CHAPTER 13

NUCLEI

Nuclear Radius

- The nuclear radius is defined as the distance between the center of the nucleus and the outermost nucleons (protons or neutrons) within the nucleus.

  $R = R_0 A^{1/3}$ where $R_0 = 1.1 \times 10^{-15}$ m or 1.1 fm.

Nuclear Density

- The ratio of the mass of the nucleus to its volume is called nuclear density. As the masses of proton and neutron are roughly equal, the mass of a nucleus is roughly proportional to A.

  As volume of a nucleus is $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$; $V \propto A$

  Thus, the density within a nucleus is independent of A.

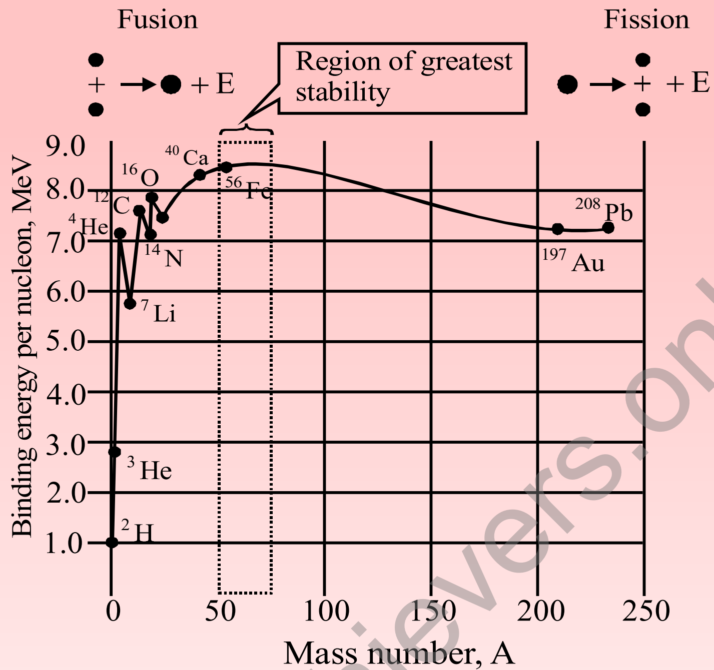
Binding Energy

- $B = (Zm_p + Nm_n - M)c^2$ Where M is mass of nucleus. The terms in the bracket is called mass defect. $\Delta m = Zm_p + Nm_n - M$

Binding energy/nucleon $= \frac{B}{A} = \left(\frac{Zm_p}{A} + \frac{Nm_n}{A} - \frac{M}{A}\right)c^2 = \frac{\Delta mc^2}{A}$.

Packing fraction $p.f. = \frac{(m - A)}{A}$.

(i) **Binding energy curve:** A graph between the binding energy per nucleon and the mass number of nuclei is called as the binding energy curve.



- **The following points may be noted from the biding energy curve:**

(a) The binding energy per nucleon is maximum (≈ 8.8 MeV) for the nucleus having mass number 56. So, this nucleus is most stable i.e., iron is the most stable element of periodic table.

(b) The light nuclei with $A < 20$ are least stable.

(c) The curve has certain peaks indicating that certain nuclei like ${}_2^4He$, ${}_6^{12}C$ and ${}_8^{16}O$ are much more stable than the nuclei in their vicinity.

(d) For atomic number $Z > 56$, the curve takes a downside turn indicating lesser stability of these nuclei.

(e) Nuclei of intermediate mass are most stable. This means maximum energy is needed to break them into their nucleons.

(f) The binding energy per nucleon has a low value for both very light and very heavy nuclei. Hence, if we break a very heavy nucleus (like uranium) into comparatively lighter nuclei then the binding energy per nucleon will increase. Hence a large quantity of energy will be liberated in this process. This phenomenon is called nuclear fission.

(g) Similarly, if we combine two or more very light nuclei (e.g., nucleus of heavy hydrogen ${}_1H^2$) into a relatively heavier nucleus (e.g., ${}_2He^4$), then also the binding energy per nucleon will increase i.e., again energy will be liberated. This phenomenon is called nuclear fusion.

- **Expression for biding energy per nucleon:** In order to compare the stability of various nuclei, we calculate binding energy per nucleon. Higher is the binding energy per nucleon more stable is the nucleus.

We have seen that the mass defect during the formation of a nucleus:

$\Delta m = Zm_p + (A - Z)\, m_n - m$, where m_p, m_n and m are masses of proton, neutron and nucleus respectively.

$\therefore$ Total binding energy of nucleus $\quad \Delta E = \Delta mc^2 = [Zm_p + (A - Z)\, m_n - m] \times c^2$

$\therefore$ Mean binding energy per nucleon $\quad = \frac{\Delta E}{A} = \frac{\Delta mc^2}{A} = \left[\frac{Z}{A}(m_p - m_n) + m_n - \frac{m}{A}\right] \times c^2$

If the mass m of the nuclease is found experimentally, we can find mean binding energy per nucleon since all other factors are known to us.

Radioactive Decay

➢ Unstable nuclides decay by alpha emission or β-emission. When the residual nucleus gets de-excited X-rays are also produced.

❖ ***Q* value of the reaction:**

➢ $Q = u_i - u_f = (M_R - M_P)c^2$ were,

$M_R \rightarrow$ mass of reactants, $M_P \rightarrow$ mass of products.

(a) For α-decay $Q = [m({}_zX^4) - m({}_{Z-2}Y^{A-4}) - m({}_2He^4)]c^2$.

$${}_ZX^A \xrightarrow[\alpha\,\text{decay}]{} {}_{Z-2}Y^{A-4} + {}_2He^4.$$

(b) Three types of β decay

(i) β^- (or electron emission). Ex.: ${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}\beta^0 + \bar{\nu}$.

(ii) β^+ (Positron emission). Ex.: ${}_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}\beta^0 + \nu$.

(iii) Electron capture. Ex.: ${}_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + \nu\,(+X\text{ ray})$

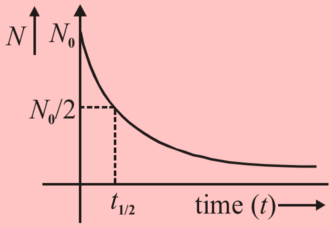
❖ **Laws of radioactive decay:**

$\frac{dN}{dt} = -\lambda N$ where λ is decay constant or disintegration constant or

$$\int_{N_0}^{N} \frac{dN}{N} = \int_0^t -\lambda dt \text{ or } N = N_0 e^{-\lambda t}.$$

The quantity $-\frac{dN}{dt}$ gives the number of decays per second and is called activity.

Thus, $-\frac{dN}{dt} = \lambda N = A$ (activity) or $A = A_0 e^{-\lambda t}$ (Bq.) or dps (disintegrations/sec.)



1 curie (Ci) = 3.7×10^{10} dps

1 Rutherford (R) = 10^6 dps

Half life $\quad t_{1/2} = \dfrac{0.693}{\lambda}.$

Average life $\quad t_{av} = \dfrac{1}{\lambda} = \dfrac{t_{1/2}}{0.693} = 1.44 t_{1/2}.$

Example 1: **What is the activity of one gram of $^{226}_{88}\text{Ra}$, whose half-life is 1622 years?**

Solution: The number of atoms in 1g of radium is

$$N = (1\,\text{g})\left(\frac{1\text{g}-\text{mole}}{226\ \text{g}}\right)\left(6.025\times10^{23}\ \frac{\text{atoms}}{\text{g}-\text{mole}}\right) = 2.666\times10^{21}$$

The decay constant is related to the half-life by

$$\lambda = \frac{0.693}{T_{1/2}} = \left(\frac{0.693}{1622\ \text{year}}\right)\left(\frac{1\,\text{year}}{365\ \text{day}}\right)\left(\frac{1\ \text{day}}{8.64\times10^4\,\text{s}}\right) = 1.355\times10^{-11}\,\text{s}^{-1}$$

The activity is then found from

Activity = $\lambda N = (1.355 \times 10^{-11}\ \text{s}^{-1})\ (2.666 \times 10^{21}) = 3.612 \times 10^{10}$ disintegration/s

The definition of the curie is 1Curie = 3.7×10^{10} disintegrations/s. This is approximately equal to the value found above.

Example 2: **The half-life of radon is 3.8 days. After how many days will only one-twentieth of the sample be left over?**

Solution: Disintegration constant, $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{3.8}\text{day}^{-1}$

Now $N = \dfrac{N_0}{20}$ As $N = N_0 e^{-\lambda t}$ $\therefore$ $\dfrac{N_0}{20} = N_0 e^{-\lambda t}$ or $e^{\lambda t} = 20.$

Taking natural logarithm, $\lambda t = \log_e 20$

$$\lambda t = 2.303 \log 20$$

$$t = \frac{2.303\times1.3010}{\lambda} = \frac{2.303\times1.3010\times3.8}{0.693} = 16.43\,\text{day}.$$

Example 3: **After a certain lapse of time, the fraction of radioactive polonium undecayed is found to be 12.5% of the initial quantity. What is the duration of this time lapse, if the half-life of polonium is 138 days?**

Solution: Here, half time, $T = 138$ days

Therefore, $\lambda = \frac{0.693}{T} = \frac{0.693}{138} = 5.022 \times 10^{-3}\ \text{day}^{-1}$

Also, $\frac{N}{N_o} = 12.5\% = \frac{12.5}{100} = 0.125$

Let t be the required time. Then, $\frac{N}{N_o} = e^{-\lambda t}$

or $0.125 = e^{-5.022 \times 10^{-3} t}$

or $e^{-5.022 \times 10^{-3} t} = \frac{1}{0.125} = 8$

or $5.022 \times 10^{-3} t = \log_e 8 = 2.303 \times 0.9031 = 2.08$

or $t = \frac{2.08}{5.022 \times 10^{-3}} = 414.18\,\text{days}$

Example 4: **The half-life of U^{238} against alpha decay is 1.42×10^{17} s. How many disintegrations per second occur in 1g of U^{238}? Given, Avogadro number $= 6.02 \times 10^{23}\ \text{mol}^{-1}$.**

Solution: Here, $T = 1.42 \times 10^{17}$ s

Avogadro number $= 6.02 \times 10^{23}\ \text{mol}^{-1}$

Number of U^{238} atoms in 1g, $N = \frac{6.02 \times 10^{23}}{238} = 2.53 \times 10^{21}$

Now, $T = \frac{0.693}{\lambda}$ or $\lambda = \frac{0.693}{T} = \frac{0.693}{1.42 \times 10^{17}} = 4.88 \times 10^{-18}\ \text{s}^{-1}$

Now, $\frac{dN}{dt} = \lambda N = 4.88 \times 10^{-18} \times 2.53 \times 10^{21} = 1.235 \times 10^{4}\ \text{s}^{-1}$

Example 5: **In an experiment, the activity of 1.2 milligram (mg) of radioactive potassium chloride (chloride of isotope K-40) was found to be 170s^{-1}. Taking molar mass of K-40Cl to be 0.075 kg mole^{-1}, find the number of K-40 atoms in the sample and hence find the half-life of K-40. Given, Avogadro number $= 6.0 \times 10^{23}\ \text{mole}^{-1}$.**

Solution: Here, molar mass of K-40Cl, M = 0.075 kg mole^{-1}

Mass of K-40Cl, m = 1.2 mg $= 1.2 \times 10^{-6}\,\text{kg}$

Since 1 mole of a substance contains molecules equal to Avogadro number (6.0×10^{23})

The number of atoms in the K-40Cl sample, $N = \frac{6.0 \times 10^{23}}{0.075} \times 1.2 \times 10^{-6} = 9.6 \times 10^{18}$

Also. activity of the given sample, $\frac{dN}{dt} = 170\,s^{-1}$

If λ is disintegration constant, then $\frac{dN}{dt} = \lambda N$ or $170 = \lambda \times 9.6 \times 10^{18}$

or $\lambda = \frac{170}{9.6 \times 10^{18}} = 1.77 \times 10^{-17}\,s^{-1}$

Therefore, half-life of the sample $T = \frac{0.693}{\lambda} = \frac{0.693}{1.77 \times 10^{-17}} = 3.915 \times 10^{16}\,s = 1.241 \times 10^{9}$ years

Example 6: **Biologically useful technetium nuclei (with atomic weight 99) have a half-life of 6 hrs. A solution containing 10^{-12} gram of this is injected into the bladder of a patient. Find the activity at the beginning and after one hour.**

Solution: Here, half-life $T = 6h$

Therefore, $\lambda = \frac{0.693}{T} = \frac{0.693}{6} h^{-1} = 0.1155 h^{-1}$

Now, number of technetium atoms in 10^{-12} g in the beginning.

$$N_o = \frac{\text{Avogadro number}}{\text{atomic weight}} \times 10^{-12} = \frac{6.025 \times 10^{23} \times 10^{-12}}{99} = 6.086 \times 10^{9}$$

Activity of technetium nuclei in the beginning $(t = 0)$

$$R_o = \lambda N_o = 0.1155 \times 6.086 \times 10^{-9} = 7.03 \times 10^{8}\,h^{-1} = 7.03 \times 10^{8}\,h^{-1}$$

Now, the number of atoms left after time t, $N = N_o e^{-\lambda t}$

Here, $t = 1h$

$$\therefore N = 6.086 \times 10^{9} e^{-0.1155 \times 1} = 6.086 \times 10^{9} \times 0.891 = 5.423 \times 10^{9}$$

Therefore, activity of technetium after 1 hr $R = \lambda N = 0.1155 \times 5.423 \times 10^{9} = 6.264 \times 10^{8} h^{-1}$

Example 7: **Calculate the half-life period of a radioactive substance, if its activity drops to 1/16 of its initial value in 30 years.**

Solution: Let R_o and R be the values of initial $(t = 0)$ and present $(t = 30$ years$)$ activities of the radio-active sample

Here; $R = \frac{1}{16} R_o$; $t = 30$ years

Now, $R = R_o e^{-\lambda t}$

$$\frac{1}{16} R_o = R_o e^{-\lambda \times 30} \quad \text{or} \quad e^{30\lambda} = 16$$

$$\text{or} \quad \lambda = \frac{\log_e 16}{30} = \frac{2.3026 \log 16}{30} = \frac{2.3026 \times 1.2041}{30} = 0.0924 \text{ years}^{-1}$$

If T is half-life period, then $T = \frac{0.693}{\lambda} = \frac{0.693}{0.0924} = 7.5$ years

Nuclear Fission

- It occurs when a heavy nucleus split (usually A > 230) into two lighter nuclei of nearly equal mass.

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236}$$

$$_{92}U^{236} \rightarrow {}_{53}I^{137} + {}_{39}Y^{97} + 2{}_on^1$$

$$_{39}Y^{97} \xrightarrow{\beta^-} {}_{40}Zr^{97} \xrightarrow{\beta^{-1}} {}_{41}Nb^{97} \xrightarrow{\beta^-} {}_{42}MO^{97}$$

$$_{53}I^{137} \xrightarrow{\beta^-} {}_{54}Xe^{137} \xrightarrow[\text{delayed neutron}]{n} {}_{54}Xe^{136}$$

$$_{92}U^{236} \rightarrow {}_{56}Ba^{140} + {}_{30}Kr^{94} + 2n$$

Nuclear Fusion

- It occurs when two light nuclei unite or fuse together to form a heavy nucleus. To carry out nuclear fusion, the temperature should be of the order of 10^7K.

$$\left.\begin{array}{l} _1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + \nu \\ _1H^2 + {}_1H^1 \rightarrow {}_1H^3 + e^+ + \nu \text{ or } \frac{3}{2}He \\ _1H^3 + {}_1H^1 \rightarrow {}_2He^4 \\ _2He^3 + {}_1H^1 \rightarrow {}_2He^4 + e^+ + 2\nu \end{array}\right\} \text{ overall energy released is of the order of 27 MeV.}$$

Example 8: **Determine the approximate density of a nucleus.**

Solution: If the nucleus is treated as a uniform sphere,

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \approx \frac{A \times (\text{mass of a nucleon})}{\frac{4}{3}\pi R^3} = \frac{A(1.7\times10^{-27}\,\text{kg})}{\frac{4}{3}\pi(1.4\times10^{-15}A^{1/3}\text{m})^3} = 1.5\times10^{-17}\frac{\text{kg}}{\text{m}^3}$$

A cube inch of nuclear material would weigh about 1 billion tons!

Example 9: **Calculate the binding energy per nucleon (B.E./nucleon) in the nuclei of $_{26}Fe^{56}$.**
Given: m ($_{26}Fe^{56}$) = 55.934939 u, m [$_0n^1$] = 1.00865 u, m [$_1H^1$] = 1.00782 u.

Solution: The $_{26}Fe^{56}$ nucleus has 26 protons and 30 neutrons.

Mass of 26 protons = 26 × 1.00782 = 26.20332 u
Mass of 30 neutron = 30 × 1.00865 = 30.25950 u
Total mass = 56.46282 u
Mass of $_{26}Fe^{56}$ nucleus = 55.934939 u
Mass defect, Δm = 0.527881 u
B.E. of $_{26}Fe^{56}$ nucleus = $\Delta m \times 931\,\text{MeV} = 0.527881 \times 931 = 491.457\,\text{MeV}$

$$\text{B.E./nucleon} = \frac{491.457}{56} = 8.776\,\text{MeV}.$$

Example 10: **Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently, the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days, the room will be safe for occupation?**

Solution: Here, half-life of the radioactive substance $T = 10$ days

Consider that the total number of radioactive atoms spread in the room are N_o in number and due to their presence, the level of radiation is 50 times the permissible level. Suppose that the room becomes worth normal occupancy in time t. Obviously, in time, t the number of radioactive atoms in the room will become

$$N = \frac{N_o}{50} \quad \text{or} \quad \frac{N}{N_o} = \frac{1}{50} \qquad \ldots(i)$$

If T is half-life of the radiation sample, then

$$\left(\frac{N}{N_o}\right) = \left(\frac{1}{2}\right)^{t/T} \qquad \ldots(ii)$$

From equation (i) and (ii), we have $\left(\frac{1}{2}\right)^{t/T} = \frac{1}{50}$ or $(0.5)^{t/T} = 0.02$

Taking logarithm of both sides, we have

$$\frac{t}{T}\log 0.5 = \log 0.02 \quad \text{or} \quad t = \frac{\log 0.02}{\log 0.5} \times T \quad \text{or} \quad t = \frac{2.3010}{1.6910} \times T = \frac{-1.6990}{-0.3010} \times 10 = 56.45 \text{ days}$$

Example 11: **A count rate meter is used to measure the activity of a given sample. At one instant, the meter shows 4750 counts $\min^{-1}$. Five minutes later, it shows 2700 counts $\min^{-1}$**

Find: **(a) the decay constant**

(b) the half-life of the sample. (Given $\log_{10} 1.760 = 0.2455$)

Solution: (a) Let N_o and N be the number of the nuclei present in the sample at $t = 0$ and at $t = 5$ min respectively.

Here activity at $t = 0$, $R_o = 4750$ counts $\min^{-1}$ and activity at $t = 5$ min, R =2700 counts $\min^{-1}$.

According to radioactive decay law, activity at $t = 0$, $R_o = 1N_o$

$$4750 = \lambda N_o \qquad \ldots(i)$$

Also, activity at $t = 5$ min, $R = \lambda N$

$$2700 = \lambda N \qquad \ldots(ii)$$

Dividing equation (i) by (ii) we have

$$\frac{4750}{2700} = \frac{\lambda N_o}{\lambda N} \quad \text{or} \quad \frac{N_o}{N} = 1.760 \qquad \ldots(iii)$$

But the number of nuclei left after $t = 5$ min is given by $N = N_o e^{-\lambda \times 5}$

or $\frac{N_o}{N} = e^{5\lambda}$

From equations (iii) and (iv) we have $e^{5}\lambda = 1.760$

$$\text{or} \quad \lambda = \frac{1}{5}\log_e 1.760 = \frac{1}{5} \times 2.3036 \log_{10} 1.760 = \frac{2.3026 \times 0.2455}{5} = 0.113 \min^{-1}$$

(b) Half-life $T = \frac{0.693}{\lambda} = \frac{0.693}{0.113} = 6.13$ min.

Example 12: **Obtain approximately the ratio of the nuclear radii of the gold isotope $_{79}Au^{197}$ and the silver isotope $_{47}Ag^{107}$.**

Solution: We know that $R \propto A^{1/3}$

Therefore, $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$

Here, $A_1 = 197$; $A_2 = 107$

$\frac{R_1}{R_2} = \left(\frac{197}{107}\right)^{1/3} = 1.226$

Example 13: **Assuming that protons and neutrons have equal masses, calculate how many times nuclear matter is denser than water. Given that nuclear radius is given by $R = 1.2 \times 10^{-15} A^{1/3}$ meter and mass of a nucleon $= 1.67 \times 10^{-27}$ kg.**

Solution: Here, $R = 1.2 \times 10^{-15} A^{1/3}$m

Mass of one nucleon $= 1.67 \times 10^{-27}$ kg

Let us calculate the density of the nucleus of mass number A.

Then, mass of the nucleus $= 1.67 \times 10^{-27} \times A$ kg

Density of the nucleus, $\rho_{nuc} = \frac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi(1.2 \times 10^{-15} A^{1/3})^3} = 2.307 \times 10^{17}$ kg m^{-3}

Now, density of water $\rho_{watt} = 10^3$ kg m^{-3} so, $\frac{\rho_{nuc}}{\rho_{watt}} = 2.307 \times 10^{14}$

Example 14: **Find out the binding energy per nucleon of an α-particle in MeV. It is given that the masses of α particle, proton and neutron are respectively 4.00150 amu 1.00728 amu and 1.00867 amu.**

Solution: An α particle i.e., nucleus of helium, consists of two protons and two neutrons.

Here, $m_{He} = 4.00150$ amu; $m_p = 1.00728$ amu; $m_n = 1.00867$ amu

Therefore, mass of protons and neutrons constituting an α particle

$$2m_p + 2m_n = 2 \times 1.00728 + 2 \times 1.00687 = 4.0319 \text{ amu}$$

Mass defect, $\Delta m = (2m_p + 2m_n) - m_{He} = 4.0319 - 4.00150 = 0.0304$ amu

Now, 1 amu $= 931.5$ MeV.

Therefore, binding energy of α particle $= \Delta m \times 931.5 = 0.0304 \times 931.5 = 28.32$ MeV

Example 15: **The mass of deuteron $({}_{1}\mathrm{H}^{2})$ nucleus is 2.013553 amu. If the masses of proton and neutron are 1.007275 amu and 1.008665 amu respectively, calculate the mass defect, the packing fraction, binding energy and binding energy per nucleon.**

Solution: Here, $m_p = 1.007275$ amu; $m_n = 1.008665$ amu and mass of ${}_{1}\mathrm{H}^{2}$ nucleus, $m_N({}_{1}H^{2}) = 2.013553$ amu. The deuteron nucleus contains one proton and one neutron. Therefore, mass of nucleons constituting deuteron, $m_p + m_n = 1.007275 + 1.008665 = 2.15940$ amu.

Therefore, mass defect, $\Delta m = (m_p + m_n) - m_N({}_{1}H^{2}) = 2.015940 - 2.013553 = 0.002387$ amu

Packing fraction, $\frac{\Delta m}{A} = \frac{0.002387}{2} = 0.0011935$ amu

Now, binding energy of deuteron $= \Delta m \times 931.5 = 0.002387 \times 931.5 = 2.22$ MeV

Therefore, binding energy per nucleon of deuteron $= 2.22/2 = 1.11\,\mathrm{MeV}$

Example 16: **Complete the following nuclear reactions:**

(i) ${}_{13}\mathrm{Al}^{27} + {}_{0}\mathrm{n}^{1} \rightarrow ? + {}_{2}\mathrm{He}^{4}$

(ii) ${}_{15}\mathrm{P}^{31} + \gamma \rightarrow ? + {}_{0}\mathrm{n}^{1}$

(iii) ${}_{9}\mathrm{F}^{19} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{8}\mathrm{O}^{16} + ?$

Solution: ${}_{13}\mathrm{Al}^{27} + {}_{0}\mathrm{n}^{1} \rightarrow ? + {}_{2}\mathrm{He}^{4}$

Let the residual nucleus be ${}_{Z}\mathrm{Y}^{A}$.

Then $A = (24+1) - 4 = 24$ and $Z = (13+0) - 2 = 11$

Therefore, residual nucleus is ${}_{11}\mathrm{Na}^{24}$ and the complete nuclear reaction is

(ii) ${}_{13}\mathrm{P}^{27} + {}_{0}\mathrm{n}^{1} \rightarrow {}_{11}\mathrm{Na}^{24} + {}_{2}\mathrm{He}^{4}$

${}_{15}\mathrm{P}^{31} + \gamma \rightarrow ? + {}_{0}\mathrm{n}^{1}$

Let the residual nucleus be ${}_{Z}\mathrm{Y}^{A}$

Then $A = (31+0) - 1 = 30$ and $Z = (15+0) - 0 = 15$

Therefore, residual nucleus is ${}_{15}\mathrm{P}^{30}$ and the complete nuclear reaction is

Example 17: **Write down the nuclear reaction equation**

(a) When neutrons bombard $_{16}S^{32}$ nuclei, radioactive phosphorus $_{15}S^{23}$ is produced.

(b) When $_{24}Si^{28}$ nuclei are bombarded by neutrons, protons issue out from the target.

Solution: (a) Let the outgoing particle be $_{Z}O^{A}$.

Then, nuclear reaction becomes $_{16}S^{32} + _{0}n^{1} \rightarrow _{15}P^{32} + _{Z}O^{A}$

Now, $A = (32+1) - 32 = 1$ and $Z = (16+0) - 15 = 1$

Therefore, the outgoing particle is $_{1}H^{1}$ (proton) and the complete nuclear reaction is

$$_{14}S^{32} + _{0}n^{1} \rightarrow _{15}P^{32} + _{1}H^{1}$$

(b) Let the residual nucleus be $_{Z}Y^{A}$. Then nuclear reaction becomes $_{16}Si^{28} + _{0}n^{1} \rightarrow _{Z}Y^{A} + _{1}H^{1}$

Now, $A = (28+1) - 1 = 28$ and $Z = (14+0) - 1 = 13$

Therefore, the residual nucleus is $_{13}Al^{28}$. The complete nuclear reaction is

$$_{14}Si^{28} + _{0}n^{1} \rightarrow _{13}Al^{28} + _{1}H^{1}$$

Example 18: **Calculate the energy released in the reaction $_{3}Li^{6} + _{0}n^{1} \rightarrow _{2}He^{4} + _{1}H^{3}$**

Given that mass of $_{3}Li^{6} = 6.015126$ amu; mass of $_{2}He^{4} = 4.002603$ amu mass of $_{1}H^{3} = 3.016049$ amu; mass of $_{0}n^{1} = 13.008665$ amu and 1 amu = 931 MeV.

Solution: The nuclear reaction is $_{3}Li^{6} + _{0}n^{1} \rightarrow _{2}He^{4} + _{1}H^{3}$

Total mass of reactants $(_{3}Li^{6}$ and $_{0}n^{1}) = 6.015126 + 1.008665 = 7.023791$ amu.

Total mass of products $(_{2}He^{4}$ and $_{1}H^{3}) = 4.002603 + 3.016049 = 7.018652$ amu

Decrease in mass $= 7.023791 - 7.018652 = 0.005139$ amu

Therefore, energy released $= 0.005139 \times 931 = 4.78$ MeV

Example 19: **How much U^{235} is consumed in a day in an atomic power house operating at 400 MW provided the whole of the mass of U^{235} is converted into energy?**

Solution: Here, rate of production of energy at atomic power house, $P = 400MW = 400 \times 10^{6}\,\text{Js}^{-1}$

Therefore, total energy produced in a day i.e., $24 \times 60 \times 60s$

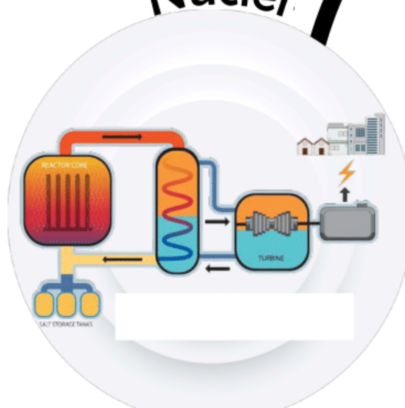
$$E = P \times 24 \times 60 \times 60 = 400 \times 10^{6} \times 24 \times 60 \times 60 = 3.456 \times 10^{13}\,\text{J}$$

If mass of U^{235} consumed per day is m (kg) so as to produce the required amount of energy, then $E = mc^2$

or $3.456 \times 10^{13} = mc^2$

or $m = \dfrac{3.456 \times 10^{13}}{c^2} = \dfrac{3.456 \times 10^{13}}{(3 \times 10^{8})^2} = 0.384 \times 10^{-3}\,\text{kg} = 0.384\,g$.

Nuclei



Composition
- Nuclei consist of protons and neutrons.
- Common name of protons & neutrons in the nucleus is nucleons
- ${}_ZX^A$, $A = Z + N$ = Mass Number
  Z = Atomic Number
  N = No. of neutrons

Contribution
1n 1932, James Chadwick discovered neutrons – elementary particles devoid of any electrical charge.

Mass-Energy Relation
Einstein showed that mass is an another form of energy and gave a relation $E = mc^2$

Mass Defect
$\Delta M = [Z m_p + (A - Z) m_n] - M$
m_p = mass of proton,
m_n = mass of neutron
M = mass of nucleus

Binding Energy
- $B.E. = \Delta Mc^2 = [Z m_p + (A - Z) m_n - M] \times c^2$
- $B.E.$ per nucleon $= B.E/A$

Nuclear fission
- Splitting of a heavy nucleus into two or more lighter nuclei.
  e.g., ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + 200$ meV

Size of Nucleus
Nuclear Volume
$V = \frac{4}{3}\pi R^3$
$= \frac{4}{3}\pi R_0^3 A$
$V \propto A$ (mass no.)

Nuclear Radius
$R = R_0 A^{1/3}$
$R_0 = 1.2 \times 10^{-15}$ m
A = Mass No.

Nuclear Density
$\rho = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$
$= \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi \times (1.2 \times 10^{-15})^3 \times A}$
$\approx 2.9 \times 10^{17}$ kgm^{-3} (independent of A)

Types of Nuclei
Isotopes
- Same atomic No. (Z)
- Different mass No. (A)
  e.g., ${}_6C^{12}$, ${}_6C^{14}$

Isobars
- Same mass No. (A)
- Different atomic No. (Z)
  e.g., ${}_6C^{14}$, ${}_7N^{14}$

Isotones
Atoms whose nuclei having same number of neutrons (N)
e.g., ${}_{11}Na^{23}$, ${}_{12}Mg^{24}$

Nuclear fusion
- Combining two lighter nuclei to form a heavy nucleus
  e.g., ${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \nu + 0.42$ MeV (energy)

Trace the Mind Map
First Level · Second Level · Third Level

PRACTICE QUESTIONS

1. Determine the energy released during the fusion of three alpha particles to form a carbon-12 ^{12}C nucleus. Given that the atomic mass of a helium-4 $_2He^4$ particle is 4.002603 atomic mass units (u), calculate the corresponding mass defect.

   a) 0.007809 u b) 0.002603 u c) 4.002603 u d) 0.5 u

2. The energy equivalent to 2 mg of matter in MeV is

   a) 11.2×10^{22} b) 56.25×10^{24} c) 11.2×10^{26} d) 56.25×10^{28}

3. The mass defect in particular nuclear reaction is 0.4 g. The amount of energy liberated is (Velocity of light= 3×10^8 ms^{-1})

   a) 1.5×10^6 b) 2.5×10^7 c) 3×10^6 d) 10^7

4. The electron in the hydrogen atom jumps from excited state ($n = 2$) to its ground state ($n = 1$) and the photons thus emitted irradiate a photosensitive material. If the work function of the material is 6.2 eV, the stopping potential is estimated to be (the energy of the electron in n^{th} state $E_n = -\frac{13.6}{n^2} eV$)

   a) 5.1 V b) 12.1 V c) 17.2 V d) 16.4 V

5. A mercury atom in its first excited state is brought to its ground state by a collision with an electron that passes through a potential difference of 3.5 V. Determine the wavelength of the photon emitted during the transition of the mercury atom to its ground state.

   a) 2050 Å b) 2240 Å c) 3535 Å d) 2935 Å

6. The half -life period of a radioactive substance is 5 days. Three fourth of substance decays in

   a) 3 days b) 10 days c) 5 days d) 12 days

7. Starting with an initial activity of one curie for a radioactive substance with a half-life of 12 hours, what will be the approximate remaining activity after one week?

   a) 1 curie b) 120 micro curies c) 60 micro curies d) 8 milli curie

8. For a radioactive substance with a half-life of 30 minutes, what is the time interval between 20% and 80% decay?

   a) 60 *minutes* b) 40 *minutes* c) 30 *minutes* d) 25 *minutes*

9. Which nucleus has the maximum average binding energy per nucleon?

$a)$ $_{2}He^{4}$ b) $_{8}O^{16}$ c) $_{26}Fe^{56}$ d) $_{92}He^{238}$

10. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to 2% of its original value

a) 3.2T year b) 4.6 T year c) 8.7 T year d) 9.2 T year

11. A radioactive sample S_1 having an activity of 10μCi has twice the number of nuclei as another sample S_2 which has an activity of 20 μCi. The half lives of S_1 and S_2 can be

a) 20 yr. and 5 yr., respectively b) 20 yr. and 10 yr., respectively

c) 10 yr. each d) 5 yr. each

12. The rest mass of an electron as well as that of positron is 0.6 MeV. When an electron and positron are annihilated, they produce gamma-rays of wavelength(s)

$a)$ 0.010 Å b) 0.024 Å c) 0.010 Å to ∞ d) 0.024 Å to ∞

13. In the Bohr model of the hydrogen atom, which of the following quantities is directly proportional to the principal quantum number (n) - the radius of the orbit (R), the speed of the electron (v), or the total energy of the electron (E)?

$a)$ R/E b) E/v c) RE d) uR

14. If r_1 and $r_2$are the radii of the atomic nuclei of mass numbers 125 and 343 respectively, then the ratio (r_1/r_2) is

$a)$ $\frac{343}{125}$ b) $\sqrt{\frac{343}{125}}$ c) $\frac{5}{7}$ d) $\frac{7}{5}$

15. Given that the maximum wavelength in the Balmer series is 6563 Å, what is the wavelength of the first line in the Balmer series?

$a)$ $\lambda = \frac{16}{3R}$ b) $\lambda = \frac{36}{5R}$ c) $\lambda = \frac{4}{3R}$ d) None of the above

16. Given that the rest mass energy of an electron is 0.54 MeV and its velocity is 0.9 times the speed of light (c), what is the kinetic energy (K.E.) of the electron?

$a)$ 0.06 MeV b) 0.41 MeV c) 0.48 MeV d) 1.32 MeV

17. If two radioactive materials, X_1 and X_2, start with the same number of nuclei but have different decay constants 9λ and λ respectively, after what time will the ratio of the number of nuclei of X_1 to X_2 decrease to 1/e?

a) $\frac{1}{10\lambda}$ b) $\frac{1}{9\lambda}$ c) $\frac{9}{10\lambda}$ d) $\frac{1}{8\lambda}$

18. How are isobars formed?

a) α −decay b) β −decay c) γ −deacy d) h −decay

19. An electron jumps from 4th orbit of 3rd orbit of hydrogen atom. Taking the Rydberg constant as $10^7 per\ metre$ what will be the frequency of radiation emitted

a) $0.25 \times 10^{12} Hz$ b) $0.25 \times 10^{14} Hz$ c) $0.25 \times 10^{13} Hz$ d) None of these

20. The half-life of a radioactive element is 4 days. The fraction left after 16 days will be

a) 0.124 b) 0.062 c) 0.093 d) 0.031

21. A mixture consists of two radioactive materials A_1 and A_2 with half lives of 30 s and 10 s respectively. Initially the mixture has 60 g of A_1 and 150 g of A_2. The amount of the two in the mixture will become equal after

a) 60 s b) 80 s c) 30 s d) 40 s

22. What is the nature of the phenomenon of radioactivity?

a) Exothermic change which increases or decreases with temperature
b) Increases on applied pressure
c) Nuclear process does not depend on external factors
d) None of the above

23. A radioactive sample is α-emitter with half life 148 days is observed by a student to have 1000 disintegration/s. The number of radioactive nuclei for given activity are

a) 1.84×10^{10} b) 1×10^{10} c) 3.45×10^{15} d) 2.75×10^{11}

24. What did Bohr utilize to elucidate his theory?

a) Conservation of linear momentum b) Conservation of angular momentum
c) Conservation of quantum frequency d) Conservation of energy

25. What is the approximate ratio of the radii of the nuclei of ${}_{13}Al^{27}$ and ${}_{52}Te^{125}$?

a) 6: 10 b) 13: 52 c) 40: 17 d) 14: 73

26. What is the fraction of the initial number of radioactive nuclei that remain undecayed after one-third of a half-life of the radioactive sample?

a) $\frac{1}{1.25}$ b) $\frac{1}{2}$ c) $\frac{1}{2\sqrt{2}}$ d) $\frac{1}{4}$

27. The masses of two radioactive substances are same and their half-lives are 2yr and 4yr respectively. The ratio of their activities after 4 yr. will be

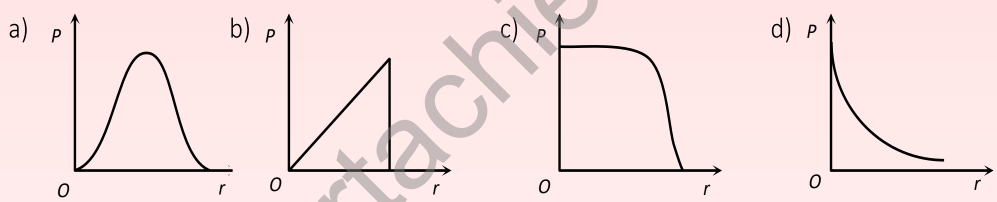
a) 1:4 b) 1:2 c) 1:3 d) 1:6

28. If a rock sample contains a radioactive isotope X and its stable decay product Y in a ratio of 1:16, and the half-life of X is 30 years, what is the estimated age of the rock?

a) 100 years b) 120 years c) 200 years d) 250 years

29. The variation of change density in a nucleus with distance from its centre is depicted by the curve 30.

1.



30. Radon (Rn) decays into Polonium (Po) by emitting an α −particle with half-life of 5 days. A sample contains 5.4×10^{10} atoms of Rn. After 15 days, the number of atoms of Rn left in the sample will be

a) 3.2×10^{10} b) 0.53×10^{10} c) 2.1×10^{10} d) 0.67×10^{10}

31. For two nuclei of the same radioactive nuclide, where one nucleus was formed in a supernova explosion 6 billion years ago, what is the probability of decay occurring during the upcoming time interval?

a) Different for each nucleus
b) Nuclei created in explosion decays first
c) Nuclei created in the reactor decays first
d) Independent of the time of creation

32. The count rate of a Geiger-Muller counter for the radiation of a radioactive material of half-life of 20 *minutes* decreases to 4 s^{-1} after 2 hours. The initial count rate was

a) 25 s^{-1} b) 80 s^{-1} c) 625 s^{-1} d) 20 s^{-1}

33. In the first Bohr orbit of hydrogen, what is the ratio of the speed of the electron to the speed of light, considering the usual meanings of e, h, and c?

a) $2\pi hc/e^2$ b) $e^2h/2\pi c$ c) $e^2c/2\pi h$ d) $2\pi e^2/hc$

34. If the radius of a nucleus of mass number 6 is R, then the radius of a nucleus of mass number 72 is

a) $3R$ b) $9R$ c) $(4/3)^{1/3}R$ d) $4R/3$

35. In an atomic power nuclear reactor that can generate 400 MW of power, where the energy released per fission of each nucleus of a uranium atom U^{238} is 170 MeV, how many uranium atoms undergo fission per hour?

a) 30×10^{25} b) 5.3×10^{22} c) 10×10^{20} d) 5.3×10^{15}

36. In the reaction identify X

$$_7N^{14} + \alpha \rightarrow {}_8X^{17} + {}_1p^1$$

a) An oxygen nucleus with mass 17
b) An oxygen nucleus with mass 16
c) A nitrogen nucleus with mass 17
d) A nitrogen nucleus with mass 16

37. Excitation energy of a hydrogen like ion in its first excitation state is 38 eV. Energy needed to remove the electron from the ion in ground state is

a) 54.4 eV b) 13.6 eV c) 40.8 eV d) 27.2 eV

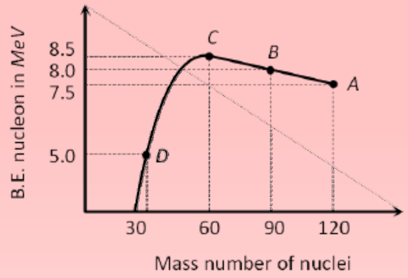
38. In artificial radioactivity, 1.73×10^6 nuclei are disintegrated into 10^6 nuclei in 15 min. The half-life in minutes must be

a) 7.5 b) 3.5 c) 15 d) 30

39. The activity of a radioactive sample is 6000 dps after 280 days and further decreases to 3000 dps after an additional 140 days. What was the initial activity of the sample in dps?

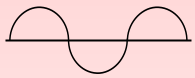
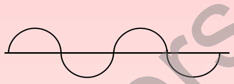
a) 6000 b) 9000 c) 3000 d) 24000

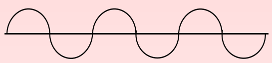
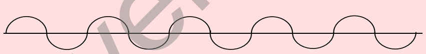
40. Binding energy per nucleon verses mass number curve for nuclei is shown in the figure. A, B, C and D are four nuclei indicated on the curve. The process that would release energy is



a) $C \rightarrow 2D$ b) $A \rightarrow B + D$ c) $A \rightarrow 2C$ d) $B \rightarrow C + D$

41. The de Broglie wave present in third Bohr orbit is

a)  b) 

c)  d) 

--------------------------------ANSWER KEY--------------------------------

1)	a	2)	a	3)	d	4)	d
5)	c	6)	b	7)	c	8)	a
9)	c	10)	c	11)	a	12)	a
13)	d	14)	d	15)	b	16)	a
17)	d	18)	b	19)	c	20)	b
21)	c	22)	c	23)	a	24)	b
25)	a	26)	a	27)	a	28)	b
29)	c	30)	d	31)	d	32)	b
33)	d	34)	c	35)	b	36)	a
37)	a	38)	a	39)	d	40)	c
		41)	c				

HINTS AND SOLUTIONS

1. (a)

Mass defect

Δm=Total mass of α-particles-mass of ^12 C nucleus

$= 3 \times 4.002603 - 12$

$= 12.007809 - 12$

$= 0.007809$ unit

2. (a)

1amu (or 1u) =$1.6605402 \times 10^{-27}$ kg

$= 1.6 \times 10^{-24}$ g

Moreover 1 amu is equivalent to 931 MeV

0r 1.6×10^{-24} g is equivalent to 931 MeV

∴ 2g is equivalent to $\frac{931}{1.6\times10^{-24}} \times 2$ MeV

and 10^{-3} g is equivalent to $\frac{931}{1.6\times10^{-24}} \times 2 \times 10^{-3}$MeV

$= 11.2 \times 10^{23}$ MeV

3. (d)

$\Delta m = 0.4g$

$= 0.4 \times 10^{-3}$ kg $= 4 \times 10^{-4}$ kg

Energy liberated, $E = \Delta mc^2$

$= 4 \times 10^{-4} \times (3 \times 10^8)^2$

$= 4 \times 10^{-4} \times 9 \times 10^{16}$

$= 36 \times 10^{12}$ J $= \frac{36 \times 10^{12}}{3.6 \times 10^6}$ kWh

$= 10^7$ kWh

4. (d)

The formula for the energy of an electron in the n^th state of a hydrogen atom is given as:

E_n = -13.6/n^2 eV

Plugging in the values, we have:

Initial = -13.6/2^2 = -13.6/4 = -3.4 eV Final = -13.6/1^2 = -13.6 eV

Now, we can calculate the energy difference:

ΔE = Final - Initial = -13.6 eV - (-3.4 eV) = -10.2 eV

The energy of the emitted photon is equal to the energy difference ΔE. According to the photoelectric effect, the energy of a photon is given by:

Photon = hν

where h is Planck's constant (h = 4.135667696 × 10^-15 eVs) and ν is the frequency of the photon.

Since we know the energy of the photon (-10.2 eV), we can calculate its frequency:

E_photon = hν -10.2 eV = (4.135667696 × 10^-15 eV·s)ν

Solving for ν:

ν = (-10.2 eV) / (4.135667696 × 10^-15 eV·s)

Now, we can calculate the stopping potential using the equation:

eV_stop = E_photon - work function

Plugging in the values:

eV_stop = -10.2 eV - 6.2 eV = -16.4 eV

The stopping potential is approximately -16.4 eV.

5. (c)

$\frac{hc}{\lambda} = E = eV$

$\Rightarrow \lambda = \frac{hc}{eV} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 3.5}$

$= 3535$ Å

6. (b)

$N = N_0 \left(\frac{1}{2}\right)^n$

Remaining part $= N_0 - \frac{3}{4}N_0$

$= \frac{1}{4}N_0$

$\frac{N_0}{4} = N_0 \left(\frac{1}{2}\right)^n$

$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n$

n=2

Time = Half year × Number of half year= 5×2 = 10 days

7. (c)

1 week = 7 days = $7 \times 24hr \simeq 14$ half lives

Number of atoms left $= \frac{N_0}{(2)^{14}}$, Activity $= N\lambda$

∴ Activity left is $\frac{1}{(2)^{14}}$ times the initial

$\Rightarrow \frac{1}{(2)^{14}} \times 1curie = \frac{1}{16384} \times 1$ curie $\cong 61 \times 10^{-6}$ curie

$\approx 60\mu\ curie$

8. (a)

Here $T_{1/2} = 30\ minutes$, we know $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$

For 20% decay $\frac{N}{N_0} = \frac{80}{100} = \left(\frac{1}{2}\right)^{t_1/30}$ ...(i)

For 80% decay $\frac{N}{N_0} = \frac{20}{100} = \left(\frac{1}{2}\right)^{t_2/30}$ ...(ii)

Dividing (ii) by (i)

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{(t_2-t_1)}{30}}$$

On solving we get $t_2 - t_1 = 60\ min$

9. (c)

The stability of a nucleus is closely related to its binding energy per nucleon. The binding energy per nucleon increases with the atomic number, meaning that heavier nuclei tend to have greater binding energy per nucleon. In the case of iron-56, which has 56 nucleons, it is considered the most stable nucleus. This is because it requires the maximum amount of energy to remove a nucleon from it compared to other nuclei. The high binding energy per nucleon in56 contributes to its stability, as the nucleons are strongly bound together.

10. (c)

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \frac{2}{100}N_0 = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{2}{100} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow n = \frac{2}{\log 1.69}$$

$$\Rightarrow \frac{t}{T} = \frac{2}{\log 1.69} \Rightarrow t = 8.7T\ year$$

11. (a)

Activity of $S_1 = \frac{1}{2}$ (activity of S_2)

Or $\lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$

Or $\frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$

Or $\frac{T_1}{T_2} = \frac{2N_1}{N_2}$

Given $N_1 = 2N_2$

$\therefore \frac{T_1}{T_2} = 4$

12. (a)

Since electron and positron annihilate

$$\lambda = \frac{hc}{E_{Total}}$$

$$= \frac{6.6\times10^{-34}\times3\times10^8}{(0.6+0.6)\times10^6\times1.6\times10^{-19}}$$

$$= 1.21\times10^{-12}m = 0.0103\text{Å}$$

13. (d)

Rydberg constant $R = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$

Velocity $v = \frac{Ze^2}{2\varepsilon_0 nh}$ and energy

$$E = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$$

Now, it is clear from above expressions $R.v \propto n$

14. (d)

$r = r_0(A)^{1/3}$

$$\therefore \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{125}{343}\right)^{1/3}$$

$$= \left[\left(\frac{5}{7}\right)^3\right]^{1/3} = \frac{5}{7}$$

15. (b)

For Balmer series $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

where $n = 3, 4, 5$

For second line $n = 3$

So $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}R \Rightarrow \lambda = \frac{36}{5R}$

16. (a)

$m_0c^2 = 0.54$ MeV and K.E. $= mc^2 - m_0c^2$

Also $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-(0.8)^2}} = \frac{m_0}{0.6}$

$\therefore E = mc^2 = \frac{m_0}{0.9}c^2 = \frac{0.54}{0.9} = 0.6$ MeV

$\therefore$ K.E. $= (0.6 - 0.54) = 0.06$ MeV

17. (d)

Here, $\frac{N_{x_1}(t)}{N_{x_2}(t)} = \frac{1}{e}$

or $\frac{N_0e^{-9\lambda t}}{N_0e^{-\lambda t}} = \frac{1}{e}$

(Because initially, both have the same number of nuclei, N_0).

or $e = \frac{e^{-\lambda t}}{e^{-9\lambda t}} = e^{8\lambda t}$

$8\lambda t = 1$

$$t = \frac{1}{8\lambda}$$

18. (b)

19. (c)

By using $v = Rc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\Rightarrow v = 10^7 \times (3 \times 10^8)\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$

$= 0.25 \times 10^{13}$ Hz

20. (b)

In time $t = T$, $N = \frac{N_0}{2}$

In another half-life, (i e, after 2 half-lives)

$N = \frac{1}{2}\frac{N_0}{2} = \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^2$

After yet another half-life, (ie, after 3 half-lives)

$N = \frac{1}{2}\left(\frac{N_0}{4}\right) = \frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^3$ and so on. Hence, after n half-lives

$N = N_0\left(\frac{1}{2}\right)^n$

$= N_0\left(\frac{1}{2}\right)^{t/T}$

where $t = n \times T$ = total time of n half-lives.

Here, $n = \frac{t}{T} = \frac{16}{4}$

$= 4$

$\therefore$ The fraction left

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$= 0.0625$

21. (c)

$N_1 = \frac{N_{01}}{(2)^{t/}}, N_2 = \frac{N_{02}}{(2)^{t/10}}$

$N_1 = N_2$

$\frac{40}{(2)^{t/30}} = \frac{160}{(2)^{t/10}} \Rightarrow 2^{t/30} = 2^{\left(\frac{t}{10}-2\right)}$

$\Rightarrow \frac{t}{30} = \frac{t}{10} - 2 \Rightarrow \frac{t}{30} - \frac{t}{10} = -2$

$\Rightarrow \frac{2t}{30} = 2 \Rightarrow t = 30$

22. (c)

Radioactivity is a natural phenomenon that occurs when atomic nuclei, due to their inherent instability, emit particles. This process is independent of external factors and occurs because of the intense conflict between the fundamental forces within the nucleus. As a result, there are numerous unstable nuclear isotopes that undergo radioactive decay, emitting different types of radiation.

23. (a)

Activity of substance that has 1000 disintegrations/sec

$= \frac{1000}{3.7 \times 10^{10}} = 0.027 \times 10^{-6}$ci $= 0.027$ µci

The number of radioactive nuclei having activity A

$N = \frac{A}{\lambda} = \frac{1000 \times T_{1/2}}{\log_e 2}$

$= \frac{1000 \times 148 \times 24 \times 3600}{0.693}$

$= 1.84 \times 10^{10}$

24. (b)
Bohr postulated that the angular momentum of the electron is conserved

25. (a)
$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5} = 6:10$$

26. (a)
$$N = \frac{N_0}{2^n} = \frac{N_0}{2^{\frac{1}{3}}}$$

27. (a)
Let initial activity of both substances are same.
$$R = R_0\left(\frac{1}{2}\right)^n = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$$
$$\therefore \frac{R_1}{R_2} = \frac{\left(\frac{1}{2}\right)^{4/2}}{\left(\frac{1}{2}\right)^{4/4}} = \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^1} = \left(\frac{1}{2}\right)^2$$
$$\Rightarrow \frac{R_1}{R_2} = \frac{1}{4}$$

28. (b)
After t second fractional amount of X left is $\frac{1}{16}$ or $\left(\frac{1}{2}\right)^4$
$$\therefore t = 4 \times T_{1/2} = 4 \times 30 = 120\ years$$

29. (c)
Charge density is uniform inside and then falls rapidly near the surface of the nucleus

30. (d)
In the given case, 15 days = 3 half-lives
Number of atoms left after 3 half live
$$= 5.4 \times 10^{10} \times \frac{1}{2^3} = 0.675 \times 10^{10}$$

31. (d)
Radioactive decay does not depend upon the time of creation.

32. (b)
$$A = A_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow 6 = A_0\left(\frac{1}{2}\right)^{\frac{2\times 60}{30}} = \frac{A_0}{16} \Rightarrow A_0 = 96s^{-1}$$

33. (d)
Speed of electron in n^{th} orbit (in CGS)
$v_n = \frac{2\pi Ze^2}{nh}(k = 1)$
For first orbit of H_1; $n = 1$ and $Z = 1$
So $v = \frac{2\pi e^2}{h} \Rightarrow \frac{v}{c} = \frac{2\pi e^2}{hc}$

34. (c)
The radius of a nucleus is often approximated using a formula based on the mass number (A) of the nucleus:
R = R₀ * A$^{1/3}$,
where R₀ is a constant and A is the mass number of the nucleus.
Given that the radius of a nucleus with mass number 6 is R, we can use this information to find the radius of a nucleus with mass number 72.
Let's assume R₀ = R/6$^{1/3}$, which corresponds to the radius of a nucleus with mass number 6.
Now, substituting A = 72 into the formula:
R' = R₀ * 72$^{1/3}$ = (R/6$^{1/3}$) * 72$^{1/3}$ = R * (8/6)$^{1/3}$ = R * (4/3)$^{1/3}$.
Therefore, the radius of a nucleus with mass number 72, denoted as R', is approximately R times the cube root of (4/3).

35. (b)
$$\text{Power} = \frac{\text{energy}}{\text{time}} = 400 \times 10^6 \text{ watt} = 4 \times 10^8 \text{ J/s}$$
$$170 \text{ MeV} = 170 \times 10^6 \times 1.6 \times 10^{-19} = 27.2 \times 10^{-12} \text{ J}$$
Number of atoms fissioned per second
$$= \frac{4 \times 10^8}{27.2 \times 10^{-12}} = \frac{4 \times 10^{20}}{27.2}$$

Number of atoms fission per hour

$$= \frac{4 \times 10^{20} \times 3600}{27.2}$$
$$= \frac{4 \times 36}{27.2} \times 10^{22}$$
$$= 5.3 \times 10^{22} \text{ m}$$

36. (a)

From the reaction, we can see that an alpha particle (α), which consists of two protons and two neutrons (helium nucleus), is being added to a nitrogen-14 (^14N) nucleus. The resulting products are an unknown nucleus with mass number 17 and atomic number 8 (X^17) and a proton (p^1).

37. (a)

Excitation energy $\Delta E = E_2 - E_1 =$
$13.6\, Z^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right]$
$\Rightarrow 38 = 13.6 \times \frac{3}{4} \times Z^2 \Rightarrow Z = 2$

Now required energy to remove the electron from ground state $= \frac{+13.6 Z^2}{(1)^2} =$
$13.6(Z)^2 = 54.4$ Ev

38. (a)

From Rutherford-Soddy law

$$N = N_0 \left(\frac{1}{2}\right)^n$$
$$n = \frac{t}{T}$$
$$\therefore\ 10^6 = 1.73 \times 10^6 \left(\frac{1}{2}\right)^{t/T}$$
$$\Rightarrow \frac{100}{173} = \left(\frac{1}{2}\right)^{t/T}$$
$$\Rightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{10}{13}\right)^2 \quad \text{(Approximately)}$$
$$\Rightarrow n = 2$$
$$\Rightarrow n = \frac{t}{T} = 2$$
$$\Rightarrow T = \frac{15}{2} = 7.5 \text{ min}$$

39. (d)

Activity reduces from 6000dps to 3000dps in 140 days. It implies that half-life of the radioactive sample is 140 days. In 280 days (or two half-lives) activity will remain $\frac{1}{4}$ th of the initial activity. Hence, the initial activity of the sample is

4×6000 dps = 24000 dps

40. (c)

Energy is released in a process when total binding energy (B.E.) of the nucleus is increased or we can say when total B. E. of products is more than the reactants. By calculation we can see that only in case of option (c), this happens

Given $A \rightarrow 2C$

B. E. of reactants $= 120 \times 7.5 =$ 900 MeV

and B. E. of products $= 2 \times (60 \times 8.5) = 1020$ MeV

i. e., B. E. of products > B. E. of reactants

41. (c)