INDEFINITE INTEGRALS

CHAPTER 07

INTEGRALS

PRIMITIVE OR ANTIDERIVATIVE:

A function $\varphi(x)$ is called a primitive (or an antiderivative or an integral) of a function f(x) if $\varphi'(x) = f(x)$.

INDEFINITE INTEGRALS:

Let f(x) be a function. Then the family of all its primitives(or antiderivatives) is called the indefinite integral of f(x) and is denoted by $\int f(x)dx$.

The symbol $\int f(x)dx$ is read as the indefinite integral of f(x) with respect to x. Thus,

$$\frac{d}{dx}[\varphi(x) + C] = f(x) \Leftrightarrow \int f(x)dx = \varphi(x) + C$$

where $\varphi(x)$ is the primitive of f(x) and C is an arbitrary constant known as constant of integration.

IMPORTANT INTEGRATION FORMULAS:

I. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \, , n \neq -1$

II. $\int dx = x + C$

III. $\int \frac{1}{x} \, dx = \log|x| + C \, , x \neq 0$

IV. $\int e^x \, dx = e^x + C$

V. $\int a^x dx = \frac{a^x}{\log_e a} + C, a > 0, a \neq 1$

VI. $\int sinx \, dx = -cosx + C$

VII. $\int cosx\, dx = sinx + C$

VIII. $\int tanx\, dx = \log\,|secx| + C$

IX. $\int cotx\, dx = \log\,|sinx| + C$

X. $\int secx\, dx = \log|secx + tanx| + C = log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

XI. $\int cosecx\, dx = \log|cosecx - cotx| + C = log\left|\tan\left(\frac{x}{2}\right)\right| + C$

XII. $\int sec^2\, dx = tanx + C$

XIII. $\int cosec^2\, dx = -cotx + C$

XIV. $\int secx.tanx\, dx = \sec x + C$

XV. $\int cosecx.cotx\, dx = -\mathrm{cosec}\,x + C$

XVI. $\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$

XVII. $\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C$

XVIII. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

XIX. $\int \frac{-dx}{1+x^2} = \cot^{-1} x + C$

XX. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

XXI. $\int \frac{-dx}{x\sqrt{x^2-1}} = cosec^{-1} x + C$

XXII. $\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}(\frac{x}{a}) + C$

XXIII. $\int \frac{-1}{\sqrt{a^2-x^2}}\, dx = \cos^{-1}(\frac{x}{a}) + C$

XXIV. $\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$

XXV. $\int \frac{-1}{a^2+x^2}\, dx = \frac{1}{a}\cot^{-1}(\frac{x}{a}) + C$

XXVI. $\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}(\frac{x}{a}) + C$

XXVII. $\int \frac{-1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\mathrm{cosec}^{-1}(\frac{x}{a}) + C$

XXVIII. $\int \frac{dx}{(x^2-a^2)} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

XXIX. $\int \frac{dx}{(a^2-x^2)} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

XXX. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$

XXXI. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

XXXII. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

XXXIII. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

XXXIV. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

XXXV. $\int e^x\, [f(x) + f'(x)]dx = e^x f(x) + C$

SOME STANDARD RESULTS ON INTEGRATION:

I. The differentiation of an integral is the integrand itself or differentiation and integration are inverse operations.

i.e. $\frac{d}{dx}(\int f(x)\, dx) = f(x)$

II. $\int k\, f(x)\, dx = k \int f(x)\, dx$, where k is a constant.

i.e. the integral of the product of a constant and a function = the constant×integral of the function.

III. $\int\{f(x) \pm g(x)\}\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

i.e. the integral of the sum or difference of a finite number of functions is equal to the sum or difference of the integrals of the various functions.

IV. $\int\{k_1\, f_1(x) \pm k_2 f_2(x) \pm \ldots \pm k_n f_n(x)\}dx = \; k_1 \int f_1\,(x)dx \pm$
$k_2 \int 2\,(x)dx \pm \ldots k_n \int f_n\,(x)dx$

i.e. the integration of the linear combination of a finite number of functions is equal to the linear combination of their integrals.

INTEGRATION BY SUBSTITUTION:

The method of evaluating an integral by reducing it to standard form by a proper substitution is called integration by substitution. To evaluate an integral of the type $\int f\{g(x)\}.\, g'(x)\, dx$, we substitute g(x)=t, so that $g'(x)dx = dt$.

SOME STANDARD SUBSTITUTIONS:

Expression	*Substitution*
$a^2 - x^2$ or $\sqrt{a^2 - x^2}$	x= $a\sin\theta$ *or* $a\cos\theta$
$a^2 + x^2$ or $\sqrt{a^2 + x^2}$	x= $a\tan\theta$ *or* $a\cot\theta$
$x^2 - a^2$ or $\sqrt{x^2 - a^2}$	x= $a\sec\theta$ *or* $a\operatorname{cosec}\theta$
$\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$	x= $a\cos 2\theta$
$\sqrt{\frac{x-\alpha}{\beta-x}}$ or $\sqrt{(x-\alpha)(\beta-x)}$	x=$\alpha\cos^2\theta + \beta\sin^2\theta$
$\sqrt{\frac{x}{a-x}}$ or $\sqrt{\frac{a-x}{x}}$	$x = a\sin^2\theta$ or $x = a\cos^2\theta$
$\sqrt{\frac{x}{a+x}}$ or $\sqrt{\frac{a+x}{x}}$	$x = a\tan^2\theta$ or $x = a\cot^2\theta$
$\int \frac{dx}{a \pm b\cos x}$	Put cosx =$\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$, then put tan$\frac{x}{2} = t$
$\int \frac{dx}{a \pm b\sin x}$	Put sinx =$\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$, then put tan$\frac{x}{2} = t$
$\int \frac{dx}{a\sin x + b\cos x}$	Put a = rcos θ and b=rsin θ where,r=$\sqrt{a^2+b^2}$ and$\theta = \tan^{-1}\frac{a}{b}$
$\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a+b\cos^2 x}$, $\int \frac{dx}{a\cos^2 x + b\sin^2 x}$, $\int \frac{dx}{(a\sin x + b\cos x)^2}$	Divide numerator and denominator by $\cos^2 x$. Reduce $\sec^2 x$ in denominator as 1+$\tan^2 x$.Put tanx=t and proceed For perfect square,
$\int \frac{\varphi(x)dx}{(ax+b)\sqrt{px+q}}$ or $\int \frac{\varphi(x)dx}{(ax^2+bx+c)\sqrt{px+q}}$	Put $\sqrt{px+q} = t$.
$\int \frac{\varphi(x)dx}{(px+q)(\sqrt{ax^2+bx+c})}$	Put px+q=$\frac{1}{t}$
$\int \frac{\varphi(x)dx}{(px^2+q)(\sqrt{ax^2+b})}$	Put x=$\frac{1}{t}$ and then put $\sqrt{a^2+bt^2} = u$

INTEGRATION BY PARTIAL FRACTIONS:

Suppose, given integral is of the form $\int \frac{p(x)}{q(x)}$, where p(x) and q(x) are polynomials in x and $q(x) \neq 0$.Then, to solve such integrals by partial fractions, we firstly take the given integrand $\frac{p(x)}{q(x)}$ and decompose it into suitable partial fraction form and then integrate each term by using suitable method.

INTEGRATION BY PARTS:

If u and v are two functions of x, then

$$\int uv\,dx = u\int v\,dx - \int (\frac{d}{dx}\,u \int v\,dx)dx$$

If two functions are of different types, then consider the 1^{st} function (i.e u) which comes first in word ILATE, where

I : Inverse Trigonometric Function

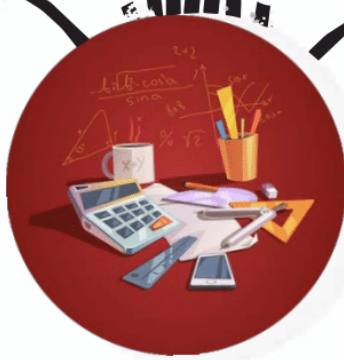
L : Logarithmic Function

A : Algebraic Function

T : Trigonometric Function

E : Exponential Function

Integrals



Integration

It is the inverse of differentiation. Let, $\frac{d}{dx}F(x)=f(x)$. Then, $\int f(x)dx=F(x)+c$, 'c': constant of integral. These integrals are called indefinite or general integrals. Properties of indefinite integrals are

(i) $\int [f(x)\pm g(x)]\,dx=\int f(x)dx\pm\int g(x)dx$, **(ii)** $\int kf(x)dx=k\int f(x)dx$,

eg : $\int(3x^2+2x)dx=x^3+x^2+c$, where c is real.

Example

$$\int_{-\pi/4}^{\pi/4}\sin^2 x\,dx=2\int_0^{\pi/4}\sin^2 x\,dx=2\int_0^{\pi/4}\left(\frac{1-\cos 2x}{2}\right)dx=\int_0^{\pi/4}(1-\cos 2x)dx=\left[x-\frac{\sin 2x}{2}\right]_0^{\pi/4}=\frac{\pi}{4}-\frac{1}{2}$$

Some standard integrals

(i) $\int x^n dx=\frac{x^{n+1}}{n+1}+c, n\neq -1$ like, $\int dx=x+c$

(ii) $\int\cos x\,dx=\sin x+c$ (iii) $\int\sin x\,dx=-\cos x+c$

(iv) $\int\sec^2 x\,dx=\tan x+c$ (v) $\int\text{cosec}^2 x\,dx=-\cot x+c$

(vi) $\int\sec x\tan x\,dx=\sec x+c$ (vii) $\int\text{cosec}\,x\cot x\,dx=-\text{cosec}\,x+c$

(viii) $\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x+c$ **(ix)** $\int\frac{dx}{\sqrt{1-x^2}}=-\cos^{-1}x+c$

(x) $\int\frac{dx}{1+x^2}=\tan^{-1}x+c$ **(xi)** $\int\frac{dx}{1+x^2}=-\cot^{-1}x+c$

(xii) $\int e^x dx=e^x+c$ (xiii) $\int a^x dx=\frac{a^x}{\log a}+c$

(xiv) $\int\frac{dx}{x\sqrt{x^2-1}}=\sec^{-1}x+c$ **(xv)** $\int\frac{dx}{x\sqrt{x^2-1}}=-\text{cosec}^{-1}x+c$

(xvi) $\int\frac{1}{x}dx=\log|x|+c$

Integration by substitution

The method in which we change the variable to some other variable is called the method of substitution. Below problems can be solved by substitution.

$\int\tan x\,dx=\log|\sec x|+c$ $\int\cot x\,dx=\log|\sin x|+c$

$\int\sec x\,dx=\log|\sec x+\tan x|+c$ $\int\text{cosec}\,x\,dx=\log|\text{cosec}\,x-\cot x|+c$.

Integration of some special functions

(i) $\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$ (ii) $\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

(iii) $\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$ (iv) $\int\frac{dx}{\sqrt{x^2-a^2}}=\log\left|x+\sqrt{x^2-a^2}\right|+c$

(v) $\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$ (vi) $\int\frac{dx}{\sqrt{x^2+a^2}}=\log\left|x+\sqrt{x^2+a^2}\right|+c.$

(vii) $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c.$

(viii) $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c.$

(ix) $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c.$

Integration by partial fractions

A rational function of the form $\frac{P(x)}{Q(x)}[Q(x)\neq 0]=T(x)+\frac{P_1(x)}{Q(x)}$, $P_1(x)$ has degree less than that of $Q(x)$. We can integrate $\frac{P_1(x)}{Q(x)}$ by expressing it in the following forms –

(i) $\frac{px+q}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}, a,b\neq 0.$

(ii) $\frac{px+q}{(x-a)^2}=\frac{A}{x-a}+\frac{B}{(x-a)^2}$ **(iii)** $\frac{px^2+qx+r}{(x-a)^2(x-b)}=\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$

(iv) $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}=\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

(v) $\frac{Px+q}{ax^2+bx+c}=\frac{A\frac{d}{dx}(ax^2+bx+c)}{ax^2+bx+c}+\frac{B}{ax^2+bx+c}$

Integration by parts

$$\int f_1(x)f_2(x)dx=f_1(x)\int f_2(x)dx-\int\left[\frac{d}{dx}f_1(x)\int f_2(x)dx\right]dx$$

First fundamental theorem of integral calculus

Let the area function be defined by $A(x)=\int_a^x f(x)dx\,\forall x\geq a$, where f is continuous on $[a,b]$ then $A'(x)=f(x)\forall x\in[a,b]$.

Second fundamental theorem of integral calculus

Let f be a continuous function of x defined on $[a, b]$ and let F be another function such that $\frac{d}{dx}F(x)=f(x)\forall x\in$ domain of f_1 then $\int_a^b f(x)dx=[F(x)+c]_a^b=F(b)-F(a)$. This is called the definite integral of f over the range $[a, b]$, where a and b are called the limits of integration, a being the lower limit and b be the upper limit.

Trace the Mind Map

- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS

1. $\int \frac{x}{4+x^4} dx$ is equal to

   a) $\frac{1}{4}\tan^{-1} x^2$+C

   b) $\frac{1}{4}\tan^{-1}\frac{x^2}{2}$

   c) $\frac{1}{2}\tan^{-1}\frac{x^2}{2}$

   d) None of these

2. $\int \frac{1}{cosx+\sqrt{3}sinx}\ dx$ is equal to

   a) $\log\tan\left(\frac{\pi}{3}+\frac{x}{2}\right)+C$

   b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{3}\right)+C$

   c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)+C$

   d) None of these

3. $\int x\ sec\ x^2\ dx$ is equal to

   a) $\frac{1}{2}\log(secx^2+tanx^2)+C$

   b) $\frac{x^2}{2}\log(secx^2+tanx^2)+C$

   c) $2\log(secx^2+tanx^2)+C$

   d) None of these

4. If $\int \frac{1}{5+4sinx} dx = A\tan^{-1}(B\tan\frac{x}{2}+\frac{4}{3})+C$,then

   a) $A=\frac{2}{3}, B=\frac{5}{3}$

   b) $A=\frac{1}{3}, B=\frac{2}{3}$

   c) $A=-\frac{2}{3}, B=\frac{5}{3}$

   d) $A=\frac{1}{3}, B=-\frac{5}{3}$

5. $\int x^{sinx}(\frac{sinx}{x} + cosx.logx)\,dx$ is equal to

a) $x^{sinx} + C$

b) $x^{sinx}cosx + C$

c) $\frac{(x^{sinx})^2}{2} + C$

d) None of these

6. Integration of $\frac{1}{1+(\log_e x)^2}$ with respect to $\log_e x$ is

a) $\frac{\tan^{-1}(\log_e x)}{x} + C$

b) $\tan^{-1}(\log_e x) + C$

c) $\frac{\tan^{-1} x}{x} + C$

d) None of these

7. If $\int \frac{cos8x+1}{tan2x-cot2x}\,dx = acos8x + C$,then a=

a) $-\frac{1}{16}$

b) $\frac{1}{8}$

c) $\frac{1}{16}$

d) $-\frac{1}{8}$

8. If $\int \frac{sin^8x-cos^8x}{1-2sin^2xcos^2x}\,dx = asin2x + C$,then a=

a) $-\frac{1}{2}$

b) $\frac{1}{2}$

c) -1

d) 1

9. $\int (x-1)e^{-x}\,dx$ is equal to

a) $-xe^{x} + C$

b) $xe^{x} + C$

c) $-xe^{-x} + C$

d) $xe^{-x} + C$

10. If $\int \frac{2^{1/x}}{x^2} dx = k\ 2^{1/x} + C$,then k is equal to

a) $-\frac{1}{\log_e 2}$

b) $-\log_e 2$

c) -1

d) ½

11. $\int \frac{1}{1+tanx} dx =$

a) $\log_e(x + sinx) + C$

b) $\log_e(sinx + cosx) + C$

c) $2sec^2\frac{x}{2} + C$

d) $\frac{1}{2}(x + \log(sinx + cosx)) + C$

12. $\int |x|^3\, dx$ is equal to

a) $\frac{-x^4}{4} + C$

b) $\frac{|x|^4}{4} + C$

c) $\frac{x^4}{4} + C$

d) None of these

13. The value of $\int \frac{cos\sqrt{x}}{\sqrt{x}} dx$ is

a) 2cos$\sqrt{x}$+C

b) $\sqrt{\frac{cosx}{x}}$+C

c) sin$\sqrt{x}$+C

d) 2sin$\sqrt{x}$+C

14. $\int e^x(1 - cotx + cot^2x)dx =$

a) $e^x cotx$+C

b) $-e^x cotx$+C

c) $e^x cosecx$+C

d) $-e^x cosecx$+C

15. $\int \frac{sin^6x}{cos^8x}dx =$

a) tan7x+C

b) $\frac{tan^7x}{7}$+C

c) $\frac{tan7x}{7}$+C

d) $sec^7x + C$

16. $\int \frac{1}{7+5Cosx}dx =$

a) $\frac{1}{\sqrt{6}}\tan^{-1}(\frac{1}{\sqrt{6}}tan\frac{x}{2}) + C$

b) $\frac{1}{\sqrt{3}}\tan^{-1}(\frac{1}{\sqrt{3}}tan\frac{x}{2}) + C$

c) $\frac{1}{4}\tan^{-1}(tan\frac{x}{2}) + C$

d) $\frac{1}{7}\tan^{-1}(tan\frac{x}{2}) + C$

17. $\int \frac{1}{1-cosx-sinx}dx =$

a) $\log|1 + cot\frac{x}{2}| + C$

b) $\log|1 - tan\frac{x}{2}| + C$

c) $\log|1 - cot\frac{x}{2}| + C$

d) $\log|1 + tan\frac{x}{2}| + C$

18. $\int \frac{x+3}{(x+4)^2}e^xdx =$

a) $\frac{e^x}{x+4} + C$

b) $\frac{e^x}{x+3} + C$

c) $\frac{1}{(x+4)^2} + C$

d) $\frac{e^x}{(x+4)^2} + C$

19. $\int \frac{\sin x}{3+4\,cos^2 x}\,dx =$

a) $\log(3 + 4cos^2 x) + C$

b) $\frac{1}{2\sqrt{3}}\,tan^{-1}\left[\frac{\cos x}{\sqrt{3}}\right] + C$

c) $-\frac{1}{2\sqrt{3}}\,tan^{-1}\left[\frac{2\cos x}{\sqrt{3}}\right] + C$

d) $\frac{1}{2\sqrt{3}}\,tan^{-1}\left[\frac{2\cos x}{\sqrt{3}}\right] + C$

20. $\int e^x \left[\frac{1-sinx}{1-cosx}\right] dx =$

a) $-e^x tan\frac{x}{2} + C$

b) $-e^x cot\frac{x}{2} + C$

c) $-\frac{1}{2}e^x tan\frac{x}{2} + C$

d) $-\frac{1}{2}e^x cot\frac{x}{2} + C$

21. $\int \frac{2}{(e^x+e^{-x})^2}\,dx =$

a) $\frac{-e^{-x}}{e^x+e^{-x}} + C$

b) $-\frac{1}{e^x+e^{-x}} + C$

c) $\frac{-1}{(e^x+1)^2} + C$

d) $\frac{1}{e^x-e^{-x}} + C$

22. $\int \frac{e^x(1+x)}{cos^2(xe^x)}\,dx =$

a) $2\log_e \cos(xe^x) + C$

b) $\sec(xe^x) + C$

c) $\tan(xe^x) + C$

d) $\tan(x + e^x) + C$

23. $\int \frac{sin^2 x}{cos^4 x}\,dx =$

a) $\frac{1}{3}tan^2 x + C$

b) $\frac{1}{2}tan^2 x + C$

c) $\frac{1}{3}tan^3 x + C$

24. The primitive of the function

$f(x) = (1 - \frac{1}{x^2})a^{x+\frac{1}{x}}$, a>0 is

a) $\frac{a^{x+\frac{1}{x}}}{\log_e a}$

b) $\log_e a \,.\, a^{x+\frac{1}{x}}$

c) $\frac{a^{x+\frac{1}{x}}}{x}\log_e a$

d) $X\frac{a^{x+\frac{1}{x}}}{\log_e a}$

25. The value of $\int \frac{1}{x+xlogx}dx$ is

a) 1+logx

b) x+logx

c) xlog(1+logx)

d) log(1+logx)

26. $\int \sqrt{\frac{x}{1-x}}dx$ is equal to

a) $\sin^{-1}\sqrt{x}$+C

b) $\sin^{-1}(\sqrt{x} - \sqrt{x(1-x)}$)+C

c) $\sin^{-1}\sqrt{x(1-x)}$ +C

d) $\sin^{-1}\sqrt{x} - \sqrt{x(1-x}$+C

27. $\int e^x\{f(x) + f'(x)\}dx =$

a) $e^x f(x) + C$

b) $e^x + f(x) + C$

c) $2e^x f(x) + C$

d) $e^x - f(x) + C$

28. The value of $\int \frac{sinx+cosx}{\sqrt{1-sin2x}}dx$ is equal to

a) $\sqrt{sin2x}$+C

b) $\sqrt{cos2x}$+C

c) $\pm(sinx - cosx)$+C

d) $\pm log(sinx - cosx)$+C

29. If $\int xsinxdx = -xcosx + \alpha$,then α is equal to

a) sinx+C

b) cosx+C

c) C

d) None of these

30. $\int \frac{cos2x-1}{cos2x+1} dx =$

a) tanx-x+C

b) x+tanx+C

c) x-tanx+C

d) –x-cotx+C

31. $\int \frac{cos2x-cos2\theta}{cosx-cos\theta}\ dx$ is equal to

a) $2(sinx + xcos\theta) + C$

b) $2(sinx - xcos\theta) + C$

c) $2(sinx + 2xcos\theta) + C$

d) $2(sinx - 2xcos\theta) + C$

32. $\int \frac{x^9}{(4x^2+1)^6} dx$ is equal to

a) $\frac{1}{5x}[4 + \frac{1}{x^2}]^{-5}$ +C

b) $\frac{1}{5}[4 + \frac{1}{x^2}]^{-5}$ +C

c) $\frac{1}{10x}[4 + \frac{1}{x^2}]^{-5}$ +C

d) $\frac{1}{10}[4 + \frac{1}{x^2}]^{-5}$ +C

33. $\int \frac{x^3}{\sqrt{1+x^2}} dx = a(1 + x^2)^{3/2} + b\sqrt{1 + x^2}$ +C , then

a) $a = \frac{1}{3}, b = 1$

b) $a = -\frac{1}{3}, b = 1$

c) $a = -\frac{1}{3}, b = -1$

d) $a = \frac{1}{3}, b = -1$

34. $\int \frac{x^3}{x+1} dx$ is equal to

a) $x+\frac{x^2}{2}+\frac{x^3}{3}-\log|1-x|+C$

b) $x+\frac{x^2}{2}-\frac{x^3}{3}-\log|1-x|+C$

c) $x-\frac{x^2}{2}-\frac{x^3}{3}-\log|1-x|+C$

d) $x-\frac{x^2}{2}+\frac{x^3}{3}-\log|1-x|+C$

35. If $\int \frac{1}{(x+2)(x^2+1)} dx = alog\ |1+x^2|+b\tan^{-1}x+\frac{1}{5}\log|x+2|+C, then$

a) $a=-\frac{1}{10}, b=-\frac{2}{5}$

b) $a=\frac{1}{10}, b=-\frac{2}{5}$

c) $a=-\frac{1}{10}, b=\frac{2}{5}$

d) $a=\frac{1}{10}, b=\frac{2}{5}$

36. $\int e^x(cosx-sinx)dx$ is equal to

a) $e^x cosx+C$

b) $-e^x cosx+C$

c) $e^x sinx+C$

d) $-e^x sinx+C$

37. $\int \frac{3e^x-5e^{-x}}{4e^x+5e^{-x}} dx = ax+b\log_e|4e^x+5e^{-x}|+C$, then

a) $a=-\frac{1}{8}, b=\frac{7}{8}$

b) $a=\frac{1}{8}, b=\frac{7}{8}$

c) $a=-\frac{1}{8}, b=-\frac{7}{8}$

d) $a=\frac{1}{8}, b=-\frac{7}{8}$

38. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to

a) $\frac{e^x}{1+x^2}$+C

b) $-\frac{e^x}{1+x^2}+C$

c) $\frac{e^x}{(1+x^2)^2}+C$

d) $-\frac{e^x}{(1+x^2)^2}+C$

39. $\int \frac{x+sinx}{1+cosx} dx$ is equal to

a) log|1+cosx|+C

b) log|x+sinx|+C

c) x-tan$\frac{x}{2}$ +C

d) xtan$\frac{x}{2}$ +C

40. $\int \tan^{-1}\sqrt{x}\, dx$ is equal to

a) $(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}+C$

b) $x\tan^{-1}\sqrt{x}-\sqrt{x}+C$

c) $\sqrt{x}-x\tan^{-1}\sqrt{x}+C$

d) $\sqrt{x}-(x+1)\tan^{-1}\sqrt{x}+C$

41. $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$ is equal to

a) $\sin(b-a)\, log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

b) $\text{cosec}(b-a)\, log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

c) $\text{cosec}(b-a)\, log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

d) $\sin(b-a)\, log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

42. $\int x^2 e^{x^3} dx$ is equal to

a) $\frac{1}{3}e^{x3} + C$

b) $\frac{1}{3}e^{x4} + C$

c) $\frac{1}{2}e^{x3} + C$

d) $\frac{1}{2}e^{x2} + C$

43. $\int \frac{e^x}{x+1}\{1 + (x+1)\log(x+1)\}dx$ equals

a) $\frac{e^x}{x+1} + C$

b) $\frac{xe^x}{x+1} + C$

c) $e^x \log(x+1) + e^x + C$

d) $e^x \log(x+1) + C$

44. $\int \frac{dx}{x(x^7+1)}$ is equal to

a) $\log\left(\frac{x^7}{x^7+1}\right) + c$

b) $\frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right) + c$

c) $\log\left(\frac{x^7+1}{x^7}\right) + c$

d) $\log\left(\frac{x^7+1}{x^7}\right) + c$

45. $\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is equal to

a) $\frac{1}{8}(x^2 - 1) + c$

b) $\frac{x^2}{4} + c$

c) $\frac{x}{2} + c$

d) $\frac{x^2}{2} + c$

46. $\int e^{\tan^{-1}x}\left|\frac{1+x+x^2}{1+x^2}\right|dx$ is equal to

a) $xe^{\tan^{-1}x}+c$

b) $x^2e^{\tan^{-1}x}\ +c$

c) $\frac{1}{x}e^{\tan^{-1}x}+c$

d) None of these

47. $\int\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$ is equal to

a) $2\ \text{cosec}\ \alpha\sqrt{\cos\alpha+\sin\alpha\tan x}+C$

b) $-2\ \text{cosec}\ \alpha\sqrt{\cos\alpha+\sin\alpha\cot x}+C$

c) $\text{cosec}\ \alpha\sqrt{\cos\alpha+\sin\alpha\cot x}+C$

d) None of these

48. If $I=\int\frac{x^5}{\sqrt{1+x^3}}\ dx$, then I is equal to

a) $\frac{2}{9}(1+x^3)^{\frac{5}{2}}+\frac{2}{3}(1+x^3)^{\frac{3}{2}}+c$

b) $\log\left|\sqrt{x}+\sqrt{1+x^3}\right|+c$

c) $\log\left|\sqrt{x}-\sqrt{1-x^3}\right|+c$

d) $\frac{2}{9}(1+x^3)^{\frac{3}{2}}-\frac{2}{3}(1+x^3)^{\frac{1}{2}}+c$

49. $\int\frac{(\sin\theta+\cos\theta)}{\sqrt{\sin 2\theta}}d\theta$ is equal to

a) $\log|\cos\theta-\sin\theta+\sqrt{\sin 2\theta}\,|+c$

b) $\log|\sin\theta-\cos\theta+\sqrt{\sin 2\theta}\,|+c$

c) $\sin^{-1}(\sin\theta-\cos\theta)+c$

d) $\sin^{-1}(\sin\theta+\cos\theta)+c$

50. $\int\frac{\cos x+x\sin x}{x(x+\cos x)}dx$ is equal to

a) $\log\big(x(x+\cos x)\big)+C$

b) $\log\left(\frac{x}{x+\cos x}\right)+C$

c) $\log\left(\frac{x+\cos x}{x}\right)$

d) None of these

ANSWER KEY

1. B	26.D
2. C	27. A
3. A	28.D
4. A	29.A
5. A	30.C
6. B	31.A
7. C	32.D
8. A	33.D
9. C	34.D
10. A	35.C
11. D	36.A
12. D	37.C
13. D	38.C
14. B	39.D
15. B	40.A
16. A	41.C
17. C	42.A
18. A	43.D
19. C	44.B
20. B	45.D
21. A	46.A
22. C	47.B
23. C	48.D
24. A	49.C
25. D	50.B

HINTS AND SOLUTIONS

1) $I = \int \frac{x}{4+x^4} dx$

Put $x^2 = t$

$\Rightarrow 2xdx = dt$

$\Rightarrow xdx = \frac{dt}{2}$

$I = \int \frac{\frac{dt}{2}}{4+t^2} dt \Rightarrow I = \frac{1}{2}\tan^{-1}(\frac{t}{2}) + C \Rightarrow I = \frac{1}{2}\tan^{-1}(\frac{x^2}{2}) + C$

2) $I = \int \frac{1}{cosx+\sqrt{3}sinx} dx \Rightarrow I = \frac{1}{2}\int \frac{1}{\frac{cosx}{2}+\frac{\sqrt{3}}{2}sinx} dx \Rightarrow I = \frac{1}{2}\int \frac{1}{COS(x-\frac{\pi}{6})} dx$

$I = \frac{1}{2}\int \sec(x-\frac{\pi}{6})\ dx \Rightarrow I = \frac{1}{2}\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)\right| + C$

3) $I = \int x\ sec\ x^2\ dx$

Put $x^2 = t \Rightarrow x = \sqrt{t}$

$\Rightarrow 2xdx = dt$

$\Rightarrow xdx = \frac{dt}{2}$

$I = \int sect\ \frac{dt}{2} \Rightarrow I = \frac{1}{2}\log(sect + tant) + C \Rightarrow I = \frac{1}{2}\log(secx^2 + tanx^2) + C$

4) $\int \frac{1}{5+4sinx} dx = A\tan^{-1}\left(Btan\frac{x}{2}+\frac{4}{3}\right) + C$

Put $\tan\frac{x}{2} = t$

$\Rightarrow x = 2\tan^{-1}t$

$\Rightarrow dx = \frac{2dt}{1+t^2}$

Put values

$\int \frac{\frac{2dt}{1+t^2}}{5+4\times\frac{2t}{1+t^2}} = \int \frac{2dt}{5t^2+8t+5} = \frac{2}{5}\int \frac{dt}{t^2+\frac{8}{5}t+1}$

Using completing square method we get

$I = \frac{2}{3}\tan^{-1}(\frac{5}{3}\tan\frac{x}{2}+\frac{4}{3}) + C \Rightarrow A = \frac{2}{3} and\ B = \frac{5}{3}$

5) $I = \int x^{sinx}(\frac{sinx}{x} + cosx.logx)\, dx$

Put $x^{sinx} = t$

Taking log on both sides,

logt=sinxlogx

$\frac{1}{t}dt = \frac{sinx}{x} + cosxlogx$

$\Rightarrow I = \int t \times \frac{dt}{t}$ ⇒I=t+C⇒I=$x^{sinx} + C$

6) $\int \frac{1}{1+(\log_e x)^2} d\,(\log_e x)$

Put $\log_e x) = t$

$\int \frac{dt}{1+t^2} = \tan^{-1} t + C = \tan^{-1}(\log_e x) + C$

7) $\int \frac{cos8x+1}{tan2x-cot2x} dx$

$=\int \frac{2cos^2 4x}{\frac{sin2x}{cos2x}-\frac{cos2x}{sin2x}} dx = \int \frac{2cos^2 4x}{sin^2 2x-cos^2 2x} \times sin2xcos2xdx = \int \frac{-cos^2 4xsin4x}{cos4x} dx$

$=\frac{-1}{2}\int sin8xdx = \frac{cos8x}{16} + C$

a=$\frac{1}{16}$

8) $\int \frac{sin^8 x-cos^8 x}{1-2sin^2 xcos^2 x} dx = \int \frac{(sin^4 x+cos^4 x)(sin^4 x-cos^4 x)}{1-2sin^2 xcos^2 x} dx$

$\int \frac{(sin^4 x+cos^4 x)(sin^2 x+cos^2 x)(sin^2 x-cos^2 x)}{(sin^2 x+cos^2 x)-2sin^2 xcos^2 x} dx = \int \frac{(sin^4 x+cos^4 x)(sin^2 x-cos^2 x)}{(sin^4 x+cos^4 x)} dx$

$= \int -cos2x\, dx = \frac{-sin2x}{2} + C$

a=$\frac{-1}{2}$

9) $\int (x-1)e^{-x} dx$

(x-1) $\int e^{-x} dx$ -$\int \left(\left[\frac{d(x-1)}{dx}\right] e^{-x}dx\right) dx$ =(x-1) $\frac{e^{-x}}{-1}$-- $\int \frac{(e^{-x})}{-1} dx$

$= -(x-1)e^{-x} + \frac{e^{-x}}{-1} + C = -xe^{-x} + e^{-x} - e^{-x} + C = -xe^{-x} + C$

10) $I = \int \frac{2^{1/x}}{x^2} dx$

Put $\frac{1}{x} = t$

$\Rightarrow \frac{-1}{x^2} dx = dt \;\Rightarrow \frac{1}{x^2} dx = -dt$

I= $\int 2^t\,(-dt) \Rightarrow$I=$\frac{-2^t}{\log_e 2} + c \Rightarrow$I=$\frac{-2^{\frac{1}{x}}}{\log_e 2} + c \Rightarrow$k=$\frac{-1}{\log_e 2}$

11) $I = \int \frac{1}{1+tanx} dx$

$= \int \frac{cosx}{sinx+cosx} dx$

Numerator can be written as:

$cosx = A(sinx + cosx) + B\frac{d(sinx+cosx)}{dx}$

$cosx = (A - B)sinx + (A + B)cosx$

$\Rightarrow A - B = 0\ and\ A + B = 1$

$\Rightarrow A = \frac{1}{2} = B$

I= $\int \frac{\frac{1}{2}(sinx+cosx)+\frac{1}{2}(cosx-sinx)}{sinx+cosx} dx \Rightarrow$I=$\frac{1}{2}\int(1 + \frac{cosx-sinx}{sinx+cosx})\,dx$

I=$\frac{1}{2}[1 + \ln(sinx + cosx)] + C$

12) $\int |x|^3\, dx$

If x>0

$\Rightarrow \int x^3\, dx = \frac{x^4}{4} + C$

If x<0

$\Rightarrow \int -x^3\, dx = -\frac{x^4}{4} + C$

13) $I = \int \frac{cos\sqrt{x}}{\sqrt{x}} dx$

Put $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} dx = dt$

$\frac{1}{\sqrt{x}} dx = 2dt$

I = $\int cost\ 2dt \Rightarrow$I= 2sint+C =2sin$\sqrt{x}$+C

14) $I=\int e^x(1-cotx+cot^2x)dx \Rightarrow I=\int e^x(cosec^2x-cotx)dx$

Here, $f(x)=-cotx \Rightarrow f'(x)=cosec^2x$

I=$-e^x cotx + C$

15) $I=\int \frac{sin^6x}{cos^8x}dx$

$I=\int tan^6x sec^2x\, dx$

Put tanx=t $\Rightarrow sec^2xdx=dt$

I= $\int t^6\, dt \Rightarrow$I $=\frac{t^7}{7}+C \Rightarrow$ I $=\frac{tan^7x}{7}+C$

16) $I=\int \frac{1}{7+5Cosx}$

Put $\tan\frac{x}{2}=t \Rightarrow dx=\frac{2dt}{1+t^2}$

$\Rightarrow cosx=\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}=\frac{1-t^2}{1+t^2} \Rightarrow I=\int \frac{\frac{2dt}{1+t^2}}{7+5\times\frac{1-t^2}{1+t^2}} \Rightarrow I=\int \frac{1}{t^2+6}dt$

$I=\frac{1}{\sqrt{6}}\tan^{-1}(\frac{t}{\sqrt{6}})+C \Rightarrow \frac{1}{\sqrt{6}}\tan^{-1}(\frac{\tan\frac{x}{2}}{\sqrt{6}})+C$

17) $I=\int \frac{1}{1-cosx-sinx}dx$

Put $\tan\frac{x}{2}=t \Rightarrow dx=\frac{2dt}{1+t^2}$

$cosx=\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}=\frac{1-t^2}{1+t^2}$

$sinx=\frac{2\tan\frac{x}{2}}{1+tan^2\frac{x}{2}}=\frac{2t}{1+t^2}$

Put values,

$I=\int \frac{\frac{2dt}{1+t^2}}{1-\frac{1-t^2}{1+t^2}-\frac{2t}{1+t^2}} \Rightarrow I=\int \frac{dt}{t^2-t} \Rightarrow I=\int \frac{dt}{t^2-t-\frac{1}{4}+\frac{1}{4}}$

$\Rightarrow I=\ln|1-\cot\frac{x}{2}|+C$

18) $I=\int \frac{x+3}{(x+4)^2}e^x dx \Rightarrow I=\int \left(\frac{(x+4)-1}{(x+4)^2}\right)e^x dx$

$I=\int \left(\frac{1}{(x+4)}-\frac{1}{(x+4)^2}\right)e^x dx$

$f(x)=\frac{1}{x+4} \Rightarrow f'(x)=\frac{-1}{(x+4)^2}$

$I=\frac{e^x}{x+4}+C$

19) $I = \int \frac{\sin x}{3+4\,cos^2 x} dx$

Put cosx=t

⇒-sinx dx=dt

⇒sinx dx=-dt

$I = \int \frac{-dt}{3+4t^2} \Rightarrow I = \frac{-1}{2\sqrt{3}} \tan^{-1}(\frac{2t}{\sqrt{3}}) + C \Rightarrow I = \frac{-1}{2\sqrt{3}} \tan^{-1}(\frac{2cosx}{\sqrt{3}}) + C$

20) $I = \int e^x \left[\frac{1-sinx}{1-cosx}\right] dx \Rightarrow I = \int e^x \left[\frac{1}{1-cosx} - \frac{sinx}{1-cosx}\right] dx$

$I = \int e^x [\frac{1}{2sin^2\frac{x}{2}} - \frac{2sin\frac{x}{2}cos\frac{x}{2}}{2sin^2\frac{x}{2}}]\, dx \Rightarrow I = \int e^x \left(\frac{cosec^2\frac{x}{2}}{2} - cot\frac{x}{2}\right) dx$

$f(x) = -cot\frac{x}{2} \Rightarrow f'(x) = \frac{cosec^2\frac{x}{2}}{2} \Rightarrow I = -e^x \cot\frac{x}{2} + C$

21) $I = \int \frac{2}{(e^x+e^{-x})^2} dx \Rightarrow I = \int \frac{2e^{2x}}{(e^{2x}+1)^2} dx$

Put $t = e^{2x} + 1 \Rightarrow dt = 2e^{2x} dx$

$I = \int \frac{dt}{t^2} \Rightarrow I = \frac{-1}{t} + C \Rightarrow I = \frac{-1}{e^{2x}+1} + C \Rightarrow I = \frac{\frac{-1}{e^x}}{e^x+e^{-x}} + C = \frac{-e^{-x}}{e^x+e^{-x}} + C$

22) $I = \int \frac{e^x(1+x)}{cos^2(xe^x)} dx$

Put $xe^x = t$

$\Rightarrow e^x(1+x)dx = dt$

$I = \int \frac{dt}{cos^2 t} \Rightarrow I = \int sec^2 t dt$ ⇒I = tant+C⇒I = tan(xe^x)+C

23) $I = \int \frac{sin^2 x}{cos^4 x} dx$

$I = \int tan^2 x sec^2 x dx$

Put $tanx = t \Rightarrow sec^2 x dx = dt$

$I = \int t^2 dt \;\Rightarrow I = \frac{t^3}{3} + C \Rightarrow I = \frac{tan^3 x}{3} + C$

24) $\mathrm{f(x)} = (1 - \frac{1}{x^2})a^{x+\frac{1}{x}} \Rightarrow \int \mathrm{f(x)\,dx} = \int (1 - \frac{1}{x^2})a^{x+\frac{1}{x}} dx$

Put $x + \frac{1}{x} = t$

$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt \Rightarrow I = \int a^t dt \Rightarrow I = \frac{a^t}{\log_e a} + C \Rightarrow I = \frac{a^{x+\frac{1}{x}}}{\log_e a} + C$

25) $I = \int \frac{1}{x+xlogx} dx \ \Rightarrow I = \int \frac{1}{x(1+logx)} dx$

Put 1+logx = t

$\Rightarrow \frac{1}{x} dx = dt$

$I = \int \frac{1}{t} dt \Rightarrow I = \log|t| + C \Rightarrow I = \log(1 + logx) + C$

26) $I = \int \sqrt{\frac{x}{1-x}} dx \Rightarrow I = \int \sqrt{\frac{x}{1-x} \times \frac{x}{x}} dx \Rightarrow I = \int \frac{xdx}{\sqrt{x-x^2}}$

Consider,

$x = A \frac{d(x-x^2)}{dx} + B$

$x = A(1 - 2x) + B$

$x = -2Ax + A + B$

$-2A = 1 \Rightarrow A = \frac{-1}{2}$

$\Rightarrow A + B = 0 \Rightarrow B = \frac{1}{2}$

$I = \int \frac{\frac{-1}{2}(1-2x)+\frac{1}{2}}{\sqrt{x-x^2}} dx \Rightarrow I = \int (\frac{-1}{2} \frac{1-2x}{\sqrt{x-x^2}} + \frac{1}{2\sqrt{x-x^2}}) dx$

$I = \frac{-1}{2} \times 2\sqrt{x - x^2} + \frac{1}{2} \int \frac{1}{\sqrt{x-x^2}} dx$

Second term after completing square method you will get as

$I = -\sqrt{x - x^2} + \sin^{-1} \sqrt{x} + C$

27) $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

28) $I = \int \frac{sinx+cosx}{\sqrt{1-sin2x}} dx \Rightarrow I = \int \frac{sinx+cosx}{cosx-sinx} dx$

Put cosx-sinx=t $\Rightarrow (sinx + cosx)dx = \pm dt$

$I = \pm \int \frac{dt}{t} \Rightarrow I = \pm \log|t| + C \Rightarrow I = \pm \log|\text{sinx} - \text{cosx}| + C$

29) $\int xsinxdx = -xcosx + \alpha$

$I = \int xsinxdx \Rightarrow I = x \int sinxdx - \int (\frac{dx}{dx} \int sinxdx) dx$

$I = -xcosx + \int cosxdx \Rightarrow I = xcosx + sinx + C$

$\Rightarrow \alpha = sinx + C$

30) $I = \int \frac{cos2x-1}{cos2x+1}dx \Rightarrow I = -\int \frac{1-cos2x}{1+cos2x}dx \Rightarrow I = -\int \frac{2sin^2x}{2cos^2\frac{x}{2}}dx$

$I = -\int tan^2xdx \Rightarrow I = -\int (sec^2x - 1)dx \Rightarrow I = -(tanx - x) + C$

$I = x - tanx + C$

31) $I = \int \frac{cos2x-cos2\theta}{cosx-cos\theta}dx \Rightarrow I = \int \frac{2cos^2x-1-(2cos^2\theta-1)}{cosx-cos\theta}dx$

$I = \int \frac{2(cos^2x-cos^2\theta)}{cosx-cos\theta}dx \Rightarrow I = 2\int (cosx + cos\theta)dx$

$I = 2(sinx + xcos\theta) + C$

32) $I = \int \frac{x^9}{(4x^2+1)^6}dx \Rightarrow I = \frac{x^9}{x^{12}[4+\frac{1}{x^2}]^6}dx$

$I = \int \frac{dx}{x^3[4+\frac{1}{x^2}]^6}$

Put $\left[4 + \frac{1}{x^2}\right] = t \Rightarrow \frac{-2}{x^3}dx = dt$

$I = \frac{1}{2}\int -t^{-6}dt \Rightarrow I = \frac{1}{2}\left[\frac{-t^{-5}}{-5}\right] + C$

$I = \frac{1}{10}\frac{1}{t^5} + C \Rightarrow I = \frac{1}{10}[4 + \frac{1}{x^2}]^{-5} + C$

33) $I = \int \frac{x^3}{\sqrt{1+x^2}}dx$

$1 + x^2 = t \Rightarrow 2xdx = dt$

$\Rightarrow xdx = \frac{dt}{2}$

$I = \int \frac{x^2}{\sqrt{1+x^2}}xdx \Rightarrow I = \int \frac{t-1}{\sqrt{t}}\frac{dt}{2} \Rightarrow I = \frac{1}{2}\int [\sqrt{t} - t^{\frac{-1}{2}}]dt$

$I = \frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}} - 2\sqrt{t}\right] + C \Rightarrow I = \frac{1}{3}[1 + x^2]^{\frac{3}{2}} - \sqrt{1 + x^2} + C$

$a = \frac{1}{3}, b = -1$

34) $I = \int \frac{x^3}{x+1}dx \Rightarrow I = \int \frac{x^3+1-1}{x+1}dx$

$I = \int \frac{(x+1)(x^2-x+1)-1}{x+1}dx \Rightarrow I = \int [x^2 - x + 1 - \frac{1}{x+1}]dx$

$I = \frac{x^3}{3} - \frac{x^2}{2} + \log|x + 1| + C$

35) SOLVE IT YOURSELF.

36) $I = \int e^x(cosx - sinx)dx = \int e^x cosxdx - \int e^x sinxdx$

$= cosxe^x + \int e^x sinxdx - \int e^x sinxdx = cosxe^x + C$

37) $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = ax + b\log_e|4e^x + 5e^{-x}|$

Differentiate both sides

$\frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} = a + b.\frac{1}{(4e^x + 5e^{-x})}(4e^x - 5e^{-x})$

$\frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} = \frac{a(4e^x + 5e^{-x}) + b(4e^x - 5e^{-x})}{4e^x + 5e^{-x}}$

$3e^x - 5e^{-x} = 4ae^x + 5ae^{-x} + 4be^x - 5be^{-x}$

$3e^x - 5e^{-x} = (4a + 4b)e^x + (5a - 5b)e^{-x}$

Comparing coefficients we get,

$a = -\frac{1}{8}, b = \frac{7}{8}$

38) $I = \int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx = \int \frac{e^x(1-2x+x^2)}{(1+x^2)^2} dx$

$= \int \frac{e^x(1+x^2)}{(1+x^2)^2} dx - \int \frac{e^x 2x}{(1+x^2)^2} dx \ = \int \frac{e^x}{(1+x^2)} dx - \int \frac{e^x 2x}{(1+x^2)^2} dx$

$= \frac{e^x}{1+x^2} + \int \frac{e^x 2x}{(1+x^2)^2} dx - \int \frac{e^x 2x}{(1+x^2)^2} dx = \frac{e^x}{1+x^2} + C$

39) $I = \int \frac{x + sinx}{1 + cosx} dx = \int \frac{x}{1+cosx} dx + \int \frac{sinx}{1+cosx} dx = \int \frac{x}{2cos^2\frac{x}{2}} dx + \int \frac{2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}} dx$

$= \frac{1}{2}\int x \sec^2\frac{x}{2} dx + \int \tan\frac{x}{2} dx$

Put $\frac{x}{2} = t \Rightarrow x = 2t \Rightarrow dx = 2dt$

$= \frac{1}{2}\int 2t \sec^2 t\, 2dt + \int \tan t\, 2dt$ $=2\int t \sec^2 t\, dt + 2\int \tan t\, dt$

= 2t tant+C=$x \tan\frac{x}{2} + C$

40) $I = \int \tan^{-1}\sqrt{x}\,dx$

$I = \int \tan^{-1} t\, 2tdt$

$I = 2\int \tan^{-1} t\,.\,tdt$

$= 2\tan^{-1} t\,.\frac{t^2}{2} - \int(\frac{t^2+1-1}{t^2+1})\,dt = \tan^{-1} t\,.\,t^2 - \int 1dt + \int \frac{1}{1+t^2}dt$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

41) $I = \int \frac{1}{\sin(x-a)\sin(x-b)}dx$

Multiplying by sin(b-a) in Num.&den.

$= \frac{1}{\sin(b-a)}\int \frac{\sin((x-a)-(x-b))}{\sin(x-a)\sin(x-b)}dx$

$\frac{1}{\sin(b-a)}\int \frac{\sin(x-a)\cos(x-b)-\cos(x-a)sin(x-b)}{\sin(x-a)\sin(x-b)}dx$

$= \frac{1}{\sin(b-a)}\left[\int \frac{\cos(x-b)}{\sin(x-b)}dx - \int \frac{\cos(x-a)}{\sin(x-a)}\right]dx = cosec(b-a)\log\left|\frac{sin(x-b)}{sin(x-a)}\right|$

42) $I = \int x^2 e^{x^3}dx$

$= \int e^t \frac{dt}{3} = \frac{1}{3}e^{x^3} + C$

43) $\int \frac{e^x}{x+1}\{1 + (x+1)\log(x+1)\}dx$

$\int e^x\left[\frac{1}{x+1} + \log(x+1)\right]dx$

$\log(x+1)\,e^x + C$

44) Let $I = \int \frac{dx}{x(x^7+1)}$

Put $x^7 = t \Rightarrow dx = \frac{1}{7x^6}dt$

$\therefore\ I = \frac{1}{7}\int \frac{dt}{t(t+1)} = \frac{1}{7}\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$= \frac{1}{7}[\log t - \log(t+1)] + c = \frac{1}{7}\log\left(\frac{t}{t+1}\right) + c = \frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right) + c$

45) Let $I = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$

Put $x = \cos\theta$, then

$$I = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right\}dx = \int \cos\left\{2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right\}dx$$

$$= \int \cos\theta\, dx = \int x\, dx = \frac{x^2}{2} + c$$

46) DO IT YOURSELF.

47) $I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}dx$

$$\Rightarrow I = \int \frac{1}{\sqrt{\sin^4 x(\cos\alpha + \cot x \sin\alpha)}}dx \Rightarrow I = \int \frac{1}{\sqrt{\cos\alpha + \cot x \sin\alpha}}\operatorname{cosec}^2 x\, dx$$

$$\Rightarrow I = -\frac{1}{\sin\alpha}\int \frac{1}{\sqrt{\cos\alpha + \cot x \sin\alpha}}d(\cos\alpha + \cot x \sin\alpha)$$

$$\Rightarrow I = -\frac{2}{\sin\alpha}\sqrt{\cos\alpha + \cot x \sin\alpha} + C \Rightarrow I = -2\operatorname{cosec}\alpha\sqrt{\cos\alpha + \cot x \sin\alpha} + C$$

48) Given, $I = \int \frac{x^5}{\sqrt{1+x^3}}dx$

Let $1 + x^3 = t \Rightarrow 3x^2 dx = dt$

$$\therefore I = \int \frac{(t-1)}{\sqrt{t}}.\frac{dt}{3} = \frac{1}{3}\int\left(\sqrt{t} - t^{-1/2}\right)dt = \frac{1}{3}\left[\frac{2t^{3/2}}{3} - 2t^{1/2}\right] + c$$

$$= \frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + c$$

49) Let $I = \int \frac{\sin\theta + \cos\theta}{\sqrt{1+\sin 2\theta - 1}}d\theta = \int \frac{\sin\theta + \cos\theta}{\sqrt{1-(\sin\theta - \cos\theta)^2}}d\theta$

Put $\sin\theta - \cos\theta = t$

$\Rightarrow (\cos\theta + \sin\theta)d\theta = dt$

$$\therefore I = \int \frac{1}{\sqrt{1-t^2}}dt = \sin^{-1} t + c = \sin^{-1}(\sin\theta - \cos\theta) + c$$

50) We have,

$$\frac{\cos x + x\sin x}{x(x+\cos x)} = \frac{(x+\cos x) - x + x\sin x}{x(x+\cos x)} = \frac{1}{x} - \frac{(1-\sin x)}{x+\cos x}$$

$$\therefore I = \int \frac{\cos x + x\sin x}{x(x+\cos x)}dx = \int \frac{1}{x} - \frac{1-\sin x}{x+\cos x}dx$$

$$\Rightarrow I = \log x - \log(x+\cos x) + C = \log\left(\frac{x}{x+\cos x}\right) + C$$