CHAPTER 08

Percentage

Percentage is a method of expressing fractions or parts of any quantity into a equal form. It is expressed in terms of hundredths.

Percent

The word per cent means per hundred. Thus 21 per cent means 21 parts out of 100 parts, which can also be written as $\frac{21}{100}$. Therefore, per cent is a fraction whose denominator is 100 and the numerator of this fraction is called the rate per cent.

So, $\frac{21}{100} = 21\%$. Here 21 per cent is the rate. The sign for per cent is %.

(i) To convert a fraction or a decimal or a whole number into a per cent, multiply if by 100.

(ii) To convert a per cent to a fraction or a decimal, divided it by 100 and delete the % sign.

Also, any fraction or decimal can be converted into its equivalent percentage by multiplying with 100%.

$1\% = \frac{1}{100}$	$12\frac{1}{2}\% = \frac{1}{8}$	$16\% = \frac{4}{25}$	$83\frac{1}{3}\% = \frac{5}{6}$
$2\% = \frac{1}{50}$	$16\frac{2}{3}\% = \frac{1}{6}$	$64\% = \frac{16}{25}$	
$4\% = \frac{1}{25}$	$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$133\frac{1}{3}\% = \frac{4}{3}$
$5\% = \frac{1}{20}$	$25\% = \frac{1}{4}$	$60\% = \frac{3}{5}$	$66\frac{2}{3}\% = \frac{2}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$33\frac{1}{3}\% = \frac{1}{3}$	$80\% = \frac{4}{5}$	$8\% = \frac{2}{25}$
$8\frac{1}{3}\% = \frac{1}{12}$	$50\% = \frac{1}{2}$	$37\frac{1}{2}\% = \frac{3}{8}$	
$10\% = \frac{1}{10}$	$100\% = \frac{1}{1}$	$87\frac{1}{2}\% = \frac{7}{8}$	

Note: Every number is 100% of itself.

Important Tips/Formulae

- If $A's$ income is $x\%$ more than that of B, B' s income is less than that of A by $\left[\frac{100 \cdot x}{100+x}\right]\%$.
- If $A's$ income is $x\%$ less than that of $B, B's$ income is more than that of A by $\left[\frac{100 \cdot x}{100-x}\right]\%$.
- Suppose the price of a certain commodity increases by $R\%$ and expenditure on this commodity remains the same. Then reduction percentage in consumption $= \left\{\frac{R}{100+R} \times 100\right\}\%$
- Suppose the price of commodity decreases by $R\%$ and expenditure on this commodity remains the same. Increase percentage in consumption $= \left\{\frac{R}{100-R} \times 100\right\}\%$

Solved Examples:

1. 60% of a number is 24 less than $3/4$ th of that number. Find the number

(a) 150
(b) 180
(c) 160
(d) 175

Sol. (c) Let the required number be x.

Then, by given condition

$$60\% \text{ of } x = \frac{3}{4}x - 24$$

$$\Rightarrow \frac{60}{100} \times x = \frac{3x}{4} - 24 \Rightarrow \left(\frac{3x}{4} - \frac{3x}{5}\right) = 24$$

$$\Rightarrow 15x - 12x = 480 \Rightarrow \ \Rightarrow x = 160$$

2. If A earns 10% more than B ' s, then how many per cent less does B earn than A ?

(a)$11\frac{1}{3}\%$
(b)$11\frac{1}{9}\%$
(c)13%
(d) $12\frac{1}{2}\%$

Sol. (b) Required percentage $= \left(\frac{R}{100-R} \times 100\right)\%$

$$= \frac{10}{100-10} \times 100$$
$$= \frac{10}{90} \times 100 = 11\frac{1}{9}\%$$

3. If the price of the cooking gas increased by 15%, by how many per cent should a family reduce its consumption so as not to exceed its budget on cooking gas?

(a) 14%
(b) 13%
(c) 17%
(d) $13\frac{1}{23}\%$

Sol. $(\boldsymbol{d})$ Suppose the price of cooking gas $= ₹100$

Increased price $= ₹115$

$\Rightarrow$ ₹115, he should reduce ₹ 15

$\Rightarrow$ ₹1, he should reduce ₹ $\frac{15}{115}$

$\Rightarrow$ ₹100 he should reduce ₹ $\frac{15}{115} \times 100 = 13\frac{1}{23}\%$

Alternate Method

Reduction percentage in consumption

$$= \left\{\frac{R}{(100+R)} \times 100\right\}\%$$
$$= \frac{15}{115} \times 100 = 13\frac{1}{23}\%$$

Practice Questions

1. The length of a rectangle is increased by 40% and its breadth decreased by 30%. The change in the area of the rectangle is

(a) 2% increase
(b) 2% decrease
(c) 10% increase
(d) 10% decrease

2. The ratio of the number of boys and girls in a government aided school is 3: 2.20% of the boys and 25% of the girls are not scholarship holders. The percentage of students who are scholarship holders is?

(a) 70%
(b) 48%
(c) 60%
(d) 78%

3. The side of a square increases by 10%, then find, by what per cent, does its area increase?

(a) 27
(b) 30
(c) 19
(d) 21

4. A number increased by $137\frac{1}{2}\%$ and the increment is 33. The number is

(a) 27
(b) 22
(c) 24
(d) 25

5. *A* 's salary is 20% less than *B* 's salary. Then, *B* 's salary is more than *A* 's salary by

(a) $33\frac{1}{3}\%$
(b) $16\frac{2}{3}\%$
(c) 20%
(d) 25%

6. In an office 40% of the staff is female, 40% of the female and 60% of the male voted for me. The percentage of votes I got was

(a) 24
(b) 42
(c) 50
(d) 52

7. The cost of an article worth ₹100 is increased by 10% first and again increased by 10%. The total increase in rupees is

(a) 20
(b) 21
(c) 110
(d) 121

8. In measuring the sides of a rectangle, there is an excess of 5% on one side and 2% deficit on the other. Then, the error per cent in the area is

(a) 3.3
(b) 3
(c) 2.9
(d) 2.7

9. In an examination, 1100 boys and 900 girls appeared. 50% of the boys and 40% of the girls passed the examination. The percentge of candidates who failed is

(a) 45
(b) 45.5
(c) 50
(d) 54.5

10. When the price of cloth was reduced by 25%, the quantity of cloth sold increased by 20%. What was the effect on gross receipt of the shop?

(a) 5% increase
(b) 5% decrease
(c) 10% increase
(d) 10% decrease

11. 20% of (50% of 5000) is

(a) 400
(b) 600
(c) 250
(d) 500

12. 23% of 8040 + 42% of 545 =? % of 3000

(a) 56.17
(b) 63.54
(c) 69.27
(d) 71.04

13. ? % of $8745 = 5159.55$

(a) 47
(b) 49
(c) 54
(d) None of these

14. If 16% of x is same as 12% of 48, then $x = ?$

(a) 24
(b) 36
(c) 32
(d) 40

15. 80% of 1450 is

(a) 1160
(b) 1235
(c) 1045
(d) 1250

16. 0.008 is what per cent of 0.2?

(a) 0.4
(b) 2
(c) 40
(d) 4

17. If x% of $\frac{25}{2}$ is 150, then the value of x is

(a) 1000
(b) 1200
(c) 1400
(d) 1500

18. 10% of 15% of 20% of ₹500 is

(a) ₹ 225
(b) ₹ 150
(c) ₹ 67
(d) ₹ 1.50

19. If 35% of a number is 12 less than 50% of that number, then the number is

(a) 80
(b) 60
(c) 50
(d) 40

20. What per cent of 400 is 60?

(a) 6
(b) 12
(c) 15
(d) 20

21. A reduction of 20% in the price of sugar enable a purchaser to obtain 3kg more for ₹ 120. The original price of sugar per kg is

(a) ₹ 15
(b) ₹ 12
(c) ₹ 8
(d) ₹ 10

22. If x earns 25% less than y, what per cent more does y earn, then x ?

(a) 25%
(b) 15%
(c) $33\frac{1}{3}\%$
(d) $16\frac{2}{3}\%$

23. A number increased by $137\frac{1}{2}\%$ gives 33, then the number is

(a) 16
(b) 24
(c) 20
(d) 28

24. An ore contains 26% copper. To get 91 kg of copper the quantity of the ore required is

(a) 350 kg
(b) 250 kg
(c) 240 kg
(d) 450 kg

25. The price of petrol went up by 25%. In order that expenses on petrol should not increase. One must reduce travel by

(a) 25%
(b) 20%
(c) 18%
(d) 15%

26. In an examination, 35% of the students passed and 455 failed. How many students appeared for the examination?

(a) 490
(b) 700
(c) 1300
(d) 845

ANSWERS

1. (b)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (d)
11. (d)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (c)
21. (d)	**22.** (c)	**23**. (b)	**24.** (a)	**25.** (b)	**26.** (b)				

Hints & Solutions

1. Here, $x = 40, y = 30$

Required change in the area of rectangle

$$= \left(x - y - \frac{xy}{100}\right)\%$$

$$= \left(40 - 30 - \frac{40 \times 30}{100}\right)\%$$

$$= -2\%$$

Where negative sign shows the decrement.

2. Let number of boys and girls in the school be 3x and 2x, respectively making 5x as the total number of students.

Now, number of students who are scholarship holders

$= 80\%$ of $3x + 75\%$ of $2x$

$= 2.4x + 1.5x = 3.9x$

Therefore, required percentage

$$= \frac{3.9x}{5x} \times 100\%$$
$$= 78\%$$

3. Required percentage increment $= \left(2x + \frac{x^2}{100}\right) = \left(2 \times 10 + \frac{(10)^2}{100}\right) = 20 + 1 = 21\%$

4. Let the number be x.

Then, 137.5% of $x = 33$

$\therefore\ x = \frac{33 \times 100}{137.5} = 24$

5. Required percentage $= \frac{20}{100-20} \times 100\%$

$= \frac{20}{80} \times 100\% = 25\%$

6. Let the total number of employes $= x$

Number of female employes $= 40\%$ of $x = \frac{4x}{10}$ and number of male employes $= \frac{6x}{10}$

∴ Number of my votes

$$= 40\% \text{ of } \frac{4x}{10} + 60\% \text{ of } \frac{6x}{10}$$
$$= \frac{16x}{100} + \frac{36x}{100} = \frac{52x}{100}$$

∴ Required percentage $= 52\%$

7. $x = 10, y = 10$

∴ Total increase

$$= \left[10 + 10 + \frac{10 \times 10}{100}\right]\% = 21\%$$

8. $x = 5\%, y = -2\%$

∴ Required error per cent in the area

$$= \left[x + y + \frac{xy}{100}\right]$$
$$= 5 - 2 - \frac{5 \times 2}{100} = 2.9\%$$

9. Number of total candidates $= 1100 + 900 = 2000$

Number of boys passed the examination $= 50\%$ of $1100 = 550$

Number of girls passed the examination $= 40\%$ of $900 = \frac{900 \times 40}{100} = 360$

Total number of candidates passed the examination

$= 550 + 360 = 910$

Total number of candidates failed the examination $= 2000 - 910 = 1090$

∴ Required percentage $= \frac{1090}{2000} \times 100 = 54.5\%$

10. $x = -25\%, y = 20\%$

∴ Required effect

$$= \left[-25 + 20 - \frac{25 \times 20}{100}\right]\%$$
$$= -10\% \text{ or } 10\% \text{ decrease}$$

11. 20% of (50% of 5000)

$= 20\%$ of $\left(\frac{50}{100} \times 5000\right)$

$= 20\%$ of 2500

$$= \frac{20 \times 2500}{100} = 500$$

12. ? % of $3000 = 23\%$ of $8040 + 42\%$ of 545

$$\Rightarrow ?\times\frac{3000}{100}$$

$$= 8040 \times \frac{23}{100} + 545 \times \frac{42}{100}$$

$$\Rightarrow ?\times \frac{3000}{100} = 1849.2 + 228.9$$

$$\Rightarrow ?\times 30 = 2078.1$$

$$\Rightarrow ? = \frac{2078.1}{30}$$

$$\Rightarrow ? = 69.27$$

13. ? % of $8745 = 5159.55$

$$\Rightarrow 8745 \times \frac{?}{100} = 5159.55$$

$$\Rightarrow ? = \frac{515955}{8745} = 59$$

14. 16% of $x = 12\%$ of 48

$$\Rightarrow x \times \frac{48}{100} = \frac{48 \times 12}{100}$$

$$\therefore x = \frac{48 \times 12}{16} = 3 \times 12 = 36$$

15. $\frac{80}{100} \times 1450 = 1160$

16. Let $x\%$ of $0.2 = 0.008$

$$\Rightarrow \frac{x}{100} \times 0.2 = 0.008$$

$$\Rightarrow x = \frac{0.008 \times 100}{0.2} \Rightarrow x = 4$$

17. $\frac{x}{100} \times \frac{25}{2} = 150$

$$\Rightarrow x = \frac{150 \times 100 \times 2}{25} \Rightarrow x = 1200$$

18. 10% of 15% of 20% of ₹500

$= \frac{10}{100} \times \frac{15}{100} \times \frac{20}{100} \times 500 = ₹1.50$

19. Let the number be x. Then,

35% of x = 50% of $x - 12$

$\Rightarrow \frac{35}{100} \times x = \frac{x}{100} \times 50 - 12$

$\Rightarrow 35x = 50x - 1200$

$\Rightarrow x = 80$

20. Let x% of 400 is 60

i.e., $\frac{x}{100} \times 400 = 60$

$\Rightarrow x = \frac{60 \times 100}{400} = 15 \Rightarrow x = 15$

21. Let the original price be ₹ x per kg.

Reduced price = ₹(80% of x)

$= ₹\frac{80}{100} \times x = ₹\frac{4x}{5}$

$\frac{120}{\frac{4x}{5}} - \frac{120}{x} = 3 \Rightarrow \frac{120 \times 5}{4x} - \frac{120}{x} = 3$

$\Rightarrow 3x = (150 - 120) = 30$

$\Rightarrow x = ₹10$ per kg

22. Let y earns a% more than x.

$a\% = \left(\frac{25}{100 - 25} \times 100\right)\%$

$a\% = \left(\frac{25}{75} \times 100\right)\%$

$a\% = \frac{100}{3}\%$

$\Rightarrow a\% = 33\frac{1}{3}\%$

23. Let the number be x, then according to question

$$x + 37.5\% \text{ of } x = 3$$

$$x + \frac{3}{8} \times x = 33$$

$$x\left(1 + \frac{3}{8}\right) = 33$$

$$x \times \frac{11}{8} = 33 \Rightarrow x = 24$$

$$\Rightarrow x\left(1 + \frac{3}{8}\right) = 33$$

24. Since $26\% \equiv 91$ kg

Hence, $100\% = \frac{91}{26} \times 100$

$100\% = 350$ kg

So, 350 kg ore is required.

25. One must reduce travel by $x\%$.

Then $x\% = \left(\frac{25}{100+25} \times 100\right)\%$

$$x\% = \left(\frac{25}{125} \times 100\right)\%$$

$$x\% = 20\%$$

26. Total number of failed students is

$$445 \equiv (100 - 35)\%$$

$$445 \equiv 65\%$$

So, the total number of students appeared for the examination is given by $100\% = \frac{455}{65} \times 100$

$$= \frac{91}{13} \times 100 = 700 \text{ student}$$