CHAPTER 01

ELECTRIC FIELD AND CHARGES

Charge

➢ "Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects".

❑ **Properties:**

(i) Charge is a scalar quantity and exists in two forms named as positive and negative.

(ii) Charge is conserved i.e., the charge can neither be created nor destroyed but it may simply be transferred from one body to other.

(iii) Charge is relativistic invariant.

(iv) Similar charges repel and opposite charges attract each other.

(v) Charge on anybody exists as an integral multiple of electronic charge i.e.,

$$q = \pm ne$$

where $n = 0, 1, 2, \ldots\ldots$

Coulomb's Law

➢ Coulomb had observed that force between two-point charges at rest

(i) varies directly as the product of the magnitude of charges i.e., $F \propto q_1 q_2$

(ii) varies inversely as the square of distance between them, i.e., $F \propto \frac{1}{r^2}$

(iii) depends on the nature of medium between the charges.

(iv) always along the line joining of the charges

(v) attractive if charges are unlike and repulsive if they are like

$$\therefore \quad \vec{F} \propto \frac{q_1 q_2}{r^2}\hat{r}\,; \qquad \vec{F} = \frac{1}{4\pi\varepsilon}\cdot\frac{q_1 q_2}{r^3}\vec{r}$$

where ε is the permittivity of the medium between the charges.

Note:

■ If ε_o is the permittivity of free space, then relative permittivity of medium or dielectric constant is given by $\varepsilon_r \quad or \quad K = \frac{\varepsilon}{\varepsilon_0}$

- The permittivity of free space $\varepsilon_0 = 8.86 \times 10^{-12}\ C^2N^{-1}m^{-2}$ and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$.

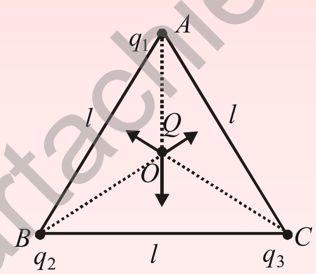
Example-1: **Coulomb's law for electrical force between two charges and Newton's law for gravitational force between two masses, both have inverse-square dependence on the distance between the charges/masses. Compare the strength of these forces by determining the ratio of their magnitude for an electron –proton system.**

Solution: The electric force between an electron and a proton at a distance r apart is $F_e = \frac{-ke^2}{r^2}$ where the negative sign indicates that the force is attractive. The corresponding gravitational force (always attractive) is

$$F_G = -G\frac{m_p m_e}{r^2}$$

where m_p and m_e are the masses of the proton and electron $\left|\frac{F_e}{F_G}\right| = \frac{ke^2}{Gm_p m_e} = 2.4 \times 10^{39}$

Example-2: **Consider three charges q_1, q_2 and q_3 each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q placed at the centroid of the triangle?**



Solution: Force on Q due to $q_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{AO^2}\hat{AO}$

Force on Q due to $q_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{BO^2}\hat{BO}$

Force on Q due to $q_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{CO^2}\hat{CO}$

Total force on Q = $= \frac{Qq}{4\pi\varepsilon_0 AO^2}(\hat{AO} + \hat{BO} + \hat{CO}) = 0$

It is clear by symmetry that the three forces will sum to zero.

Electric Field

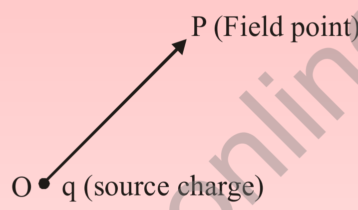
➢ The region surrounding a charge in which its electrical effects are perceptible is called electric field. Thus, electric field is a region around a point charge, a group of point charges or a distribution of charges in which a charged particle experiences force.

❖ ELECTRIC FIELD INTENSITY ($\vec{E}$)

It is defined as the force experienced by a unit positive charge (called test charge) supposed to be placed at that point i.e.,

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

where the limit implies that the test charge itself has no field.



Note:

- If the source charge is a point charge, then $\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q q_0}{r^3} \vec{r}$ $\because$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^3} \vec{r}$$

- The direction of electric field due to positive charge is from charge to point while due to negative charge is from point to charge.
- As in free space $E_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$ and hence in a medium of permittivity ε, $E = \frac{1}{4\pi\varepsilon} \cdot \frac{q}{r^2}$

so, $\frac{E}{E_0} = \frac{\varepsilon}{\varepsilon_0} = \frac{1}{K}$ or $E = \left(\frac{E_0}{K}\right)$

i.e., by presence of a dielectric, electric field decreases and becomes 1/K times of its value in free space.

❖ ELECTRIC LINES OF FORCE

The path traced by a unit positive charge when placed in the electric field of another charge is represented by electric lines of force.

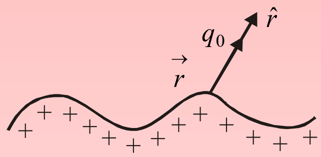
Properties of lines of force

- The tangent at a point on the line of force gives the direction of the electric field at that point.
- The lines of force diverge out from a positive charge and converge at a negative charge.
- The electric lines of force contract lengthwise and expand laterally.
- Two lines of force never intersect. If they are assumed to be intersecting, there will be two tangents and hence implies two directions of electric field at that point of intersection, which is impossible.
- The number of electrical lines of force in a region represents intensity of electric field.

❖ FIELD DUE TO A GROUP OF CHARGES

To find the field due to a group of point charges $q_1, q_2 ....q_n$ are $\vec{E}_1, \vec{E}_2, \vec{E}_3........\vec{E}_n$ respectively, then according to superposition principle resultant electric field $\vec{E}$ is given as

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...\vec{E}_n$$



- **Linear charge distribution:** When the charge is distributed uniformly along line, e.g., a straight wire or circumference of a circle. This is represented by λ, which is equal to charge per unit length $\lambda = dq/dl$.

  If the charge distribution is continuous then the electrical field strength at any point may be calculated by dividing the charge into infinitesimal elements. If dq is small element of the charge within the charge distribution, then the electric field $d\vec{E}$ at point P at a distance r from the charge element dq is

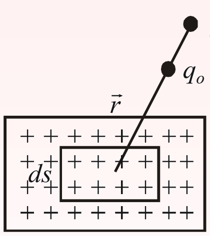
$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}.\frac{dq}{r^2}\hat{r}$$

$\therefore$ The net field strength due to linear charge distribution is given by $\vec{E} = \frac{1}{4\pi\varepsilon}\int\frac{\lambda}{r^2}\hat{r}$

- **Surface charge distribution:** When charge is distributed continuously over some area. This is represented by σ and which is equal to charge per unit area $\sigma = \frac{dq}{ds}$

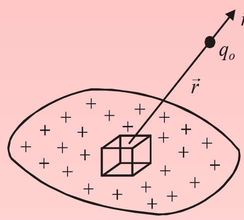
  Therefore, the net field strength due to surface charge distribution is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}.\int\frac{\sigma ds}{r^2}\hat{r}$$



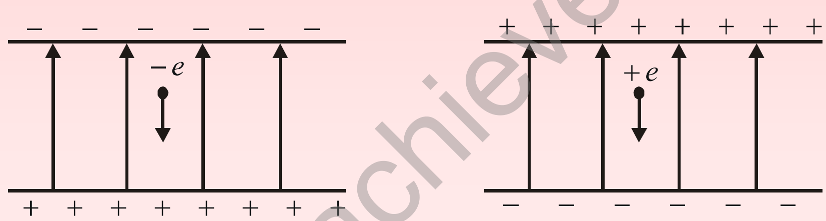
- **Volume charge distribution:** When a charge is continuously distributed over a volume e.g., a sphere or a cube. It is represented in terms of ρ, which is equal to charge per unit volume $\rho = \frac{dq}{dV}$.

  Therefore, the net field strength due to volume charge distribution is given by $\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{\rho ds}{r^2} \hat{r}$



Example-3: **An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude $2.0 \times 10^4 NC^{-1}$. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance. Compute the time of fall in each case. Contrast the situation with that of 'free fall under gravity'.**

Solution:



(*a*) (*b*)

In figure (a) the field is upward, so the negatively charged electron experiences a downward force of magnitude *eE* where *E* is the magnitude of the electric field. The acceleration of the electron is

$$a_e = eE / m_e$$

where m_e is the mass of the electron.

Starting from rest, the time required by the electron to fall through a distance *h* is given by $t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$

For $e = 1.602 \times 10^{-19} C$, $m_e = 9.110 \times 10^{-31} Kg$, $E = 2.0 \times 10^4 N C^{-1}$ and $h = 1.5 \times 10^2 m$,

$$t_e = 2.9 \times 10^{-9} s$$

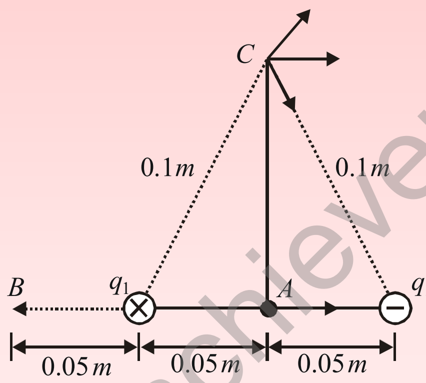
In figure (b), the field is downward, and the positively charged proton experiences a downward force of magnitude *eE*. The acceleration of the proton is $a_p = eE / m_p$

The time of fall for the proton is $t_p = \sqrt{\dfrac{2h}{a_p}} = \sqrt{\dfrac{2hm_p}{eE}} = 1.3 \times 10^{-7}\, s$

$$a_p = \frac{eE}{m_p} = \frac{(1.602\times10^{-19}C)\times(2.0\times10^{4}\,NC^{-1})}{1.673\times10^{-27}\,kg} = 1.9\times10^{12}\,ms^{-2}$$

which is enormous compared to the value of $g(9.8\ ms^{-2})$. The acceleration of the electron is even greater. Thus, the effect of acceleration due to gravity can be ignored in this example.

Example-4: **Two-point charges q_1 and q_2 of $+10^{-8}C$ and $-10^{-8}C$ respectively are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in figure.**



Solution: The electric field vector E_1 at A due to the positive charge q_1 points towards the right and has a magnitude

$$E_1 = \frac{(9\times10^{9}\,Nm^2C^{-2})\times(10^{-8}C)}{(0.05\ m^2)} = 3.6\times10^{4}\,NC^{-1}$$

The electric field vector E_2 at A due to the negative charge q_2 points towards the right and has a magnitude

$$E_2 = \frac{(9\times10^{9}\,Nm^2C^{-2})\times(10^{-8}C)}{(0.05\ m^2)} = 3.6\times10^{4}\,NC^{-1}$$

The magnitude of the total electric field at A is $E_A = E_1 + E_2 = 7.2\times10^{4}\,NC^{-1}$

E_A is directed toward the right.

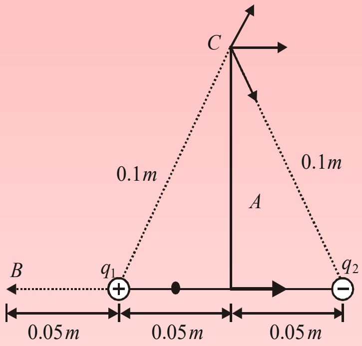
The electric field vector E_1 at B due to the positive charge q_1 points towards the left and has a magnitude

$$E_1 = \frac{(9\times10^{9}\,Nm^2C^{-2})\times(10^{-8}C)}{(0.05\ m^2)} = 3.6\times10^{4}\,NC^{-1}$$

The electric field vector E_2 at B due to the negative charge q_2 points towards the right and has a magnitude.

$$E_2 = \frac{(9\times10^9\, Nm^2C^{-2})\times(10^{-8}C)}{(0.15m)^2} = 4\times10^3\, NC^{-1}$$

The magnitude of the total electric field at B is $E_B = 3.2\times10^4\, NC^{-1}$



E_B is directed towards the left. The magnitude of each electric field vector at point C, due to charge q_1 and q_2 is:

$$E_1 - E_2 = \frac{(9\times10^9\, Nm^2C^{-2})\times(10^{-8}C)}{(0.10m)^2} = 9\times10^3\, NC^{-1}$$

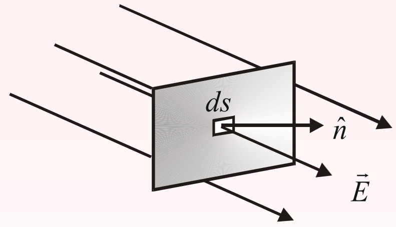
The directions in which these two-vector point are indicated in the figure. The resultant of these two vectors is

$$E_C = E_1\cos\frac{\pi}{3} + E_2\cos\frac{\pi}{3} = 9\times10^3\, NC^{-1}$$

E_C point towards the right.

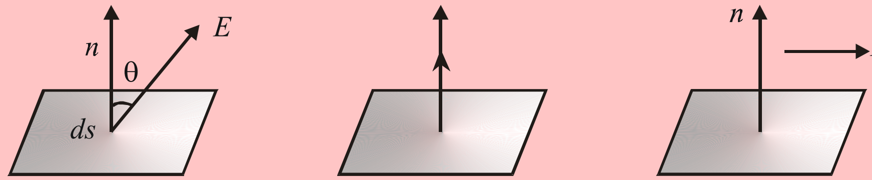
Electric Flux

➢ The total number of electric lines of force through a given surface area is called the electric flux. It is represented by ϕ. The electric flux through a surface element $\Delta\vec{S}$ is $\Delta\phi = \vec{E}.\Delta\vec{S}$



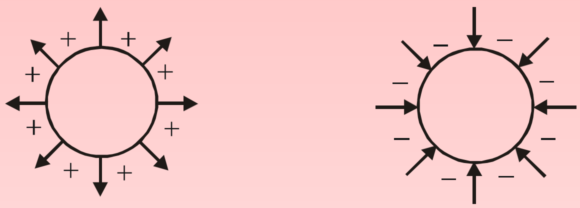
Net electric flux through the whole surface is $\phi = \sum\vec{E}.\Delta\vec{S} = \int\vec{E}.d\vec{S}$ or

It is scalar quantity with unit $(V\times m)$ and dimensions $[ML^3T^{-3}A^{-1}]$



$d\phi = E\,ds\cos\theta$ $\quad (d\phi)_{max} = E\ ds$ $\quad d\,\phi_{min} = 0$

For a closed surface outward flux is taken to be positive while inward negative



$+$ve flux $\qquad -$ve flux

Gauss Theorem

The net electric flux through a closed surface in vacuum is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

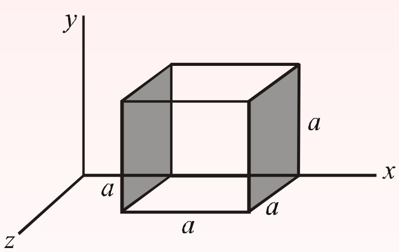
i.e., $\phi_s = \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0} \times q$ (net charge enclosed within the surface)

Example-5: **The electric field components in the following figure are $E_x = ax^{1/2}$, $E_y = E_z = 0$, in which $\alpha = 800\ N/C\ m^{1/2}$.**

Calculate:

(1) flux ϕ_ε through the cube and

(2) the charge within the cube. Assume that $\alpha = 0.1\ m$.



Solution: (1) Since the electric field has only an x component, for faces perpendicular to x direction, the angle between E and ΔS is 0°. Therefore, the flux $\phi = E.\Delta S$ is separately zero for each face of the cube except the two shaded ones. Now the magnitude of the electric field at the left face is

$E_L = \alpha x^{1/2} = \alpha a^{1/2}$ ($x = a$ at the left face)

The magnitude of electric field at the right face is

$E_L = \alpha x^{1/2} = \alpha(2a)^{1/2}$ ($x = 2a$ at the right face)

The corresponding fluxes are

$\phi_L = E_L.\Delta S = E_L \Delta S \cos\theta = -E_L.\Delta S$, since $\theta = 180°$ $= -E_L.a^2$

$\phi_R = E_R.\Delta S = E_R \Delta S \cos\theta = -E_R.\Delta S$, since $\theta = 0°$ $= E_R.a^2$

Net flux through the cube $= \phi_L + \phi_R = E_R a^2 - E_L a^2$

$= a^2(E_R - E_L) = \alpha a^2[(2a)^{1/2} - a^{1/2}] = \alpha a^{5/2}(\sqrt{2} - 1) = 800(0.1)^{5/2}(\sqrt{2} - 1)$

$= 1.05 Nm^2C^{-1}$

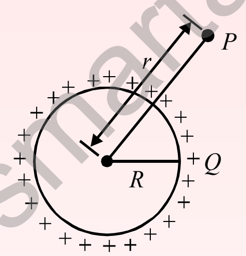
(2) We can use Gauss's theorem to find the total charge q inside the cube. We have $\phi = q/\varepsilon_0$ Therefore $q = 1.05 \times 8.854 \times 10^{-12} C = 9.27 \times 10^{-12} C$

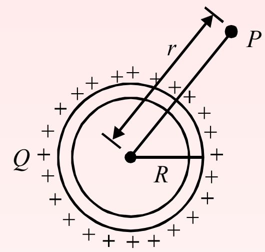
❖ ELECTRIC FIELD INTENSITY DUE TO SOME COMMON CONFIGURATIONS

(I) Charged sphere:

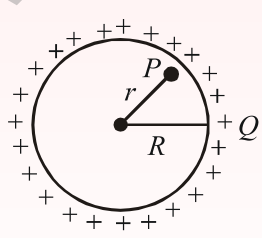
- For an external point, a solid or hollow, conducting or non-conducting charged spheres behaves as if whole of its charge is concentrated at its center, i.e.,

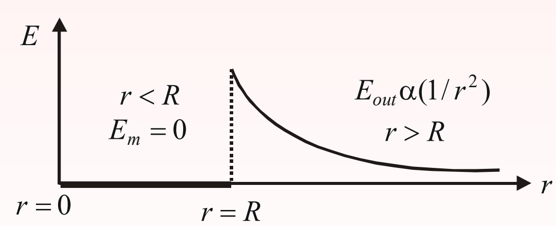
$$E_{out} = \frac{1}{4\pi\varepsilon_0} . \frac{Q}{r^2}$$ (r = distance from center of point)





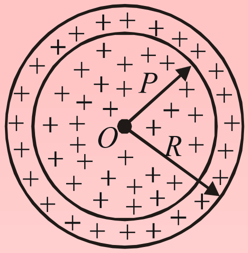
- In case of hollow or solid conducting sphere for an internal point (i.e., r < R) i.e., $E_{in} = 0$

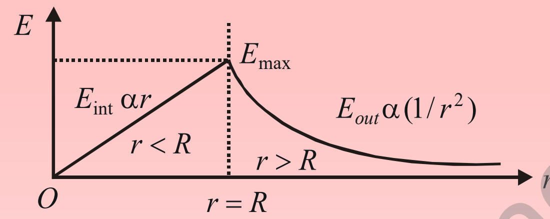




- In case of spherical charged conductor, the electric field is discontinuous at the surface.
- In case of spherical volume distribution of charge, intensity at internal point $P(r < R)$ will be

$$E_{in} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^3} r$$





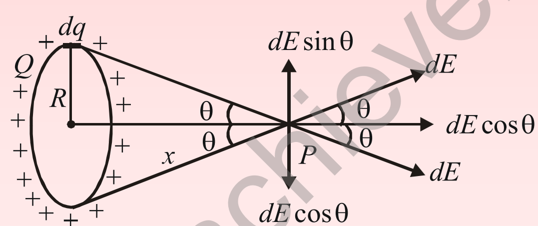
From figure, at the center, E is minimum and zero.

- Inside the sphere, E increases linearly with r as moves from the center to the surface.

For $r < R$, $\vec{E}$ is continuous and $E_{\max} = \dfrac{Q}{4\pi\varepsilon_0 R^2}$

- Outside the sphere $E \propto \dfrac{1}{r^2}$ and is zero at infinity

(II) A charged ring



Let a ring of radius R is charged with positive charge Q. Consider a point P on the axis at 'x' distance from center. Let a small charge element 'dq', due to which electric field at P is

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{(R^2 + x^2)}$$

Due to symmetry, element $dE\sin\theta$ cancels each other. So net field is given by $\int dE\cos\theta$ only.

Net field at p, $E = \int dE \cos\theta$

$$= \int_0^Q \frac{dq}{4\pi\varepsilon_0 (R^2 + x^2)} \frac{x}{\sqrt{R^2 + x^2}}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(R^2 + x^2)^{\frac{3}{2}}}}$$

The direction E is along the axis

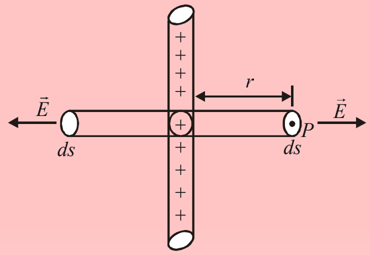
- **Special cases:**

(a) At center, $x = 0$, $E = 0$

(b) At $x = \dfrac{R}{\sqrt{2}}$, E is maximum

(III) A charged sheet

Let 'σ' is surface charge density of thin long sheet. To find electric field at 'r' distance from sheet at P, consider a small element on sheet of charge dq, of surface area ds.



Assume small element is enclosed by Gaussian surface,

According to Gaussian theorem

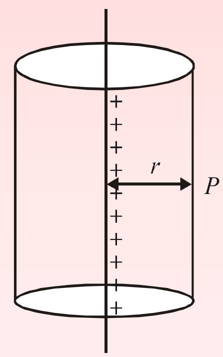
$$\phi = \int \vec{E}.ds = \frac{q}{\varepsilon_o} \; ; \quad E\,ds = \frac{dq}{\varepsilon_o} \; ; \quad E = \frac{dq}{ds\,\varepsilon_o} \qquad \boxed{E = \frac{\sigma}{\varepsilon_o}}$$

Similarly, if sheet is thick and conducting then net electric field inside is zero and outside is given by

$$\boxed{E = \frac{\sigma}{2\varepsilon_o}}$$

(IV) A charged wire

Let a long linear charge of density is given as 'λ' due to which field has to be find out at a distance 'r'. Let the wire is enclosed by a cylinder of radius 'r'



$$\therefore \int \vec{E}.d\vec{s} = \frac{q}{\varepsilon_o}$$

$$E(2\pi rl) = \frac{q}{\varepsilon_o} \qquad (l = \text{ length of wire})$$

$$E = \frac{q}{2\pi\varepsilon_o rl} \qquad \boxed{E = \frac{\lambda}{2\pi\varepsilon_o r}} \qquad \therefore \; E \propto \frac{1}{r}$$

Electric Potential

➤ The electric potential at any point in an electric field is defined as the work done in moving unit positive test charge from infinity to that point. If W is the work done in bringing the charge q_0 from infinity to a point P, then potential at P

$$\boxed{V = \lim_{q_0 \to 0} \frac{W}{q_0}}$$

■ Alternatively, the electric potential at any point in an electric field is defined as the negative line integral for the electric field vector $\vec{E}$ from a point infinitely away, from all charges, to the point, i.e.,

$$V = -\int_{\infty}^{r} \vec{E}.d\vec{r}$$

$$V = -\int_{\infty}^{r} \frac{\vec{F}}{q_o}.d\vec{r} \qquad V = -\frac{W}{q_0}$$

- It is scalar quantity, $V = \frac{W}{q_0} = [ML^2T^{-3}A^{-1}]$

  And SI unit is J/C or volt

  As $V = -\int \vec{E}.d\vec{r}$ i.e., $E = -\frac{dV}{dr}$

- If the field is produced by a point charge, then, $\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^3}\vec{r}}$

  and $V = -\int_{\infty}^{r} \vec{E}.d\vec{r}$

  so, $V = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^3}\vec{r}.d\vec{r}$

  i.e., $\boxed{V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}}$

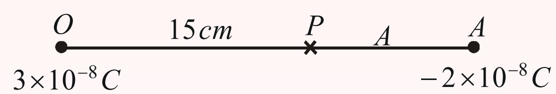
- **Equipotential surfaces:** For a given charge distribution, locus of all points having same potential is called 'equipotential surface'.
  - Equipotential surfaces can never cross each other.
  - Equipotential surfaces are always perpendicular to the lines of force.

  If a charge is moved from one point to the other over an equipotential surface, work done will be zero as

  $$W_{AB} = -U_{AB} = q(V_B - V_A) = 0 \quad [V_B = V_A]$$

Example-6: **Two charges $3\times10^{-8}C$ and $-2\times10^{-8}C$ are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero**.

Solution: Let us take the origin O at the location of the positive charge. The line joining the two charges is taken to be the x–axis, the negative charge is taken to be on the right side of the origin.



Let P be the required point on the x–axis where the potential is zero. If x is the x-coordinate of P, obviously x must be positive. (There is no possibility of potential due to the two charges adding up to zero for $x < 0$). If x lies between O and A, we have

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{3\times10^{-8}}{x\times10^{-2}} - \frac{2\times10^{-8}}{(15-x)\times10^{-2}}\right] = 0$$

where x is in cm. That is $\frac{3}{x}-\frac{2}{15-x}=0$

which gives $x = 9$ cm

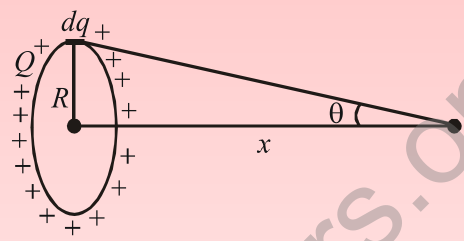
If x lies on the extended line OA, the required condition is

$$\frac{3}{x}-\frac{2}{x-15}=0$$

which gives $x = 45$ cm

❖ ELECTRIC POTENTIAL DUE TO SOME COMMON CONFIGURATIONS

(I) A charged ring

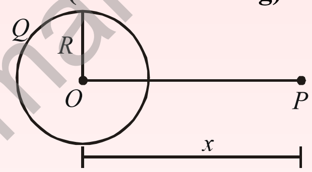


■ Let a ring of radius R is charged with positive charge Q. Consider a point P on its axis at a distance 'x' from the centre. Electric potential at P due to a small charge $'dq'$ is

$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R^2+x^2}}$$

$\therefore$ Net potential at P is $\boxed{V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{(R^2+x^2)^{\frac{1}{2}}}}$

(II) A solid charged sphere (non-conducting)



At Centre

$$V_o=\int_0^{\infty}dV=-\int_0^{\infty}Edx$$

$$V_{(\infty)}-V_{(0)}=-\left[\int_0^R Edx+\int_R^{\infty}Edx\right]$$

$$0-V_{(0)}=-\left[\int_0^R\frac{1}{4\pi\varepsilon_0}\frac{Qx}{R^3}dx+\int_R^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}dx\right]$$

$$V_{(0)}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\frac{R^2}{2}-\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\infty}-\frac{1}{R}\right]\quad=\frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}+\frac{Q}{4\pi\varepsilon_0 R}$$

Potential at center = $=\frac{3}{2}\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

At any point inside sphere

Now if any point 'A' is at 'x' distance from O inside sphere

$$V_A - V_o = -\left[\int_0^x E dx\right]$$

$$V_A - V_0 = -\int_0^x \frac{1}{4\pi\varepsilon_0}\frac{Qx}{R^3}dx$$

$$V_A - \frac{3}{2}\frac{Q}{4\pi\varepsilon_0 R} = \frac{-Q}{4\pi\varepsilon_0 R^3}\frac{x^2}{2}$$

$$V_A = \frac{Q}{(4\pi\varepsilon_0)^2 R^3}[3R^2 - x^2]$$

$$\therefore \quad \boxed{V_{inside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R^3}(3R^2 - x^2)}$$

Electric Potential Energy

➢ The electric potential energy of a system of point charges is defined as the work done in arranging this system of charges from an infinite separation from each other to their respective position.

It is assumed that their initial kinetic energy is zero.

If a point charge q_1 is present in an electric field where potential is V. By definition

$$V = \frac{U}{q_1} \Rightarrow U = q_1 V$$

The electric potential energy of two-point changes q_1 and q_2 at separation r_{12} is

$$\boxed{U_{12} = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r_{12}}}$$

So, in case of discrete distribution of charges $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{13}} + ...\right]$

and in case of continuous distribution of charge as $dU = Vdq$ or $U = \int Vdq$

■ **Electron volt (unit of energy):** When a charge q is accelerated through a potential difference of V volt, the energy gained by charge = qV.

"An electron volt is defined as the energy acquired by a particle having one electronic charge (e) when accelerated by 1 volt i.e., $1\ eV = 1.6 \times 10^{-19}\ J$.

Example-7: **A particle, of positive charge $Q = Ze$ where Z is an integer, having a fixed position P. Another charged particle of mass m and charge $q = e$ moves at a constant speed in a circle of radius r_1 with Centre at P. Find the work that must be done to increase the radius of circle to r_2.**

Solution: Let q orbit round Q in a circle of radius r.

K.E. of orbiting particle = $\frac{1}{2}mv^2$. ... (i)

where v is orbital velocity.

Potential energy of $q = -\frac{1}{4\pi\varepsilon_o}\frac{qQ}{r}$ .... (ii)

P.E. is negative since q is negative.

Electrostatic attraction on $q = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r^2}$ .... (iii)

This is used as centripetal force required for circular motion.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}.\frac{Qq}{r^2}$$

$$mv^2 = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r} \quad .... \text{(iv)}$$

From (1) and (4)

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_o}.\frac{Qq}{2r}$$

Total energy of the orbiting charge

$$= \text{K.E.} + \text{P.E.} = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_o}.\frac{Qq}{r}\right) - \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r} = -\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_o}.\frac{Qq}{r}$$

The total energy of q when in orbit of radius r_1

$$E_1 = -\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_o}.\frac{Qq}{r_1}$$

When it is in orbit of radius r_2

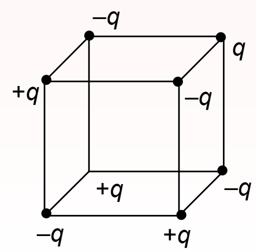
$$E_2 = -\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_o}.\frac{Qq}{r_2}$$

The work done on q = change in energy

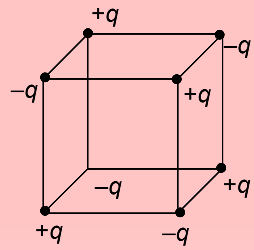
$$= E_2 - E_1 = -\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_o}\cdot\frac{Qq}{r_2} - \left(-\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_o}\cdot\frac{Qq}{r_1}\right) = \frac{Qq}{8\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{Ze^2}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad (\because \ Q = Z\,e \text{ and } q = e)$$

Example-8: **In the figure shown alternative positive and negative charges of magnitude *q* are placed at the corners of a cube of side length *a*. What is the work done in moving the charges far from each other?**



Solution: Initial electrostatics potential energy of the system



$$U_i = \frac{-12Kq^2}{a} + \frac{12Kq^2}{\sqrt{2}a} - \frac{4Kq^2}{\sqrt{3}\,a} = \frac{Kq^2}{a}\left[-12 + 6\sqrt{2} - \frac{4}{\sqrt{3}}\right] = \frac{Kq^2}{\sqrt{3}a}\left[6\sqrt{6} - 12\sqrt{3} - 4\right]$$

Final electrostatics potential energy of the system is zero.

i.e., $U_F = 0$

$$\therefore\ W = \frac{Kq^2}{\sqrt{3}a}\left[-6\sqrt{6} + 12\sqrt{3} + 4\right]$$

$$\therefore\ \text{Work done} = \frac{2Kq^2}{\sqrt{3}}\left[6\sqrt{3} + 2 - 3\sqrt{6}\right]$$

Example-9: **A charge q is placed on the surface of an originally uncharged soap bubble of radius R_0. Because of the mutual repulsion of the charged surface, the radius is increased to a somewhat larger value R. Show that $q = \left[\frac{32}{3}\pi^2\varepsilon_0 pR_0R(R^2 + R_0R + R_0^2)\right]^{1/2}$ in which p is the pressure of the atmosphere.**

Solution: Initial potential energy $= \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 R_0} = \frac{q^2}{8\pi\varepsilon_0 R_0}$

Potential energy after expansion $= \frac{q^2}{8\pi\varepsilon_0 R}$

Decrease of potential energy $= \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_0} - \frac{1}{R}\right) = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{R - R_0}{RR_0}\right)$

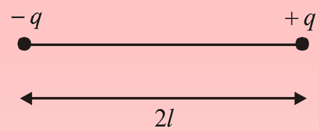
Work done against pressure $= 4\pi p\int_{R_0}^{R} r^2\,dr = \frac{4}{3}\pi p\left[r^3\right]_{R_0}^{R} = \frac{4}{3}\pi p\left[R^3 - R_0^3\right]$

Hence, $\frac{q^2}{8\pi\varepsilon_0}\left(\frac{R - R_0}{RR_0}\right) = \frac{4}{3}\pi p\left(R^3 - R_0^3\right)$

$$q = \left[\frac{32}{3}\pi^2\varepsilon_0 pRR_0\left(R^2 + R_0R + R_0^2\right)\right]^{1/2}$$

Electric Dipole

Two equal and opposite charges separated by a finite distance constitute an electric dipole. If $-q$ and $+q$ are charges at distance $2l$ apart, then dipole moment



$\vec{p} = q \times 2\vec{l}$ and $p = 2ql$

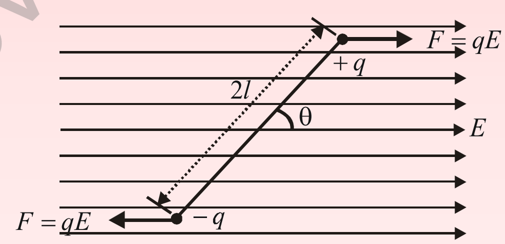
It has dimension $[LTA]$ and SI unit (coulomb $\times$ m).

It is a vector quantity whose direction is from –ve to +ve charge.

❖ DIPOLE IN ELECTRIC FIELD

- **Couple:** When a dipole is placed in a uniform electric field as shown in figure, the net force on it is zero.

  Force, $F = qE$ is acting on each charge but in opposite direction so they form a couple.



Net torque = moment of the couple

= Force $\times$ arm of couple

$= qE \times 2l \sin\theta$

$\therefore$ $\boxed{\tau = pE\sin\theta}$

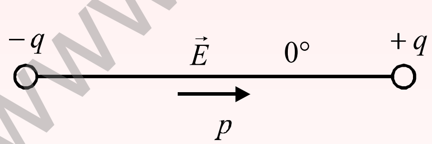
$\because$ $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$ when $\theta = 0$, $\tau = 0$,

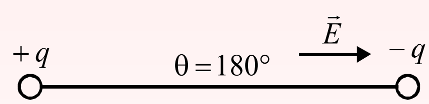
$\tau = \tau_{min}$

when $\theta = 90°$, $\tau = PE$

$\tau_{max} = PE$

- **Work:** Work done in rotating a dipole in uniform electric field through a small angle $d\theta$ will be





$dW = \tau \, d\theta$

$\int dW = \int_{\theta_1}^{\theta_2} pE\sin\theta \, d\theta$ $\quad W = -pE[\cos\theta_1 - \cos\theta_2]$

if $\theta_1 = 0$ and $\theta_2 = \theta$, then $W = pE[1 - \cos\theta]$

$\Rightarrow$ $W_{min} = 0$, when $\theta = 0$ i.e., dipole is parallel to the field.

$W_{max} = 2pE$ when $\theta = 180°$ i.e., dipole is antiparallel to the field.

When electric field is non-uniform net force on dipole is non-zero.

- **Potential Energy:** In case of a dipole, potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to any angle θ.

  If $\theta_1 = 90°, \theta_2 = \theta$

  $U = pE(0 - \cos\theta)$ i.e., $\boxed{U = -pE\cos\theta = -\vec{p}.\vec{E}}$

Electric Charges and Fields

Electric charges

Quantization of charge
All charges must be a multiple of charge (e)
$Q = ne$
Where n is an integer

- Two kinds of charges + ve and – ve
- S.I. unit is Coulomb (C)
  1 C = charge flowing through a wire in 1s if current in it is 1A
- $e = 1.602 \times 10^{-19}$C.

Conservation of charge
It is not possible to create or destroy net charge of an isolated system.

Electric Field

Electric field lines
- Always normal to conducting surface
- Lines originating from +ve charge
- Terminating at –ve charge
- Never intersect each other
- Do not form closed loop
- Are imaginary lines.

Electric field due to point charge
$\vec{E} = \frac{\vec{F}}{q} = k\frac{Q}{r^2}$ (N/C)

- Due to discrete distribution of charges
  $\vec{E} = \sum_{i}^{n} \vec{E}_i$
- Due to continuous distribution of charge
  $\vec{E} = k\int \frac{dQ}{r^3}\vec{r}$
  $|\vec{E}| = k\int \frac{dQ}{r^2}$

Coulomb's Law
- Force between two charged particles
  $\vec{F} = \frac{k\, q_1\, q_2 \vec{r}}{r^3} = \frac{kq_1\, q_2 \hat{r}}{r^2}$, force is attractive when q_1 and q_2 are of opposite sign, otherwise repulsive
  $|\vec{F}| = \frac{k\, q_1\, q_2}{r^2}$
  $k = \frac{1}{4\pi\varepsilon_0} \approx 9\times10^9\,\text{Nm}^2\text{C}^{-2}$
- Force between multiple charges
  $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \ldots\ldots = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$
  where $\hat{r}$ is the unit vector along charge particle under consideration
  $\hat{r}_{1i}$ = distance between 1st and i^{th} particle
  "The electric force with which two charges attract or repel one another are not affected by the presence of other charges". It is known as Superposition Principle.

Electric Dipole moment, $\vec{p} = q\vec{d}$
- Electric field due to dipole at axial position,
  $\vec{E} = \frac{1}{4\pi\,\varepsilon_0}\left(\frac{2p}{r^3}\right)\hat{r}$
- Electric field due to dipole at an equatorial position,
  $|E| = \frac{1}{4\pi\,\varepsilon_0}\left(\frac{p}{r^3}\right)$
- Torque on an electric dipole placed in an electric field $(\vec{E})$
  $\vec{\tau} = \vec{p}\times\vec{E} = p\,E\sin\theta$

Gauss' Theorem
$\oint \vec{E}.\vec{ds} = q_{in}/\varepsilon_0 = \phi$
where, q_{in} = net charge enclosed
ϕ = flux through a closed surface

Applications of Gauss' Law
- Electric Field due to linear charge distribution
  $E = \lambda/2\pi\,\varepsilon_0 r$,
  λ = linear charge density
- Electric Field due to a plane sheet of charge
  $E = \sigma/2\varepsilon_0$,
  σ = charge per unit area
- Electric Field due to a charged conducting plate
  $E = \sigma/\varepsilon_0$,
  σ = charge per unit area
- Electric Field inside a conductor = 0

Electric field due to uniformly charged spherical shell at :
- Outside point,
  $E = Q/4\pi\varepsilon_0 r^2$
- Internal point,
  $E = 0$
- The surface,
  $E = Q/4\pi\varepsilon_0 R^2$

Electric flux
$\Delta\phi = \vec{E}.\Delta\vec{S}$, $\Delta\vec{S}$ = area vector
$\vec{E}$ = electric field
$\phi = \int \vec{E}.\vec{ds}$

In 1985, Charles Augustin de Coulomb gave Coulomb's law

Trace the Mind Map
- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS

1. An electric shock is experienced on opening the door of a car due to
a) heating of car engine
b) motion of car
c) discharge of electric charge
d) None of the above 2

2. A soap bubble is given a negative charge, then its radius
a) decreases
b) increases
c) remains unchanged
d) nothing can be predicted as information is insufficient

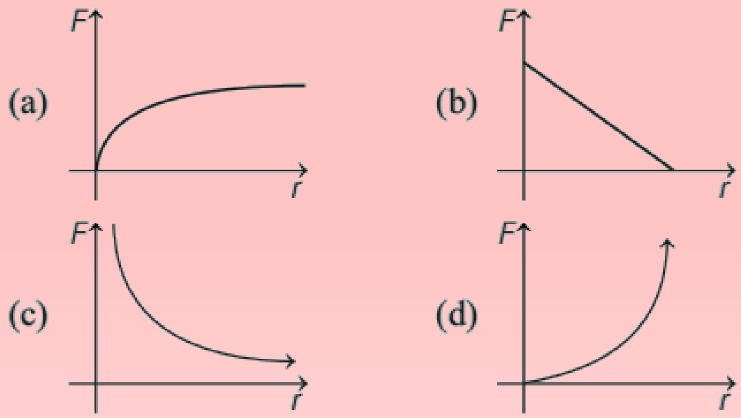
3. If two bodies are rubbed and one of them acquires q1 charge and another acquires q2 charge, then ratio of q1: q 2 is
a) 1: 2 b) 2: 1 c) – 1: 1 d) 1: 4

4. In charging by induction,
a) body to be charged must be an insulator
b) body to be charged must be a semiconductor
c) body to be charged must be a conductor
d) any type of body can be charged by induction

5. The Coulomb's law is based on the assumption that charges are treated as
a) positive charges
b) negative charges
c) neutral charges
d) point charges

6. The ratio of electric fields on the axis and at equator of an electric dipole will be
a) 1: 1 b) 2: 1 c) 4: 1 d) None of these

7. Force between two charges varies with distance between them as



8. If point charges Q1 7 = ′ 2 10- C and Q2 7 = ′ 3 10- C are at 30 cm separation, then find electrostatic force between them.

a) 2×10^{-3} N b) 6×10^{-3} N c) 510^{-3} N d) 1×10^{-3} N

9. The electric field at a point is

a) always continuous
b) continuous, if there is no charge at that point
c) continuous, if there is a charge at that point
d) None of the above

10. When the charge of a body becomes half, the electric field becomes

a) half b) twice c) thrice d) No change

11. A charged particle is free to move in an electric field. It will travel

a) always along a line of force
b) along a line of force, if its initial velocity is zero
c) along a line of force, if it has some initial velocity in the direction of an acute angle with the line of force
d) None of the above

12. Two field lines can never cross each other because

a) field lines are closed curves
b) field lines repel each other
c) field lines crowded only near the charge
d) field has a unique direction at each point

13. The surface charge density s of an area element ignores
a) the quantisation of charge
b) discontinuity in charge distribution at macroscopic level
c) discontinuity in charge distribution at microscopic level
d) Both (a) and (c)

14. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80 $\mu C m^{-2}$. What is the charge on the sphere?
a) 0.7×10^{-1} C b) 1.4×10^{-4} C c) 1.4×10^{-3} C d) 1.7×10^{-4} C

15. If ∫E. d's = 0 over a surface, then
a) the electric field inside the surface and on it is zero
b) the electric field inside the surface is necessarily uniform
c) the number of flux lines entering the surface must be equal to the number of flux lines leaving it
d) None of the above

16. Total electric flux coming out of a unit positive charge put in air is
a) ε_0 b) ε_0 ^-1 c) $(4p\ \varepsilon 0)^{-1}$ d) $4p\varepsilon_0$

17. If same charge q is placed inside a sphere and cube having radius 1m and side 2m, respectively. What will be the ratio of flux passing through them?
a) 1: 1 b) 1: 8 c) 8: 1 d) 1: 2

18. A hollow metal sphere of radius R is uniformly charged. The electric field due to the sphere at a distance r from the centre
a) zero as r increases for r < R, decreases as r increases for r >R
b) zero as r increases for r <R, increases as r increases for r> R
c) decreases as r increases for r <R and for r >R
d) increases as r increases for r <R and for r R> 20

19. What is the force between two small charged spheres having charges of $2 * 10^{-7} C$ and $3 * 10^{-7} C$ placed 30 cm apart in air?
a) $5 \times 10^{-2} N$
b) $6 \times 10^{-3} N$
c) $7 \times 10^{-3}\ N$
d) $8 \times 10^{-4} N$

20. Two-point charges q_A = 3mC and q_B = -3mC are located 20 cm apart in vacuum. What is the electric field at the mid-point O of the line AB joining the two charges?

a) 0

b) $2.7 * 10^6$ (N/C)

c) $5.4 * 10^6$ (N/C)

d) $10.2 * 10^6$ (N/C)

21. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

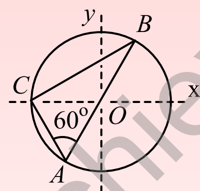
a) 0

b) $2 * 10^2 Nm^2/C$

c) $2 * 10^4 Nm^2/C$

d) $2 * 10^5 Nm^2/C$

22. Consider a system of three charges $\frac{q}{3}, \frac{q}{3}$ and $-\frac{2q}{3}$ placed at point *A*, *B* and *C*, respectively, as shown in the figure. Take *O* to be the centre of the circle of radius *R* and angle *CAB*=60°.



a) The electric field at point *O* is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis

b) The potential energy of the system is zero

c) The magnitude of the force between the charges at *C* and *B* is

$$\frac{q^2}{54\pi\varepsilon_0 R^2}$$

d) The potential at point *O* is

$$\frac{q}{12\pi\varepsilon_0 R}$$

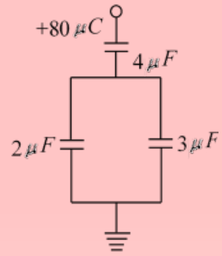
23. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is

a) $-\frac{Q}{4}(1 + 2\sqrt{2})$ b) $\frac{Q}{4}(1 + 2\sqrt{2})$ c) $-\frac{Q}{2}(1 + 2\sqrt{2})$ d) $\frac{Q}{2}(1 + 2\sqrt{2})$

24. Two conducting spheres of radii 5 cm and 10 cm are given a charge of 15μC each. After the two spheres are joined by a conducting wire, the charge on the smaller sphere is

a) 5μC b) 10μC c) 15μC d) 20μC

25. In the given circuit, a charge of $+80\ \mu C$ is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



a) $+32\ \mu C$ b) $+40\ \mu C$ c) $+48\ \mu C$ d) $+80\ \mu C$

26. A hollow sphere of charge does not produce an electric field at any

a) Point beyond 2 m
b) Point beyond 10 m
c) Interior point
d) Outer point

27. A point Q lies on the perpendicular bisector of an electrical dipole of dipole moment p. If the distance of Q from the dipole is r (much larger than the size of the dipole), then the electric intensity E at Q is proportional to

a) r^{-2} b) r^{-4} c) r^{-1} d) r^{-3}

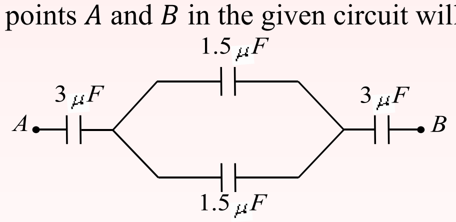
28. If an insulated non-conducting sphere of radius R has charge density ρ. The electric field at a distance r from the centre of sphere $(r < R)$ will be

a) $\frac{\rho R}{3\varepsilon_0}$ b) $\frac{\rho r}{\varepsilon_0}$ c) $\frac{\rho r}{3\varepsilon_0}$ d) $\frac{3\rho R}{\varepsilon_0}$

29. A soap bubble is given a negative charge, then its radius

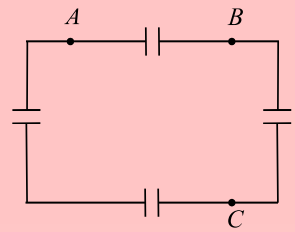
a) Decreases
b) Remains unchanged
c) Increases
d) Nothing can be predicted as information is insufficient

30. The capacitance between the points A and B in the given circuit will be



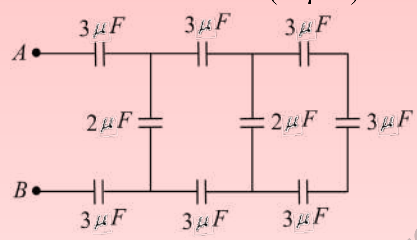
a) $1\ \mu F$ b) $2\ \mu F$ c) $3\ \mu F$ d) $4\ \mu F$

31. Four capacitors of each of capacity $3\mu F$ are connected as shown in the adjoining figure. The rati equivalent capacitance between A and B and between A and C will be



a) 4: 3 b) 3: 4 c) 2: 3 d) 3: 2

32. The equivalent capacitance between A and B is (in μF)



a) 25 b) $\frac{84}{25}$ c) 9 d) 1

33. For a dipole $q = 2 \times 10^{-6} C$ and $d = 0.01\ m$. Calculate the maximum torque for this dipole if E $5 \times 10^5\ N/C$

a) $1 \times 10^{-3} Nm^{-1}$ b) $10 \times 10^{-3} Nm^{-1}$ c) $10 \times 10^{-3} Nm$ d) $1 \times 10^2 Nm^2$

34. If the electric field given by $(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 9\hat{\mathbf{k}})$, the electric flux through a surface of area 20 unit lying in the *Y-Z* plane will be

a) 100 unit b) 80unit c) 180 unit d) 20 unit

35. An electron of mass m_e initially at rest moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p also initially at rest takes time t_2 to move through and equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio of t_2/t_1 is nearly equal to

a) 1 b) $(m_p/m_e)^{1/2}$ c) $(m_e/m_p)^{1/2}$ d) 1836

36. Two unlike charges of the same magnitude Q are placed at a distance d. The intensity of the electric field at the middle point in the line joining the two charges.

a) Zero b) $\frac{3Q}{4\pi\varepsilon_0 d^2}$ c) $\frac{6Q}{2\pi\varepsilon_0 d^2}$ d) $\frac{4Q}{4\pi\varepsilon_0 d^2}$

37. A solid conducting sphere of radius R_1 is surrounded by another concentric hollow conducting sphere of radius R_2. The capacitance of this assembly is proportional to

a) $\frac{R_2 - R_1}{R_1 R_2}$ b) $\frac{R_2 + R_1}{R_1 R_2}$ c) $\frac{R_1 R_2}{R_1 + R_2}$ d) $\frac{R_1 R_2}{R_2 - R_1}$

38. A total charge Q is broken in two parts Q_1 and Q_2 and they are placed at a distance R from each other. The maximum force of repulsion between them will occur, when

a) $Q_2 = \frac{Q}{R}, Q_1 = Q - \frac{Q}{R}$

b) $Q_2 = \frac{Q}{4}, Q_1 = Q - \frac{2Q}{3}$

c) $Q_2 = \frac{Q}{4}, Q_1 = \frac{3Q}{4}$

d) $Q_1 = \frac{Q}{2}, Q_2 = \frac{Q}{2}$

39. Two-point charges + 8q and -2q are located at $x = 0$ and $x = L$ respectively. The location of a point on the x- axis at which the net electric field due to these two-point charges is zero is

a) 2L b) L/4 c) 8L d) 4L

40. An electric dipole in a uniform electric field experiences (When it is placed at an angle θ with the field)

a) Force and torque both

b) Force but no torque

c) Torque but no force

d) No force and no torque

------------------------ ANSWER KEY ------------------------

1)	b	2)	b	3)	c	4)	d
5)	d	6)	c	7)	b	8)	b
9)	a	10)	a	11)	d	12)	b
13)	a	14)	c	15)	c	16)	a
17)	a	18)	b	19)	b	20)	c
21)	b	22)	c	23)	b	24)	b
25)	c	26)	c	27)	d	28)	c
29)	b	30)	a	31)	a	32)	d
33)	a	34)	b	35)	b	36)	d
37)	d	38)	d	39)	a	40)	c

HINTS AND SOLUTIONS

1. **(b)**
The reason for these experiences is the discharge of electric charges through our body, which was accumulated due to the rubbing of insulating surface

2. **(b)**
Whether the bubble is given negative charge or positive charge, the radius will increase in both cases because when positive charge is given to it, again the charges will repel each other and this will expand the bubble and the radius will increase.

3. **(c)**
Based on formula

4. **(d)**
Theory

5. (d)
Coulomb's Law is applicable only for point charges.

6. **(c)**
The force between the electrons can be determined from the Coulomb's law of electric charges. This law is also called the inverse square law. Where, the force is inversely proportional to the square of the distance. And the force is also called the electrostatic force.

7. (b)
∵ Electrostatic force,
$F = \frac{kQ1Q2}{r2}$

$= 9\times10^{9}\times2\times10^{-7}\times3\times10^{-7} / (30\times10^{-2})^{2}$
$= 6\times10^{-3}$ N

8. (b) and (c)

The region of space in which an electric charge experiences a force is known as the Electric field.

The direction of the electric field is along the direction in which the positive test charge placed in the field would tend to move if free to do so.

The electric field at a point will be continuous only if it is unaffected by other charges at that point.

If a charged particle is placed in its pathway, the electric field lines would start to converge or diverge by losing their continuity.
The electric field because of any charge will be discontinuous if there is any other charge present in that medium.

9. (b)
As E = $q/4\pi\varepsilon0r^{2}$
If q′=2q, then E′=2E
So electric field is doubled

10. (a)

When we apply an electric field(E) across a charge particle, then force acting on it is given by

F=qE

∴ The force acting in the same direction of electric field.

∴ The charge particle always moves along electric line of force.

11. (d)
Electric lines of force never intersect each other because at the point of intersection, two tangents can be drawn to the two lines of force. This means two directions of electric field at the point of intersection, which is not possible.

12. (b)

13. (a)
The surface charge density as stated ignores charge quantification and charge distribution discontinuities at the microscopic level, whereas the macroscopic surface charge density is smoothed out average of the microscopic charge density across an area element S, which is huge microscopically but small macroscopically.

14. (c)
d=2.4m

r=1.2m

Surface charge density, σ=80.0 μ C/m^2

=80×10^{-6} μ C/m^2

Total charge on surface of sphere,

Q= Charge density × Surface area

= $\sigma\times4\pi r2$

= $80\times10^{-6}\times 4 \times 3.14 \times (1.2)^2$

=1.447×10^{-3} C

Therefore, the charge on the sphere is 1.447×10^{-3} C

15. (c) and (d)
As ∫→E.→ ds = qϵ0
Where q is charge enclosed by the surface.
when ∫→E.→ ds =0 ,q=0
i.e. net charge enclosed by the surface must be zero.
Therefore, charges may be inside the surface.
Example: +q and -q.

16. (a)
Total flux coming out from unit charge

$$= \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0}\times 1 = \varepsilon_0^{-1}$$

17. (a)
The total flux linked with a closed surface is ε01 times the charge enclosed by the closed surface.
enclosed Flux, $\phi=\varepsilon_0$ q enclosed
Since enclosed charge is same, so the flux will be same.
flux through sphere = flux through cube ⇒1:1

18. (b)
Charge Q will be distributed over the surface of hollow metal sphere.
(i) For r < R (inside)
By Gauss law,

$$\oint Ein^{-}.dS = \varepsilon0qen = 0$$

$$\Rightarrow Ein = 0 \qquad (\because qen = 0)$$

(ii) For r > R (out side)

$$\oint E^{-}0.dS = \varepsilon0qen$$

$$Here,\ qen = Q(\because qen = Q)$$

$$\therefore E04\pi r2 = \varepsilon0Q$$

$$\therefore E0 \propto r21$$

19. (b)

Charge on the first sphere, q1 = 2×10^{-7} C

Charge on the second sphere, q2 = 3×10^{-7} C

Distance between the spheres, r=30cm=0.3m

F = $q1q2/4\pi\varepsilon0r^2$

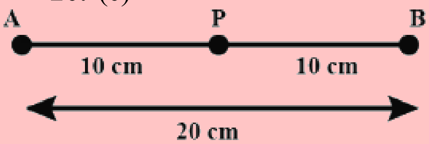
Where, ε0= Permittivity of free space

F = $2\times10^{-7}\times3\times10^{-7}/(0.3)^2\ 9\times10^9$

= 6×10^3 N

Hence, force between the two small charged spheres is 6×10^{-3} N. The charges are of same nature. Hence, force between them will be repulsive.

20. (c)



$$E_A = \frac{1}{4\pi\epsilon_0}\frac{q_A}{r^2}$$

$$E_A = \frac{9\times 10^9 \times 3 \times 10^{-6}}{(0.1)^2}$$

$$= \frac{27\times 10^3}{(0.1)^2} = \frac{27\times 10^3}{0.01}$$

$$E_A = 2700 \times 10^3 NC^{-1} = 2.7 \times 10^6 NC^{-1}$$

$$E_A = E_B = 2.7 \times 10^6 NC^{-1}$$

Resultant field,
ER = EA + EB= $2.7 \times 10^6 + 2.7 \times 10^6$
ER = 5.4×10^6 N/C

21. (b)
Net electric flux (ΦNet) through the cubic surface is
∈0 = Permittivity of free space
= 8.854×10^{-12} N^{-1} C^2 m^{-2} q
= Net charge contained inside the cube
= 2.0 μC = 2×10^{-6} C
∴ = 2.2 × 105 N m^2 C^{-1}
The net electric flux through the surface is 2.2×10^5 N m^2 C^{-1}.

22. (c)
Distance

$$BC = AB\sin 60° = (2R)\frac{\sqrt{3}}{2} = \sqrt{3}R$$

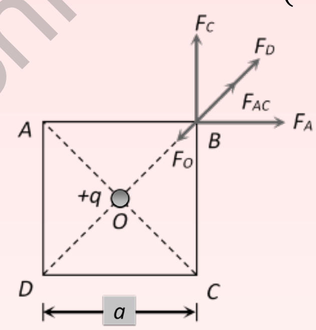
$$|F_{BC}| = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{\left(\sqrt{3}R\right)^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}$$

23. (b)

If all charges are in equilibrium, system is also in equilibrium.
Charge at centre : charge q is in equilibrium because no net force acting on it corner charge :
If we consider the charge at corner B. This charge will experience following forces

$F_A = k\frac{Q^2}{a^2}, F_C = \frac{kQ^2}{a^2}, F_D = \frac{kQ^2}{\left(a\sqrt{2}\right)^2}$ and

$$F_O = \frac{KQq}{\left(a\sqrt{2}\right)^2}$$



Force at B away from the centre $= F_{AC} + F_D$

$$= \sqrt{F_A^2 + F_B^2} + F_D = \sqrt{2}\frac{kQ^2}{a^2} + \frac{kQ^2}{2a^2}$$

$$= \frac{kQ^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

Force at B towards the centre $= F_O = \frac{2kQq}{a^2}$
For equilibrium of charge at $B, F_{AC} + F_D = F_O$

$$\Rightarrow \frac{kQ^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)=\frac{2KQq}{a^2} \Rightarrow q = \frac{Q}{4}(1+2\sqrt{2})$$

24. (b)

Charge on smaller sphere

$$= \text{Total charge}\left(\frac{r_1}{r_1+r_2}\right)$$

$$= 30\left(\frac{5}{5+10}\right) = 10\mu C$$

25. (c)

$$q_3 = \frac{C_3}{C_2+C_3}.Q$$

$$q_3 = \frac{3}{3+2}\times 80 = \frac{3}{5}\times 80$$

$$= 48\ \mu C$$

26. (c)

Inside the hollow charged spherical conductor electric field is zero

27. (d)

On equatorial line of electric dipole, $\propto \frac{1}{r^3}$

28. (c)

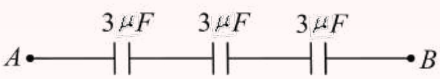
For non-conducting sphere

$E_{in} = \frac{k.Qr}{R^3} = \frac{\rho r}{3\varepsilon_0}$

29. (b)

Every system tends to decrease its potential energy to attain more stability when we increase charge on soap bubble its radius increases

30. (a)



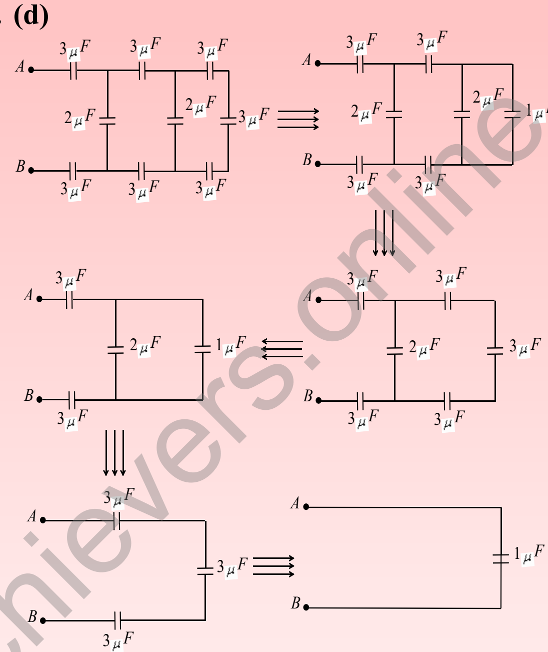
$$\frac{1}{C_{AB}} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1 \Rightarrow C_{AB} = 1\mu F$$

31. a)

$$C_{AB} = 3+\frac{3}{3} = 4\mu F\ , C_{AC} = \frac{3}{2}+\frac{3}{2} = 3\mu F$$

$$\therefore C_{AB} : C_{AC} = 4 : 3$$

32. (d)



Starting from the right end of the network, three $3\mu F$ capacitors are connected in series. The equivalent capacitance of these three capacitors is

$$\frac{1}{C_{AB}} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} \Rightarrow C_{AB} = 1\mu F$$

33. (c)

$$\tau_{\max} = pE = q(2l)E = 2\times 10^{-6}\times 0.01\times 5\times 10^5$$

$$= 10\times 10^{-3} N-m$$

34. (a)

Electric flux is equal to the product of an area element and the perpendicular component of **E**. As the surface is lying in Y-Z plane

$\therefore \boldsymbol{E}.d\boldsymbol{A} = \phi = (5)(20)$

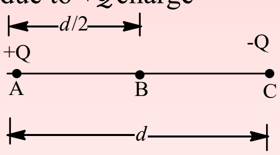
$= 100$ unit.

35. (b)

For electron $s = \frac{eE}{m_e} \times t_1^2$, For proton $s = \frac{eE}{m_p} \times t_2^2$

$$\frac{t_2^2}{t_1^2} = \frac{m_p}{m_e} \Rightarrow \frac{t_2}{t_1} = \sqrt{\frac{m_p}{m_e}} = \left(\frac{m_p}{m_e}\right)^{1/2}$$

at a distance *d*. Electric field at Centre (B) due to $+Q$charge



$$(E_1) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\frac{d}{2}\right)^2}$$

36. (b)

Two equal and opposite charges are placed

Similarly, electric field due to $-Q$charge

$$(E_2) = \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\left(\frac{d}{2}\right)^2}$$

Therefore, net electric field at point

$E = E_1 + E_2$

$$\frac{1}{4\pi\varepsilon_0}\frac{8Q}{d^2}$$

37. (d)

Capacitance of the given assembly

$$C = 4\pi\varepsilon_0\left(\frac{R_1R_2}{R_2 - R_1}\right) \Rightarrow C \propto \frac{R_1R_2}{(R_2 - R_1)}$$

38. (d)

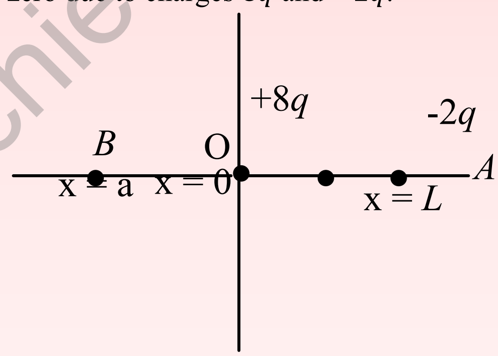
$Q_1 + Q_2 = Q$ ...(i) and $F = k\frac{Q_1Q_2}{r^2}$ ...(ii)

From (i) and (ii) $F = \frac{kQ_1(Q-Q_1)}{r^2}$

For F to be maximum $\frac{dF}{dQ_1} = 0 \Rightarrow Q_1 = Q_2 = \frac{Q}{2}$

39. (a)

Suppose that a point B, where net electric field is zero due to charges $8q$ and $-2q$.



$$\boldsymbol{E}_{BO} = \frac{-1}{4\pi\varepsilon_0}.\frac{8q}{a^2}\hat{\boldsymbol{i}}$$

$$\boldsymbol{E}_{BA} = \frac{1}{4\pi\varepsilon_0}.\frac{+2q}{(a+L)^2}\hat{\boldsymbol{I}}$$

According to condition $\boldsymbol{E}_{BO} + \boldsymbol{E}_{BA} = 0$

$$\therefore \frac{1}{4\pi\varepsilon_0}\frac{8q}{a^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{(a+L)^2}$$

$$\Rightarrow \frac{2}{a} = \frac{1}{a+L}$$

$\Rightarrow 2a + 2L = a$

$\therefore \;\; 2L = -a$

Thus, at distance $2L$ from origin, net electric field will be zero.

40. (c)

In uniform electric field dipole experience only torque but no force