

CHAPTER 04

Indices and Surds

The term indices refer to the power to which a number is raised. Thus, n^2 is a number with an index of 2. It prefers the phrase ' n ' to the power of ' 2 '. Term surd is not often used, instead roots are used, occasionally you will be asked to give an answer in surd form, this implies that you should provide the answer in terms of constant and square roots instead of working out an imprecise decimal approximation.

Indices

Let a be a real number and m be a positive integer, then $a \times a \times a \times a \times \dots$ m times $= a^m$ where m is known as indices to the base a .

Rules of Indices

Let a and b be two real numbers and m and n be two positive integers, then

- $a^m \times a^n = a^{m+n}$. $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{m \times n}$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $(ab)^m = a^m \times b^m$
- $a^{-m} = \frac{1}{a^m}$
- $(a)^0 = 1$

e.g. If $4^a = 5$, $5^b = 6$, $6^c = 7$, $7^d = 8$, then find the value of $abcd$.

Sol. We have,

$$\begin{aligned}4^a &= 5 \\5^b &= 6 \\6^c &= 7 \\7^d &= 8\end{aligned}$$

On putting value of 5 from Eq. (i) in Eq. (ii), we get

$$\begin{aligned}
 & (4^a)^b = 6 \\
 \Rightarrow & 4^{ab} = 6 \\
 \text{Likewise } & (4^{ab})^c = 7 \\
 \Rightarrow & 4^{abc} = 7 \\
 \text{and } & (4^{abc})^d = 8 \Rightarrow 4^{abcd} = 8 \\
 \Rightarrow & (2^2)^{abcd} = 2^3 \\
 \Rightarrow & 2^{2abcd} = 2^3
 \end{aligned}$$

Since, there exist equality between them with same base, then their power must be equal.

$$2abcd = 3 \Rightarrow abcd = \frac{3}{2} = 15$$

Surds

Let a is a rational number and m is a positive integer then if $\frac{1}{a^m}$ be a irrational number, then $\frac{1}{a^m}$ is known as surds of power m . i.e. $\frac{1}{a^m} = \sqrt[m]{a} = m$ th root of a .

Rules of Surds

Let a is a rational number and m and n are two positive integers, then

- $(\sqrt[n]{a})^n = a$
- $\sqrt[m]{ab} = \sqrt[m]{a} \cdot \sqrt[m]{b} \cdot \sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$
- $(\sqrt[m]{a})^n = \sqrt[m]{a^n} \cdot \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Similarity

If the product of two surds is rational, then each of the two surds is called a rationalizing factor of the other.

- Rationalizing factor of $\frac{1}{\sqrt{a}} = \sqrt{a}$
- Rationalizing factor of $\frac{1}{a \pm \sqrt{b}} = a \mp \sqrt{b}$

- Rationalizing factor of $\frac{1}{\sqrt{a} \pm \sqrt{b}} = \sqrt{a} \mp \sqrt{b}$

Addition and Subtraction of Surds

For addition and subtraction of surds, first make the possible factors of the terms, then add or subtract the equivalent surds.

e.g. Find the value of $\sqrt{80} + 3\sqrt{245} - \sqrt{125}$

Sol. $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

$$3\sqrt{245} = 3\sqrt{49 \times 5} = 21\sqrt{5}$$

$$\sqrt{125} = \sqrt{25 \times 5} = 5\sqrt{5}$$

$$\therefore \sqrt{80} + 3\sqrt{245} - \sqrt{125} = 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5} = 20\sqrt{5}$$

Multiplication and Division of Surds

For multiplication and division of surds, first make the denominates of the powers, same as we did while arranging them in increasing or decreasing order (or in comparison). Then, multiply or divide as usual.

e.g. Find the product of $\sqrt{5}$, $\sqrt[6]{6}$ and $\sqrt[3]{4}$.

Sol. LCM of 2, 6 and 3 = 6

$$\therefore \sqrt{5} = 5^{1/2} = 5^{3/6} = (125)^{1/6}, \sqrt[6]{6} = 6^{1/6} = (6)^{1/6}$$

$$\sqrt[3]{4} = 4^{1/3} = 4^{2/6} = (16)^{1/6}$$

$$\therefore \text{Required product} = (125 \times 6 \times 16)^{1/6} = (12000)^{1/6}$$

e.g., Divide $12 \times 4^{1/3}$ by $3\sqrt{2}$.

Sol. $\frac{12 \times 4^{1/3}}{3 \times 2^{1/2}} = \frac{4 \times 4^{2/6}}{1 \times 2^{3/6}} = \frac{4 \times (16)^{1/6}}{(8)^{1/6}}$

$$= 4 \left(\frac{16}{8}\right)^{1/6} = 4(2)^{1/6} = 4\sqrt[6]{2}$$

Solved Examples:

1. If $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$, then x is equal to

- (a) -2
- (b) 2

- (c) -1
(d) 1

Sol. (c) $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$

$$\Rightarrow \left(\frac{3}{5}\right)^{3-6} = \left(\frac{3}{5}\right)^{2x-1}$$

$$\Rightarrow 2x - 1 = -3 \Rightarrow 2x = -3 + 1$$

$$\therefore x = \frac{-2}{2} = -1$$

2. The value of the expression $\frac{15^3 \times 21^2}{35^2 \times 3^4}$ will be

- (a) 2
(b) 0
(c) 1
(d) 3

Sol. (c) $\frac{15^3 \times 21^2}{35^2 \times 3^4} = \frac{(5 \times 3)^3 \times (7 \times 3)^2}{(5 \times 7)^2 \times 3^4}$

$$= \frac{5^3 \times 3^3 \times 7^2 \times 3^2}{5^2 \times 7^2 \times 3^4} = \frac{3^4}{3^4} = 1$$

3. If $\left[3^{m^2} \div (3^m)^2\right]^{\frac{1}{m}} = 81$, then find the value of m .

- (a) 0,6
(b) 2,4
(c) 1,4
(d) 5,1

Sol. (a) $\left[\frac{3^{m^2}}{3^{2m}}\right]^{\frac{1}{m}} = 81$

$$\Rightarrow 3^{m^2-2m} = 81^m$$

$$\Rightarrow 3^{m^2-2m} = 3^{4m}$$

$$\Rightarrow m^2 - 2m - 4m = 0 \Rightarrow m(m - 6) = 0$$

$$\Rightarrow m = 0,6$$

4. If $\sqrt{5 + \sqrt[3]{x}} = 3$, then find the value of x .

- (a) 216
- (b) 64
- (c) 125
- (d) 27

Sol. (b) $\sqrt{5 + \sqrt[3]{x}} = 3$

$$\Rightarrow 5 + \sqrt[3]{x} = 9$$

$$\Rightarrow \sqrt[3]{x} = 4 \Rightarrow x = (4)^3 = 64 \Rightarrow x = 64$$

5. Simplify $\left[\sqrt[3]{\sqrt[6]{2^9}}\right]^4 \times \left[\sqrt[6]{\sqrt[3]{2^9}}\right]^4$.

- (a) 2^{15}
- (b) 2^{10}
- (c) 2^{12}
- (d) 2^{16}

Sol. (d) $\left[\sqrt[3]{\sqrt[6]{2^9}}\right]^4 \times \left[\sqrt[6]{\sqrt[3]{2^9}}\right]^4$

$$= \left[\left(\sqrt[6]{2^9}\right)^{\frac{1}{3}}\right]^4 \times \left[\left(\sqrt[3]{2^9}\right)^{\frac{1}{6}}\right]^4 = \left[(2^9)^{\frac{1}{18}}\right]^4 \times \left[(2^9)^{\frac{1}{18}}\right]^4$$
$$= (2^2)^4 \times (2^2)^4 = (2^2 \times 2^2)^4 = 2^{16}$$

Practice Questions

1. What should come in place of the question mark (?) in the following equation?

$$31^{7.5} \div 31^{3/2} \times 31^{-3} = (\sqrt{31})^?$$

- (a) $9/2$
- (b) 6
- (c) $7/2$
- (d) 4

2. $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$ is equal to

- (a) 5
- (b) 3
- (c) 1
- (d) 0

3. In the equation $(x/21) \times (x/189) = 1$, which of the following number will replace both the x ?

- (a) 21
- (b) 63
- (c) 3969
- (d) None of these

4. The number of boys raised ₹ 400 for a relief fund, each boy giving as many 25 paise coin as there were boys. The number of boys was

- (a) 40
- (b) 16
- (c) 20
- (d) 100

5. If 6440 soldiers were asked to stand in rows to form a perfect square, it was found that 40 soldiers were left out. What was the number of soldiers in each row?

- (a) 40
- (b) 80
- (c) 64
- (d) 60

6. The value of $\sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + \sqrt{49}}}}}$ is

- (a) 3
- (b) 9
- (c) 7
- (d) 5

7. Each member of picnic party contributed twice as many rupees as the total number of members and the total collection was ₹ 3042. The number of members present in the party was
- (a) 2
(b) 32
(c) 40
(d) 39
8. If cube root of 175616 is 56 , then the value of $\sqrt[3]{175.616} + \sqrt[3]{0.175616} + \sqrt[3]{0.000175616}$ is equal to
- (a) 0.168
(b) 62.16
(c) 6.216
(d) 6.116
9. The value of $\frac{3 \cdot 9^{n+1} + 9 \cdot 3^{2n-1}}{9 \cdot 3^{2n} - 6 \cdot 9^{n-1}}$ is
- (a) $3\frac{3}{5}$
(b) $3\frac{2}{5}$
(c) $3n + 1$
(d) $3n - 16$
10. The least number which is a perfect square and has 7936 as one of its factors is equal to
- (a) 12.008
(b) 246016
(c) 61504
(d) 240616
11. Each student of class 10 contributed some money for a picnic. The money contributed by each student was equal to the cube of the total number of students. If the total collected amount was ₹ 29791, find the total number of students.
- (a) 15
(b) 27
(c) 31
(d) 34

12. $17^{3.5} \times 17^{7.3} \div 17^{4.2} = 17^2$, then find the value of (?).

- (a) 6.5
- (b) 7.2
- (c) 6.6
- (d) 15.8

13. If $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = 2^x$, then x is equal to

- (a) 2
- (b) 0
- (c) 3
- (d) 1

14. $(-\frac{1}{343})^{-\frac{2}{3}}$ is equal to

- (a) $-\frac{1}{49}$
- (b) $\frac{1}{49}$
- (c) -49
- (d) 49

15. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{x+2}$, then the value of x is

- (a) 4
- (b) 5
- (c) -3
- (d) -6

16. Which is greater $\sqrt[3]{4}$, $\sqrt[3]{6}$, $\sqrt[6]{15}$, $\sqrt[12]{245}$?

- (a) $\sqrt[3]{4}$
- (b) $\sqrt[3]{6}$
- (c) $\sqrt[6]{15}$
- (d) $\sqrt[12]{245}$

17. If $x = 7 - 4\sqrt{3}$, then the value of $(x + \frac{1}{x})$ is

- (a) $3\sqrt{3}$
- (b) $8\sqrt{3}$
- (c) $14 + 8\sqrt{3}$
- (d) 14

18. $(16)^{0.16} \times (16)^{0.04} \times (2)^{0.2}$ is equal to

- (a) 1
- (b) 2
- (c) 4
- (d) 16

19. Given $\sqrt{2} = 1.414$, then the value of $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$ is

- (a) 8.484
- (b) 8.526
- (c) 8.426
- (d) 8.876

20. The value of $\left(2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2}\right)$ is

- (a) 2
- (b) $2 - \sqrt{2}$
- (c) $2 + \sqrt{2}$
- (d) $2\sqrt{2}$

21. Simplify $\frac{(6.25)^{\frac{1}{2}} \times (0.0144)^{\frac{1}{2}} + 1}{(0.027)^{\frac{1}{3}} \times (81)^{\frac{1}{4}}}$

- (a) 0.14
- (b) 1.4
- (c) 1
- (d) $1.\bar{4}$

22. If $\frac{(x^3)^2 \times x^4}{x^{10}} = x^p$, then the value of p is

- (a) 26
- (b) 2
- (c) 1
- (d) 0

23. The value of x is $25^{7.5} \times 5^{2.5} \div 125^{1.5} = 5^x$

- (a) 16
- (b) 17.5
- (c) 8.5
- (d) 13

24. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then $n = ?$

- (a) 3
- (b) 2
- (c) 1
- (d) 0

25. Which of the two is greater 2^{300} or 3^{200} ?

- (a) 3^{200}
- (b) 2^{300}
- (c) Both are equal
- (d) Can't say

26. If $A = 5 + 2\sqrt{6}$, then the value of $\sqrt{A} + \frac{1}{\sqrt{A}}$ is

- (a) $2\sqrt{3}$
- (b) $\sqrt{3}$
- (c) $\sqrt{2}$
- (d) 7

27. The value of $\left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\right) + \left(\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)$ is

- (a) $\sqrt{5} + \sqrt{6}$
- (c) $\sqrt{5} - \sqrt{6}$
- (b) $2\sqrt{5} + \sqrt{6}$
- (d) $2\sqrt{5} - 3\sqrt{6}$

ANSWERS

1.	(b)	2.	(a)	3.	(b)	4.	(a)	5.	(b)	6.	(a)	7.	(d)	8.	(c)	9.	(a)	10.	(b)
11.	(c)	12.	(c)	13.	(d)	14.	(d)	15.	(a)	16.	(b)	17.	(d)	18.	(b)	19.	(a)	20.	(a)
21.	(d)	22.	(d)	23.	(d)	24.	(a)	25.	(a)	26.	(a)	27.	(c)						

Hints & Solutions

$$\begin{aligned} 1. \quad & 31^{7.5} \div 31^{3/2} \times 31^{-3} = (\sqrt{31})^? \\ & \Rightarrow 31^{7.5} \div 31^{1.5} \times 31^{-3} = (\sqrt{31})^? \\ & \Rightarrow 31^{7.5-1.5-3} = (\sqrt{31})^? \\ & \Rightarrow 31^3 = (31)^{\frac{?}{2}} \Rightarrow \frac{?}{2} = 3 \Rightarrow ? = 6 \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{1}{3-\sqrt{8}} = \frac{1}{\sqrt{9}-\sqrt{8}} \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = (\sqrt{9} + \sqrt{8}) \\ & \therefore \frac{1}{-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} \\ & = (3 + \sqrt{8}) - (\sqrt{8} + \sqrt{7}) + (\sqrt{7} + \sqrt{6}) - (\sqrt{6} + \sqrt{5}) + (\sqrt{5} + 2) \end{aligned}$$

$$\begin{aligned} 3. \quad & \therefore \frac{x}{21} \times \frac{x}{189} = 1 \\ & \Rightarrow x^2 = 21 \times 189 \\ & \quad \quad \quad x = \sqrt{21 \times 21 \times 3 \times 3} = 21 \times 3 = 63 \end{aligned}$$

$$\begin{aligned} 4. \quad & \text{Let the number of boys} = x \\ & \text{According to question,} \\ & \frac{25}{100} x^2 = 400 \Rightarrow x^2 = 1600 \Rightarrow x = 40 \end{aligned}$$

$$\begin{aligned} 5. \quad & \text{Total number of soldiers} = 6440 \text{ and number of left soldiers} = 40 \therefore \text{Number of} \\ & \text{soldiers in square} = 6440 - 40 = 6400 \\ & \therefore \text{Number of soldiers in a row} = \sqrt{6400} = 80 \end{aligned}$$

$$6. \sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + 7}}}}$$

$$\begin{aligned}
 &= \sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{36}}}} \\
 &= \sqrt{5 + \sqrt{11 + \sqrt{19 + 6}}} \\
 &= \sqrt{5 + \sqrt{11 + 25}} \\
 &= \sqrt{5 + \sqrt{11 + 5}} = \sqrt{5 + \sqrt{16}} \\
 &= \sqrt{5 + 4} = \sqrt{9} = 3
 \end{aligned}$$

7. Let the number of members = x According to question,

$$\begin{aligned}
 &2x^2 = 3042 \\
 \Rightarrow &x^2 = 1521 \\
 \Rightarrow &x = \sqrt{39 \times 39} = 39
 \end{aligned}$$

$$8. \because \sqrt[3]{175616} = 56$$

Now,

$$\begin{aligned}
 &\sqrt[3]{175.616} + \sqrt[3]{0.175616} \\
 &\quad + \sqrt[3]{0.000175616} \\
 &= \sqrt[3]{\frac{175616}{1000}} + \sqrt[3]{\frac{175616}{1000000}} \\
 &\quad + \sqrt[3]{\frac{175616}{1000000000}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{56}{10} + \frac{56}{100} + \frac{56}{1000} \\
 &= 5.6 + 0.56 + 0.056 = 6.216
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \frac{3 \cdot 9^{n+1} + 9 \cdot 3^{2n-1}}{9 \cdot 3^{2n} - 6 \cdot 9^{n-1}} \\
 &= \frac{3 \cdot 3^{2n+2} + 3^2 \cdot 3^{2n-1}}{3^2 \cdot 3^{2n} - 3 \times 3^{2n-2} \times 2} \\
 &= \frac{3^{2n+3} + 3^{2n+1}}{3^{2n+2} - 2 \cdot 3^{2n-1}} = \frac{3^{2n+1}[9 + 1]}{3^{2n-1}[27 - 2]} \\
 &= \frac{3^2 \times 10}{25} = \frac{18}{5} = 3\frac{3}{5}
 \end{aligned}$$

11. Total collected amount = ₹29791 \therefore Total number of students

$$\begin{aligned}
 &= \sqrt[3]{29791} \\
 &= \sqrt[3]{31 \times 31 \times 31} = 31
 \end{aligned}$$

12. $17^{3.5} \times 17^{7.3} \div 17^{4.2} = 17^?$

$$\Rightarrow \frac{17^{3.5+7.3}}{17^{4.2}} = 17^?$$

$$\Rightarrow 17^{10.8-4.2} = 17^?$$

$$\Rightarrow 17^{6.6} = 17^?$$

Since, base are same.

$$\therefore ? = 6.6$$

13. Given, $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = 2^x$

$$\Rightarrow \sqrt{2}^{\sqrt{2} \times \sqrt{2}} = 2^x$$

$$\Rightarrow (\sqrt{2})^2 = 2^x$$

$$\Rightarrow 2^{\frac{1}{2} \times 2} = 2^x$$

$$\Rightarrow 2^1 = 2^x \Rightarrow x = 1$$

14. $(-\frac{1}{343})^{-\frac{2}{3}} = (-343)^{\frac{2}{3}}$

$$= (-7^3)^{2/3} = (-7)^2 = 49$$

$$15. 5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{x+2}$$

$$\Rightarrow \frac{5 \times 5^{\frac{1}{2}} \times 5^3}{5^{-\frac{3}{2}}} = 5^{x+2}$$

$$\Rightarrow 5 \times 5^{\frac{1}{2}} \times 5^3 \times 5^{\frac{3}{2}} = 5^{x+2}$$

$$\Rightarrow 5^{1+\frac{1}{2}+3+\frac{3}{2}} = 5^{x+2}$$

$$\Rightarrow 5^{4+2} = 5^{x+2}$$

$$\Rightarrow x + 2 = 6$$

$$\therefore x = 4$$

$$16. \sqrt[3]{4}, \sqrt[3]{6}, \sqrt[6]{15}, \sqrt[12]{245}$$

$$\text{LCM of } 3, 3, 6, 12 = 12$$

$$\therefore \sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$$

$$\sqrt[3]{6} = \sqrt[12]{6^4} = \sqrt[12]{1296}$$

$$\sqrt[6]{15} = \sqrt[12]{15^2} = \sqrt[12]{225}$$

$$\sqrt[12]{245} = \sqrt[12]{245} = \sqrt[12]{245}$$

Clearly, $\sqrt[3]{6}$ is greater

$$17. x = 7 - 4\sqrt{3}$$

$$\therefore \frac{1}{x} = 7 + 4\sqrt{3} \text{ (Conjugate)}$$

$$\therefore x + \frac{1}{x} = 7 - 4\sqrt{3} + 7 + 4\sqrt{3} = 14$$

$$18. (16)^{0.16} \times (16)^{0.04} \times (2)^{0.2}$$

$$= (2)^{0.64} \times (2)^{0.16} \times (2)^{0.20}$$

$$= (2)^{1.00} = 2$$

$$19. \sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$$

$$= 2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2}$$

$$= 6\sqrt{2}$$

$$= 6 \times 1.414 = 8.484$$

$$\begin{aligned} 20. & 2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2} \\ &= 2 + \sqrt{2} + \frac{2-\sqrt{2}}{2} + \frac{2+\sqrt{2}}{-2} \\ &= \frac{4 + 2\sqrt{2} + 2 - \sqrt{2} - 2 - \sqrt{2}}{2} \\ &= \frac{4}{2} = 2 \end{aligned}$$

$$\begin{aligned} 21. & \frac{(6.25)^2 \times (0.0144)^2 + 1}{1} \\ & (0.027)^3 \times (81)^4 \\ &= \frac{2.5 \times 0.12 + 1}{0.3 \times 3} = \frac{1.3}{0.9} = \frac{13}{9} \\ &= \frac{13}{9} = 1.\bar{4} \end{aligned}$$

$$\begin{aligned} 22. & \frac{(x^3)^2 \times x^4}{x^{10}} = x^p \Rightarrow \frac{x^{6+4}}{x^{10}} = x^p \\ & \Rightarrow x^p = 1 = x^0 \\ & \Rightarrow x^p = x^0 \Rightarrow p = 0 \end{aligned}$$

$$\begin{aligned} 23. & 25^{7.5} \times 5^{25} \div 125^{1.5} = 5^x \\ & \Rightarrow 5^{15} \times 5^{25} \div 5^{4.5} = 5^x \\ & \Rightarrow 5^{15} \times \frac{1}{5^2} = 5^x \\ & \Rightarrow 5^x = 5^{13} \Rightarrow x = 13 \end{aligned}$$

$$\begin{aligned} 24. & \frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27 \\ & \Rightarrow \frac{(3^2)^n \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3 \end{aligned}$$

$$\Rightarrow \frac{3^{2n} \times 3^5 \times 3^9}{3 \times 3^{16}} = 3^3$$

$$\Rightarrow \frac{3^{2n} \times 3^{14}}{3^{17}} = 3^3$$

$$\Rightarrow 3^{2n} \times 3^{14} = 3^{17} \times 3^3 \Rightarrow 3^{2n} = \frac{3^{20}}{3^{14}}$$

$$\Rightarrow 3^{2n} = 3^6 \Rightarrow 2n = 6 \Rightarrow n = 3$$

25. In these types of questions, either the base or the exponent is made same.

$$2^{300} = (2^3)^{100} = 8^{100}$$

$$3^{200} = (3^2)^{100} = 9^{100}$$

Now, clearly $9^{100} > 8^{100}$

So, $3^{200} > 2^{300}$

$$26. A = 5 + 2\sqrt{6} = \sqrt{(\sqrt{2} + \sqrt{3})^2}$$

$$= \sqrt{3} + \sqrt{2}$$

$$\therefore \frac{1}{A} = \sqrt{3} - \sqrt{2}$$

$$\text{So, } \sqrt{A} + \frac{1}{\sqrt{A}}$$

$$= \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}$$

$$= 2\sqrt{3}$$

$$27. \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$\frac{\sqrt{5} - \sqrt{3} + \sqrt{10} - \sqrt{6} + \sqrt{5} - \sqrt{10}}{+\sqrt{3} - \sqrt{6}}$$

$$= \frac{2\sqrt{5} - 2\sqrt{6}}{2} = \sqrt{5} - \sqrt{6}$$