DEFINITE INTEGRALS

CHAPTER 08

DEFINITE INTEGRALS

DEFINITE INTEGRAL:

An integral of the form $\int_a^b f(x)dx$ is known as definite integral and is given by $\int_a^b f(x) = g(b) - g(a)$, where f(x) is derivative of g(x), a and b are lower and upper limits of a definite integral.

DEFINITE INTEGRAL AS A LIMIT OF SUM:

Let us define a continuous function f(x) in [a, b]. Divide interval [a, b] into an equal sub-interval, each of length h, so that $h = \frac{b-a}{n}$.

Then, $\int_a^b f(x)dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h) + \cdots + f\{a+(n-1)h\}]$

were, $h = \frac{b-a}{n}$

FUNDAMENTAL THEOREM OF CALCULUS:

(i) Let f be a continuous function defined on the closed interval [a,b] and A(x) be the area of function , i.e. $A(x) = \int_a^x f(x)dx$

Then, A'(x)=$f(x), \forall\ x \in [a, b]$

(ii) Let f be a continuous function defined on a closed interval [a,b] and F be an anti-derivative off. Then , $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

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Integrals

$$\int_{-\pi/4}^{\pi/4} \sin^2 x\,dx = 2\int_0^{\pi/4} \sin^2 x\,dx = 2\int_0^{\pi/4}\left(\frac{1-\cos 2x}{2}\right)dx = \int_0^{\pi/4}(1-\cos 2x)\,dx = \left[x-\frac{\sin 2x}{2}\right]_0^{\pi/4} = \frac{\pi}{4}-\frac{1}{2}$$

Integration

It is the inverse of differentiation. Let, $\frac{d}{dx}F(x)=f(x)$. Then, $\int f(x)dx=F(x)+c$, 'c': constant of integral. These integrals are called indefinite or general integrals.

Properties of indefinite integrals are

(i) $\int [f(x)\pm g(x)]\,dx=\int f(x)dx\pm\int g(x)dx$, **(ii)** $\int kf(x)dx=k\int f(x)dx$,

eg : $\int(3x^2+2x)dx=x^3+x^2+c$, where c is real.

Example

(i) $\int x^n dx=\frac{x^{n+1}}{n+1}+c, n\neq -1$ like, $\int dx=x+c$

(ii) $\int \cos x\,dx=\sin x+c$ **(iii)** $\int \sin x\,dx=-\cos x+c$

(iv) $\int \sec^2 x\,dx=\tan x+c$ **(v)** $\int \operatorname{cosec}^2 x\,dx=-\cot x+c$

(vi) $\int \sec x\tan x\,dx=\sec x+c$ **(vii)** $\int \operatorname{cosec} x\cot x\,dx=-\operatorname{cosec} x+c$

(viii) $\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x+c$ **(ix)** $\int\frac{dx}{\sqrt{1-x^2}}=-\cos^{-1}x+c$

(x) $\int\frac{dx}{1+x^2}=\tan^{-1}x+c$ **(xi)** $\int\frac{dx}{1+x^2}=-\cot^{-1}x+c$

(xii) $\int e^x dx=e^x+c$ **(xiii)** $\int a^x dx=\frac{a^x}{\log a}+c$

(xiv) $\int\frac{dx}{x\sqrt{x^2-1}}=\sec^{-1}x+c$ **(xv)** $\int\frac{dx}{x\sqrt{x^2-1}}=-\operatorname{cosec}^{-1}x+c$

(xvi) $\int\frac{1}{x}dx=\log|x|+c$

Some standard integrals

The method in which we change the variable to some other variable is called the method of substitution. Below problems can be solved by substitution.

$\int\tan x dx=\log|\sec x|+c$ $\int\cot x dx=\log|\sin x|+c$

$\int\sec x dx=\log|\sec x+\tan x|+c$ $\int\operatorname{cosec} x dx=\log|\operatorname{cosec} x-\cot x|+c.$

Integration by substitution

(i) $\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$ **(ii)** $\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

(iii) $\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$ **(iv)** $\int\frac{dx}{\sqrt{x^2-a^2}}=\log\left|x+\sqrt{x^2-a^2}\right|+c$

(v) $\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$ **(vi)** $\int\frac{dx}{\sqrt{x^2+a^2}}=\log\left|x+\sqrt{x^2+a^2}\right|+c.$

(vii) $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c.$

(viii) $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c.$

(ix) $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c.$

Integration of some special functions

A rational function of the form $\frac{P(x)}{Q(x)}\left[Q(x)\neq 0\right]=T(x)+\frac{P_1(x)}{Q(x)}$, $P_1(x)$ has degree less than that of $Q(x)$. We can integrate $\frac{P_1(x)}{Q(x)}$ by expressing it in the following forms –

(i) $\frac{px+q}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}, a,b\neq 0.$

(ii) $\frac{px+q}{(x-a)^2}=\frac{A}{x-a}+\frac{B}{(x-a)^2}$ **(iii)** $\frac{px^2+qx+r}{(x-a)^2(x-b)}=\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$

(iv) $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}=\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

(v) $\frac{Px+q}{ax^2+bx+c}=\frac{A\frac{d}{dx}(ax^2+bx+c)}{ax^2+bx+c}+\frac{B}{ax^2+bx+c}$

Integration by partial fractions

$$\int f_1(x)f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left[\frac{d}{dx}f_1(x)\int f_2(x)dx\right]dx$$

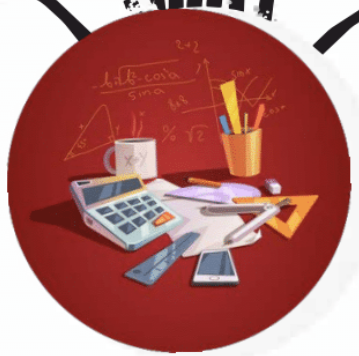
Integration by parts

Let the area function be defined by $A(x)=\int_a^x f(x)dx\,\forall x\geq a$, where f is continuous on $[a,b]$ then $A'(x)=f(x)\,\forall x\in[a,b]$.

First fundamental theorem of integral calculus

Let f be a continuous function of x defined on $[a, b]$ and let F be another function such that $\frac{d}{dx}F(x)=f(x)\,\forall x\in$ domain of f_1 then $\int_a^b f(x)dx=[F(x)+c]_a^b=F(b)-F(a)$. This is called the definite integral of f over the range $[a, b]$, where a and b are called the limits of integration, a being the lower limit and b be the upper limit.

Second fundamental theorem of integral calculus



Trace the Mind Map

- First Level
- Second Level
- Third Level

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PROPERTIES OF DEFINITE INTEGRALS:

(i) $\int_a^b f(x)dx = \int_a^b f(t)dt$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where a < c < b.

(iv) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

(v) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(vi) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

(vii) $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(2a-x) = f(x) \\ 0, & if\ f(2a-x) = -f(x) \end{cases}$

(viii) $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, if\ f(-x) = f(x), i.e. even \\ 0, if\ f(-x) = -f(x), i.e. odd \end{cases}$

PRACTICE QUESTIONS

1. $\int_0^1 \sqrt{x(1-x)}\,dx$ equals
   a) $\frac{\pi}{2}$
   b) $\frac{\pi}{4}$
   c) $\frac{\pi}{6}$
   d) $\frac{\pi}{8}$

2. $\int_0^{\pi} \frac{1}{1+sinx}dx$equals
   a) 0
   b) 1/2
   c) 2
   d) 3/2

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3. The value of $\int_0^{\pi} \frac{xtanx}{secx+cosx} dx$ is

   a) $\frac{\pi^2}{4}$

   b) $\frac{\pi^2}{2}$

   c) $\frac{3\pi^2}{2}$

   d) $\frac{\pi^2}{3}$

4. The value of $\int_0^{2\pi} \sqrt{1+\sin\frac{x}{2}} dx$ is

   a) 0

   b) 2

   c) 8

   d) 4

5. The value of integral $\int_0^{\frac{\pi}{2}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}} dx$ is

   a) 0

   b) $\pi/2$

   c) $\pi/4$

   d) None of these

6. $\int_0^{\infty} \frac{1}{1+e^x}$ dx equals

   a) log2-1

   b) log2

   c) log4-1

   d) –log2

7. $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}}$ dx equals

   a) 2

   b) 1

   c) $\frac{\pi}{4}$

   d) $\frac{\pi^2}{8}$

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8. $\int_0^{\pi/2} \frac{cosx}{(2+sinx)(1+sinx)}$ dx equals

a) $\log\frac{2}{3}$

b) $\log\frac{3}{2}$

c) $\log\frac{3}{4}$

d) $\log\frac{4}{3}$

9. $\int_0^{\pi/2} \frac{1}{2+cosx}$ dx equals

a) $\frac{1}{3}\tan^{-1}(\frac{1}{\sqrt{3}})$

b) $\frac{2}{\sqrt{3}}\tan^{-1}(\frac{1}{\sqrt{3}})$

c) $\sqrt{3}\tan^{-1}(\sqrt{3})$

d) $2\sqrt{3}\tan^{-1}(\sqrt{3})$

10. $\int_0^{\pi} \sqrt{\frac{1-x}{1+x}}$ dx=

a) $\frac{\pi}{2}$

b) $\frac{\pi}{2} - 1$

c) $\frac{\pi}{2} + 1$

d) $\pi + 1$

11. $\int_0^{\pi} \frac{1}{a+bcosx} dx =$

a) $\frac{\pi}{\sqrt{a^2-b^2}}$

b) $\frac{\pi}{ab}$

c) $\frac{\pi}{a^2+b^2}$

d) $\pi(a+b)$

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12. $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{cotx}} dx$ is

a) $\frac{\pi}{3}$

b) $\frac{\pi}{6}$

c) $\frac{\pi}{12}$

d) $\frac{\pi}{2}$

13. Given that $\int_0^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} dx = \frac{\pi}{2(a+b)(b+c)(c+a)}$

The value of $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$, is

a) $\frac{\pi}{60}$

b) $\frac{\pi}{20}$

c) $\frac{\pi}{40}$

d) $\frac{\pi}{80}$

14. $\int_1^e \frac{logx}{x} dx$=

a) $\frac{1}{2}$

b) $\frac{e^2}{2}$

c) 1

d) ∞

15. $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$ is equal to

a) $\frac{\pi}{12}$

b) $\frac{\pi}{6}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{3}$

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16. $\int_0^3 \frac{3x+1}{x^2+9} dx =$

a) $\frac{\pi}{12} + \log(2\sqrt{2})$

b) $\frac{\pi}{2} + \log(2\sqrt{2})$

c) $\frac{\pi}{6} + \log(2\sqrt{2})$

d) $\frac{\pi}{3} + \log(2\sqrt{2})$

17. The value of the integral $\int_0^\infty \frac{x}{(1+x)(1+x^2)} dx$ is

a) $\frac{\pi}{2}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{6}$

d) $\frac{\pi}{3}$

18. $\int_{-\pi/2}^{\pi/2} \sin|x| dx$ is equal to

a) 1

b) 2

c) -1

d) -2

19. $\int_0^{\pi/2} \frac{1}{1+tanx} dx$ is equal to

a) $\frac{\pi}{4}$

b) $\frac{\pi}{3}$

c) $\frac{\pi}{2}$

d) π

20. The value of $\int_0^{\pi/2} cosxe^{sinx} dx$ is

a) 1

b) e-1

c) 0

d) -1

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21. If $\int_0^a \frac{1}{1+4x^2}\,dx = \frac{\pi}{8}$, then a equals

a) $\frac{\pi}{2}$

b) $\frac{1}{2}$

c) $\frac{\pi}{4}$

d) 1

22. If $\int_0^1 f(x)dx = 1, \int_0^1 xf(x)dx = a\,, \int_0^1 x^2 f(x)dx = a^2$, then $\int_0^1 (a-x)^2\,dx$ equals

a) $4a^2$

b) 0

c) $2a^2$

d) None of these

23. The value of $\int_{-\pi}^{\pi} sin^3\,x\,cos^2 x\,dx$ is

a) $\frac{\pi^4}{2}$

b) $\frac{\pi^4}{4}$

c) 0

d) None of these

24. $\int_{\pi/6}^{\pi/3} \frac{1}{\sin 2x}\,dx$is equal to

a) $\log_e 3$

b) $\log_e \sqrt{3}$

c) $\frac{1}{2}\log(-1)$

d) log(-1)

25. $\int_{-1}^{1} |1-x|dx$ is equal to

a) -2

b) 2

c) 0

d) 4

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26. The derivative of $f(x) = \int_{x^2}^{x^3} \frac{1}{\log_e t}\, dt$,(x>0), is

a) $\frac{1}{3lnx}$

b) $\frac{1}{3lnx} - \frac{1}{2lnx}$

c) $(lnx)^{-1}x(x-1)$

d) $\frac{3x^2}{lnx}$

27. If $I_{10} = \int_0^{\pi/2} x^{10} sinx\, dx$, then the value of $I_{10} + 90I_8$ is

a) $9\left(\frac{\pi}{2}\right)^9$

b) $10\left(\frac{\pi}{2}\right)^9$

c) $\left(\frac{\pi}{2}\right)^9$

d) $9\left(\frac{\pi}{2}\right)^8$

28. $\int_0^1 \frac{x}{(1-x)^{5/4}}\, dx =$

a) $\frac{15}{16}$

b) $\frac{3}{16}$

c) $-\frac{3}{16}$

d) $-\frac{16}{3}$

29. $\int_0^{\pi/2} \sqrt{1 - sin2x}\, dx$ is equal to

a) $2\sqrt{2}$

b) $2(\sqrt{2}+1)$

c) 2

d) $2(\sqrt{2}-1)$

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30. The value of the integral $\int_{-2}^{2}|1-x^2|dx$ is

a) 4

b) 2

c) -2

d) 0

31. $\int_0^{\pi/2}\frac{1}{1+cot^3x}dx$ is equal to

a) 0

b) 1

c) $\frac{\pi}{2}$

d) $\frac{\pi}{4}$

32. $\int_0^{\pi/2}\frac{sinx}{sinx+cosx}dx$ equals to

a) π

b) $\pi/2$

c) $\pi/3$

d) $\pi/4$

33. $\int_0^1\frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}dx$ is equal to

a) 0

b) π

c) $\pi/2$

d) $\pi/4$

34. $\int_0^{\pi/2} xsinx$ dx is equal to

a) $\frac{\pi}{4}$

b) $\frac{\pi}{2}$

c) π

d) 1

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35. $\int_0^{\pi/2} \sin 2x \log tanx \, dx$ is equal to

a) π

b) $\frac{\pi}{2}$

c) 0

d) 2π

36. The value of $\int_0^{\pi} \frac{1}{5+3cosx} dx$ is

a) $\frac{\pi}{4}$

b) $\frac{\pi}{8}$

c) $\frac{\pi}{2}$

d) 0

37. $\int_0^{\infty} \log\left(x+\frac{1}{x}\right)\frac{1}{1+x^2} dx=$

a) $\pi ln2$

b) $-\pi ln2$

c) 0

d) $-\frac{\pi}{2} ln2$

38. $\int_0^{2a} f(x)dx$ is equal to

a) $2\int_0^{a} f(x)dx$

b) 0

c) $\int_0^{a} f(x)dx + \int_0^{a} f(2a-x)dx$

d) $\int_0^{a} f(x)dx + \int_0^{2a} f(2a-x)dx$

39. If $f(a+b-x) = f(x)$, then $\int_a^b x\, f(x)dx$ is equal to

a) $\frac{a+b}{2} \int_a^b f(b-x)\, dx$

b) $\frac{a+b}{2} \int_a^b f(b+x)\, dx$

c) $\frac{b-a}{2} \int_a^b f(x)\, dx$

d) $\frac{a+b}{2} \int_a^b f(x)\, dx$

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40. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$, is

a) 1

b) 0

c) -1

d) $\frac{\pi}{4}$

41. The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is

a) 2

b) 3/4

c) 0

d) -2

42. The value of $\int_{-\pi/2}^{\pi/2}(x^3 + xcosx + tan^5 x + 1)$ dx, is

a) 0

b) 2

c) π

d) 1

43. $\int_{-\pi/4}^{\pi/4} \frac{1}{1+cos2x} dx$ is equal to

a) 1

b) 2

c) 3

d) 4

44. $\int_{a+c}^{b+c} f(x)dx$ is equal to

a) $\int_a^b f(x-c)dx$

b) $\int_a^b f(x+c)dx$

c) $\int_a^b f(x)dx$

d) $\int_{a-c}^{b-c} f(x)dx$

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45. If f and g are continuous functions in [0,1] satisfying f(x)=f(a-x) and g(x)+g(a-x)=a, then $\int_0^a f(x)g(x)\,dx$ is equal to

a) $\frac{a}{2}$

b) $\frac{a}{2}\int_0^a f(x)dx$

c) $\int_0^a f(x)dx$

d) $a\int_0^b f(x)dx$

46. $\int_0^1 \tan\sin^{-1}x\ dx$ equals

a) 2

b) 0

c) -1

d) 1

47. $\int_{-1/7}^{20/7} \sin(x-[x])dx$ (where [·] denotes greatest integer) is

a) 0

b) $3\,(\cos 1-1)$

c) $3(1-\cos 1)$

d) None of these

48. Let $f(x)$ be an integrable function defined on $[a,b], b>a>0$. If $I_1=\int_{\pi/6}^{\pi/3} f(\tan\theta+\cot\theta)\sec^2\theta\ d\theta$ and, $I_2=\int_{\pi/6}^{\pi/3} f(\tan\theta+\cot\theta)\,\mathrm{cosec}^2\theta\ d\theta$, then $\frac{I_1}{I_2}=$

a) A positive integer

b) A negative integer

c) An irrational number

d) None of these

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49. If $(-1, 2)$ and $(2, 4)$ are two points on the curve $y = f(x)$ and if $g(x)$ is the gradient of the curve at point (x, y), then the value of the integral $\int_{-1}^{2} g(x)\,dx$, is

a) 2

b) -2

c) 0

d) 1

50. The value of the integral $\int_{-1}^{1} \sin^{11} x \; dx$, is

a) $\frac{10}{11}, \frac{8}{9}, \frac{6}{7}, \frac{4}{5}, \frac{2}{3}$

b) $\frac{10}{11}, \frac{8}{9}, \frac{6}{7}, \frac{4}{5}, \frac{2}{3}, \frac{\pi}{2}$

c) 1

d) 0

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ANSWER KEY

1. D
2. C
3. A
4. C
5. C
6. B
7. A
8. D
9. B
10. B
11. A
12. C
13. A
14. A
15. A
16. A
17. B
18. B
19. A
20. B
21. B
22. B
23. C
24. B
25. B
26. C
27. B
28. D
29. D
30. A
31. D
32. D
33. C
34. D
35. C
36. A
37. B
38. C
39. D
40. B
41. C
42. C
43. A
44. B
45. B
46. D
47. C
48. A
49. A
50. D

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HINTS AND SOLUTIONS

1. $I = \int_0^1 \sqrt{x(1-x)}\,dx \Rightarrow I = \int_0^1 \sqrt{x-x^2}\,dx$

   $I = \int_0^1 \sqrt{\frac{1}{4}+x-x^2-\frac{1}{4}}dx \Rightarrow I = \int_0^1 \sqrt{\frac{1}{4}-\left(x^2-x+\frac{1}{4}\right)}dx$

   $I = \int_0^1 \sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}\,dx \Rightarrow I = \left[\frac{x-\frac{1}{2}}{2}.\sqrt{x(1-x)}+\frac{1}{2}\times\frac{1}{4}\sin^{-1}(2x-1)\right]_0^1$

   $I = 0+\frac{1}{8}(\sin^{-1}(1)-\sin^{-1}(-1)) \Rightarrow I = \frac{1}{8}\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right)$

   $I = \pi/8$

2. $I = \int_0^{\pi}\frac{1}{1+sinx}dx \Rightarrow I = \int_0^{\pi}\frac{1}{1+sinx}\times\frac{1-sinx}{1-sinx}dx$

   $I = \int_0^{\pi}\frac{1-sinx}{1-sin^2x}dx \Rightarrow I = \int_0^{\pi}\frac{1-sinx}{cos^2x}dx$

   $I = \int_0^{\pi}(sec^2x - tanxsecx)\,dx \Rightarrow I = [tanx - secx]_0^{\pi}$

   $I = -(-1-1) \Rightarrow I = 2$

3. $I = \int_0^{\pi}\frac{xtanx}{secx+cosx}dx \Rightarrow I = \int_0^{\pi}\frac{(\pi-x)tan(\pi-x)}{sec(\pi-x)+cos(\pi-x)}dx$

   $I = \int_0^{\pi}\frac{-(\pi-x)tanx}{-secx-cosx}dx \Rightarrow I = \int_0^{\pi}\frac{(\pi-x)tanx}{secx+cosx}dx$

   $2I = \int_0^{\pi}\frac{xtanx}{secx+cosx}dx + \int_0^{\pi}\frac{(\pi-x)tanx}{secx+cosx}dx \Rightarrow 2I = \int_0^{\pi}\frac{\pi tanx}{secx+cosx}dx$

   $2I = \int_0^{\pi}\frac{\pi\frac{sinx}{cosx}}{\frac{1}{cosx}+cosx}dx \Rightarrow 2I = \int_0^{\pi}\frac{\pi sinx}{1+cos^2x}dx$

   $Put\ cosx = t \Rightarrow -sinxdx = dt$

   $\Rightarrow sinxdx = -dt \Rightarrow 2I = \int_1^{-1}\frac{-\pi dt}{1+t^2}$

   $I = \frac{-\pi}{2}[\tan^{-1}t]_1^{-1} \Rightarrow I = \frac{\pi^2}{4}$

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4. $I = \int_0^{2\pi} \sqrt{1+\sin\frac{x}{2}}\, dx$

$\Rightarrow I = \int_0^{2\pi} [\cos\frac{x}{4} + \sin\frac{x}{4}]^2\, dx \Rightarrow I = \int_0^{2\pi} [\cos\frac{x}{4} + \sin\frac{x}{4}] dx \Rightarrow I = \left[\frac{\sin\frac{x}{4}}{\frac{1}{4}} - \frac{\cos\frac{x}{4}}{\frac{1}{4}}\right]_0^{2\pi}$

$\Rightarrow I = 4\left[\sin\frac{\pi}{2} - \left(\cos\frac{\pi}{2} - \cos 0\right)\right] \Rightarrow I = 8$

5. $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$.....(1)

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos(\frac{\pi}{2}-x)}}{\sqrt{\cos(\frac{\pi}{2}-x)}+\sqrt{\sin(\frac{\pi}{2}-x)}} dx \Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$ .....(2)

Adding 1 and 2

$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx \Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} 1\, dx \Rightarrow I = \frac{1}{2}[x]_0^{\pi/2} \Rightarrow I = \frac{\pi}{4}$

6. $I = \int_0^{\infty} \frac{1}{1+e^x} dx$

$\Rightarrow I = \int_0^{\infty} \frac{\frac{1}{e^x}}{\frac{1+e^x}{e^x}} dx \Rightarrow I = \int_0^{\infty} \frac{e^{-x}}{1+e^{-x}} dx$

$Put\ 1 + e^{-x} = t \Rightarrow -e^{-x}dx = dt$

$\Rightarrow e^{-x}dx = -dt \Rightarrow I = \int_1^2 \frac{-dt}{t} \Rightarrow I = -[log|t|]_2^1 \Rightarrow I = -(\log 1 - \log 2) \Rightarrow I = \log 2$

7. $I = \int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

$Put\ \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt \Rightarrow I = \int_0^{\pi/2} 2sint\, dt \Rightarrow I = 2[-cost]_0^{\pi/2} \Rightarrow I = -2(0-1) \Rightarrow I = 2$

8. $I = \int_0^{\pi/2} \frac{cosx}{(2+sinx)(1+sinx)} dx$

$Put\ sinx = t \Rightarrow cosx dx = dt$

$\Rightarrow I = \int_0^1 \frac{dt}{(1+t)(2+t)} \Rightarrow I = \int_0^1 \frac{[(t+2)-(t+1)]dt}{(1+t)(2+t)} \Rightarrow I = \int_0^1 \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt$

$I = [\log|t+1| - \log|t+2|]_0^1 \Rightarrow I = 2log2 - log3 \Rightarrow I = log4 - log3 \Rightarrow I = \log\left(\frac{4}{3}\right)$

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9. $I = \int_0^{\pi/2} \frac{1}{2+cosx} dx$

$Put \tan\frac{x}{2} = t \Rightarrow x = 2\tan^{-1} t$

$\Rightarrow dx = \frac{2dt}{1+t^2} \Rightarrow cosx = \frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$

$I = \int_0^1 \frac{\frac{2dt}{1+t^2}}{2+\frac{1-t^2}{1+t^2}} \Rightarrow I = \int_0^1 \frac{2dt}{3+t^2} \Rightarrow I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right]_0^1 \Rightarrow I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

10. $I = \int_0^{\pi} \sqrt{\frac{1-x}{1+x}} dx \Rightarrow I = \int_0^{\pi} \sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}} dx \Rightarrow I = \int_0^{\pi} \frac{1-x}{\sqrt{1-x^2}} dx$

Consider,

$1 - x = A\frac{d(1-x^2)}{dx} + B \Rightarrow 1 - x = -2Ax + B$

Comparing both sides,

$-2A = -1 \Rightarrow A = \frac{1}{2}\ and\ B = 1$

$\Rightarrow I = \int_0^{\pi} \frac{\left(\frac{1}{2}\right)(-2x)+\frac{1}{2}}{\sqrt{1-x^2}} dx \Rightarrow I = \frac{1}{2}\int_0^{\pi} \frac{-2x}{\sqrt{1-x^2}} dx + \int_0^{\pi} \frac{1}{2\sqrt{1-x^2}} dx$

$I = \left[\frac{1}{2}\left(2\sqrt{1-x^2}\right) + \frac{1}{2}\sin^{-1} x\right]_0^{\pi} \Rightarrow I = \sqrt{1-\pi^2} - 1$

11. $I = \int_0^{\pi} \frac{1}{a+bcosx} dx$

$Put \tan\frac{x}{2} = t \Rightarrow x = 2\tan^{-1} t \Rightarrow dx = \frac{2dt}{1+t^2} \Rightarrow cosx = \frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$

$I = \int_0^{\infty} \frac{\frac{2dt}{1+t^2}}{a+b\left(\frac{1-t^2}{1+t^2}\right)} \Rightarrow I = 2\int_0^{\infty} \frac{dt}{a+b+(a-b)t^2} \Rightarrow I = 2\int_0^{\infty} \frac{dt}{(\sqrt{a+b})^2+\left(\sqrt{(a-b)}t\right)^2}$

$\Rightarrow I = \frac{2}{\sqrt{(a+b)(a-b)}}\left[\tan^{-1}\left(\frac{t}{\sqrt{\frac{a+b}{a-b}}}\right)\right]_0^{\infty} \Rightarrow I = \frac{\pi}{\sqrt{(a^2-b^2)}}$

12. $I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{cotx}} dx \Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\frac{cosx}{sinx}}} dx \Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}} dx$ .....(1)

$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}} dx \Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}} dx$ .....(2)

Adding 1 and 2

$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}} dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{cosx}}{\sqrt{sinx}+\sqrt{cosx}} dx \Rightarrow 2I = \int_{\pi/6}^{\pi/3} dx \Rightarrow I = \frac{\pi}{12}$

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13. $I = \int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} \Rightarrow I = \frac{1}{5}\int_0^\infty \frac{5dx}{(x^2+4)(x^2+9)}$

$\Rightarrow I = \frac{1}{5}\left(\int_0^\infty \frac{1}{x^2+4}dx - \int_0^\infty \frac{1}{x^2+9}dx\right) \Rightarrow I = \frac{1}{5}\left(\frac{1}{2}\tan^{-1}\frac{x}{2} - \frac{1}{3}\tan^{-1}\frac{x}{3}\right)_0^\infty \Rightarrow I = \frac{\pi}{60}$

14. $I = \int_1^e \log x\, dx \Rightarrow I = \log x \int_1^e dx - \int_1^e \left(\frac{d(\log x)}{dx}\int dx\right)dx \Rightarrow I = [x\log x]_1^e -$

$\int_1^e \frac{1}{x}x\, dx$

$I = e\log e - \log 1 - [x]_1^e \Rightarrow I = e - 0 - (e-1) \Rightarrow I = 1$

15. $I = \int_1^{\sqrt{3}} \frac{1}{1+x^2}dx \Rightarrow I = \tan^{-1}\sqrt{3} - \tan^{-1}1 \Rightarrow I = \frac{\pi}{3} - \frac{\pi}{4} \Rightarrow I = \frac{\pi}{12}$

16. $I = \int_0^3 \frac{3x+1}{x^2+9}dx$

Consider,

$3x + 1 = A\frac{d(x^2+9)}{dx} + B \Rightarrow 3x + 1 = 2Ax + B$

Comparing both sides,

$2A = 3 \Rightarrow A = \frac{3}{2}, B = 1$

$I = \int_0^3 \frac{2x.\frac{3}{2}+1}{x^2+9}dx \Rightarrow I = \frac{3}{2}\int_0^3 \frac{2x}{x^2+9}dx + \int_0^3 \frac{1}{x^2+9}dx$

$\Rightarrow I = \frac{3}{2}(log18 - log9) + \frac{1}{3}\tan^{-1}(1) \Rightarrow I = \log 2\sqrt{2} + \frac{\pi}{12}$

17. $I = \int_0^\infty \frac{x}{(1+x)(1+x^2)}dx$

$Put\ x = tan\theta \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow d\theta = \frac{1}{1+x^2}dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{tan\theta}{1+tan\theta}d\theta \Rightarrow I = \int_0^{\pi/2} \frac{sin\theta}{cos\theta+sin\theta}d\theta$ ……..(1)

$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2}-\theta)}{cos(\frac{\pi}{2}-\theta)+sin(\frac{\pi}{2}-\theta)}d\theta \Rightarrow I = \int_0^{\pi/2} \frac{cos\theta}{cos\theta+sin\theta}d\theta$ …..(2)

Adding 1 and 2,

$I = \int_0^{\pi/2} d\theta \Rightarrow I = \frac{\pi}{4}$

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18. $I = \int_{-\pi/2}^{\pi/2} \sin|x|dx \Rightarrow I = \int_{-\pi/2}^{0} \sin(-x)dx + \int_{0}^{\pi/2} sinxdx$

$\Rightarrow I = -\int_{-\pi/2}^{0} sinxdx + \int_{0}^{\pi/2} sinxdx \Rightarrow I = -[-cosx]_{-\frac{\pi}{2}}^{0} - [cosx]_{0}^{\frac{\pi}{2}}$

$\Rightarrow I = 1 - 0 - (0 - 1) \Rightarrow I = 2$

19. $\int_{0}^{\pi/2} \frac{1}{1+tanx} dx \Rightarrow I = \int_{0}^{\pi/2} \frac{cosx}{cosx+sinx} dx$ .....(1)

$I = \int_{0}^{\pi/2} \frac{\cos(\frac{\pi}{2}-x)}{cos(\frac{\pi}{2}-x)+sin(\frac{\pi}{2}-x)} dx \Rightarrow I = \int_{0}^{\pi/2} \frac{sinx}{cosx+sinx} dx$ .....(2)

Adding 1 and 2

$2I = \int_{0}^{\pi/2} dx \Rightarrow I = \frac{\pi}{4}$

20. $I = \int_{0}^{\pi/2} cosxe^{sinx} dx$

$Put\ sinx = t \Rightarrow cosxdx = dt$

$I = \int_{0}^{1} e^{t}\, dt \Rightarrow I = e - e^{0} \Rightarrow I = e - 1$

21. $\int_{0}^{a} \frac{1}{1+4x^2} dx = \frac{\pi}{8}$

$\frac{1}{2}[\tan^{-1}(2x)]_{0}^{a} = \frac{\pi}{8} \Rightarrow \tan^{-1}(2a) = \frac{2\pi}{8} \Rightarrow a = \frac{1}{2}$

22. $I = \int_{0}^{1} (a - x)^2 f(x)dx$

$I = \int_{0}^{1} (a^2 - 2ax + x^2) f(x)dx \Rightarrow I = \int_{0}^{1} (a^2 f(x) - 2af(x) + x^2 f(x))dx$

$I = a^2 \int_{0}^{1} f(x)dx - 2a \int_{0}^{1} xf(x)dx + \int_{0}^{1} x^2 f(x)dx \Rightarrow I = a^2 \times 1 - 2a \times a + a^2 \Rightarrow$

$I = 0$

23. $I = \int_{-\pi}^{\pi} sin^3\, x\, cos^2 x\, dx$

Here,

$f(x) = sin^3 x cos^2 x \Rightarrow f(-x) = -sin^3 x cos^2 x \Rightarrow f(-x) = -f(x)$

Function is odd.

$\int_{-\pi}^{\pi} sin^3\, x\, cos^2 x\, dx = 0$

24. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin 2x} dx \Rightarrow I = \int_{\pi/6}^{\pi/3} cosec\ 2x\, dx \Rightarrow I = \frac{1}{2}[\log_e |tanx|]_{\pi/6}^{\pi/3} \Rightarrow I = log\sqrt{3}$

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25. $I = \int_{-1}^{1} |1-x| dx \Rightarrow I = \int_{-1}^{0} (1-x) dx + \int_{0}^{1} (1-x) dx \Rightarrow I = \left[x - \frac{x^2}{2}\right]_{-1}^{0} \left[x - \frac{x^2}{2}\right]_{0}^{1}$

$I = 1 + \frac{1}{2} + 1 - \frac{1}{2} \Rightarrow I = 2$

26. $f(x) = \int_{x^2}^{x^3} \frac{1}{\log_e t} dt \Rightarrow f(x) = [\log(\log_e t)]_{x^2}^{x^3} \Rightarrow f(x) = \log(\log_e x^3) - \log(\log_e x^2)$

$f'(x) = \frac{1}{3\log_e x} \times 3x^2 - \frac{1}{2\log_e x} \times 2x \Rightarrow f'(x) = (logx)^{-1} \times (x-1)$

27. $I_{10} = \int_0^{\pi/2} x^{10} sinx\, dx$

$I_{10} = x^{10} \int_0^{\pi/2} sinx\, dx - \int_0^{\frac{\pi}{2}} (\frac{dx^{10}}{dx} \int sinx\, dx)\, dx$

$I_{10} = (-x^{10} cosx)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (10x^9(-cosx)\, dx) \Rightarrow I_{10} = -[0] + 10 \int_0^{\pi/2} x^9 cosx\, dx$

$I_{10} = [10x^9 sinx]_0^{\frac{\pi}{2}} - 10 \int_0^{\frac{\pi}{2}} (\frac{dx^9}{dx} \int cosx\, dx) dx \Rightarrow I_{10} = 10\left(\frac{\pi}{2}\right)^9 - 90I_8$

$I_{10} + 90I_8 = 10\left(\frac{\pi}{2}\right)^9$

28. $I = \int_0^1 \frac{x}{(1-x)^{5/4}} dx$

$Put\ 1 - x = t \Rightarrow x = 1 - t$

$\Rightarrow dx = -dt$

$\Rightarrow I = \int_0^1 \frac{(1-t)(-dt)}{t^{5/4}} \Rightarrow I = \int_0^1 (t^{-5/4} - t^{-\frac{1}{4}}) dt \Rightarrow I = -4 - \frac{4}{3} \Rightarrow I = -\frac{16}{3}$

29. Solve it yourself.

30. $I = \int_{-2}^{2} |1-x^2| dx \Rightarrow I = \int_{-2}^{-1} -(1-x^2) dx + \int_{-1}^{1} (1-x^2) dx + \int_{-2}^{2} -(1-x^2) dx$

$I = \left[-x + \frac{x^3}{3}\right]_{-2}^{-1} + \left[x - \frac{x^3}{3}\right]_{-1}^{1} + \left[-x + \frac{x^3}{3}\right]_{1}^{2} \Rightarrow I = 4$

31. $I = \int_0^{\pi/2} \frac{1}{1+cot^3 x} dx$….(1)

$I = \int_0^{\pi/2} \frac{1}{1+cot^3\left(\frac{\pi}{2}-x\right)} dx \Rightarrow I = \int_0^{\pi/2} \frac{1}{1+tan^3 x} dx$ …..(2)

Adding 1 and 2

$2I = \int_0^{\pi/2} \frac{1}{1+cot^3 x} dx + \int_0^{\pi/2} \frac{1}{1+tan^3 x} dx \Rightarrow 2I = \int_0^{\pi/2} \frac{2+tan^3 x+cot^3 x}{(1+cot^3 x)(1+tan^3 x)} dx$

$2I = \int_0^{\pi/2} \frac{2+tan^3 x+cot^3 x}{1+tan^3 x+cot^3 x+1} dx \Rightarrow 2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{\pi}{4}$

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32. $I = \int_0^{\pi/2} \frac{sinx}{sinx+cosx} dx$.....(1)

$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)+\cos(\frac{\pi}{2}-x)} dx \Rightarrow I = \int_0^{\pi/2} \frac{cosx}{sinx+cosx} dx$

Adding 1 and 2,

$2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{\pi}{4}$

33. $I = \int_0^1 \frac{d}{dx} \left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\} dx \Rightarrow I = \left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1 \Rightarrow I = \sin^{-1} 1 - \sin^{-1} 0$

$I = \frac{\pi}{2}$

34. $I = \int_0^{\pi/2} xsinxdx \Rightarrow I = x\int_0^{\pi/2} sinxdx - \int_0^{\frac{\pi}{2}} (\frac{dx}{dx}\int sinxdx)\, dx$

$I = [-xcosx]_0^{\frac{\pi}{2}} - \int_0^{\pi/2} -cosxdx \Rightarrow I = [sinx]_0^{\pi/2} \Rightarrow I = 1$

35. $I = \int_0^{\pi/2} \sin 2x \log tanx\, dx$....(1)

$I = \int_0^{\pi/2} \sin\left[2\left(\frac{\pi}{2}-x\right)\right] \log\tan\left(\frac{\pi}{2}-x\right) dx \Rightarrow I = \int_0^{\pi/2} \sin 2x \log cotx\, dx$ .....(2)

$2I = \int_0^{\pi/2} \sin 2x \log tanx\, dx + \int_0^{\pi/2} \sin 2x \log cotx\, dx$

$2I = \int_0^{\frac{\pi}{2}} \sin 2x\, (\log(\tan x\ .\cot x))dx \Rightarrow 2I = \int_0^{\pi/2} sin2x(\log(1))dx = 0$

36. $I = \int_0^{\pi} \frac{1}{5+3cosx} dx \Rightarrow I = \int_0^{\pi} \frac{1}{5+3\left(\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)} dx \Rightarrow I = \int_0^{\pi} \frac{sec^2\frac{x}{2}}{8+2tan^2\frac{x}{2}} dx$

$Put\ tan\frac{x}{2} = t \Rightarrow sec^2\frac{x}{2}dx = dt$

$I = \int_0^{\infty} \frac{dt}{4+t^2} \Rightarrow I = \frac{1}{2}\left[\tan^{-1}\left(\frac{1}{2}\right)\right]_0^{\infty} \Rightarrow I = \frac{1}{2}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{4}$

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37. $I = \int_0^{\infty} \log\left(x + \frac{1}{x}\right) \frac{1}{1+x^2} dx$

$Put\ x = tan\theta \Rightarrow dx = sec^2\theta d\theta$

$I = \int_0^{\pi/2} \log\left(tan\theta + \frac{1}{tan\theta}\right) \frac{1}{1+tan^2\theta} sec^2\theta\ d\theta \Rightarrow I = \int_0^{\pi/2} \log\left(\frac{sec^2\theta}{tan\theta}\right) \frac{1}{sec^2\theta} sec^2\theta d\theta$

$I = \int_0^{\pi/2} \log\left(\frac{1}{\sin\theta \cos\theta}\right) d\theta \Rightarrow I = -\int_0^{\pi/2} \log(sin\theta cos\theta)\ d\theta$

$I = -\int_0^{\frac{\pi}{2}} (\log\sin\theta + \log\cos\theta) d\theta$

$Let\ I_1 = \int_0^{\pi/2} (\log\sin\theta) d\theta$

$I_1 = \int_0^{\pi/2} (\log\sin\left(\frac{\pi}{2} - \theta\right)) d\theta \Rightarrow I_1 = \int_0^{\pi/2} (\log\cos\theta) d\theta$

Adding I and I1

$2I = \int_0^{\pi/2} (\log cos\theta + \log\sin\theta) d\theta \Rightarrow 2I = \int_0^{\pi/2} [\log\sin 2\theta - \log 2] d\theta$

$2\theta = t \Rightarrow 2d\theta = dt$

$I = \left(\frac{1}{2} - \frac{\pi}{2}\log 2\right) \Rightarrow I = -\frac{\pi}{2}\log 2$

$-\int_0^{\frac{\pi}{2}} (\log\sin\theta + \log\cos\theta) d\theta = -2I = \pi log 2$

38. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a - x)dx$

39. $I = \int_a^b x\, f(x)dx \ldots.(1)$

$I = \int_a^b (a + b - x)\, f(a + b - x)dx \ldots..(2)$

Adding 1 and 2

$2I = \int_a^b x\, f(x)dx + \int_a^b (a + b - x)\, f(a + b - x)dx \Rightarrow 2I = \int_a^b (a + b)\, f(x)dx$

$I = \frac{a+b}{2} \int_a^b f(x)dx$

40. $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx \ldots.(1)$

$I = \int_0^1 \tan^{-1}\left(\frac{2(1-x)-1}{1+1-x-(1-x)^2}\right) dx \Rightarrow I = \int_0^1 \tan^{-1}(\frac{1-2x}{1+x-x^2})\ dx \ldots..(2)$

Adding 1 and 2,

$2I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx + \int_0^1 \tan^{-1}(\frac{1-2x}{1+x-x^2})\ dx$

$2I = \int_0^1 \tan^{-1}(\frac{2x-1}{1+x-x^2})\ dx - \int_0^1 \tan^{-1}(\frac{2x-1}{1+x-x^2})\ dx \Rightarrow I = 0$

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41. $I = \int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx \ldots.(1)$

$I = \int_0^{\pi/2} \log\left(\frac{4+3\sin(\frac{\pi}{2}-x)}{4+3\cos(\frac{\pi}{2}-x)}\right) dx \Rightarrow I = \int_0^{\pi/2} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) dx \ldots.(2)$

Adding 1 and 2

$2I = \int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx + \int_0^{\pi/2} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) dx \Rightarrow I = 0$

42. $I = \int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\, dx$

Applying simple formulas and integration by parts,

$I = \pi$

43. $\int_{-\pi/4}^{\pi/4} \frac{1}{1+\cos 2x} dx = \int_{-\pi/4}^{\pi/4} \frac{1}{2\cos^2 x} dx \Rightarrow \int_{-\pi/4}^{\pi/4} \sec^2 x dx = \frac{1}{2}[\tan x]_{-\pi/4}^{\pi/4} = 1$

44. $I = \int_{a+c}^{b+c} f(x)dx$

$I = [f(x)]_{a+c}^{b+c}$ =f(b+c)-f(a+c)

Now solve by options.

45. $I = \int_0^a f(x)g(x)\, dx \Rightarrow I = \int_0^a f(a-x)g(a-x)\, dx$

$I = \int_0^a f(x)\big(a - g(x)\big)dx \Rightarrow I = a\int_0^a f(x)dx - \int_0^a f(x).g(x)dx$

$I = a\int_0^a f(x)dx - I \Rightarrow 2I = a\int_0^a f(x)\, dx \Rightarrow I = \frac{a}{2}\int_0^a f(x)dx$

46. $I = \int_0^1 \tan \sin^{-1} x\ dx$

$Put\ \sin^{-1} x = t$

$x = \sin t \Rightarrow dx = costdt$

$I = \int_0^{\pi/2} tant.cost\, dt \Rightarrow I = \int_0^{\pi/2} \sin t\, dt \Rightarrow I = [-cost]_0^{\pi/2}$

$I = 0 - (-1) \Rightarrow I = 1$

47. $I = \int_{-1/7}^{20/7} \sin(x - [x])dx = \int_{-1/7}^{3-\frac{1}{7}} \sin(\{x\})dx$

$= 3\int_0^1 \sin x\, dx = 3[-\cos x]_0^1 = 3(1 - \cos 1)$

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48. We have, $I_1 - I_2 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta)\,(\sec^2\theta - \operatorname{cosec}^2\theta)d\theta$

$\Rightarrow I_1 - I_2 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta)d(\tan\theta + \cot\theta)$

$\Rightarrow I_1 - I_2 = \int_{\frac{4}{\sqrt{3}}}^{\frac{4}{\sqrt{3}}} f(t)\,dt$, where $t = \tan\theta + \cot\theta \Rightarrow I_1 - I_2 = 0$

$\Rightarrow I_1 = I_2 \Rightarrow \frac{I_1}{I_2} = 1$

49. We have, $g(x) = f'(x), f(-1) = 2$ and $f(2) = 4$

$\therefore \int_{-1}^{2} g(x)\,dx = \int_{-1}^{2} f'(x)\,dx = f(2) - f(-1) = 4 - 2 = 2$

50. We have, $[\sin(-x)]^{11} = (-\sin x)^{11} = -\sin^{11} x$

$\therefore \sin^{11} x$ is an odd function x

Hence, $\int_{-1}^{1} \sin^{11} x\ dx = 0$

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