MATRICES

CHAPTER 03

TYPES OF MATRIX

DEFINITION OF A MATRIX:

A matrix is an ordered rectangular array of numbers or functions. The number of functions is called the elements or the entries of the matrix.

ORDER OF MATRIX:

A matrix of order $m \times n$ is of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

where, m represents number of rows and n represents number of columns.

In notation form, it can be rewritten as $A = [a_{ij}]_{m \times n}$

where, $1 \leq i \leq m$, $1 \leq j \leq n$ and $i, j \in N$.

Here, a_{ij} is an element lying in the ith row and jth column.

TYPES OF MATRICES:

I **ROW MATRIX:** A matrix having only one row, is called a row matrix.

II **COLUMN MATRIX:** A matrix having only one column, is called a column matrix.

III **ZERO OR NULL MATRIX:** If all the elements of a matrix are zero, then it is called a zero matrix or null matrix. It is denoted by symbol O.

IV **SQUARE MATRIX:** A matrix in which number of rows and number of columns are equal, is called a square matrix.

V **DIAGONAL MATRIX:** A square matrix is said to be a diagonal matrix, if all the elements lying outside the diagonal elements are zero.

VI **SCALAR MATRIX:** A diagonal matrix in which all the diagonal elements are equal, is called a scalar matrix.

VII UNIT OR IDENTITY MATRIX: A diagonal matrix in which all the diagonal elements are equal to unity(one), is called an identity matrix. It is denoted by I.

EQUALITY OF MATRICES:

Two matrices are said to be equal, if their order are same and their corresponding elements are also equal i.e. $a_{ij} = b_{ij} \forall i, j$

ADDITION OF MATRICES:

Let A and B be two matrices each of same order $m \times n$. Then, the sum of matrices A+B is a matrix whose elements are obtained by adding the corresponding elements of A and B.

i.e. if $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$

Then, $A + B = [a_{ij} + b_{ij}]_{m\times n}$

PROPERTIES OF MATRIX ADDITION:

Let A,B and C are three matrices of same order $m \times n$, then

i. Matrix addition is commutative

i.e A+B = B+A

ii. Matrix addition is associative

i.e. (A+B)+C = A+(B+C)

iii. **Existence of additive identity** Zero matrix(O) of order $m \times n$ (same as of A) is called additive identity as A+O=A=O+A.

iv. **Existence of additive inverse** For the square matrix, the matrix(-A) is called additive inverse if A+(-A)=O=(-A)+A

MULTIPLICATION OF A MATRIX BY A SCALAR:

Let $A = [a_{ij}]_{m\times n}$ be a matrix and k be any scalar. Then, kA is another matrix which is obtained by multiplying each element of A by k

i.e. $kA = k[a_{ij}]_{m\times n} = [k(a_{ij})]_{m\times n}$

NEGATIVE OF A MATRIX:

If we multiply a matrix A by a scalar quantity (-1), then the negative of a matrix (i.e. –A) is obtained. In negative of A, each element is multiplied by (-1).

PROPERTIES OF SCALAR MULTIPLICATION:

Let A and B be the two matrices of same order, then

i. k(A+B)=kA+kB, where k is a scalar
ii. $(k_1 + k_2)A = k_1A + k_2A$, where $k_1$and k_2 are scalars.
iii. (kl)A=k(lA)=l(kA), where l and k are scalars.

DIFFERENCE OF MATRICES:

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ be two matrices each of same order $m \times n$, then difference of these matrices A-B is defined as a matrix $D = [d_{ij}]$, where $d_{ij} = a_{ij} - b_{ij}, \forall i, j$

MULTIPLICATION OF MATRICES:

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$ be two matrices such that the number of columns of A is equal to the number of rows of B, then multiplication of A and B is denoted by AB and it is given by $c_{ik} = \sum_{k=1}^{n} a_{ij}b_{jk}$, where c_{ik} is the (i,k)th element of matrix C, where C=AB.

PROPERTIES OF MULTIPLICATION OF MATRICES:

i. Let A,B and C be three matrices of same order. Then, matrix multiplication is associative i.e. (AB)C = A(BC).
ii. **Existence of multiplicative identity:** For every square matrix A, there exists an identity matrix I of same order such that A.I=A=I.A
iii. Matrix multiplication is distributive over addition. i.e. A(B+C) = AB+AC
iv. **Non-commutativity:** Generally, matrix multiplication is not commutative i.e. if A and B are two matrices and AB,BA both exist, then it is not necessary that AB = BA
v. If the product of two matrices is a zero matrix, then it is not necessary that one of the matrices is zero matrix.

TRANSPOSE OF A MATRIX:

The matrix obtained by interchanging the rows and columns of a given matrix A is called transpose of a matrix A. It is denoted by $A' or A^T or A^c$.

PROPERTIES OF TRANSPOSE OF MATRICES:

i. $(A')' = A$
ii. $(A \pm B)' = A' \pm B'$
iii. $(kA)' = kA'$, where k is any constant
iv. $(AB)' = B'A'$

SYMMETRIC AND SKEW-SYMMETRIC MATRICES:

A square matrix A is called symmetric matrix, if $A' = A$ and a square matrix A is called skew-symmetric, if $A' = -A$

PROPERTIES OF SYMMETRIC AND SKEW-SYMMETRIC MATRICES:

i. For a square matrix A with real number entries, $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix.
ii. Any square matrix A can be expressed as the sum of a symmetric and skew-symmetric matrices. i.e. $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

ELEMENTARY OPERATIONS OF A MATRIX:

There are six operations(transformations) on a matrix, three of which are due to rows and three are due to columns. These operations are known as elementary operations or transformations.

i. **The interchange of any two rows or two columns** Symbolically, the interchange of ith and jth rows is denoted by $R_i \leftrightarrow R_j$ and interchange of ith and jth columns is denoted by $C_i \leftrightarrow C_j$

ii. **The multiplication of the elements of any row or column by a non-zero number Symbolically**, the multiplication of each element of the itch row by k, where $k \neq 0$, is denoted by $R_i \to kR_i$. The corresponding column operation is denoted by $C_i \to kC_i$.

iii. **The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non-zero number** Symbolically, the addition to the elements of itch row, the corresponding elements of jth row multiplied by k is denoted by $R_i \to R_i + kR_i$. The corresponding column operation is denoted by $C_i \to C_i + kC_i$

INVERTIBLE MATRIX:

A square matrix A of order m is said to be invertible, if there exists another square matrix B of the same order m such that AB=BA=I, where I is a unit matrix of same order m. The matrix B is called the inverse matrix of A and it is denoted by A^{-1}.

PROPERTIES OF INVERTIBLE MATRICES:

Let A and B be two non-zero invertible matrices of same order.

i. **Uniqueness of inverse** If inverse of a square matrix exists, then it is unique.

ii. $AA^{-1} = A^{-1}A = I$

iii. $(AB)^{-1} = B^{-1}A^{-1}$

iv. $(A^{-1})^{-1} = A$

v. $(A')^{-1} = (A^{-1})'$ where A' is transpose of a matrix A.

Matrices

Definition and its types

A matrix of order $m \times n$ is an ordered rectangular array of numbers or functions having 'm' rows and 'n' columns. The matrix $A=[a_{ij}]_{m\times n}$, $1 \le i \le m$, $1 \le j \le n$; $i, j \in N$ is given by

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2n} \\ a_{m_1} & a_{m_2} & \dots a_{mn} \end{bmatrix}_{m\times n}$$

- **Column matrix :** It is of the form $[a_{ij}]_{m\times 1}$
- **Row matrix :** It is of the form $[a_{ij}]_{1\times n}$
- **Square matrix :** Here, $m = n$ (no. of rows = no. of columns)
- **Diagonal matrix :** All non-diagonal entries are zero i.e. $a_{ij} = 0 \, \forall \, i \ne j$
- **Scalar matrix :** $a_{ij} = 0 \cdot i \ne j$ and $a_{ij} = k$ (Scalar), $i = j$, for some constant k.
- **Identity matrix :** $a_{ij} = 0, i \ne j$ and $a_{ij} = 1, i = j$
- **Zero matrix :** All elements are zero.

Equality of two matrices

$A=[a_{ij}]=[b_{ij}]=B$ if, A and B are of same order and $a_{ij}=b_{ij} \, \forall \, i$ and j; $i, j \in N$

Transpose of a Matrix

If $A=[a_{ij}]_{m\times n}$, then its transpose $A' = (A^T) = [a_{ji}]_{n\times m}$ i.e. if $A=(2 \;\; 1)$ then $A^T = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Also, $(A')' = A$, $(kA)' = kA'$, $(A+B)' = A'+B'$, $(AB)' = B'A'$.

- A is symmetric matrix if $A=A'$ i.e. $A'=A$.
- A is skew – symmetric if $A=-A'$ i.e. $A'=-A$.
- A is any square matrix, then–

$A=\frac{1}{2}\{(A+A')+(A-A')\}$ = sum of a symmetric and a skew-symmetric matrix.
($A+A'$: S.M.; $A-A'$: Skew S.M.)

For example if $A=\begin{pmatrix} 2 & 8 \\ 6 & 4 \end{pmatrix}$, then $A=\frac{1}{2}\left\{\begin{pmatrix} 4 & 14 \\ 14 & 8 \end{pmatrix}+\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}\right\}$.

Properties for applying the operations

$R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$

$R_i \to kR_i$ or $C_i \to kC_i$

$R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Operations on matrices

Multiplication

If $A=[a_{ij}]_{m\times n}$ and $B=[b_{jk}]_{n\times p}$, then $AB = C = [C_{ik}]_{m\times p}$, $[C_{ik}] = \sum_{j=1}^{n} a_{ij} b_{jk}$.

Also, $A(BC) = (AB)C$, $A(B+C) = AB + AC$ and $(A+B)C = AC + BC$, but $AB \ne BA$ (always).

Addition

If A, B are two matrices of same order, then $A+B=[a_{ij}+b_{ij}]$. The addition of A and B follows: $A+B=B+A$, $(A+B)+C = A+(B+C)$, $-A=(-1)A$, $k(A+B) = kA + kB$, k is scalar and $(k+I)A = kA + IA$, k and I are constants.

- If $A=\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$ then $A+B=\begin{bmatrix} -1 & 5 \\ -2 & 9 \end{bmatrix}$
- If $A=[2 \; 3]_{1\times 2}$, $B=\begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2\times 1}$, then $AB = [2\times 4+3\times 5] = [23]_{1\times 1}$

Trace the Mind Map
- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS:

1. Multiplication of real valued square matrices of same dimension:
   a) Associative
   b) Commutative
   c) Always positive definite
   d) Not always possible to commute
2. If a matrix A is symmetric as well as Skew symmetric, then:
   a) A is a diagonal matrix
   b) A is a unit matrix
   c) A is a triangular matrix
   d) A is a null matrix
3. If A is a Skew Symmetric matrix, then A^2 is
   a) Symmetric
   b) Skew-Symmetric
   c) Diagonal
   d) Scalar
4. If the order of A is 4×3, the order of B is 4×5 and the order of C is 7×3, then the order of $(A^T B^T)C^T$ is
   a) 5×3
   b) 4×5
   c) 5×7
   d) 4×3

5. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then the value of A^4 is

   a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

   c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

6. Consider the following question and decide which of the statements is sufficient to answer the question.

   Find the value of n , if

   Statements:

   i. AB = A

   ii. $A = \begin{bmatrix} n & 9 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   a) Only i. is Sufficient

   b) Only ii. is Sufficient

   c) Either i. or ii. is Sufficient

   d) Both i. and ii. are not sufficient

7. A square matrix A is called orthogonal if ____ where A' is the transpose of A

   a) $A = A^2$

   b) $A' = A^{-1}$

   c) $A = A^{-1}$

   d) $A' = A$

8. If a matrix has p elements, where p is a prime number then what is the number of possible orders it can have ?

   a) 1

   b) 2

   c) P

   d) None of these

9. If $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} = \frac{1}{2}(P+Q)$

Where P is a symmetric and Q is a skew- symmetric matrix then P and Q are ?

a) $P = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$

b) $P = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ 5 & 3 & 0 \end{bmatrix}$

c) $P = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$

d) $P = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 0 \end{bmatrix}$

10. Which one of the following matrices is an elementary matrix?

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 2 \end{bmatrix}$

11. If A and B are two $n \times n$ non singular matrix then______

a) AB is non singular

b) AB is singular

c) $(AB)^{-1} = A^{-1}B^{-1}$

d) $(AB)^{-1}$ does not exist

12. A is a scalar matrix with $k \neq 0$ of order 3. Then A^{-1} is

a) $\frac{1}{k^2}I$

b) $\frac{1}{k^3}I$

c) $\frac{1}{k}I$

d) kI

13. If $A = \frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$ then $(AA^T)^{-1}$=??

a) I

b) 5I

c) 3I

d) -3I

14. If A= $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$, then A^{-1}=?

a) $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

b) $\begin{bmatrix} cos\theta & sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

c) $\begin{bmatrix} cos\theta & -sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

d) $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

15. From the matrix equation $AB = AC$ we can conclude $B = C$ provided

a) A is singular

b) A is non-singular

c) A is symmetric

d) A is square

16. Let A and B be two non-zero square matrices and AB and BA both are defined. It means

a) No. of columns of A ≠No. of rows of B

b) No. of rows of A ≠No. ofcolumns of B

c) Both matrices (A) and (B) have same order

d) Both matrices (A) and (B) does not have same order

17. If A $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$, then which of the following statements are correct?

A. A is a square matrix

B. A^{-1} exists

C. A is a symmetric matrix

D. |A|=19

E.A is a null matrix.

Choose the correct answer from the options given below

a) A,B,C only

b) A,D,E only

c) A,B,D only

d) C,D,E only

18. The number of all possible matrices of order 2×2 with each entry 0 and 1 is :

a) 27

b) 18

c) 16

d) 81

19. If $\begin{bmatrix} x & y & z \\ 2 & u & v \\ -1 & 6 & w \end{bmatrix}$ is skew symmetric matrix, then value of $x^2 + y^2 + z^2 + u^2 + v^2 + w^2\, is$:

a) 1

b) 4

c) 36

d) 41

20. If the system of equations $x + 2y - 3z = 1, \left(p + 2\right)z = 3, \left(2p + 1\right)y + z = 2$ is consistent, then the value of p is

a) -2

b) -1/2

c) 0

d) 2

21. If for a matrix $A, A^2 + I = O$, where I is the identity matrix, then A equals

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

22. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, I is the unit matrix of order 2 and a, b are arbitrary constants, then $(aI + bA)^2$ is equal to

a) $a^2 I - abA$

b) $a^2 I + 2abA$

c) $a^2 I + b^2 A$

d) None of the above

23. Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to

a) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$

b) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

d) $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$

24. If $A(\theta) = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ and $AB = I$, then $(\sec^2\theta)B$ is equal to

a) $A(\theta)$

b) $A\left(\frac{\theta}{2}\right)$

c) $A(-\theta)$

d) $A\left(-\frac{\theta}{2}\right)$

25. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = (\text{adj } A)$, and $C = 5A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to

a) 5

b) 25

c) -1

d) 1

26. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, then A is equal to

a) Idempotent

b) Involuntary

c) Nilpotent

d) Scalar

27. Let A=$\begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and A^{-1}=$xA + yI$, then the value of x and y are

a) $x = \frac{-1}{11}, y = \frac{2}{11}$

b) $x = \frac{-1}{11}, y = \frac{-2}{11}$

c) $x = \frac{1}{11}, y = \frac{2}{11}$

d) $x = \frac{1}{11}, y = \frac{-2}{11}$

28. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(y) = \begin{bmatrix} \cos x & 0 & \sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix}$, then $[F(x)G(y)]^{-1}$ is equal to

a) $F(-x)G(-y)$

b) $F(x^{-1})G(y^{-1})$

c) $G(-y)F(-x)$

d) $G(y^{-1})F(x^{-1})$

29. If X and Y are 2×2 matrices such that $2X+3Y=O$ and $X+2Y=I$, where O and I denote the 2×2 zero matrix and the 2×2 identity matrix, then X is equal to

a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

b) $\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$

c) $\begin{bmatrix}-3 & 0\\ 0 & -3\end{bmatrix}$

d) $\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

30. If $\begin{bmatrix}1 & 1 & 1\\ 1 & -2 & -2\\ 1 & 3 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 3\\ 4\end{bmatrix}$, then $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ is equal to

a) $\begin{bmatrix}0\\ 1\\ 1\end{bmatrix}$

b) $\begin{bmatrix}1\\ 2\\ -3\end{bmatrix}$

c) $\begin{bmatrix}5\\ -2\\ 1\end{bmatrix}$

d) $\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}$

31. Consider the system of equations

$a_1 x + b_1 y + c_1 z = 0$

$a_2 x + b_2 y + c_2 z = 0$

$a_3 x + b_3 y + c_3 z = 0$

if $\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$, then the system has

a) More than two solutions

b) One trivial and one non-trivial solutions

c) No solution

d) Only trivial solution (0,0,0)

32. If $a_1, a_2, a_3, a_4, a_5, a_6$ are in AP with common difference $d \neq 0$,then the system of equations $a_1 x + a_2\, y = a_3, a_4 x + a_5 y = a_6$has

a) Infinite number of solutions

b) Unique solution

c) No solution

d) Cannot say any thing

33. Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If $AX = B$, then X is equal to

a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$

d) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

34. The real value of k for which the system of equations $2k\,x - 2y + 3z = 0, x + ky + 2z = 0, 2x + kz = 0$, has non-trivial solution is

a) 2

b) -2

c) 3

d) -3

35. If $1, \omega, \omega^2$ are the cube roots of unity and if

$\begin{bmatrix} 1+\omega & 2\omega \\ -2\omega & -b \end{bmatrix} + \begin{bmatrix} a- & \omega \\ 3\omega & 2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$, then $a^2 + b^2$ is equal to

a) $1+\omega^2$

b) $\omega^2 - 1$

c) $1 + \omega$

d) $(1+\omega)^2$

36. If a system of the equations $(\alpha+1)^3 x + (\alpha+2)^3 y - (\alpha+3)^3 = 0$, $(\alpha+1)x + (\alpha+2)y - (\alpha+3) = 0$, and $x + y - 1 = 0$ is consistent. What is the value of α?

a) 1

b) 0

c) -3

d) -2

37. Let $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in R$. Then, $\left(F(\alpha)\right)^{-1}$ is equal to

a) $F(-\alpha)$

b) $F(\alpha^{-1})$

c) $F(2\,\alpha)$

d) None of these

38. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A-B)(A+B)$ then which of the following will be always true?

a) $AB = BA$

b) Either of A or B is a zero matrix

c) Either of A or B is an identity matrix

d) $A = B$

39. If the system of linear equations $x + 2ay + az = 0, x + 3by + bz = 0$ and $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c

a) Are in AP

b) Are in GP

c) Are in HP

d) Satisfy $a + 2b + 3c = 0$

40. If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric and skew-symmetric matrix, then the symmetric matrix is

a) $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

41. If $A = [a_{ij}]_{m \times n}$ is a matrix of rank r, then

a) $r = \min(m, n)$

b) $r < \min(m, n)$

c) $r \leq \min(m, n)$

d) None of these

42. If ω is a root of unity and $A=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}$, then A^{-1} is equal to

a) $\begin{bmatrix}1 & \omega & \omega^2\\ \omega^2 & 1 & \omega\\ \omega & \omega^2 & 1\end{bmatrix}$

b) $\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega^2 & \omega\\ 1 & \omega & \omega^2\end{bmatrix}$

c) $\begin{bmatrix}1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\\ 1 & 1 & 1\end{bmatrix}$

d) $\frac{1}{2}\begin{bmatrix}1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\\ 1 & 1 & 1\end{bmatrix}$

43. If $A = [a_{ij}]$ is a scalar matrix, then trace of A is

a) $\sum_i \sum_j a_{ij}$

b) $\sum_i a_{ij}$

c) $\sum_j a_{ij}$

d) $\sum_i a_{ii}$

44. If $\begin{bmatrix}2+x & 3 & 4\\ 1 & -1 & 2\\ x & 1 & -5\end{bmatrix}$ is a singular matrix, then x is

a) $\frac{13}{25}$

b) $-\frac{25}{13}$

c) $\frac{5}{13}$

d) $\frac{25}{13}$

45. A square matrix P satisfies $P^2 = I - P$, where I is the identity matrix. If $P^n = 5I - 8P$, then n is equal to

a) 4

b) 5

c) 6

d) 7

46. If ω is a complex cube root of unity, then the matrix $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$ is a

a) Singular matrix
b) Non-symmetric matrix
c) Skew-symmetric matrix
d) None of these

47. Matrix A such that $A^2 = 2A - I$, where I is the identity matrix. Then, for $n \geq 2, A^n$ is equal to

a) $nA - (n-1)I$
b) $nA - I$
c) $2^{n-1}A - (n-1)I$
d) $2^{n-1}A - I$

48. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true?

a) If det (A)=± 1, then A^{-1} need not exist
b) If det (A)=± 1, then A^{-1} exists but all its entries are not necessarily integers
c) If det $(A)$$\neq \pm 1$, then A^{-1} exists and all its entries are non-integers.
d) If det (A) =± 1,then A^{-1} exists and all its entries are integers

49. If A is a non-zero column matrix of order $m \times 1$ and B is a non-zero row matrix of order $1 \times n$, then rank of AB equals

a) 1
b) 2
c) 3
d) 4

50. If $f(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then$\{f(\theta)^{-1}\}$ is equal to

a) $f(-\theta)$
b) $f(\theta)^{-1}$
c) $f(2\theta)$
d) None of these

ANSWER KEY

1. A
2. D
3. A
4. C
5. A
6. D
7. B
8. B
9. A
10. B
11. A
12. C
13. A
14. A
15. B
16. C
17. C
18. C
19. D
20. B
21. B
22. C
23. C
24. C
25. D
26. C
27. A
28. C
29. C
30. B
31. A
32. B
33. D
34. A
35. C
36. D
37. A
38. A
39. C
40. A
41. C
42. B
43. D
44. B
45. C
46. A
47. A
48. D
49. C
50. A

HINTS AND SOLUTIONS

1. Let A, B and C be the matrices a, b, and c be scalars and the sizes of matrices are such that the operations can be performed then The multiplication of real-valued square matrices of the same dimension is: Associative, which means that for any three matrices A, B, and C of the same dimension, the following holds true: (AB)C = A(BC) Not Commutative, which means that the order of the matrices matters. In other words, AB is not necessarily equal to BA, unless one or both of the matrices are diagonal or scalar matrices. Not always positive definite, since the determinant of the product of two matrices is equal to the product of the determinants of the matrices, and the determinant of a matrix is positive if and only if the matrix is invertible. Therefore, the product of two invertible matrices is invertible, and hence positive definite. However, the product of two non-invertible matrices may not be invertible, and hence not positive definite. The order of the multiplication matters, so it is not always possible to compute the matrices. That is, AB is not necessarily equal to Baubles one or both the matrices are diagonal or scalar matrices
2. Consider a matrix A is skew symmetric ,

   Then $A^T = -A$

   And A is a symmetric, then $A^T = A$

   Calculations:

   Since, A is symmetric

   $A^T = A$

   →-A=A

   →2A=0 → $A = 0$

   Hence, A is a null matrix
3. Given : A is a skew symmetric matrix

   → $A^T = -A$

   Now take $(A^2)^T = (A \cdot A)^T = A^T A^T$ = A x –A =A^2

   → $(A^2)^T = A^2$

   Hence $A^2 is\ Symmetric$.

4. Given:

Order of A is 4 × 3, the order of B is 4 × 5 and the order of C is 7 x 3

The transpose of the matrix obtained by interchanging the rows and columns of the original matrix.

So, order of A^T is 3 × 4 and order of C^T is 3 x 7

Now, A^TB = {3 × 4} {4 × 5} = 3 × 5

→ Order of A^TB is 3 × 5

Hence order of $(A^T\mathrm{B})^T$ is 5 × 3

Now order of $(A^T\mathrm{B})^T C^T$ = {5 × 3} {3 × 7} = 5 × 7

Hence Order of $(A^T\mathrm{B})^T C^T$ is 5 x 7

5. Given $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

A^2=AA=$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, A^4=$A^2A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Hence 1st option is correct answer .

6. From Statement1:

AB=A

We cannot find anything from this statement

From statement 2 :

$A = \begin{bmatrix} n & 9 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

We cannot find anything from this statement

Combining statement 1 and 2

AB = $\begin{bmatrix} (n \times 1 + 9 \times 0) & (n \times 0 + 9 \times 1) \\ (2 \times 1 + 1 \times 0) & (2 \times 0 + 1 \times 1) \end{bmatrix}$

AB = $\begin{bmatrix} n & 9 \\ 1 & 2 \end{bmatrix}$

Also, A= $\begin{bmatrix} n & 9 \\ 1 & 2 \end{bmatrix}$

We cannot find the value of n from both statements together

7. Suppose A is a square matrix with real elements and of $n \times n$ *order and* A^T or A' is the transpose of Athen according to the definition

   $AA^T = I$

   Pre multiplication by A^{-1}

   $A^{-1} AA^T = A^{-1}I \Rightarrow IA^T = A^{-1} \Rightarrow A^T = A^{-1} OR\ A' = A^{-1}$

   Then A is a orthogonal matrix

8. Let A be the matrix of order $m \times n$ such that if it has p elements where p is a prime

   $$m \cdot n = p$$

   As we know that prime factorization of a prime number is p = p x 1.So the possible orders of matrix A are (p×1),(1×p). Hence, the no. possible orders it can have is 2.

9. Given : $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} = \frac{1}{2}(P+Q)$

   Where P is symmetric and Qis a skew symmetric matrix.

   Here we have to find the matrix P and Q As we know, any square matrix can be expressed as the sum of the symmetric and skew symmetric matrices. If A is a square matrix then A can be expressed as Where A+A' is symmetric and A –A' is skew symmetric

   By comparing

   $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} = \frac{1}{2}(P+Q)$ with $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

   We get P =A+A' and Q=A-A'

   $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}\ AND\ A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$

   $P = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$

   Similarly, $Q = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$

   Hence,

   $P = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$ *and* $Q = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$

10. Let's check option b

Let E= $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Apply $R_{1\rightarrow}$ R_1-5R_2

E= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$=$I_{3\times 3}$

We can see that option b can be converted in to identity matrix by one elementary operation So, b option is correct

11. A and B is nonsingular matrices of order nxn

|A|≠0 and |B|≠0. A and B are of the same order , so AB is defined and is on the same order Thus, |AB|=|A||B|

|AB|≠ 0

Thus AB is non singular

12. Since A is a scalar matrix, we have $A = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$

|A|=k^3

Adj(A)= $\begin{pmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{pmatrix}$

A^{-1}=$\frac{1}{k^3}\begin{pmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{pmatrix} = \frac{1}{k}\begin{pmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{pmatrix}$=$\frac{1}{k}I$

13. Given matrix $A = \frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$

Now its transpose will be$A^T = \frac{1}{3}\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{pmatrix}$

The product will be $AA^T = \frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix} \cdot \frac{1}{3}\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$

, $AA^T = I$

$(AA^T)^{-1}$=$(I)^{-1} = I$

14. Given A=$\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

Det(A)=|A|=(cos$\theta\times$ $cos\,\theta$)-($sin\theta\times -sin\theta$) $= 1$

$A_{11} = (-1)^{1+1}\, cos\theta = cos\theta\,, A_{12} = (-1)^{1+2}\, sin\theta = -sin\theta, A_{21} = (-1)^{2+1}(-sin\theta) = sin\theta$

$A_{22} = (-1)^{2+2}\, cos\theta = cos\theta$

Cofactors of matrix A =$A_{ij} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

Now, Adj(A)= $[A_{ij}]^T = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

Hence,$A^{-1} = \frac{Adj(A)}{|A|} = \frac{\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}}{1} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

Option 1 is correct

15. Since,|A|$\neq$ 0. So, A^{-1} exists.

$AB = AC \Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$(A^{-1}A)B = (A^{-1}A)C \Rightarrow B = C$

16. A and B be two non zero square matrices

Calculations :Matrix AB is defined means Columns is equal to the rows of B and BA is defined means Columns of B is equal to the rows of A .Hence , Both matrices (A) and (B) have some order is correct.

17. In the above matrix A = $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$, we can see the number of rows and columns are 2 respectively. Since the order of the matrix is 2 × 2,henche A is a square matrix

B. The given we can 2 × 2 matrix A = $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$.We first find the determinant of A.

Det A = (2 × 5) - (3 × -3) = 10 + 9 = 19⇒|A| = 19

Since, |A| = 0 ⇒ A^{-1} exists.

C. To know if a matrix is symmetric, find thetranspose of that matrix. If the transpose of that matrix is equal to itself, it is a symmetric matrix. That is A = A^T

Here A = $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$ $then\ A^T = \begin{bmatrix} 2 & 3 \\ -3 & 5 \end{bmatrix}$. Here , A≠ A^T. Thus A is not a symmetric matrix

D. We have already derived |A|=19

E. Null Matrix: If in a null matrix all the elements are zero then it is called a null matrix . Here we can see A is not a null matrix. Thus A,B,D is the correct answer

18. The number of all possible entries of 2×2 $matrix\ is$ 4 Every entry has two choice 0 or 1 Thus , the total no. of choices is $2 \times 2 \times 2 \times 2 = 2^4$ =16

19. An odd order Skew Symmetric matrix having 0 at its diagonal and a_{ij}=$-a_{ij}$

Calculations:

2= -y ⇒y=-2

z= -(-1)=1

v=-6

Hence, the value of $x^2 + y^2 + z^2 + u^2 + v^2 + w^2 = 0 + 0 + 0 + 36 + 4 = 41$

Hence the correct option is 41

20. (b) is the correct option

21. We have,

$$A = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow A^2 = -I \Rightarrow A^2 + I = O$$

22. $(aI + bA)^2 = (aI + bA)(aI + bA)$

$$= a^2I^2 + aI(bA) + bA(aI) + (bA)^2$$

Now, $I^2 = I$ and $IA = A$

$\therefore (aI + bA)^2 = a^2I + 2abA + b^2(A^2)$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore (aI + bA)^2 = a^2I + 2abA$

23. We know that, if $A = \text{diag.}(d_1\ d_2, \dots \dots .., d_n)$ is a diagonal matrix, then for any $k \in N$

$A^k = \text{diag}\ (d_1^k, d_2^k, \dots, d_n^k)$

Here, $A = \text{diag.}(a, a, a)$

$$\therefore A^n = \text{diag}\ (a^n, a^n, a^n) = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

24. $AB = I \Rightarrow \quad B = A^{-1}$

$= \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$\Rightarrow \quad (\sec^2\theta)\, \mathrm{B} = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \mathrm{A}(-\theta)$

25. $B = \operatorname{adj}(A) = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$

Therefore, $\operatorname{adj}(B) = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix}$

Now, $|\operatorname{adj} B| = \begin{vmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{vmatrix} = 625$

and $|C| = 125|A| = 125\begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 5 & 0 \end{vmatrix} = 625$

$\therefore \frac{|\operatorname{adj}(B)|}{|C|} = \frac{625}{625} = 1$

26. $A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$= \begin{bmatrix} = a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$

$\Rightarrow \qquad A^2 = 0$

$\therefore$ A is nilpotent matrix of order 2

27. $A^{-1} = \frac{1}{1+10}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$

Also, $A^{-1} = xA + yI$

$\Rightarrow \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} x & 2x \\ -5x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$

$\Rightarrow \quad x + y = \frac{1}{11}, 2x = \frac{-2}{11}$

$\Rightarrow \quad x = \frac{-1}{11}, \quad y = \frac{2}{11}$

28. We have,

$[F(x)\, G(y)]^{-1} = [G(y)]^{-1}\, [f(x)]^{-1}$

$\Rightarrow [F(x)\, G(y)]^{-1} = G(-y)\, F(-x)$

29. Given, $2X + 3Y = 0$ ... (i)

and $X + 2Y = I$ ...(ii)

where $O=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ and $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

On solving Eqs. (i) and (ii), we get

$X = -3I = \begin{bmatrix}-3 & 0\\0 & -3\end{bmatrix}$

30. Given, $\begin{bmatrix}1 & 1 & 1\\1 & -2 & -2\\1 & 3 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\3\\4\end{bmatrix} \Rightarrow \begin{bmatrix}x+y+z\\x-2y-2z\\x+3y+z\end{bmatrix} = \begin{bmatrix}0\\3\\4\end{bmatrix}$

On Comparing both sides, we get

$x + y + z = 0$ ...(i)

$x - 2y - 2z = 3$ ...(ii)

and $x + 3y + z = 4$ ...(iii)

On solving Eqs. (i), (ii) and (iii) , we get

$x = 1, y = 2$ and $z = -3$

31. DO IT YOURSELF

32. Let$\Delta= \begin{bmatrix}a_1 & a_2\\a_4 & a_5\end{bmatrix}$

$= a_1a_5 - a_2a_4$

$= a_1(a_1 + 4d) - (a_1 + d)(a_1 + 3d)$

$= a_1^2 + 4a_1d - a_1^2 - 4a_1d - 3d^2 = -3d^2 \neq 0$

Hence, given system of equations has unique solution.

33. Since, $A = \begin{bmatrix}1 & -1 & 2\\2 & 0 & 1\\3 & 2 & 1\end{bmatrix}$

Now, $|A| = 1(0-2) + 1(2-3) + 2(4-0) = 5$

$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix}-2 & 5 & -1\\1 & -5 & 3\\4 & -5 & 2\end{bmatrix}$

Now, $A^{-1}B = \frac{1}{5}\begin{bmatrix}-2 & 5 & -1\\1 & -5 & 3\\4 & -5 & 2\end{bmatrix}\begin{bmatrix}3\\1\\4\end{bmatrix} \Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$

34. Given, $2kx - 2y + 3z = 0, x + ky + 2z = 0, 2x + kz = 0$

For non-trivial solution $\begin{vmatrix} 2k & -2 & 3 \\ 1 & k & 2 \\ 2 & 0 & k \end{vmatrix} = 0$

$\Rightarrow 2k(k^2 - 0) + 2(k - 4) + 3(0 - 2k) = 0 \quad \Rightarrow 2k^3 - 4k - 8 = 0$

$\Rightarrow (k - 2)(2k^2 + 4k + 4) = 0 \Rightarrow \quad k = 2$

35. Given, $\begin{bmatrix} 1+\omega & 2\omega \\ -2\omega & -b \end{bmatrix} + \begin{bmatrix} a & -\omega \\ 3\omega & 2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+\omega+a & \omega \\ \omega & 2-b \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$

$\Rightarrow \quad 1 + \omega + a = 0, 2 - b = 1 \Rightarrow \quad a = -1 - \omega, b = 1$

$\therefore \quad a^2 + b^2 = (-1 - \omega)^2 + 1^2 = 1 + \omega^2 + 2\omega + 1^2$

$= 0 + \omega + 1 = 1 + \omega$

36. Given equations are

$(\alpha + 1)^3 x + (\alpha + 2)^3 y - (\alpha + 3)^3 = 0$

$(\alpha + 1)x + (\alpha + 2)y - (\alpha + 3) = 0$ and $x + y - 1 = 0$

Since, this system of equations is consistent.

$\therefore \begin{vmatrix} (\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+3)^3 \\ (\alpha+1) & (\alpha+2) & -(\alpha+3) \end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 + C_1$

$\Rightarrow \begin{bmatrix} (\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+1)^3 \\ & (\alpha+1)^3 - (\alpha+3)^3 & \\ (\alpha+1) & (\alpha+2) & -(\alpha+1) \\ & -(\alpha+3) + (\alpha+1) & \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} (\alpha+1)^3 & 3\alpha^2 + 9\alpha + 7 & -6\alpha^2 - 24\alpha - 26 \\ (\alpha+1) & 1 & -2 \\ 1 & 0 & 0 \end{bmatrix} = 0$

$\Rightarrow \quad -2(3\alpha^2 + 9\alpha + 7) + 6\alpha^2 + 24\alpha + 26 = 0 \Rightarrow 6\alpha + 12 = 0 \quad \Rightarrow \alpha = -2$

37. We have,

$F(\alpha)F(-\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow F(\alpha)F(-\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \Rightarrow F(-\alpha) = [F(\alpha)]^{-1}$

38. Solve on your own.

39. Since, $A^2 - B^2 = (A-B)(A+B)$

$=A^2 - B^2 + AB - BA$

$\Rightarrow \quad AB = BA$

Since, the system of linear equations has a non-zero solution , then

$$\begin{bmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{bmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow \quad (3b-2a)(c-a) - (4c-2a)(b-a) = 0$

$\Rightarrow \quad 3bc - 3ba - 2ac + 2a^2 = 4bc - 2ab - 4ac + 2a^2$

$\Rightarrow \quad 2ac = bc + ab$

On dividing by abc both sides ,we get

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$\Rightarrow \quad a, b, c$ are in HP.

40. We know that

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

Clearly, $\frac{1}{2}(A + A^T)$ is a symmetric matrix and $\frac{1}{2}(A - A^T)$ is a skew-symmetric matrix

Now,

$$\frac{1}{2}(A + A^T) = \frac{1}{0}\left\{\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}\right\}$$

$$\Rightarrow \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$$

41. It is a direct consequence of the definition of rank

42. Since, $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$

43. In a square matrix, the trace of A is defined as the sum of the diagonal elements

Hence, trace of $A = \sum_{i=1}^{n} a_{ii}$

44. Since, $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix

$\therefore \begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$

$\Rightarrow (2+x)(5-2) - 3(-5-2x) + 4(1+x) = 0$

$\Rightarrow 6 + 3x + 15 + 6x + 4 + 4x = 0 \Rightarrow 13x + 25 = 0 \Rightarrow x = -\frac{25}{13}$

45. $\because P63 = P(I-P) \quad \because P^2 = I - P)$

$= PI - P^2 = PI - (I - P)$

Now, $P^4 = P \cdot P^3$

$\Rightarrow P^4 = P(2P - I) \Rightarrow P^4 = 2P^2 - P$

$\Rightarrow P^4 = 2I - 2P - P \Rightarrow P^4 = 2I - 3P$

And $P^5 = P(2I - 3P)$

$\Rightarrow P^5 = 2P - 3(I - P) \Rightarrow P^5 = 5P - 3I$

Also, $P^6 = P(5P - 3I)$

$\Rightarrow P^6 = 5P^2 - 3P \Rightarrow P^6 = 5(I-P) - 3P \Rightarrow P^6 = 5I - 8P$

So, $n = 6$

46. Now, $\begin{vmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{vmatrix}$

$= 1(\omega^3 - 1) - \omega^2(\omega^4 - \omega) + \omega(\omega^2 - \omega^2)$

$= 1(1-1) - \omega^2(\omega - \omega) + 0 = 0$

Hence, matrix A is singular

47. Given, $A^2 = 2A - I$

Now, $A^3 = A^2 \cdot A = 2A^2 = -IA$

$= 2A^2 - A = 2(2A - I) - A$

$= 3A - 2I = 3A - (3 - 1)I$

… … … … …

… … … … …

$A^n = nA - (n - 1)I$

48. As $\det(A) = \pm 1, A^{-1}$exists

and $A^{-1} = \frac{1}{\det(A)}(\text{adj } A) = \pm(\text{adj } A)$

All entries in adj (A) are integers.

$\therefore A^{-1}$ has integer entries

49. As $\det(A) = \pm 1, A^{-1}$exists

and $A^{-1} = \frac{1}{\det(A)}(\text{adj } A) = \pm(\text{adj } A)$

All entries in adj (A) are integers. $\therefore A^{-1}$ has integer entries.

Let $A = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ and $B = [b_{11}\ b_{12}\ b_{13} \cdots b_{1n}]$ be two non-zero column and row matrices

respectively.We have, $A\,B = \begin{bmatrix} a_{11}\,b_{11} & a_{11}\,b_{12} & a_{11}\,b_{13} & \cdots a_{11}\,b_{1\,n} \\ a_{21}\,b_{11} & a_{21}\,b_{12} & a_{21}\,b_{13} & a_{21}\,b_{1\,n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}\,b_{11} & a_{m1}\,b_{12} & a_{m1}\,b_{13} & a_{m1}\,b_{1\,n} \end{bmatrix}$

Since A and B are non-zero matrices. Therefore, the matrix AB will also be a non-zero matrix. The matrix AB will have at least one non-zero element obtained by multiplying corresponding non-zero elements of A and B. All the two-rowed minors of A obviously vanish. But, A is a non-zero matrix. Hence, rank $(A = 1)$

50. $|f(\theta)| = 1(\cos^2\theta + \sin^2\theta) = 1$

Now, adj$\{f(\theta)\} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \ \{f(\theta)\}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-\theta)$