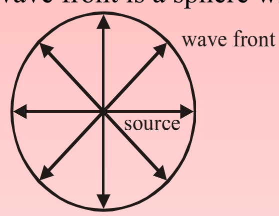
WAVE OPTICS

CHAPTER 10

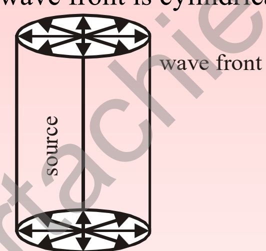
Wave fronts

➢ The focus of all the point which are in the same phase is defined as wave front. It is perpendicular to the rays.

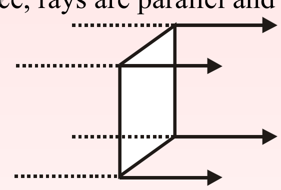
- If the source is a point source, wave front is a sphere with center at the source.



- If the source is extended object, wave front is cylindrical.



- If the source is at infinite distance, rays are parallel and hence wave front is plane.



❖ **Huygen's principle of secondary wavelets:**

➢ Each point on a wave front acts as a fresh source of disturbance. The disturbances from these points are called secondary wavelets. These wavelets spread out in the medium with the velocity of light in that medium. The new wave front at any later time is obtained by taking the forward envelope of the secondary wavelets at that time.

Interference

➢ It is the phenomena of redistribution of light energy in a medium on account of superposition of light waves from different coherent sources.

❖ **Conditions for sustained interference:**

➢ The light should be monochromatic. If this is not so, the fringes of different colors will overlap. The two sources producing interference must be coherent. The wave trains causing interference must have light waves of the same wavelength and be traveling must be coherent. The two interfering wave trains must have the same plane of polarization. To observe interference fringes clearly, it is necessary that the fringe width β is sufficiently large. This is possible if the two coherent sources are narrow, parallel and close to each other. Also, these should be observed at sufficiently large distance.

❖ **Coherent sources:**

➢ Two sources are said to be coherent if they produce waves of same frequency with a constant phase difference. Unlike sound waves, two independent sources of light cannot be coherent. Since sound is a bulk property of matter, therefore, two independent sources of sound can be identical in all respect and can produce coherent waves. On the contrary, light is not a bulk property of matter, it is a property of each individual atom. As the individual atoms emit light randomly and independently, therefore, two independent sources of light cannot be coherent.

Young's double slit experiment

(i) For a bright fringe, $y = \frac{nD\lambda}{d}$, where $n = 0, 1, 2, 3$........

(ii) For a dark fringe, $y = \frac{(2n+1)D\lambda}{2d}$, where $n = 0, 1, 2, 3$,.........

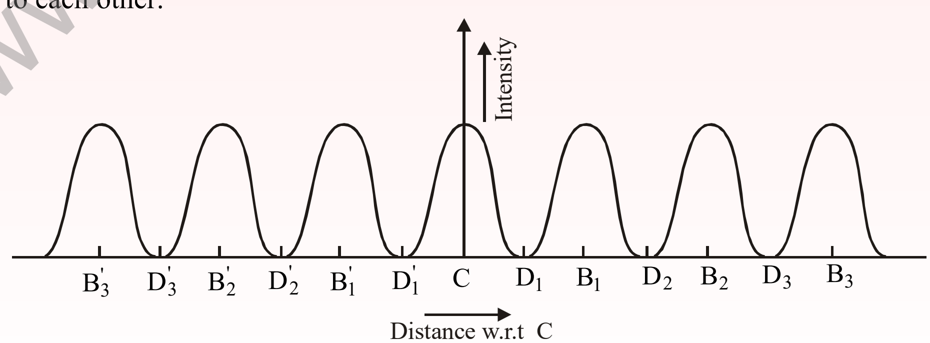
(iii) This distance between any two consecutive bright fringes is equal to the width of a dark fringe. It is denoted by β

$$\beta = \frac{D\lambda}{d}[n-(n-1)] = \frac{D\lambda}{d}$$

β is directly proportional to λ. So, the fringes produced by light of shorter wavelength will be narrow as compared to these produced by light of shorter wavelength will be narrow as compared to those produced by light of longer wavelength.

β is directly proportional to D. Farther the screen from the slits, more is the fringe width.

β is inversely proportional to the distance d between the sources. Lesser the separation between two coherent sources, more is the fringe width. If d becomes large, then the fringe width becomes too small to be detected. This explains as to why the two coherent sources must be very close to each other.



❖ **Maxima and Minima:**

➢ From the interference of waves, we know that when two coherent waves of intensity I_1 and I_2 are superimposed with a constant phase difference ϕ_1, then intensity of the resultant wave obtained is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \text{(Interference term)}$$

In the above equation, the last term is called the interference term. For incoherent sauces, this term is equal to zero. That is,

$$I = I_1 + I_2$$

The phase difference produced between waves arriving at a common point of two parts:

Initial phase difference (ϕ_o).

Phase difference produced due to path difference (ϕ'),

i.e., $\phi' = \frac{2\pi}{\lambda} p$ where p is the path difference.

Thus, the total phase difference is

$$\phi = \phi_o + \phi'$$

$$\phi = \phi_o + \frac{2\pi}{\lambda} p$$

❖ **Maxima:**

When $\phi = (2n)\pi$, where $\phi = (2n)\pi$

Then, $\cos\phi = +1$

Hence, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}$

Since intensity $\propto$ (amplitude)^2

So, $A_{max} = A_1^2 + A_2^2 + 2A_1A_2 = (A_1 + A_2)^2$

Example-1: **Two sources of intensity *I* and 4*I* are used in an interference experiment. Find the intensity at a point where the waves from the two sources superimpose with a phase difference of**

(i) zero

(ii) $\frac{\pi}{2}$

(iii) π

Solution: In case of interference,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(i) As $\phi = 0, \cos\phi = 1$

$\therefore I = 4I + I + 2\sqrt{4I \times I} \times 1 = \mathbf{9I}$

(ii) As $\phi = \frac{\pi}{2}, \cos\phi = 0$

$\therefore I = 4I + I + 2\sqrt{4I \times I} \times 0 = \mathbf{5I}$

(iii) As $\phi = \pi$ $\cos\phi = -1$

$\therefore I = 4I + I + 2\sqrt{4I \times I} \times (-1)) = \mathbf{I}$

Example-2: **A double slit arrangement produces fringes for λ = 5890 Å that are 0.4° apart. What is the angular width if the entire arrangement is immersed in water? (μ_W = 4/3)**

Solution: Let θ° be the angular width in water.

We know angular width = $\frac{\lambda}{d}$

$\Rightarrow$ angular width $\propto \lambda$

$$\frac{\theta\pi}{180}\times\frac{180}{0.4\pi} = \frac{\lambda_{water}}{\lambda_{air}} \text{ or, } \frac{\theta}{0.4}=\frac{\lambda_W}{\lambda_a} \quad \text{… (i)}$$

Now, $_a\mu_W = \frac{\lambda_a}{\lambda_W} \Rightarrow \frac{\lambda_a}{\lambda_W} = 4/3$

Hence from equation (i), we have

$$\frac{\theta}{0.4}=\frac{3}{4} \Rightarrow \theta = \mathbf{0.3°}$$

Example-3: **Find the maximum intensity in case of interference of *n* identical waves each of intensity I_0 if the interference is**

(i) coherent

(ii) incoherent

Solution: (i) In case of two coherent sources,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

I_R will be maximum when cosϕ = maximum = 1

$\therefore (I_{max})_{co} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

So, for *n* identical waves each of intensity I_0,

$(I_{max})_{co} = (\sqrt{I_0}+\sqrt{I_0}\ \ldots)^2 = (n\sqrt{I_0})^2 = n^2 I_0$

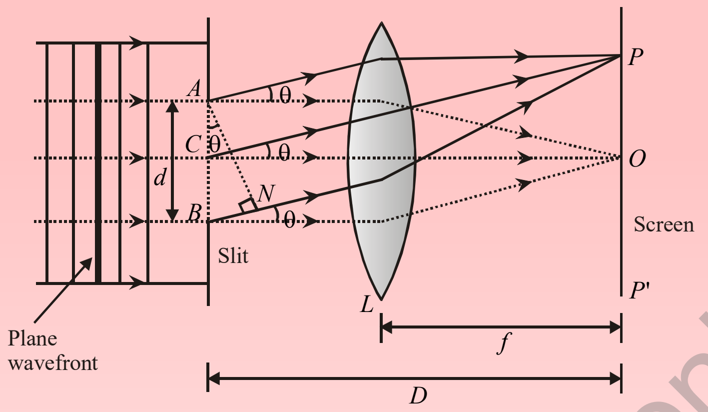
(ii) In case of incoherent sources,

$I_R = I_1 + I_2 + \ldots + I_n = I_0 + I_0 + \ldots (n \text{ times}) = \boldsymbol{nI_0}$

Diffraction

- The phenomena of bending of wave around the edge of an obstacle is called diffraction. This is only observable when obstacles are comparable with the wavelength of wave.

Diffraction at a single slit



Maxima:

When $\phi = (2n-1)\pi$

Then $\cos\phi = -1$

Hence $I = I_1 + I_2 + 2\sqrt{I_1 I_2}$ and $A_{min} = (A_1 - A_2)^2$

(i) The condition for the first secondary minimum is $\theta_1 = \dfrac{\lambda}{d}$

(ii) The condition for the second secondary minimum is $\theta_2 = \dfrac{2\lambda}{d}$

(ii) A point P on the screen will be the position of nth minimum if

$\theta_n = \dfrac{n\lambda}{d}$ (Assuming that $\sin\theta_n \approx \theta_n$)

Here $n = 1,2,3,.....$

(iv) Proceeding in the same manner, we can establish the condition for nth secondary maximum. A point P on the screen will be the position of nth maximum if

$$\theta_n = (2n+1)\frac{\lambda}{2d}$$

(v) **Width of secondary maximum:**

Width of secondary maximum, $\beta = y_n - y_{n-1} = \dfrac{nD\lambda}{d} - \dfrac{(n-1)D\lambda}{d} = \dfrac{D\lambda}{d}$

$\therefore$ Width of central maximum, $\beta_o = \dfrac{2D\lambda}{d}$

If the lens is held very close to the slit or the screen is far away from the lens, then

$\beta = \dfrac{f\lambda}{d}$ and $\beta_o = \dfrac{2f\lambda}{d}$ where f is the focal length of the lens.

The central maximum is twice as wide as a secondary maximum.

The width of a central or secondary maximum is proportional to the wavelength λ of light used.

If the width d of the slit is increased, the secondary maxima will get narrow.

(vi) **Comparison between interference and diffraction**

Interference	Diffraction
Interference is due to the superposition of two different wave trains coming from two coherent sources	Diffraction is due to the superposition of secondary wavelets from different parts of the same wavefront.
In the case of interference, fringe width is generally constant.	In the case of diffraction, the fringes are of varying width.
All the maxima in the interference pattern have the same intensity.	The maxima in diffraction pattern are of varying intensities.
In interference pattern, there is a good contrast between maxima and minima.	In diffraction pattern, there is a comparatively poor contrast between maxima and minima.

Example-4: **A slit 4.0 cm wide is irradiated with micro waves of wave length 2.0 cm. Find the angular spread of central maxima, assuming incidence normal to the plane of the slit.**

Solution: Here $d = 4.0\text{ cm}, \lambda = 2.0\text{ cm}$.

Now, the half angular spread of central maxima is given by

$d \sin\theta_1 = 1 \times \lambda = \lambda$

$$\therefore \quad \sin\theta_1 = \frac{\lambda}{d} = \frac{2}{4} = 0.5. \quad \text{or} \quad \theta_1 = 30^\circ$$

As the central maxima spreads on both sides of the Centre of screen, so the angular spread of central maximum = 60°.

Example-5: **A laser operates at a frequency of 5 × 10^{14} Hz and has an aperture of 10^{-2} m. What will be the angular spread?**

Solution: Here $f = 5 \times 10^{14}\text{ Hz}, d = 10^{-2}\text{ m}$.

Angular spread, $\theta = \frac{\lambda}{d} = \frac{c}{fd}$ $\quad \left(\because \lambda = \frac{c}{f}\right)$

Now $c = 3 \times 10^8\text{ ms}^{-1}$

$$\therefore \quad \theta = \frac{3 \times 10^8}{5 \times 10^{14} \times 10^{-2}} = 0.6 \times 10^{-4}\text{ radian}$$

Example-6: **In a single slit diffraction experiment, first minima for red light (660 nm) coincide with first maxima of some other wavelength λ'. Calculate λ'.**

Solution: Using, $\sin\theta = \dfrac{n\lambda}{d}$ (For minima)

For red light, $\sin\theta = \dfrac{1\times\lambda_r}{d}$

For wave length λ', position of first maxima is midway between first and second minima

$$\therefore \quad \sin\theta' = \frac{\lambda' + 2\lambda'}{2d} = \frac{3}{2}\frac{\lambda'}{d}$$

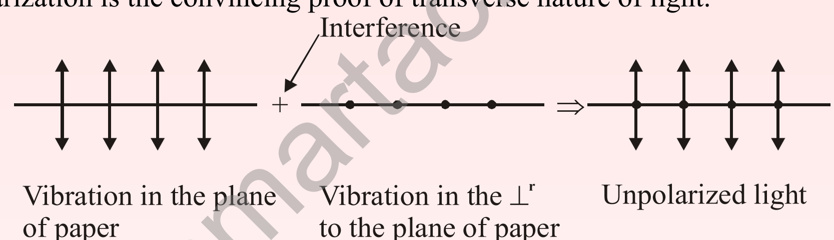
But as per statement, $\sin\theta = \sin\theta'$ *i.e.*, $\dfrac{2}{3}\lambda_r = \dfrac{2}{3}\times 660 = 440\,\text{nm}$.

Polarization

- The phenomena of restricting the vibration of light in a particular direction, direction perpendicular to the direction of wave motion, is called polarization of light.

An ordinary beam of light consists of a large number of waves emitted by atoms or molecules of the light source. Each atom produces a wave with its own orientation of $\vec{E}$ in which all directions are equally probable.

Polarization is the convincing proof of transverse nature of light.

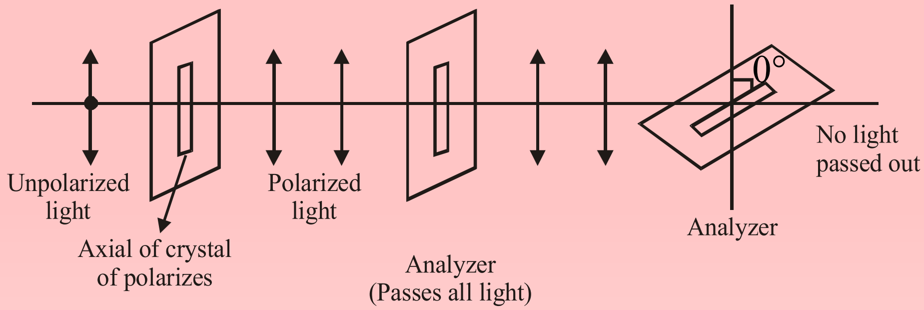


i.e., in case of interference of plane-polarized light, the interfering waves must have same plane of polarization otherwise unpolarized or partially polarized light will result.
Elliptically or circularly polarized lights result due to superposition of two mutually perpendicular plane polarized lights (differing in phase by $\dfrac{\pi}{2}$) with unequal or equal amplitudes respectively.

The devices which convert unpolarized light into polarized light is called **polarizer.**

Example:

Tourmaline crystal, Nicol prism and calcite crystals acts as polarizer as well as analyzer.



❖ **Malus's law:**

When a beam of plane polarized light is incident on an analyzer, the intensity I of light transmitted from the analyzer varies directly as the square of the cosine of the angle θ between the planes of transmission of analyzer and polarizer.

$$I \propto \cos^2\theta$$

Intensity of light transmitted from the analyzer is given by

$$I = I_o \cos^2\theta$$

where $I_o = (ka^2)$ is the intensity of light transmitted from polarizer.

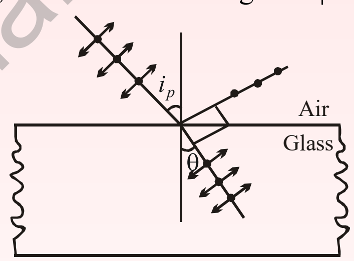
Case-I: When $\theta = 0°$ or 180°, $\cos\theta = \pm 1$ $\qquad I = I_o$

Case-II: When $\theta = 90°$, $\cos\theta = 0$ $\qquad I = 0$

Case-III: If unpolarized light is incident on the analyzer, then intensity of transmitted light of one-half of the intensity of incident light (the average value of $\cos^2\theta =$ is 0.5).

❖ **Brewster's law:**

When unpolarized light is incident at polarizing angle i_p and an interface separating air from medium of refractive index μ, then the reflected light is ϕ fully polarized, provided



$$\mu = \tan i_p$$

This is Brewster's law.

Example-7: **At what angle should the axes of two polaroids be placed so as to reduce the intensity of incident unpolarized light to 1/3.**

Solution: According to the law of Malus $I = I_0 \cos^2\theta$

$$\Rightarrow \cos^2\theta = \frac{I}{I_0} = \frac{1}{3} \qquad \Rightarrow \cos\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$\text{or } \cos\theta = \frac{1.732}{3} = 0.5773 \qquad \text{or } \theta = 54.7°$$

Example-8: **A ray of light is incident on the surface of a glass plate of refractive index 1.536 at the polarizing angle. Calculate the angle of refraction.**

Solution: Since $\tan i_p = \mu = 1.536$

$\therefore i_p = \tan^{-1}(1.536) = 56°56'$, $r = 90° - i_p = 90° - 56°56' = 33°4'$

Example-9: **The intensity of the polarized light becomes 1/20th of its initial intensity after passing through the analyzer. What is the angle between the axis of the analyzer and the initial amplitude of the light beam?**

Solution: Here $I = \frac{1}{20} I_0 = 0.05 I_0$.

Using $I = I_0 \cos^2\theta$, we get

$0.05 I_0 = I_0 \cos^2\theta$

$\cos^2\theta = 0.05$ or $\cos\theta = \sqrt{0.05} = 0.2236$

$\therefore \quad \theta = \cos^{-1}(0.2236) = 76°29'$.

Example-10: **A beam of polarized light makes an angle of 60° with the axis of the Polaroid sheet. How much is the intensity of light transmitted through the sheet?**

Solution: Here $\theta = 60°$

Using $I = I_0 \cos^2\theta$, we get

$I = I_0(\cos 60°)^2 = \frac{1}{4} I_0 \quad \left(\because \cos 60° = \frac{1}{2}\right)$

$\therefore$ Intensity of transmitted light $= \frac{1}{4} \times 100 = 25\%$

Thus, the intensity of the transmitted light is 25% of the intensity of incident light.

Example-11: **The spectral line emitted by a star, known to have a wavelength of 6500 Å, when observed in the laboratory appears to has a wave length 6525 Å. What is the speed of the star in the line of sight relative to the earth? Is the star approaching or receding? Speed of light in air = 3 × 10^8 ms^{-1}.**

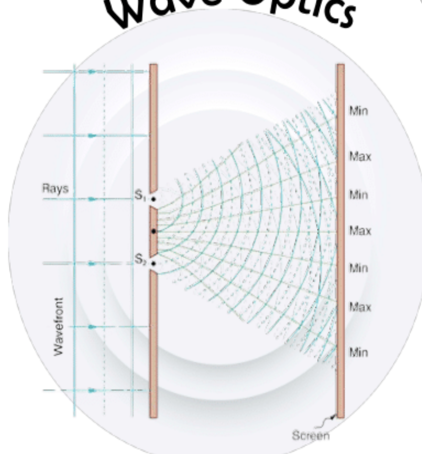
Solution: Here $\lambda = 6500Å, \lambda' = 6525Å$; $\quad c = 3 \times 10^8 \, \text{m s}^{-1}$

Using $\Delta\lambda = \frac{v}{c}\lambda$, we get

$$v = \frac{\Delta\lambda \times c}{\lambda} = \left(\frac{\lambda' - \lambda}{\lambda}\right)c = \left(\frac{6525 - 6500}{6500}\right) \times 3 \times 10^8 = 1.15 \times 10^6 \, \text{ms}^{-1}$$

Since wavelength is increasing, so the star is receding.

Wave Optics



Interference of Light

Two waves superimpose to form a resultant wave of greater or lower or same amplitude.

$I=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2=\beta\left(A_1+A_2\right)^2$

- For constructive Interference, $A=A_1+A_2$
- For destructive interference, $A=\left(A_1 \sim A_2\right)$

Coherent source

Two sources of light are said to be coherent if the initial phase difference between the waves emitted by the sources remains constant with time, otherwise they are called incoherent source of light.

Diffraction of light by single slit

$b\sin\theta = n\lambda$ (dark fringe)

Linear width of central maxima

Width of central maxima $=\frac{2\lambda}{b}$

Angular width $=\frac{2D\lambda}{b}$

$b\sin\theta=(2n+1)\frac{\lambda}{2}$

(for maxima bright fringe)

Young's Double Slit Experiment

Path difference

- For bright fringe $\Delta\phi=n\lambda$
- For dark fringe $\Delta\phi=\left(n+\frac{1}{2}\right)\lambda$

Based on interference of light

Distance between n[th] dark fringe and central fringe

$x_n=\frac{(2n+1)\lambda D}{2d}$

d = distance between slits

Distance between n^{th} bright fringe and central fringe

$x_n=\frac{n\lambda D}{d}$

D = distance between source and screen

Fringe-width

$\beta=\frac{D}{d}\lambda$

It is the distance between two consecutive, bright or dark fringes.

Huygens' Principle

Each point on the primary wavefront is the source of a secondary wavelets

Wave front

- Locus of all particles vibrating in some phase

For spherical wavefront

$I\propto\frac{1}{r^2}$

$A\propto\frac{1}{r}$

Trace the Mind Map

- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS

1. What is the wavelength of electromagnetic waves transmitted from a source that emits waves at a frequency of 7.6×10^6 Hz?

a) 39.5 m b) 40.5 m c) 42.3 m d) 50.9 m

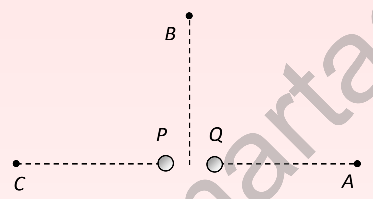
2. What is the phase difference between the wavelets from the opposite edges of a single slit at normal incidence, when observing the Fraunhofer diffraction pattern at the angular position of the first diffraction minimum?

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) 2π

3. In Young's double slit experiment, carried out with light of wavelength $\lambda = 6000$ Å, the distance between the slits is 0.5 mm and the screen is at 100 cm from the slits. The central maximum is at $x = 0$. The third maximum (taking the central maximum as zeroth maximum) will be at x equal to

a) 1.67 cm b) 3.6 cm c) 0.5 cm d) 5.0 cm

4. In the given figure, P and Q represent two coherent sources with equal intensity, emitting radiation of wavelength 20 m. The separation between P and Q is 5.0 m, and the phase of P is 90° ahead of the phase of Q. A, B, and C are three observation points located equidistantly from the midpoint of PQ. What will be the ratio of the intensities of radiation at points A, B, and C?

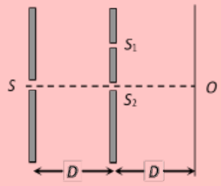


a) 0: 1: 4 b) 4: 1: 0 c) 0: 1: 2 d) 2: 1: 0

5. When a beam of plane-polarized light, with an incident flux of energy of 2×10^{-3} W, falls normally on a polarizer of cross-sectional area 2×10^{-4} m^2, the polarizer begins to rotate with an angular frequency of 31.4 rad/s. What will be the energy of the light passing through the polarizer per revolution?

a) 3×10^{-4}Joule b) 10^{-3}Joule c) 3×10^{-2}Joule d) 10^{-1}Joule

6. In the given setup, there are two ideal slits, S_1 and S_2, separated by a distance d. The slits are illuminated by light of wavelength λ, which passes through an ideal source slit, S, positioned on the same line as S_2. The planes of the slits and the source slit are separated by a distance D. A screen is placed at a distance D from the plane of the slits. What is the minimum value of d required for achieving darkness at point O?



a) $\sqrt{\frac{3\lambda D}{2}}$ b) $\sqrt{\lambda D}$ c) $\sqrt{\frac{\lambda D}{2}}$ d) $\sqrt{3\lambda D}$

7. A sugar solution has a specific rotation of 0.7 dig m^2 k/g. A sample of impure sugar solution with a density of 200 kg m^{-3} is taken in a polarimeter tube with a length of 20 cm. The observed optical rotation in the polarimeter is found to be 19°. What is the percentage of purity of the sugar in the solution?

a) 20% b) 80% c) 95% d) 97%

8. The angle of polarization for any medium is 30°, what will be critical angle for this

a) $\sin^{-1}\sqrt{3}$ b) $\tan^{-1}\sqrt{3}$ c) $\cos^{-1}\sqrt{3}$ d) $\sin^{-1}\frac{1}{\sqrt{3}}$

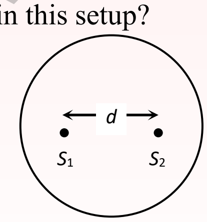
9. The two coherent sources of equal intensity produce maximum intensity of 50 units at a point. If the intensity of one of the sources is reduced by 53% by reducing its width, then the intensity of light at the same point will be

a) 30 b) 35.5 c) 67 d) 60.5

10. The velocity of a moving galaxy is 200 $km\ s^{-1}$ and the apparent change in wavelength of a spectral line emitted from the galaxy is observed as 0.2 nm. Then, the actual wavelength of the spectral line is

a) 3000 Å b) 5000 Å c) 6000 Å d) 4500Å

11. Consider a situation where two coherent sources are located at a distance d from each other and radiate in phase with a wavelength λ. A detector moves in a circular path around the sources within the same plane. What is the angular position of the 4th interference maximum (n=4) in this setup?



a) $\sin^{-1}\frac{n\lambda}{d}$ b) $\cos^{-1}\frac{\lambda}{d}$ c) $\tan^{-1}\frac{d}{\lambda}$ d) $\cos^{-1}\frac{\lambda}{nd}$

12. If we assume that the universe is expanding, and we examine the spectrum of light emitted by a star that is moving away from Earth, what will happen to the wavelength of the light?

a) There will be no change
b) The spectrum will move to infrared region
c) The spectrum will seem to shift to ultraviolet side
d) None of the above

13. A light wave is incident normally over a slit of width 30×10^{-5}cm. The angular position of second dark fringe from the central maxima is 45°. What is the wavelength of light

a) 6000 Å b) 5000 Å c) 2355 Å d) 1500 Å

14. The Brewster angle for the glass-air interface is 50.16°. If a ray of light going from air to glass strikes at an angle of incidence 40°, then the angle of refraction is
Hint- tan (50.16°) = 1.198)
a) 60° b) 40° c) 39.71° d) 50.16°

15. The observation in the Newton's rings experiment states that when white light is used, the colour seen in the reflected light is complementary to the colour observed in the transmitted light through the same point. This phenomenon can be attributed to what underlying reason?

a) 90° change of phase in one of the reflected waves
b) 180° change of phase in one of the reflected waves
c) 145° change of phase in one of the reflected waves
d) 45° change of phase in one of the reflected waves

16. In the spectrum of light of a luminous heavenly body the wavelength of a spectral line is measured to be 5353Å while actual wavelength of the line is 5300Å. The relative velocity of the heavenly body with respect to earth will be (velocity of light is 3×10^8m/s)

a) 3×10^5m/s moving towards the earth b) 3×10^5m/s moving away from the earth
c) 3×10^6m/s moving towards the earth d) 3×10^6m/s moving away from the earth

17. In an electromagnetic wave, the amplitude of electric field is 0.5 V/m, the frequency of wave is The wave is propagating along z-axis. The average energy density of electric field, in Joule/m^3,
a) 1.1×10^{-11} b) 6.25×10^{-12} c) 3.3×10^{-13} d) 6.25×10^{-14}

18. When a single slit with a width of 0.50 mm is illuminated by light with a wavelength of 400 nm, and the observing screen is positioned at a distance of 80 cm from the slit, what will be the width of the central bright fringe?

a) 1.28 mm b) 2 mm c) 4 mm d) 5.6 mm

19. The rms value of the electric field of the light coming from the Sun is 360 N/C. The average total energy density of the electromagnetic wave is

a) $6.37 \times 10^{-9} J/m^3$ b) $81.35 \times 10^{-12} J/m^3$ c) $3.3 \times 10^{-3} J/m^3$ d) $1.14 \times 10^{-6} J/m^3$

20. In a medium with an electric dielectric constant of 1 and a relative magnetic permeability of 20, what is the wave impedance of the medium?

a) $2\ \Omega$ b) $376.6\ \Omega$ c) $1684\ \Omega$ d) $3776\ \Omega$

21. Which of the following statements is correct regarding the propagation of electromagnetic waves for communication purposes?

a) Space wave propagation is achieved by ionospheric reflection
b) Sky wave propagation is used for line-of-sight communication
c) Electromagnetic waves of frequencies higher than 30 MHz penetrate ionosphere
d) Satellite communication uses sky wave mode of propagation

22. In a Young's double-slit experiment, if the separation of the two slits is doubled, what adjustment should be made to the distance D between the screen and the slits in order to maintain the same fringe spacing?

a) $\frac{D}{2}$ b) $\frac{D}{\sqrt{2}}$ c) 2D d) 4D

23. In hydrogen spectrum the wavelength of H_α line is 656 nm whereas in the spectrum of a distant galaxy, H_α line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is

a) $2 \times 10^8 m/s$ b) $2 \times 10^7 m/s$ c) $2 \times 10^6 m/s$ d) $2 \times 10^5 m/s$

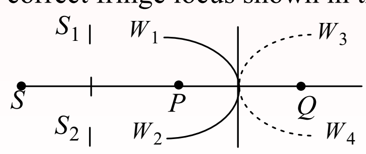
24. In a Young's double-slit experiment, if the source illuminating the slits is changed from blue to violet light, what will happen to the width of the fringes?

a) Increases b) Decreases c) Becomes unequal d) Remains constant

25. When a circular disc is positioned in front of a narrow source, and the point of observation is at a distance of 1.8 meter from the disc, the disc covers the first half-period zone (HPZ). At this point, the intensity is measured as I_0. If the ratio of consecutive amplitudes of the half-period zones is 0.9, what will be the intensity at a point located 30 cm away from the disc?

a) $I_1 = 0.531 I_0$ b) $I_1 = 0.053 I_0$ c) $I_1 = 53 I_0$ d) $I_1 = 5.03 I_0$

26. Out of the interference fringes produced on screen XY by the slits S_1 and S_2, identify the correct fringe locus shown in the figure.



a) PQ b) W_1W_2 c) W_3W_4 d) XY

27. To observe the coloured effect caused by interference when oil is flowing on water, what should be the approximate thickness of the oil film?

a) 100 Å b) 10000 Å c) 10 mm d) 1 cm

27. When a circular disc is positioned in front of a narrow source, and the point of observation is 1 m away from the disc, it covers the first half-period zone (HPZ) and the intensity at this point is denoted as I. If the point of observation is moved to a distance of 22 cm from the disc, what will be the intensity at this new position?

a) $\left(\frac{R_6}{R_2}\right)^2 I$ b) $\left(\frac{R_7}{R_2}\right)^2 I$ c) $\left(\frac{R_8}{R_2}\right)^2 I$ d) $\left(\frac{R_9}{R_2}\right)^2 I$

28. The refractive index of air is 1.0003. What will be the thickness of an air column, such that it contains one additional wavelength of yellow light (wavelength: 6000 Å), compared to the same thickness of vacuum?

a) 2 mm b) 2 cm c) 2 m d) 2 km

29. In Young's double-slit experiment, the intensity on the screen at a point where the path difference between the two slits is λ is denoted as K. What will be the intensity at the point on the screen where the path difference is λ/2?

a) K/4 b) K/2 c) K d) zero

30. The wavelength of light observed on the earth, from a moving star is found to decrease by 0.01%. Relative to the earth the star is

a) Moving away with a velocity of $1.5 \times 10^5 m/s$
b) Coming closer with a velocity of $1.5 \times 10^5 m/s$
c) Moving away with a velocity of $3\times 10^4 m/s$
d) Coming closer with a velocity of $3\times 10^4 m/s$

31. Red light of wavelength 600 nm is incident normally on an optical diffraction grating with 2.5×10^5lines/m. Including central principal maxima, how many maxima may be observed on a screen which is far from the grating

a) 15 b) 17 c) 8 d) 16

32. In a biprism experiment, the 4th dark fringe is observed at a particular point. However, when a thin transparent film is inserted in the path of one of the waves, the 7th bright fringe is seen at the same point. How can the thickness of the film be described in terms of the wavelength of light and the refractive index of the film?

a) $\frac{1.5\lambda}{(\mu-1)}$ b) $1.5(\mu-1)\lambda$ c) $3.5(\mu-1)\lambda$ d) $\frac{3.5\lambda}{(\mu-1)}$

34. In Young's double-slit experiment, if the separation between the slits is reduced to 1/5 of its original value, the fringe width increases by a factor of n. What is the value of n?

a) 5 b) 1/5 c) 10 d) $\frac{1}{10}$

35. For a scattering substance, the intensities of the scattered light for two beams of light in a given direction are in the ratio of 1296:81. What is the ratio of the frequency of the first beam to the frequency of the second beam?

a) 64 : 127 b) 2 : 1 c) 64 : 27 d) None of these

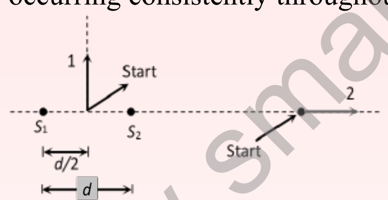
36. A mixture of light, comprising a known wavelength of 490 nm and an unknown wavelength, illuminates Young's double-slit apparatus, resulting in two overlapping interference patterns on the screen. The central maximum of both lights aligns with each other. Additionally, it is observed that the third bright fringe of the known light coincides with the fourth bright fringe of the unknown light. Based on this information, what is the wavelength of the unknown light?

a) 300 nm b) 885.0 nm c) 360 nm d) 720 nm

37. To facilitate the skywave propagation of a 20 MHz signal, what should be the maximum electron density in the ionosphere?

a) $\sim 4.4 \times 10^{12} m^{-3}$ b) $\sim 10^{6} m^{-3}$ c) $\sim 10^{14} m^{-3}$ d) $\sim 10^{22} m^{-3}$

38. In the given figure, sources S_1 and S_2 emit light of wavelength λ in all directions. The sources are in phase and separated by a distance equal to 2λ. When starting from the indicated start point and traveling along paths 1 and 2, the interference results in a maximum (maxima) occurring consistently throughout the journey.



a) Path 1 b) Path 2 c) Any path d) None of these

39. Critical angle for certain medium is $\sin^{-1}(0.3)$. The polarizing angle of that medium is

a) $\tan^{-1}[1.5]$ b) $\sin^{-1}[0.8]$ c) $\tan^{-1}[3.333]$ d) $\tan^{-1}[0.6667]$

40. The equations of displacement of two waves are given as $y_1 = 20\sin(3\pi t + \pi/3)$ $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$, then what is the ratio of their amplitude?

a) 1:2 b) 2:1 c) 1:1 d) None of these

41. Among the following types of radiations, identify the one with the shortest wavelength.

a) Blue light b) γ-rays c) X-rays d) Red light

---------------------------ANSWER KEY----------------------------------

1)	a	2)	d	3)	b	4)	d
5)	a	6)	c	7)	d	8)	a
9)	b	10)	a	11)	b	12)	b
13)	c	14)	c	15)	b	16)	d
17)	d	18)	a	19)	d	20)	c
21)	d	22)	c	23)	b	24)	b
25)	a	26)	c	27)	c	28)	d
29)	a	30)	d	31)	d	32)	b
33)	d	34)	a	35)	b	36)	c
37)	a	38)	b	39)	c	40)	b
			41)	b			

HINTS AND SOLUTIONS

1. (a)

$$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{7.6 \times 10^6} = 39.5m$$

2. (d)

The angular position of first diffraction minimum is

$\sin\theta = \dfrac{\lambda}{a}$

The phase difference is,

$$\phi = \frac{2\pi}{\lambda} \times \Delta x$$

$$\phi = \frac{2\pi}{\lambda} \times \lambda$$

$$\phi = 2\pi$$

3. (b)

Distance of third maxima from central maxima is

$$x = \frac{3\lambda D}{d} = \frac{3 \times 6000 \times 10^{-10} \times (100 \times 10^{-2})}{0.5 \times 10^{-3}} = 3.6cm$$

4. (d)

Since P is ahead of Q by 90° and path difference between P and Q is $\lambda/4$. Therefore at A, phase difference is zero, so intensity is $4I$. At C it is zero and at B, the phase difference is 90°, so intensity is $2I$

5. (a)

Using Malus law, $I = I_0 \cos^2\theta$

As here polarizer is rotating, *i.e.*, all the values of θ are possible

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} I\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} I_0 \cos^2\theta\, d\theta$$

On integration we get $I_{av} = \frac{I_0}{2}$

Where $I_0 = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{p}{A} = \frac{2 \times 10^{-3}}{2 \times 10^{-4}} = 10\ \frac{Watt}{m^2}$

$\therefore I_{av} = \dfrac{1}{2} \times 10 = 5\, Watt$

and Time period $T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{31.4} = \frac{1}{5} s$

$\therefore$ Energy of light passing through the polarizer per revolution $= I_{av} \times \text{Area} \times T = 5 \times 3 \times 10^{-4} \times \frac{1}{5} = 3 \times 10^{-4} J$

6. (c)

Path difference between the waves reaching at O, $\Delta = \Delta_1 + \Delta_2$ where Δ_1= Initial path difference Δ_2= Path difference between the waves after emerging from slits

$\Delta_1 = SS_1 - SS_2 = \sqrt{D^2 + d^2} - D$ and

$\Delta_2 = S_1O - S_2O = \sqrt{D^2 + d^2} - D$

$\therefore \Delta = 2\left\{(D^2 + d^2)^{\frac{1}{2}} - D\right\}$

$$= 2\left\{\left(D + \frac{d^2}{2D}\right) - D\right\}$$

$= \frac{d^2}{D}$ (From Binomial expansion)

For obtaining dark at O, Δ must be equals to $(2n-1)\frac{\lambda}{2}$ *i.e* $\frac{d^2}{D} = (2n-1)\frac{\lambda}{2} \Rightarrow \sqrt{\frac{(2n-1)\lambda D}{2}} = d$

For minimum distance $n = 1$ so $d = \sqrt{\frac{\lambda D}{2}}$

7. (d)

The strength of solution is given by

$c = \dfrac{\theta}{l \times s}$

Where the symbols have their usual meanings.

Here, $\theta = 19°$, $l = 14$ cm $= 0.14$ m

$S = 0.7$ deg m^{2}kg^{-1}

$\therefore c = \dfrac{19}{0.14 \times 0.7} = 194$ kg $-$ m^{-3}

The sugar sample dissolved in a m^3 of water is 200 kg in which 194 kg is pure sugar.

Therefore, purity is $\frac{194}{200} \times 100 = 97\%$

8. (a)

By using $\mu = \tan\theta_p \Rightarrow \mu = \tan 30 = 1/\sqrt{3}$

Also $C = \sin^{-1}\left(\frac{1}{\mu}\right) \Rightarrow C = \sin^{-1}(\sqrt{3})$

9. (b)

Intensity of each source $= I_0 = \frac{50}{4} = 12.5$ unit

If the intensity of one of the sources is reduced by 53% then $I_1 = 25$ unit and $I_2 = 12.5 - 12.5 \times \frac{53}{100} = 5.875$ (unit)

Hence resultant intensity at the same point will be

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= 12.5 + 5.875 + 2\sqrt{12.5 \times 5.875} = 35.5\text{unit}$$

10. (a)

Here, $\Delta\lambda = 0.2\text{nm} = 0.2 \times 10^{-9}\text{m}$

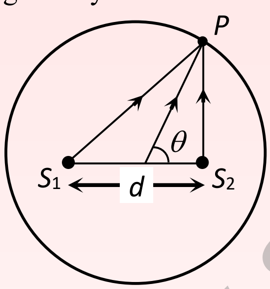
$v = 200\text{kms}^{-1} = 300 \times 10^3\text{ms}^{-1}$

As $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \lambda = \frac{\Delta\lambda c}{v}$

$$\therefore \lambda = \frac{(0.2 \times 10^{-9}\text{m})(3 \times 10^8\text{ms}^{-1})}{(200 \times 10^3\text{ms}^{-1})}$$
$$= 3 \times 10^{-7}\text{m}$$
$$= 3000 \times 10^{-10}\text{m} = 3000\ \text{Å}$$

11. (b)

Here path difference at a point P on the circle is given by



$\Delta x = d\cos\theta$ ...(i)

For maxima at P

$\Delta x = n\lambda$ ...(ii)

From equation (i) and (ii)

$$n\lambda = d\cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{n\lambda}{d}\right) = \cos^{-1}\left(\frac{\lambda}{d}\right)$$

12. (b)

As a result of the universe's expansion, stars will move away from Earth, causing an increase in the observed wavelength. Consequently, the star's spectrum will shift towards the infrared region.

13. (c)

In a single-slit diffraction pattern, the angular position of the nth dark fringe is given by the formula:

$\theta = n\lambda / w$

where: θ is the angular position of the fringe, n is the order of the fringe (in this case, n = 2 for the second dark fringe), λ is the wavelength of light, and w is the width of the slit.

We can rearrange the formula to solve for the wavelength λ:

$\lambda = \theta * w / n$

Given: $\theta = 45° = 45 \times (\pi / 180)$ radians (converting to radians),

$w = 30 \times 10^{-5}$ cm

$= 30 \times 10^{-7}$ m (converting to meters),

n = 2.

Substituting these values into the formula, we can calculate the wavelength λ:

$\lambda = (45 \times \pi / 180) \times (30 \times 10^{-7}) / 2$

$= \pi \times (15 \times 10^{-7}) / 2$

$\approx 3.14 \times (15 \times 10^{-7})) / 2$

$\approx 2.355 \times 10^{-7}$ meters

14. (c)

$\mu = \tan i_p = \tan 50.16° = 1.198$

$$\because 1.198 = \frac{\sin 40°}{\sin r}$$
$$\Rightarrow \sin r = 0.63 \Rightarrow r = 39.71°$$

15. (b)

16. (d)

$$\Delta\lambda = \frac{v_s}{c}\lambda \Rightarrow v_s = \frac{\Delta\lambda . c}{\lambda} = \frac{53 \times 3 \times 10^8}{5300}$$
$$= 3 \times 10^6 \text{m/s away from earth}$$

17. (d)

Average energy density of electric field is given by

$$u_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$
$$= \frac{1}{4} \times 10^{-12}(0.5)^2$$
$$= 0.06 \times 10^{-12}\ J/m^3$$

18. (a)

Width of central bright fringe

$$= \frac{2\lambda D}{d} = \frac{2 \times 400 \times 10^{-9} \times 80 \times 10^{-2}}{0.50 \times 10^{-3}}$$
$$= 1.28 \times 10^{-3}\text{m}$$
$$= 1.28\text{mm}$$

19. (d)

Average energy density

$$v_{AV} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0\left(\sqrt{2}E_{rms}\right)^2 = \varepsilon_0 E_{rms}^2$$
$$= 8.85 \times 10^{-12} \times (360)^2$$
$$= 1.146 \times 10^{-6}\text{J/m}^3$$

20. (c)

$$\text{Wave impedance } Z = \sqrt{\frac{\mu_r}{\varepsilon_r}} \times \sqrt{\frac{\mu_0}{\varepsilon_0}}$$
$$= \sqrt{\frac{20}{1}} \times 376.6 = 1684\Omega$$

21. (d)

22. (c)

Use Formula,

$$\beta = \frac{\lambda D}{d}$$

23. b)

$$v = \frac{c\Delta\lambda}{\lambda} = \frac{3 \times 10^8 \times (706 - 656)}{656}$$
$$= \frac{1500}{656} \times 10^7$$
$$= 2 \times 10^7 \text{m/s}$$

24. (b)

$$\beta \propto \lambda$$

25. (a)

$$I_0 = R^2 = \frac{R_2^2}{3.3}$$

Number of HPZ covered by the disc at b = 30cm

1. $(n_1 b_1 = n_2 b_2$

$$n_2 = \frac{n_1 b_1}{b_2} = \frac{1 \times 1}{0.3} = 3.3$$

Hence the intensity at this point is

$$I = R'^2 = \left(\frac{R_5}{1.8}\right)^2 = \left(\frac{R_5}{R_4} \times \frac{R_4}{R_3} \times \frac{R_3}{R_2}\right)^2 \times \left(\frac{R_2}{1.8}\right)^2$$
$$I = (0.9)^6 I_0$$
$$I_1 = 0.531 I_0$$

Hence the correct answer will be (a)

26. (c)

Interference fringes are bands on screen XY running parallel to the length of slits. Therefore, the locus of fringes is represented correctly byW_3W_4.

27. (c)

To observe the coloured effect caused by interference when oil is flowing on water, the approximate thickness of the oil film should be in the range of a few (μm) to tens of μm.

$$= \left(\frac{R_9}{R_2}\right)^2 I$$

Interference effects occur when there is a significant difference in the path length travelled by light waves reflecting off different interfaces. In the case of oil flowing on water, the oil film thickness determines the path length difference and thus influences the interference pattern observed.

28. (d)

$I = \frac{R_2^2}{4}$ given $n_1 b_1 = n_2 b_2 \Rightarrow 1 \times 100 = n_2 \times 25$

$\therefore n_2 = 4$ HPZ

$\therefore I = \left(\frac{R_9}{2}\right)^2$

$= \left(\frac{R_9}{R_8} \times \frac{R_8}{R_7} \times \frac{R_7}{R_6} \times \frac{R_6}{R_5} \times \frac{R_5}{R_4} \times \frac{R_4}{R_3} \times \frac{R_3}{R_2} \times \frac{R_2}{R_2}\right)^2$ Then

$$= \left(\frac{R_9}{R_2}\right)^2 I$$

29. (a)

The wavelength of yellow light in vacuum is

$\lambda_0 = 6000Å$

and that in air is

$$p = \frac{t}{\lambda_0} = \frac{t}{6000\,Å}$$

and $p + 1 = \frac{t}{\lambda_a} = \frac{t}{6000\,Å/1.0003}$.

from these two equations, we have

$$\frac{t}{6000\,Å} + 1 = \frac{1.0003\,t}{6000\,Å}$$

or $t + 6000\,Å = 1.0003\,t$

or $t = \frac{6000\,Å}{0.0003} = 2 \times 10^7 Å = \mathbf{2\ mm}$.

30. (d)

When path difference is $\lambda, I_{max} = 4I = K$

When path difference is $\frac{\lambda}{2}$, phase difference, $\phi = \pi$

$\therefore \; I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

$= I + I - 2I = 0$

31. (d)

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \frac{0.01}{100} = \frac{v}{3 \times 10^8} \Rightarrow v = 3 \times 10^4 m/s$$

(Since wavelength is decreasing, so star is coming closer)

32. (b)

Here, wavelength, $\lambda = 600nm = 600 \times 10^{-9} m$

Number of lines per meter, $N = 2 \times 10^5$

For principal maxima is grating spectra

$$\frac{\sin\theta}{N} = n\lambda,$$

Where $n(= 1,2,3)$ is the order of principal maxima and θ is the angle of diffraction

The maximum value of $\sin\theta$ is 1

$$\therefore n = \frac{1}{N\lambda} = \frac{1}{2 \times 10^5 \times 600 \times 10^{-9}} = 8$$

$\therefore$ Number of maxima $= 2n + 1 = 2 \times 8 + 1 = 17$

33. (d)

For 4th dark fringe, $x_1 = (2n-1)\frac{\lambda}{2}\frac{D}{d} = \frac{7\lambda D}{2d}$

For 7th bright fringe, $x_2 = n\lambda\frac{D}{d} = \frac{7\lambda D}{d}$

$$x_2 - x_1 = (\mu - 1)t\frac{D}{d}$$

$$\frac{\lambda D}{d}\left[7 - \frac{7}{2}\right] = (\mu - 1)t\frac{D}{d}$$

$$t = \frac{3.5\lambda}{(\mu - 1)}$$

34. (a)

In Young's double-slit experiment, the fringe width (the distance between adjacent bright or dark fringes) is given by the formula:

w = λL / d

where: w is the fringe width, λ is the wavelength of light, L is the distance between the double-slit and the screen, and d is the separation between the two slits.

If the separation between the slits is reduced to 1/5 of its original value (d' = d/5), we can calculate the new fringe width (w') using the same formula:

$$w' = \lambda L / d'$$

To find the factor by which the fringe width increases, we can calculate the ratio of the new fringe width to the original fringe width:

n = w' / w

Substituting the value, we have:

$$n = (\lambda L / d') / (\lambda L / d) = d / d' = d / (d/5) = 5$$

Therefore, the value of n is 5. The fringe width increases by a factor of 5 when the separation between the slits is reduced to 1/5 of its original value.

35. (b)

According to Rayleigh scattering formula,

Intensity of scattered light $I \propto \frac{1}{(\lambda)^4} \propto f^4$

$$\frac{f_1}{f_2} = \left[\frac{I_1}{I_2}\right]^{-1/4}$$

$$= \left[\frac{1296}{81}\right]^{-1/4}$$

$$= \frac{6}{3}$$

$= 2$

36. (c)

$$n_1\lambda_1 = n_2\lambda_2 \Rightarrow 3 \times 480 = 4 \times \lambda_2 \Rightarrow \lambda_2 = 360nm$$

37. (a)

If maximum electron density of the ionosphere is $N_{\max}$ per m^3 then the critical frequency f_c is given by

$$f_c = 9(N_{\max})^{1/2}$$

$$\Rightarrow 20 \times 10^6 = 9(N)^{1/2} \Rightarrow N = 4.4 \times 10^{12} m^{-3}$$

38. (b)

Total phase difference

= Initial phase difference + Phase difference due to path

$$= 66° + \frac{360°}{\lambda} \times \Delta x = 66° + \frac{360°}{\lambda} \times \frac{\lambda}{2} = 66° + 180 = 246°$$

39. (c)

Critical angle, $C = \sin^{-1}(0.3)$

$\sin(C) = 0.3$

$$\mu = \frac{1}{\sin C} = \frac{1}{0.3}$$

Polarizing angle $i_p = \tan^{-1}(\mu) = \tan^{-1}\left(\frac{1}{0.3}\right)$

$= \tan^{-1}(3.33)$

40. (b)

Here, $a = 20, b = \sqrt{5^2 + \left(5\sqrt{3}\right)^2} = 10$

$$\therefore \frac{a}{b} = \frac{20}{10} = 2:1$$

41. (b)

$$\lambda_{Red} > \lambda_{Blue} > \lambda_{X-ray} > \lambda_{\gamma}$$