dispersive power is defined as ratio of angular dispersion to the mean deviation. Where

$$\boxed{\mu_y = \dfrac{\mu_v + \mu_R}{2}}$$

❖ **Chromatic Aberration:**

A white object appeared coloured in chromatic aberration.
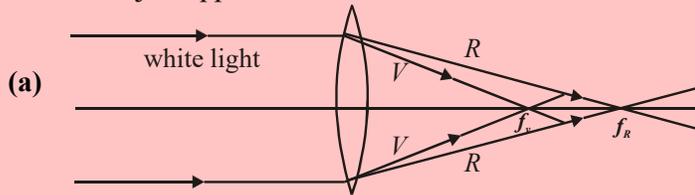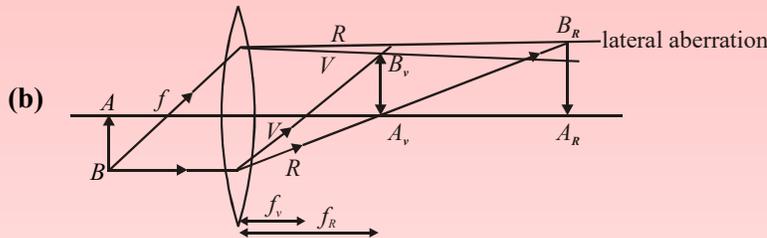
(a)

$$\therefore \quad \boxed{\text{longitudinal chromatic aberration} = (f_R - f_v)} \quad \boxed{f_R - f_v = \omega f_y}$$

(b)

$A_v\, B_v \rightarrow$ violet – violet striking point.

$A_R\, B_R \rightarrow$ Red – red striking point.

❖ **Achromatic combination:**

Two lenses in contact to produce no chromatic aberration

Condition: $\boxed{\dfrac{\omega}{f_y} + \dfrac{\omega'}{f_y'} = 0}$

(i) Both the lenses should be of different material, otherwise $\omega = \omega'$.

(ii) f_y and f'_y should be of different signs. *i.e.* if one convex then other should be concave.

$$\frac{1}{fy} + \frac{1}{f'y} = 0 \ \text{ or } \ \frac{1}{fy} = 0 \ \text{ or } \ f_y = \infty \ \text{ or plane glass sheet.}$$

(iii) The lens having lesser focal length should be of the material with lesser dispersive power.

$$\frac{f_y}{f'_y} = -\frac{\omega}{\omega'}$$

Optical Instruments

❖ **Visual angle:**

Angle subtended by an object on our eye is called visual angle. Apparent size of the object depends upon the visual angle.

❖ **Magnifying power:**

The work of telescope and microscope is to increase the visual angle.

$$\text{Magnifying Power} = m = \frac{\text{visual angle made by the image}}{\text{Visual angle made by the object without instrument for unaided eye}}$$

Astronomical telescope

objective lens
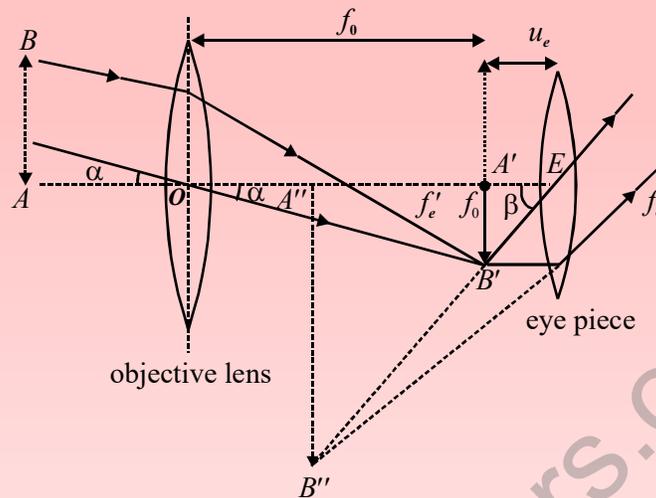
eye piece

$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{A'B'/EA'}{A'B'/OA'} = \frac{OA'}{EA'} = \frac{f_0}{-u_e} \qquad \ldots(i)$$

Case I: When the last image is at distance of distinct vision (D).

i.e., $EA'' = -D$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{(For eye piece)}$$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \; ; \qquad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Putting in equation …(i) $\boxed{L = f_0 + u_e}$ L is length of the tube.

$$\boxed{M = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

Note: Only numerical values are to be placed. (For f_0, f_e, D)

Case II: When last image is at ∞. (For relaxed eye) [normal setting]
For this the image $A'B'$ should be at f_e. So that the final image is at ∞.
Here $u_e = f_e$

∴ $\boxed{M = -\frac{f_0}{f_e}}$ $\boxed{L = f_0 + f_e}$

Example-17: **The focal length of the objective of an astronomical telescope is 75cm and that of the eye-piece is 5 cm. If the final image is formed at the least distance of distinct vision from the eye, calculate the magnifying power of the telescope.**

Solution: Here, $f_0 = 75\,\text{cm}$; $f_e = 5\text{cm}$

We know, $D = 25$

Now, $M = \dfrac{f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right) = -\dfrac{75}{5}\left(1 + \dfrac{5}{25}\right) = -15 \times 1.2 = -18$

Simple Microscope

➤ A simple microscope is nothing but a bi-convex lens of high focal power which is used to magnify the near object. It is generally known as eye-piece.

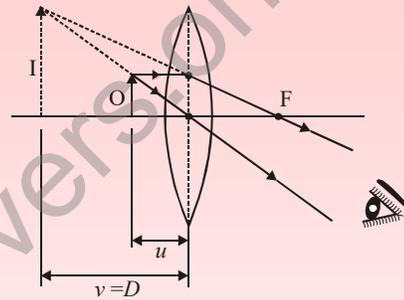**Case I:** Eye focused at near point i.e., the image is formed at a distance D from the lens/eye.

In this case $v = D$

$$M = \frac{v}{u} = v \times \frac{1}{u} = v \times \left(-\frac{1}{f} + \frac{1}{v}\right)$$

Now, according to our sign convention $V = -D$ here.

$$\therefore \quad M = (-D) \times \left(-\frac{1}{f} + \frac{1}{-D}\right)$$

or $\boxed{M = 1 + \dfrac{D}{f}}$

Eye focused at near point

Case II: Eye focused at infinity (Normal adjustment).

In this case we find out angular magnification. If the object is viewed directly by the eye, keeping the object at the near point, then

$$\therefore \quad \theta_0 = \frac{O}{D}$$

Now let us find the angle subtended by its image, if the image is formed at ∞. In this case, the object will have to be kept at the focus of the lens.
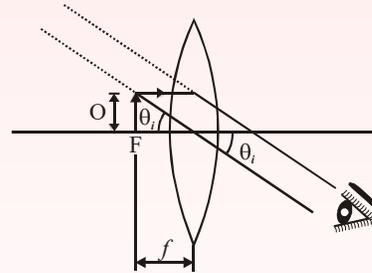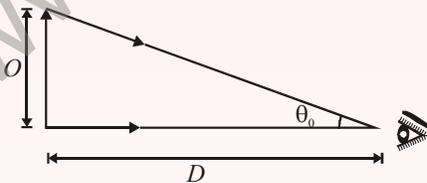
In this case, $\theta_i = \dfrac{O}{f}$

Therefore, angular magnification

$$M = \frac{\theta_i}{\theta_0} = \frac{O/f}{O/D} \qquad \text{or,} \quad \boxed{M = \frac{D}{f}}$$

This is one less than the magnification when the image is formed at the near point, but, naturally, viewing is more comfortable and the difference in magnification usually small.

For example, if we want a magnification of six, f required will be 5 cm in the first case (taking D = 25 cm) and $(D/6 = 25/6) \simeq$ 4cm in the second case.

So, magnification is approximately inversely proportional to the focal length smaller the f, smaller will be radius of the sphere out of which the lens is to be cut off. Hence, for all practical purposes, it is not possible to have magnification > 10 through a simple microscope. For higher magnifications, we need at least two lenses, one compounding the effect of the other. This is termed as compound microscope.

Example-18: **A simple microscope consists of a convex lens of power +25D and a concave lens of power –20D in contact. Find the magnifying power when final image is formed (a) at infinity (b) at a distance of distinct vision.**

Solution: Here $P_1 = +25D$, $P_2 = -20D$, $D = 25$ cm

Since lenses are in contact, so power of the combination

$P = P_1 + P_2 = 25 - 20 = +5D$

∴ Focal length of the combination, $f = \dfrac{100}{P} = \dfrac{100}{5} = 20$ cm

(a) Magnifying power when final image is formed at ∞, $M.P. = \dfrac{D}{f} = \dfrac{25}{20} = 1.25$

(b) Magnifying power when final image is formed at the distinct vision,

$M.P. = 1 + \dfrac{D}{f} = 1 + \dfrac{25}{20} = 1 + 1.25 = 2.25$.

Compound Microscope
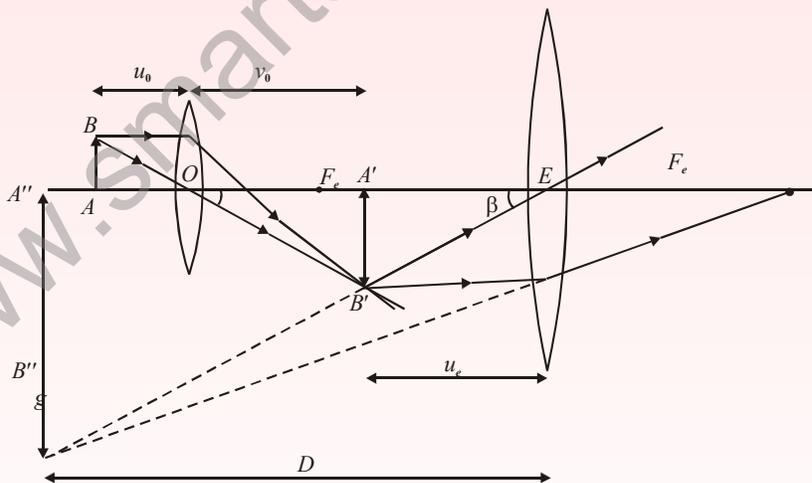
$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{A'B'/EA'}{\text{size of object/least dist. of distinct vision}} = \frac{A'B'/EA'}{AB/D} = \frac{A'B'}{AB} \times \frac{D}{EA'}$$

$$= \frac{+v_0}{-u_0} \times \left(\frac{-D}{-u_e}\right) \qquad \therefore \qquad \boxed{M = \frac{-v_0}{u_0}\left(\frac{D}{u_e}\right)} \qquad \text{...(ii)}$$

Case I: When the last image is at least distance of distinct vision 'D'. (Near point)

For eye piece $u = -u_e, v = -D, f = f_e$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \ , \ \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} \ , \ \frac{D}{u_e} = 1 + \frac{D}{f_e}$$

Putting in (ii)

$$\boxed{M = \frac{-v_0}{u_0}(1 + D/f_e)} \quad \boxed{L = v_0 + u_e} \quad \text{Here, L is length of the tube which is OE.}$$

Case II: When last image is at ∞. (Far point) for eye piece $u_e = f_e$

Putting in (ii)

$$\boxed{M = -\frac{v_0}{u_0}\left(+\frac{D}{f_e}\right)} \quad \boxed{L = v_0 + f_e} \quad \text{L is length of the tube.}$$

Note: Only numerical values are to be placed.

Example-19: **A compound microscope has a magnification of 30. The focal length of its eye-piece is 5 cm. Assuming the final image to be formed at least distance of distinct vision (25cm), calculate the magnification produced by the objective.**

Solution: Here, Normal $= -30$, (magnifying power is negative)

$f_0 = 5\,\text{cm}, D = 25\,\text{cm}$

Now, $M = m_o \times m_e = m_o\left(1 + \dfrac{F}{f_e}\right)$

$-30 = m_o\left(1 + \dfrac{25}{5}\right)$ or $m_o = -5$

Reflecting type Telescope (Newtonian)

> The objective is a large concave parabolic spherical mirror made of alloy of copper and tin. A Ramsden eyepiece is used with this model.

The rays from the distant stars/planets after reflection from the objective mirror (concave mirror of large aperture and large focal length) are reflected to one side at right angles to the axis of the objective mirror, with the help of a small plane mirror, as shown in the sketch. Thereafter viewer sees the final image through the eyepiece.
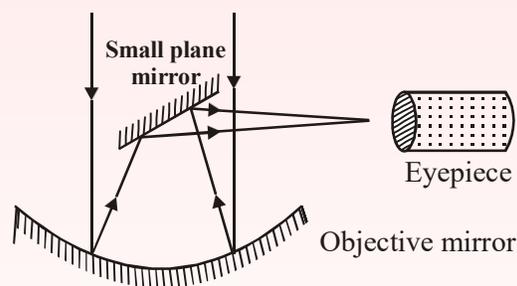
Reflecting type telescope (Newtonian model)

Example-20: A reflecting type telescope has a concave reflector of radius of curvature 120 cm. Calculate the focal length of eye piece to secure a magnification of 20.

Solution: Here, $M = -20$ (magnifying power is negative),

Here, radius of curvature of concave reflector, $R = 120\,cm$

Therefore, focal length of concave reflector, $f_o = \dfrac{R}{2} = -\dfrac{120}{2} = -60\,cm$

Now, magnifying power, $M = \dfrac{f_o}{f_e}$ or $f_e = \dfrac{f_o}{M} = \dfrac{-60}{-20} = 2\,cm$

Resolving power of optical instruments

➤ The power of an instrument to distinguish the images of the two closely spaced objects.

(a) Limit of resolution:

Minimum distance between two lines at which they are just distinct by an optical instrument is called the limit of resolution of that instrument.

Smaller the limit of resolution, higher is the resolving power.

Limit of resolution of telescope $\Delta x = \dfrac{1.22\lambda}{D}$.

[D → diameter of objective lens, λ → wavelength of light used]

(b) Resolving power of telescope:

$R.P = \dfrac{1}{\Delta x} = \dfrac{D}{1.22\lambda}$ [D is diameter of objective lens]

(c) Resolving power of microscope: $R.P. = \dfrac{\mu \sin\theta}{\lambda}$

Where, θ is the half angle of the cone of light from the point object on to the objective lens.

❖ **Vision:**

(i) Far point is at ∞ and near point is 25cm.

(ii) If the object is at ∞, eye is least strained and said to be relaxed. However, if the object is at least distance of distinct vision *i.e.*, $D = 25$cm, eye is under maximum strain.

(iii) Unit of resolution of eye is one minute.

(iv) Persistence of vision is 1/10 sec.

❖ **Myopia or near sightedness:**

Far objects are not clearly visible. Far point is at lesser than ∞ hence making image before retina. Divergent lens is used to remove this defect.
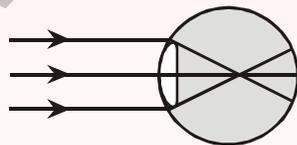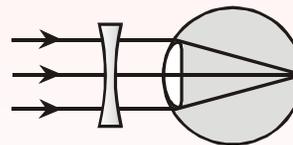
Defected eye

Corrected eye

❖ Far sightedness or Hypermetropia:

Near objects are not clearly visible. Near point is at distance greater then 25cm. Hence image will be behind retina. Convergent lens is used to remove this defect.

Defected eye

Corrected eye

Ray Optics & Optical Instruments

Lens Marker's Formula

$$\frac{1}{v} - \frac{1}{u} = \frac{\mu_2 - \mu_1}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad [If\ \mu_2 = \mu,\ \mu_1 = 1\ (air)]$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \text{(lens formula)}$$

Lateral Magnification $= \dfrac{h_2}{h_1} = \dfrac{v}{u}$
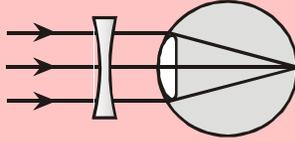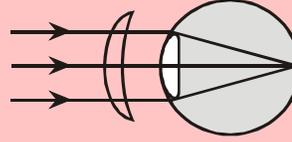
Simple Microscope

$$M = 1 + \frac{D}{f} \quad \text{[image at near point]}$$

$$M = D/f \quad \text{[image at infinity]}$$

Compound Microscope

$$M = \frac{v_0}{u_0}\left[\frac{D}{f_e}\right] \quad \text{[normal adjustment]}$$

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = -\frac{1}{f_0}\left(1 + \frac{D}{f_e}\right)$$

For final image at least distance

Telescope

$$M = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right) \quad \text{[image at near point]}$$

$$M = -\frac{f_0}{f_e} \quad \text{[image at infinity]}$$

Optical Instruments

Power of a lens

$$P = \frac{1}{f}$$

[For combination of lens when they are in contact]

$$P = \frac{1}{f_1} + \frac{1}{f_2}$$

for combination of lens when they are separated

$$P = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Prism

Dispersion

Splitting of polychromatic light into its constituent colours.

Angle of deviation $\delta = i + i' - A$

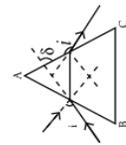$\delta_{minimum} = 2i - A \quad [i = i']$

$\delta_{minimum} = (\mu - 1)A$, if A is small

Reflection of Light

- $\angle i = \angle r$
- Incident ray reflected ray and normal to the reflecting surface are coplanar

Sign Conventions

- Pole in case of mirror and optical centre in case of lenses are taken as origin
- Principle axis as the X-axis
- All distances are measured from origin
- All distances measured in the direction of incident ray are taken + ve.
- All distances measured in the direction opposite to the incident ray are taken - ve.

Spherical Mirror

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f}$$

Lateral Magnification $= \dfrac{h_2}{h_1} = -\dfrac{v}{u}$

Refraction of light
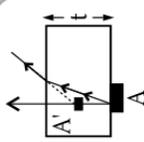
$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$$

$$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

$$\Delta t = \left(1 - \frac{1}{\mu}\right)t = \text{image shift}$$

Total Internal reflection

When ray passes from optically denser to rarer medium, if incident angle (i) greater than critical angle, entire light is then reflected back to the denser medium again, this process is called T.I.R. It is used in optical fibre.

Critical Angle

- Incident angle (θ_c) for which angle of refraction is $90°$

i.e., $\sin \theta_c = 1/\mu$

$\theta_c = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

When ray passes from optically denser to rarer medium.

Refraction on Spherical surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Lateral Magnification

$$m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u} = \frac{R - v}{R - u}$$

PRACTICE QUESTIONS

1. The length of the tube of a microscope is 5 cm. The focal lengths of the objective and eye lenses are 0.2 cm and 0.5 cm. The magnifying power of the microscope is
   a) 5
   b) 230
   c) 1250
   d) 500

2. In the given figure, a mixture of blue, green, and red-coloured rays is incident normally on a right-angled prism. The critical angles for the material of the prism are 40° for red, 41° for green, and 42° for blue. In this arrangement,
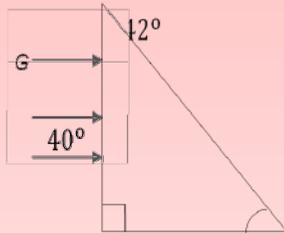
   a) Red colour from blue and green
   b) Blue colour from red and green
   c) Green colour from red and blue
   d) All the three colours

3. For a telescope with the maximum possible magnification, we would choose the lenses with the longest focal lengths. Therefore, select the lenses with focal lengths of 100 cm and 10 cm.

   a) 100 cm, 0.3 cm
   b) 10 cm, 0.3 cm
   c) 10 cm, 4 cm
   d) 100 cm, 4 cm

4. In the given setup, a double convex lens made of a material with refractive index μ_1 is positioned between two liquids with refractive indices μ_2 and μ_3, where $\mu_2 > \mu_1 > \mu_3$. When a wide, parallel beam of light is directed towards the lens from the left, the lens will cause the light rays to
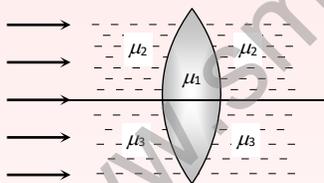
   a) A single convergent beam
   b) Two different convergent beams
   c) Two different divergent beams
   d) A convergent and a divergent beam

5. The diameter of the eye-ball of a normal eye is about 3 cm. The power of the eye lens varies from

   a) 2 D to 10 D
   b) 37 D to 33 D
   c) 9 D to 8 D
   d) 44 D to 40 D

6. A 3 cm thick layer of water covers a 5 cm thick glass slab. A coin is placed at the bottom of the slab and is being observed from the air side along the normal to the surface. Find the apparent position of the coin from

4 cm h_1 Water

6 cm h_2 Glass

Coin
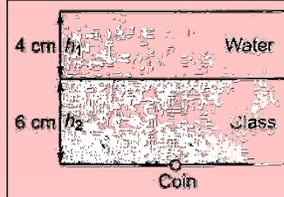
a) 5.6 cm b) 8.0 cm c) 10 cm d) 5 cm

7. If a small object is positioned 20 cm in front of a plane mirror, and you stand behind the object, 30 cm from the mirror to observe its image, you would need to focus your eyes at a distance of

a) 20 cm b) 50 cm c) 80 cm d) 40 cm

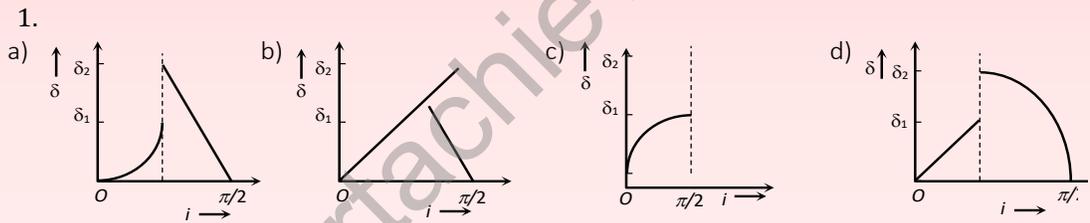**8.** A ray of light travels from a medium of refractive index μ to air. Its angle of incidence in the medium is i, measured from the normal to the boundary, and its angle of deviation is δ. δ is plotted against i which of the following best represents the resulting curve

1.

a) b) c) d)

9. If a 150 W unidirectional bulb has a luminous intensity of 80 candela, the total luminous flux emitted by the bulb is

a) 861 $lumen$ b) 1005 $lumen$ c) 1256 $lumen$ d) 1561 $lumen$

10. The dispersive powers of crown and flint glasses are 0.01 and 0.03 respectively. In an achromatic combination of lenses, the focal length of flint glass lens is $20\ cm$. The focal length of crown glass lens will be

a) $-6.67\ cm$ b) $+6.67\ cm$ c) $-10.3\ cm$ d) $+10.3\ cm$

11. A boy is trying to start a fire by focusing Sunlight on a piece of paper using an equiconvex lens of focal length $20\ cm$. The diameter of the Sun is $1.39 \times 10^9\ m$ and its means distance from the earth is $1.5 \times 10^{11} m$. What is the diameter of the Sun's image on the paper

a) $6.5 \times 10^{-5} m$ b) $12.4 \times 10^{-4} m$ c) $18.52 \times 10^{-4} m$ d) $6.5 \times 10^{-4} m$

12. The magnification produced by the objective lens and the eye lens of a compound microscope are 20 and 5 respectively. The magnification of this microscope is

   a) 25   b) 50   c) 100   d) 200

13. Given the refractive index of a denser medium with respect to a rarer medium as N_{12} and its critical angle θ_c, the angle of incidence A can be determined. At this angle, when light is traveling from the denser medium to the rarer medium, a portion of the light is reflected, and the remaining part is refracted. The angle between the reflected and refracted rays is 90 degrees. The relationship between these angles can be expressed as:
   a) $\tan^{-1}(\sin \theta_c)$
   b) $1/\tan^{-1}(\sin \theta_c)$
   c) $1/\cos^{-1}(\sin \theta_c)$
   d) $\cos^{-1}(\sin \theta_c)$

14. The refractive index of glass is 1.5 for red light and 1.51 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then,

   a) $D_1 < D_2$
   b) $D_1 = D_2$
   c) D_1 can be less than or greater than D_2 depending upon the angle of prism
   d) $D_1 > D_2$

15. When a plano-convex lens is silvered on the plane side, it behaves like a concave mirror with a focal length of 40 cm. On the other hand, when it is silvered on the convex side, it behaves like a concave mirror with a focal length of 15 cm. Find the refractive index of the lens material:

   a) 3.0   b) 2.0   c) 2.5   d) 1.5

16. An L-shaped wire is created by bending a small piece of wire, with equal lengths for its upright and horizontal portions. The wire is positioned with its horizontal portion aligned along the axis of a concave mirror, having a radius of curvature of 10 cm. The bend in the wire is located 20 cm away from the mirror's pole. In this configuration, the lengths of the images formed for the upright and horizontal portions of the wire have a ratio of

   a) 1 : 2   b) 3 : 1   c) 1 : 3   d) 2 : 1

17. A thin equiconvex lens is constructed using glass with a refractive index of 1.2, and its focal length in air is measured to be 0.3 m. However, when this lens is submerged in a liquid, it behaves as a concave lens with a focal length of 0.5 m. To determine the refractive index of the liquid,

   a) $\frac{17}{9}$ b) $\frac{10}{9}$ c) $\frac{13}{8}$ d) $\frac{9}{8}$

18. The refractive index of a certain glass is 1.5 for light whose wavelength in vacuum is 5000 Å. The wavelength of this light when it passes through glass is

a) 4000 Å   b) 3333 Å   c) 6666 Å   d) 15000 Å

19. In a room made of mirrors, an insect is moving along the diagonal of the floor. The velocity of the image of the insect on two adjacent wall mirrors is measured to be 20 cm/s. Determine the velocity of the image of the insect in the ceiling mirror.

a) 10 cms^{-1} b) 20 cms^{-1} c) $\dfrac{10}{\sqrt{2}} \text{ cms}^{-1}$ d) $20\sqrt{2} \text{ cms}^{-1}$

20. If the critical angle for total internal reflection from a medium to vacuum is 60°, the velocity of light in the medium is

a) $3 \times 10^8 \ m/s$ b) $2.59 \times 10^8 \ m/s$ c) $2 \times 10^8 \ m/s$ d) $\sqrt{3} \times 10^8 \ m/s$

21. A square cross-section glass rod with a refractive index of 1.5 is bent into the shape shown in the figure. A parallel beam of light is directed towards the flat surface A, as depicted. Determine the maximum value of d/R, where d represents the width of a side of the square and R represents the radius of the circular arc.
(This value ensures that the light entering the glass slab through surface A will exit the glass through surface B.)
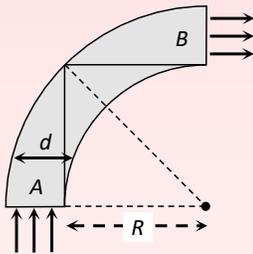
a) 1.5   b) 0.5   c) 1.3   d) None of these

22. The rate at which the apparent depth of water in a cylindrical water tank, with a diameter of R cm, is decreasing is x cm/minute. This decrease is due to water being drained out at a constant rate. Find the amount of water drained in cubic cm (cc) per minute.
($n_1 = $ refractive index of air, $n_2 = $ refractive index of water)

a) $x \, \pi \, R^2 \, n_1/n_2$ b) $x \, \pi \, R^2 \, n_2/n_1$ c) $2 \, \pi \, R \, n_1/n_2$ d) $\pi \, R^2 \, x$

23. A converging beam of light is directed towards a point I on a screen. To the path of the beam, a plane glass plate is introduced. The glass plate has a thickness t in the direction of the beam and a refractive index of μ. The glass plate in the path of the beam causes a shift in the convergence point. The amount of shift is

a) $t \left(1 - \dfrac{1}{\mu}\right)$ away   b) $t \left(1 + \dfrac{1}{\mu}\right)$ away   c) $t \left(1 - \dfrac{1}{\mu}\right)$ nearer   d) $t \left(1 + \dfrac{1}{\mu}\right)$ nearer

24. *A person is using spectacles with a power of +2 D. This indicates that the person is suffering from*

a) Myopia   b) Presbyopia   c) Astigmatism   d) Hypermetropia

25. In a grease spot photometer, light from a lamp with dirty chimney is exactly balanced by a point source distance 15 cm from the grease spot. On clearing the dirty chimney, the point source is moved 3 cm to obtain a balance again. Then the percentage of light absorbed by the dirty chimney is nearly

a) 64%   b) 89%   c) 44%   d) 56%

26. Light travels in two media A and B with speeds $2 \times 10^8 ms^{-1}$ and $2.6 \times 10^8 ms^{-1}$ respectively. Then the critical angle between them is

a) $\sin^{-1}\left(\dfrac{3}{10}\right)$ b) $\tan^{-1}\left(\dfrac{13}{4}\right)$ c) $\tan^{-1}\left(\dfrac{2}{3}\right)$ d) $\sin^{-1}\left(\dfrac{10}{13}\right)$

27. A Galilean telescope has an objective of focal length 90 cm and magnifying power 60. The distance between the two lenses in normal

a) 88.5 cm   b) 100.5 cm   c) 150 cm   d) 200 cm

28. A telescope using light having wavelength 6000 Å and using lenses of focal 2.5 and 30 cm. If the diameter of the aperture of the objective is 20 cm, then the resolving limit and magnifying power of the telescope is respectively

a) $6.1 \times 10^{-6} rad$ and 12   b) $5.0 \times 10^{-6} rad$ and 12
c) $6.1 \times 10^{-6} rad$ and 8.3×10^{-2} d) $5.0 \times 10^{-6} rad$ and 8.3×10^{-2}
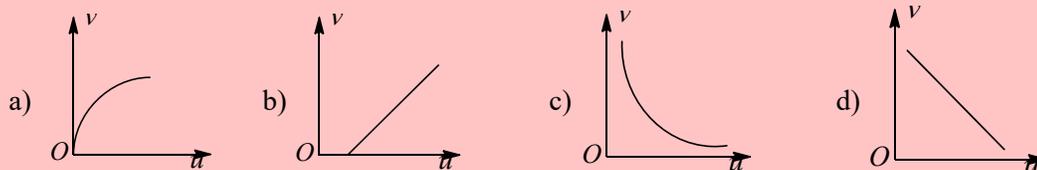
29. What is the focal length of a combination of a plano-convex lens and a plano-concave lens, where both lenses have parallel plane surfaces, are made of different materials with refractive indices µ_1 and µ_2, and have a curved surface with a radius of curvature R?

a) $\dfrac{R}{2(\mu_1 + \mu_2)}$ b) $\dfrac{R}{2(\mu_1 - \mu_2)}$ c) $\dfrac{R}{(\mu_1 - \mu_2)}$ d) $\dfrac{2R}{(\mu_2 - \mu_1)}$

30. A boy 1 m tall with his eye level at 0.8 m stands before a mirror fixed on a wall. The minimum length of mirror required to view the complete image of boy is

a) 1.09 m   b) 0.06 m   c) 0.69 m   d) 0.12 m

31. The variation of *u versus v* for a convex mirror is

a) [graph showing concave-increasing curve, v vs u]
b) [graph showing straight line through region, v vs u]
c) [graph showing decreasing curve, v vs u]
d) [graph showing straight decreasing line, v vs u]

32. A prism of a certain angle deviation the red and blue rays by 4° and 8° respectively. Another prism of the same angle deviates the red and blue rays by 8° and 12° respectively. The prisms are small angled and made of different materials. The dispersive powers of the materials of the prisms are in the ratio

a) 5 : 3　　　　　b) 9 : 11　　　　　c) 3 : 5　　　　　d) 11 : 9

33. A compound microscope has an objective and eye-piece as thin lenses of focal lengths 3 cm and 5 cm respectively. The distance between the objective and the eye-piece is 25 cm. The distance at which the object must be placed Infront of the objective if the final image is located at 30 cm from the eye-piece, it numerically

a) 95/6 cm　　　　b) 5 cm　　　　c) 95/89 cm　　　　d) 25/6 cm

34. What is the measure of angle BAC in a glass prism, where the principal section is an isosceles triangle ABC with AB = AC, and the face AC is silvered? Given that a ray of light is incident normally on the face AB and, after two reflections, it emerges from the base BC perpendicular to the base.

a) 30°　　　　b) 36°　　　　c) 60°　　　　d) 72°

35. If the refractive index of a prism's material is $1/\sqrt{2}$ and the angle of the prism is 30°, and one of its refracting faces is polished, what would be the angle of incidence at which the incident beam of light retraces back?

a) 0°　　　　b) 20.7°　　　　c) 37°　　　　d) 90°

36. When a beaker contains water up to a height h_1 and kerosene above the water up to a height h_2, with the total height of the liquid column being $(h_1 + h_2)$, and considering the refractive index of water as u_1 and the refractive index of kerosene as u_2, what is the apparent shift in the position of the bottom of the beaker when viewed from above?

a) $\left(1 - \dfrac{1}{u_1}\right)h_2 + \left(1 - \dfrac{1}{u_2}\right)h_1$

b) $\left(1 + \dfrac{1}{u_1}\right)h_1 + \left(1 + \dfrac{1}{u_2}\right)h_2$

c) $\left(1 - \dfrac{1}{u_1}\right)h_2 + \left(1 - \dfrac{1}{u_2}\right)h_2$

d) $\left(1 + \dfrac{1}{u_1}\right)h_2 - \left(1 + \dfrac{1}{u_2}\right)h_1$

37. In an achromatic convergent doublet consisting of two lenses in contact, the power of the doublet is +3D, and the convex lens in the doublet has a power of +4D. What is the ratio of the dispersive powers of the convergent and divergent lenses?

    a) 2:5   b) 1:5   c) 5:2   d) 5:1

38. If a lens with a refractive index of n is placed in a liquid with a refractive index of n^', what will be the focal length of the lens in the liquid, given that its focal length in air is f?

    a) $\dfrac{fn'(n-1)}{n'-n}$ b) $\dfrac{f(n'-n)}{n'(n-1)}$ c) $\dfrac{n'(n-1)}{f(n'-n)}$ d) $\dfrac{fn'n}{n-n'}$

39. What is the radius of curvature of a planoconvex lens with a maximum thickness of 5 cm, where the lens is placed on a horizontal table and the apparent depth of the bottommost point of the lens is 4 cm when the curved surface is in contact with the table surface? Additionally, when the lens is inverted and the plane face is in contact with the table surface, the apparent depth of the canter of the plane face is found to be (13/4) cm.

    a) 34 cm   b) 128 cm   c) 75 cm   d) 68 cm

40. If the refracting angle of a prism is A and the refractive index of the prism's material is given by tan(A/2), what is the angle of minimum deviation of the prism?

    a) $\pi + 2A$ b) $\pi - 2A$ c) $\dfrac{\pi}{2} + A$ d) $\dfrac{\pi}{2} - A$

41. If a biconvex lens forms a real image of an object placed perpendicular to its principal axis, and the radii of curvature of the lens tend to infinity, what would happen to the image?

    a) Disappear
    b) Remain as real image still
    c) Be virtual and of the same size as the object
    d) Suffer from aberrations

42. Given a concave mirror, an object is positioned at a distance d_1 from the principal focus on the principal axis. Consequently, the image is formed at a distance d_2 from the principal focus. What is the focal length of the mirror?

    a) $\sqrt{d_1 d_2}$
    b) $d_1 d_2$
    c) $(d_1 + d_2)/2$
    d) $\sqrt{\dfrac{d_1}{d_2}}$

------------------------------ANSWER KEY------------------------------

1)	c	2)	a	3)	a	4)	d
5)	a	6)	b	7)	a	8)	b
9)	a	10)	a	11)	c	12)	c
13)	a	14)	a	15)	c	16)	b
17)	b	18)	b	19)	d	20)	b
21)	b	22)	b	23)	a	24)	d
25)	b	26)	d	27)	a	28)	a
29)	c	30)	a	31)	c	32)	a
33)	d	34)	b	35)	b	36)	c
37)	b	38)	a	39)	a	40)	b
		41)	c	42)	a		

HINTS AND SOLUTIONS

1. (c)

$$m \simeq \frac{LD}{f_o f_e} \Rightarrow m = \frac{5 \times 25}{0.5 \times 0.2} = 1250$$

2. (a)

Condition of no emergence is
$A > C$

As angle of prism is greater than critical angle for blue and green coloured rays, total internal reflection will take place at second surface and hence the arrangement will separate red colour from blue and green.

3. (a)

$$m = -\frac{f_o}{f_e}$$

4. (d)

As $\mu_2 > \mu_1$, the upper half of the lens will become diverging

As $\mu_1 > \mu_3$, the lower half of the lens will become converging

5. (b)

An eye sees distant objects with full relaxation

So $\dfrac{1}{3 \times 10^{-2}} - \dfrac{1}{-\infty} = \dfrac{1}{f}$ or
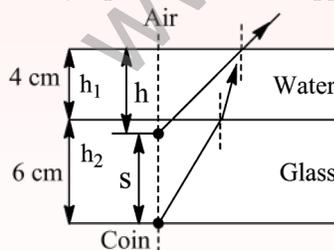
$$P = \frac{1}{f} = \frac{1}{3 \times 10^{-2}} = 33 D$$

An eye sees an object at $25\ cm$ with strain

So $\dfrac{1}{3 \times 10^{-2}} - \dfrac{1}{-25 \times 10^{-2}} = \dfrac{1}{f}$

$$\text{or } P = \frac{1}{f} = 33 + 4 = 37D$$

6. (a)

Using equation, the total apparent shift is

7. (b)

$$s = h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right)$$

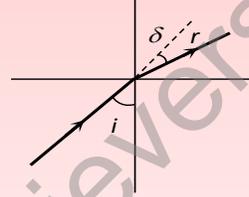$$\text{Or } s = 3\left(1 - \frac{1}{4/3}\right) + 5\left(1 - \frac{1}{3/2}\right)$$

$$= 2.4 \text{ cm}$$

Thus, $h = h_1 + h_2 - s = 3 + 5 - 2.4$
$= 5.6$ cm

8. (a)

The ray of light is refracted at the plane surface. However, since the ray is travelling from a denser to a rarer medium, for an angle of incidence (i) greater than the critical angle (c) the ray will be totally internally reflected
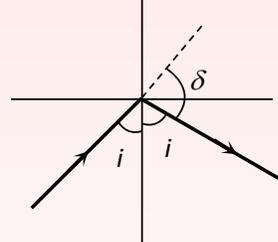

For $i < c$; deviation $\delta = r - i$ with $\dfrac{1}{\mu} = \dfrac{\sin i}{\sin r}$

Hence $\delta = \sin^{-1}(\mu \sin i) - i$

This is a non-linear relation. The maximum value of δ is $\delta_1 = \dfrac{\pi}{2} - C$

Where $i = c$ and $\mu = \dfrac{1}{\sin c}$

For $i > c$, deviation $\delta = \pi - 2i$
δ decreases linearly with i

$$\delta_2 = \pi - 2c = 2\delta_1$$

9. (b)

$$\phi = 4\pi L = 4 \times 3.14 \times 80$$
$$= 1004.8 \text{ lumen}$$

10. (a)

By using $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{0.01}{f_1} + \frac{0.03}{20} = 0$

$$f_1 = -6.67 \text{ cm}$$

11. (c)

$d = \frac{D \times f}{r_1} = \frac{1.39 \times 10^9 \times 20 \times 10^{-2}}{1.5 \times 10^{11}}$
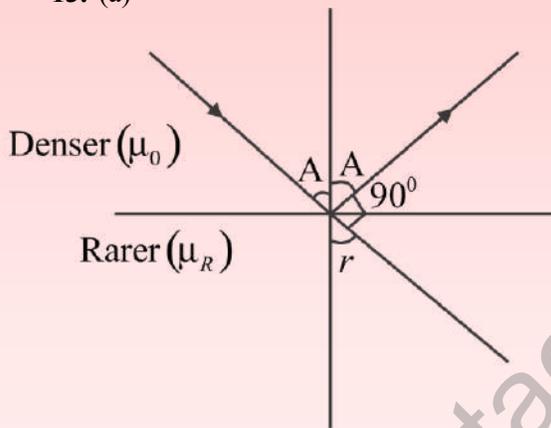
$$= 18.52 \times 10^{-4} \text{m}$$

12. (c)

Given $M_0 = 20$, $M_e = 5$

∴ magnification of this microscope is

$$M = M_0 \times M_e = 20 \times 5 = 100$$

13. (a)

Let's consider the angle of refraction as 90- A.
The critical angle can be determined using the
equation $\sin(\theta_c) = (\mu_r/\mu_i)$, where μ_r and μ_i are the
refractive indices of the medium of refraction
and the medium of incidence, respectively.
We have observed that $\sin(\theta_r) = (\mu_r/\mu_i)$
Substituting the known values, we can write:
$\sin(\theta_c) = \sin(\theta_r)$
$\sin(\theta_c) = \sin(A) * \cos(90 - A)$
$\sin(\theta_c) = \sin(A) * \cos(A)$
$\sin(\theta_c) = \tan(A)$
Therefore, $A = \arctan(\sin(\theta_c))$

14. (a)

$D = (\mu - 1)A$
For blue light μ is greater than that for red
light, So, $D_2 > D_1$

15. (c)

Here $\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$

Plano-convex lens silvered on plane side has
$f_m = \infty$

∴ $\frac{1}{F} = \frac{2}{f} + \frac{1}{\infty} \Rightarrow \frac{1}{40} = \frac{2}{f} \Rightarrow f = 80 \text{ cm}$
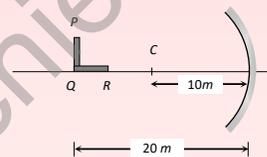
Plano-convex lens silvered on convex side
has $f_m = \frac{R}{2}$

∴ $\frac{1}{F} = \frac{2}{f} + \frac{2}{R} \Rightarrow \frac{1}{15} = \frac{2}{80} + \frac{2}{R} \Rightarrow R = 120 \text{ cm}$

Now using $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R}\right)$, we get $\mu = 2.5$

16. (b)

Focal length of mirror $f = \frac{R}{2} = \frac{10}{2} = 5cm$

For part PQ:
transverse magnification
length of image $L_1 = \left(\frac{f}{f-u}\right) \times L_0$

$= \left(\frac{-5}{-5 - (-20)}\right) \times L_0 = \frac{-L_0}{3}$

For part QR: longitudinal magnification
Length of image $L_2 = \left(\frac{f}{f-u}\right)^2 L_0$

$= \left(\frac{-5}{-5 - (-20)}\right)^2 \times L_0 = \frac{L_0}{9} \Rightarrow \frac{L_1}{L_2} = \frac{3}{1}$

17. (b)

$\frac{f_l}{f_a} = \frac{a\mu_g - 1}{l\mu_g - 1} \Rightarrow \frac{-0.5}{0.2} = \frac{1.2 - 1}{l\mu_g - 1} \Rightarrow l\mu_g - 1$

$$= -1.08$$

$\Rightarrow l\mu_g = 1.08 = \frac{54}{50} \Rightarrow \frac{a\mu_g}{a\mu_l} = \frac{54}{50} \Rightarrow \frac{1.2}{a\mu_l} = \frac{54}{50}$
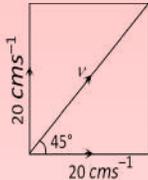
$$\Rightarrow a\mu_l = \frac{10}{9}$$

18. (b)

$$\lambda_{medium} = \frac{\lambda_{air}}{\mu} = \frac{5000}{1.5} = 3333 \text{ Å}$$

19. (d)

$$v \cos 45° = 20 \quad v = 20\sqrt{2} \text{ cms}^{-1}$$

In the ceiling mirror be original velocity will be seen

20. (b)

$$\mu = \frac{1}{\sin C} = \frac{1}{\sin 60} = \frac{2}{\sqrt{3}}$$

$$\therefore v = \frac{3 \times 10^8 \times \sqrt{3}}{2} = 2.6 \times 10^8 \text{m/s}$$

21. (b)

$$\Rightarrow \theta \geq \sin^{-1}\left(\frac{1}{\mu}\right) \Rightarrow \sin \theta \geq \frac{1}{\mu}$$

From figure $\sin \theta = \frac{R}{R+d}$

$$\Rightarrow \frac{R}{R+d} \geq \frac{1}{\mu} \Rightarrow \left(1 + \frac{d}{R}\right) \leq \mu$$

$$\Rightarrow \frac{d}{R} \leq \mu - 1 \Rightarrow \left(\frac{d}{R}\right)_{max} = 0.5$$

22. (b)

Apparent depth $h' = \dfrac{h}{_{air}\mu_{liquid}}$

$$\Rightarrow \frac{dh'}{dt} = \frac{1}{_a\mu_\omega} \frac{dh}{dt} \Rightarrow$$

$$x = \frac{1}{_a\mu_\omega} \frac{dh}{dt} \quad \left[\because \frac{dh'}{dt} = x\right]$$

$$\Rightarrow \frac{dh}{dt} = {_a\mu_\omega}x$$
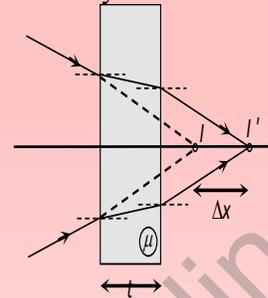
Now volume of water $V = \pi R^2 h$

$$\Rightarrow \frac{dV}{dt} = \pi R^2 \frac{dh}{dt} = \pi R^2 \cdot {_a\mu_\omega}x$$

$$= {_a\mu_\omega}\pi R^2 x = \frac{\mu_\omega}{\mu_a}\pi R^2 x = \left(\frac{n_2}{n_1}\right)\pi R^2 x$$

23. (a)

Normal shift $\Delta x = \left(1 - \frac{1}{\mu}\right)t$ and shift takes place in direction of ray

24. (d)

The person using spectacles with a power of +2 D is suffering from hypermetropia or farsightedness. Hypermetropia is a refractive error where the eye focuses light behind the retina instead of directly on it. This results in difficulty in focusing on nearby objects, causing blurred vision. The positive power of +2 D in the spectacles helps to bring the focal point forward, allowing the person to see nearby objects more clearly.

25. (b)

$$r_1 = 15 \text{ cm}, r_2 = 5 \text{ cm}$$

$$\frac{I_1}{I_2} = \frac{25}{225}, 1 - \frac{I_1}{I_2} = 1 - \frac{25}{225}$$

Or $\dfrac{I_2 - I_1}{I_2} = \dfrac{200}{225}$

Or $\dfrac{I_2 - I_1}{I_2} \times 100 = 89\%$

26. (d)

Light travels slower in denser medium. Hence medium A is a denser medium and medium B is a rarer medium. Here, light travels from medium A to medium B.

Let C be the critical angle between them

$\therefore \sin C = {}^A\mu_B = \dfrac{1}{{}^B\mu_A}$

Refractive index of medium B w.r.t medium A is

$${}^A\mu_B = \dfrac{\text{Velocity of light in medium } A}{\text{Velocity of light in medium } B} = \dfrac{v_A}{v_B}$$

$\therefore \sin C = \dfrac{v_A}{v_B} = \dfrac{2 \times 10^8}{2.6 \times 10^8} = \dfrac{10}{13}$

$\Rightarrow C = \sin^{-1}\left(\dfrac{10}{13}\right)$

27. (a)

In Galilean telescope, a convergent lens is used as the objective and a divergent lens as the eyepiece. Magnifying power and length of telescope are written as

$M = \dfrac{f_0}{u_e}$ and $L = f_0 - u_e$

In normal adjustment, ie, in relaxed eye state $u_e = f_e$

So, $M_\infty = \dfrac{f_0}{f_e} = 60$

or $f_e = \dfrac{f_0}{60} = \dfrac{90}{60} = 1.5 \ cm$

And $L_\infty = f_0 - f_e$

$\therefore L_\infty = 90\text{-}1.5 = 88.5 \ cm$

28. (a)

$m = \dfrac{f_0}{f_e} = \dfrac{30}{2.5} = 12$

Resolving limit $= \dfrac{1.22\ \lambda}{a} = \dfrac{1.22 \times (5000 \times 10^{-10})}{0.1}$

$= 6.1 \times 10^{-6} \ rad$

29. (c)

The combination of two lenses is

As $\dfrac{4}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

$\therefore \dfrac{1}{F} = (u_1 - 1)\left(\dfrac{1}{\infty} + \dfrac{1}{R}\right) + (u_2 - 1)\left(\dfrac{1}{-R} - \dfrac{1}{\infty}\right)$
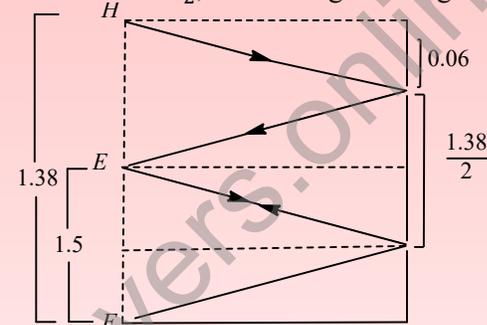
$= \dfrac{u_1 - 1}{R} + \dfrac{u_2 - 1}{R}$

$\dfrac{1}{F} = \dfrac{u_1 - u_2}{R}$

Or $F = \dfrac{R}{u_1 - u_2}$

30. (a)

From the figure, it is clear that eye is at 1.38 m from the foot. Rays from foot can enter eye after reflection at M_2, whose height from ground

Again, eye is at $1 - 0.8 = 0.2$ m from heat. Rays from head can enter eye after reflection at M_1, whose height above eye is

$\dfrac{0.2}{2} = 0.4 \ m$

$\therefore$ Minimum length of mirror

$= 0.69 + 0.4 = 1.09 \ m$

31. (c)
32. (a)

For one prism, $\omega_1 = \dfrac{\delta_B - \delta_R}{\delta} = \dfrac{8-4}{6}$ $\left[\because \delta = \dfrac{\delta_B + \delta_R}{2}\right]$

$\Rightarrow \omega_1 = \dfrac{4}{6}$

For other prism, $\omega_2 = \dfrac{\delta_B - \delta_R}{\delta} = \dfrac{12-8}{10}$ $\left[\because \delta = \dfrac{12+8}{2}\right]$

$\Rightarrow \omega_2 = \dfrac{4}{10} =$

$\therefore \dfrac{\omega_1}{\omega_2} = \dfrac{4 \times 10}{6 \times 4}$ or $\dfrac{\omega_1}{\omega_2} = \dfrac{5}{3}$

33. (d)

34. (b)

From the figure

$i_1 = 90° - (90° - A) = A$

and $\propto = 90° - 2i_1 = 90° - 2A$

$\therefore i_2 = 90° - \alpha = 90° - (90° - 2A) = 2A$

$\therefore \beta = 90° - i_2 = 90° - 2A$

From the geometry of the figure

$A + 2A + 2A = 180°$

$$\therefore A = 36°$$

35. (b)

$A = r_1 + r_2$

$\therefore 30° = r_1 + 0°$

or $r_1 = 30°$

Now, $\dfrac{\sin i}{\sin 30°} = \dfrac{1}{\sqrt{2}}$

or $\sin i = \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

Or $\sin i = \dfrac{1}{2\sqrt{2}}$

Or $i = 20.7°$

36. (c)

Apparent shift $h = \left(1 - \dfrac{1}{\mu}\right)h$

$\therefore$ Apparent shift produced by water

$\triangle h_1 = \left(1 - \dfrac{1}{\mu_1}\right)h_1$

And apparent shift produced by kerosene

$\triangle h_2 = \left(1 - \dfrac{1}{\mu_2}\right)h_2$

$\triangle h = \triangle h_1 + \triangle h_2$

$= \left(1 - \dfrac{1}{\mu_1}\right)h_1 + \left(1 - \dfrac{1}{\mu_2}\right)h_2$

37. (b)

The condition for achromatism is

$\omega_1 P_1 + \omega_2 P_2 = 0$

$\omega_1 P_1 = -\omega_2 P_2$

$\Rightarrow \dfrac{\omega_1}{\omega_2} = -\dfrac{P_2}{P_1}$

Now, $P_1 + P_2 = 3D$

Or $4 + P_2 = 3$ or $P_2 = -1D$

$$\therefore \dfrac{\omega_1}{\omega_2} = -\dfrac{-1}{5} = \dfrac{1}{5}$$

38. (a)

$\dfrac{1}{f} = \left(\dfrac{n}{1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$\dfrac{1}{f_1} = \left(\dfrac{n}{n'} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

Dividing, $\dfrac{f_1}{f} = \dfrac{(n-1)n'}{n-n'}$

Or $f_1 = -\dfrac{fn'(n-1)}{n'-n}$

39. (a)

$n = \dfrac{\text{Real depth}}{\text{Apparent depth}} = \dfrac{5}{4}$

$\dfrac{n_1}{u} + \dfrac{n_2}{v} = \dfrac{n_1 \sim n_2}{R}$

$\dfrac{1.25}{5} - \dfrac{4}{13} = \dfrac{1.25 - 1}{R}$

$R = 34$ cm

40. (b)

$\mu = \tan\dfrac{A}{2} = \dfrac{\sin(\frac{A + \delta m}{2})}{\sin A/2}$

or $\dfrac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \dfrac{\sin(\frac{A + \delta m}{2})}{\sin A/2}$

or $\sin\left(90° - \dfrac{A}{2}\right) = \sin\left(\dfrac{A + \delta m}{2}\right)$

or $90° - \dfrac{A}{2} = \left(\dfrac{A + \delta m}{2}\right)$

or $180° - A = A + \delta m$

$\delta m = 180° - 2A = \pi - 2A$

41. (c)

$\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

For biconvex lens $R_2 = -R_1$ $\therefore \dfrac{1}{f} = (\mu - 1)\left(\dfrac{2}{R}\right)$

Given $R = \infty$ $\therefore f = \infty$, so no focus at real distance

42. (a)

$1 = v1 + u1 \Rightarrow f1 = vu\, v + u \Rightarrow vu$

$= f(v + u)$

Substituting the values of v and u, we get

$(f + d2)(f + d1) = f(f + d2 + f + d1)$

$\Rightarrow f2 + fd1 + fd2 + d1d2$

$= 2f2 + fd1 + fd2 \Rightarrow 2f2 - f2$

$= d1d2 \Rightarrow f2 = d1d2 \Rightarrow f$

$= \sqrt{(d1d2)}$