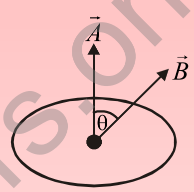
ELECTROMAGNETIC INDUCTION

CHAPTER 06

Magnetic flux

- Magnetic flux ϕ through an area A in a magnetic field B is defined as

$\phi = \vec{B}.\vec{A}$, $\phi = BA\cos\theta$



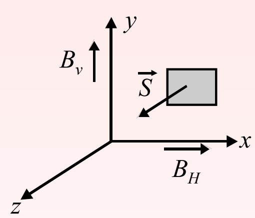
For $\theta < 90°, \cos\theta > 0$ and hence ϕ is positive.

For $\theta > 90°$, $\cos\theta < 0$ and hence ϕ is negative.

- **Units of Magnetic flux:**

CGS unit of magnetic flux is Maxwell. SI unit is weber (Wb). $1\text{Wb}=10^8$ Maxwell.

Example-1: **At a given place, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S, if the area S is in x-y plane?**



Solution: As here $\vec{B} = \hat{i}B_H - \hat{j}B_V$ = constant, so $\phi = \int \vec{B}.\vec{ds} = \vec{B}.\vec{S}$

[as $\vec{B}$ = constant]

So, as for area in x-y plane $\vec{S} = S\hat{k}$;

$\phi_{xy} = (\hat{i}B_H - \hat{j}B_V)\cdot(\hat{k}S) = 0$

[as $\hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$]

Faraday's Law

- **Whenever there is a change in magnetic flux, an induced emf is produced in a closed loop. The induced emf is directly proportional to the time rate of change of magnetic flux through the circuit,**

$e = -\frac{d\Phi_B}{dt}$ [where $\Phi_B = \int \vec{B}.\vec{dA}$ is the magnetic flux.

By the definition of flux, we know that $\Phi_B = \vec{B}.\vec{A} = BA\cos\theta$, thus flux may change with time in several ways

- The magnitude of B can change with time.
- The area A enclosed by the loop can change with time.
- The angle θ between B and normal to the loop can change with time.
- Any combination of the above

If a single loop is replaced by a coil of N turns, then the net emf induced is given by

$$e = \frac{-Nd\Phi_B}{dt}$$

Example-2: **A closed coil of copper whose area is (1.0 metre×1.0 metre) is free to rotate about an axis. The coil is placed perpendicular to a magnetic field of 0.10 weber/metre^2. It is rotated through 180° in 0.01 second. Calculate the magnitude of the induced emf and the induced current in the coil. The resistance of the coil is 2.0 ohm. If the coil be opened then what will be the value of the induced emf and the current?**

Solution: The change in flux linked with the coil on rotating it through 180° is

$= nAB - (-nAB) = 2nAB$

$\therefore$ Induced emf. $= -\frac{d\phi}{dt} = \frac{2nAB}{dt}(\text{numerically}) = \frac{2\times1\times1\times0.1}{0.01} = 20$ volt

The coil is closed and has a resistance of 2.0 ohm. Therefore $i = (20/2) = 10$A.

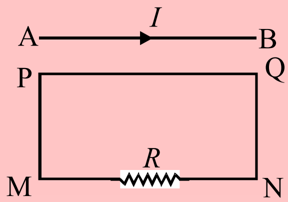
Lenz's law

- **The effect of the induced emf on the direction of the induced current in a conductor is such as to oppose the cause producing it.**

The negative sign in Faraday's law indicates that the induced emf opposes the change in magnetic flux that produces it.

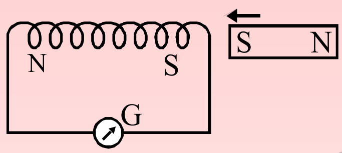
Though in general the direction of induced emf or current is determined by Lenz's law, in case of motion of a straight conductor in a magnetic field it can also be determined by the **Fleming's right-hand rule.**, According to it if the **fore finger points in the direction of field and the thumb in the direction of motion, the central finger will point in the direction of the induced emf or current.**

Example-3: **If the current *I* in conductor AB is decreasing to zero, find the direction of induced current in resistance MN.**

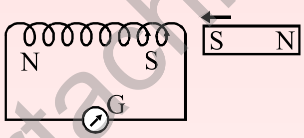


***Solution*:** According to Lenz's law, the induced emf opposes both the growth and decay of current. So, induced current in PQ is along PQ and so in resistance it is along NM.

Example-4: **Give the direction of deflection in galvanometer when S–pole is moved towards coil.**



Solution: According to Lenz's Law, when S-pole of magnet is moved towards the coil, coil being at rest, the current flows in clock-wise direction in the face of the coil towards the S-pole of the approaching magnet.



Motional EMF

- **When a conductor moves in a magnetic field, it cuts the magnetic lines of force and an emf is hence induced across its ends which is known as Motional emf.**

  If a conductor cuts the lines of force in uniform magnetic field, the emf induced is given as

  Emf induced $= \vec{B}.\frac{d\bar{A}}{dt} = \vec{B}.(d\bar{l} \times \bar{v})$ across a length $d\bar{l}$. Net induced emf $= \int \vec{B}.(d\vec{l} \times \vec{v})$

Example-5: **An air-plane with 20m wing span is flying at 250 MS^{-1} straight south parallel to the earth's surface. The earth's magnetic field has a horizontal component of 2 × 10^{-5} Wb m^{-2} and the dip angle is 60°. Calculate the induced emf between the plane tips.**

Solution: As the plane is flying horizontally, it will cut the vertical component of earth's field B_v. So, the emf induced between its tips, $e = B_V vl$

But as by definition of angle of dip, $\tan\phi = \dfrac{B_V}{B_H}$

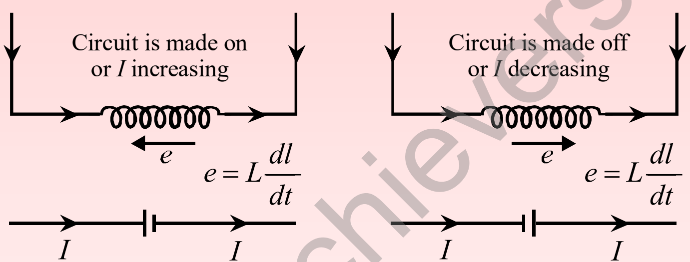
$$i.e.,\quad B_V = B_H \tan\phi$$

$$\text{so,}\quad e = (B_H \tan\phi)vl = 2\times10^{-5}\times\sqrt{3}\times250\times20\ (\text{volt})$$

$$i.e.,\quad e = (\sqrt{3})\times10^{-1}\,\text{V} = 0.173\ (\text{volt})$$

Self- Induction

- Whenever the electric current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, in accordance with Faraday's law of electromagnetic induction, an emf is induced in the coil or the circuit which **opposes** the change that causes it. This phenomenon is **called self-induction** and the emf induced, is called back emf.



(A) (B)

So, if I is the current flowing through a circuit, $\phi \propto I$ i.e., $\phi = LI$ $\therefore$ $L = \dfrac{\phi}{I}\left(\dfrac{Wb}{\text{amp}} = \text{Henry}\right)$

Where L is a constant of proportionality and is called coefficient of self-induction or simply inductance. The configuration having inductance is called **inductor** and is represented by the symbol ⌒0000000⌒.

Self-inductance of a coil: $L_C = \dfrac{\mu_o}{4\pi}(2\pi^2N^2R) = \dfrac{1}{2}\mu_o\pi N^2R$

Self-inductance of a solenoid: $L_S = \mu_o\left(\dfrac{N^2l}{l^2}\right)A = \mu_o n^2 Al$, were, $n = N/l$

Example-6: **Calculate the inductance of a 25 cm long solenoid if it has 1000 turns and radius of its circular cross-section is 5 cm.**

Solution: As for a solenoid $B = \dfrac{\mu_0}{4\pi}\left(4\pi\dfrac{N}{L}I\right) = 10^{-7}\left(4\pi\times\dfrac{1000}{0.25}\times I\right) = 16\pi\times10^{-4}I$

So, $\phi = B(NS) = 16\pi\times10^{-4}I\times10^3\times\pi\times(0.05)^2 = 4\pi^2\times10^{-3}I$

and hence $L = (\phi/I) = 4\pi^2\times10^{-3} = 0.04\text{H}$

Mutual-Induction

- Whenever the current passing through a coil in circuit changes, the magnetic flux linked with a neighbouring coil or circuit changes. This phenomenon is called **'mutual induction'.**

  $\phi_s \propto I_p$ or $\phi_s = MI_p$ (unit of M is same as that of coefficient of self-induction).

  where M is a constant of proportionality and is called coefficient of mutual induction or simply mutual inductance.

  The mutual inductance M of two coils or circuits having self-inductance L_1 and L_2 is given by

  $M = k\sqrt{L_1L_2}$

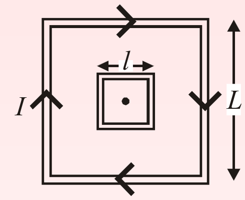
  where k is a constant called **'coefficient of coupling'**. If the coils are wound over each other, the coupling is said to be tight otherwise loose. For tight coupling $k = 1$ and so $M = \sqrt{L_1L_2}$ while for loose coupling $0 < k < 1$ and hence $M < \sqrt{L_1L_2}$.

Example-7: **A small square loop of wire of side *l* is placed inside a large square loop of wire of side *L* (>> *l*). The loops are coplanar and their centers coincide. What is the mutual inductance of the system?**

Solution: Considering the larger loop to be made up of four rods each of length L, the field at the center, *i.e.*, at a distance (L/2) from each rod, will be

$$B = 4\times\frac{\mu_0}{4\pi}\frac{I}{d}[\sin\alpha+\sin\beta] \quad i.e., \; B = 4\times\frac{\mu_0}{4\pi}\frac{I}{(L/2)}\times 2\sin 45$$

$$i.e., \; B_1 = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{L}I .$$



So, the flux linked with smaller loop $\phi_2 = B_1S_2 = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{L}l^2 I$

and hence, $M = \frac{\phi_2}{I} = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{l^2}{L}$

Coils in series and Parallel:

Coils in series. $L_S = L_1 + L_2 + ...$

Coils in parallel. $\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$ i.e., $L_p = \frac{L_1L_2}{L_1+L_2}$

Example-8: **The equivalent inductance of two inductors is 2.4 H when connected in parallel and 10 H when connected in series. What is the value of the inductances of the individual inductor?**

Solution: As inductors obey laws similar to 'grouping of resistance',

$\therefore \quad L_1 + L_2 = 10 \quad$ ...(i)

and $\quad \frac{L_1L_2}{(L_1+L_2)} = 2.4 \quad$ ...(ii)

Substituting the value of ($L_1 + L_2$) from first expression into second, we get

$L_1L_2 = (2.4)(L_1 + L_2) = 2.4\times 10 = 24$

so that $(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2$ *i.e.*, $L_1 - L_2 = [(10)^2 - 4 \times 24]^{1/2} = 2$

and as $L_1 + L_2 = 10,\ L_1 = 6\text{H}$ and $L_2 = 4\text{H}$

Energy Stored in Capacitor

- **ENERGY STORED IN INDUCTOR:**

  $U = \frac{1}{2} L i^2$

  Energy stored per unit volume $(u) = \frac{1}{2} \frac{B^2}{\mu_o}$. This is also called as magnetic energy density.

  Ideal inductor has zero resistance.

Example-9: **The current in a coil of self-inductance 2.0 Henry is increasing according to $I = 2 \sin^2 t$ A. Find the amount of energy spent during the period when the current changes from 0 to $2A$.**

Solution: When the current in the coil is changing, work done in time dt

$$dW = Pdt = eI\ dt = L\frac{dI}{dt} \times Idt \qquad \left[\text{as } e = L\frac{dI}{dt}\right]$$

or, $$W = L\int I\,dI = 2\int_0^2 I\,dI \quad i.e., \quad W = [I^2]_0^2 = 4\text{J}$$

Electromagnetic Induction

Faraday's laws of electromagnetic induction

1. Whenever magnetic flux through an area bounded by a closed conducting loop changes, an emf is produced in the loop.
2. The emf is given by $\varepsilon = -d\phi/dt$ where, $\phi = \int \vec{B}.\vec{ds}$ is the magnetic flux through the area.

Lenz's law

The direction of the induced current is such that it opposes the change that has induced it.

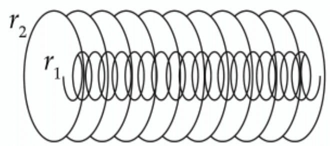
Mutual induction

$\phi = Mi$

$$\frac{d\phi}{dt} = -M\frac{di}{dt}$$

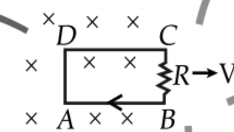
$M_{12} = \mu_0 N_1 N_2 \pi r_1^2 / l$

$M_{21} = \mu_0 N_1 N_2 \pi r_2^2 / l$



Emf induced in an a.c. generator, $\varepsilon = NBA\omega \sin \omega t$

Rectangular loop



Thermal power developed in the loop is

$$P = \frac{v^2B^2l^2}{R}$$

EMF induced

$E = vBl$

$$i = \frac{vBl}{R}$$

Magnetic force on the loop

$F = B^2l^2 v/R$

= Force required to move the loop with constant velocity (v)

In 1831, Michael Faraday discovered electromagnetic induction and James Clerk Maxwell mathematically described it.

Self inductance of long solenoid

$L = \mu_0 n^2 \pi r^2 l$

n = Number of turns per unit length

ϕ = Flux = $(\mu_0 ni)\pi r^2$

r = Radius of each loop of solenoid

- **Growth of current in LR Circuit**

$$i = \frac{\varepsilon}{R}(1 - e^{tR/L}) = i_0(1 - e^{-t/\tau})$$

- **Decay of current**

$$i = i_0 e^{-t/\tau}$$

- **Energy stored in an Inductor**

$$U = \frac{1}{2}Li^2$$

Induced EMF

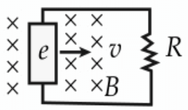
$$\varepsilon = \frac{-d\phi}{dt}$$

Motional EMF

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \left|Bl\frac{dx}{dt}\right|$$

$= Blv$

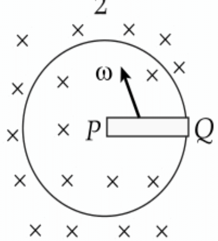
$i = Blv/(R+r)$



r = Resistance of rod moving with velocity v in uniform magnetic field $\vec{B}$

EMF induced in a rotating conductor

$$\varepsilon = \frac{1}{2}B\omega l^2$$



Where l = Length of rod

Induced current

$$I = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt}$$

Self induction

If we consider a solenoid of N turns, the flux through each turn, $\phi = \int \vec{B}.\vec{ds}$.

EMF induced between the ends of coil, $\varepsilon = -N\frac{d}{dt}\int \vec{B}.\vec{ds}$

Trace the Mind Map
- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS

1. A wire measuring 3m in length is being moved at a velocity of 2 m/s through a magnetic field with an intensity of 0.75 Wb/m^2. The motion of the wire is perpendicular to the direction of the magnetic field. The induced emf in the wire is

a) 2.5 V b) 1 V c) 0.1V d) 4.5 V

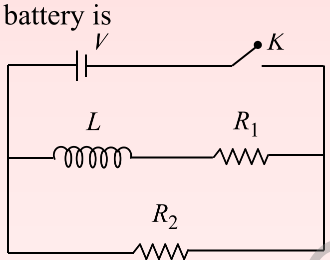
2. A coil of 500 turns is wound on a book and this book is lying on the table. The vertical component of earth's magnetic field is 0.9×10^{-4} T and the area of the coil is 0.1 m^{-2}. The book is turned over once about a horizontal axis is 0.5 s. This average emf induced in the coil is

a) 0.03 V b) 0.018 V c) Zero d) 0.1 V

3. The self-inductance of a circular coil with 200 turns of wire and a radius of 5 cm is

a) 12.5×10^{-3}mH b) 25 mH c) 50×10^{-3}H d) 50×10^{-3}mH

4. When the key K in the circuit shown below is closed at t=0, the resulting current through the battery is



a) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ b) $\frac{V(R_1+R_2)}{\sqrt{R_1^2R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = \infty$ d) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{\sqrt{R_1^2R_2^2}}$ at $t = \infty$

5. When a square loop with a side length of 20 cm is transformed into a circular loop within a time of 0.5 seconds, and there is a uniform magnetic field of 0.2 T directed perpendicular to the loop, calculate the induced electromotive force (emf) in the loop

a) 17.2×10^{-3} V b) 6.6×10^{-5} V c) 17.2×10^{-4} V d) 4.60×10^{-8} V

6. When a pair of coils is subjected to a 70 Hz AC current with a peak value of 3 A flowing through one of the coils, and the mutual inductance between the pair of coils is 150 MH, determine the peak value of the voltage induced in the second coil.

a) 30π V b) 63π V c) 15π V d) 300π V

7. In the region between the pole faces of an electromagnet, the magnetic induction is measured to be 0.9 Weber/m^2. In this scenario, if a straight conductor with a length of 10 cm is perpendicular to the magnetic field (B) and is moving with a velocity of 5 m/s, both perpendicular to the magnetic induction and its own length, determine the induced e m f in the conductor.

a) 0.08 V b) 0.45 V c) 0.35 V d) 0.07 V

8. In the presence of a uniform magnetic field directed perpendicularly into the plane of the paper, an irregularly shaped conducting loop is undergoing a gradual transformation into a circular loop within the plane of the paper. Then,

a) Current is induced in the loop in the anticlockwise direction
b) Current is induced in the loop in the clockwise direction
c) AC is induced in the loop
d) No current is induced in the loop

9. Given that the coefficient of mutual inductance between two coils is 6 MH and the current flowing through one of the coils is 2 amperes, determine the induced (e.m.f.) in the second coil.

a) 3 mV b) 2 mV c) 3 V d) Zero

10. If in a coil rate of change of area is $\frac{7\ \text{metre}^2}{\text{milli second}}$, current becomes 1 amp form 2 amp in 3.5×10^{-3} sec magnetic field is 2 tesla, the self-inductance of the coil is

a) 2H b) 55 H c) 20 H d) 49 H

11. Faraday's laws arise from the principle of conservation of a which specific physical quantity?

a) Energy b) Energy and magnetic field
c) Charge d) Magnetic field

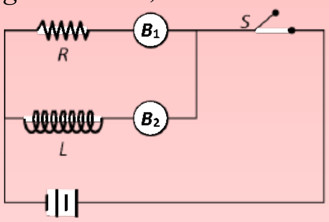
12. If the oscillating frequency of a cyclotron is 30 MHz and the radius of its Dee's is 0.6 m, determine the kinetic energy of a proton that is accelerated by the cyclotron.

a) 10.2 MeV b) 2.55 MeV c) 20.4 MeV d) 6.6 MeV

13. Flux ϕ (in weber) in a closed circuit of resistance 10Ω varies with time t (in second) according to equation $\phi = 5t^2 - t + 1$.
The magnitude of the induced current at t =0.2 s is

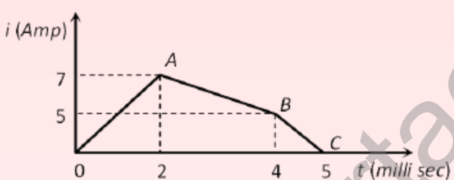
a) 1.2 A b) 0.8 A c) 0.6 A d) 0.1 A

14. *The figure presented illustrates two bulbs, B1 and B2, a resistor R, and an inductor L. In the given circuit, when the switch S is turned off*



a) Both B_1 and B_2 die out promptly
b) Both B_1 and B_2 die out with some delay
c) B_1 dies out promptly but B_2 with some delay
d) B_2 dies out promptly but B_1 with some delay

15. The current through a 7.6 H inductor is shown in the following graph. The induced emf during the time interval $t = 4\, milli - sec$ to $5\, milli - sec$ will be



a) $10^3 V$ b) $-38 \times 10^3\ V$ c) $38 \times 10^3\ V$ d) Zero

16. When an aircraft with a wing-span of 50 m flies at a constant altitude in the northern hemisphere, moving eastward at a speed of 540 km/h, and considering the vertical component of the Earth's magnetic field as 1.6×10^{-5} T, determine the resulting electromotive force (emf) between the tips of the wings.

a) 0.5 V b) 0.35 V c) 0.12 V d) 2.1 V

17. According to Lenz's law of electromagnetic induction
a) The induced emf is not in the direction opposing the change in magnetic flux.
b) The relative motion between the coil and magnet produces change in magnetic flux
c) Only the magnet should be moved towards coil
d) Only the coil should be moved towards magnet

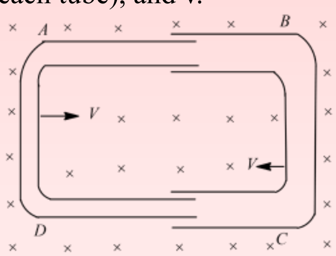
18. Consider a situation where a circular metal plate with a radius of R is rotating uniformly at an angular velocity of ω. The plane of the plate is perpendicular to a uniform magnetic field B. In this setup, calculate the induced emf between the center and the rim of the plate.

a) $\pi\omega BR^2$ b) ωBR^2 c) $\pi\omega BR^2/2$ d) $\omega BR^2/2$

19. Two concentric coils each of radius equal to π cm are placed at right angles to each other. 4 A and 6 A are the currents flowing in each coil respectively. The magnetic induction in $\mathrm{Wb/m^2}$ at the centre of the coils will be $(\mu_0 = 4\pi \times 10^{-7}\mathrm{Wb/Am})$

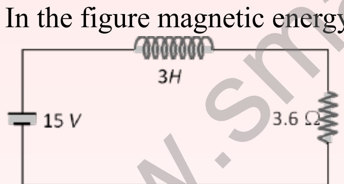
a) 1.2×10^{-5} b) 10^{-5} c) 1.44×10^{-5} d) 5×10^{-5}

20. In the given configuration, there are two conducting U-tubes where one can slide inside the other, ensuring electrical contacts between them. The magnetic field B is perpendicular to the plane represented in the figure. As both tubes move towards each other at a constant velocity v, find the induced emf in the circuit. This emf can be expressed in terms of B, l (the width of each tube), and v.



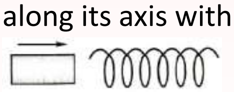
a) Blv b) −Blv c) Zero d) 2 Blv

21. In the figure magnetic energy stored in the coil is

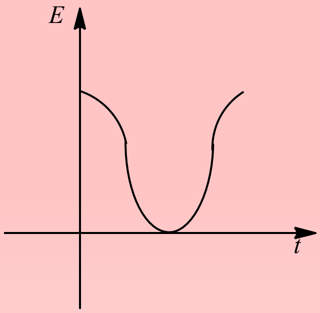


a) Zero b) 25 J c) 25.9 joules d) None of the above

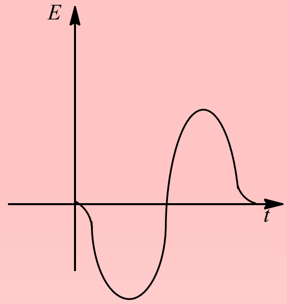
22. The variation of induced emf(ε) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as



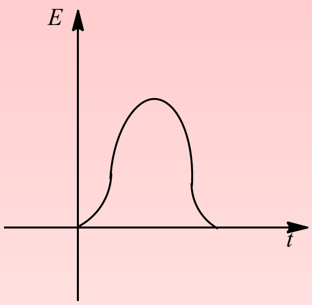
a)



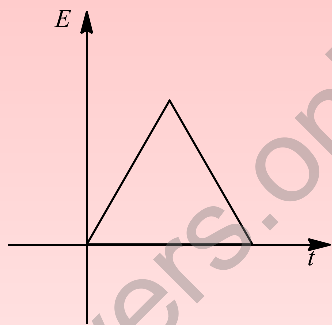
b)



c)



d)



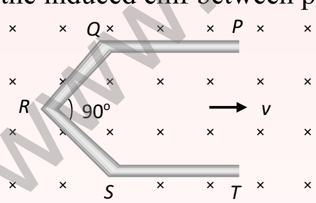
23. Given two circuits with a coefficient of mutual induction of 0.09 henry, the average electromotive force (e.m.f.) induced in the secondary circuit by a change of current from 0 to 30 amperes in 0.005 seconds in the primary circuit is

a) 120 V b) 540 V c) 200 V d) 300V

24. If two solenoids have equal numbers of turns and their lengths and radii are in the same ratio of 2: 1, determine the ratio of their self-inductances.

a) 1: 2 b) 2: 1 c) 1: 1 d) 1: 4

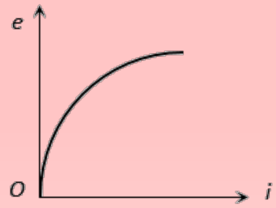
25. A conductor PQRST moves along its bisector with a velocity of 2 m/s through a perpendicular magnetic field of 0.5 wb/m^2, as shown in fig. If all the four sides are of 1m length each, then the induced emf between points P and T is



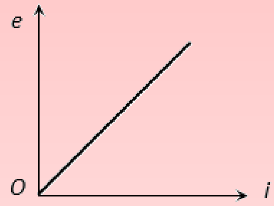
a) 0 b) 1.41 volt c) 0.71 volt d) None of the above

26. *Two identical induction coils with an inductance of L are connected in series. They are positioned in close proximity to each other, and the winding direction of one coil is precisely opposite to that of the other. Thus,* which of the following graphs is correct

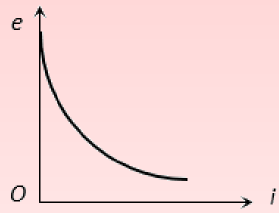
a)



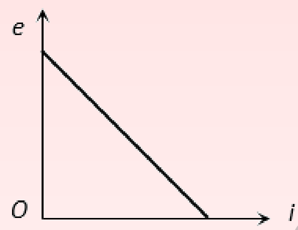
b)



c)



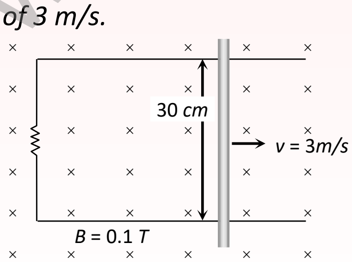
d)



27. Induced emf in the coil depends upon

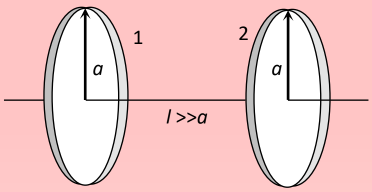
a) Conductivity of coil
b) Amount of flux
c) Rate of change of linked flux
d) Resistance of coil

28. *In the given figure, a metal rod establishes contact and forms a closed circuit. The circuit is perpendicular to a magnetic field with a magnitude of B = 0.1Tesla. With a resistance of 5Ω, determine the force required to move the rod in the indicated direction with a constant speed of 3 m/s.*



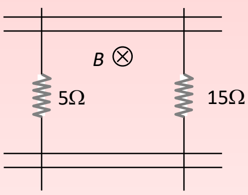
a) 0.54×10^{-3} N b) 0.54×10^{-2} N c) 3.75×10^{2} N d) 3.75×10^{-4} N

29. Determine the mutual inductance of a two-loop system, as depicted in the figure, with a center separation denoted as l.



a) $\frac{\mu_0 \pi a^4}{8l^3}$ b) $\frac{\mu_0 \pi a^4}{4l^3}$ c) $\frac{\mu_0 \pi a^4}{6l^3}$ d) $\frac{\mu_0 \pi a^4}{2l^3}$

30. A pair of parallel conducting rails lie at right angles to a uniform magnetic field of 5.0 T as shown in the fig. Two resistors 15 Ω and 5 Ω are to slide without friction along the rail. The distance between the conducting rails is 0.2 m. Then



a) Induced current = $\frac{1}{100}$ A directed clockwise if 15Ω resistor is pulled to the right with speed 0.2 ms^{-1} and 5Ω resistor is held fixed
b) Induced current = $\frac{1}{300}$ A directed anti-clockwise if 15 Ω resistor is pulled to the right with speed 0.2 ms^{-1} and 5Ω resistor is held fixed
c) Induced current = $\frac{1}{300}$ A directed clockwise if 5 Ω resistor is pulled to the left at 0.2ms^{-1} and 15Ω resistor is held at rest
d) Induced current = $\frac{1}{100}$ A directed anti-clockwise if 5 Ω resistor is pulled to the left at 0.2 ms^{-1} and 15Ω resistor is held at rest

31. The average energy stored in a pure inductance L when a current i flows through it can be expressed as

a) Li^2 b) $2Li^2$ c) $\frac{Li^2}{4}$ d) $\frac{2i^2}{2}$

32. When an athlete runs towards the east at a speed of 25 km/h while holding a horizontally positioned 4 m metallic rod, and considering a horizontal component of the Earth's magnetic field in the region as 5×10^{-4} Tesla with an angle of dip of 30 degrees, determine the induced emf across the ends of the rod.

a) 7.5 mV
b) 4.3 mV
c) 6.7 mV
d) 8 mV

33. For a circular coil with a radius of 6 cm and 200 turns of wire, estimate the value of the coefficient of self-induction of the coil.

a) 4.7 millihenry b) 25×10^{-3} millihenry c) 47×10^{-3} millihenry d) 50×10^{-3} millihenry

34. The quantity of electric charge Q passing through any point of the circuit during the time interval At, where the magnetic flux through the circuit changes by an amount $\Delta\phi$, is determined by the following relationship:

a) $Q=\frac{\Delta\phi}{\Delta t}$ b) $Q=\frac{\Delta\phi}{\Delta t}\times R$ c) $Q=-\frac{\Delta\phi}{\Delta t}+R$ d) $Q=\frac{\Delta\phi}{R}$

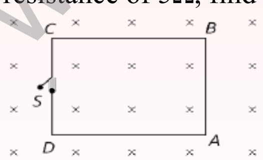
35. Coefficient of coupling between two coils of self-inductances L_1 and L_2 is unity. i.e

a) 50% flux of L_1 is linked with L_2
b) 100% flux of L_1 is linked with L_2
c) $\sqrt{L_1}$ time of flux of L_1 is linked with L_2
d) None of the above

36. The mutual inductance of a system consisting of a small square loop of wire with side length l placed inside a larger square loop of wire with size L (L > l), where the loops are coplanar and their canters coincide, is

a) $2\sqrt{2}\mu\frac{l^1}{\pi L}$
b) $2\sqrt{2}\mu\frac{l^2}{\pi L}$
c) $2\sqrt{2}\mu\frac{l^2 L}{\pi}$
d) $2\sqrt{2}\mu\frac{l^2 L^2}{\pi}$

37. If the magnetic field in the cylindrical region, as depicted in the figure, increases at a constant rate of 10 MT/sec, and each side of the square loop ABCD has a length of 2 cm with a resistance of 5Ω, find the current in the wire AB once the switch S is closed.



a) 1.25×10^{-7} A, (anti-clockwise)
b) 1.25×10^{-7} A, (clockwise)
b) 2.5×10^{-7} A, (anti-clockwise)
d) 2.5×10^{-7} A, (clockwise)

38. What does Lenz's law give

a) The magnitude of the induced emf

b) The direction of the induced current

c) Both the magnitude and direction of the induced current

d) The magnitude of the induced current

39. Given that the axle of a truck is 2.5 m long and the truck is moving due north at a speed of 50 m/s, while located in an area where the vertical component of the Earth's magnetic field is 70 ft., calculate the potential difference between the two ends of the axle.

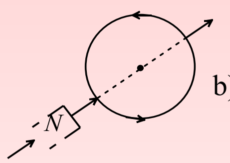
a) 8.75 mV with west end positive

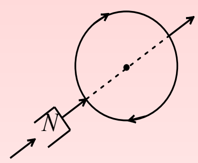
b) 8.75 mV with east end positive

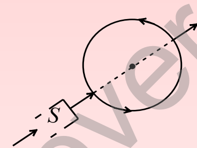
c) 8.75 mV with north end positive

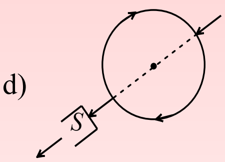
d) 8.75 mV with south end positive

40. Among the given options, which figure accurately represents Lenz's law? The figures depict the movement of a labelled pole of a bar magnet into a closed circular loop, and the arrows on the loop indicate the direction of the induced current as per Lenz's law.

a) 

b) 

c) 

d) 

41. In a DC motor, the armature has a resistance of 30Ω. When the motor is operated using a 110-volt DC power supply, it draws a current of 1.2 amperes. Calculate the value of the back (EMF) induced in the motor.

a) 150 V

b) 170 V

c) 181 V

d) 146 V

-----------------------------ANSWER KEY------------------------------------

1)	d	2)	b	3)	a	4)	c
5)	a	6)	b	7)	b	8)	a
9)	d	10)	d	11)	a	12)	d
13)	d	14)	c	15)	c	16)	c
17)	b	18)	d	19)	c	20)	d
21)	c	22)	b	23)	b	24)	b
25)	b	26)	d	27)	c	28)	a
29)	d	30)	d	31)	d	32)	d
33)	a	34)	c	35)	b	36)	b
37)	a	38)	b	39)	a	40)	a
41)	d						

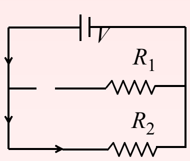
HINTS AND SOLUTIONS

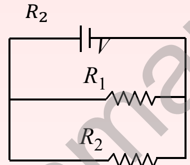
1. (d)
The emf induced will be
$$e = vBl = 2 \times 0.75 \times 3 = 4.5 \text{ V}$$

2. (b)
$$e = \frac{-d\phi}{dt} = \frac{-NBA(\cos 0° - \cos 180°)}{dt}$$
$$= \frac{2\,NBA}{dt} = \frac{2 \times 500 \times 0.9 \times 10^{-4} \times 0.1}{0.5}$$
$= 0.018$ V

3. (a)
Self-inductance of coil is
$$L = \frac{\mu_0 n^2 \pi r}{2}$$
$$= \frac{4\pi \times 10^{-7}}{2} \times (200)^2 \times \pi \times (5 \times 10^{-2})$$
$$= 12.5 \times 10^{-3}\text{H} = 12.5 \text{ mH}$$

4. (c)
At $t = 0$ inductor behaves as broken wire then $i = \frac{V}{R_2}$





At $t = \infty$ Inductor behaves as conducting wire
$$i = \frac{V}{R_2R_2/(R_1 + R_2)} = \frac{V(R_1 + R_2)}{R_1R_2}$$

5. (a)
$$e = \frac{d\phi}{dt} = \frac{BdA}{dt} = \frac{0.2(\pi r^2 - L^2)}{dt}$$
$$= 17.2 \times 10^{-3}\ V$$

6. (b)
The current flows through the coil 1 is $I_1 = I_0 \sin \omega t$
Where I_0 is the peak value of current
Magnetic flux linked with the coil 2 is
$\phi_2 = MI_1 = MI_0 \sin \omega t$
Where M is the mutual inductance between the two coils
The magnitude of induced emf in coil 2 is
$$|\varepsilon_2| = \frac{d\phi_2}{dt} = \frac{d}{dt}(MI_0 \sin \omega t) = MI_0\omega \cos \omega t$$
∴ Peak value of voltage induced in the coil 2 is
$$= MI_0\omega = 150 \times 10^{-3} \times 3 \times 2\pi \times 70$$
$$= 63\pi \text{ V}$$

7. (b)
$$e = Bvl \Rightarrow e = 0.9 \times 5 \times (10 \times 10^{-2})$$
$$= 0.45 \text{ V}$$

8. (a)
When the shape of a loop is altered, causing a change in the flux linked with the loop, an induced electromotive force (EMF) is generated. Consequently, an induced-current flows through the coil. By applying the right-hand screw rule, we determine that the induced current circulates in an anticlockwise direction.

9. (d)
In secondary emf induces only when current through primary changes

10. (d)
$$e = B.\frac{dA}{dt} = L\frac{di}{dt} \Rightarrow 2 \times \frac{7}{10^{-3}}$$
$$= L \times \frac{(2-1)}{3.5 \times 10^{-3}} \Rightarrow L = 49H$$

11. (a)

Faraday's laws involve conversion of mechanical energy into electrical energy. This is in accordance with the law of conservation of energy

12. (d)

KE of charged possible in a cyclotron,

$$E_k = \frac{q^2B^2r^2}{2m}$$

But frequency $f = \frac{qB}{2\pi m}$

$$\therefore E_k = \frac{(2\pi mf)^2r^2}{2m} = 2\pi^2 mf^2r^2$$

Or $E_k = 2 \times (3.14)^2 \times 1.67 \times 10^{-27} \times (30 \times 10^6)^2 \times (0.6)^2$

$= 10.66 \times 10^{-13}$J

$$\therefore \quad E_k = \frac{10.66 \times 10^{-13}}{1.6 \times 10^{-19}}$$

$= 6.66 \times 10^6$ eV

$= 6.66$ MeV

13. (d)

From, Faraday's second law, $e = -\frac{d\phi}{db}$

$= -[10t - 1]$

$= -[10 \times (0.2) - 1] = +1$

Now, $i = \frac{e}{R} = \frac{1}{10} = 0.1$ A

14. (c)

Current in B_1 will promptly become zero while current in B_2 will slowly tend to zero

15. (c)

Rate of decay of current between $t = 4\ ms$ to $5\ ms = \frac{di}{dt} = -$ (Slope of the line BC)

$= -\left(\frac{5}{1\times10^{-3}}\right) = -5 \times 10^3 A/s$. Hence induced emf

$$e = -L\frac{di}{dt} = -7.6 \times (-5 \times 10^3)$$

$= 38 \times 10^3 V$

16. (c)

$L = 50\ m, v = 540 km\ h^{-1} = 150 m\ sec^{-1}$ and $B = 1.6 \times 10^{-5} T$

$\Rightarrow e = Blv = 1.6 \times 10^{-5} \times 50 \times 150$

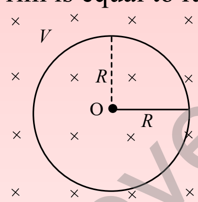
$= 0.12\ V$

17. (b)

According to Lenz's law of electromagnetic induction, the relative motion between the coil and magnet produces change in magnetic flux

18. (d)

From Faraday's law of electromagnetic induction, the emf induced between center and rim is equal to rate of change of magnetic flux.



$$e = -\frac{d\phi}{dt}$$

Where, $d\phi = B\ dA$, where B is magnetic field and dA the area.

$$\therefore \quad e = -\frac{B\int_0^R dA}{T}$$

$$e = -\frac{B \times \pi R^2}{T}$$

Also, $\omega = \frac{2\pi}{T}$, where T is periodic time,

$$e = -\frac{B\pi R^2}{2\pi/\omega}$$

$$= -\frac{BR^2\omega}{2}$$

19. (c)

$$B_p = \frac{\mu_0 I_2}{2R}$$

$$= \frac{4\pi \times 10^{-7} \times 6}{2 \times 0.01\pi} = 1.2 \times 10^{-5} Wb/m^2$$

$$B_Q = \frac{\mu_0 I_1}{2R}$$

$$= \frac{4\pi \times 10^{-7} \times 4}{2 \times 0.01\pi} = 0.8 \times 10^{-5} Wb/m^2$$

$$\therefore B = \sqrt{B_p^2 + B_Q^2} = 1.44 \times 10^{-5} Wb/m^2$$

20. (d)

Relative velocity $= v - (-v) = 2v = \frac{dl}{dt}$

Now, $e = \frac{d\phi}{dt}$

$$e = \frac{Bldl}{dt} \qquad \left(\frac{dl}{dt} = 2v\right)$$

Induced emf $e = 2\,Blv$

21. (c)

$$i = \frac{V}{R} = \frac{15}{3.6} = 4.16A$$

$$U = \frac{1}{2}Li^2 = \frac{1}{2} \times 3 \times 17.3 = 25.95\,J$$

22. (b)

Polarity of emf will be opposite in the two cases while entering and while leaving the coil. Only in option (b) polarity is changing.

23. (b)

$$e_0 = M\frac{di}{dt}$$

=0.09 × 30/0.005

=540 V

24. (b)

Self-inductance of a solenoid

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l}$$

Where l is the length of the solenoid, N is the total number of turns of the solenoid and A is the area of cross-section of the solenoid

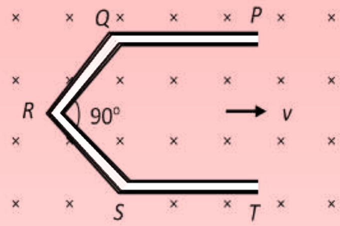
$$\therefore \frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{r_1}{r_2}\right)^2 \left(\frac{l_2}{l_1}\right)$$

Here, $N_1 = N_2, \frac{l_1}{l_2} = \frac{2}{1}, \frac{r_1}{r_2} = \frac{2}{1}$

$$\therefore \frac{L_1}{L_2} = \left(\frac{2}{1}\right)^2 \left(\frac{1}{2}\right) = \frac{2}{1}$$

25. (b)

There is no induced emf in the part PQ and ST because they are moving along their length while emf induced between Q and S *i.e.*, between P and T can be calculated as follows



Induced emf between Q and S = Induced emf between P and $Q = Bv(\sqrt{2}l) = 0.5 \times 2 \times 1 \times \sqrt{2} = 1.41\ volt$

26. (d)

When the two coils are connected in series with opposite windings, the electromotive force (EMF) produced in the first coil is 180° out of phase with the EMF produced in the second coil. This means that the EMF in the first coil is negative, while the EMF in the second coil is positive.

Regarding the net inductance, when the coils are connected in series, their inductances add up. Thus, the total inductance (L) of the series combination is the sum of the individual inductances (L_1 and L_2):

$L = L_1 + L_2$.

From Faraday's law of electromagnetic induction

$\phi = Li$, where ϕ is flux and i the current

$$\therefore \qquad L = -\frac{\phi}{i} + \frac{\phi}{i}$$

$$\Rightarrow \qquad L = 0$$

27. (c)

According to Faraday's law, "the induced emf in a closed loop equals the time rate of change of magnetic flux through the loop."

ie., $e = -\frac{d\phi_B}{dt}$

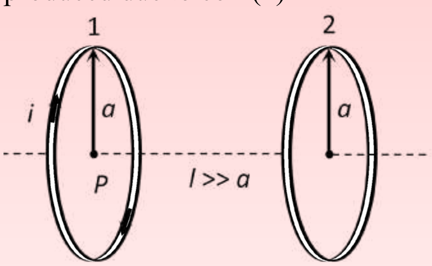
Hence, induced emf in a coil depends on rate of change of flux.

28. (a)
Induced current in the circuit $i = \frac{Bvl}{R}$
Magnetic force acting on the wire $F_m = Bil = B\left(\frac{Bvl}{R}\right)l$

$\Rightarrow F_m = \frac{B^2vl^2}{R}$. External force needed to move the rod with constant velocity

$$(F_m) = \frac{B^2vl^2}{R} = \frac{(0.1)^2 \times (3) \times (0.3)^2}{5}$$
$$= 0.54 \times 10^{-3} N$$

29. (d)
Magnetic field at the location of coil (2) produced due to coil (1)



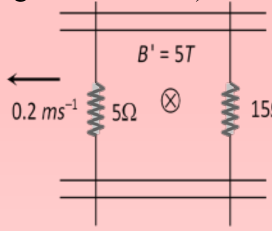
$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{l^3}$
Flux linked with coil (2)

$\phi = B_1 A_2 = \frac{\mu_0}{4\pi} \frac{2i(\pi a^2)}{l^3} \times (\pi a^2)$

Also $\phi_2 = Mi \Rightarrow M = \frac{\mu_0 \pi a^4}{2l^3}$

30. (d)
When 5Ω resistor is pulled left at $0.2\ m/sec$ induced emf in the said resistor $= e = vBl = 0.2 \times 5 \times 0.2 = 0.2\ V$
Resistor 15Ω is at rest so induced emf in it $(e = vBl)$ be zero
Now net emf in the circuit $= 0.2V$ and equivalent resistance of the circuit $R = 20\Omega$

Hence current $i = \frac{0.2}{20} amp = \frac{1}{100} amp$
And its direction will be anti-clockwise (according to Lenz's law)



31. (d)
As we know $e = -\frac{d\phi}{dt} = -L\frac{di}{dt}$
Work done against back emf e in time dt and current i is

$$dW = -eidt = L\frac{di}{dt}idt = Li\,di \Rightarrow W$$
$$= L\int_0^i i\,di = \frac{1}{2}Li^2$$

32. (d)
Component of field in the vertical direction = B=5× 10^{-4} tan (30)
vertical component of field is perpendicular to both velocity and length of rod
emf = Blv=8mV

33. (a)
$\phi = Li \Rightarrow NBA = Li$
Since magnetic field at the Centre of circular coil carrying current is given by $B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Ni}{r}$

$\therefore N.\frac{\mu_0}{4\pi} \cdot \frac{2\pi Ni}{r} \cdot \pi r^2 = Li \Rightarrow L = \frac{\mu_0 N^2 \pi r}{2}$
Hence self-inductance of a coil

$$= \frac{4\pi \times 10^{-7} \times 200 \times 200 \times \pi \times 0.06}{2}$$
$$= 4.73mH$$

34. c)
The relationship that determines the quantity of electric charge passing through any point of a circuit during a time interval At, given a change in magnetic flux Δϕ, is given by Faraday's law of electromagnetic induction.

According to Faraday's law, the induced electromotive force (EMF) in a circuit is equal to the rate of change of magnetic flux through the circuit.

Mathematically, Faraday's law can be expressed as:
EMF = -dϕ/dt

where EMF is the induced electromotive force, dϕ/dt represents the rate of change of magnetic flux (Δϕ/Δt), and the negative sign indicates the direction of the induced EMF.

To determine the quantity of electric charge passing through a point in the circuit, we can integrate the induced EMF over the time interval Δt:

Q = ∫(EMF) dt

Substituting the expression for EMF from Faraday's law, we have:

Q = ∫(-dϕ/dt) dt

Integrating both sides of the equation give:

Q = -∫dϕ

Q = -Δϕ/ Δt

35. (b)
Two coils are said to be magnetically coupled if full or a part of the flux produced by one links with the other. Let L_1 and L_2 be the self-inductances of the coils and M be their mutual inductances, then

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

When 100% flux produced by one coil links with the other, then mutual inductance between the two is maximum and is given by

$$M = \sqrt{L_1 L_2}$$

In that case, k = 1(unity)

36. (b)
Field at the center of outer square loop $= B = 2\sqrt{2}\mu\frac{i}{\pi L}$ along the axis.
flux linking the smaller square loop=

$$Bl^2 = 2\sqrt{2}\mu\frac{i}{\pi L}l^2$$

mutual inductance $= 2\sqrt{2}\mu\frac{l^2}{\pi L}$

37. (a)

$$i = \frac{e}{R} = \frac{A}{R}\cdot\frac{dB}{dt} = \frac{(1\times10^{-2})^2}{16}$$

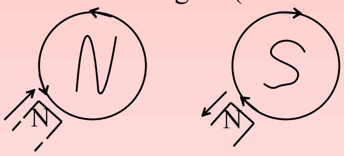
$$\times 20 \times 10^{-3} = 1.25 \times 10^{-7}A$$

38. (b)
The Lenz's law gives the direction of induced current. According to this law, the induced current opposes the cause that produces it.

39. (a)
While moving due north, the truck intercepts vertical component of earth’s field.
∴ $e = Blv = (70 \times 10^{-6})2.5 \times 50$
$= 8.75 \times 10^{-3}V = 6.75$ mV
According to Lenz’s law, west end of the axle will be positive.

40. (a)

When a bar magnet's north pole approaches a coil, the induced current in the coil circulates in an anticlockwise direction, causing the coil to present its north pole to the magnet (as depicted in figure (a)). Conversely, when the north pole of a bar magnet moves away from the coil, the induced current in the coil flows in a clockwise direction, resulting in the coil presenting its south pole to the magnet (as shown in figure (b)).



41. (d)

$$i = \frac{E - e}{R} \Rightarrow 1.2 = \frac{110 - e}{30} \Rightarrow e = 146\ V$$