CHAPTER 01

Number System

Numbers are everywhere in our daily life for counting, measuring, labeling, ordering (sequencing) and coding. In the most popular, the Hindu-Arabic system, we use the symbol 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These symbols are called digits. Out of these ten digits, 0 is called an insignificant digit whereas others are called significant digits.

Number

A mathematical symbol represented by a set of digits called a number. Every digit has a face value which equals the value of the digit itself, irrespective of its place in the numeral. Each digit in a number or numeral has a place value besides its face value. We can express a number in two ways i.e., in words and in figures. e.g., If a number figure is 3290, then we can write this number in words as 'three thousand two hundred and ninety'.

Face Value and Place Value of the Digits in a Number

Face Value (Real Value)

In a number, the face value of a digit is the value of the digit itself. The place of the digit is not considered in the number. e.g., In the number 43857, the face value of 4 is 4 , the face value of 3 is 3 and so on.

Place Value (Local Value)

In a number, the place value of a digit changes according to the change of its place.

Place value of unit digit = (Unit digit) $\times$ 1

Place value of tens digit = (Tens digit) $\times$ 10

Place value of hundreds digit = (Hundreds digit) $\times$ 100

Place value of thousands digit = (Thousands digit) $\times$ 1000 and so on.

e.g., In the number 48379, the place value of 4 is 4×10000 i.e., 40000, the place value of 8 is 8×1000 i.e., 8000 and so on.

Ascending Order

A number is said to be in ascending order, if they are arranged with smallest number to greatest number. **e.g.,** 2, 5, 7, 9, 13 are in ascending order.

Descending Order

A number is said to be in descending order, if they are arranged with greatest number to smallest number.

e.g., 13, 9,7, 5, 2 are in descending order.

Successor

A number (say b) is said to be a successor of another number (say a) if $a+1=b$

Predecessor

A number (say a) is said to be a predecessor of another number (say b) if $b-1=a$

Type of Numbers

Natural Numbers: Counting number is known as natural number. **e.g.,** 1, 2, 3, 4, ..., ∞

Whole Numbers: Natural number including zero (0) is known as whole numbers. **e.g.,** 0,1,2,3, ... , ∞

Integers: The set of positive and negative whole numbers including 0 is known as integers.

e.g., ..., - 3, - 2, - 1, 0, 1, 2, 3…,

Rational Numbers: The numbers of the form $\frac{p}{q}$, where $q \neq 0$ and p and q is integer is known as rational number. **e.g.,** $\frac{5}{1}, -\frac{2}{1}, -\frac{5}{7}, \frac{0}{1}, \frac{1}{5}, \frac{3}{5}$ etc.

Irrational Numbers: The number, which is non-repeating and non-terminating, is called irrational number.

e.g., $\sqrt{3}, \sqrt{5}, \sqrt{2}$ etc.

Real Numbers: All rational and irrational numbers are known as real numbers.

e.g., $5,7,3,\frac{1}{2},\frac{1}{5},\frac{12}{17},\sqrt{3},\sqrt{5}$ etc.

π is an irrational number.

Even Numbers: The number, which is multiple of or divisible by 2 , is called even number.

e.g., 2,4,6,8, ...

Odd Numbers: The number, which is neither multiple of 2 nor divisible by 2 , is called odd number.

e.g., 1,3,5,7, ...

Prime Numbers: The number, which has only two factors i.e., 1 and itself, is called prime number.

e.g., 2, 3, 5, 7, 11, 13, ...

2 is only even number which is prime.

Composite Numbers: The number, which has more than two factors, is known as composite number.

e.g., 4,8,9,10, ...4 is composite number because it is divisible by 1,2 and 4.

Consecutive Numbers: These are series of numbers differing by 1 in ascending or descending order.

e.g., $12,13,14,15, \ldots$

Similarly, example of consecutive even numbers are $4,6$, $8,10, \ldots ,22,24,26,28$ ... and so on.

e.g. of consecutive prime numbers are $7,11,13,17, \ldots$

Decimal Numbers: A collection of digits $(0,1,2,3, \ldots ,9)$ after a point (called the decimal point) is called a decimal fraction.

A number containing a decimal point is called a decimal number.

Every decimal fraction represents a fraction. These

fractions have denominators with powers of 10. As

$$\begin{aligned} 35.467 &= (3 \times 10^1) + (5 \times 10^0) + \frac{4}{10^1} + \frac{6}{10^2} + \frac{7}{10^3} \\ &= (3 \times 10) + (5 \times 1) + \frac{4}{10} + \frac{6}{100} + \frac{7}{1000} \\ &= 30 + 5 + \frac{4}{10} + \frac{6}{100} + \frac{7}{1000} \\ &= 35 + \frac{400 + 60 + 7}{1000} = 35 + \frac{467}{1000} = \frac{35467}{1000} \end{aligned}$$

Perfect Square Numbers: The numbers whose square root can be determined, are called perfect square numbers.

e.g., $4,36,225, \ldots$ etc.

Two Digits Numbers If unit place digit is y and ten digit is x, then the two digits numbers can be expressed as $10x + y$.

e.g., $87 = 10 \times 8 + 7$

Divisibility

A number (dividend) is said to be divisible by another number (called divisor) when the quotient is a natural number and the remainder is zero. In other words, we can say that when a number (dividend) is divisible by another number (divisor), the dividend can be expressed as multiple of the divisor. The relation so obtained can be given by

$$\text{Dividend} = \text{Divisor} \times \text{Quotient}$$

Here, remainder equals zero.

Divisibility Tests

Divisibility by 2 A number is divisible by 2, when its unit's digit is either even or zero.

e.g., The numbers 4, 6, 8, 10, 24, 28, 322, 4886 etc., are divisible by 2.

$3,5,7,11,13,221,995,4637$ etc., are not divisible by 2. The rule emanates form the fact that any number multiplied by 2 gives product whose unit's digit is either even or zero.

Divisibility by 3 A number is divisible by 3 when the sum of its digits is divisible by 3.

$426 \Rightarrow 4 + 2 + 6 = 12$, which is divisible by 3.

Hence, 426 is also divisible by 3.

$8349 \Rightarrow 8 + 3 + 4 + 9 = 24$, which is divisible by 3.

Hence, 8349 is also divisible by 3.

$7493 \Rightarrow 7 + 4 + 9 + 3 = 23.23$ is not divisible by 3.

Hence, 7943 is also not divisible by 3].

Note This rule comes from the fact that the multiples of 3 are numbers, the sum of whose digits is divisible by 3.

3×6	$= 18;$	$1 + 8$	$= 9$ 9 is divisible by 3
3×27	$= 81;$	$8 + 1$	$= 9$
3×14	$= 42;$	$4 + 2$	$= 6$
3×220	$= 660;$	$6 + 6 + 0$	$= 12$
3×12547	$= 37641;$		$3 + 7 + 6 + 4 + 1 = 21$ and so on.

In the above examples 9,6,12 and 21 all are divisible by 3.

Divisibility by 4 A number is divisible by 4, when the number formed by its two extreme right digits is either divisible by 4 or both these digits are zero.

e.g., $3\underline{24}, 56\underline{32}, 35\underline{00}, 43\underline{20}$ and $894\underline{12}$ are divisible by 4 as they satisfy the above mentioned conditions.

$3\underline{23}, 51\underline{31}, 35\underline{10}$ and $546\underline{45}$ are not divisible by 4 as 23, 31,10 and 45 respectively, are not divisible by 4 nor is the condition of double zero fulfilled.

Note This rule is derived from the fact that 100 or multiples of 100 are always divisible by 4. So, if any such two digits number which is divisible by 4, is added to the multiples of 100, the sum will also be divisible by 4.

Divisibility by 5 A number is divisible by 5 when its unit's digit is either 5 or zero.

e.g., 2145,630,735 and 4500 are divisible by 5 as they have either 5 or 0 at their unit's place.

1546, 243, 11, 19647 are not divisible by 5 as they do not satisfy the above condition.

Divisibility by 6 A number is divisible by 6, when it is divisible by 2 as well as 3. This rule emanates from the fact that 2 and 3 are the two factors or sub-multiples of 6. Hence, both the conditions i.e., for divisibility by 2 as well as divisibility by 3 must be satisfied simultaneously. Alternatively, for a number to be divisible by 6 it must have either zero or an even digit at the unit's place and also simultaneously the sum of its digits must be divisible by 3.

- 72

(i) Its unit's digit i.e., 2 is an even number and hence it is divisible by 2.

(ii) Also, the sum of its digits equals $9(= 7 + 2)$, which is divisible by 3.

(iii) Therefore, 72 is divisible by 6.

- 2247

(i) Its unit's digit is an odd number and hence it is not divisible by 2.

(ii) The sum of its digits equals $15(= 2 + 2 + 4 + 7)$, which is divisible by 3.

(iii) We see that both the conditions i.e., divisibility by 2 and divisibility by 3 are not satisfied simultaneously. Therefore, 2247 is not divisible by 6, even though the number is divisibly by 3.

Divisibility by 7 A number is divisible by 7, when the difference between twice the digit at ones and the number formed by other digits is either zero or a multiple of 7.

e.g., 658 is divisible by 7

because $65 - 2 \times 8 = 65 - 16 = 49$

As 49 is divisible by 7, the number 658 is also divisible by 7.

Divisibility by 8 A number is divisible by 8, when the number formed by its three extreme right digits is divisible by 8 or when these last three digits are zeros.

- 3648 Since, 648 is divisible by 8,3648 is also divisible by 8.
- $11\underline{600}$ Since, 600 is divisible by 8,11600 is also divisibly by 8.
- 216000As, the last three digits of the given number are zeros, 216000 is divisible by 8.
- 21700 Since, 700 is not divisible by 8,21700 is also not divisible by 8.

Divisibility by 9 A number is divisible by 9, when the sum of its digits is divisible by 9.

- 39537

Sum of the digits $= 3 + 9 + 5 + 3 + 7 = 27$.

Since, 27 is divisible by 9,39537 is also divisible by 9.

Divisibility by 10 A number is divisible by 10, when it has zero at its unit's place.

e.g., The numbers 150, 7250, 1900, 35450 etc., are divisible by 10.

8564, 7509, 29005 etc., are not divisible by 10.

Divisibility by 11 A number is divisible by 11 when the difference between when the sum of the digits at odd places and the sum of the digits at even places is either 0 or divisible by 11.

- 9851833

Difference = Sum of digits at odd places

- Sum of digits at even places

$= (9 + 5 + 8 + 4) - (8 + 1 + 3 + 3) = 26 - 15 = 11$

So, the given number is divisible by 11.

-602613 Difference $= (6 + 2 + 1) - (0 + 6 + 3) = 9 - 9 = 0$

So, the given number is divisible by 11.

Divisibility by 12 A number is divisible by 12, when it is divisible by both the numbers 3 and 4 . This is so because 4 and 3 are the two factors or sub-multiples of 12. Therefore, conditions of divisibility by 4 as well as 3 must be satisfied simultaneously.

Alternatively, for a number to be divisible by 12, its last two digits must be either zero or the number formed by them, must be divisible by 4 and at the same time the sum of all the digits of the number must be divisible by 3.

- $96\underline{12}$

(i) 12 is divisible by 4. So, the number is divisible by 4.

(ii) $9 + 6 + 1 + 2 = 18$, which is divisible by 3. Hence, 9612 is divisible by 3.

(iii) Therefore, 9612 is divisible by 12.

- $76\underline{23}$

(i) Its last two digits are neither zeros nor is 23 divisible by 4. So, the number is not divisible by 4.

(ii) $7 + 6 + 2 + 3 = 18$, which is divisible 3. So, the number is divisible by 3.

(iii) We find that conditions for divisibility by 4 and 3 are not satisfied simultaneously. Therefore, 7623 is not divisible by 12 even though it is divisibly by 3.

To Find the Unit's Place Digit of a Given Exponential In case of 0, 1, 5, 6 The unit's place digit is 0,1,5,6 respectively.

In case of 4, 9

(a) if power is odd → The unit's place digit is 4 and 9 respectively.

(b) if power is even → The unit's place digit is 6 and 1 respectively.

In case of 2, 3, 7, 8 See the following example.

To find the unit's place digit of $(134647)^{553}$.

Step I $553 \div 4$ gives 1 as remainder this remainder is taken as new power.

Step II $(134647)^{553} = (134647)^1 = 7^1 = 7$.

∴ The unit's place digit is 7.

On dividing it, the remainder obtained is zero, take 4 as new power instead of zero. e.g., $(134647)^{552}$

Sol. $(134647)^{552} = (134647)^0 = 7^0 = 7^4 = 2401$

∴ The unit's place digit is 1.

Important Tips/Formulae

- Sum of the first n natural numbers $\frac{n(n+1)}{2}$
- Sum of the first n even natural numbers $= n(n+1)$
- Sum of the first n odd natural numbers $= n^2$
- Sum of the squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
- Sum of the cubes of first n natural numbers $= \left\{\frac{n(n+1)}{2}\right\}^2$
- nth term of the seres $a, a+d, a+2d, a+3d, \ldots = a+(n-1)d$ Where a is the first term and d is common difference.
- Sum to the n terms of the series $a, a+d, a+2d \ldots$

$$= \frac{n}{2}[2a+(n-1)d] \text{ or } = \frac{n}{2}[a+l]$$

  Where l = last term
- Dividend = (Divisor × Quotient) + Remainder

Solved Examples:

1. Find the unit digit in the expression

$55^{725} + 73^{5810} + 22^{853}$

(a) 5
(b) 6
(c) 7
(d) 4

Sol. (b) The unit digit of 55^{725} is 5.

73^{5810} is 9 (Remainder is 2 ∴ $(3)^2 = 9$)

22^{853} is 2 ∴ The unit a digit of the expression $= 5 + 9 + 2 = 16$ i.e., 6

2. How many integers lie between 300 and 1000, which are exactly divisible by 13?

(a) 50
(b) 54
(c) 53
(d) 52

Sol. (c) The numbers lying between 300 and 1000, which are exactly divisible by 13 , are $312, 325, 338, \ldots, 988$

Here, $a = 312, cd = 13$

Let n be the numbers of terms.

$$\therefore\ t_n = a + (n-1)d, 988 = 312 + (n-1) \times 13$$
$$\Rightarrow\ \frac{676}{13} = n - 1$$
$$\Rightarrow\ n = 52 + 1 = 53$$

Alternate

$$\text{Required Number} = \frac{1000}{13} - \frac{300}{13}$$
$$= 76\frac{12}{13} - 23\frac{1}{13}$$
$$= 76 - 23 = 53$$

3. Find the sum of all natural numbers from 75 to 97.

(a) 2008
(b) 1985
(c) 1895
(d) 1978

Sol. (d) $1 + 2 + 3 + \cdots + 74 = \frac{74(74+1)}{2} = 37 \times 75 = 2775$

and $1 + 2 + 3 + \cdots + 97 = \frac{97(97+1)}{2} = \frac{97 \times 98}{2} = 4753$

$\therefore 75 + 76 + 77 + 78 + \cdots + 97 = 4753 - 2775 = 1978$

4. A number, when divided by 114, leaves remainder 21 . If the same number is divided by 19 , then the remainder will be

(a) 1
(b) 2
(c) 7
(d) 17

Sol. (b) Let given number $= (14K + 2)$

$$= (19 \times 6K) + 19 + 2$$
$$= 19(6K + 1) + 2$$

Hence, the required remainder is 2.

Practice Questions

1. A number of three digits when divided by 2,5,9, 11 leaves remainder 1 in each case. The number is

(a) 981
(b) 983
(c) 991
(d) 997

2. The sum of a two digits number and the number obtained by reversing its digits is a square number. How many such numbers are there?

(a) 5
(b) 6
(c) 7
(d) 8

3. When a number is divided by 24, the remainder is 16. The remainder when the same number is divided by 12 is

(a) 3
(b) 4
(c) 6
(d) 8

4. The unit's digit in the product $7^{71} \times 6^{63} \times 3^{65}$ is

(a) 1
(b) 2
(c) 3
(d) 4

5. The units digit of the expression $25^{6251} + 36^{528} + 73^{54}$ is

(a) 6
(b) 5
(c) 4
(d) 0

6. The sum of the squares of 3 consecutive positive numbers is 365. The sum of the numbers is

(a) 30
(b) 33
(c) 36
(d) 45

7. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, then the dividend

(a) 4236
(b) 4306
(c) 4336
(d) 5336

8. If x is a number midway between 10 and 16 and y is half of 78, then $\frac{y}{x}$ is equal to

(a) 6
(b) 5
(c) 4
(d) 3

9. The difference between the largest 3 digits number and the smallest 2 digits number is

(a) 989
(b) 899
(c) 998
(d) 988

10. What largest number of four digits is exactly divisible by 88?

(a) 9988
(b) 8888
(c) 9768
(d) 9944

11. In 337^{337}, the unit digit is occupied by

(a) 1
(b) 3
(c) 7
(d) 9

12. When a number is divided by 121, the remainder is 25. If the same number is divided by 11, the remainder will be

(a) 3
(b) 4
(c) 6
(d) 25

13. In a group of cows and hens, the number of legs are 14 more than twice the number of heads. The number of cows is

(a) 5
(b) 7
(c) 10
(d) 12

14. The simplified value of $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots$

$$\left(1-\frac{1}{99}\right)\left(1-\frac{1}{100}\right), \text{ is}$$

(a) $\frac{2}{99}$
(b) $\frac{1}{25}$
(c) $\frac{1}{50}$
(d) $\frac{1}{100}$

15. In a division problem, the division is 4 times the quotient and 3 times the remainder. If the remainder is 4, the dividend is

(a) 36
(b) 40
(c) 12
(d) 30

16. Which of the least number should be added at 1000 to make it exactly divisible by 45?

(a) 35
(b) 80
(c) 20
(d) 10

17. The sum of odd numbers from 10 to 60 is

(a) 775
(b) 468
(c) 921
(d) 875

18. The sum of three consecutive natural numbers each divisible by 3 is 72. What is the largest among them?

(a) 21
(b) 24
(c) 27
(d) 30

19. In a question on division with zero remainder, a candidate took 12 as divisor, instead of 21. The quotient obtained by him was 35. The correct quotient is

(a) 13
(b) 20
(c) 0
(d) 12

20. The sum of all integers between 200 and 400 divisible by 9 is

(a) 3366
(b) 6633
(c) 6336
(d) 6363

21. The unit digit of the expression

$125^{813} \times 553^{3703} \times 4532^{828}$ is

(a) 4
(b) 2
(c) 0
(d) 5

22. Find the number of divisors of 1420.

(a) 14
(b) 15
(c) 13
(d) 12

23. If $1^2 + 2^2 + 3^2 + \cdots + x^2 = \frac{x(x+1)(2x+1)}{6}$, then $1^2 + 3^2 + 5^2 + \cdots + 19^2$ is equal to

(a) 1330
(b) 2100
(c) 2485
(d) 2500

24. The difference between the largest 4 digits number and the smallest 3 digits number is

(a) 9899
(b) 8999
(c) 9989
(d) 9889

25. What can be said about the expansion of $2^{12n} - 6^{4n}$, where n is a positive integer?

(a) Last digit is 4
(b) Last digit is 8
(c) Last digit is 2
(d) Last two digits are zero

26. Find the unit digit in the product of $(268 \times 539 \times 826 \times 102)$

(a) 5
(b) 3
(c) 4
(d) 2

27. On dividing a certain number by 357, the remainder is 39. On dividing the same number by 17. What will be the remainder?

(a) 5
(b) 3
(c) 7
(d) 6

28. Each member of a picnic party contributed twice as many rupees as the total collection was ₹ 3042. The number of members present in the party was

(a) 2
(b) 32
(c) 42
(d) 39

29. A number consists of two digits. If the digits interchange places and the new number is added to the original number, the resulting number will be divisible by

(a) 11
(b) 5
(c) 3
(d) 9

30. An $85 \mathrm{~m}$ long rod is divided into two parts. If one part is $\frac{2}{3}$ of the other part, then the longer part (in m) is

(a) 34
(b) $56\frac{2}{3}$
(c) 85
(d) 51

31. The unit's place digit in the product $(3127)^{173}$ will be

(a) 1
(b) 3
(c) 7
(d) 9

32. The sum of all natural numbers between 100 and 200 which are multiples of 3 is

(a) 5000
(b) 4950
(c) 4980
(d) 4900

33. A number divided by 899 gives a remainder of 63. If the number is divided by 29, the remainder will be

(a) 2
(b) 5
(c) 13
(d) 28

ANSWERS

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(d)	**5.**	(d)	**6.**	(d)	**7.**	(d)	**8.**	(d)	**9.**	(a)	**10.**	(d)
11.	(c)	**12.**	(a)	**13.**	(b)	**14.**	(c)	**15.**	(b)	**16**.	(a)	**17.**	(d)	**18.**	(c)	**19.**	(b)	**20.**	(b)
21.	(c)	**22.**	(d)	**23.**	(a)	**24.**	(a)	**25.**	(d)	**26.**	(c)	**27.**	(a)	**28.**	(d)	**29.**	(a)	**30.**	(d)
31.	(c)	**32.**	(b)	**33.**	(b)														

Hints & Solutions

1. LCM of $2,5,9,11 = 990$

Required number $= 990 + 1 = 991$

2. Let the two digits number be $10x + y$.

By given condition,

$10x + y + 10y + x = z^2$

(Square number)

$11(x + y) = z^2$

$$x + y = \frac{z^2}{11}$$

Putting the all values of x and y which sum is 11, we find 8 such numbers.

3. When 16 is divided by 12 the remainder is 4. So, the required remainder is 4.

4. Unit digit of $[7^{71} \times 6^{63} \times 3^{65}]$

$=$ Unit digit of

$[7^{4\times17+3} \times 6 \times 3^{4\times16+1}]$

$=$ Unit digit of $[7^3 \times 6 \times 3^1]$

$=$ Unit digit of $[343 \times 6 \times 3]$

$\because 7^3 = 343]$

$=$ Unit digit of $[3 \times 6 \times 3]$

$=$ Unit digit of $[54] = 4$

5. Unit digit of
$(25^{6251} + 36^{528} + 73^{54})$

$=$ Unit digit of $[5 + 6 + (3)^{54}]$

$=$ Unit digit of $[5 + 6 + (3^4)^{13} \times 3^2]$

$=$ Unit digit of $[5 + 6 + 1 \times 9]$

$=$ Unit digit of $[20] = 0$

6. Let the three consecutive numbers be $x, x + 1$ and $x + 2$ respectively.
According to question,

$x^2 + (x + 1)^2 + (x + 2)^2 = 365$

$x^2 + x^2 + 1 + 2x + x^2 + 4 + 4x = 365$

$3x^2 + 6x + 5 - 365 = 0$

$x^2 + 2x - 120 = 0$

On solving, $x = 10$

$\therefore$ Sum of numbers

$= 10 + 11 + 12 = 33$

7. Let remainder $= 2x = 46$

$x = \frac{46}{2} = 23$

$\therefore$ Quotient $= 23$, Remainder $= 46$, Divisor $= 230$

$\because$ Dividend = Divisor × Remainde r

- Quotient

$= 230 \times 23 + 46 = 5336$

8. Value of x between 10 and 16 will be $11,12,13,14$, and 15.

According to question,

$y = \frac{78}{2} = 39$

Then, $\frac{y}{x} = \frac{39}{13}$

(Let $x = 13$ because 39 divided by 13)

$\frac{y}{x} = 3$

9. Largest three digit's number $= 999$

Smallest two digit's number $= 10$

Difference $= 999 - 10 = 989$

10. Largest number of four digits $= 9999$

88) 9999 (113
88
119
88
319
264
55

$\therefore$ Required number $= 9999 - 55 = 9944$

11. The given number is 337^{337}.

The unit's place digit is 7, so by applying the method, we divide the power by 4 i.e.,

4)37(84
32
17
16
1

The remainder is 1.

So, the new power is 1.

$\therefore\ (337)^{337} = (337)^1$

Hence, the unit's place digit is 7.

12. By Shortcut: $25 \div 11$ gives 3 as remainder

13. Let there be x cows and y hens.

Then, $(4x + 2y) - 14 = 2(x + y)$

$$4x + 2y - 14 = 2x - 2y$$
$$4x - 2x = 14$$
$$2x = 14$$
$$x = \frac{14}{2} = 7$$

14. $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)\ldots\left(1 - \frac{1}{99}\right)$

$$\left(1 - \frac{1}{100}\right)$$

$$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \ldots \times \frac{98}{99} \times \frac{99}{100} = \frac{2}{100} = \frac{1}{50}$$

15. Let division $= d$, then $d = 4q = 3r$

Since $r = 4$, then $d = 3 \times 4 = 12$

and $q = \frac{4\times3}{4} = 3$

$\therefore\ N = dq + r = 12 \times 3 + 4 = 40$

16. $45)1000(22$

$$\begin{array}{l} \underline{90} \\ 100 \\ \underline{90} \\ 10 \end{array}$$

So, the required number $= 45 - 10 = 35$

17. Sum to first n odd numbers $= \left(\frac{59+1}{2}\right)^2$

{Last odd number $= 59$}

$= (30)^2 = 900$

and sum to odd numbers from 1 to 10

$= \left(\frac{9+1}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

∴ The required odd numbers $= 900 - 25 = 875$

18. Let the three consecutive number divisible by 3 be $x, (x+3), (x+6)$, then

$$x + (x+3) + (x+6) = 72$$
$$3x + 9 = 72$$
$$3x = 63$$
$$x = 21$$

So, the largest number must be $x + 6 = 21 + 6 = 27$

19. The number is $35 \times 12 = 420$

Now, correct quotient $= 420 \div 21 = 20$

20. The numbers are $207, 210, 225, \ldots, 396$

$$T_n = a + (n-1)d$$
$$396 = 207 + (n-1) \times 9$$
$$\Rightarrow n = \frac{396 - 207}{9} + 1$$
$$n = 22$$
$$\therefore S_n = \frac{n}{2}(a + l)$$
$$= \frac{22}{2}(207 + 396)$$
$$= 6633$$

21. The unit digit in $125^{813} = 5$

The unit digit in

$(553)^{3703}$ = Divide 3703 by 4 the remainder is $0 = 0$

The unit digit in $(4532)^{828}$ = Divide 828 by 4 the remainder is $0 = 0$

Hence, unit digit will be $= 5 \times 0 \times 0 = 0$

22. $1420 = 2^2 \times 5^1 \times 71^1$

$\therefore$ Number of divisors $= (2+1)(1+1)(1+1) = 12$

23. $1^2 + 2^2 + 3^2 + \cdots + 19^2$

$$= \frac{19 \times 20 \times 39}{6} = 2470$$

$2^2 + 4^2 + \cdots + 18^2 = 1140$

$$= 2^2(1^2 + 2^2 + \cdots + 9^2)$$

$$= \frac{4 \times 9 \times 10 \times 19}{6} = 1140$$

$\therefore 1^2 + 3^2 + 5^2 + \cdots + 19^2$

$$= 2470 - 1140 = 1330$$

24. Largest four digit's number $= 9999$

Smallest three digit's number $= 100$

Difference $= 9999 - 100 = 9899$

25. $2^{12n} - 6^{4n} = (2^{12})^n - (6^4)^n$

$$= (4096)^n - (1296)^n$$

$$= (4096 - 1296)[(4096)^{n-1}$$

$$+ (4096)^{n-2}(1296)$$

$$+ \cdots + (1296)^{n-1}]$$

$$= 2800(k)$$

Hence, last two digits are always be zero.

26. Product of unit digits of the number $= 8 \times 9 \times 6 \times 2 = 864$

$\therefore$ Required digit $= 4$

27. Let the given number be $(357k + 39)$.

Then, $(357k + 39)$

$$= (17 \times 21k) + (17 \times 2) + 5$$

$$= 17 \times (21k + 2) + 5$$

$\therefore$ Required remainder $= 5$

28. Let the number of members of a picnic $= x$

∴ Contribution of each member

$$\therefore\ 2x \times x = 3042$$
$$\Rightarrow\ x^2 = \frac{3042}{2} = 1521$$
$$\therefore\ x = 39$$

29. Let the unit's place digit $= x$ and ten's place digit $= y$

∴ number $= 10y + x$

and new number $= 10x + y$

Sum of number

$= 10y + x + 10x + y$

$= 11(x + y)$

This shows that the sum of numbers is divisible by 11.

30. Let the length of first part $= x$ m

∴ The length of another part $= (85 - x)$m

According to the question,

$$x = \frac{2}{3}(85 - x)$$
$$\Rightarrow\ 3x = 170 - 2x$$
$$\Rightarrow\ x = \frac{170}{5} = 34$$

∴ The length of first part $= 34$ m and the length of second part $= 85 - 34 = 51$ m

31. Unit's place digit in $(3127)^{173}$

= unit's place digit in $(7)^{173}$

$= 7^{172+1} = (7)^1$

= unit's place digit in $1 \times 7 = 7$

32. Multiple of 3 between 100 and 200 are 102, 105, 198.

Let the number of numbers is n.

$\therefore\ 198 = 102 + (n - 1) \cdot 3$

$\Rightarrow (n - 1) = \dfrac{198 - 102}{3}$

$$= \frac{96}{3} = 32$$
$$= 33$$

$$\Rightarrow \; n = 33$$

∴ Required sum $= \frac{33}{2}(102 + 198)$

$$= \frac{33}{2} \times 300$$
$$= 4950$$

33. If a number x is divided by 899 gives a remainder 63 and quotient y.

∴ $x = y \times 899 + 63$

$\Rightarrow \; x = (31y + 2) \times 29 + 5$

It is clear from above that if x is divided by 29, then the remainder is 5.