PROBABILITY

CHAPTER 14

PROBABILITY

SOME BASIC DEFINITIONS:

I **EXPERIMENT:** An operation which can produce some well-defined outcomes, is called an experiment.

II **RANDOM EXPERIMENT:** An experiment in which total outcomes are known in advance but occurrence of specific outcome can be told only after completion of the experiment, is known as a random experiment.

III **OUTCOMES:** A possible result of a random experiment is called its outcomes.

IV **SAMPLE SPACE:** The set of all possible outcomes of a random experiment is called its sample space. It is usually denoted by S.

V **TRIAL :** When a random experiment is repeated under identical conditions and it does not have the same result each time but may result in anyone of the several possible outcomes, then such experiment is called a trial and outcomes are called cases

EVENT: A subset of the sample space associated with a random experiment is called an event.

TYPES OF EVENTS:

I **IMPOSSIBLE AND SURE EVENTS:** The empty set $\emptyset$ and the sample space S describe events (as S and $\emptyset$ are also subset of S). The empty set $\emptyset$ is called an impossible event and whole sample space S is called the sure event.

II **SIMPLE EVENT:** If an event has only one sample point of a sample space, then it is called a simple or elementary event.

III **COMPOUND EVENT:** If an event has more than one sample point, then it is called a compound event.

IV **EQUALLY LIKELY EVENT:** The given events are said to be equally likely, if none of them is expected to occur in preference to the other.

V **MUTUALLY EXCLUSIVE EVENT:** A set of events is said to be mutually exclusive, if the happening of one excludes the happening of the other, i.e. if A and B are mutually exclusive, then $(A \cap B) = \emptyset$.

VI **EXHAUSTIVE EVENT:** A set of events is said to be exhaustive, if the performance of the experiment always result in the occurrence of at least one of them. If $E_1, E_2, \dots, E_n$ are exhaustive events, then $E_1 \cup E_2 \cup \dots \cup E_n = S$.

VII **COMPLEMENT OF An EVENT :** Let A be an event in a sample space S, then complement of A is the set of all sample points of the space other than the sample point in A and it is denoted by A' or $\bar{A}$, i.e. $A' = \{n : n \in S, n \neq A\}$

PROBABILITY OF AN EVENT:

In a random experiment, let S be the sample space and E be the event. Then,

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes} = \frac{n(E)}{n(S)}$$

i. If E is an event and S is the sample space, then
   a) $0 \leq P(E) \leq 1$
   b) $P(\emptyset) = 0$
   c) P(S) =1

ii. $P(\bar{E}) = 1 - P(E)$

COIN:

A coin has two sides, head and tail. If an event consists of more than one coin, then coins are considered as distinct, if not otherwise stated.

i. Sample space of one coin = {H,T}
ii. Sample space of two coins = {(H,T),(T,H),(H,H),(T,T)}
iii. Sample space of three coins = {(H,H,H),(H,H,T),(H,T,H),(T,H,H),(H,T,T),(T,H,T),(T,T,H),(T,T,T)}

DIE:

A die has six faces marked 1,2,3,4,5 and 6. If we have more than one die, then all die is considered as distinct, if not otherwise stated.

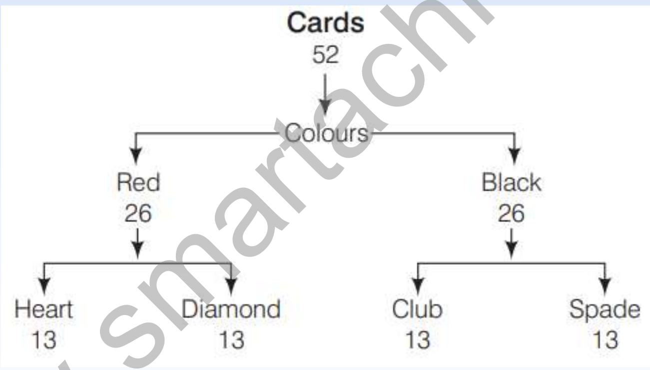
i. Sample space of a die = {1,2,3,4,5,6}

ii. Sample space of two dice $= \left\{\begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix}\right\}$

PLAYING CARDS:

A pack of playing cards has 52 cards. There are 4 suits namely spade, heart, diamond, and club, each having 13 cards. There are two colors, red (heart and diamond) and black (spade and club), each having 26 cards.

In 13 cards of each suit, there are 3 face cards namely king, queen and jack, so there are in all 12 face cards. Also, there are 16 honour cards, 4 of each suit namely ace, king, queen and jack.



IMPORTANT RESULTS ON PROBABILITY:

I ADDITION THEOREM OF PROBABILITY:

a) For two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

b) For three events A,B and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

If A,B and C are mutually exclusive events, $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

II IF A AND B ARE TWO EVENTS ASSOCIATED TO A RANDOM EXPERIMENT, THEN

a) $P(\bar{A}\cap B) = P(B) - P(A\cap B)$

b) $P(A\cap \bar{B}) = P(A) - P(A\cap B)$

c) $P[(A\cap \bar{B})\cup(\bar{A}\cap B)] = P(A) + P(B) - 2P(A\cap B)$

d) $P(\bar{A}\cap \bar{B}) = 1 - P(A\cup B)$

e) $P(\bar{A}\cup \bar{B}) = 1 - P(A\cap B)$

f) $P(A) = P(A\cap B) + P(A\cap \bar{B})$

g) $P(B) = P(A\cap B) + P(B\cap \bar{A})$

III **a)** P(exactly one of A,B occurs) $= P(A) + P(B) - 2P(A\cap B) = P(A\cup B) - P(A\cap B)$

b) P(neither A nor B) $= P(\bar{A}\cap \bar{B}) = 1 - P(A\cup B)$

IV If A,B and C are three events, then P(exactly one of A,B,C occurs)

$$= P(A) + P(B) + P(C) - 2P(A\cap B) - 2P(B\cap C) - 2P(A\cap C) + 3P(A\cap B\cap C)$$

V **a)** $P(\bar{A}) = 1 - P(A)$

b) $P(A\cup\bar{A})=P(S)$, $P(\emptyset) = 0$

CONDITIONAL PROBABILITY:

Let E and F be two events associated with a random experiment. Then, probability of occurrence of event E, when the event F has already occurred, is called conditional probability of event E over F and is denoted by P(E/F).

$$P(E/F) = \frac{P(E\cap F)}{P(F)}, where\ P(F) \neq 0.$$

PROPERTIES OF CONDITIONAL PROBABILITY:

Let A,B and C be the events of a sample space S. Then,

i. P(S/A)=P(A/A)=1

ii. $P\{(A\cup B)/C\} = P(A/C) + P(B/C) - P\{(A\cap B)/C\}\ ; P(C) \neq 0$

iii. $P(A'/B) = 1 - P(A/B)$, $where\ A'\ is\ complement\ of\ A$

MULTIPLICATION THEOREM OF PROBABILITY:

Let A and B are two events associated with a random experiment, then $P(A \cap B) = \begin{cases} P(A).P(B/A), where\ P(A) \neq 0 \\ P(B).P(A/), where\ P(B) \neq 0 \end{cases}$

MULTIPLICATION THEOREM FOR MORE THAN TWO EVENTS:

Let E,F and G be three events of sample space S, then $P(E \cap F \cap G) = P(E).P\left(\frac{F}{E}\right).P\left(\frac{G}{E \cap F}\right)$

INDEPENDENT EVENTS:

Two events A and B are said to be independent, if the occurrence or non-occurrence of one event does not affect the occurrence or non-occurrence of another event. Two events E and F are said to be independent, if $P(F/E) = P(F), P(E) \neq 0$

and $P(E/F) = P(E), P(F) \neq 0$

THEOREM OF TOTAL PROBABILITY:

Let S be the sample space and $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment.

If E is any event which occurs with $E_1, E_2, \ldots, E_n$

Then, $P(E) = P(E_1).P(E/E_1) + P(E_2).P(E/E_2) + \ldots + P(E_n).P(E/E_n)$

Or $P(E) = \sum_{i=1}^{n} P(E_i).P\left(\frac{E}{E_i}\right)$

BAYE'S THEOREM:

Let S be the sample space and $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment.

If A is any event which occurs with $E_1, E_2, \ldots, E_n$, then probability of occurrence of E_i, when A occurred. $P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i).P\left(\frac{A}{E_i}\right)}; i = 1,2, \ldots n$

RANDOM VARIABLE:

A random variable is a real valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by capital letter X.

PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE:

The system in which the value of a random variable are given along with their corresponding probability is called probability distribution.

If X is a random variable and takes the value $x_1, x_2, x_3, \dots, x_n$ with respective probabilities $p_1, p_2, p_3, \dots, p_n$. Then the probability distribution of X is represented by

X	x_1	x_2	x_3	...	x_n
$P(X)$	p_1	p_2	p_3	...	p_n

where, $p_i > 0 \text{ such that } \sum p_i = 1; i = 1,2,3, \dots, n$

MEAN AND VARIANCE OF A RANDOM VARIABLE:

Mean of a random variable is $\sum_{i=1}^{n} x_i . p_i$. It is also called expectation of X, i.e. E(X) = $\sum_{i=1}^{n} x_i . p_i$

Variance is given by V(X)= $\sum_{i=1}^{n} x_i{}^2 . p_i - (\sum_{i=1}^{n} x_i . p_i)^2$

or $V(X) = E(X^2) - [E(X)]^2$

where $E(X^2) = \sum_{i=1}^{n} x_i{}^2 . p_i$

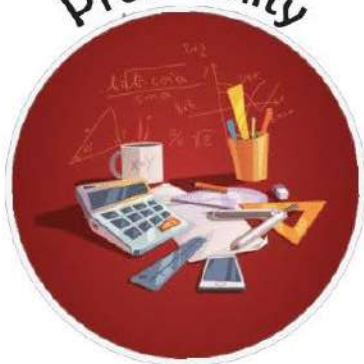
STANDARD DEVIATION:

Standard deviation of the probability distribution is the square root of variance, which is represented by σ

and it is given by

$\sigma = \sqrt{Variance} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)}$ or $\sqrt{E(X-\mu)^2}$

Probability



Conditional Probability

The probability of the event E is called the conditional probability of E given that F has already occurred, and is denoted by $P(E/F)$. Also,

$$P(E/F)=\frac{P(E\cap F)}{P(F)}, P(F)\neq 0.$$

Properties

(i) $0\leq P(E/F)\leq 1$, $P(E'/F)=1-P(E/F)$

(ii) $P((E\cup F)/G)=P(E/G)+P(F/G)-P((E\cap F)/G)$

(iii) $P(E\cap F)=P(E)P(F/E), P(E)\neq 0$

(iv) $P(F\cap E)=P(F)P(E/F), P(F)\neq 0$

e.g., : if $P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$, then

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}$$

Independent Event

If E and F are independent, then

$P(E\cap F)=P(E)P(F)$, $P(E-F)=P(E)$, $P(F)\neq 0$

and $P(F-E)=P(F)$, $P(E)\neq 0$.

If A,B,C are mutually independent events then

(i) $P(A\cap B)=P(A).P(B)$

(ii) $P(A\cap C)=P(A).P(C)$

(iii) $P(B\cap C)=P(B).P(C)$

(iv) $P(A\cap B\cap C)=P(A).P(B).P(C)$

Theorem of total probability

Let $\{E_1, E_2 \ldots E_n\}$ be a partition of a sample space 'S' and suppose that each of $E_1, E_2, \ldots E_n$ has non-zero probability. Let 'A' be any event associated with S, then $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \ldots + P(E_n)P(A/E_n)$.

$$P\left(\frac{E_i}{A}\right)=\sum_{i=1}^{n}P(E_i)P(A/E_i)$$

Probability Distribution

The probability distribution of a random variable x is the system of numbers

$x: x_1\ x_2 \ldots\ldots x_n$, $P(x): p_1\ p_2 \ldots\ldots p_n$

where, $p_i>0$,

$$\sum_{i=1}^{n}p_i=1, i=1,2,\ldots,n.$$

Bayes' Theorem

If $E_1, E_2, \ldots, E_n$ are events which constitute a partition of sample space S, i.e., $E_1, E_2, \ldots E_n$ are pairwise disjoint and $E_1\cup E_2\cup\ldots\cup E_n=S$ and A be any event with non-zero probability

then $P(E_i/A)=\frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n}P(E_i)P(A/E_i)}, n=1,2,3\ldots\ldots n$

Mean of a random variable

Let x be a random variable whose possible values $x_1, x_2, \ldots, x_n$ occur with probabilities $p_1, p_2, \ldots, p_n$ respectively. Then, mean of x is, $\mu=\sum_{i=1}^{n}x_ip_i$. It is also called the expectation of x, denoted by $E(x)$.

Random Variable

Real valued function whose domain is the sample space of a random experiment.

Trace the Mind Map

- First Level
- Second Level
- Third Level

PRACTICE QUESTIONS:

1. The probability that a man will live 10 more years is $1/4$ and the probability that his wife will live 10 more years is $1/3$. Then the probability that neither will be alive in 10 years, is
   a) 5/12
   b) 1/2
   c) 7/12
   d) 11/12
2. A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

   For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$,then $P(E \cup F)$ is
   a) 0.77
   b) 0.87
   c) 0.35
   d) 0.50
3. Two friends A and B have equal number of daughters. There are three cinema tickets which are to be distributed among the daughters of A and B. The probability that all the tickets got to daughters of A is $1/20$. The number of daughters each of them have is
   a) 4
   b) 5
   c) 6
   d) 3

4. A and B stand in a ring with 10 other persons. If the arrangement of the persons is at random, then the probability that there are exactly 3 persons between A and B is
   a) 2/11
   b) 9/11
   c) 1/11
   d) None of these
5. If M and N are any two events. The probability, that exactly one of them occurs, is
   a) $P(M) + P(N) - P(M \cap N)$
   b) $P(M) + P(N) + P(M \cap N)$
   c) $P(M) + P(N)$
   d) $P(M) + P(N) - 2P(M \cap N)$
6. If A and B are independent events such that $P(A) > 0, P(B) > 0$, then
   a) A and B are mutually exclusive
   b) A and $\bar{B}$ are independent
   c) $P(A \cup B) = P(\bar{A})P(\bar{B})$
   d) $P\left(\frac{A}{B}\right) = P\left(\frac{\bar{A}}{B}\right)$
7. Past records reveal that during a particular was, out of 9 vessels expected to arrive at the Mumbai harbour exactly 7 reached the harbour safely. If 3 vessels were expected to arrive there on a particular data, the probability that exactly two would arrive at the harbour safely, is
   a) $\frac{91}{243}$
   b) $\frac{92}{243}$
   c) $\frac{95}{243}$
   d) None of these

8. In a binomial distribution the probability of getting a success is 1/4 and standard deviation is 3, then its mean is
   a) 6
   b) 8
   c) 12
   d) 10
9. A coin is tossed $2n$ times. The chance that the number of times one gets head is not equal to the number of times one gets tail, is
   a) $\frac{(2n!)}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$
   b) $1-\frac{(2n!)}{(n!)^2}$
   c) $1-\frac{(2n!)}{(n!)^2}.\frac{1}{4^n}$
   d) None of these
10. If $P(B)=\frac{3}{4}, P(A\cap B\cap \bar{C})=\frac{1}{3}$ and $P(\bar{A}\cap B\cap \bar{C})=\frac{1}{3}$, then $P(B\cap C)$ is
   a) $\frac{1}{12}$
   b) $\frac{1}{6}$
   c) $\frac{1}{15}$
   d) $\frac{1}{9}$
11. In a college, 25% of the boys and 10% of the girls offer Mathematics. The girls constitute 60% of the total number of students. If a student is selected at random and is found to be studying Mathematics. The probability that the student is a girl is
   a) 1/6
   b) 3/8
   c) 5/8
   d) 5/6

12. Among 15 players, 8 are batsman and 7 are bowlers. The probability that a team is chosen of 6 batsman and 5 bowlers, is

a) $\frac{{}^8C_6 \times {}^7C_5}{{}^{15}C_{11}}$

b) $\frac{{}^8C_6 + {}^7C_5}{{}^{15}C_{11}}$

c) $\frac{15}{28}$

d) None of these

13. Three mangoes and three apples are kept in a box. If two fruits are selected at random from the box, the probability that the selection will contain one mango and one apple, is

a) $\frac{3}{5}$

b) $\frac{5}{6}$

c) $\frac{1}{36}$

d) None of these

14. An ordinary cube has four blank faces, one face marked 2 and another marked 3. Then the probability of obtaining 9 in 5 throws, is

a) $\frac{31}{7776}$

b) $\frac{5}{2592}$

c) $\frac{5}{1944}$

d) $\frac{5}{1296}$

15. Which one of the following is not correct?

a) The probability that in a family of 4 children, there will be at least one boy, is $\frac{15}{16}$

b) Two cards are drawn with replacement from a well shuffled pack. The probability of drawing both aces, is $\frac{1}{169}$

c) The probability guessing at least 8 out 10 answers in a true false examination, is $\frac{7}{64}$

d) A coin is tossed three times. The probability of getting exactly two heads, is $\frac{3}{8}$

16. If $\frac{1+4p}{p}, \frac{1-p}{4}, \frac{1-2p}{2}$ are probabilities of three mutually exclusive events, then

a) $\frac{1}{3} \leq p \leq \frac{1}{2}$

b) $\frac{1}{2} \leq p \leq \frac{2}{3}$

c) $\frac{1}{6} \leq p \leq \frac{1}{2}$

d) None of these

17. In a binomial distribution, mean is 3 and standard deviation is $\frac{3}{2}$, then the probability distribution is

a) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$

b) $\left(\frac{1}{4}+\frac{3}{4}\right)^{12}$

c) $\left(\frac{1}{4}+\frac{3}{4}\right)^{9}$

d) $\left(\frac{3}{4}+\frac{1}{4}\right)^{9}$

18. If n positive integers are taken at random and multiplied together, the probability that the last digit of the product is 2, 4, 6 or 8, is

a) $\frac{4^n+2^n}{5^n}$

b) $\frac{4^n \times 2^n}{5^n}$

c) $\frac{4^n-2^n}{5^n}$

d) None of these

19. In shuffling a pack of playing cards, four are accidently dropped. The probability that missing cards should be one from each suit, is

a) $\frac{1}{256}$

b) $\frac{1}{270725}$

c) $\frac{2197}{20825}$

d) None of these

20. In a series of three trials the probability of exactly two successes in nine times as large as the probability of three successes. Then, the probability of success in each trial is

a) 1/2

b) 1/3

c) 1/4

d) 3/4

21. A bag contains 50 tickets numbered 1,2,3, … ,50 of which five are drawn at random and arranged in ascending order of magnitude $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_3 = 30$, is

a) $\frac{^{20}C_2}{^{50}C_5}$

b) $\frac{^{29}C_2}{^{50}C_5}$

c) $\frac{^{20}C_2 \times {}^{29}C_2}{^{50}C_5}$

d) None of these

22. There are 5 duplicate and 10 original items in an automobile shop and 3 items are brought at random by a customer. The probability that none of the items is duplicate, is

a) 20/91

b) 22/91

c) 24/91

d) 89/91

23. A box contains 15 transistors, 5 of which are defective. An inspector takes out one transistor at random, examines it for defects and replaces it. After it has replaced another inspector does the same thing and then so does a third inspector. The probability that at least one of the inspectors finds a defective transistor, is equal to

a) 1/27

b) 8/27

c) 19/27

d) 26/27

24. A bag contains $(2n+1)$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $n+1$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is $31/42$, then n is equal to

a) 10
b) 11
c) 12
d) 13

25. In an entrance test there are multiple choice questions. There are four possible answers to each question, of which one is correct. The probability that a student knows the answer to a question is 90%. If be gets the correct answer to a question, then the probability that he was guessing, is

a) $\frac{37}{40}$
b) $\frac{1}{37}$
c) $\frac{36}{37}$
d) $\frac{1}{9}$

26. 4 five-rupee coins, 3 two-rupee coins and 2 one-rupee coins are stacked together in a column at random. The probability that the coins of the same denominator are consecutive is

a) $\frac{13}{9!}$
b) $\frac{1}{210}$
c) $\frac{1}{35}$
d) None of these

27. A random variable X takes values 0, 1, 2, 3,… with probability

$P(X = x) = k(x + 1)\left(\frac{1}{5}\right)^x$, where k is constant, then $P(X = 0)$ is

a) $\frac{7}{25}$

b) $\frac{18}{25}$

c) $\frac{13}{25}$

d) $\frac{16}{25}$

28. A random variable X follows binomial distribution with mean α and variance β.Then

a) $0 < \alpha < \beta$

b) $0 < \beta < \alpha$

c) $\alpha < 0 < \beta$

d) $\beta < 0 < \alpha$

29. Suppose $f(x) = x^3 + ax^2 + bx + c,$ where a, b, c are chosen respectively by throwing a dice three times. Then, the probability that $f(x)$ is an increasing function, is

a) $\frac{4}{9}$

b) $\frac{3}{8}$

c) $\frac{2}{5}$

d) $\frac{16}{34}$

30. One Indian and four American men and their wife's are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{2}{5}$

d) $\frac{1}{5}$

31. For $k = 1,2,3$ the box B_k contains k red balls and $(k+1)$ white balls, Let $P(B_1) = \frac{1}{2}, P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$

A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from box B_2,is

a) $\frac{35}{78}$

b) $\frac{14}{39}$

c) $\frac{10}{13}$

d) $\frac{12}{13}$

32. A number n is chosen at random from $S = \{1, 2, 3, \dots, 50\}$.

Let $A = \left\{n \in S: n + \frac{50}{n} > 27\right\}, B = \{n \in S: n \text{ is a prime}\}$ and $C = \{n \in S: n \text{ is a square}\}$. Then, correct order of their probabilities is

a) $P(A) < P(B) < P(C)$

b) $P(A) > P(B) > P(C)$

c) $P(B) < P(A) < P(C)$

d) $P(A) > P(C) > P(B)$

33. Four positive integers are taken at random and are multiplied together. Then the probability that the product ends in an odd digit other than 5, is

a) 609/625

b) 16/625

c) 2/5

d) 3/5

34. A bag contains four tickets marked with numbers $112, 121, 211, 222$. One ticket is drawn at random from the bag. Let $E_i (i = 1,2,3)$ denote the event that ith digit on the ticket is 2. Then, which one of the following is incorrect?

a) E_1 and E_2 are independent

b) E_2 and E_3 are independent

c) E_3 and E_1 are independent

d) E_1, E_2, E_3 are independent

35. The probability distribution of a random variable X is given by

$X = x$:	0	1	2	3	4
$P(X = x)$:	0.4	0.3	0.1	0.1	0.1

The variance of X is

a) 1.76
b) 2.45
c) 3.2
d) 4.8

36. Fifteen coupons are numbered 1 to 15. Seven coupons are selected at random, one at a time with replacement. The probability that the largest number appearing on a selected coupon be 9, is

a) $\left(\frac{1}{15}\right)^7$
b) $\left(\frac{8}{18}\right)^7$
c) $\left(\frac{3}{5}\right)^7$
d) None of these

37. If $A_1, A_2, \dots, A_n$ are n independent events such that $P(A_i) = \frac{1}{i+1}, i = 1,2, \dots, n$. The probability that none of the n events occurs, is

a) $\frac{n}{n+1}$
b) $\frac{1}{n+1}$
c) $\frac{n}{(n+1)(n+2)}$
d) None of these

38. In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. The minimum number of bombs which should be dropped to give a 99% chance or better of completely destroying the target is

a) 10
b) 11
c) 12
d) None of these

39. In $x = 33^n, n$ is a positive integral value, then what is the probability that x will have 3 at its units place?

a) 1/3
b) 1/4
c) 1/5
d) 1/2

40. A, B, C are any three events. If $P(S)$ denotes the probability of S happening, then $P(A \cap (B \cup C)) =$

a) $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$
b) $P(A) + P(B) + P(C) - P(B)P(C)$
c) $P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$
d) $P(A) + P(B) + P(C)$

41. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane, is

a) 0.06
b) 0.14
c) 0.32
d) 0.7

42. There is an objective type question with 4 answer choices exactly one of which is correct. A student has not studied the topic on which the question has been set. The probability that the student guesses the correct answer, is

a) $\frac{1}{2}$
b) $\frac{1}{4}$
c) $\frac{1}{8}$
d) None of these

43. An anti-aircraft gun can take a maximum of four shots at any plane moving away from it. The probabilities of hitting the plane at the 1st, 2nd ,3rd and 4th shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

a) 0.6976

b) 0.3024

c) 0.72

d) 0.6431

44. Among the workers in a factory only 30% receive bonus and among those receiving bonus only 20% are skilled. The probability that a randomly selected worker is skilled and is receiving bonus is

a) 0.03

b) 0.02

c) 0.06

d) 0.015

45. A random variable X can attain only the value 1, 2, 3, 4, 5 with respective probabilities $k, 2k, 3k, 2k, k$. If m is the mean of the probability distribution, then$(k, m)$is equal to

a) $\left(3, \frac{1}{9}\right)$

b) $\left(\frac{1}{9}, 3\right)$

c) $\left(\frac{1}{8}, 4\right)$

d) (1,3)

46. If $4P(A) = 6P(B) = 10P(A \cap B) = 1$, then$P\left(\frac{B}{A}\right)$ is equal to

a) $\frac{2}{5}$

b) $\frac{3}{5}$

c) $\frac{7}{10}$

d) $\frac{19}{60}$

47. The distribution of a random variable X is given below

$\boldsymbol{X}$	-2	-1	0	1	2	3
$\boldsymbol{P(X)}$	$\frac{1}{10}$	k	$\frac{1}{5}$	2k	$\frac{3}{10}$	k

The value of k is

a) $\frac{1}{10}$
b) $\frac{2}{10}$
c) $\frac{3}{10}$
d) $\frac{7}{10}$

48. A and B are the independent events. The probability that both occur simultaneously is $\frac{1}{6}$ and the probability that neither occur is $\frac{1}{3}$. The probability of occurrence of the events A and B is

a) $\frac{1}{2}, \frac{3}{2}$
b) $\frac{1}{2}, \frac{1}{3}$
c) Not possible
d) None of these

49. The probability distribution of a random variable X is given as

$\boldsymbol{X}$	-5	-4	-3	-2	-1	0	1	2	3	4	5
$\boldsymbol{P(X)}$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of P is

a) $\frac{1}{72}$
b) $\frac{3}{73}$
c) $\frac{5}{72}$
d) $\frac{1}{74}$

50. There are 9999 tickets bearing numbers 0001, 0002,….,9999. If one ticket is selected from these tickets at random, the probability that the number on the ticket will consists of all different digits, is

a) $\frac{5040}{9999}$
b) $\frac{5000}{9999}$
c) $\frac{5030}{9999}$
d) None of these

ANSWER KEY

1. B
2. A
3. D
4. A
5. D
6. B
7. D
8. C
9. C
10. A
11. B
12. A
13. A
14. D
15. C
16. D
17. A
18. C
19. C
20. C
21. C
22. C
23. C
24. A
25. B

26.B
27. D
28.B
29.A
30.C
31.B
32.B
33.B
34.D
35.A
36.C
37.B
38.A
39.B
40.C
41.C
42.B
43.A
44.C
45.B
46.A
47.A
48.B
49.A
50.A

HINTS AND SOLUTIONS

1. Required probability $= \left(1-\frac{1}{4}\right)\left(1-\frac{1}{3}\right) = \frac{1}{2}$
2. $P(E) = P(X=2) + P(X=3) + P(X=5) + P(X=7)$

   $= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

   $P(F) = P(X=1) + P(X=2) + P(X=3) = 0.15 + 0.23 + 0.12 = 0.5$

   $P(E \cap F) = P(X=2) + P(X=3) = 0.23 + 0.12 = 0.35$

   $\therefore\ P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.62 + 0.5 - 0.35 = 0.77$
3. Let each of the friends has x daughters. Then,

   Probability that all the tickets go to the daughters of $A = \frac{{}^{x}C_3}{{}^{2x}C_3}$

   $\therefore \frac{{}^{x}C_3}{{}^{2x}C_3} = \frac{1}{20} \Rightarrow \frac{x-2}{4(2x-1)} = \frac{1}{20} \Rightarrow 5x - 10 = 2x - 1 \Rightarrow x = 3$
4. The total number of ways in which 12 persons can stand in a ring $= 11\,!$. Three persons between A and B can be selected in ${}^{10}C_3$ ways. A and B can interchange their positions in 2 ! ways. Also, 3 persons between A and B can stand in 3 ! ways and the other in 7 in 7 ! ways

   $\therefore$ Favourable number of ways $= 2\,!\,3\,!\,7\,! \cdot {}^{10}C_3 = 2!\,10!$

   Hence, required probability $= \frac{2\,!10\,!}{11\,!} = \frac{2}{11}$
5. $\because$ Probability $= P(M \cap \overline{N}) + P(\overline{M} \cap N)$

   $\Rightarrow\ P = P(M) - P(M \cap N) + P(N) - P(M \cap N)$

   $\Rightarrow\ P = P(M) + P(N) - 2P(M \cap N)$
6. Solve on your own
7. On the basis of past records, the probability of safe arrival of a vessel on Mumbai harbor is 7/9. Since, arrival of all the vessels is independent and there are only two possibilities on every namely safe arrival and unsafe arrival. We can use the binomial distribution.

   Here $n = 3, p = \frac{7}{9}, q = \frac{2}{9}$ and $r = 2$

   $\therefore P(2) = {}^{3}C_2 \left(\frac{7}{9}\right)^2 \left(\frac{2}{9}\right) = \frac{98}{243}$

8. Given that, probability of success $p = \frac{1}{4}$ and probability of unsuccess $q = \frac{3}{4}$

$\therefore$ Mean $= np$ and standard deviation $= \sqrt{\text{variance}}$

$\Rightarrow 3 = \sqrt{\text{Variance}} \Rightarrow$ variance = 9

$\Rightarrow npq = 9 \Rightarrow n \cdot \frac{1}{4} \cdot \frac{3}{4} = 9 \Rightarrow n = \frac{9 \cdot 4 \cdot 4}{3} \Rightarrow n = 48$

$\therefore$ Mean $= np = \frac{1}{4} \times 48 = 12$

9. The required probability $= 1 -$ probability of equal number of heads and tails.

Out of $2n$ tossed n times heads and n times tails. $= 1 - {}^{2n}C_n \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{2n-n}$

$= 1 - \frac{(2n)!}{n!n!} \left(\frac{1}{2}\right)^{2n} = 1 - \frac{(2n)!}{(n!)^2} \cdot \frac{1}{4^n}$

10. $P(\bar{A} \cap (B \cap \bar{C})\} = P(B \cap \bar{C}) - P(A \cap B \cap \bar{C})$

$= P(B) - P(B \cap C) - P(A \cap B \cap \bar{C})$

$\Rightarrow -P(\bar{A} \cap B \cap \bar{C}) - P(A \cap B \cap \bar{C}) + P(B) = P(B \cap C)$

$\Rightarrow P(B \cap C) = \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{1}{12}$

11. Let total number of students be 100 in which 60% girls and 40% boys.

Number of boys =40, number of girls =60

25% of boys offer Mathematics $= \frac{25}{100} \times 40 = 10$ boys

10% girls offer Mathematics $= \frac{10}{100} \times 60 = 6$ girls

It means, 16 students offer Mathematics.

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$

12. Total number of ways of selecting 11 players $= {}^{15}C_{11}$

Favorable cases $= {}^{8}C_6 \times {}^{7}C_5$

$\therefore$ Required probability $= \frac{{}^{8}C_6 \times {}^{7}C_5}{{}^{15}C_{11}}$

13. Solve on your own.

14. A total of 9 can be obtained in the following mutually exclusive ways:

(I) 2 occurs in 3 throws out of 5 and 3 occurs in one out of the remaining 2 throws. The number of such ways is $^5C_3 \cdot {}^2C_1$

(II) 3 occurs three times out of 5 throws. The number of such ways is $^5C_3$

So, required probability $= P(I) + P(II) = \frac{^5C_3 \times {}^2C_1}{6^5} + \frac{^5C_3}{6^5} = \frac{5}{1296}$

15. (a) P (no boy in family of 4) $= P$ (all girls in it)$= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

Hence, the probability of having at least one boy

$= 1 - \frac{1}{16} = \frac{15}{16}$

(b)P (first card is an ace)$= \frac{1}{13}$

and P (second card is an ace) $= \frac{1}{13}$

Therefore, P (both cards are aces) $= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

(c) Let guessing correctly one answer as a success. Then, we have

$p = \frac{1}{2}, q = \frac{1}{2}, n = 10$

$\therefore\ P(8) + P(9) + P(10) = {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$

$= \frac{45+10+1}{1024} = \frac{7}{128}$

(d) we have, $n = 3, p = \frac{1}{2}, q = \frac{1}{2}$

Where obtaining a head has been reckoned a success.

Now, $P(2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8}$

Hence, it is clear that option (c) is not correct.

16. Since $\frac{1+4p}{p}, \frac{1-p}{2}$ and $\frac{1-2p}{2}$ are probabilities of three mutually exclusive events

$\therefore 0 \le \frac{1+4p}{p} 1, 0 \le \frac{1-p}{2} \le 1, 0 \le \frac{1-2p}{2} \le 1$ and,$0 \le \frac{1+4p}{p} + \frac{1-p}{2} + \frac{1-2p}{2} \le 1$

$\Rightarrow -\frac{1}{4} \le p \le \frac{3}{4}, -1 \le p \le 1, -\frac{1}{2} \le p \le \frac{1}{2}$ and $\frac{1}{2} \le p \le \frac{5}{2}$

$\Rightarrow \max\left\{-\frac{1}{4}, -1, -\frac{1}{2}, \frac{1}{2}\right\} \le p \le \min\left\{\frac{3}{4}, 1, \frac{1}{2}, \frac{5}{2}\right\} \Rightarrow \frac{1}{2} \le p \le \frac{1}{2} \Rightarrow p = \frac{1}{2}$

17. Given, $np = 3 = \sqrt{npq} = \frac{3}{2}$

$\Rightarrow \; q = \frac{npq}{np} = \frac{9}{4\times 3} = \frac{3}{4} \Rightarrow p = 1 - \frac{3}{4} = \frac{1}{4}$

Also, $np = 3 \;\Rightarrow\; n = 12$

Hence, binomial distribution is $(q+p)^n = \left(\frac{3}{4} + \frac{1}{4}\right)^{12}$

18. The last digit of the product will be 1, 2, 3, 4, 5, 6, 7, 8 or 9 if and only if each of the n positive integers ends in any of these digits. Now the probability of an integer ending in 1,2,3,4,5,6,7,8 or 9 is $\frac{8}{10}$. Therefore the probability that the last digit of the product of n integer in $1, 2, 3, 4, 5, 6, 7, 8$ or 9 is $\left(\frac{4}{5}\right)^n$.The probability for an integer toned in $1, 3, 7$ or 9 is $\frac{4}{10} = \frac{2}{5}$. Therefore the probability for the product of n positive integers to end in 1, 3, 7 or 9 is $\left(\frac{2}{5}\right)^n$. Hence the required probability$= \left(\frac{4}{5}\right)^n - \left(\frac{2}{5}\right)^n = \frac{4^n - 2^n}{5^n}$

19. Total cases $= {}^{52}C_4$

Favourable cases $= \left({}^{13}C_1\right)^4$

So, probability $= \frac{\left({}^{13}C_1\right)^4}{{}^{52}C_4} = \frac{13\times13\times13\times13\times1\times2\times3\times4}{52\times51\times50\times49} = \frac{2197}{20825}$

20. Let x be the probability of success in each trial, then $(1-x)$ will be the probability of failure in each trial. Thus, probability of exactly successes in a series of three trials

$= P(\overline{E}_1\, E_2\, E_3 + E_1\overline{E}_2\, E_3 + E_1\, E_2\overline{E}_3) = (1-x)x\cdot x + x(1-x)x + x\cdot x(1-x)$

$= 3x^2(1-x)$

and the probability of three success

$P(E_1E_2E_3) = x\cdot x\cdot x = x^3$

According to question,

$9x^3 - 3x^2(1-x) \Rightarrow 3x = 1 - x \Rightarrow 4x = 1 \Rightarrow x = \frac{1}{4}$

Hence, the probability of success in each trial is $\frac{1}{4}$.

21. Five tickets out of 50 can be drawn in ${}^{50}C_5$ ways. Since $x_1 < x_2 < x_3 < x_4 < x_5$ and $x_3 = 30$. Therefore, $x_1, x_2 < 30$ i.e. x_1 and x_2 should come from tickets numbered 1 to 29 and this may happen in ${}^{29}C_2$ ways. Remaining two i.e. $x_4, x_5 < 30$, should come from 20 tickets numbered 31 to 50 in ${}^{29}C_2$ ways.

So, favorable number of elementary events $= {}^{29}C_2 \times {}^{20}C_2$

Hence, required probability $= \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$

22. Duplicate =5, original = 10

Taking 3 times.

The probability that none of the items is duplicate *ie*, all the three are original

$= \frac{{}^{10}C_3}{{}^{15}C_3} = \frac{24}{91}$

23. Probability of defective transistor $= \frac{5}{15} = \frac{1}{3}$ and probability of non-defective transistor $= 1 - \frac{1}{3} = \frac{2}{3}$

Probability that the inspectors find non-defective transistors$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

Hence, probability that at least one of the inspectors finds a defective transistor

$= 1 - \frac{8}{27} = \frac{19}{27}$

24. Let A_1 denote the event that a coin having heads on both sides is chosen, and A_2 denote the event that a fair coin is chosen. Let E denote the event that head occurs. Then

$P(A_1) = \frac{n}{2n+1}, P(A_2) = \frac{n+1}{2n+1}, P(E/A_1) = 1, P(E/A_2) = \frac{1}{2}$

Now, $P(E) = P(A_1 \cap E) + P(A_2 \cap E)$

$\Rightarrow P(E) = P(A_1)P(E/A_1) + P(A_2)P(E/A_2) \Rightarrow \frac{31}{42} = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$

$\Rightarrow \frac{31}{42} = \frac{3n+1}{2(2n+1)} \Rightarrow 124n + 62 = 126n + 42 \Rightarrow 2n = 20 \Rightarrow n = 10$

25. We define the following events:

A_1: He knows the answer;

A_2:He does not know the answer;

E:He gets the correct answer

Then, $P(A_1) = \frac{9}{10}, P(A_2) = 1 = \frac{9}{10} = \frac{1}{10}$,

$P(E|A_1) = 1, P(E|A_2) = \frac{1}{4}$

$\therefore$ Required probability $= P(A_2|E) = \frac{P(A_2)P(E/A_2)}{P(A_1)P(E|A_1)+P(A_2)P(E|A_2)} = \frac{\frac{1}{10}\cdot\frac{1}{4}}{\frac{9}{10}\cdot 1+\frac{1}{10}\cdot\frac{1}{4}} = \frac{1}{37}$

26. 4 five-rupee, 3 two-rupee and 2 one-rupee coins can be stacked together in a column in $\frac{9!}{4!3!2!}$ ways. The number of ways in which coins of the same denomination the consecutive is same as the number of ways of arranging 3 distinct items i.e. 3! Ways

Hence, required probability $= \frac{3!}{\frac{9!}{4!\,3!\,2!}} = \frac{1}{210}$

27. $P(X = 0) = k, P(X = 1)2k\left(\frac{1}{5}\right)^1$

$P(X = 2) = 3k\left(\frac{1}{5}\right)^2, \ldots$

Since, $P(X = 0) + P(X = 1) + P(X = 2)+\ldots = 1$

$\therefore k + 2k\left(\frac{1}{5}\right) + 3k\left(\frac{1}{5}\right)^2 + \ldots = 1$and $+\frac{k}{5} + 2k\left(\frac{1}{5}\right)^2 + \ldots = \frac{1}{5}$

$- \quad - \quad - \quad - \quad -$

$k + k\left(\frac{1}{5}\right) + k\left(\frac{1}{5}\right)^2 + \ldots = \frac{4}{5}$

$\Rightarrow \frac{k}{1-\frac{1}{5}} = \frac{4}{5} \Rightarrow \; k = \frac{16}{25}$

$\therefore \; P(X = 0) = \frac{16}{25}(0 + 1)\left(\frac{1}{5}\right)^0 = \frac{16}{25}$

28. For binomial distribution

0<variance<mean

$\Rightarrow \; 0 < \beta < \alpha$

29. $\because f(x) = x^3 + ax^2 + bx + c$

$\therefore f'(x) = 3x^2 + 2ax + b$

$y = f(x)$ is increasing.$\Rightarrow f'(x) \geq 0, \forall x$

And for $f'(x) = 0$ should not form an interval.

$\Rightarrow 4a^2 - 4 \times 3 \times b \leq 0 \Rightarrow a^2 - 3b \leq 0$

This is true for exactly 16 ordered pairs $(a, b), 1 \leq a, b \leq 6$ namely (1, 1), (1, 2), (1, 3), (1, 4); (1, 5), (1, 6), (2, 2),(2, 3), (2, 4), (2, 5), (2, 6); (3, 3),(3, 4),(3, 5),(3, 6) and (4, 6).Thus, required probability $= \frac{16}{36} = \frac{4}{9}$

30. Let E =event when each American man is seated adjacent to his wife

and A =event when Indian man is seated adjacent to his wife.

Now, $n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife. Again, $n(E) = (5!) \times (2!)^4 \therefore P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)} = \frac{(4!)\times(2!)^3}{(5!)\times(2!)^4} = \frac{2}{5}$

31. $P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right)+P(B_2)P\left(\frac{R}{B_2}\right)+P(B_3)P\left(\frac{R}{B_3}\right)} = \frac{\frac{1}{3}\times\frac{2}{5}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{5}+\frac{1}{6}\times\frac{3}{7}} = \frac{\frac{2}{15}}{\frac{1}{6}+\frac{2}{15}+\frac{1}{14}} = \frac{14}{39}$

32. Given, $S = \{1,2,3 \ldots ,50\}$

$A = \left\{n \in S: n + \frac{50}{n} > 27\right\}$

$= \{n \in S: n^2 - 27n + 50 > 0\} = \{n \in S: n < 2 \text{ or } n > 25\}$

$= \{1, 26, 27, \ldots , 50\} \Rightarrow n(A) = 26$

$B = \{n \in S: n \text{ is a prime}\}$

$= \{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47\} \Rightarrow n(B) = 15$

$\therefore C = \{n \in S: n \text{ is a square}\}$

$= \{1,4,9,16,25,36,49\} \Rightarrow n(C) = 7$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{26}{50}, P(B) = \frac{15}{50}, P(C) = \frac{7}{50} \Rightarrow P(A) > P(B) > P(C)$

33. If any number the last digit can be 0,1,2,3,4,5,6,7,8,9. We want that the last digit in the product is an odd digit other than 5 i.e. it is any one of the digits 1,3,7,9. This means that the product is not divisible by 2 or 5. The probability that a number is divisible by 2 or 5 is $\frac{6}{10}$, and in the case the last digit can be one of 0,2,4,5,6 or 8. The probability that the number is not divisible by 2 or 5, is $1-\frac{6}{10}=\frac{2}{5}$

In order that the product is not divisible by 2 or 5, none of the constituent numbers should be divisible by 2 or 5 and its probability is $\left(\frac{2}{5}\right)^4=\frac{16}{125}$

34. We have, $P(E_i)=\frac{1}{2}$ for $i=1,2,3$

For $i\neq j$, we have,

$$P(E_i\cap E_j)=\frac{1}{4}=P(E_i)P(E_j)$$

$\Rightarrow E_i$ and E_j are independent events for $i\neq j$

Also, $P(E_1\cap E_2\cap E_3)=\frac{1}{4}\neq P(E_1)\,P(E_2)\,P(E_3)$

$\Rightarrow E_1, E_2, E_3$ are not independent

Hence option (d) is not correct

35. Solve on your own.

36. Probability of each case $=\frac{9}{15}=\frac{3}{5}$

Required probability (with replacement)= $\left(\frac{3}{5}\right)^7$

37. Required probability $=P(\bar{A}_1\cap\bar{A}_2\cap\ldots\cap\bar{A}_n)$

$=P(\bar{A}_1)P(\bar{A}_2)\ldots P(\bar{A}_n)=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{n}{n+1}=\frac{1}{n+1}$

38. We have, $p =$ Probability that the bomb strikes the target $= 1/2$

Let n be the number of bombs which should be dropped to ensure 99% chance or better of completely destroying the target. Then, the probability that out of n bombs, at least two strike the target, is greater than 0.99 Let X denote the number of bombs striking the target. Then, $P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n, r = 0, 1, 2, \ldots, n$

Now, $P(X \geq 2) \geq 0.99$

$\Rightarrow \{1 - P(X < 2)\} \geq 0.99 \Rightarrow 1 - \{P(X = 0) + P(X = 1)\} \geq 0.99$

$\Rightarrow 1 - \left\{(1 + n)\frac{1}{2^n}\right\} \geq 0.99 \Rightarrow 0.01 \geq \frac{1+n}{2^n} \Rightarrow 2^n > 100 + 100\,n \Rightarrow n \geq 11$

Thus, the minimum number of bombs is 11

39. Given that, $x = 33^n$

Where, n is a positive integral value.

Here, only four digits may be at the unit place ie, 1, 3, 7, 9.

$\therefore n(S) = 4$

Let E be the event of getting 3 at its units place.

$n(E) = 1$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$

40. Clearly,

$P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$

$= P(A \cap B) + P(B \cap C) - P(A \cap B \cap C$

41. Let A and B are the Ist and IInd aeroplane hit the target respectively and their corresponding probabilities are

$P(A) = 0.3$ and $P(B) = 0.2 \Rightarrow P(\bar{A}) = 0.7$ and $P(\bar{B}) = 0.8$

$\therefore$ Required probability

$= P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(B) + \ldots = (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + (0.7)(0.8)(0.7)(0.8)(0.7)(0.2) + \ldots = 0.14[1 + (0.56) + (0.56)^2 + \ldots]$

$= 0.14\left(\frac{1}{1-0.56}\right) = 0.32$

42. Total cases = 4

Correct option = 1

So, probability of correct answer = $\frac{1}{4}$

43. Probability that at least one shot hits the plane

$= 1 - P$(none of the shot hits the plane)

$= 1 - 0.6 \times 0.7 \times 0.8 \times 0.9 = 1 - 0.3024 = 0.6976$

44. Consider the following events:

$A = A$ worker receives bonus, $B = A$ worker is skilled. We have,

$P(A) = \frac{30}{100}$ and $P(B/A) = \frac{20}{100}$

$\therefore$ Required probability $= P(A \cap B) = P(A)P(B/A)$

$\Rightarrow$ Required probability $= \frac{30}{100} \times \frac{20}{100} = 0.06$

45. We know sum of probability distribution is 1

$\therefore\ k + 2k + 3k + 2k + k = 1$

$\Rightarrow\ k = \frac{1}{9}$

$\therefore$ Mean, $m = \sum_{i=1}^{5} P_i x_i$

$= k(1) + 2k(2) + 3k(3) + 2k(4) + k(5)$

$= k(1 + 4 + 9 + 8 + 5) = \frac{1}{9} \times 27 = 3$

$\therefore\ (k, m) = \left(\frac{1}{9}, 3\right)$

46. Given, $4P(A) = 6P(B) = 10P(A \cap B) = 1$

$\therefore\ P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{10}}{\frac{1}{4}} = \frac{2}{5}$

47. We know, total probability distribution is 1.

$\therefore \frac{1}{10} + k + \frac{1}{5} + 2k + \frac{3}{10} + k = 1$

$\Rightarrow \frac{6}{10} + 4k = 1$

$\Rightarrow k = \frac{1}{10}$

48. Given, $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A)P(B) = \frac{1}{6}$ ...(i)

and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

$\Rightarrow P(\bar{A})P(\bar{B}) = \frac{1}{3}$

$\Rightarrow \{1 - P(A)\}\{1 - P(B)\} = \frac{1}{3}$

$\Rightarrow 1 - \frac{1}{3} + P(A)P(B) = P(A) + P(B)$

$\Rightarrow \frac{2}{3} + \frac{1}{6} = P(A) + P(B)$ [from Eq.(i)]

$\Rightarrow P(A) + P(B) = \frac{5}{6}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

or $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$

49. Sum of Probabilities=1

$\Rightarrow p + 2p + 3p + 4p + 5p + 7p + 8p + 9p$
$+10p + 11p + 12p = 1$

$\Rightarrow 72p = 1 \Rightarrow p = \frac{1}{72}$

50. Total number of cases = 9999

Favourable cases $= 10 \times 9 \times 8 \times 7 = 5040$

$\therefore$ Probability $= \frac{5040}{9999}$