VECTORS

CHAPTER 11

VECTORS

SCALARS:

A quantity which has magnitude but no direction, is called scalars.

VECTORS:

A quantity which has magnitude as well as direction, is called a vector.

MAGNITUDE OF A VECTOR:

The length of the vector **AB** or **a** is called the magnitude of **AB** or **a** and it is represented by |**AB**| or |**a**|. If $\boldsymbol{a} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$, then $|\boldsymbol{a}| = \sqrt{x^2 + y^2 + z^2}$.

TYPES OF VECTORS:

i. **Zero or Null Vector -** A vector whose initial and terminal points are coincident and whose magnitude is zero is called a zero or null vector.

ii. **Unit Vector –** A vector whose modulus is unity, is called a unit vector. The unit vector in the direction of a vector $\vec{a}$ is denoted by $\hat{a}$, read as 'a cap'.

iii. **Co-initial Vectors –** Vectors having the same initial points are called co-initial vectors.

iv. **Collinear or Parallel Vectors –** Vectors having the same or parallel supports are called collinear vectors.

v. **Equal Vectors –** Two vectors are equal, if they have same magnitude and direction.

vi. **Negative of a Vector –** The vector which has the same magnitude as the vector $\vec{a}$ but opposite direction, is called the negative of $\vec{a}$ and is denoted by $-\vec{a}$.

vii. **Coplanar Vectors –** A system of vectors is said to be coplanar, if their supports are parallel to the same plane.

viii. **Like and Unlike Vectors –** Vectors are said to be like when they have the same sense of direction and unlike when they have opposite directions.

ix. **Coterminous Vectors –** Vectors having the same terminal point are called coterminous vectors.

x. **Free Vector –** If the initial point of a vector is not specified, then it is called a free vector.

POSITION VECTOR:

Let O be any point called the origin of reference or say simply origin and a point P in space, having coordinates (x,y,z) w.r.t. origin O(0,0,0) then vector **OP(=r)** having O and P as its initial points respectively, is called the position vector of the point P w.r.t. O.

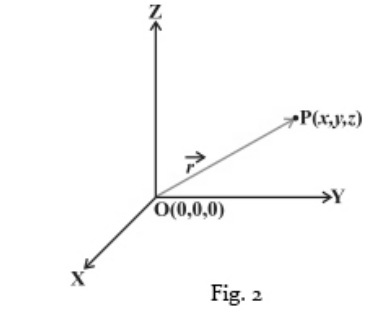


Fig. 2

The magnitude of **OP** or **r** is given by $|\mathbf{OP}|=|\mathbf{r}|=\sqrt{x^2+y^2+z^2}$

ADDITION OF VECTORS:

i. **TRIANGLE LAW OF VECTOR ADDITION :**

If two vectors are represented along two sides of a triangle taken in order, then their resultant is represented by the third side taken in opposite direction i.e. in ΔABC, by triangle law of vector addition, we have **AB+BC=AC**

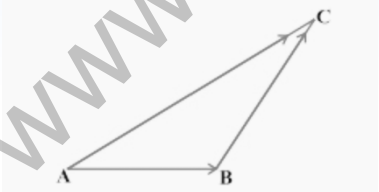


Fig. 1

ii. PARALLELOGRAM LAW OF VECTOR ADDITION:

If two vectors are represented along the two adjacent sides of parallelogram, then their resultant is represented by the diagonal of the parallelogram, which is coinitial with the given vector. If the sides OA and OC of parallelogram OABC represents **OA** and **OC,** respectively, then we get **OA+OC=OB** or **OA+AB=OB**

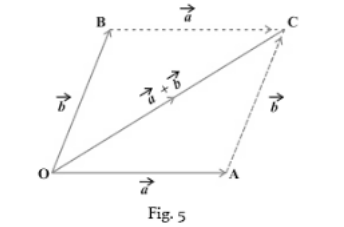


Fig. 5

PROPERTIES OF VECTOR ADDITION:

i. For any two vectors **a** and **b,**

a+b = b+a

ii. For any three vectors **a,b and c,**

a+(b+c) = (a+b)+c

iii. For any vector **a**, we have

a+0 = 0+a = a

The zero vector **0** is called the **additive identity** for the vector addition.

MULTIPLICATION OF A VECTOR BY SCALAR:

Let **a** be a given vector and λ be a scalar. Then the product of the vector a by the scalar λ denoted by $\boldsymbol{\lambda a}$**,** is called the multiplication of a vector a by the scalar λ.

PROPERTIES OF MULTIPLICATION OF A VECTOR BY A SCALAR:

Let **a** and **b** be any two vectors and k and m be any scalars. Then,

i. k**a**+m**a**=(k+m)**a**

ii. k(**a**+**b**)=k**a**+k**b**

iii. k(m**a**)=(km)**a**

COMPONENTS OF A VECTOR:

If $a = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$, then we can say that the scalar components of a along X-axis, Y-axis and Z-axis are a_1, a_2 and a_3 respectively.

IMPORTANT RESULTS IN COMPONENT FORM:

If a and b be any two vectors given in the component form as $a = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $b = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$.Then, (a_1, a_2, a_3) and (b_1, b_2, b_3) are called **direction ratios** of **a** and **b** respectively.

i. The sum (or resultant) of the vectors **a** and **b** is given by

a+b = $(a_1 + b_1)\,\hat{\imath} + (a_2 + b_2)\,\hat{\jmath} + (a_3 + b_3)\,\hat{k}$

ii. The difference of the vectors **a** and **b** is given by

a-b = $(a_1 - b_1)\,\hat{\imath} + (a_2 - b_2)\,\hat{\jmath} + (a_3 - b_3)\,\hat{k}$

iii. The vectors **a** and **b** are equal if

$a_1 = b_1\,,\ a_2 = b_2,\ a_3 = b_3$

iv. The multiplication of vector **a** by any scalar λ is given by

$\lambda a = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath} + (\lambda a_3)\hat{k}$

v. If $\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = k(constant)$

Then vectors a and b will be collinear.

vi. If it is given that l,m and n are direction cosines of a vector, then

$l\hat{\imath} + m\hat{\jmath} + n\hat{k} = (\cos\alpha)\hat{\imath} + (\cos\beta)\hat{\jmath} + (\cos\gamma)\hat{k}$

Is the unit vector in the direction of that vector, where $\alpha, \beta and\ \gamma$ are the angles which the vector makes with X,Y and Z-axes respectively.

VECTORS JOINING TWO POINTS:

If $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are any two points on theplane, then vector joining of P_1 and P_2 is $\boldsymbol{P_1P_2 = OP_2 - OP_1}$

$= (x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}) - (x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k})$

$= (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$

SECTION FORMULAE:

Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$, respectively and P be a point which divides AB internally in the ratio m:n.

Then, position vector of $P = \frac{m\boldsymbol{b}+n\boldsymbol{a}}{m+n}$.

If P divides AB externally in the ratio m:n. Then, position vector of $P = \frac{m\boldsymbol{b}-n\boldsymbol{a}}{m-n}$.

SCALAR OR DOT PRODUCT:

Let **a** and **b** be the two non-zero vectors inclined at an angle θ. Then the scalar product or dot product of **a** and **b** is represented by **a.b** and it is defined as **a.b = |a||b|cosθ,** where $0 \leq \theta \leq \pi$.

IMPORTANT RESULTS BASED ON SCALAR PRODUCT:

i. $a \perp b \Leftrightarrow a.b = 0$

ii. $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1 \; and \; \hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = \vec{0}$

iii. If $\theta = 0$, then a.b=|a||b|

If $\theta = \pi$, then a.b= -|a||b|

iv. The angle between two non-zero vectors a and b is given by $\cos\theta = \frac{a.b}{|a||b|} \; or \; \cos^{-1}\left(\frac{a.b}{|a||b|}\right)$

v. Projection of a on b = $\frac{a.b}{|b|}$

vi. If a force F displaces a particle from a point A to a point B, then work done by the force= F.AB.

PROPERTIES OF SCALAR PRODUCT:

i. a.b=b.a

ii. a.(b+c)=a.b+a.c

iii. $a.a = |a|^2 = a^2$

where, a represents magnitude of vector a.

iv. $(a + b)(a - b) = a^2 - b^2$, where a and b represent the magnitude of vectors a and b.

v. $(\lambda a).b = \lambda(a.b)$

CROSS OR VECTOR PRODUCT:

Let θ be the angle between two non-zero vectors a and b, then the vector or cross product of a and b is represented by $a \times b$ and defined as $a \times b = |a||b| \sin\theta\, \hat{n}$,

where, $\hat{n}$ is a unit vector perpendicular to the plane of a and b.

PROPERTIES OF CROSS PRODUCT:

i. $(a \times b) = -(b \times a)$

ii. $a \times (b + c) = a \times b + a \times c$

iii. $\lambda(a \times b) = (\lambda a) \times b = a \times (\lambda b)$

IMPORTANT REULTS BASED ON CROSS PRODUCT:

i. A unit vector perpendicular to both a and b is given by $\hat{n} = \frac{a \times b}{|a \times b|}$.

ii. $a \parallel b \Leftrightarrow a \times b = 0$

iii. $a \times a = 0$

iv. $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

v. $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

vi. If $a = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $b = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ then

$$a \times b = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

vii. Area of parallelogram with sides a and b $= |a \times b|$

viii. Area of a parallelogram with diagonals d_1 and $d_2 = \frac{1}{2}|d_1 \times d_2|$

ix. Area of a $\Delta ABC = \frac{1}{2}|AB \times BC| = \frac{1}{2}|BC \times BA| = \frac{1}{2}|CB \times CA|$

SCALAR TRIPLE PRODUCT:

Let a,b and c be any three vectors. The scalar product of a and (b × c), i.e. **a.(b × c)** is called the scalar triple product of a,b and c and it is denoted by [a b c] or [a,b,c].Thus **[a b c]= a.(b × c)**

IMPORTANT RESULTS BASED ON SCALAR TRIPLE PRODUCT:

i. The volume of a parallelepiped formed by adjacent sides given by the three vectors a,b and c is $|a.(b \times c)|$.

ii. If $a = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$, $b = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $c = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$, then

[a b c] $= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

iii. If a,b and c be any three vectors, then

[a b c]=[b c a]=[c a b]

iv. In scalar triple product, **a.(b × c),** then dot and cross can be interchanged,

i.e. **a.(b × c) = (a × b).c**

v. The scalar triple product of three vectors changes in sign but not in magnitude when the cyclic order of vector is changed.

i.e. **[a b c]= -[b a c]= -[c b a]= -[a c b]**

vi. If any two of three vectors a,b and c are same, then **[a a b]=[a b a]=[b a a]=0**

COPLANARITY OF THREE VECTORS:

Three non-zero, non collinear vectors a, b and c are coplanar, if and only if a. **(b × c)=0**

Vectors

Unit vector

For a given vector $\vec{a}$, the vector $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the direction of $\vec{a}$. e.g., If $\vec{a}=5\hat{i}$, then $\hat{a}=\frac{5\hat{i}}{|5|}=\hat{i}$, which is a unit vector.

Position of vectors

Position vector of a point $P(x, y, z)$ is $x\hat{i}+y\hat{j}+z\hat{k}$ and its magnitude is $OP(r)=\sqrt{x^2+y^2+z^2}$. eg: Position vector of $P(2, 3, 5)$ is $2\hat{i}+3\hat{j}+5\hat{k}$ and its magnitude is $\sqrt{2^2+3^2+5^2}=\sqrt{38}$.

Types of Vectors

(i) Zero vector (initial and terminal points coincide)
(ii) Unit vector (magnitude is unity)
(iii) Coinitial vectors (same initial points)
(iv) Collinear vectors (parallel to the same line)
(v) Equal vectors (same magnitude and direction)
(vi) Negative of a vector (same magnitude, opp. direction)

Direction ratios and direction cosines

The scalar components of a vector are its direction ratios, and represent its projections along the respective axes. The magnitude (r) direction ratios (a, b, c) and direction cosines (l,m,n) of vector $a\hat{i}+b\hat{j}+c\hat{k}$ are related as:

$l=\frac{a}{r}, m=\frac{b}{r}, n=\frac{c}{r}$ Where $r=\left|\sqrt{a^2+b^2+c^2}\right|$

e.g. : If $\overrightarrow{AB}=\hat{i}+2\hat{j}+3\hat{k}$, then $r=\left|\sqrt{1+4+9}\right|=\sqrt{14}$

Direction ratios are $(1, 2, 3)$ and direction cosines are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

Position Vector

The position vector of a point R dividing a line segment joining P, Q whose position vectors are $\vec{a}, \vec{b}$ resp., in the ratio $m : n$.

(i) internally is $\frac{n\vec{a}+m\vec{b}}{m+n}$, (ii) externally is $\frac{m\vec{b}-n\vec{a}}{m-n}$

Vector

A quantity that has both magnitude and direction is called a vector. The distance between the initial and terminal points of a vector is called its magnitude. Magnitude of vector $\overrightarrow{AB}$ is $|\overrightarrow{AB}|$.

Scalar product of two vectors

If $\vec{a}, \vec{b}$ are the vectors and 'θ', angle between them, then their scalar product $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow \cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

Cross product of two vectors

$\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, $\hat{n}$ is a unit vector perpendicular to line joining a, b.

Properties of Vector

If we have two vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and λ is any scalar, then-

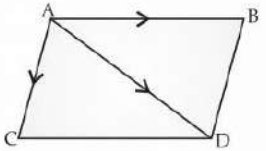
(i) $\vec{a}\pm\vec{b}=(a_1\pm b_1)\hat{i}+(a_2\pm b_2)\hat{j}+(a_3\pm b_3)\hat{k}$

(ii) $\lambda\vec{a}=(\lambda a_1)\hat{i}+(\lambda a_2)\hat{j}+(\lambda a_3)\hat{k}$

(iii) $\vec{a}.\vec{b}=a_1b_1+a_2b_2+a_3b_3$ and

(iv) $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

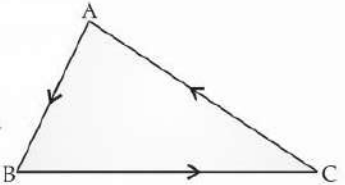
The vector sum of two co-initials vectors is given by the diagonal of the parallelogram whose adjacent sides are given vectors.



If $\overrightarrow{AB}, \overrightarrow{AC}$ are the given vectors, then $\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{AD}$

The vector sum of the three sides of a triangle taken in order is $\vec{0}$. i.e.,

If ABC is given triangle, then $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$.



Trace the Mind Map

First Level · Second Level · Third Level

PRACTICE QUESTIONS

1. Forces of magnitudes 3 and 4 units acting along $6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ respectively act on a particle and displace it from $(2,2-1)$ to $(4,3,1)$. The work done is
   a) 124/7
   b) 120/7
   c) 125/7
   d) 121/7
2. The vector $\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{\mathbf{i}} + (4x-2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The values of x is
   a) $\left\{-\frac{2}{3}, 2\right\}$
   b) $\left\{\frac{1}{3}, 2\right\}$
   c) $\left\{\frac{2}{3}, 0\right\}$
   d) $\{2,7\}$
3. The projection of the vector $2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ on the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, is
   a) $\frac{2}{\sqrt{14}}$
   b) $\frac{1}{\sqrt{14}}$
   c) $\frac{3}{\sqrt{14}}$
   d) None of these
4. If $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{\mathbf{u}}\ p\ \vec{\mathbf{v}}\ p\ \vec{\mathbf{w}}] - [p\ \vec{\mathbf{v}}\ \ \vec{\mathbf{w}}\ q\ \vec{\mathbf{u}}] - [2\vec{\mathbf{w}}\ q\ \vec{\mathbf{v}}\ q\ \vec{\mathbf{u}}] = 0$ holds for
   a) Exactly two value of (p, q)
   b) More than two but not all values of (p, q)
   c) All values of $(p.q)$
   d) Exactly one value of (p, q)

5. The equation $\mathbf{r}^{\vec{2}} - 2\,\vec{\mathbf{r}} \cdot \vec{\mathbf{c}} + h = 0, |\vec{\mathbf{c}}| > \sqrt{h}$, represent
   a) Circle
   b) Ellipse
   c) Cone
   d) Sphere
6. If three concurrent edges of a parallelopiped of volume V represent vectors $\vec{a}, \vec{b}, \vec{c}$ then the volume of the parallelopiped whose three concurrent edges are the three concurrent diagonals of the three faces of the given parallelopiped, is
   a) V
   b) 2V
   c) 3V
   d) None of these
7. $\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}} = 2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, then the area of $\Delta\, ABC$ is
   a) 3 sq units
   b) 4 sq units
   c) 16 sq units
   d) 9 sq units
8. If $\vec{a}, \vec{b}, \vec{c}$ are any three mutually perpendicular vectors of equal magnitude a, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to
   a) a
   b) $\sqrt{2}\,a$
   c) $\sqrt{3}\,a$
   d) 2a
9. For any three non-zero vectors $\vec{\mathbf{r}}_1\vec{\mathbf{r}}_2$ and $\vec{\mathbf{r}}_3$, $\begin{vmatrix} \vec{\mathbf{r}}_1 \cdot \vec{\mathbf{r}}_1 & \vec{\mathbf{r}}_1 \cdot \vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_1 \cdot \vec{\mathbf{r}}_3 \\ \vec{\mathbf{r}}_2 \cdot \vec{\mathbf{r}}_1 & \vec{\mathbf{r}}_2 \cdot \vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_2 \cdot \vec{\mathbf{r}}_3 \\ \vec{\mathbf{r}}_3 \cdot \vec{\mathbf{r}}_1 & \vec{\mathbf{r}}_3 \cdot \vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_3 \cdot \vec{\mathbf{r}}_3 \end{vmatrix} = 0$, Then, which of the following is false?
   a) All the three vectors are parallel to one and the same plane
   b) All the three vectors are linearly dependent
   c) This system of equation has a non-trivial solution
   d) All the three vectors are perpendicular to each other

10. If the volume of a parallelopiped with $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ as coterminous edges is 9 cu units, then the volume of the parallelopiped with $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c}), (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}), (\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ as coterminous edges is
    a) 9 cu units
    b) 729 cu units
    c) 81 cu units
    d) 27 cu units
11. If A, B and C are the vertices of a triangle whose position vectors are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively G is the centroid of the ΔABC, then $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$ is
    a) $\vec{0}$
    b) $\vec{a} + \vec{b} + \vec{c}$
    c) $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$
    d) $\frac{\vec{a}-\vec{b}-\vec{c}}{3}$
12. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{p} - \vec{q}$, $\vec{b} = \vec{p} + 3\vec{q}$ and also given that $|\vec{p}| = |\vec{q}| = 2$. If the vectors $\vec{p}$ and $\vec{q}$ are inclined at an angle $\pi/3$, then the ratio of the lengths of the diagonals of the parallelogram is
    a) $\sqrt{6}:\sqrt{2}$
    b) $\sqrt{3}:\sqrt{5}$
    c) $\sqrt{7}:\sqrt{3}$
    d) $\sqrt{6}:\sqrt{5}$
13. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2, \vec{w} = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ are equals
    a) 2
    b) $\sqrt{7}$
    c) $\sqrt{14}$
    d) 14

14. The vectors $2\hat{\imath} - m\hat{\jmath} + 3m\hat{k}$ and $(1+m)\hat{\imath} - 2m\hat{\jmath} + \hat{k}$ include an acute angle for

a) $m = -\frac{1}{2}$

b) $m \in \left[-2, -\frac{1}{2}\right]$

c) $m \in R$

d) $m \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$

15. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of points A, B, C, D such that no three of them are collinear and $\vec{a} + \vec{c} = \vec{b} + \vec{d}$, then $ABCD$ is a

a) Rhombus

b) Rectangle

c) Square

d) Parallelogram

16. When a right handed rectangular cartesian system $OXYZ$ rotated about z-axis through $\pi/4$ in the counter-clock-wise sense it is found that a vector $\vec{r}$ has the components $2\sqrt{2}, 3\sqrt{2}$ and 4. The components of $\vec{a}$ in the $OXYZ$ coordinate system are

a) $5, -1, 4$

b) $5, -1, 4\sqrt{2}$

c) $-1, -5, 4\sqrt{2}$

d) None of these

17. The vector equation of the line passing through the points $(3,2,1)$ and $(-2,1,3)$ is

a) $\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

b) $\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(-5\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

c) $\vec{\mathbf{r}} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \lambda(5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

d) $\vec{\mathbf{r}} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

18. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}$. If $|\vec{\alpha}| = |\vec{\beta}| = 2$ and the angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then the angle of a diagonal of the parallelogram are

    a) $4\sqrt{5}, 4\sqrt{3}$

    b) $4\sqrt{3}, 4\sqrt{7}$

    c) $4\sqrt{7}, 4\sqrt{5}$

    d) None of these

19. The vectors $\vec{u} = (al + a_1 l_1)\hat{\imath} + (am + a_1 m_1)\hat{\jmath} + (an + a_1 n_1)\hat{k}$, $\vec{v} = (bl + b_1 l_1)\hat{\imath} + (bm + b_1 m_1)\hat{\jmath} + (bn + b_1 n_1)\hat{k}, \vec{w} = (cl + c_1 l_1)\hat{\imath} + (cm + c_1 m_1)\hat{\jmath} + (cn + c_1 n_1)\hat{k}$

    a) Form an equilateral triangle

    b) Are coplanar

    c) Are collinear

    d) Are mutually perpendicular

20. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ and $\overrightarrow{AD} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

    a) $\frac{8}{9}$

    b) $\frac{\sqrt{17}}{9}$

    c) $\frac{1}{9}$

    d) $\frac{4\sqrt{5}}{9}$

21. The length of longer diagonal of the parallelogram constructed on $5\vec{\mathbf{a}} + 2\vec{\mathbf{b}}$ and $\vec{\mathbf{a}} - 3\vec{\mathbf{b}}$. If it is given that $|\vec{\mathbf{a}}| = 2\sqrt{2}, |\vec{\mathbf{b}}| = 3$ and angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\frac{\pi}{4}$, is

    a) 15

    b) $\sqrt{113}$

    c) $\sqrt{593}$

    d) $\sqrt{369}$

22. Three vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a} \times \vec{b} = 2\vec{a} \times \vec{c}, |\vec{a}| = |\vec{c}| = 1$ and $|\vec{b}| = 4$. If the angle between $\vec{b}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, then $\vec{b} - 2\vec{c}$ is equal to

a) $\pm 4\vec{a}$

b) $\pm 3\vec{a}$

c) $\pm 5\vec{a}$

d) $\pm 4\vec{a}$

23. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices of a triangle ABC, then a unit vector perpendicular to its plane is

a) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

b) $\frac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

c) $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

d) None of these

24. If the non-zero vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then the solution of the equation, $\vec{r} \times \vec{a} = \vec{b}$ is given by

a) $\vec{r} = x\vec{a} + \frac{\vec{a}\times\vec{b}}{|\vec{a}|^2}$

b) $\vec{r} = x\vec{b} - \frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$

c) $\vec{r} = x(\vec{a} \times \vec{b})$

d) $\vec{r} = x(\vec{b} \times \vec{a})$

25. Constant forces $\vec{\mathbf{P}}_1 = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{P}}_2 = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{P}}_3 = \hat{\mathbf{j}} - \hat{\mathbf{k}}$ act on a particle at point A. The work done when the particle is displaced from the point A to B where $\vec{\mathbf{A}} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ is

a) 3

b) 9

c) 20

d) None of these

26. Let $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ be the unit vectors such that $\vec{\alpha}$ and $\vec{\beta}$ are mutually perpendicular and $\vec{\gamma}$ is equally inclined to $\vec{\alpha}$ and $\vec{\beta}$ at an angle θ. If $\vec{\gamma} = x\vec{\alpha} + y\vec{\beta} + z(\vec{\alpha} \times \vec{\beta})$, then which one of the following is incorrect?

a) $z^2 = 1 - 2x^2$

b) $z^2 = 1 - 2y^2$

c) $z^2 = 1 - x^2 - y^2$

d) $x^2 + y^2 = 1$

27. If V is the volume of the parallelopiped having three coterminus edges as $\vec{a}, \vec{b}$ and $\vec{c}$, then the volume of the parallelopiped having three coterminus edges as

$\vec{\alpha} = (\vec{a} \cdot \vec{a})\vec{a} + (\vec{a} \cdot \vec{b})\vec{b} + (\vec{a} \cdot \vec{c})\vec{c}$

$\vec{\beta} = (\vec{a} \cdot \vec{b})\vec{a} + (\vec{b} \cdot \vec{b})\vec{b} + (\vec{b} \cdot \vec{c})\vec{c}$

$\vec{\gamma} = (\vec{a} \cdot \vec{c})\vec{a} + (\vec{b} \cdot \vec{c})\vec{b} + (\vec{c} \cdot \vec{c})\vec{c}$, is

a) V^3

b) $3V$

c) V^2

d) $2V$

28. The area of parallelogram constructed on the vectors $\vec{a} = \vec{p} + 2\vec{q}$ and $\vec{b} = 2\vec{p} + \vec{q}$, where $\vec{p}$ and $\vec{q}$ are unit vectors forming an angle of 30° is

a) 3/2

b) 5/2

c) 7/2

d) None of these

29. **I.** Two non-zero. Non-collinear vectors are linearly independent.

II. Any three coplanar vectors are linearly dependent. Which of the above statements is /are true?

a) Only **I**

b) Only **II**

c) Both **I** and **II**

d) Neither **I** nor **II**

30. If $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ constitute the corresponding reciprocal system of vectors then for any arbitrary vector $\vec{\alpha}$

a) $\vec{\alpha} = (\vec{\alpha} \cdot \vec{a})\vec{a} + (\vec{\alpha} \cdot \vec{b})\vec{b} + (\vec{\alpha} \cdot \vec{c})\vec{c}$

b) $\vec{\alpha} = (\vec{\alpha} \cdot \vec{p})\vec{p} + (\vec{\alpha} \cdot \vec{q})\vec{q} + (\vec{\alpha} \cdot \vec{r})\vec{r}$

c) $\vec{\alpha} = (\vec{\alpha} \cdot \vec{p})\vec{a} + (\vec{\alpha} \cdot \vec{q})\vec{b} + (\vec{\alpha} \cdot \vec{r})\vec{c}$

d) None of these

31. If $\vec{\alpha} = x(\vec{a} \times \vec{b}) + y(\vec{b} \times \vec{b}) + z(\vec{c} \times \vec{a})$ and $[\vec{a}\ \vec{b}\ \vec{c}] = \frac{1}{8}$, then x + y + z is equal to

a) $8\,\vec{\alpha} \cdot (\vec{a} + \vec{b} + \vec{c})$

b) $\vec{\alpha} \cdot (\vec{a} + \vec{b} + \vec{c})$

c) $8\,(\vec{a} + \vec{b} + \vec{c})$

d) None of these

32. $(\vec{x} - \vec{y}) \times (\vec{x} + \vec{y}) =$……. where $\vec{x}, \vec{y} \in R^3$

a) $2(\vec{x} \times \vec{y})$

b) $|\vec{x}|^2 - |\vec{y}|^2$

c) $\frac{1}{2}(\vec{x} \times \vec{y})$

d) None of these

33. A vector perpendicular to the plane containing the points $A(1.-1,2), B(2,0,-1), C(0,2,1)$ is

a) $4\hat{i} + 8\hat{j} - 4\hat{k}$

b) $8\hat{i} + 4\hat{j} + 4\hat{k}$

c) $3\hat{i} + \hat{j} + 2\hat{k}$

d) $\hat{i} + \hat{j} - \hat{k}$

34. Which one of the following is not correct?

a) If $\vec{p} \cdot \vec{a} = \vec{p} \cdot \vec{b} = \vec{p} \cdot \vec{c}$ for some non-zero vector $\vec{p}$ then $\vec{a}, \vec{b}, \vec{c}$ are coplanar

b) The vectors $\hat{i} + 3\hat{j}, 2\hat{i} + \hat{k}$ and $\hat{j} + \hat{k}$ are coplanar

c) The vector $\vec{a} \times (\vec{b} \times \vec{c})$ is coplanar with $\vec{a}$ and $\vec{b}$

d) If $\vec{a}, \vec{b}$ are unit vectors and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then $|\vec{a} + \vec{b}| < 1$

35. Let $ABCD$ be the parallelogram whose sides AB and AD are represented by the vectors $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ respectively. Then if $\vec{\mathbf{a}}$ is a unit vector parallel to $\overrightarrow{\mathbf{AC}}$, then $\vec{\mathbf{a}}$ is equal to

a) $\frac{3\hat{\mathbf{i}}-6\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{3}$

b) $\frac{3\hat{\mathbf{i}}+6\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{3}$

c) $\frac{3\hat{\mathbf{i}}-6\hat{\mathbf{j}}-3\hat{\mathbf{k}}}{7}$

d) $\frac{3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{7}$

36. The equation of the plane perpendicular to the line $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$ and passing through the point (2,3,1) is

a) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1$

b) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1$

c) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 7$

d) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 10$

37. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are three non-coplanar vectors and $\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{r}}$, are reciprocal vectors, then $(l\vec{\mathbf{a}} + m\vec{\mathbf{b}} + n\vec{\mathbf{c}}) \cdot (l\vec{\mathbf{p}} + m\vec{\mathbf{q}} + n\vec{\mathbf{r}})$ is

a) $l + m + n$

b) $l^3 + m^3 + n^3$

c) $l^2 + m^2 + n^2$

d) None of these

38. If forces of magnitudes 6 and 7 units acting in the directions $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ respectively act on a particle which is displaced from the point $P(2, -1, -3)$ to $Q(5, -1, 1)$, then the work done by the forces is

a) 4 units

b) -4 units

c) 7 units

d) -7 units

39. If the constant forces $2\hat{\imath} - 5\hat{\jmath} + 6\hat{k}$ and $-\hat{\imath} + 2\hat{\jmath} - \hat{k}$ act on a particle due to which it is displaced from a point $A(4, -3, -2)$ to a point $B(6, 1, -3)$, then the work done by the forces is

a) 15 units

b) -15 units

c) 9 units

d) -9 units

40. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be non-zero vectors such that $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = -\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$. If θ is the acute angle between vectors $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$, then the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ is equal to

a) $\frac{2\pi}{3}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{2}$

41. If the position vectors of three points A, B, C are respectively $\hat{\imath} + \hat{\jmath} + \hat{k}, 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $7\hat{\imath} + 4\hat{\jmath} + 9\hat{k}$, then the unit vector perpendicular to the plane of triangle ABC is

a) $31\hat{\imath} - 18\hat{\jmath} - 9\hat{k}$

b) $\frac{31\hat{\imath} - 38\hat{\jmath} - 9\hat{k}}{\sqrt{2486}}$

c) $\frac{31\hat{\imath} + 38\hat{\jmath} + 9\hat{k}}{\sqrt{2486}}$

d) None of these

42. Let $\vec{p}$ and $\vec{q}$ be the position vectors of P and Q respectively, with respect to O and $|\vec{p}| = p, |\vec{q}| = q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If $O\vec{R}$ and $\vec{O}S$ are perpendicular, then

a) $9p^2 = 4q^2$

b) $4p^2 = 9q^2$

c) $9p = 4q$

d) $4p = 9q$

43. Let $ABCD$ be the parallelogram whose sides AB and AD are represented by the vectors $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ respectively. Then, if $\vec{\mathbf{a}}$ is a unit vector parallel to $\overrightarrow{\mathbf{AC}}$, then $\vec{\mathbf{a}}$ equal to

a) $\frac{1}{3}(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

b) $\frac{1}{3}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

c) $\frac{1}{7}(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

d) $\frac{1}{7}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

44. The length of the longer diagonal of the parallelogram constructed on $5\vec{a} + 2\vec{b}$ and $\vec{a} - 3\vec{b}$ if it is given that $|\vec{a}| = 2\sqrt{2}, |\vec{b}| = 3$ and angle between $\vec{a}$ and $\vec{b}$ is $\pi/4$, is

a) 15

b) $\sqrt{113}$

c) $\sqrt{593}$

d) $\sqrt{369}$

45. If the position vectors of the vertices of a triangle are $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$, then the triangle is

a) Equilateral

b) Isosceles

c) Right angled isosceles

d) Right angled

46. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$, then the area of the parallelogram having diagonals $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{b}} + \vec{\mathbf{c}}$ is

a) $4\sqrt{6}$

b) $\frac{1}{2}\sqrt{21}$

c) $\frac{\sqrt{6}}{2}$

d) $\sqrt{6}$

47. If $4\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 8\hat{\mathbf{k}}, 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ are the position vectors of the vertices A, B and C respectively of triangle ABC. The position vector of the point where the bisector of angle A meets BC is

a) $\frac{1}{2}(6\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 18\hat{\mathbf{k}})$

b) $\frac{2}{3}(6\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - 8\hat{\mathbf{k}})$

c) $\frac{1}{3}(-6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 9\hat{\mathbf{k}})$

d) $\frac{2}{3}(-6\hat{\mathbf{i}} - 12\hat{\mathbf{j}} + 8\hat{\mathbf{k}})$

48. If $\vec{\mathbf{u}}_1$ and $\vec{\mathbf{u}}_2$ be vectors of unit length and θ be the angle between them, then $\frac{1}{2}|\vec{\mathbf{u}}_2 - \vec{\mathbf{u}}_1|$is.

a) $\sin\theta$

b) $\sin\frac{\theta}{2}$

c) $\cos\theta$

d) $\cos\frac{\theta}{2}$

49. If the vectors

$\vec{a} = \hat{\imath} + a\hat{\jmath} + a^2\hat{k}, \vec{b} = \hat{\imath} + b\hat{\jmath} + b^2\hat{k}, \vec{c} = \hat{\imath} + c\hat{\jmath} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is X

a) 0

b) 1

c) 2

d) -1

50. If the vectors $\alpha\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{j}} + \gamma\hat{\mathbf{k}}$ $(\alpha, \beta, \gamma \neq 1)$ are coplanar, then the value of $\frac{1}{1-\alpha} + \frac{1}{1-\beta} - \frac{1}{1-\gamma}$ is

a) -1

b) 0

c) 1

d) 1/2

ANSWER KEY

1. A
2. A
3. A
4. D
5. D
6. B
7. A
8. C
9. A
10. C
11. A
12. A
13. C
14. D
15. D
16. D
17. A
18. B
19. B
20. B
21. C
22. A
23. B
24. A
25. D
26. D
27. A
28. A
29. C
30. C
31. A
32. A
33. B
34. D
35. D
36. B
37. C
38. A
39. B
40. D
41. B
42. A
43. D
44. C
45. D
46. A
47. A
48. B
49. D
50. C

HINTS AND SOLUTIONS

1. Total force, $\vec{\mathbf{F}} = 3\left(\frac{6\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{7}\right) + 4\left(\frac{3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+6\hat{\mathbf{k}}}{7}\right) = \frac{(30\hat{\mathbf{i}}-2\hat{\mathbf{j}}+33\hat{\mathbf{k}})}{7}$

   $\therefore\ \vec{\mathbf{d}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}} - \left(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

   $\therefore$ Work done $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

   $= \left(\frac{30\hat{\mathbf{i}}-2\hat{\mathbf{j}}+33\hat{\mathbf{k}}}{7}\right) \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = \frac{60-2+66}{7} = \frac{124}{7}$

2. Since, $2\left|\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right| = \left|4\hat{\mathbf{i}} + (4x-2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right|$

   $\Longrightarrow 2\sqrt{1 + x^2 + 9} = \sqrt{4^2 + (4x-2)^2 + 2^2}$

   $\Longrightarrow\ 12x^2 - 16x - 16 = 0 \Longrightarrow (3x+2)(x-2) = 0 \Longrightarrow x = 2, -\frac{2}{3}$

3. Do it yourself.

4. Since,

   $[3\vec{\mathbf{u}}\ p\ \vec{\mathbf{v}}\ p\ \vec{\mathbf{w}}] - [p\ \vec{\mathbf{v}}\ \vec{\mathbf{w}}\ q\ \vec{\mathbf{u}}] - [2\ \vec{\mathbf{w}}\ q\ \vec{\mathbf{v}}\ q\ \vec{\mathbf{u}}] = 0$

   $\therefore 3p^2[\vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} \times \vec{\mathbf{w}})] - pq[\vec{\mathbf{v}} \cdot (\vec{\mathbf{w}} \times \vec{\mathbf{u}})]$

   $-2q^2[\vec{\mathbf{w}} \cdot (\vec{\mathbf{v}} \times \vec{\mathbf{u}})] = 0 \Longrightarrow (3p^2 - pq + 2q^2)[\vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} \times \vec{\mathbf{w}})] = 0$

   But $[\vec{\mathbf{u}}\ \vec{\mathbf{v}}\ \vec{\mathbf{w}}] \neq 0$

   $\Longrightarrow 3p^2 - pq + 2q^2 = 0 \Longrightarrow p = q = 0$

5. The given equation is

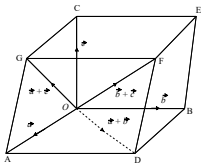
   $\overrightarrow{\mathbf{r^2}} - 2\vec{\mathbf{r}} \cdot \vec{\mathbf{c}} + h = 0, |\vec{\mathbf{c}}| > \sqrt{h}$

   This is the equation of sphere in diameter form. *ie*, $(\vec{\mathbf{r}} - \vec{\mathbf{a}}) \cdot \left(\vec{\mathbf{r}} - \vec{\mathbf{b}}\right) = 0$

6. We have, $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right| = V$

   Volume V_1 of the parallelopiped having diagonals of the given parallelopiped as three concurrent edges is given by

   $V_1 = \left|\left[\vec{a} + \vec{b}\ \vec{b} + \vec{c}\ \vec{c} + \vec{a}\right]\right| = \left|2\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right| = 2V$



7. Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{4+16+16} = 3$ sq units

8. Do it yourself

9. Do it yourself

10. Let $\vec{A} = \vec{a} \times \vec{b}, \vec{B} = \vec{b} \times \vec{c}, \vec{C} = \vec{c} \times \vec{a}$

    Given, $\left[\vec{A}\ \vec{B}\ \vec{C}\right] = 9$ cu units

    Using the relation $\left[\vec{A} \times \vec{B}\ \vec{B} \times \vec{C}\ \vec{C} \times \vec{A}\right] = \left[\vec{A}\ \vec{B}\ \vec{C}\right]^2 = (9)^2 = 81$ cu units

11. Position vectors of vertices A, B and C of the triangle ABC are $\vec{a}, \vec{b}$ and $\vec{c}$

    ∴Centroid of triangle

    $$(G) = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

    Now, $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$

    $$= \left(\vec{a} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) + \left(\vec{b} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) + \left(\vec{c} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) = \vec{0}$$

12. Do it yourself

13. Since, $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$

    The projection of $\vec{v}$ along $\vec{u} = \frac{\vec{v}\cdot\vec{u}}{|\vec{u}|}$

    and the projection of $\vec{w}$ along $\vec{u} = \frac{\vec{w}\cdot\vec{u}}{|\vec{u}|}$

    according to given condition,

    $$\frac{\vec{v}\cdot\vec{u}}{|\vec{u}|} = \frac{\vec{w}\cdot\vec{u}}{|\vec{u}|} \Rightarrow \vec{v}\cdot\vec{u} = \vec{w}\cdot\vec{u} \quad \ldots.(\text{i})$$

    Also, $\vec{v}\cdot\vec{w} = 0$

    Now, $|\vec{u} - \vec{v} + \vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2$

    $-2\vec{u}\cdot\vec{v} - 2\vec{v}\cdot\vec{w} + 2\vec{u}\cdot\vec{w} = 1 + 4 + 9 - 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u}$ [from Eq. (i)

    $\Rightarrow |\vec{u} - \vec{v} + \vec{w}|^2 = 14 + 0 \Rightarrow |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$

14. Do it yourself

15. Do it yourself

16. Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ in $OXYZ$ system

Also, let $\vec{r} = X\hat{\imath} + Y\hat{\jmath} + Z\hat{k}$ in the new coordinate system

Since the right handed rectangular system $OXYZ$ is rotated about z-axis through $\frac{\pi}{4}$ in anticlockwise direction. Therefore,

$x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$

$\Rightarrow x = X\cos\frac{\pi}{4} - Y\sin\frac{\pi}{4}, y = X\sin\frac{\pi}{4} + Y\cos\frac{\pi}{4}$ and, $z = Z$

It is given that $X = 2\sqrt{2}, Y = 3\sqrt{2}$ and $Z = 4$

$\therefore x = 2 - 3 = -1, y = 5$ and $z = 4$

Hence, $\vec{r} = -\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

17. The vector equation of line passing through points $(3, 2, 1)$ and $(-2, 1, 3)$

$\mathbf{\vec{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda[(-2-3)\hat{\mathbf{i}} + (1-2)\hat{\mathbf{j}} + (3-1)\hat{\mathbf{k}}]$

$= 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

18. Do it yourself

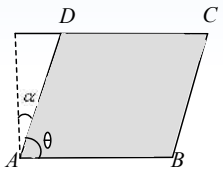
19. We have,

$$[\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} al + a_1l_1 & am + a_1m_1 & an + a_1n_1 \\ bl + b_1l_1 & bm + b_1m_1 & bn + b_1n_1 \\ cl + c_1l_1 & cm + c_1m_1 & cn + a_1n_1 \end{vmatrix}$$

$$\Rightarrow [\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} a & a_1 & 0 \\ b & b_1 & 0 \\ c & c_1 & 0 \end{vmatrix} \begin{vmatrix} l & l_1 & 0 \\ m & m_1 & 0 \\ n & n_1 & 0 \end{vmatrix} = 0$$

Hence, the given vectors are coplanar

20. $\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$, $\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$



$\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{AD}} = -2 + 20 + 22 = 40$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{4 + 100 + 120} = \sqrt{225} = 15$

$|\overrightarrow{\mathbf{AD}}| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore \cos\theta = \frac{40}{45} = \frac{8}{9} \therefore \theta + \alpha = 90° \Rightarrow \alpha = 90° - \theta \Rightarrow \cos\alpha = \sin\theta = \sqrt{1 - \frac{64}{81}} = \frac{\sqrt{17}}{9}$

21. Given that, $|\vec{a}| = 2\sqrt{2}, |\vec{b}| = 3$

The longer vectors are $5\vec{a} + 2\vec{b} + \vec{a} - 3\vec{b} = 6\vec{a} - \vec{b}$

Length of one diagonal

$= |6\vec{a} - \vec{b}|$

$= \sqrt{36\vec{a}^2 + \vec{b}^2 - 2 \times 6|\vec{a}|\,|\vec{b}| \cos 45^\circ} = \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}}$

$= \sqrt{288 + 9 - 12 \times 6} = \sqrt{225} = 15$

Other diagonal is $4\vec{a} + 5\vec{b}$.

Its length $= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

22. Do it yourself

23. Do it yourself

24. Since $\vec{a}, \vec{b}$ and $a \times \vec{b}$ are non-coplanar vectors

$\therefore \vec{r} = x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})$ for some scalars x, y, z …(i)

Now,

$\vec{b} = \vec{r} \times \vec{a}$

$\Rightarrow \vec{b} = \{x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})\} \times \vec{a}$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) + z\left((\vec{a} \times \vec{b}) \times \vec{a}\right) \Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) - z\left(\vec{a} \times (\vec{a} \times \vec{b})\right)$

$\Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) - z\{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\} \Rightarrow \vec{b} = y(\vec{b} \times \vec{a}) + z(\vec{a} \cdot \vec{a})\vec{b}$ $[\because \vec{a} \cdot \vec{b} = 0]$

Comparing the coefficients, we get

$y = 0, z = \frac{1}{\vec{a} \cdot \vec{a}} = \frac{1}{|\vec{a}|^2}$

Putting the values of y and z in (i), we get

$$\vec{r} = x\vec{a} + \frac{1}{|\vec{a}|^2}(\vec{a} \times \vec{b})$$

25. $\therefore$ Total force $\vec{\mathbf{P}} = \vec{\mathbf{P}}_1 + \vec{\mathbf{P}}_2 + \vec{\mathbf{P}}_3 = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} - \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} + \hat{\mathbf{j}} - \hat{\mathbf{k}} = 2\hat{\mathbf{j}}$

and displacement $\overrightarrow{\mathbf{AB}} = 6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}} - (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\therefore$Work done $= \vec{\mathbf{P}} \cdot \overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{j}}(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 8$

26. We have, $|\vec{\alpha}| = 1 = |\vec{\beta}|$ and $\vec{\alpha} \cdot \vec{\beta} = 0$

Now, $\vec{\gamma} = x\vec{\alpha} + y\vec{\beta} + z(\vec{\alpha} \times \vec{\beta}) \Rightarrow \vec{\alpha} \cdot \vec{\gamma} = x(\vec{\alpha} \cdot \vec{\alpha}) + y(\vec{\alpha} \cdot \vec{\beta}) + z\{\vec{\alpha} \cdot (\vec{\alpha} \times \vec{\beta})\}$

$\vec{\beta} \cdot \vec{\gamma} = x(\vec{\beta} \cdot \vec{\alpha}) + y(\vec{\beta} \cdot \vec{\beta}) + z\{\vec{\beta} \cdot (\vec{\alpha} \times \vec{\beta})\}$

And,$(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\gamma} = x\{\vec{\alpha} \cdot (\vec{\alpha} \times \vec{\beta}) + y\{\vec{\beta} \cdot (\vec{\alpha} \times \vec{\beta})\} + z\{(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\beta})\}$

$\Rightarrow \cos\theta = x, \cos\theta = y$ and $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = z|\vec{\alpha} \times \vec{\beta}|^2 \Rightarrow x = \cos\theta, y = \cos\theta$ and

$[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = z \quad [\because |\vec{\alpha} \times \vec{\beta}| = |\vec{\alpha}|\,|\vec{\beta}| \sin 90° = 1]$

Now,$[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]^2 = \begin{vmatrix} \vec{\alpha} \cdot \vec{\alpha} & \vec{\alpha} \cdot \vec{\beta} & \vec{\alpha} \cdot \vec{\gamma} \\ \vec{\beta} \cdot \vec{\alpha} & \vec{\beta} \cdot \vec{\beta} & \vec{\beta} \cdot \vec{\gamma} \\ \vec{\gamma} \cdot \vec{\alpha} & \vec{\gamma} \cdot \vec{\beta} & \vec{\gamma} \cdot \vec{\gamma} \end{vmatrix}$

$\Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]^2 = \begin{vmatrix} 1 & 0 & \cos\theta \\ 0 & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix} = 1 - 2\cos^2\theta \Rightarrow z^2 = 1 - 2x^2$

Also, $z^2 = 1 - 2y^2$ and $z^2 = 1 - x^2 - y^2$

27. We have, $|[\vec{a}\ \vec{b}\ \vec{c}]| = V$

Let V_1 be the volume of the parallelopiped formed by the vectors $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$. Then,

$V_1 = |[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]|$

Now,$[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} & \vec{c} \cdot \vec{c} \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$

$\Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = [\vec{a}\ \vec{b}\ \vec{c}]^2 [\vec{a}\vec{b}\ \vec{c}] \Rightarrow [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = [\vec{a}\ \vec{b}\ \vec{c}]^3$

$\therefore V_1 = |[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]| = |[\vec{a}\ \vec{b}\ \vec{c}]^3| = V^3$

28. Do it yourself

29. **I.** It is true that non-zero, non-collinear vectors are linearly independent.

II. It is also true that any three coplanar vectors are linearly dependent.

$\therefore$ Both **I** and **II** are true.

30. Let $\vec{\alpha} = \lambda\,\vec{a} + \mu\vec{b} + t\vec{c}$ …(i)

Now, $\vec{a}\cdot\vec{p} = \vec{b}\cdot\vec{q} = \vec{c}\cdot\vec{r} = 1$

$\Rightarrow\ \vec{\alpha}\cdot\vec{p} = \lambda\,(\vec{a}\cdot\vec{p}) + 0 + 0 \Rightarrow\ \lambda = \vec{\alpha}\cdot\vec{p}$

Similarly, $\mu = \vec{\alpha}\cdot\vec{q}$ and $t = \vec{\alpha}\cdot\vec{r}$

From Eq. (i), we get

$\vec{\alpha} = (\vec{\alpha}\cdot\vec{p})\vec{a} + (\vec{\alpha}\cdot\vec{q})\vec{b} + (\vec{\alpha}\cdot\vec{r})\vec{c}$

31. We have ,$\vec{\alpha} = x(\vec{a}\times\vec{b}) + y(\vec{b}\times\vec{c}) + z(\vec{c}\times\vec{a})$

Taking dot product with $\vec{a}, \vec{b}, \vec{c}$ respectively, we get

$\vec{\alpha}\cdot\vec{a} = y[\vec{a}\,\vec{b}\,\vec{c}] \implies y = 8(\vec{\alpha}\cdot\vec{a})$

$\vec{\alpha}\cdot\vec{b} = z\left((\vec{c}\times\vec{a})\cdot\vec{b}\right) \implies \vec{\alpha}\cdot\vec{b} = z[\vec{a}\,\vec{b}\,\vec{c}] \implies z = 8(\vec{\alpha}\cdot\vec{b})$

and $\vec{\alpha}\cdot\vec{c} = x(\vec{a}\times\vec{b}.\vec{c})$

$\vec{\alpha}\cdot\vec{c} = x[\vec{a}\,\vec{b}\,\vec{c}] \implies x = 8(\vec{\alpha}\cdot\vec{c}) \therefore x + y + z = 8\vec{\alpha}\cdot(\vec{a} + \vec{b} + \vec{c})$

32. $(\vec{x} - \vec{y}) \times (\vec{x} + \vec{y})$

$= \vec{x}\times\vec{x} + \vec{x}\times\vec{y} - \vec{y}\times\vec{x} - \vec{y}\times\vec{y} = \vec{0} + \vec{x}\times\vec{y} + \vec{x}\times\vec{y} - \vec{0} = 2(\vec{x}\times\vec{y})$

33. We know that a vector perpendicular to the plane containing the points $\vec{A}, \vec{B}, \vec{C}$ is given by $\vec{A}\times\vec{B} + \vec{B}\times\vec{C} + \vec{C}\times\vec{A}$.

Given, $\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} + 0\hat{j} - \hat{k}$ and $\vec{C} = 0\hat{i} + 2\hat{j} + \hat{k}$

Now, $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \hat{i} + 5\hat{j} + 2\hat{k}$

$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{C}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{i} + \hat{j} - 2\hat{k}$

Thus,

$\vec{A}\times\vec{B} + \vec{B}\times\vec{C} + \vec{C}\times\vec{A} = (\hat{i} + 5\hat{j} + 2\hat{k}) + (2\hat{i} - 2\hat{j} + 4\hat{k}) + (5\hat{i} + \hat{j} - 2\hat{k})$ $= 8\hat{i} + 4\hat{j} + 4\hat{k}$

34. (a) Let $\vec{\mathbf{p}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ where at least one of x, y, z is non-zero. Let

$\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{l}} + a_3\hat{\mathbf{k}}$

$\vec{\mathbf{b}} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{l}} + b_3\hat{\mathbf{k}}$

$\vec{\mathbf{c}} = c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{l}} + c_3\hat{\mathbf{k}}$

$\therefore$ By given conditions

$a_1x + a_2y + a_3z = 0$

$b_1x + b_2y + b_3z = 0$

$c_1x + c_2y + c_3z = 0$

$\Rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_3 & c_3 \end{vmatrix} = 0 \Rightarrow [\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,] = 0 \Rightarrow \vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar.

(b) Vectors are coplanar, if

$$\begin{vmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$ie, -7 = 0$Which is not possible.

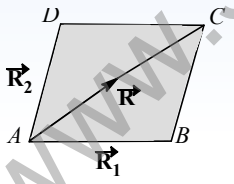
(c) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}}$

$\Rightarrow \vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ is coplanar with $\vec{b}$ and $\vec{c}$.

(d) $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1 \therefore |\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = (\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}})$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 1 + 1 = 2 \cdot 1 \cdot 1 \cos\frac{\pi}{3} = 3 \Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3} > 1$

35. Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$



and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{R}}$ (along $\overrightarrow{\mathbf{AC}}$) $= \vec{\mathbf{R}}_1 + \vec{\mathbf{R}}_2 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{a}}$ (unit vector along AC) $= \dfrac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|}$

$= \frac{3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{\sqrt{9+36+4}} = \frac{1}{7}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

36. The given line is parallel to the vector $\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The required plane passing through the point $(2, 3, 1) ie, 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and is perpendicular to the vector $\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Its equation is

$$[(\vec{\mathbf{r}} - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})] \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 0 \Rightarrow \vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1$$

37. $\because$ $\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{r}}$ are reciprocal vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ respectively.

$\therefore \vec{\mathbf{p}} \cdot \vec{\mathbf{a}} = 1, \vec{\mathbf{p}} \cdot \vec{\mathbf{b}} = 0, \vec{\mathbf{p}} \cdot \vec{\mathbf{c}}$ etc.

$$\therefore (l\vec{\mathbf{a}} + m\vec{\mathbf{b}} + n\vec{\mathbf{c}}) \cdot (l\vec{\mathbf{p}} + m\vec{\mathbf{q}} + n\vec{\mathbf{r}}) = l^2 + m^2 + n^2$$

38. The resultant force is given by

$$\vec{F} = 6\frac{(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})}{\sqrt{1+4+4}} + 7\frac{(2\hat{\imath} - 3\hat{\jmath} - 6\hat{k})}{\sqrt{4+9+36}} = 4\hat{\imath} - 7\hat{\jmath} - 2\hat{k}$$

$\vec{d}$ = Displacement $= \vec{P}Q$

$$\vec{d} = (5\hat{\imath} - \hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) = 3\hat{\imath} + 4\hat{k}$$

$\therefore$ Work done $= \vec{F} \cdot \vec{d} = 12 + 0 - 8 = 4$ units

39. Resultant force $\vec{F}$ is given by

$$\vec{F} = (2\hat{\imath} - 5\hat{\jmath} + 6\hat{k}) - (-\hat{\imath} + 2\hat{\jmath} - \hat{k}) = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$$

Let $\vec{d}$ be the displacement vector. Then,

$$\vec{d} = A\vec{B}$$

$$\Rightarrow \vec{d} = (6\hat{\imath} + \hat{\jmath} - 3\hat{k}) - (4\hat{\imath} - 3\hat{\jmath} - 2\hat{k}) = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$$

$\therefore W =$ Work done

$$\Rightarrow W = \vec{F} \cdot \vec{d} \Rightarrow W = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) \cdot (2\hat{\imath} + 4\hat{\jmath} - \hat{k})$$

$$\Rightarrow W = 2 - 12 - 5 = -15 \text{ units}$$

40. Given, $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = -\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$

$$\Rightarrow (\vec{\mathbf{c}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}} - (\vec{\mathbf{c}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} = -\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$$

On comparing both sides, we get

$$(\vec{\mathbf{c}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}} = 0$$

$$|\vec{\mathbf{c}}||\vec{\mathbf{a}}| \cos\theta = 0$$

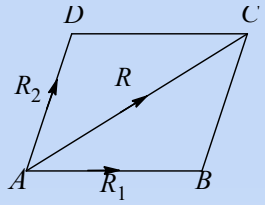
$$\Rightarrow \theta = \frac{\pi}{2}$$

41. The required vector is given by $\hat{n} = \frac{\vec{AB}\times\vec{AC}}{|\vec{AB}\times\vec{AC}|} = \frac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

42. Do it yourself

43. Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$



$\therefore\ \vec{\mathbf{R}}\ (\text{along } \overrightarrow{\mathbf{AC}}) = \vec{\mathbf{R}}_1 + \vec{\mathbf{R}}_2 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{a}}\ (\text{unit vector angle } \overrightarrow{\mathrm{AC}}) = \frac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|} = \frac{3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{\sqrt{9+36+4}} = \frac{1}{7}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

44. Do it yourself

45. Do it yourself

46. Given that,

$\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

Let$\vec{\mathbf{A}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and

$\vec{\mathbf{B}} = \vec{\mathbf{b}} + \vec{\mathbf{c}} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + (7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}) = 8\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 16\hat{\mathbf{k}}$

If $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are diagonals, then area of parallelogram

$$= \frac{1}{2}|\vec{\mathbf{A}} \times \vec{\mathbf{B}}| = \frac{1}{2}\begin{Vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{Vmatrix} = \frac{1}{2}|\hat{\mathbf{i}}(64-72) - \hat{\mathbf{j}}(32-48) + \hat{\mathbf{k}}(24-32)|$$

$$= \frac{1}{2}|-8\hat{\mathbf{i}} + 16\hat{\mathbf{j}} - 8\hat{\mathbf{k}}| = |-4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}|$$

$$= \sqrt{(-4)^2 + (8)^2 + (-4)^2} = \sqrt{16 + 64 + 16} = \sqrt{96} = 4\sqrt{6}$$

47. $\overrightarrow{AB} = 2\hat{i} + 3\hat{j} + 4\hat{k} - 4\hat{i} - 7\hat{j} - 8\hat{k} = -2\hat{i} - 4\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = 2\hat{i} + 5\hat{j} + 7\hat{k} - 4\hat{i} - 7\hat{j} - 8\hat{k} = -2\hat{i} - 2\hat{j} - \hat{k}$

$\therefore |\overrightarrow{AB}| = 6$ and $|\overrightarrow{AC}|=3$

$\therefore$ Position vector of required bisector

$= \frac{6(2\hat{i}+5\hat{j}+7\hat{k})+3(2\hat{i}+3\hat{j}+4\hat{k})}{6+3} = \frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

48. $\left[\frac{1}{2}|\vec{u}_2 - \vec{u}_1|\right]^{-2} = \frac{1}{4}[|\vec{u}_2|^2 + |\vec{u}_1|^2 - 2\vec{u}_2 \cdot \vec{u}_1]$

$= \frac{1}{4}[1 + 1 - 2|\vec{u}_2||\vec{u}_1|\cos\theta] = \frac{1}{4}[2 - 2\cos\theta] = \sin^2\frac{\theta}{2}$

$\Rightarrow \frac{1}{2}|\vec{u}_2 - \vec{u}_1| = \sin\frac{\theta}{2}$

49. Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors. Therefore,

$[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$

$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \Rightarrow \Delta \neq 0$, where $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Now,

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$\Rightarrow \Delta(1 + abc) = 0 \Rightarrow abc = -1 \qquad [\because \Delta \neq 0]$

50. Given vectors are coplanar, if $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 0 & c & \gamma \end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$

$$\Rightarrow \begin{vmatrix} \alpha & 1-\alpha & 0 \\ 1 & \beta-1 & 1-\beta \\ 1 & 0 & \gamma-1 \end{vmatrix} = 0 \Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\begin{vmatrix} \frac{\alpha}{1-\alpha} & 1 & 0 \\ \frac{1}{1-\beta} & -1 & 1 \\ \frac{1}{1-\gamma} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\left[\frac{\alpha}{1-\alpha}(1) - 1\left(-\frac{1}{1-\beta} - \frac{1}{1-\gamma}\right)\right] = 0$$

But $\alpha \neq 1$, $\beta \neq 1$ and $\gamma \neq 1$

$\therefore \frac{1}{(1-\alpha)} - 1 + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = 0 \Rightarrow \frac{1}{1-\alpha} + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = 1$