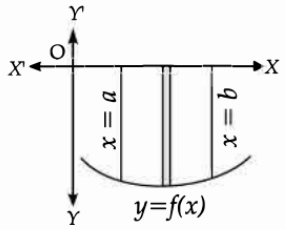
Applications of the Integrals

Area under simple curves

If the curve under consideration lies below X-axis, then $f(x) < 0$ from $x=a$ to $x=b$, the area bounded by the curve $y=f(x)$ and the ordinates $x=a$, $x=b$ and X-axis is negative

$$A=\left|\int_a^b f(x)dx\right|$$

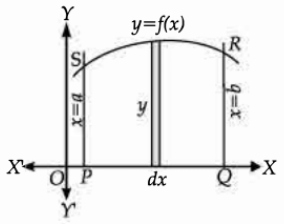


The area of the region bounded by the curve $y=f(x)$, X- *axis* and the lines $x=a$ and $x=b\,(b>a)$ is given by

$$A=\int_a^b y\,dx \text{ or } \int_a^b f(x)dx$$

eg : The area bounded by $y=x^2$, X- axisin I quadrant and the lines $x=2$ and $x=3$ is -

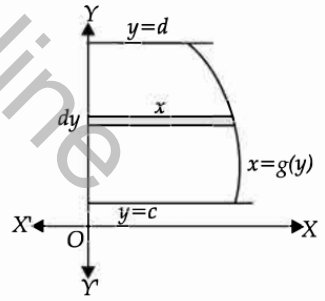
$$A=\int_2^3 y\,dx=\int_2^3 x^2\,dx=\left[\frac{x^3}{3}\right]_2^3=\frac{1}{3}(27-8)=\frac{19}{3}\text{ Sq.units.}$$



The area of the region bounded by the curve $x=f(y), Y-axis$ and the lines $y=c$ and $y=d\,(d>c)$ is given by $A=\int_c^d x\,dy$ or $\int_c^d f(y)dy$

eg : The area bounded by $x=y^3$, $Y-axis$ in the I quadrant and the lines $y=1$ and $y=2$ is

$$\int_a^b x\,dy=\int_1^2 y^3\,dy=\left[\frac{1}{4}y^4\right]_1^2=\frac{1}{4}(2^4-1^4)=\frac{15}{4}\text{ Sq. units}$$



Trace the Mind Map

First Level Second Level Third Level

PRACTICE QUESTIONS

1. The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is

   a) $\frac{4+3\log 3}{2}$ sq unit

   b) $\frac{4-3\log 3}{2}$ sq unit

   c) $\frac{3}{2}\log 3$ sq unit

   d) $\frac{1}{2} + \log 3$ sq unit

2. The area bounded by the curve $y = x^4 - 2x^3 + x^2 + 3$ with x-axis and ordinates corresponding to the minima of y, is

   a) 1

   b) $\frac{91}{30}$

   c) $\frac{30}{9}$

   d) 4

3. The area bounded by $y = |\sin x|$, x-axis and the lines $|x| = \pi$ is

   a) 2 sq units

   b) 3 sq units

   c) 4 sq units

   d) None of these

4. The area bounded by curve

   $x^2 + y^2 = 25, 4y = |4 - x^2|$ and $x = 0$ above the x-axis is

   a) $24\sin^{-1}\left(\frac{4}{5}\right)$

   b) $25\sin^{-1}\left(\frac{4}{5}\right)$

   c) $4 + 25\sin^{-1}\left(\frac{4}{5}\right)$

   d) None of these

5. Let $f(x) = \min\{x+1, \sqrt{(1-x)}\}$, then area bounded by $f(x)$ and x-axis is

   a) $\frac{1}{6}$ sq unit

   b) $\frac{5}{6}$ sq unit

   c) $\frac{7}{6}$ sq unit

   d) $\frac{11}{6}$ sq unit

6. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the line $x = 0$ and $x = \frac{\pi}{4}$

   a) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

   b) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

   c) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

   d) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

7. The area bounded by the x-axis, part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates $x = 2$ and $x = 4$, is divided into two equal parts by the ordinate $x = a$, then the value of $'a'$ is

   a) $2\sqrt{2}$

   b) $\pm 2\sqrt{2}$

   c) $\pm\sqrt{2}$

   d) ± 2

8. In the interval $[0, \pi/2]$, area lying between the curves $y = \tan x, y = \cot x$ and x-axis is

   a) $\log 2$

   b) $\frac{1}{2}\log 2$

   c) $2\log\left(\frac{1}{\sqrt{2}}\right)$

   d) $\frac{3}{2}\log 2$

9. The areas of the figure into which curve $y^2 = 6x$ divides the circle $x^2 + y^2 = 16$ are in the ratio

   a) $\frac{2}{3}$

   b) $\frac{4\pi-\sqrt{3}}{8\pi+\sqrt{3}}$

   c) $\frac{4\pi+\sqrt{3}}{8\pi-\sqrt{3}}$

   d) None of these

10. Area lying in the first quadrant and bounded by the curve $y = x^3$ and the line $y = 4x$, is

   a) 2

   b) 3

   c) 4

   d) 5

11. The area cut off from a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex, then k is

   a) $\frac{2}{3}$

   b) $\frac{1}{3}$

   c) $\frac{3}{2}$

   d) 3

12. If A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0, y = 0$ and $x = \pi/4$, then for $x > 2$

   a) $A_n + A_{n-2} = \frac{1}{n-1}$

   b) $A_n + A_{n-2} < \frac{1}{n-1}$

   c) $A_n - A_{n-2} = \frac{1}{n-1}$

   d) None of these

13. Area of the region bounded by the curve $y = \begin{cases} x^2, x < 0 \\ x, \ x \geq 0 \end{cases}$ and the line $y = 4$ is

a) $\frac{10}{3}$ sq unit

b) $\frac{20}{3}$ sq unit

c) $\frac{40}{3}$ sq unit

d) None of these

14. The line $x = \frac{\pi}{4}$ divides the area of the region bounded by $y = \sin x$, $y = \cos x$ and x-axis $\left(0 \leq x \leq \frac{x}{2}\right)$ into two regions of areas A_1 and A_2. Then $A_1 : A_2$ equals

a) 4:1

b) 3:1

c) 2:1

d) 1:1

15. The area bounded by $x = 1, x = 2, xy = 1$ and x-axis is

a) $(\log 2)$ sq unit

b) 2 sq unit

c) 1 sq unit

d) None of these

16. The sine and cosine meet each other at number of points and develop the symmetrical area number of times, area of one such region is

a) $4\sqrt{2}$

b) $3\sqrt{2}$

c) $2\sqrt{2}$

d) $\sqrt{2}$

17. The sine and cosine meet each other at number of points and develop the symmetrical area number of times, area of one such region is

a) 1:1

b) 2:1

c) 1:2

d) None of these

18. Let A_1 be the area of the parabola $y^2 = 4ax$ lying between vertex and latusrectum and A_2 be the area between latusrectum and double ordinate $x = 2a$. Then, $A_1/A_2 =$

    a) $2\sqrt{2} - 1$

    b) $(2\sqrt{2} + 1)/7$

    c) $(2\sqrt{2} - 1)/7$

    d) None of these

19. The area of the region bounded by the curve $a^4y^2 = (2a - x)x^5$ is to that of the circle whose radius is a, is given by the ratio

    a) 4:5

    b) 5:8

    c) 2:3

    d) 3:2

20. Ratio of the area cut off a parabola by any double ordinate is that corresponding rectangle contained by that double ordinate and its distance from the vertex is

    a) 1/2

    b) 1/3

    c) 2/3

    d) 1

21. The area bounded by the curves $f(x) = ce^x (c > 0)$, the x-axis and the two ordinates $x = p$ and $x = q$, is proportional to

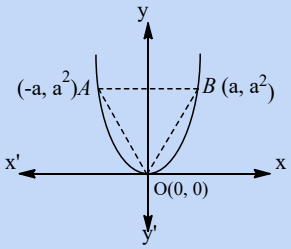
    a) $f(p)f(q)$

    b) $|f(p) - f(q)|$

    c) $f(p) + f(q)$

    d) $\sqrt{f(p)f(q)}$

22. The figure shows a ΔAOB and the parabola $y = x^2$. The ratio of the area of the ΔAOB to the area of the region AOB of the parabola $y = x^2$ is equal to



a) 3/5

b) 3/4

c) 7/8

d) 5/6

23. The area of the quadrilateral formed by the tangents at the end points of latus rectum to ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is

a) 27/4 sq unit

b) 9 sq unit

c) 27/2 sq unit

d) 27 sq unit

24. The volume of the solid obtained by revolving about y-axis the area enclosed between the ellipse $x^2 + 9y^2 = 9$ and the straight line $x + 3y = 3$, in the first quadrant is

a) 3π

b) 4 π

c) 6 π

d) 9 π

25. If the ordinate $x = a$ divides the area bounded by x-axis part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates $x = 2, x = 4$ into two equal parts, then a is equal

a) $\sqrt{2}$ sq unit

b) $2\sqrt{2}$ sq unit

c) $3\sqrt{2}$ sq unit

d) None of these

26. The positive value of the parameter 'a' for which the area of the figure founded by $y = \sin ax, y = 0, x = \pi/a$ and $x = \pi/3a$ is 3, is equal t0

a) 2

b) $1/2$

c) $\frac{2+\sqrt{3}}{3}$

d) $\sqrt{3}$

27. The area between the curves

$y = xe^x$ and $y = xe^{-x}$ and line $x = 1$, in square unit, is

a) $2\left(e + \frac{1}{e}\right)$ sq units

b) 0 sq unit

c) $2e$ sq units

d) $\frac{2}{e}$ sq unit

28. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$ is given by

a) $\frac{45}{7}$

b) $\frac{25}{4}$

c) $\frac{\pi}{18}$

d) $\frac{9}{2}$

29. The positive value of the parameter $'a'$ for which the area of the figure bounded by $y = \sin a\,x, y = 0, x = \frac{\pi}{a}$ and $x = \frac{\pi}{3a}$ is 3, is equal to

a) 2

b) 1/2

c) $\frac{2+\sqrt{3}}{3}$

d) 3/2

30. The line $y = mx$ bisects the area enclosed by the lines $x = 0, y = 0, x = 3/2$ and the curve $y = 1 + 4x - x^2$. The value of m, is

    a) $13/8$

    b) $13/32$

    c) 13/16

    d) 13/4

31. The area bounded by the curve $y = x + \sin x$ and its inverse function between the ordinates $x = 0$ and $x = 2\pi$, is

    a) 8π sq unit

    b) 4π sq unit

    c) 8 sq unit

    d) None of these

32. The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is

    a) $\frac{4+3\ln 3}{2}$

    b) $\frac{4-3\ln 3}{2}$

    c) $\frac{3}{2}\ln 3$

    d) $\frac{1}{2} + \ln 3$

33. The value of m for which the area included between the curves $y^2 = 4ax$ and $y = mx$ equals, $a^2/3$ is

    a) 1

    b) 2

    c) 3

    d) $\sqrt{3}$

34. The area bounded by $y = 4 - x^2$ and $y = \left[3 + \frac{x^2}{4}\right]$, where $[\cdot]$ denotes greatest integer function, is

a) 1 sq unit

b) $\frac{1}{3}$ sq unit

c) $\frac{2}{3}$ sq unit

d) $\frac{4}{3}$ sq unit

35. The area formed by triangular shared region bounded by the curves $y = \sin x, y = \cos x$ and $x = 0$ is

a) $(\sqrt{2} - 1)$ sq unit

b) 1 sq unit

c) $\sqrt{2}$ sq unit

d) $(1 + \sqrt{2})$ sq unit

36. The area bounded by the curve $x = a\cos^3 t,\ y = a\sin^3 t$, is

a) $\frac{3\pi a^2}{8}$

b) $\frac{3\pi a^2}{16}$

c) $\frac{3\pi a^2}{32}$

d) $3\pi a^2$

37. The value of c for which the area of the figure bounded by the curve $y = 8x^2 - x^5$, the straight lines $x = 1$ and $x = c$ and the x-axis is equal to $\frac{16}{3}$ is

a) 2

b) $\sqrt{8 - \sqrt{17}}$

c) 3

d) -1

38. The parabola $y^2 = 4x$ and $x^2 = 4\,y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is

a) 1:1:1

b) 2:1:2

c) 1:2:3

d) 1:2:1

39. Area enclosed by the curve

$\pi[4(x - \sqrt{2})^2 + y^2] = 8$ is

a) π sq units

b) 2 sq units

c) 3π sq units

d) 4 sq units

40. If the area above the x-axis bounded by the curves $y = 2^{kx}$ and $x = 0$ and 2 is $\frac{3}{\log 2}$ then the value of k is

a) 1/2

b) 1

c) -1

d) 2

41. If a curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area bounded by the curves, line $x = 4$ and x-axis is 8 sq unit, then

a) $a = 3, b = -1$

b) $a = 3, b = 1$

c) $a = -3, b = 1$

d) $a = -3, b = -1$

42. The area of the figure bounded by the curve $|y| = 1 - x^2$ is

a) 2/3

b) 4/3

c) 8/3

d) -5/3

43. Line $x = 1$ divides A enclosed by circle $x^2 + y^2 = 16$ in two portions A_1 and $A_2 (A_1 > A_2)$, then $\frac{A_1}{A_2}$ is

a) 4

b) 3

c) 2

d) None of these

44. The area bounded by $y = \sin^{-1}x, x = \frac{1}{\sqrt{2}}$ and x-axis is

a) $\left(\frac{1}{\sqrt{2}} + 1\right)$ sq units

b) $\left(1 - \frac{1}{\sqrt{2}}\right)$ sq units

c) $\frac{\pi}{4\sqrt{2}}$ sq units

d) $\left(\frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right)$ sq units

45. The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1$ and $x = b$ is equal to $(\sqrt{(b^2 + 1)} - \sqrt{2})$ for all $b > 1$, then $f(x)$ is

a) $\sqrt{(x - 1)}$

b) $\sqrt{(x + 1)}$

c) $\sqrt{(x^2 + 1)}$

d) $\frac{x}{\sqrt{(1+x^2)}}$

46. The area bounded by $|x - 1| \leq 2$ and $x^2 - y^2 = 1$, is

a) $6\sqrt{2} + \frac{1}{2}\log|3 + 2\sqrt{2}|$

b) $6\sqrt{2} + \frac{1}{2}\log|3 - 2\sqrt{2}|$

c) $6\sqrt{2} - \log|3 + 2\sqrt{2}|$

d) None of these

47. The area between the parabola $y^2 = 4ax$ and the line $y = mx$ in square units is

a) $\frac{5a^2}{3m}$

b) $\frac{8a^2}{3m^3}$

c) $\frac{7a^2}{4m^2}$

d) $\frac{3a^2}{5m}$

48. The length of the parabola $y^2 = 12x$ cut off by the latusretum is

a) $6\left[\sqrt{2} + \log(1+\sqrt{2})\right]$

b) $3\left[\sqrt{2} + \log(1+\sqrt{2})\right]$

c) $6\left[\sqrt{2} - \log(1+\sqrt{2})\right]$

d) $3\left[\sqrt{2} - \log(1+\sqrt{2})\right]$

49. If A_1 is the area enclosed by the curve $xy = 1, x$-axis and the ordinates $x = 1, x = 2$; and A_2 is the area enclosed by the curve $xy = 1, x$-axis and the ordinates $x = 2, x = 4$, then

a) $A_1 = 2\,A_2$

b) $A_2 = 2\,A_1$

c) $A_2 = 3\,A_1$

d) $A_1 = A_2$

50. The slope of tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point (1, 2), then the area of the region bounded by the curve, the x-axis and the line $x = 1$ is

a) $\frac{5}{6}$ sq unit

b) $\frac{6}{5}$ sq unit

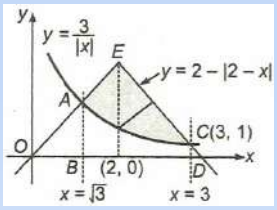
c) $\frac{1}{6}$ sq unit

d) 6 sq unit

ANSWER KEY

1. Required area $= \int_{\sqrt{3}}^{3}(2-12-x)dx - \int_{\sqrt{3}}^{3}\frac{3}{|x|}dx$

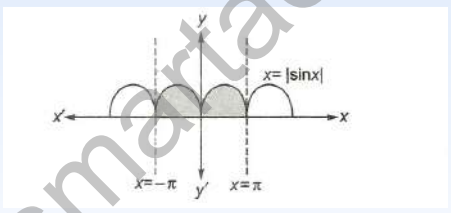
$= \int_{\sqrt{3}}^{2} x\,dx + \int_{2}^{3}(4-x)dx - \int_{\sqrt{3}}^{3}\frac{3}{x}dx = \left[\frac{x^2}{2}\right]_{\sqrt{3}}^{2} + \left[4x - \frac{x^2}{2}\right]_{2}^{3} - [3\log x]_{\sqrt{3}}^{3}$



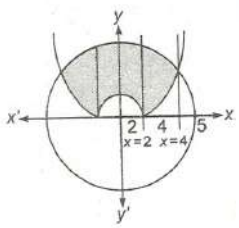
$= \frac{1}{2}[4-3] + \left[12 - \frac{9}{2} - (8-2)\right] - 3[\log 3 - \log\sqrt{3}] = \frac{1}{2} + \frac{3}{2} - 3\log\frac{3}{\sqrt{3}} = \frac{4}{2} - \frac{3}{2}\log 3$

2. Required area $= \frac{91}{30}$

3. Required area $= 2\int_0^{\pi}\sin x\,dx = 2[-\cos x]_0^{\pi} = 2[1+1] = 4$ sq units



4. Required area $= 2\left[\int_0^4 \sqrt{(25)-x^2)}dx - \int_0^2 \frac{4-x^2}{4}dx - \int_2^4 \frac{x^2-4}{4}dx\right]$



$= 2\left[\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_0^4 - \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3} - 4x\right]_2^4\right]$

$= 2\left[\left[2\times 3 + \frac{25}{5}\sin^{-1}\left(\frac{4}{5}\right) - \frac{1}{4}\left(\frac{64}{3} - 16\right) + \frac{1}{4}\left(\frac{8}{3} - 8\right)\right]\right] = 4 + \sin^{-1}\left(\frac{4}{5}\right)$

5. $f(x) = \min\{x+1, \sqrt{(1-x)}\} = \begin{cases} x+1, -1 \le x < 0 \\ \sqrt{1-x}, 0 < x \le 1 \end{cases}$

$\therefore$ Required area $= \left|\int_{-1}^{0}(x+1)dx\right| + \left|\int_{0}^{1}\sqrt{(1-x)}dx\right| = 7/6$ sq unit

6. Required area $= \int_0^{\pi/4}\left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right)dx$

$\because \left[\frac{1+\sin x}{\cos x} > \frac{1-\sin x}{\cos x} > 0\right]$

$$= \int_0^{\pi/4}\left(\sqrt{\frac{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} - \sqrt{\frac{1-\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}}\right)dx$$

$$= \int_0^{\pi/4}\frac{1+\tan\frac{x}{2}-1+\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx = \int_0^{\pi/4}\frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx$$

put $\tan\frac{x}{2} = t \Longrightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$\therefore$ Required area $= \int_0^{\tan\frac{\pi}{8}}\frac{4t\ dt}{(1+t^2)\sqrt{1-t^2}} = \int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

7. Required value of a=$\pm 2\sqrt{2}$
8. Required area=$\log 2$
9. We have,

$A_1 =$ Area bounded by the two curves

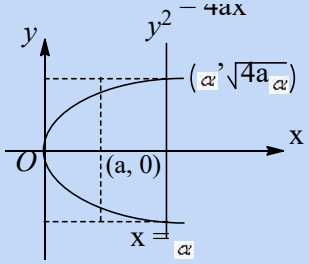
$\Rightarrow A_1 = \int_0^2\sqrt{6\,x}\,dx + \int_2^4\sqrt{16-x^2}\,dx = \frac{4\sqrt{3}+16\,\pi}{3}$

$A_2 =$ Area bounded by $x^2+y^2 = 16$ and outside $y^2 = 6\,x$

$\Rightarrow A_2 = 16\,\pi - \frac{4\sqrt{3}+16\,\pi}{3} = \frac{32\,\pi-4\sqrt{3}}{3}$

10. Area required = 4

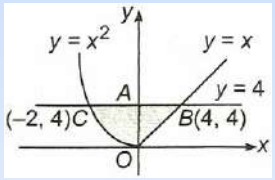
11. Required area $= 2\int_0^{\alpha}\sqrt{4ax}dx = k(\alpha)(2\sqrt{4a\alpha})$



$\frac{8\sqrt{a}}{3}\alpha^{3/2} = 4\sqrt{a}k\alpha^{3/2} \Rightarrow k = \frac{2}{3}$

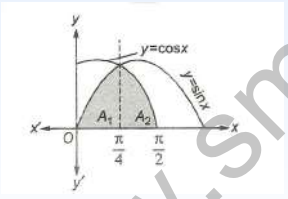
12. $A_n + A_{n-2} = \frac{1}{n-1}$

13. Required area $= \int_{-2}^{0}(4 - x^2)dx + \int_0^4(4 - x)dx$



$= \left[4x - \frac{x^3}{3}\right]_{-2}^{0} + \left[4x - \frac{x^2}{2}\right]_0^4 = 8 - \frac{8}{3} + 8 = \frac{40}{3}$ sq unit

14. Area, $A_1 = \int_0^{\frac{\pi}{4}} \sin x\, dx$



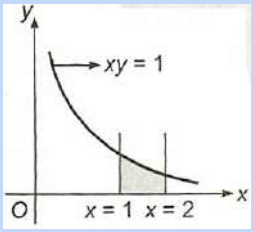
$= -[\cos x]_0^{\pi/4} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$

and area, $A_2 = \int_{\pi/4}^{\pi/2} \cos x\, dx$

$= [\sin x]_{\pi/4}^{\pi/2} = \left[1 - \frac{1}{\sqrt{2}}\right] = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore A_1 : A_2 = \frac{\sqrt{2}-1}{\sqrt{2}} : \frac{\sqrt{2}-1}{\sqrt{2}} = 1:1$

15. Required area $= \int_1^2 \frac{1}{x} dx$



$= [\log|x|]_1^2 = \log 2$ sq unit

16. Area=$3\sqrt{2}$

17. Given area bounded by the curve, $y = \sqrt{3x+4}$, x-axis and the line $x = -1$ and $x = 4$ is A and area bounded by the curve $y = \sqrt{3x+4} ie, y = \pm(3x+4)^{1/2}$ x-axis and the line $x = -1$ and $x = 4$ is B

$\therefore B = 2A$ [Since, it is the area of both sides about x-axis]Now, $A: B = A: 2A = 1: 2$

18. We have,

$A_1 = 2\int_0^a \sqrt{4ax}\, dx$ and, $A_2 = 2\int_0^{2a} \sqrt{4ax} - 2\int_0^a \sqrt{4ax}\, dx$

$\Rightarrow A_1 = \frac{8a^2}{3}$ and, $A_2 = \frac{16}{3}\sqrt{2}a^2 - \frac{8}{3}a^2 \Rightarrow \frac{A_1}{A_2} = \frac{1}{2\sqrt{2}-1} - \frac{2\sqrt{2}+1}{7}$

19. Given curve $a^4y^2 = (2a-x)x^5$

Cut off x-axis, when $y = 0$

$0 = (2a-x)x^5$

$\therefore x = 0, 2a$

Hence, the area bounded by the curve

$a^4y^2 = (2a-x)x^5$ is $A_1 = \int_0^{2a} \frac{\sqrt{(2a-x)}x^{5/2}}{a^2} dx$

Put $x = 2a\sin^2\theta$

$\therefore dx = 4a\sin\theta\cos\theta d\theta$

$\therefore A_1 = \int_0^{\pi/2} \frac{\sqrt{2a}\cos\theta(2a)^{5/2}\sin^5\theta\, 4a\sin\theta\cos\theta}{a^2} d\theta = 32a^2\int_0^{\pi/2} \sin^6\theta\cos^2\theta\, d\theta$

$= 32a^2 \cdot \frac{(5\cdot3\cdot1)(1)}{8\cdot6\cdot4\cdot2} \cdot \frac{\pi}{2}$ (by walli's formula)$= \frac{5\pi a^2}{8}$

Area of circle, $A_2 = \pi a^2$

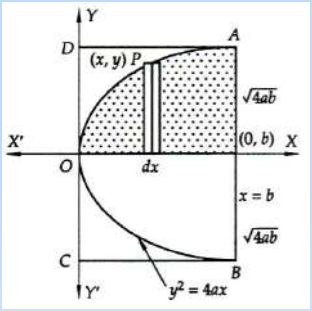
$\therefore \frac{A_1}{A_2} = \frac{5}{8}$

20. Let $y^2 = 4ax$ be a parabola and let $x = b$ be a double ordinate. Then,

A_1 = Area enclosed by the parabola $y^2 = 4ax$ and the double ordinate $x = b$

$\Rightarrow A_1 = 2\int_0^b y\,dx = 2\int_0^b \sqrt{4ax}\,dx = 4\sqrt{a}\int_0^b \sqrt{x^3}\,dx$

$\Rightarrow A_1 = 4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^b = 4\sqrt{a}\times\frac{2}{3}b^{3/2} = \frac{8}{3}a^{1/2}b^{3/2}$

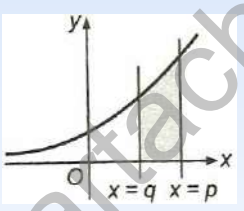


And, A_2 = Area of the rectangle $ABCD$

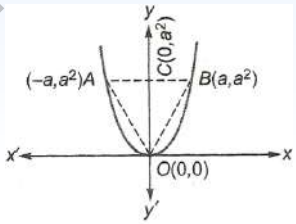
$\Rightarrow A_2 = AB \times AD = 2\sqrt{4ab}\times b = 4\,a^{1/2}b^{3/2}$

$\therefore A_1 : A_2 = 8/3\,a^{1/2}b^{3/2} : 4\,a^{1/2}b^{3/2} = 2/3 : 1 = 2 : 3$

21. Required area $= \int_q^p ce^x dx = [ce^x]_q^p = c[e^p - e^q] = f(p) - f(q)$



22. Area of curve $OAB = 2\int_0^{a^2} x\,dy$

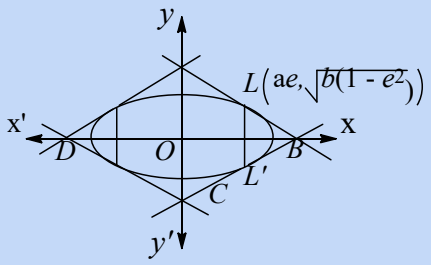


$= 2\int_0^{a^2}\sqrt{y}\,dy = 2\left[\frac{y^{3/2}}{3/2}\right]_0^{a^2} = \frac{4}{3}[a^3]$

Now, Area of $\Delta OAB = \frac{1}{2}\times AB \times OC = \frac{1}{2}\times 2a \times a^2 = a^3$

$\therefore \frac{\text{Area of } \Delta\, AOB}{\text{Area of curve } AOB} = \frac{a^3}{\frac{4}{3}a^3} = \frac{3}{4}$

23. Given equation of ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$



To find tangents at the end points of latus rectum we find $ae, ie, ae=\sqrt{a^2-b^2}=\sqrt{4}=2$

By symmetry the quadrilateral is rhombus. So, area of rhombus is four times the area of the right angled formed by the tangent and axes in the first quadrant

$\Rightarrow$ Equation of tangent at $\left[ae,\sqrt{b(1-e^2)}\right]=\left(2,\frac{5}{3}\right)$ is $\frac{2}{9}x+\frac{5}{3}\cdot\frac{y}{5}=1$

$\Rightarrow \frac{x}{9/2}+\frac{y}{3}=1$

$\therefore$ Area of quadrilateral $ABCD=4$(area of ΔAOB) $=4\left(\frac{1}{2}\cdot\frac{9}{2}\cdot 3\right)=27$ sq unit

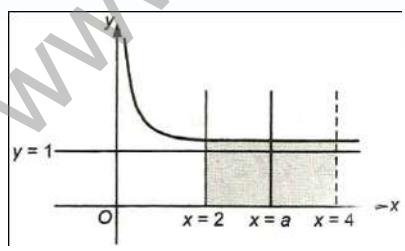
24. Volume $=\int_0^1 \pi\ x^2\,dy-\int_0^1 \pi\ x^2\,dy=\pi\int_0^1 9(1-x^2)dy-\pi\int_0^1 9(1-y)^2\,dy$

$$=9\pi\left[\left(y-\frac{y^3}{3}\right)+\frac{(1-y)^3}{3}\right]_0^1=9\pi\left[\left(1-\frac{1}{3}\right)+\left(0-\frac{1}{3}\right)\right]=3\pi$$

25. $\therefore$ Area $=\int_2^4\left(1+\frac{8}{x^2}\right)dx$

Since, the ordinate $x=a$ divides area into two equal parts, therefore,

$\int_2^a\left(1+\frac{8}{x^2}\right)dx=\frac{1}{2}\int_2^4\left(1+\frac{8}{x^2}\right)dx \Rightarrow \left[x-\frac{8}{x}\right]_2^a=\frac{1}{2}\left[x-\frac{8}{x}\right]_2^4$

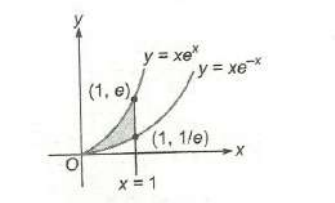


$\Rightarrow \left(a-\frac{8}{a}\right)-(2-4)=\frac{1}{2}[(4-2)-(2-4)] \Rightarrow a-\frac{8}{a}+2=2$

$\Rightarrow a=\sqrt{8}=2\sqrt{2}$ sq unit

26. A=1/2

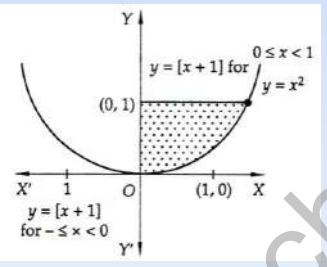
27. Required area $= \int_0^1 xe^x \ dx - \int_0^1 xe^{-x} \ dx$



$= [xe^x - e^x]_0^1 - [-xe^{-x}e^{-x}]_0^1 = \frac{2}{e}$ sq unit

28. Area of required region=$\frac{9}{2}$

29. We have, $\int_{\pi/3}^{\pi/a} \sin ax \ dx = 3 \Rightarrow \frac{1}{a}\left[1 + \frac{1}{2}\right] = 3 \Rightarrow 2\,a = 1 \Rightarrow a = \frac{1}{2}$



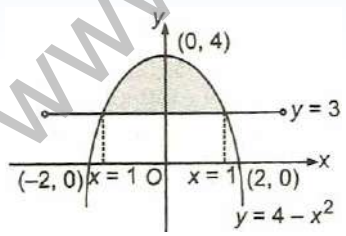
30. M=13/16

31. Area = 8 sq. unit

32. Area bounded = $\frac{4-3\ln 3}{2}$

33. M=2

34. $y = \left[3 + \frac{x^2}{4}\right] = 3, -2 < x < 2$



Required area $= 2\left\{\int_0^1 (4 - x^2)dx - 3\right\} = 2\left\{\left[4x - \frac{x^3}{3}\right]_0^1 - 3\right\}$

$= 2\left\{4 - \frac{1}{3} - 3\right\} = \frac{4}{3}$ sq unit

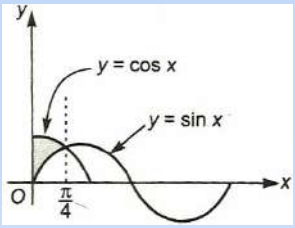
35. The given equation of curves are

$y = \sin x$ …(i)

and $y = \cos x$ …(ii)

From Eqs. (i) and (ii), we get $\sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

$\therefore$ Required area $= \int_0^{\pi/4} (\cos x - \sin x)dx$



$= [\sin x + \cos x]_0^{\pi/4} = \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \cos 0\right) = \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$

$= \frac{2}{\sqrt{2}} - 1 = (\sqrt{2} - 1)$ sq unit

36. Area$=\frac{3\pi a^2}{8}$

37. For $c < 1$, $\int_c^1 (8x^2 - x^5)dx = \frac{16}{3}$

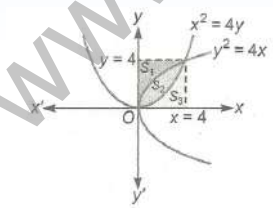
$\Rightarrow \frac{8}{3} - \frac{1}{6} - \frac{8c^3}{3} + \frac{c^6}{6} = \frac{16}{3} \Rightarrow c^3\left[-\frac{8}{3} + \frac{c^3}{6}\right] = \frac{16}{3} - \frac{8}{3} + \frac{1}{6} = \frac{17}{6}$

$\Rightarrow c = -1$ satisfy the above equation

For $c \geq 1$, none of the values of c satisfy the required condition that

$$\int_1^c (8x^2 - x^5)dx = \frac{16}{3}$$

38. $S_1 = S_3 = \int_0^4 \frac{x^2}{4} dx$



$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{16}{3}$ sq units

Now, $S_2 + S_3 = \int_0^4 \sqrt{4x}\ dx = 2 \times \left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{32}{3}$ sq units

$\Longrightarrow s_2 = \frac{16}{3}$ sq units $\therefore$ $S_1 : S_2 : S_3 = \frac{16}{3} : \frac{16}{3} : \frac{16}{3} = 1 : 1 : 1$

39. The given equation can be rewritten as

$\frac{(x-\sqrt{2})^2}{2/\pi}+\frac{y^2}{8/\pi}=1$

Which represent an ellipse.

Here, $a=\sqrt{\frac{2}{\pi}}$

and $b=\sqrt{\frac{8}{\pi}}$

Area enclosed by an ellipse$=\pi ab$

$=\pi\sqrt{\frac{2}{\pi}}\sqrt{\frac{8}{\pi}}=4$ sq units

40. Given. area $=\int_0^2 2^{kx}\,dx=\frac{3}{\log 2}$

$\Rightarrow \left[\frac{2^{kx}}{\log_e 2}\right]_0^2=\frac{3}{\log 2}\Rightarrow \frac{2^{2k}}{\log_e 2}-\frac{1}{\log_e 2}=\frac{3}{\log 2}$

$\Rightarrow 2^{2k}-1=3\Rightarrow 2^{2k}=2^2$

$\Rightarrow 2k=2\Rightarrow k=1$

41. Given equation of curve is $y=a\sqrt{x}+bx$. This curve passes through (1, 2)

$\therefore 2=a+b$ …(i)

and area bounded by the curve and line $x=4$ and x-axis is 8 sq unit, then

$\int_0^4(a\sqrt{x}+bx)dx=8$

$\Rightarrow \frac{2a}{3}\left[x^{3/2}\right]_0^4+\frac{b}{2}[x^2]_0^4=8$

$\Rightarrow \frac{2a}{3}\cdot 8+8b=8\Rightarrow 2a+3b=3$ …(ii)

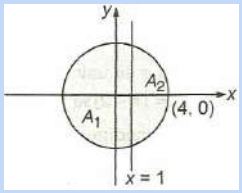
On solving Eqs. (i) and (ii), we get

$a=3$ and $b=-1$

42. Area=8/3

43. Equation of circle is $x^2 + y^2 = 16$

$\therefore$ Total area of circle $= A_1 + A_2 = 16\pi$ …(i)



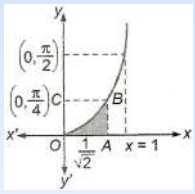
$\frac{A_1}{A_2} = \frac{16\pi}{A_2} - 1$ [on dividing Eq. (i) by A_2]

and $A_2 = 2\int_1^4 \sqrt{16 - x^2}dx$

$A_2 = 2\left\{\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right\}_1^4 = 2\left\{4\pi - \frac{\sqrt{15}}{2} - 8\sin^{-1}\left(\frac{1}{4}\right)\right\}$

$= 8\pi - \sqrt{15} - 16\sin^{-1}\left(\frac{1}{4}\right) \therefore \frac{A_1}{A_2} = \frac{16\pi}{8\pi - \sqrt{15} - 16\sin^{-1}\left(\frac{1}{4}\right)} - 1$

44. Required area = area of rectangle $OABC$ − area of curve $OBCO$



$= \frac{\pi}{4\sqrt{2}} - \int_0^{\frac{\pi}{4}} \sin y\, dy = \frac{\pi}{4\sqrt{2}} + [\cos y]_0^{\frac{\pi}{4}} = \left[\frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - 1\right)\right]$ sq unit

45. $\int_1^b f(x)dx = \sqrt{(b^2 + 1)} - \sqrt{2}$

On differentiating both sides w. r. t. b, we get $f(b) = \frac{b}{\sqrt{(b^2+1)}}$. Hence, $f(x) = \frac{x}{\sqrt{(x^2+1)}}$

46. Required area $= 2\int_{-1}^3 \sqrt{x^2 - 1}\, dx = 2\left[\frac{x\sqrt{x^2-1}}{2} - \frac{1}{2}\log|x + \sqrt{x^2 - 1}\right]_{-1}^3$

$= \left(x\sqrt{x^2 - 1} - \log|x + \sqrt{x^2 - 1}|\right)_{-1}^3 = 6\sqrt{2} - \log|3 + 2\sqrt{2}|$

47. Area $= \frac{8a^2}{3m^3}$

48. The equation of latus rectum of the parabola $y^2 = 12x$ is $x = 3$

Coordinates of end points of latus rectum are (3,6) and (3,-6)

Required length $= 2\int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= 2\int_0^3 \sqrt{1 + \left(\frac{6}{y}\right)^2} = 2\int_0^3 \sqrt{\frac{12x+36}{12x}}\, dx = 2\int_0^3 \frac{x+3}{\sqrt{x^2+3x}}\, dx$

$= 2\left[\int_0^3 \frac{2x+3}{2\sqrt{x^2+3x}}\, dx + \frac{3}{2}\int_0^3 \frac{1}{\sqrt{\left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}}\, dx\right]$

$= 2\left[\sqrt{x^2+3x} + \frac{3}{2}\log\left|\left(x+\frac{3}{2}\right) + \sqrt{x^2+3x}\right|\right]_0^3$

$= 2\left[3\sqrt{2} + \frac{3}{2}\log\left(\frac{9}{2} + 3\sqrt{2}\right) - \frac{3}{2}\log\left(\frac{3}{2}\right)\right] = 2\left[3\sqrt{2} + 3\log\left(3 + 2\sqrt{2}\right)^{\frac{1}{2}}\right]$

$= 2\left[3\sqrt{2} + 3\log\left(\sqrt{2}+1\right)\right] = 6\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$

49. $A_1 = A_2$

50. We have, $\frac{dy}{dx} = 2x + 1$

$\Rightarrow\ y = x^2 + x + c$, it passes through (1, 2) $\therefore\ c = 0$

Then, $y = x^2 + x$

$\therefore$ Required area $= \int_0^1 (x^2 + x)dx = \frac{5}{6}$ sq unit