MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

CHAPTER 04

Introduction

- Inside a current carrying conductor there are innumerable charged particle in motion, namely, electrons. Hence a current carrying conductor can exert force on a moving charged particle or another current carrying conductor. This effect of a current carrying conductor is called the **magnetic effect of current.** The earliest record of the magnetic effect of a current is that of Oersted, who discovered that a pivoted magnetic needle beneath a wire tends to set itself at right angles to the wire when a current is passed through it. Later, experiments by Biot and Savart led to the formulation of this effect (i.e., magnetic field) due to a '**current-element**' for a current carrying conductor.
- In SI, a magnetic field of strength B is said to exist at a point, if a current element placed at that point or a moving charge passing through that point, experiences a sideways force given by $\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$ or $\Delta\vec{F} = q_o\vec{v} \times \vec{B}$
- That is the force is given in magnitude by $\Delta F = I\Delta l B\sin\theta$ or $q_o v B\sin\theta$, where θ is the angle between $\Delta\vec{l}$ and the direction of $\vec{B}$ which is taken as the direction of no-deflection of a moving charged particle or a current element, and in direction it is given by the direction of a screw perpendicular to $\Delta\vec{l}$ and $\vec{B}$ and rotating from $\Delta\vec{l}$ to $\vec{B}$ through the shorter angle.

Biot-Savart Law

This is a law based on experimental facts. This establishes the magnitude and direction of the magnetic field at a point due to a current element of a current-carrying conductor. This law is

$$\Delta\vec{B}_o = \frac{\mu_o}{4\pi}\frac{I\Delta\vec{l} \times \vec{r}}{r^3}$$

This is the magnitude of the field in vacuum is $\Delta B_o = \frac{\mu_o}{4\pi}\frac{I\Delta l\sin\theta}{r^2}$, where θ is the angle between $\Delta\vec{l}$ and $\vec{r}$ and in direction it is along a screw perpendicular to $\Delta\vec{l}$ and $\vec{r}$ which is rotating from $\Delta\vec{l}$ to $\vec{r}$ through the shorter angle. We will apply this law to find the magnetic field due to the conductors of definite geometrical shapes.

S.I Unit of $B = $ Tesla and $\frac{\mu_o}{4\pi} = 10^{-7}$ SI unit

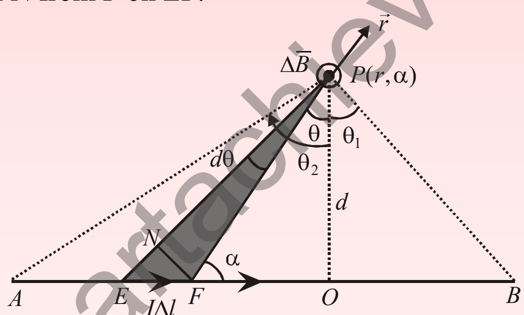
Magnetic Field

Calculation of Magnetic field due to Current-Carrying Conductors of simple Geometrical shapes.

❖ Due to a straight conductor

Suppose AB is a straight conductor carrying a current I. P is a point at a perpendicular distance d from the conductor. Draw perpendicular PO from P on AB. Then $OP = d$. Suppose $FE = \Delta l$ is an element of the conductor. Let P be at a distance r from the element in direction α counted anti clockwise from $\Delta \vec{l}$. Then according to Biota-Savart law, we have

$dB = \frac{\mu_o}{4\pi} \cdot \frac{I \Delta l \sin\alpha}{r^2}$ in magnitude, and in direction, it is along a screw perpendicular to $\Delta \vec{l}$ and $\vec{r}$ rotating from $\Delta \vec{l}$ too $\vec{r}$ through the smaller angle. After the operation of rotating of the screw, the direction is found to the perpendicular out of the plane of paper. The direction of the magnetic field due to all other elements is exactly the same and hence the field due to all other elements is exactly the same, and hence the field due to the entire conductor will be along the normal to the plane containing P and the conductor AB.
Draw a perpendicular FN from F on EP.



From ΔEFN, $FN = \Delta l \sin\theta$.

Also, $FN = r d\theta$ where $\theta = \angle FPO$

$\therefore \quad \Delta l \sin\theta = r d\theta$

$$\therefore \quad dB = \frac{\mu_o}{4\pi} \frac{I r d\theta}{r^2} = \frac{\mu_o I}{4\pi} \cdot \frac{d\theta}{r}$$

$$\text{In} \quad \Delta PFO, \frac{d}{r} = \cos\theta$$

$$\therefore \quad dB = \frac{\mu_o I}{4\pi} \frac{\cos\theta \, d\theta}{d} = \frac{\mu_o I}{4\pi d} \cos\theta \, d\theta$$

If θ_1 and θ_2 be the extreme values of θ when the element is at the ends of the conductor, the total field at P becomes.

$$B = \frac{\mu_o l}{4\pi d} \int_{-\theta_1}^{+\theta_2} \cos\theta \, d\theta = \frac{\mu_o I}{4\pi d} [\sin\theta]_{-\theta_1}^{+\theta_2}; \qquad B = \frac{\mu_o I}{4\pi d} (\sin\theta_1 + \sin\theta)_2$$

❖ **Special cases**

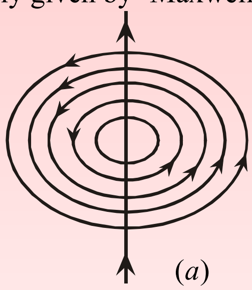
(a) When the conductor is infinitely long when the conductor is made larger and larger, θ_1 and θ_2 become bigger and bigger and tend to $\pi/2$.

$\therefore \quad B = \frac{\mu_o I}{4\pi d}(\sin\pi/2 + \sin\pi/2)$ or $B = \frac{\mu_o I}{2\pi d}$

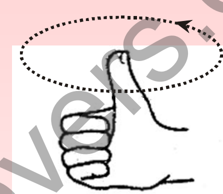
The corresponding H-vector is $H = \frac{1}{2\pi d}$. $\quad (H = \frac{B}{\mu_o})$

(b) When a conductor is finite, but the point is very close to it in this case also θ_1 and θ_2 tend to $\pi/2$. Hence in this case also $B = \frac{\mu_o I}{2\pi d}$ and $H = \frac{B}{\mu_o} = \frac{I}{2\pi d}$

The lines of $\vec{B}$ around a long, straight current carrying wire are shown in figure. These lines are circular and are in a plane perpendicular to the wire. The direction is more conveniently given by 'Maxwell's right-hand rule'.



(*a*)

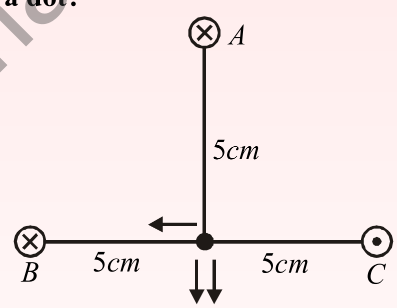


(*b*)

(c) When the point is found to be on the conductor magnetic field at that point is zero.

(d) When the point is found to be on the extension of the conductor (backward/forward) magnetic field is zero.

Example-1: **Three very ling wires perpendicular to the plane of the paper, the carrying 5A each in the direction shown in the figure. What is the B-field at the point midway between B and C shown by a dot?**



Solution: The magnetic field due to B

$= \frac{\mu_o I}{2\pi d} = \frac{4\pi\times10^{-7}}{2\pi}\frac{I}{d} \qquad = \frac{2I}{d}\times10^{-7}$

$= \frac{2\times5}{0.05}\times10^{-7} \qquad = 2\times10^{-5}$ T perpendicular to BC (downward)

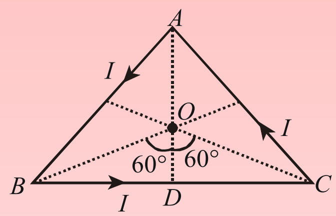
The magnetic field due to the conductor at C is of the same magnitude and direction. The field due to the conductor at A is of the same magnitude, but its direction is along CB.

$\therefore$ The result field $= \sqrt{4^2+2^2}\times10^{-5} = \sqrt{20}\times10^{-5} \quad = 4.48\times10^{-5}$ Tesla.

Example-2: **A current of 1A is flowing in the sides of an equilateral triangle of side 4.5×10^{-2} m. Find the magnetic field at the centroid of the triangle.**

Solution: Let I be the current flowing in the sides of the triangle. The magnetic field induction at the centroid O due to current through one side BC of the triangle will be

$$B_1 = \frac{\mu_o}{4\pi}\frac{I}{R}(\sin\phi_1 + \sin\phi_2)$$



Total magnetic field induction at O due to current through all the three sides of a triangle will be

$$B = 3B_1 = \frac{3\mu_o}{4\pi}\frac{I}{R}(\sin\phi_1 + \sin\phi_2)$$

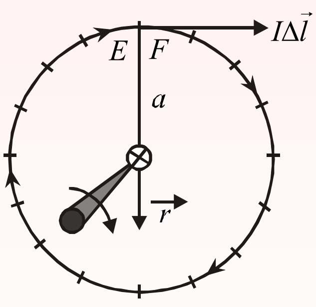
Here, $I = 1A;\ \phi_1 = 60° = \phi_2$ and $R = OD = BD/\tan 60° = \frac{a/2}{\sqrt{3}} = \frac{a}{2\sqrt{3}} = \frac{4.5\times10^{-2}}{2\sqrt{3}} m$

$$B = 3\times10^{-7}\times\frac{2\sqrt{3}}{4.5\times10^{-2}}(\sin 60° + \sin 60°) = \frac{3\times2\sqrt{3}}{4.5}\times10^{-5}\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = 4\times10^{-5}T$$

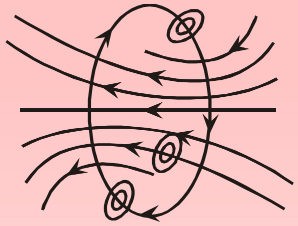
❖ **Magnetic field due to a circular coil:**

Consider a circular conductor of radius a and carrying a current I. Divide the conductor into elements and consider one such element $EF = \Delta l$. The distance of the field point (center of the coil) is the radius of the coil.

$\therefore$ By Biot-Savart law, $\Delta B = \frac{\mu_O}{4\pi}.\frac{I\Delta l\sin 90°}{a^2} = \frac{\mu_o}{4\pi}.\frac{I\Delta l}{a^2}$



➢ The direction of $\Delta\vec{B}$ is along a screw perpendicular to $\Delta\vec{l}$ and $\vec{r}$ rotating from $\Delta\vec{l}$ or $\vec{r}$ through the smaller angle. This direction, as found by actually rotating screw, is perpendicular into the plane of the coil. The field due to all other elements are also in the same direction and so direction of the field due to the entire conductor is perpendicular into the plane of the coil. The magnitude of the field is obviously given by



$B = \Sigma\Delta B$ where Σ extends overall the elements

$$= \Sigma \frac{\mu_o}{4\pi} \cdot \frac{I\Delta l}{a^2} = \frac{\mu_o I}{4\pi a^2} \Sigma\Delta l$$

Now, $\Sigma\Delta l =$ entire length of the conductor $= 2\pi a N$, $N =$ number of turns of the coil,

$$\therefore \quad B = \frac{\mu_o I}{4\pi a^2} \times 2\pi a N = \frac{\mu_o N I}{2a}$$

The lines of B due to a circular conductor are shown in the figure. These lines are circular at the conductor and are almost parallel at the center. This is why the magnetic field due to a circular conductor is uniform over a very small region surrounding the center.

Example-3: **In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius $5.1 \times 10^{-11}\, m$ at a frequency of $6.8 \times 10^{15}\, rev/\sec$. What value of B is set up at the center of the orbit? What is the equivalent magnetic dipole moment?**

Solution: The current is the rate at which charge passes through any point on the orbit. Therefore, the current constituted by the motion of the electron is

$I = e/T = ve \qquad (\because \ 1/T = v, \ frequency)$

$= 6.8 \times 10^{15} \times 1.6 \times 10^{-19}$

$= 1.1 \times 10^{-3}$ ampere

B at the center of a circular pattern is given by

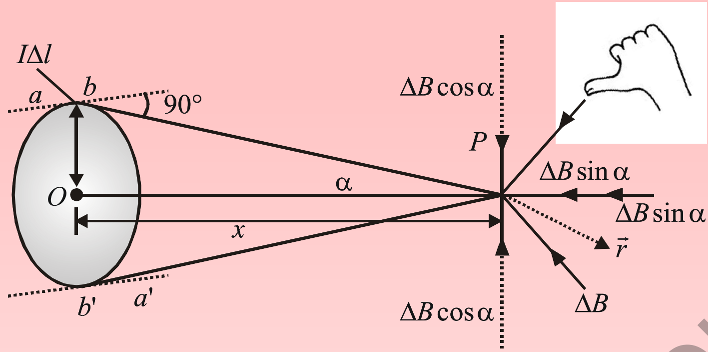
$$B = \frac{\mu_0 I}{2a} = \frac{4\pi \times 10^{-7} \times 1.1 \times 10^{-3}}{2 \times 5.1 \times 10^{-11}} \text{ Tesla} = 1.36 \times 10 = 13.6 \text{ Tesla}$$

The equivalent dipole moment is $m = IS = 1.1 \times 10^{-3} \times \pi (5.1 \times 10^{-11})^2 = 9.0 \times 10^{-24}\ Am^2$.

❖ Magnetic field intensity at any point on the axis of a circular conductor:

Let P be any point on the axis of a circular conductor of radius a and carrying current I at a distance x from the Centre of the coil.

Let $ab = \Delta l$ be any current element. Here angle between $\Delta\vec{l}$ and $\vec{r} = 90°$.



$$\Delta B = \frac{\mu_o}{4\pi}\frac{1\Delta l \sin 90°}{r^2} = \frac{\mu_o I\,\Delta l}{4\pi r^2}$$

and its direction is along a screw perpendicular to $\Delta\vec{l}$ and $\vec{r}$, rotating from $\Delta\vec{l}$ and $\vec{r}$ though the smaller angle. By actually rotating a screw, it is found to be perpendicular to $\vec{r}$ and towards the point P.

Let us resolve ΔB along and perpendicular to the axis. If the inclination of $\vec{r}$ to the axis be α, then the two resolved components are $\Delta B \cos\alpha$ along and perpendicular to the axis respectively. Because of symmetry of the conductor, the normal component of various elements will add upto zero and the axial components will not add up to zero. Hence the resultant field is along the axis and its magnitude is given by

$$B = \Sigma\Delta B \sin\alpha = \Sigma\frac{\mu_o}{4\pi}\frac{I\Delta l}{r^2}\sin\alpha$$

or $$B = \frac{\mu_o}{4\pi}\frac{I\sin\alpha}{r^2}\Sigma\Delta l$$

$$= \frac{\mu_o}{4\pi}\frac{I\sin\alpha}{r^2}\,l$$, where $l =$ length of the conductor.

If $N =$ number of turns of the coil then

$$l = 2\pi a N$$

and $r^2 = a^2 + x^2$ and $\sin\alpha = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$

$$B = \frac{\mu_o o}{4\pi}\frac{I\frac{a}{\sqrt{a^2+x^2}}.2\pi aN}{a^2+x^2} \quad \text{or} \quad B = \frac{\mu_o N a^2 I}{2(a^2+x^2)^{3/2}}$$

The corresponding H-vector is $H = \frac{\text{N}a^2 I}{2(a^2+x^2)^{3/2}}$ $\left(\therefore \frac{B}{\mu_o} = H\right)$

Special cases:

(a) When the point is at the Centre, $x = 0$

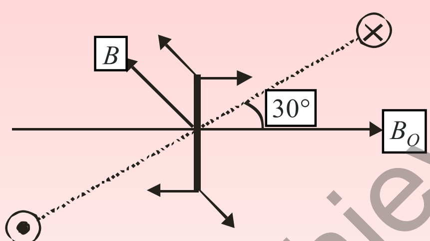
$\therefore$ B-field at the Centre of the circular coil $= \dfrac{\mu_o NI}{2a}$ and H-field at the Centre $= \dfrac{NI}{2a}$

or $B_{centre} = \dfrac{\mu_o NI}{2a}$ and $H_{centre} = \dfrac{NI}{2a}$

(b) When the point is very far off $x = \infty$, B-field is zero.

Example-4: **A small magnet is suspended at the center of a vertical circular coil. When the coil carries a current of 1.25 ampere and makes an angle of 30° with the magnetic meridian, the suspended magnet points east-west. If the number of turns in the coil is 10 and its radius 20 cm, find the horizontal component of the earth's field.**

Solution:



The torque on the needle due to the earth's field $= MB_o \sin 90° = MB_0$

where M is the magnetic moment of the needle. The torque on the needle due to the B-field of the current $= MB \sin 30° = MB/2$

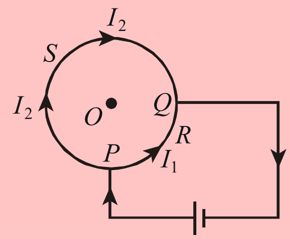
For equilibrium, we have $MB_0 = \dfrac{1}{2} MB$

or, $B_0 = \dfrac{1}{2} \cdot \dfrac{\mu_0 NI}{2a}$

or, $B_0 = \dfrac{4\pi \times 10^{-7} \times 10 \times 1.25}{4 \times 0.2} = 1.9 \times 10^{-5}$ Tesla

Example-5: **A cell is connected across two points P and Q of a uniform circular conductor. Prove that the magnetic field at its center O will be zero.**

Solution: Let l_1, l_2 be the lengths of the two parts PRQ and PSQ of the conductor and ρ be the resistance per unit length of the conductor.



The resistance of the portion PRQ will be, $R_1 = l_1\rho$

The resistance of the portion PRQ will be $R_2 = l_2\rho$. Let I_1, I_2 be the current in PRQ and PSQ respectively. Since these parts are in parallel, the potential difference across their ends will be same

$\therefore \quad I_1R_1 = I_2R_2$

or $\quad I_1l_1\rho = I_2l_2\rho \quad , I_1l_1 = I_2l_2 \qquad \ldots$(i)

According to Biota -Savart's law, the magnetic fields at the center O due to current through circular conductors PRQ and PSQ will be

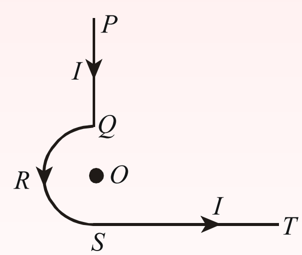
$$B_1 = \frac{\mu_1}{4\pi}\frac{I_1l_1\sin 90°}{r^2} \text{ and } B_2 = \frac{\mu_0}{4\pi}\frac{I_2l_2\sin 90°}{r^2}$$

Since, $\quad I_1l_1 = I_2l_2$

$\therefore \quad B_1 = B_2$

According to right hand rule, the directions of B_1 and B_2 are opposite to each other, Hence, the resultant magnetic field at O will be zero.

Exmaple-6: **In the given figure find the magnetic field induction at O when PQ and ST are infinite long and radius of curved path $QRS = r$.**



Solution: Magnetic field induction at O due to current through PQ is zero as the point O lies on the axis of conductor PQ, i.e. $B_1 = 0$

Magnetic field induction at O due to current through semicircular coil QRS will be

$$B_2 = \frac{\mu_o}{4\pi}\frac{\pi I}{r}$$

It is directed upwards, perpendicular to the plane of coil.

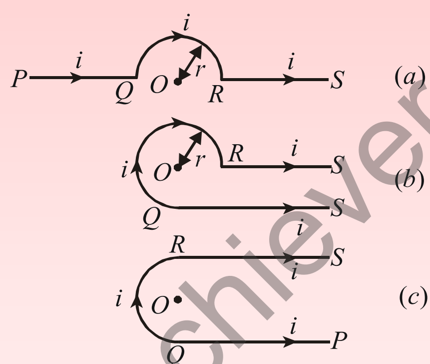
Magnetic field at O due to current through straight conductor ST, will be

$$B = \frac{\mu_o}{4\pi}\frac{I}{r}[\sin 0° + \sin 90°] = \frac{\mu_o}{4\pi}\frac{I}{r}$$

Its direction is $\perp r$ to the plane of coil upwards.

Thus, total magnetic field induction at O due to current through the entire conductor will be $B = B_1 + B_2 + B_3 = 0 + \frac{\mu_o}{4\pi}\frac{\pi I}{r} + \frac{\mu_o}{4\pi}\frac{I}{r} \;\; = \frac{\mu_o}{4\pi}\frac{I}{r}(\mu + 1)$

Example-7: **Find the magnetic field induction at a point O if the current carrying wire has the shape shown in figure (a), (b), (c). The radius of the curved part of the wire is r, the linear parts are assumed to be very long.**



Solution: (a) Magnetic field induction at O due to current through straight wires PQ and RS is zero. The magnetic field induction at O due to current through semicircular part QR of wire is given by

$$B_1 = \frac{\mu_o}{4\pi} \times \frac{\pi i}{r}$$ (It acts $\perp r$ to the plane of the current loop downwards).

$$\therefore \quad B = 0 + \frac{\mu_o}{4} \times \frac{i}{r} = \frac{\mu_o}{4} \times \frac{i}{r}$$

(b) Magnetic field induction at O due to current through straight portion PQ.

$$B_1 = \frac{\mu_o}{4\pi} \times \frac{i}{r}(\sin 0° + \sin 90°) = \frac{\mu_o}{4\pi}\frac{i}{r}$$ acting $\perp r$ to loop downwards.

Magnetic field induction at O due to current through curved part QR of the wire is

$$B_2 = \frac{\mu_o}{4\pi} \times \frac{3\pi}{2} \times \frac{i}{r} = \frac{\mu_o}{4\pi}\frac{i}{r} \times \frac{3\pi;}{2}$$ acting $\perp r$ to loop downwards

Magnetic field induction at O due to current through straight wire RS is zero.

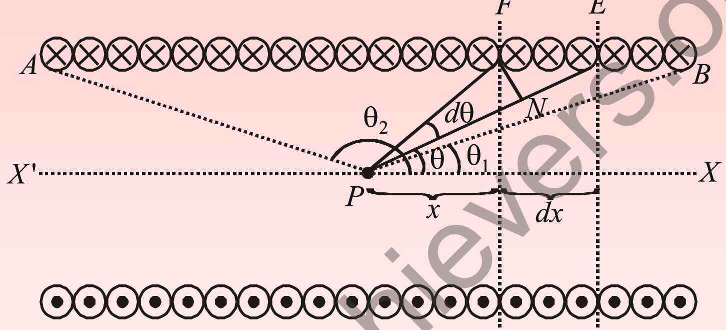
$$B = B_1 + B_2 = \frac{\mu_o}{4\pi}\frac{i}{r} + \frac{\mu_o}{4\pi}\frac{i}{r} \times \frac{3\pi}{2} = \frac{\mu_o}{4\pi}\frac{i}{r}\left(1 + \frac{3\pi}{2}\right)$$

(c) In this case, the effective magnetic field induction at O due to current through the entire wire will be $B = \frac{\mu_o}{4\pi}\frac{i}{r} + \frac{\mu_o}{4\pi}\frac{\pi i}{r} + \frac{\mu_o}{4\pi}\frac{i}{r} = \frac{\mu_o}{4\pi}\frac{i}{r}(2+\pi)$

❖ **Magnetic field due to solenoid:**

Suppose that a long wire is wound on a cylinder in such a way that its turns lies side (not one above the other) on the cylinder. A wire shaped in this way is called a solenoid. A solenoid has a uniform distributed of turns and so linear density of turns is a constant.

Consider a solenoid have n turns per unit length, radius a and carrying current I. Let P be any point on the axis of the solenoid.



At a distance x along the axis from P, consider an elementary length of the solenoid of width $dx = EF$. This element may be considered as a circular conductor of (ndx) turns and P as a point at a distance x from its centre. Hence the field at P due to this elementary length of the solenoid is given by the formula for the magnetic field at a point on the axis of a circular conductor and is

$$dB = \frac{\mu_o (ndx) h^2}{2(a^2 + x^2)^{3/2}} = \frac{\mu_o n I a^2 dx}{2r^3} \quad \text{where } r = \sqrt{a^2 + x^2}$$

Draw a perpendicular En from E on FP.

From the triangle ENF, $EN = dx \sin\theta$ and again from ΔPNE, $EN = rd\theta$

Where $PE = r$

$$\therefore \quad dx \sin\theta = rd\theta \text{ and } r = \sqrt{a^2 + x^2} \quad ; \ dB = \frac{\mu_o n I a^2 \left(\frac{rd\theta}{\sin\theta}\right)}{2rh2} = \frac{\mu_o n I a^2 d\theta}{2r^2 \sin\theta}$$

Again $\frac{a}{r} = \sin\theta$ or $r = \frac{a}{\sin\theta}$

$$dB = \frac{\mu_o I a^2 d\theta}{2\sin\theta}\frac{a^2}{\sin^2\theta} = \frac{1}{2}\mu_o n I \sin\theta \, d\theta$$

If θ_1 and θ_2 be the values of θ when the element is at the ends of the solenoid, the total field at P is given by

$$B = \frac{1}{2}\mu_o n I \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \frac{1}{2}\mu_o n I(\cos\theta_1 - \cos\theta_2)$$

Special cases:

(1) When the solenoid is very long

In this case $\theta_1 = 0$ and $\theta_2 = \pi$

$\therefore \quad B = \mu_o nI$ and $H = nI$

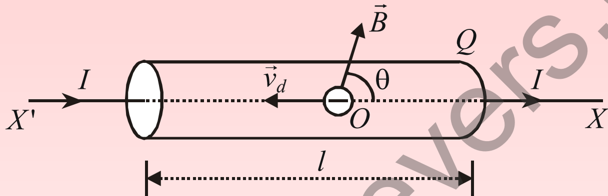
(ii) When the solenoid is short, but it is very thin

Here, also $\theta_1 = 0$ and $\theta_2 = \pi$

$\therefore \quad B = \mu_o nI$ and $H = nI$

❖ FORCE ON CONDUCTOR CARRYING CURRENT PLACED IN A MAGNETIC FIELD.

Consider a straight conductor PQ of length l, area of cross-section A, carrying current I placed in a uniform magnetic field of induction, $\vec{B}$. Let the conductor be placed along X-axis and magnetic field be acting in XY plane making an angle θ with X-axis. Suppose the current *I* flow through the conductor from the end P to Q. Since the current in a conductor is due to motion of electrons, therefore, electrons are moving from the end Q to P (along X' axis)



Let $\vec{v}_d =$ drift velocity of electron

$-e =$ charge on each electron.

Then, magnetic Lorentz force on an electron is given by

$$\vec{f} = -e(\vec{v}_d \times \vec{B})$$

If n is the number density of free electrons i.e., number of free electrons per unit volume of the conductor, then total number of free electrons in the conductor will be given by

$N = n(Al) = nAl$

$\therefore$ Total force on the conductor is equal to the force acting on all the free electrons inside the conductor while moving in the magnetic field and is given by

$$\vec{F} = N\vec{f} = nAl[-e(\vec{v}_d \times \vec{B})]$$

$$= -nAle(\vec{v}_d \times \vec{B})$$

We know that current through a conductor is related with drift velocity by the relation

$I = nAev_d$

$\therefore \quad Il = nAev_d.l$

We represent $I\vec{l}$ as current element vector. It acts in the direction of flow of current i.e., along OX. Since $I\vec{l}$ and $\vec{v}_d$ have opposite directions, hence we can write

$I\vec{l} = -n Ale\vec{v}_d$

$\vec{F} = I\vec{l} \times \vec{B} \qquad |\vec{F}| = I|\vec{l} \times \vec{B}|$

$F = I l B \sin\theta$

where θ is the smaller angle between $I\vec{l}$ and $\vec{B}$.

Special Cases:

(a) If θ° or 180°, $\sin\theta = 0$, from $F = IlB(0) = 0$

It means a linear conductor carrying a current if be placed parallel to the direction of magnetic field, it experiences no force.

(b) If $\theta = 90°, \sin\theta = 1;$ from $F = I\ lB \times 1 = IlB$

It means a linear conductor carrying current if be placed perpendicular to the direction of magnetic field, it experiences maximum force. The direction of which can be given by right-handed screw rule of Fleming's left-hand rule. Accordingly, the direction of $\vec{F}$ is perpendicular to the plane of paper directed upwards.

It is important to note that the relation is valid only if magnetic field $\vec{B}$ is uniform over the whole length of the conductor.

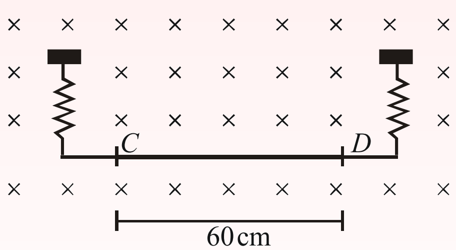
Example-8: **A conductor of length a = 30 cm and carrying current $I = 10\ A$ is placed perpendicular to a long straight conductor carrying $I' = 15\ A$ with its nearer end at a distance d = 12 cm. Calculate the force experienced by the conductor.**

Solution: Consider an element dx at a distance x. Field at the element $= \dfrac{\mu_0 I'}{2\pi x}$ and dF, force on the element $= \left(\dfrac{\mu_0 I'}{2\pi x}\right) I dx$ in the direction parallel to the long conductor and perpendicular to the short conductor.

$$\therefore \qquad F = \frac{\mu_0 I I'}{2\pi}\int_a^{d+a}\frac{dx}{x} = \frac{\mu_0 I I'}{2\pi}\ln\left(\frac{d+a}{d}\right)$$

$$\text{Here} \qquad F = \left(\frac{4\pi \times 10^{7} \times 10 \times 15}{2\pi}\right)\ln\left(\frac{12+30}{12}\right) = 3.75 \times 10^{-5}\ N.$$

Example-9: **A wire of 60 cm length and mass 10 gram is suspended by a pair of flexible leads in a magnetic field of induction 0.40 weber/meter^2. What are the magnetic and direction of the current required to remove the tension in the supporting leads?**



Solution: We know that the magnetic force experienced by the linear wire carrying current, placed in magnetic field is given by

$$F = i|\vec{l} \times \vec{B}| = ilB\sin\theta$$

Here, $\vec{l}$ and $\vec{B}$ are perpendicular, so, $\theta = 90°$

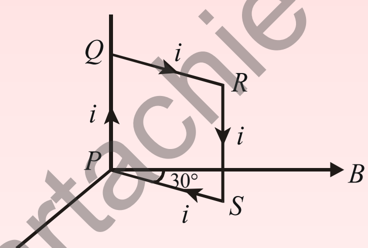
$\therefore \; F = ilB$

If this force F balances the weight of wire $(= mg)$, then the tension in the supporting leads will be removed.

So, $ilB = mg$ or $i = \frac{mg}{lB} = \frac{(10\times10^{-3})\times9.8}{(60\times10^{-2})\times0.4} = 0.41A$

The force $F(= ilB)$ must be acting vertically upwards. It will be so if the direction of current is from left to right.

Example-10: **Figure shows, rectangular twenty turn loop of wire, $10cm \times 5cm$. It carries a current of 0.10 ampere and is hanged at on side. What torque (direction and magnitude (acts on the loop if it is mounted with its plane at an angle of 30° to the direction of a uniform field to magnetic induction 0.50 tesla?**



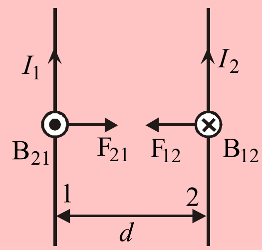
Solution: Here, $n = 20$, $i = 0.1A$; $B = 0.5T$

Torque acting on the coil is

$$\tau = niBA\cos\theta = 20\times0.1\times0.5\times(0.1\times0.05)\cos30° = 4.33\times10^{-3}\,N-m$$

3. INTERACTION BETWEEN TWO LONG STRAIGHT CONDUCTORS.

Let us consider two long, straight parallel conductors separated by a distance 'd' and carrying currents I_1 and I_2 respectively in the same direction. Since each conductor lies in the magnetic field of the other each will experience a force, exerted on it by the magnetic field set up by the current in the other conductor. B_{12} (Magnetic field due to the first conductor at the site of the second conductor) $= \frac{\mu_o I_1}{2\pi d}$. The screw rule for the direction of the magnetic field indicates that this field is perpendicular into the paper shown usually by the symbol '$\otimes$', a direction perpendicular out of the paper is shown by symbol $\odot$.



The force exerted on a current element in a magnetic field is given by that is $\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$

$\therefore$ The force exerted by the first on unit length of the second

$= I_2 B_{12} \sin 90°$ $\quad (\because \theta = 90° \text{ here})$

$= I_2 \frac{\mu_o I_1}{2\pi d} = \frac{\mu_o I_1 I_2}{2\pi d}$.

The screw rule for the direction of the force of indicates that is force is towards the first. The force exerted by the second conductor on the first per unit length of it, is the same in magnitude but opposite in direction as may be seen by considering the field due to the second on the first or by Newton's third law. Hence the two conductors attract each other.

$\therefore$ The mutual force of attraction per unit length between two long parallel conductors is

$$E = \frac{\mu_o I_1 I_2}{2\pi d} \quad \text{or} \quad 2 \times 10^{-7} \frac{I_1 I_2}{d}$$

This formula is made the basis for the definition of the 'ampere' in SI.

Put $F = 2 \times 10^{-7}$ Newton

$I_1 = I_2$ and $d = 1m$ in the above formula.

Then $2 \times 10^{-7} = 2 \times 10^{-7} \frac{I_1^2}{1}$ or $I_1 = 1 = I_2$.

Thus, one ampere is that steady current which, if maintained in each of two infinitely long, straight parallel wires of negligible cross-section placed 1 metre apart, in vacuum, will produce between the wires a force of 2×10^{-7} Newton per metre length of the wire.

The reason for defining the ampere in terms of a force of attraction between two long straight parallel current carrying conductors is that it has now got the same footage as the definitions of the units of electric charge and magnetic charge (magnetic pole).

4. INTERACTION BETWEEN ISOLATED CURRENT ELEMENTS

Let us consider two isolated current elements $i_1 dl_1$ and $i_2 dl_2$. The magnetic field due to 1 at the site of 2 is given by Biota-Savart law is $d\vec{B}_{12} = \frac{\mu_o}{4\pi} \frac{i_1 d\vec{l}_1 \times \vec{r}_{12}}{r_{12}^3}$

The force experienced by a current element in a magnetic field is given by Lorentz force $d\vec{F} = id\vec{l} \times \vec{B}$

$\therefore$ Force experienced by 2 due to $1 = i_2 d\vec{l}_2 \times \frac{\mu_o}{4\pi} \frac{i_1 d\vec{l}_1 \times \vec{r}_{12}}{r_{12}^3}$

$$\Rightarrow \quad d\vec{F}_{12} = \frac{\mu_o}{4\pi} i_1 i_2 \frac{d\vec{l}_2 \times (d\vec{l}_1 \times \vec{r}_{12})}{r_{12}^3}$$

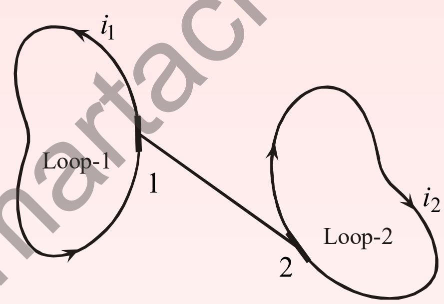
Similarly, $dF_{12} = \frac{\mu_o}{4\pi} i_1 i_2 \frac{d\vec{l}_1 \times (d\vec{l}_2 \times \vec{r}_{21})}{r_{21}^3}$; $|d\vec{F}_{12}| = \frac{\mu_o}{4\pi} i_1 i_2 \frac{|d\vec{l}_1||d\vec{l}_2|}{r_{21}^3} \cos\theta_2$

$$|d\vec{F}_{12}| = \frac{\mu_o}{4\pi} i_1 i_2 \frac{|d\vec{l}_1||d\vec{l}_2|}{r_{21}^2} \cos\theta_1; \qquad \therefore \quad \theta_1 \neq \theta_2 \; |d\vec{F}_{12}| \neq d\vec{F}_{21}|$$

Thus, we see that the forces are not equal and opposite. Hence there is apparent violation of the third law of motion. This does not mean that there is breakdown of Newton's third law.

This apparent violation is due to acceleration and retardation of charge carriers at the ends of the elements. In the isolated elements charge carries are accelerated at the starting end from zero to a uniform velocity and at the other end from that uniform velocity to zero.

This acceleration and retardation produce and electromagnetic field, which carries away momentum from charge carriers, that is, charge carriers experience additional force. This force compounded with the above force shows that the force exerted by the elements are equal and opposite. So, violation is only apparent. This situation arises only in case of isolated current element where there are acceleration and retardation of the charge carriers at the ends. In case of loops where there is no such acceleration or retardation of charge carriers there should be no violation of third law. In fact, such difficulty does not arise. For loops shown in the figure here.



$$\vec{F}_{12} = \frac{\mu_o}{4\pi} i_1 i_2 \oint_1 \oint_2 \frac{d\vec{l}_2 \times (d\vec{l}_1 \times \vec{r}_{12})}{r_{12}}$$

By mathematical laws this can be transformed into symmetrical from

$$\vec{F}_{12} = \frac{\mu_o}{4\pi} i_1 i_2 \oint_1 \oint_2 \frac{(d\vec{l}_1 . d\vec{l}_2)\vec{r}_{12}}{r_{12}}$$

This equation shows that $\vec{F}_{12} = -\vec{F}_{21}$.

Lorentz Force

The force experienced by a charged particle moving in space where both electric and magnetic fields exist is called **Lorentz force**.

Force due to electric field: When a charge particle carrying charge +q is subjected to an electric field of strength $\vec{E}$, it experiences a force given by $\vec{F}_e = q\vec{E}$.

Whose direction is the same at that of $\vec{E}$.

Force due to magnetic field: IF the charged particle is moving in a magnetic field $\vec{B}$, with a velocity $\vec{v}$ it experiences a force given by

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

The direction of this force is in the direction of $(\vec{v} \times \vec{B})$ i. perpendicular to the plane containing $\vec{v}$ and $\vec{B}$ is directed as given by right hand screw rule.

Due to both the electric and magnetic fields, the total force experienced by the charged particle will be given by

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$= q(\vec{E} + \vec{v} \times \vec{B})$$

This is called **Lorentz force.**

Special case:

(a) When $\vec{v}, \vec{E}$ and $\vec{B}$, all the three are collinear. In this situation the charged particle is moving parallel or antiparallel to the fields, the magnetic force on the charged particles is zero.

The electric force on the charged particle will produce acceleration $\vec{a} = \frac{q\vec{E}}{m}$,

Along the direction of electric field. As a result of which there will be change in the speed of charged particle along the direction of the field. In this situation there will be no change in the direction of motion of the charged particle but, the speed, velocity, momentum and kinetic energy all will change.

(b) When $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular to each other. In this situation if $\vec{E}$ and $\vec{B}$ are such that $\vec{F} = \vec{F}_e + \vec{F}_m = 0$ It means the particle will pass through the fields without any change in its velocity. Here,

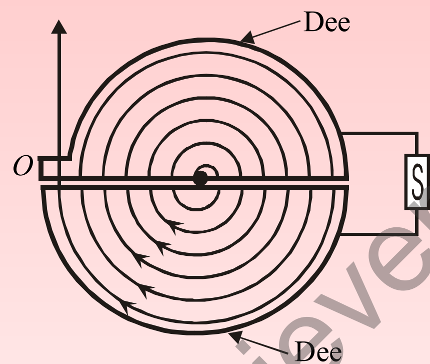
$F_e = F_m$ so, $qE = qvB$ or $v = E/B$

This concept has been used on velocity-selector to get a charged beam having a definite velocity.

Cyclotron

The above facts regarding the motion of a charged moving particle in a uniform magnetic field are utilized in accelerating charged particles to very high velocities in a machine called a Cyclotron.

A cyclotron, developed by EO Lawrence in 1932, consists of two D-shaped copper chambers placed side by side leaving a small gap. A magnetic field perpendicular to the plane of these Dees (Ds) is maintained with the help of an electromagnet having flat, circular pole pieces. An altering electric field is applied across the gap between the Dees with the help of an oscillator.



A source of ion is placed at the center of the dees. Ions coming out of the source with some velocities are bent by the magnetic field and move in a circle with a constant time period depending on the magnetic field and the mass and charge of the ions. If the frequency of the applied electric field and the strength of the magnetic field are such that the moment the ions reach the gap, the electric field between the dees exerts an impulsive force on them, they gain velocity. Thus, they go to a path of higher radius, but their angular velocity remains unchanged. Again, on reaching the gap, the ions will be accelerated and will move to a still higher radius. Thus, they keep on adding in velocity and spiraling around, till they come out at opening O with tremendous speed. The condition for acceleration at the opportune moment is that the angular frequency of the ions in the magnetic field be the same as the frequency of the electric field. Therefore, the condition for resonance is that

$$v = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

be the required frequency of the oscillating field. This is called cyclotron frequency.

Example-11: **In a cyclotron the frequency of the applied voltage is $1.2 \times 10^7\ Hz$. Find the value of the B–field for resonance with deuterons of mass $1.67 \times 10^{-27}\ kg$. If the diameter of the cyclotron is 1 m, what is the velocity of the particle, when it comes out of the cyclotron?**

Solution: The force experience by a deuteron = Bev = centripetal force = mv^2 / r

;

$$\therefore \qquad Be = \frac{mv}{r} = \frac{m\omega r}{r} = m\omega$$

$$\therefore \qquad B = \frac{m\omega}{e} = \frac{2\pi v m}{e} = \frac{2\pi(1.2\times10^{7})\times(1.67\times10^{-27})}{1.6\times10^{-19}}$$

or B = 1.57 Tesla

When the deuteron comes out, the radius of the orbit is equal to the radius of the Dees.

$$\therefore \qquad Be = \frac{mv_{\max}}{R} \quad ; \quad v_{\max} = \frac{BeR}{m} = \frac{1.57\times1.6\times10^{-19}\times0.5}{1.67\times10^{-27}} = 7.5\times10^{7}\ ms^{-1}.$$

Example-12: **An electron enters into a uniform magnetic field of 8.0 Gauss (1 gauss (G) =** $10^{-4}T$ **) with velocity** $4.0\times10^{6}\ m/s$ **at an angle 30° with the field. Calculate the radius, period of revolution and pitch of the helical path.**

Solution: We know $T = \frac{2\pi m}{Bq}$

$$\therefore \quad \text{Here } T = \frac{2\pi\times9.1\times10^{-31}}{8\times10^{-4}\times1.6\times10^{-19}} = 4.47\times10^{-8}\ s$$

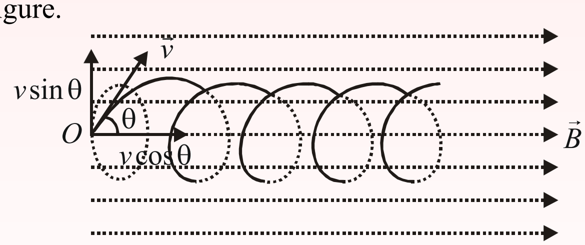
Now, $v\sin30^\circ = \omega R$ or $R = \frac{Tv\sin30^\circ}{2\pi}$

$$R = \frac{4.47\times10^{-8}\times4.0\times10^{6}\times0.5}{2\pi} = 1.42\times10^{-2}\ m$$

$\text{Pitch} = v\cos30^\circ\times T = 4.0\times10^{6}\times0.866\times4.47\times10^{-8} = 15.48\times10^{-2}\ m$.

❖ MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Suppose a particle of mass m and charge q, entering a uniform magnetic field induction $\vec{B}$ at O, with velocity $\vec{v}$, making an angle θ with the direction of magnetic field acting in the plane of paper as shown in figure.



Resolving $\vec{v}$ into two rectangular components we have:

$v\cos\theta(=v_1)$ acts in the direction of the magnetic field and $v\sin\theta(=v_2)$ acts perpendicular to the direction of magnetic field.

For component velocity $\vec{v}_2$, the force acting on the charged particle due to magnetic field is

$$\vec{F} = q(\vec{v}_2 \times \vec{B})$$

or $$F = q|\vec{v}_2 \times \vec{B}| = qv_2 B\sin 90°$$

$$= q(v\sin\theta)B \qquad \ldots(i)$$

The direction of this force $\vec{F}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{v}_2$ hence acts perpendicular to the plane of the paper. As this force is to remain always perpendicular to $\vec{v}_2$, it cannot change the magnitude of velocity $\vec{v}_2$. It changes only the direction of motion. Hence the charged particle is made to move on a circular path in the magnetic field, as shown in figure with dotted circle. The force F on the charged particle due to magnetic field provides the required centripetal force $(= mv_2^2/r)$ necessary for motion along the circular path of radius r.

$\therefore$ $$Bqv_2 = mv_2^2/r$$

or $$v_2 = Bqr/m$$

or $$v\sin\theta = Bqr/m \qquad \ldots(ii)$$

The angular velocity of rotation of the particle in magnetic field will be

$$\omega = \frac{v\sin\theta}{r} = \frac{Bqr}{mr} = \frac{Bq}{m}$$

The frequency of rotation of the particle in magnetic field will be

$$v = \frac{\omega}{2\pi} = \frac{Bq}{2\pi m} \qquad \ldots(iii)$$

The time period of revolution of the particle in the magnetic field will be

$$T = \frac{1}{v} = \frac{2\pi m}{Bq} \qquad \ldots(iv)$$

From (iii) and (iv) we note that v and T do not depend upon velocity $\vec{v}$ of the particle. It means, all the charged particles having the same specific charge (charge/mass) but moving with different velocities at a point, will complete their circular paths due to component velocities perpendicular to the magnetic fields in the same time.

For component velocity v_1 $(v\cos\theta)$, there will be no force on the charged particle in the magnetic field, because, the angle between $\vec{v}_1$ and $\vec{B}$ is zero. Thus, the charged particle covers the linear distance in direction of the magnetic field with a constant speed $v\cos\theta$.

Therefore, under the combined effect of the two component velocities, the charged particle in magnetic field will cover linear path of the charged particle will be **helica**l, whose axis is parallel to the direction of magnetic field.

Resolving $\vec{v}$ into two rectangular components we have:

$v\cos\theta(=v_1)$ acts in the direction of the magnetic field and $v\sin\theta(=v_2)$ acts perpendicular to the direction of magnetic field.

For component velocity $\vec{v}_2$, the force acting on the charged particle due to magnetic field is

$$\vec{F} = q(\vec{v}_2 \times \vec{B})$$

or $$F = q\,|\vec{v}_2 \times \vec{B}| = qv_2\,B\sin 90^\circ$$

$$= q(v\sin\theta)\,B \qquad \ldots(i)$$

The direction of this force $\vec{F}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{v}_2$ hence acts perpendicular to the plane of the paper. As this force is to remain always perpendicular to $\vec{v}_2$, it cannot change the magnitude of velocity $\vec{v}_2$. It changes only the direction of motion. Hence the charged particle is made to move on a circular path in the magnetic field, as shown in figure with dotted circle. The force F on the charged particle due to magnetic field provides the required centripetal force $(= mv_2^2/r)$ necessary for motion along the circular path of radius r.

$\therefore$ $$B\,q\,v_2 = mv_2^2/r$$

or $$v_2 = B\,qr/m$$

or $$v\sin\theta = Bqr/m \qquad \ldots(ii)$$

The angular velocity of rotation of the particle in magnetic field will be

$$\omega = \frac{v\sin\theta}{r} = \frac{B\,qr}{mr} = \frac{B\,q}{m}$$

The frequency of rotation of the particle in magnetic field will be

$$\nu = \frac{\omega}{2\pi} = \frac{B\,q}{2\pi\,m} \qquad \ldots(iii)$$

The time period of revolution of the particle in the magnetic field will be

$$T = \frac{1}{\nu} = \frac{2\pi\,m}{B\,q} \qquad \ldots(iv)$$

From (iii) and (iv) we note that ν and T do not depend upon velocity $\vec{v}$ of the particle. It means, all the charged particles having the same specific charge (charge/mass) but moving with different velocities at a point, will complete their circular paths due to component velocities perpendicular to the magnetic fields in the same time.

For component velocity v_1 $(v\cos\theta)$, there will be no force on the charged particle in the magnetic field, because, the angle between $\vec{v}_1$ and $\vec{B}$ is zero. Thus, the charged particle covers the linear distance in direction of the magnetic field with a constant speed $v\cos\theta$.

Therefore, under the combined effect of the two component velocities, the charged particle in magnetic field will cover linear path of the charged particle will be **helical**, whose axis is parallel to the direction of magnetic field.

The linear distance covered by the charge particle in the magnetic field in time equal to one revolution of its circular path (known as pitch of helix) will be $d = v_1 T = v\cos\theta\,\frac{2\pi m}{Bq}$.

Note:

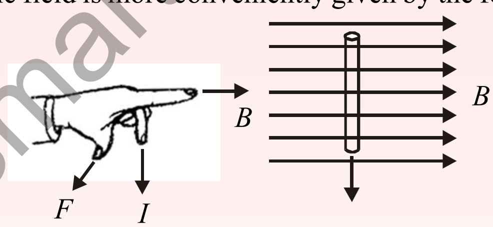
(a) If a charged particle having charge q is at rest on a magnetic field $\vec{B}$, it experiences no force, as $v = 0$ and $F = qvB\sin\theta$

(b) If charged particle is moving parallel to the direction of $\vec{B}$, it also does not experience any force because angle θ between $\vec{v}$ and $\vec{B}$ is 0° or 180° and $0°\sin 180° = 0$. Therefore, the charged particle in this situation will continue moving along the same path with the same velocity.

(c) If charged particle is moving perpendicular to the direction of $\vec{B}$, it experiences a maximum force which acts perpendicular to the direction of $\vec{B}$ as well as $\vec{v}$. Hence this force will provide the required centripetal force and the charged particle will describe a circular path in the magnetic field to radius r, given by $\frac{mv^2}{r} = Bqv$

When a charged a particle is projected perpendicular to the magnetic field, (i) its path is circular in a plane perpendicular to the plane of magnetic field and direction of motion of the charged particle. (ii) the speed and kinetic energy of the particle remain constant. (iii) The velocity of the charged particle changes only in direction. The force acting on the charged particle is independent of the radius of circular path but depends upon the speed of the charged particle i.e., $F \propto r°$ (but $F \propto v$). (v) The time period of revolution of a charged particle in the magnetic field is independent of velocity of particle and radius of the circular path i.e., $T \propto r^o v^o$.

Fleming's left-hand rule: The direction of the force experienced by a linear conductor placed at right angles to a magnetic field is more conveniently given by the following rule by Prof. Fleming.



If the forefinger, middle finger and thumb of left hand are extended so as to make them mutually perpendicular then on pointing the forefinger and the middle finger in the directions of the field and the current respectively, the thumb indicates the direction of the force experienced by the conductor.

Example-13: **A proton is moving with a velocity of 10^5 ms^{-1} perpendicular to a magnetic field of strength 1.5 Tesla. What is the magnetic force on the proton? Compare it with the gravitational force. (Charge carried by a proton $= 1.6\times10^{-19}$C mass of a proton $= 1.67\times10^{-27}$ kg).**

Solution: We have $\Delta F = q_o vB\sin(v\hat{,}B)$

Since the angle between v and B is 90°, $\sin(v\hat{,}B)$

$\therefore\ \Delta F = 1.6\times10^{-19}\times10^{5}\times1.5 = 2.40\times10^{-14}$ N

The gravitational pull $= 1.67\times10^{-27}\times9.8 = 1.6\times10^{-26}$ N

Thus, the magnetic force is 10^{12} times greater than the gravitational force.

Example-14: **A beam of ions with velocity $2\times10^{5}\ ms^{-1}$ enters normally into a uniform magnetic field of 0.04 tesla. If the specific charge of ion is $5\times10^{7}\,C\,kg^{-1}$, find the radius of the circular path described.**

Solution: Here, $v = 2\times10^{5}\ ms^{-1}$

$B = 0.04T;\ q/m = 5\times10^{7}\ C/kg$

As $qvB = mv^{2}/r$

or $r = mv/qB = \dfrac{b}{(q/m)B} = \dfrac{2\times10^{5}}{(5\times10^{7})\times0.04} = 0.01\,m$

Example-15: **An electron of mass $0.90\times10^{-30}\ kg$ under the action of magnetic field moves in a circle of radius 2.0 cm at a speed of $3.0\times10^{6}\ ms^{-1}$. If a proton of mass $1.8\times10^{-27}\,kg$ were to move in a circle of the same radius in the same magnetic field , find the speed.**

Solution: Here, $m_e = 9\times10^{-31}\,kg$

$r_e = 0.02\,m;\ v_e = 3\times10^{6}\ ms^{-1}$

$m_p = 1.8\times10^{-27}\ kg;\ v_p = ?;\ r_p = 0.02\,m$

We know that $qvB = mv^{2}$

or $v = qrB/m$

so, $\dfrac{v_p}{v_e} = \dfrac{q_p\,r_p\,B/m_p}{q_e\,r_e\,B/m_e} = \dfrac{m_e}{m_p}$ [as $q_p = q_e;\ \ r_p = r_e$]

$\therefore\ v_p = v_e\times\dfrac{m_e}{m_p} = \dfrac{3\times10^{6}\times9\times10^{-31}}{1.8\times10^{-27}} = 1.5\times10^{3}\ \text{ms}^{-1}$

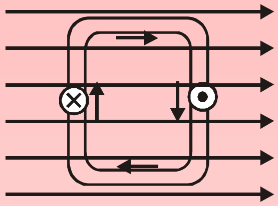
Torque

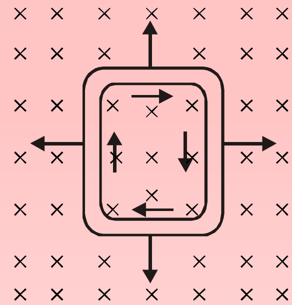
❖ TORQUE ON A CURRENT CARRYING LOOP IN A MAGNETIC FIELD

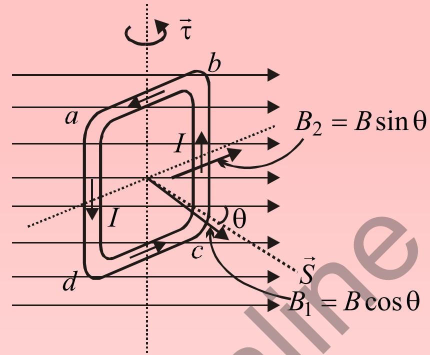
(i) Consider a rectangular loop of wire with sides l and b the loop is in the plane of the paper and the magnetic field is parallel to the plane of the paper. Sides ad and bc of the loop are perpendicular to the field in all the figures. In figure the magnetic field is perpendicular into the paper and in figure the magnetic field is neither perpendicular nor parallel to the plane of the loop, but it is inclined to it. In figure equal and opposite magnetic forces of magnitude, $F = BIl$ are exerted on ad and bc perpendicular into the paper and out of the paper respectively. Since ab and cd are parallel to the magnetic field $(\alpha = 0)$ no magnetic force acts on them. Thus, the net translatory force on the loop is zero. The torque, however, is not zero, because the forces on the sides ad and bc constitute a couple of moment (torque = force × perpendicular distance).

$\tau = BIl \times b = IBS$

$\because \quad S = l \times b$ area of the loop.







In second figure the forces on ad and bc are equal and opposite in the plane on the paper, the forces on ab and cd are also equal and opposite. Hence the loop experiences torque and no translatory force in this position. Therefore, this is the equilibrium position of a current loop in a magnetic field.

In the last figure the magnetic field is inclined to the normal to the loop by θ. A magnetic field is a vector. It may be resolved into two resolved components, along and perpendicular to the plane of the loop. The resolved components are $B_2 = B\sin\theta$ and $B_1 = B\cos\theta$ respectively. We have just seen that a magnetic field perpendicular to the loop exerts no torque, no force and hence due to the normal component, the loop experiences neither torque nor a translatory force. But the other component, being parallel to the plane of the loop exerts a torque given by through its also exerts no force.

Thus τ (torque on the loop in the inclined position) $= IB_2S$

or $\quad \tau = IB.S\sin\theta$

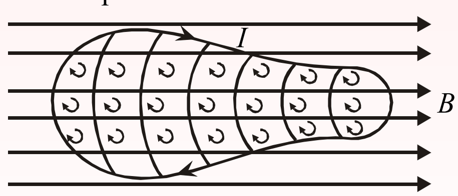
If the loop is a closely wound one, having N turns then evidently $\quad \tau = NIB\sin\theta$

An area S can be represented be vector $\vec{S}$ of magnitude S and along the motion of a screw held along the normal to S and rotating in the direction of the current round the loop. We may then write

$$\vec{\tau} = I\vec{S} \times \vec{B} \text{ or } \qquad \vec{\tau} = IS\,\vec{n} \times \vec{B}$$

where $\vec{\tau}$ is the vector torque on the loop and $\hat{n}$ is unit vector along the normal to S.

(ii) When the loop is of any shape carrying a current in the clockwise direction. Divide the loop into a large number of smaller loops.



Imagine current I to circulate in each loop in the same direction. Along the sides of adjacent loops, the currents cancel out, since each side in traversed twice in the opposite direction, and only the current along the external boundary itself remains. Hence the entire loop is equivalent to a summation of the smaller loops.

τ (torque on the loop)

$= \tau_1 + \tau_2 + \tau_3 + ...$ where τ_1, τ_2, τ_3, ... are the torques on the elementary loops

$= IBS_1 + IBS_2 + IBS_3 + ...$

$= IB(S_1 + S_2 + S_3 + ...)$

$= IBS$ where $S = S_1 + S_2 + S_3 + ... =$ area of the loop.

The force is zero, because the force on each elementary loop is zero. Thus the shape of the loop has nothing to do with the torque on a current loop.

Magnetic Potential Energy

❖ MAGNETIC POTENTIAL ENERGY OF A CURRENT LOOP IN A MAGNETIC FIELD

In placing a current loop in a magnetic field, an external agent must have done some work, i.e., during the process of placing it in the magnetic field there is a flow of energy from the external agent to the current loop. This energy is called the **magnetic potential energy** of the loop. Thus, a current loop possesses potential energy in any position. Let it be U_o, when $\theta = 0$. The total potential energy in any position is the work done by the external agent from this position to the assigned position plus the potential energy at $\theta = 0$.

$$\therefore \quad U = U_o + \int_0^\theta \tau d\theta = U_o + \int_0^\theta MB\sin\theta d\theta = U_o + MB\,[-\cos\theta]_0^\theta$$

or $\quad U = U_o + MB(1-\cos\theta)$

If we take $\theta = \pi/2$ as the zero-energy position, then

$0 = U_0 + MB$ or $U_0 = -MB$

or $\quad U = -\vec{M}.\vec{B}$

Equation provides an alternative unit and definition of magnetic moment. According to this equation,

ΔU (change in potential energy) $= U - U_o = MB(1-\cos\theta)$.

This is the work done in turning the normal to the loop through θ in the magnetic field.

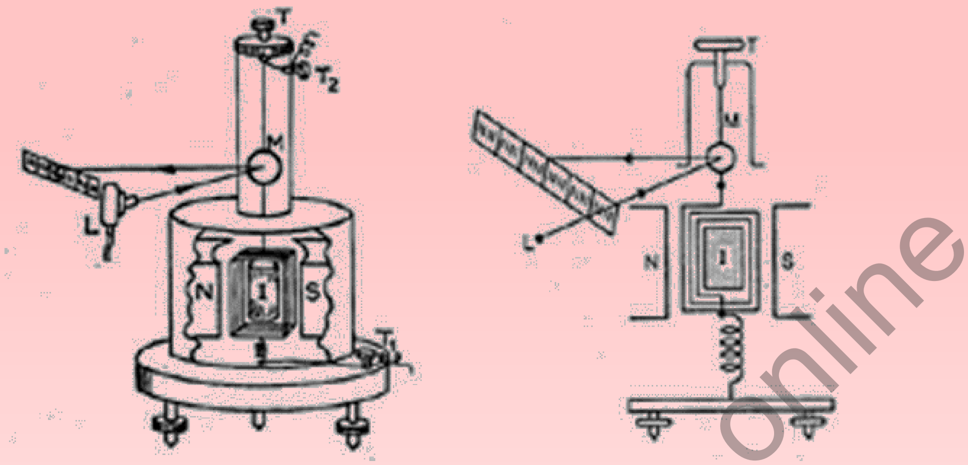
$\therefore$ Work done in turning through θ by an external angle $= MB(1-\cos\theta)$

Put $\theta = \pi/2$ and $B = 1$ Tesla

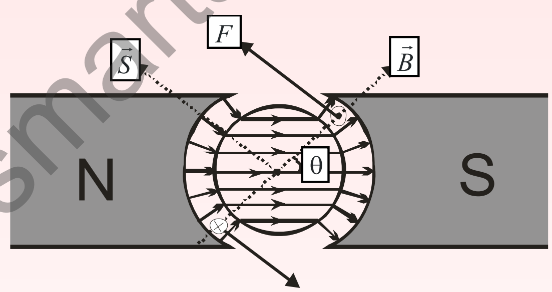
$\therefore$ Work done in turning though $\pi/2 = MB$.

Thus the magnetic moment may be defined as the energy required to turn the normal to the loop through $\pi/2$ in a unit uniform magnetic B-field of 1 tesla. An alternative unit of magnetic moment is obviously, joule per tesla (JT^{-1}), as can be seen easily.

Moving Coil Galvanometer



A suspended type moving-coil galvanometer consists of a rectangular (or circular) metallic frame. It is suspended between the semi-cylindrical poles of a permanent magnet by means of a fine phosphor bronze wire from a torsion head T. Co-axially with the cylindrical space between the poles, a soft iron cylinder is placed in such a way that a small air gap is left all around for the free turning of the coil. The cylindrical poles of the magnet give rise to a radial field and the soft iron cylinder intensifies this field by virtue of its high permeability. The advantage of the radial field the horizontal section is shown in figure is that the plane of the coil is always parallel to the magnetic field. One end of the coil is soldered to the phosphor bronze wire and the other end to a loosely wound spiral of the same material, namely, phosphor bronze. This is connected to a binding screw T_1.



Which forms one terminal of the galvanometer. The other terminal T_2 is connected to the torsion head. The whole instrument is enclosed in a flat cylindrical box supported on a heavy circular base provided with three leveling screws. The small angle of rotation of the coil is measured by a lamp and scale arrangement. For this, a small concave mirror M is attached to the suspension wire. Light from a source L is directed on to the mirror through a glass window and the reflected light is received on a translucent scale graduated in millimeter.

THEORY:

When a current is passed through the coil, it experiences a magnetic torque due to the magnetic field, which is given by $\tau = NISB \sin\varphi$, where N = number of turns, $I =$ current through the coil, $S =$ area of the coil, $B =$ magnetic field and $\varphi =$ angle between the normal to the coil and the magnetic field. Due to the use of cylindrical poles, the magnetic field between the poles is radial in nature. In such a field the plane of the coil is always parallel to the magnetic field. Hence, $\varphi = \frac{\pi}{2}$ always in such a field.

$\therefore \quad \tau = NISB$

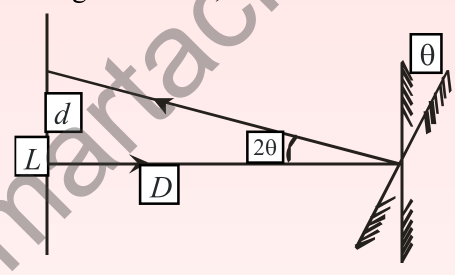
By the action of this torque, the coil is set in rotation about the axis of suspension producing a torsion in the suspension wire. A mechanical torque is then called into play in opposite direction, which increases with an increase of the angle of twist, and eventually when this torsional torque equals the magnetic torque, the coil comes to rest.

If $c =$ the torsional torque per unit twist, then for equilibrium $NISB = c\theta$, where θ is the angle of twist.

or $I = \frac{c}{NSB} \times \theta = K\theta$

where $K = \frac{c}{NSB}$, called the current reduction factor of the galvanometer.

When the coil rotates through θ, the attached mirror will also rotate through θ, but the reflected ray from it will rotate through 2θ. Let D be the distance of the scale from the mirror and d be the distance through which the image is shifted, then



$$\tan 2\theta = \frac{d}{D}$$

or $2\theta = \frac{d}{D}$ (since θ is small)

or $\theta = \frac{d}{2D}$ $\quad\quad \therefore \quad\quad I = \frac{c}{NSB}\frac{d}{2D}$ $\quad\quad\quad I \propto d$.

Current sensitivity: These galvanometers are extremely sensitive because, K is very small. The current sensitivity (S_i) is defined as the deflection in millimeters produced on a scale 1 metre away by a current of one *micro ampere*. The order of sensitivity of this type of galvanometer is $10^{-11}\, A/mm$.

Factors governing the sensitivity of a galvanometer: Since $I = K\theta = K\frac{d}{2D}$, for a given current, the greater the deflection, the smaller is the reduction factor K. Hence the sensitivity of a galvanometer is inversely proportional to its reduction factor. That is, the current sensitivity $S_i \,\alpha\, \frac{NSB}{c}$. Thus, the factors governing sensitivity are:

1. The sensitivity is proportional to the number of turns, the area of the coil and the intensity of the magnetic field.

2. The sensitivity is inversely proportional to the torsional torque per unit twist (c). The complete expression for torsional rigidity, i.e., torsional torque per unit twist is

$$c = \frac{n\pi r^4}{2l},$$

where n = modulus of rigidity of phosphor bronze, r = radius of the phosphor bronze wire and l = length of the phosphor bronze wire.

All these factors are controlled to contribute their maximum towards the sensitivity of the galvanometer. This is why these galvanometers are extremely sensitive.

Example-16: **A moving coil galvanometer consists of a coil of area $2.5\times10^{-4}\,m^2$ and 100 turns. It is suspended in a radial B–field of strength 0.1 tesla. Calculate the current required for a deflection of one degree. The torsional rigidity of the suspension wire is $1.5\times10^{-7}\,Nm/radian$.**

Solution: We have, $I = K\theta$

Where $K = \frac{c}{NSB}$

Here $\theta = 1^\circ = \frac{\pi}{180}$ *radian* $\quad I = \frac{c}{NSB}.\frac{\pi}{180} \quad = \frac{1.5\times10^{-7}}{100\times2.5\times10^{-4}\times0.1}\times\frac{\pi}{180} = 10^{-6}$ ampere

Example-17: **A milliammeter of resistance 10 ohm gives a full-scale deflection for 50 milliampere of current. How will you convert it into (a) voltmeter reading up to 100 Volt, (b) ammeter measuring up to 1 ampere?**

Solution: To convert the milliammeter into a voltmeter of 100 V a high resistance of such value should be connected in series with the coil of the ammeter that a current of 50 milliamperes is produced through the ammeter

By ohm's law

$$50\times10^{-3} = \frac{100}{R+10} \quad \text{or} \quad R = 1990\,\Omega$$

To convert the milliammeter into an ammeter if 1 ampere, a shunt of such resistance should be connected in parallel with the ammeter coil that a current of 50 milliampere is passed through the coil of the ammeter and the rest is passed through the shunt. $S(1-50\times10^{-3}) = 10\times50\times10^{-3}$

or $\quad S(1-0.05) = 0.5 \quad S = \frac{0.5}{0.95} = 0.526\,\Omega$

❖ COMPARISON OF ELECTRICAL AND MAGNETIC FORCES:

Form coulomb's law, the magnitude of electrostatic force between two charges q_1 and q_2 separated by distance r is given by

$$F_e = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}$$

Consider two parallel conducting wires of length dl_1 and dl_2 carrying currents I_1 and I_2 separated by distance r.

The magnitude of the magnetic force acting between these two current elements is given by

$$F_m = \frac{\mu_o}{4\pi}\frac{I_1 I_2}{r^2} dl_1\, dl_2$$

Let q_1 and q_2 be the charges flowing for time t in wires for currents I_1 and I_2. Then

$$I_1 dl_1 = \frac{q_1}{t}.dl_1 = q_1 v_1 \quad \text{and} \quad I_2 dl_2 = \frac{q_2}{t} \quad dl_2 = q_2 v_2$$

$$\therefore F_m = \frac{\mu_o}{4\pi}\frac{q_1 q_2}{r^2} v_1 v_2$$

Dividing by $\frac{F_m}{F_e} = v_1 v_2 (\mu_o \epsilon_o)$.

Since left hand side in dimensionless, therefore, right hand side of must also be dimensionless. It means the quantity $\mu_o \epsilon_o$ must have the dimensions of $(\text{velocity})^{-2}$. Here v_1 and v_2 are the drift velocities of electrons in current elements. As drift velocities are of the order of $10^{-5}\, ms^{-1}$, therefore

$$v_1 v_2 = 10^{-5} \times 10^{-5} = 10^{-10}\, m^2 s^{-2}$$

It also found that

$$c^2 = \frac{1}{\mu_o \epsilon_o} \qquad \text{or} \qquad \mu_o \epsilon_o = c^{-2}$$

where c is the velocity of light i. e. $c = 3\times 10^8\ ms^{-1}$

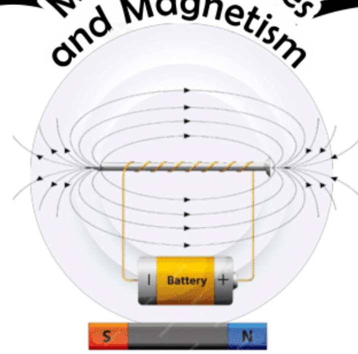
Putting values in $\frac{F_m}{F_e} = \frac{10^{-10}}{(3\times 10^8)^2} \cong 10^{-27}$

It shows that the magnetic forces are very much smaller than the electrostatic forces in current carrying conductors. However, the electric forces never dominate the magnetic forces in current carrying conductors because

(i) the matter is electrically neutral to very high degree of accuracy. The flowing charges do not disturb the neutrality of the matter. Thus, the coulomb's electric forces are very rarely in evidence.

(ii) in an electric current, the large number of electrons are drifting towards the same direction. This adds up weak magnetic fields, which become evident.

Moving Charges and Magnetism



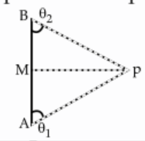
Magnetic Field ($\vec{B}$)

- It is a region around a magnet or current carrying conductor or a moving charge in which its magnetic effect can be felt
- SI unit is Tesla(T) = weber/m^2
- 1 Gauss = 10^{-4} Tesla where gauss is the CGS unit

Magnetic field due to straight current carrying wire

$B = \frac{\mu_0 i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

where θ_1 and θ_2 are the angles corresponding to the lower and upper ends respectively



i.e $\theta_1 = 0$

$\theta_2 = \pi$

$B = \frac{\mu_0 i}{2\pi d}$

Oersted's Law

In April 1820, Hans Christian Oersted discovered that flow of current in a wire can deflect a nearby magnetic compass needle.

Solenoid

- Magnetic field at a point inside due to a long solenoid, $B = \mu_0 ni$
- At a point on one end $B = \frac{\mu_0 ni}{2}$

  where n = number of turns per unit length along the length of solenoid.

Magnetic Force on a moving charge

$\vec{F} = q(\vec{v} \times \vec{B})$

$= qvB \sin\theta$

- For $\theta = 0, \vec{F} = 0$ along the magnetic field
- For $\theta = 90°$, *i.e.* if charge's velocity is perpendicular to field direction, force is perpendicular to both field and velocity

$F = qvB = \frac{mv^2}{r}$

$r = \frac{mv}{qB}$ = Radius of the circle in which charge rotates

Time period $(T) = \frac{2\pi m}{qB}$

Frequency $(\nu) = \frac{1}{T} = \frac{qB}{2\pi m}$

If $\theta \neq 0, 180°, 90°$

Then, $F = qvB \sin\theta$

Force between parallel current carrying wires

$F = \frac{\mu_0 i_1 i_2}{2\pi d}$

Torque experienced by a loop in uniform magnetic field

$\vec{\tau} = \vec{M} \times \vec{B}$

$\tau = NBiA$

Definition of Ampere

If two parallel wires carrying same current are kept 1 m apart, if experience a force $F = 2\times10^{-7}$ N, then current = 1A in each wire.

Sensitivity

Voltage sensitivity = NBA/CG

current sensitivity = NBA/C

Galvanometer

- Ammeter: $S = \frac{I_g}{I - I_g} G$
- Voltmeter: $R = \frac{V}{I_g} - G$

Field due to a current carrying circular ring

- Field at a point far away from the centre: i.e., for $d >> a$, $B = \frac{\mu_0 i a^2}{2d^3}$
- Field at an axial point: $B = \frac{\mu_0 i a^2}{2(a^2 + d^2)^{3/2}}$
- Field at the centre: $B = \frac{\mu_0 i}{2a}$ ⊗ ⊙

Force on a current carrying conductor

$d\vec{F} = i(d\vec{l} \times \vec{B}) F = ilB$

Biot-Savart's Law

⊙ ⊗

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^3}$ [θ = angle between $d\vec{l}$ and $\vec{r}$]

$dB = \frac{\mu_0}{4\pi} \frac{i dl \sin\theta}{r^2}$ [Direction of field will be perpendicular to the plane containing current element and the point of observation]

Ampere's Law

$\oint \vec{B}.d\vec{l} = \mu_0 i$

where i = Total current crossing the area bounded by closed curve.

Trace the Mind Map

- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS

1. To determine the minimum magnitude of the magnetic field, we consider a scenario where a particle with a charge of 2×10^{-12} C moves along the x-axis with a velocity of 10^5 m/s. The particle experiences a force of 10^{-9} N in the y-direction due to the magnetic field. Calculate the minimum value of the magnetic field.

a) 5×10^{-3} T in z − direction
b) 2×10^{-3} T in z − direction
c) 4×10^{-3} T in z − direction
d) 9×10^{-3} T in z − direction

2. When a proton with an energy of 1 MeV moves in a plane perpendicular to a uniform magnetic field of 6.28×10^(-4) T, and considering its mass as 1.7×10^(-27) kg, the cyclotron frequency of the proton is approximately equal to.

a) 10^7 Hz   b) 10^5Hz   c) 10^6 Hz   d) 10^4 Hz

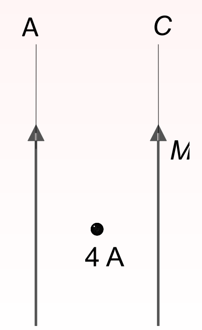
3. Given that a magnetic pole with a strength of 9×10^{-2} A-m experiences a force of 2N, determine the magnetic field at the point where the pole is located.

a) 20 Wbm^{-2} b) 50 Wbm^{-2} c) 112.5 T   d) 22.2 T

4. Consider a charged particle with mass 'm' and charge 'q' entering a region of uniform magnetic field 'B' perpendicular to its velocity 'v'. Prior to entering the magnetic field region, the particle was accelerated by a potential difference 'V' (in volts). Calculate the diameter of the circular path traced by the charged particle within the magnetic field region.

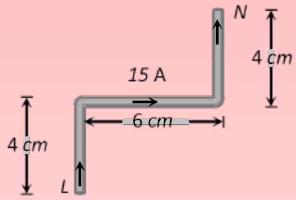
a) $\frac{2}{B}\sqrt{\frac{mV}{q}}$ b) $\frac{2}{B}\sqrt{\frac{2mV}{q}}$ c) $B\sqrt{\frac{2mV}{q}}$ d) $\frac{B}{q}\sqrt{\frac{2mV}{B}}$

5. AB and CD are long parallel conductors separated by certain distance. X is the mid-point between them (see the figure). The net magnetic field at X is B. Now, the current 4 A is switched off. The field at X now becomes



a) 2B b) B c) $\frac{B}{2}$ d) 3B

6. When a wire AB carrying a current of 15 A is bent into the shape depicted below and placed in a magnetic field of 3 T, which is perpendicular to the plane of the paper and directed outward, the wire will undergo a force. Determine the value of force.

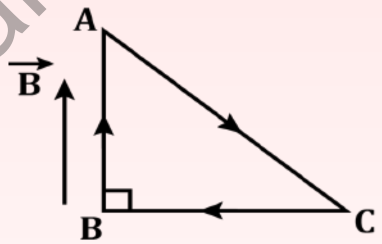


a) Zero b) 5 N c) 3 N d) 4.5 N

7. When a beam of electrons passes through mutually perpendicular electric and magnetic fields, it remains undeflected. However, if the electric field is switched off while maintaining the same magnetic field, the electrons will continue to move in

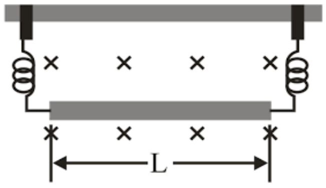
a) In an elliptical orbit b) In a circular orbit
c) Along a parabolic path d) Along a straight line

8. Consider a closed loop in the shape of a right-angled isosceles triangle ABC, carrying a current. The loop is placed in a uniform magnetic field that is aligned with the side AB. If the magnetic force experienced by the side BC is denoted as F, determine the force exerted on the side AC of the triangle.



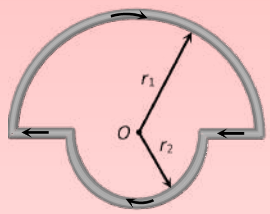
a) $-\sqrt{2}$F b) -F c) F d) $\sqrt{2}$F

9. A 44.0g wire of length L=98.0cm is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (see figure). What is the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads?



b) *10A, right* *b) 20A left* *c) 10 A left* *d) none of these*

10. Consider the figure shown, which consists of two semicircles with radii r_1 and r_2, through which a current i is flowing. Find the magnetic induction at the centre point O.



a) $\frac{\mu_0 i}{r}(r_1 + r_2)$ b) $\frac{\mu_0 i}{4}(r_1 - r_2)$ c) $\frac{\mu_0 i}{4}\left(\frac{r_1 + r_2}{r_1 r_2}\right)$ d) $\frac{\mu_0 i}{4}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$

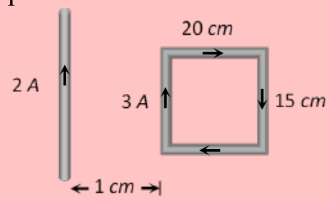
11. Let's consider a wire loop PQR in the shape of a right-angled triangle, where PQ = 2x, PR = 5x, and QR = 7x. A steady current I flows through this wire loop. The magnitude of the magnetic field at point P, caused by this wire loop, is given as: K (μ_o I)/(24πx). Determine the value of k.

a) 8 b) 3 c) 7 d) None of these

12. Consider two wires, A and B, with lengths 50 cm and 30 cm, respectively. Wire A is bent into a circle of radius r, while wire B is bent into an arc of radius r. A current passes through wire A, and a current i_2 passes through wire B. To achieve the same magnetic inductions at the centre, determine the ratio of i_1 to i_2.

a) 3: 4 b) 3: 5 c) 2: 3 d) 4: 3

13. The net force on the square coil is:

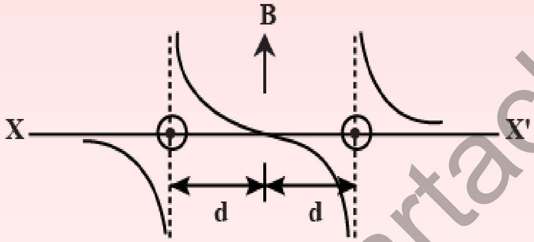


a) 25×10^{-7}N moving towards wire
b) 25×10^{-7}N moving away from wire
c) 32×10^{-7}N moving towards wire
d) 32×10^{-7}N moving away from wire

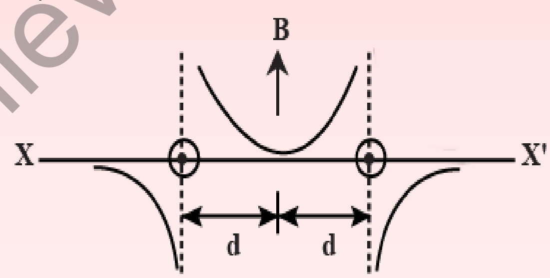
14. Let's consider a situation where two long parallel wires are placed 2d units apart. These wires carry equal and steady currents flowing out of the plane of the paper, as shown. The magnetic field along the line XX' varies according to the given expression or function.

1.

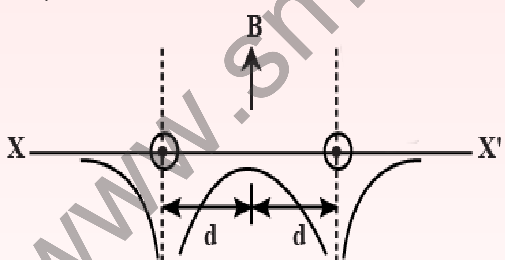
a)



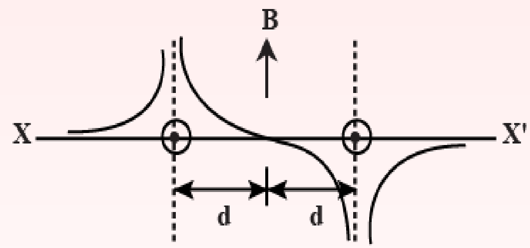
c)



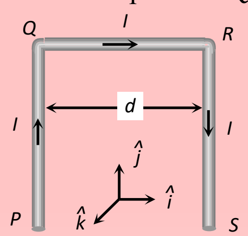
b)



d)



15. PQ and RS are long straight conductors, distance d apart, carrying a current I. The magnetic field at the midpoint of QR is



a) $\frac{-\mu_0 I}{2\pi d}\hat{k}$ b) $\frac{-\mu_0 I}{\pi d}\hat{k}$ c) $\frac{-\mu_0 I}{4\pi d}\hat{k}$ d) $\frac{-\mu_0 I}{8\pi d}\hat{k}$

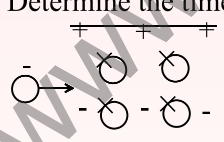
16. At a specific location, the magnetic induction of the Earth is measured to be 6×10^{-6} Wb/m^2. To counteract this magnetic induction, we need to generate an equal and opposite magnetic induction at the centre of a circular conducting loop with a radius of 5 cm. Find the current value required in the loop.

a) 0.56 A b) 5.6 A c) 0.28 A d) 2.8 A

17. A horizontal rod, weighing 15 gm and measuring 20 cm in length, is placed on a smooth inclined plane tilted at a 60° angle with respect to the horizontal. The rod is oriented parallel to the edge of the inclined plane. To maintain the rod in a stationary position on the inclined plane, a uniform magnetic field of induction B is applied vertically downward. The current passing through the rod is set to 1.96 amperes. Find the specific value of B that allows the rod to remain motionless on the inclined plane.

a) 1.96 tesla b) $\frac{1}{1.96}$ tesla c) 0.66 tesla d) None of the above

18. Consider where an electron enters the region between the plates of a charged capacitor, as depicted. The plates have a charge density of σ, generating an electric field intensity E in the space between them. Additionally, a uniform magnetic field B exists in the same region, perpendicular to both the direction of the electric field E and the electron's motion. The electron moves perpendicular to both E and B, maintaining its path without any deviation. Determine the time required for the electron to cover a distance of l in this region.



a) $\frac{\sigma l}{\varepsilon_0 B}$ b) $\frac{\sigma B}{\varepsilon_0 l}$ c) $\frac{\varepsilon_0 l B}{\sigma}$ d) $\frac{\varepsilon_0 l}{\sigma B}$

19. If a straight wire carrying a current is transformed into a circular loop, and the magnitude of the magnetic moment associated with the loop is represented by M in MKS units, determine the length of the wire used in creating the loop.

a) $\frac{4\pi}{M}$ b) $\sqrt{\frac{4\pi M}{i}}$ c) $\sqrt{\frac{r\pi i}{M}}$ d) $\frac{M\pi}{4i}$

20. When two parallel wires in free space are situated 15 cm apart and each wire carries a current of 5 A in the same direction, calculate the force per meter of length that one wire exerts on the other.

a) 2×10^{-4}N, attractive
b) 3.33×10^{-4}N, repulsive
b) 3.33×10^{-5}N, attractive
d) 2×10^{-7}N, repulsive

21. Consider two long solenoids. The first solenoid consists of 100 turns per cm and carries a current of magnitude resulting in a magnetic field of 8.3×10^{-2} Wb/m^2 at its centre. The second solenoid is comprised of 200 turns per cm and carries a current of magnitude i/3. Find the value of the magnetic field at the centre of the second solenoid.

a) $5.5 \times 10^{-2}\ \mathrm{Wbm^{-2}}$
b) $5.5 \times 10^{-5}\ \mathrm{Wbm^{-2}}$
c) $5.5 \times 10^{-3}\ \mathrm{Wbm^{-2}}$
d) $5.5 \times 10^{-4}\ \mathrm{Wbm^{-2}}$

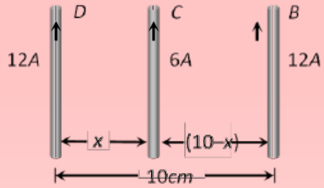
22. In the given situation, there exists a homogeneous electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$, both pointing in the same direction. A proton is projected with a velocity parallel to $\vec{E}$. Determine the behaviour or the path followed by the proton under these conditions.

a) Go on moving in the same direction with increasing velocity
b) Go on moving in the same direction with constant velocity
c) Turn to its right
d) Turn to its left

23. A large magnet is broken into two pieces so that their lengths are in the 4:1. The pole strengths of the two pieces will have ratio

a) 2: 1 b) 1: 2 c) 4: 1 d) 1: 1

24. Consider a configuration where three long, straight, and parallel wires carrying currents are arranged as shown in the figure. Wire C, carrying a current of 6.0 A, is positioned in such a way that it experiences no force. Find the distance between wire C and wire D in this arrangement.



a) 9 cm b) 7 cm c) 5 cm d) 3 cm

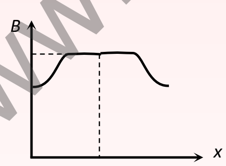
25. A wire along x-axis carries a current 5 A. Find the force in newton on a 2 cm section of the wire exerted by a magnetic field $\vec{B} = (0.85\,\hat{j} + 0.24\,\hat{k})$ T.

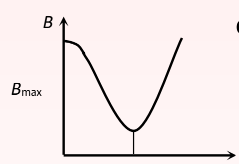
a) $(1.26\,\hat{k} - 2.59\,\hat{j})10^{-2}$ N
b) $(-1.26\,\hat{k} + 2.59\,\hat{j}) \times 10^{-2}$ N
c) $(-2.97\,\hat{k} + 0.84\,\hat{j}) \times 10^{-2}$ N
d) $2.97\,(\hat{k} - 0.84\,\hat{j}) \times 10^{-2}$ N

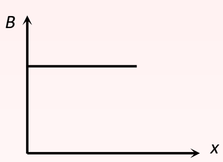
26. A solenoid of 2.5 m length and 5 cm diameter has 20 turns per cm. A current of 8 A is flowing through it. The magnetic field induction at axis inside the solenoid is

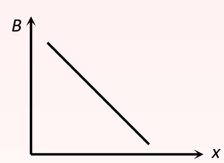
a) $2\pi \times 10^{-4}$ T b) $2\pi \times 10^{-5}$ T c) $12.8\,\pi$ G d) 1.28π G

27. The graph depicting the relationship between the magnetic induction (B) along the axis of a long solenoid, caused by the current flow (i) within it, and the distance (x) from one end is
1.

a) 

b) 

c) 

d) 

28. To extend the range of a voltmeter, which has a resistance of $35\times10^3\ \Omega$, to three times its original value, an additional resistance needs to be connected in series. Determine the value of the additional resistance required

a) $9\times10^6\ \Omega$ b) $0.55\times10^5\ \Omega$ c) $1.5\times10^5\ \Omega$ d) $9\times10^5\ \Omega$

29. When a vertical circular coil carrying current is situated in a way that a neutral point is achieved at its centre, find the angle between the plane of the coil and the magnetic meridian.

a) 0 b) 45° c) 60° d) 90°

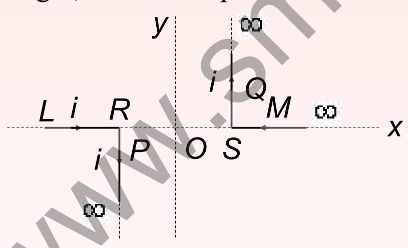
30. When comparing three magnets with cross-sectional areas A, 3A, and 5A, all having the same length, determine the ratio of their magnetic moments.

a) 5: 3: 1 b) 1: 3: 5 c) 3: 6: 1 d) 1: 1: 1

31. In a mass spectrometer designed to measure the masses of ions, the ions undergo initial acceleration through an electric potential V. Subsequently, they are directed to move along semicircular paths of radius R by a constant magnetic field B. Keeping V and B constant, the ratio of the charge on the ion to its mass remains proportional.

a) 1/R b) $1/R^2$ c) R^2 d) R

32. Consider a pair of stationary, infinite long bent wires positioned in the x-y plane. These wires carry currents of 12 A each, as shown in the figure. The wire segments L and M run parallel to the x-axis, while the segments P and Q are parallel to the y-axis. Additionally, the points OS and OR are both located at a distance of 0.03 m. Determine the magnetic field induction at the origin, denoted as point O.



a) 10^{-3} T b) 4×10^{-3} T c) 1.6×10^{-4} T d) 10^{-4} T

33. When a 10 Ω shunt is applied, the deflection in a galvanometer decreases from 50 divisions to 20 divisions. Determine the resistance of the galvanometer.

a) 15 Ω b) 36 Ω c) 24 Ω d) 30 Ω

34. Consider a long straight wire positioned along the z-axis, carrying a current I in the negative z direction. The magnetic vector field $\vec{B}$ at a point located in the z=0 plane, with coordinates (x, y) is

a) $\frac{\mu_o I(y\hat{i}-x\hat{j})}{2\pi(x^2+y^2)}$ b) $\frac{\mu_o I(x\hat{i}+y\hat{j})}{2\pi(x^2+y^2)}$ c) $\frac{\mu_o I(x\hat{i}-y\hat{j})}{2\pi(x^2+y^2)}$ d) $\frac{\mu_o I(x\hat{i}-y\hat{j})}{2\pi(x^2+y^2)}$

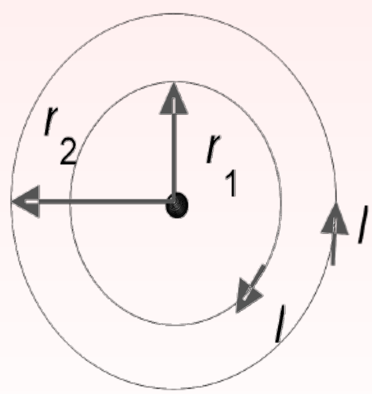
35. A cable, carrying a direct current, is buried in a wall that lies in a north-south plane. On the west side of the wall, a horizontal compass needle deviates from its usual northward orientation and points towards the south instead. This occurs because the coil is positioned in a manner that

a) Vertically upwards and the current is also flowing upwards
b) Vertically upwards and the current is flowing down downwards
c) Horizontal with current from south to north
d) Horizontal with current from north to south

36. In Nebraska, where the horizontal component of Earth's magnetic field is 0.5 G, a vertical wire carries a current of 10 A in an upward direction. Find the magnitude and direction of the force exerted on 1 meter of the wire.
(1 G = 10^{-4} T)

a) 5 E to W b) 5×10^{-3} E to W c) 5×10^{-3} E to W d) 5×10^{-4} E to W

37. Two circular concentric loops of radii $r_1 = 10$ cm and $r_2 = 20$ cm are placed in the $X - Y$ plane as shown in the figure. A current $I = 10$ A is flowing through them. The magnetic moment of this loop system is



a) $+ 0.4\,\hat{\mathbf{k}}\,(\mathrm{Am^2})$ b) $-1.5\,\hat{\mathbf{k}}\,(\mathrm{Am^2})$ c) $+2.51\,\hat{\mathbf{k}}\,(\mathrm{Am^2})$ d) $+ 1.3\,\hat{\mathbf{j}}\,(\mathrm{Am^2})$

38. A linear conductor with a length of 50 cm carries a current of 4A. The conductor is positioned within a magnetic field of strength 400 G, making an angle of 30° with the field's direction. Under these conditions, the conductor experiences a force of a certain magnitude. Determine this magnitude of force.

a) 4×10^{-4} N b) 4×10^{-2} N c) 3×10^{2} N d) 3×10^{4} N

39. To convert an ammeter with a resistance of G Ω and a range of ampere into an ammeter with a different range in ampere, a parallel resistance needs to be introduced. Determine the value of this parallel resistance.

a) $\mathrm{n\,G}$ b) $(\mathrm{n}-1)\mathrm{G}$ c) $\frac{\mathrm{G}}{\mathrm{n}}$ d) $\frac{\mathrm{G}}{\mathrm{n}-1}$

40. The magnetic potential due to a magnetic dipole at a point on its axis distant 30 cm from its centre is found to be $1.4 \times 10^{-5}\ \mathrm{JA^{1}m^{-1}}$. The magnetic moment of the dipole will be

a) 28.6 $\mathrm{Am^2}$ b) 32.2 $\mathrm{Am^2}$ c) 12.6 $\mathrm{Am^2}$ d) None of these

41. Let's consider a galvanometer with a resistance of G and a current that produces a full-scale deflection. To convert it into an ammeter with a range of 0 to a shunt with a value S_1 is utilized. Similarly, for a range of 0 to 2i, a shunt with a value S_2 is employed. The ratio S_1/S_2.

a) $\left(\frac{2\mathrm{i}-\mathrm{i_g}}{\mathrm{i}-\mathrm{i_g}}\right)$ b) $\frac{1}{2}\left(\frac{\mathrm{i}-\mathrm{i_g}}{2\mathrm{i}-\mathrm{i_g}}\right)$ c) 2 d) 1

42. When a proton and an α-particle are projected perpendicular to a magnetic field, determine the ratio of their trajectory radii.

a) 2: 1 b) 1: 2 c) 4: 1 d) 1: 4

---------------------ANSWER KEY----------------------

1)	a	2)	d	3)	d	4)	b
5)	c	6)	d	7)	b	8)	b
9)	a	10)	c	11)	c	12)	b
13)	c	14)	a	15)	b	16)	b
17)	c	18)	c	19)	b	20)	b
21)	a	22)	a	23)	d	24)	c
25)	d	26)	c	27)	a	28)	b
29)	d	30)	b	31)	b	32)	c
33)	a	34)	a	35)	a	36)	d
37)	c	38)	b	39)	d	40)	c
		41)	a	42)	b		

HINTS AND SOLUTIONS

1. (a)

The minimum value of magnetic field

$$B = \frac{F}{qv \sin 90°}$$

$$= \frac{10^{-9}}{2 \times 10^{-12} \times 10^5}$$

$= 5 \times 10^{-3}$ T in z – direction

2. (d)

Cyclotron frequency is given by

$$v = \frac{qB}{2\pi m}$$

$$\therefore v = \frac{1.6 \times 10^{-19} \times 6.28 \times 10^{-4}}{2 \times 3.14 \times 1.7 \times 10^{-27}}$$

$$= 0.94 \times 10^4 \approx 10^4 \text{ Hz}$$

3. (d)

$$B = \frac{F}{m} = \frac{2}{9 \times 10^{-2}}$$

$= 22.2$ T or 22.2 Wbm^{-2}

4. (b)

$$Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq} \quad \text{... (i)}$$

Since particle was initially at rest and gained a velocity v due to a potential difference of V volt. So,

KE of particle $= \frac{1}{2} mv^2 = qV$

$$v = \sqrt{\frac{2qV}{m}} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$r = \frac{m}{Bq}\sqrt{\frac{2qV}{m}}$$

$$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

∴ Diameter of the circular path

$$d = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}$$

5. (c)

Magnetic field at mid-point M in first case is $B = B_{PQ} - B_{RS}$

($\because B_{PQ}$ and B_{RS} are in opposite directions)

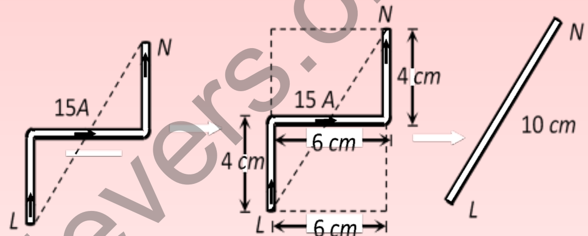
$$= \frac{4\,\mu_0}{4\pi d} - \frac{2\,\mu_0}{4\pi d} = \frac{2\,\mu_0}{4\pi d}$$

When the current 4 A is switched off, the net magnetic field at M is due to current 1 A

$$B' = \frac{\mu_0 \times 4 \times 1}{4\pi d} = \frac{B}{2}$$

6. (d)

The given wire can be replaced by a straight wire as shown below

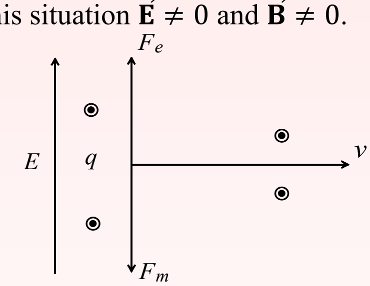


Hence force experienced by the wire

$$F = Bil = 3 \times 15 \times 0.1 = 4.5N$$

7. (b)

If both electric and magnetic fields are present and perpendicular to each other and the particle is moving perpendicular to both of them with $F_e = F_m$. In this situation $\vec{\mathbf{E}} \neq 0$ and $\vec{\mathbf{B}} \neq 0$.



But if electric field becomes zero, then only force due to magnetic field exists. Under this force, the charge moves along a circle

8. (b)

Force along AB is zero as magnetic field is along AB.

The net force on a current carrying loop is always zero.

Fnet=F on AB→+F on BC→+F

Hence on AC,

⟹0=0+F+ (F on AC)

Hence, F on AC is −F

9. (a)

The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by FB=iLB, where L is the length of the wire. Thus,

iLB = mg ⟹ i= mg/LB

= 10A

(b) Applying the right-hand rule reveals that the current must be from left to right.

10. (c)

The magnetic induction due to both semicircular parts will be in the same direction perpendicular to the paper inwards

$$\therefore B = B_1 + B_2 = \frac{\mu_0 i}{4r_1} + \frac{\mu_0 i}{4r_2} = \frac{\mu_0 i}{4}\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

11. (c)

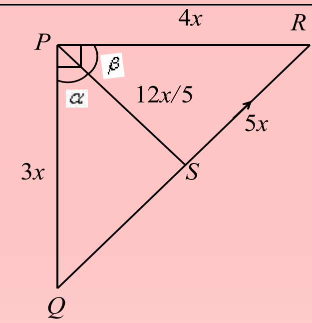
Magnetic field at P due to PQ & PR is zero

∴ Magnetic field at P due to QR

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{PS}(\sin\alpha + \sin\beta)$$

Were, $B = \frac{\mu_0}{4\pi} \cdot \frac{1}{\frac{12x}{5}}\left[\frac{3}{5} + \frac{4}{5}\right]$

$$B = \frac{\mu_0}{4\pi} \times \frac{1}{12x} \times 7 = \frac{7\mu_0 I}{48\,\pi x} \quad \therefore k = 7$$



12. (b)

For wire A,

$B_1 = \frac{\mu_0 i_1}{2r}$

Where $r = \frac{50}{2\pi}$

For wire B,

Circumference = length

$n\pi r = 30$

or $n\pi = \frac{30}{r} = \frac{30}{40/2\pi} = \frac{3}{2}\pi$

or $\theta = n\pi = \frac{3}{2}\pi$

$\therefore B_2 = \frac{\mu_0}{4\pi}\left(\frac{i_2}{r}\right)\theta$

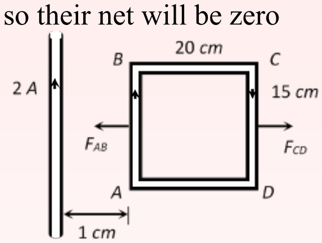
But $B_1 = B_2$

or $\frac{\mu_0 i_1}{2r} = \frac{\mu_0}{4\pi}\left(\frac{i_2}{r}\right)\theta$

or $\frac{i_1}{i_2} = \frac{3}{5}$

13. (c)

Force on side BC and AD are equal but opposite so their net will be zero



But $F_{AB} = 10^{-7} \times \frac{2\times2\times1}{1\times10^{-2}} \times 15 \times 10^{-2} = 6 \times 10^{-6} N$

and $F_{CD} = 10^{-7} \times \frac{2\times2\times1}{(21\times10^{-2})} \times 15 \times 10^{-2} = 2.8 \times 10^{-6} N$

$\Rightarrow F_{net} = F_{AB} - F_{CD} = 3.2 \times 10^{-6} N$

$= 32 \times 10^{-7} N$, towards the wire

14. (a)

If the current flows out of the paper, the magnetic field at points to the right of the wire will be upwards and to the left will be downward. Now magnetic field at C, is zero. The field in the region BX 'will be upwards (+ve) because all points lying in this region are to the right of both the wires. Similarly, magnetic field in the region AX will be downwards (– ve). The field in the region AC will be upwards (+ve) because points are closer to A compared to B. Similarly magnetic field in region BC will be downward (–ve). Graph 2) satisfies all these conditions.

15. (b)

The field at the midpoint of BC due to AB is $\left(-\frac{\mu_0}{4\pi}.\frac{i}{d/2}\hat{k}\right)$ and the same is due to CD.

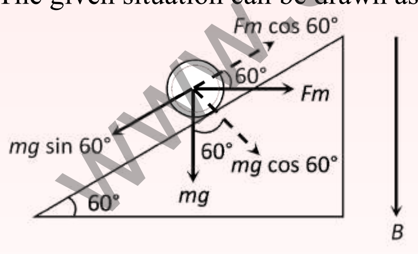
Therefore, the total field is $\left[-\left(\frac{\mu_0 i}{\pi d}\right)\hat{k}\right]$

16. (b)

$$\frac{\mu_0}{4\pi}\times\frac{2\pi i}{r}=H\Rightarrow\frac{(10^{-7})\times 2\times 3.142\times i}{0.05}$$
$$=7\times 10^{-5}$$
$$\therefore i=\frac{7\times 0.05\times 10^{-5}}{2\times 3.142\times 10^{-7}}=\frac{35}{2\times 3.142}$$
$$=5.6\,A$$

17. (c)

The given situation can be drawn as follows



$$F=ilB\Rightarrow mg\sin 60°=ilB\cos 60°$$
$$\Rightarrow B=\frac{0.015\times 10\times\sqrt{3}}{0.2\times 1.96}=0.66T$$

18. (c)

Force on the electron due to the electric field E is $F_E=(-e)E$

Force on the electron due to the magnetic field B is $F_B=(-e)vB$ The electron will move in the fields undeflected, if these two forces are equal and opposite

$eE=evB$ or $v=\frac{E}{B}$

Electric field between the plates is $E=\frac{\sigma}{\varepsilon_0}$

$$\therefore v=\frac{\sigma}{\varepsilon_0 B}$$

The time take by the electron to travel a distance l in the space is $t=\frac{l}{v}=\frac{l}{\frac{\sigma}{\varepsilon_0 B}}=\frac{l\varepsilon_0 B}{\sigma}$

19. (b)

$l=2\pi r$ or $r=l/2\pi$

Area of circular loop, $A=\pi r^2$

Magnetic moment $M=lA=i\pi r^2$

$=i\pi\times l^2/4\pi^2$ or $l=\sqrt{4\pi M/i}$

20. (b)

$$F=\frac{\mu_0}{4\pi}\frac{2i_1 i_2}{a}=10^{-7}\times\frac{2\times 5\times 5}{0.15}$$
$$=3.33\times 10^{-5}N$$

Direction of current is same, so force is attractive

21. (a)

Magnetic field due to a long solenoid is given by

$$B=\mu_0 ni$$

From given data,

$8.3\times 10^{-2}=\mu_0\times 100\times 10^2\times i$

…(i)

and $B=\mu_0\times 200\times 10^2\times\left(\frac{i}{3}\right)$

…(ii)

Solving Eqs. (i) and (ii), we get

$$B=5.53\times 10^{-2}\ \text{Wb/m}^2$$

22. a)

Here magnetic force is zero, but the velocity increases due to electric force

23. (d)

Pole strength does not depend on length.

24. (c)

For no force on wire C, force on wire C due to wire D = force on wire C due to wire B

$$\Rightarrow \frac{\mu_0}{4\pi} \times \frac{2 \times 12 \times 6}{x} \times l = \frac{\mu_0}{4\pi} \times \frac{2 \times 6 \times 12}{(10 - x)} \times l \Rightarrow x = 5\ cm$$

25. (d)

$\vec{F} = i[\vec{l} \times \vec{B}]$

$= 3.5\ [10^{-2}\hat{i} \times (0.85\ \hat{j} - 0.24\ \hat{k})$

$= (2.97\hat{k} - 0.84\ \hat{j}) \times 10^{-2}$ N.

26. (c)

$B = \mu_0\ ni$

$= 4\pi \times 10^{-7} \times (20 \times 20) \times 8$

$= 1.28\pi \times 10^{-3}$ T

$= 1.28\pi \times 10^{-3} \times 10^{4}$ G $= 12.8\pi$G.

27. (a)

Magnetic field in the middle of the solenoid is maximum, magnetic field at it's one end is half of the $M.F.$ at the centre

$$i.e. B_{end} = \frac{1}{2} B_{\text{centre}}$$

28. (b)

$V = i_g R$ and $V' = i_g R'$ or $\frac{R'}{R} = \frac{V'}{V}$

Or $R' = \frac{V'}{V} R = \frac{3V}{V} \times 35 \times 10^3 = 1.05 \times 10^5 \Omega$

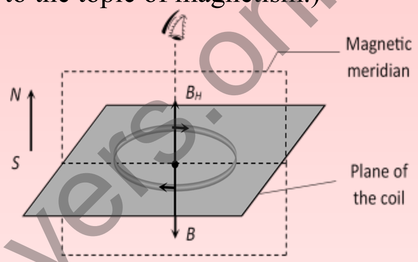
$\therefore$ Additional resistance

$= 1.05 \times 10^5 - 0.5 \times 10^5$

$= 0.55 \times 10^5\ \Omega$

29. (d)

The magnetic meridian refers to a vertical plane aligned in the north-south direction, within which the Earth's magnetic field (B_H) is present. To achieve a neutral point at the centre of a coil, it is necessary for the magnetic field resulting from the current (B) and B_H to counteract each other. As a result, the plane of the coil and the magnetic meridian must be positioned perpendicular to one another, as depicted below. (For additional information, refer to the topic of magnetism.)



30. (b)

As magnetic moment $\propto$ pole strength $\propto$ area of cross section and $M \propto m \propto A$

$$M_1 : M_2 : M_3 = 1 : 3 : 5$$

31. (b)

Radius of path

$$R = \frac{1}{B}\sqrt{\frac{2MV}{q}} \Rightarrow \frac{q}{m} = \frac{2V}{B^2R^2} \Rightarrow \frac{q}{m} \propto \frac{1}{R^2}$$

32. (c)

Total magnetic field induction at O is

$\vec{\mathbf{B}} = \vec{\mathbf{B}}_{LR} + \vec{\mathbf{B}}_{RP} + \vec{\mathbf{B}}_{MS} + \vec{\mathbf{B}}_{SQ}$

$= 0 + \frac{\mu_0}{2\pi}\frac{i}{r} + 0 + \frac{\mu_0}{2\pi}\frac{i}{r} = \frac{\mu_0}{2\pi}\frac{2i}{r}$

$= \frac{2\times10^{-7}\times2\times12}{0.03} = 1.6 \times 10^{-4}$ T

33. (a)

$i = 50\,k;\ i_g = 20k$, where k is the figure of metit of galvanometer; $S = i_g G/(i - i_g)$;

So, $10 = \frac{20k.G}{(50k-20k)}$

On solving we get $G = 15\Omega$.

34. (a)

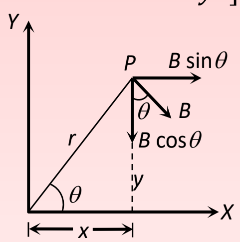
Magnetic field at P is $\vec{B}$, perpendicular to OP in the direction shown in figure

So, $\vec{B} = B\sin\theta\hat{\imath} - B\cos\theta\,\hat{\jmath}$

Here $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

$\therefore \vec{B} = \frac{\mu_0 I}{2\pi}.\frac{1}{r^2}(y\hat{\imath} - x\hat{\jmath}) = \frac{\mu_0 I(y\hat{\imath} - x\hat{\jmath})}{2\pi(x^2+y^2)}$ [as $r^2 = x^2 + y^2$]



35. (a)

The magnetic needle will be deflected towards south if initially the direction of force on the needle due to current carrying conductor is towards west. It will be so if the direction of the current is vertically upwards and the wire is held vertically upwards (according to Fleming's left-hand rule).

36. (d)

$F = Bil = (0.5 \times 10^{-4}) \times 10 \times 1$
$= 5 \times 10^{-4}$ east to west.

37. (c)

Here, magnetic moment due to loop 1

$M_1 = iA_1$
$= i\,\pi r_1^2$
$= 10 \times \pi\,(0.30)^2 = 0.9\,\pi$

Similarly magnetic moment due to loop 2

$M_2 = iA_2 = i\pi r_2^2$
$= 10 \times \pi\,(0.10)^2 = 0.1\,\pi$

Net magnetic moment

$= M_1 - M_2$
$= 0.9\,\pi - 0.1\,\pi = 2.512\text{Am}^2$

38. (b)

$F = ilB\sin\theta = 4 \times 0.50 \times (400 \times 10^{-4}) \times \sin 30°$
$= 4 \times 10^{-2}$ N

39. (d)

$$S = \frac{i_g\,G}{i - i_g} = \frac{iG}{ni - i} = \frac{G}{n-1}$$

40. (c)

Here, $r = 30$ cm $= 0.3$ m

$\theta = 0°$ (Axial line)

$V = 1.4 \times 10^{-5}$J/A –m; M =?

As $V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$

$1.4 \times 10^{-5} = 10^{-7}\frac{M \times 1}{(0.3)^2}$

$M = 12.6$ Am^2

41. (a)

$$S_1 = \frac{i_g\,G}{i - i_g};\ S_2 = \frac{i_g\,G}{2i - i_g};\ \text{so,}\ \frac{S_1}{S_2} = \left(\frac{2i - i_g}{i - i_g}\right)$$

42. (b)

In perpendicular magnetic field

Magnetic force = centripetal force

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

For proton $q_1 = e, m_1 = m$

For α-particle $q_2 = 2e, m_2 = 4m$

$$\therefore \frac{r_1}{r_2} = \frac{m_1 v}{q_1 B} \times \frac{q_2 B}{m_2 v}$$

$$\frac{r_1}{r_2} = \frac{m}{e} \times \frac{2e}{4m}$$

$$\frac{r_1}{r_2} = \frac{1}{2}$$