

CHAPTER 02

HCF and LCM

HCF of any given set of number is the greatest factor common to them and LCM of two or more numbers is the smallest number which is common multiple of the given number.

HCF (Highest Common Factor)

A common factor of two or more numbers is a number which divides each of them exactly.

e.g., 2 is a common factor of 18 and 24.

It is also known as GCD (Greatest Common Divisor) or GCM (Greatest Common Multiple)

Methods of Finding HCF

- Factorization Method Break the given numbers into prime factors and then find the product of common prime factors with least powers gives the HCF.

e.g., Find the HCF of 24, 45 and 60.

Sol.

$$24 = 2 \times 2 \times 2 \times 3$$

$$45 = 3 \times 3 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$\text{HCF of 24, 45, 60} = 3$$

- Division Method Divide the larger number by smaller number and then divide the divisor by the remainder and repeat this process till the remainder becomes zero and in the last divisor is the require HCF.

e.g., Find the HCF of 13281 and 15844.

$$\begin{array}{r}
 13281 \overline{)15844} (1 \\
 \underline{13281} \\
 2563 \overline{)13281} (5 \\
 \underline{12815} \\
 466 \overline{)2563} (5 \\
 \underline{2330} \\
 233 \overline{)466} (2 \\
 \underline{466} \\
 \times
 \end{array}$$

∴ HCF = 233

If the successive divisions continue until remainder of 1 is obtained this means that the original numbers have no common divisor.

LCM (Least Common Multiple)

The least number which is exactly divisible by each one of the given numbers is called their LCM.

Methods of Finding LCM

- Factorization Method Break the given numbers into their prime factors and then find the product of highest powers of all the factors, which occur in the given number and this product is the required LCM.

e.g., Find the LCM of 24, 45 and 60.

$$\text{Sol. } 24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$$45 = 3 \times 3 \times 5 = 3^2 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$$

The product of the divisors and the undivided numbers is the required LCM.

- Division Method Write down the given numbers in a line, separating them by commas and then divide by anyone of the prime numbers which exactly divides two of the given numbers. Now, write down the quotients and the undivided numbers in the line below the first and repeat this process until you get a line of numbers which are prime to one-another. In the last the product of all the divisors and the numbers in the last line is the required LCM.

e.g., Find the LCM of 24, 45 and 60.

Sol.

2	12, 30, 40
2	6, 15, 20
3	3, 15, 10
5	1, 5, 10
2	1, 1, 2
	1, 1, 1

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

Important Tips/Formulae

- Product of two numbers = (their HCF) \times (their LCM)
- **Co-prime** Two numbers are coprime if their HCF is 1.
 - (i) $\text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$
 - (ii) $\text{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$
- The greatest number that will divide x, y, z leaving remainders a, b and c respectively is given by HCF of $(x - a), (y - b)$ and $(z - c)$.
- The least number which when divided by x, y and z leaves the same remainder R in each case is given by LCM of $[(x, y, z) + R]$.
- The greatest number that will divide x, y and z leaving the same remainder in each case is given by HCF of $|x - y|, |y - z|$ and $|z - x|$.

Solved Examples:

1. Among how many children may 429 mangoes and also 715 oranges be equally divided?

- (a) 143
- (b) 123
- (c) 152
- (d) 160

Sol. (a) HCF (429, 715)

$$\begin{array}{r} 426 \) \ 715 \ (\ 1 \\ \underline{429} \\ 286 \) \ 429 \ (\ 1 \\ \underline{286} \\ 143 \) \ 286 \ (\ 2 \\ \underline{286} \\ \times \end{array}$$

∴ The number of children must be 143.

2. Find the HCF of $\frac{35}{12}$, $\frac{49}{30}$ and $\frac{21}{40}$.

- (a) $\frac{4}{38}$
- (b) $\frac{7}{120}$
- (c) $\frac{5}{132}$
- (d) $\frac{7}{142}$

Sol. (b) HCF of $\frac{35}{12}$, $\frac{49}{30}$ and $\frac{21}{40} = \frac{\text{HCF of } 35, 49 \text{ and } 21}{\text{LCM of } 12, 30 \text{ and } 40}$

Now, LCM

$$\begin{array}{r} 2 \overline{) 12, 30, 40} \\ \underline{2 \ 6, 15, 20} \\ 3 \ 3, 15, 10 \\ \underline{5 \ 1, 5, 10} \\ 2 \ \underline{1, 1, 2} \\ \quad \underline{1, 1, 1} \end{array}$$

$$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 2 = 120$$

$$\text{HCF} = 35 = 5 \times 7$$

$$49 = 7 \times 7 \quad 21 = 3 \times 7$$

$$\therefore \text{HCF} = 7$$

$$\text{Required HCF} = \frac{7}{120}$$

3. Two numbers are in the ratio 5:7. If their HCF is 4, find the numbers.

- (a) 15,21
- (b) 20,28
- (c) 25,35
- (d) 30,42

Sol. (b) Let the required numbers be $5x$ and $7x$. Then, their HCF is x .

So, $x = 4$

∴ The numbers are (5×4) and (7×4) i.e. 20 and 28.

4. Find the least number which when divided by 16, 24 and 36 leaves the remainders 7, 25 and 27 respectively.

- (a) 140
- (b) 125
- (c) 130
- (d) 135

Sol. (d) Here $16 - 7, 24 - 25, 36 - 27 = 9, 9, 9$

Required number = LCM of (16,24,36) – 9

$$= 144 - 9 = 135$$

2	16,	24,	36
2	8,	12,	18
3	4,	6,	9
2	4,	2,	3
2	2,	1,	3
3	1,	1,	3
	1,	1,	1

Practice Questions

1. HCF of the reciprocals of the fractions $\frac{3}{4}, \frac{9}{10}$ and $\frac{15}{16}$ is

- (a) $\frac{3}{15}$
- (b) $\frac{3}{80}$
- (c) $\frac{1}{15}$
- (d) $\frac{2}{45}$

2. The LCM of two numbers is 48. The numbers are in the ratio 2: 3. The sum of the numbers is

- (a) 28
- (b) 32
- (c) 40
- (d) 64

3. Which is the smallest number of five digits which is divided by 41?

- (a) 10045
- (b) 10004
- (c) 10041
- (d) 10025

4. Which of the following number has the highest divisor?
- (a) 99
 - (b) 101
 - (c) 176
 - (d) 182
5. In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?
- (a) 12220 cm
 - (b) 12230 cm
 - (c) 12240 cm
 - (d) 12250 cm
6. Mayank, Manoj and Ankit begin to jog around a circular stadium. They complete their revolutions in 425, 565 and 635, respectively. After how many seconds will they be together at the starting point?
- (a) 520 s
 - (b) 504 s
 - (c) 380 s
 - (d) 480 s
7. The HCF of 1056, 1584, 2178 is
- (a) 66
 - (b) 56
 - (c) 62
 - (d) 64
8. The LCM of 22, 54, 108, 135 is
- (a) 5940
 - (b) 5490
 - (c) 5405
 - (d) 5095

9. The HCF of $\frac{5}{6}, \frac{10}{18}, \frac{25}{36}$ is

- (a) $\frac{5}{36}$
- (b) $\frac{25}{6}$
- (c) $\frac{25}{36}$
- (d) $\frac{5}{18}$

10. The LCM of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$ is

- (a) $\frac{1}{54}$
- (b) $\frac{10}{27}$
- (c) $\frac{20}{3}$
- (d) None of these

11. Find the ratio between LCM and HCF of 5, 15 and 20.

- (a) 8: 1
- (b) 14: 3
- (c) 12: 2
- (d) 12: 1

12. The HCF and LCM of two numbers are 18 and 3780 respectively. If one of them is 540, then the second one is

- (a) 142
- (b) 126
- (c) 118
- (d) 112

13. The least number of square tiles required to pave the ceiling of a room 15 m 17 cm long and 9 m 2 cm broad is

- (a) 656
- (b) 814
- (c) 902
- (d) 738

14. What is the largest number which can divide 1356, 1868 and 2764 leaving 12 as remainder in each case?

- (a) 64
- (c) 156
- (b) 124
- (d) 260

15. If the LCM of three numbers is 9570, then their HCF will be

- (a) 11
- (b) 12
- (c) 19
- (d) 21

16. The product of the LCM and HCF of two numbers is 24. The difference of the two numbers is 2. Find the numbers.

- (a) 8 and 6
- (b) 8 and 10
- (c) 12 and 4
- (d) 6 and 4

17. Find out the HCF of 3^8 , 3^5 , 3^9 and 3^{14}

- (a) 3^{14}
- (b) 3^8
- (c) 3^5
- (d) 3^9

18. Find out of LCM of 4^5 , 4^{-18} , 4^{12} and 4^7

- (a) 4^{-81}
- (c) 4^7
- (b) 4^5
- (d) 4^{12}

19. HCF of ₹ 1.20, ₹ 3.40, ₹ 4.80

- (a) 80 paise
- (b) 60 paise
- (c) 40 paise
- (d) 20 paise

20. The HCF and LCM of two numbers are 21 and 4641 respectively. If one of the numbers lies between 200 and 300, the two numbers are
- (a) 273,357
 - (b) 210,340
 - (c) 215,314
 - (d) 210,252
21. Find the largest number which divides 62,132 and 237 to leave the same remainder in each case.
- (a) 21
 - (b) 30
 - (c) 35
 - (d) 40
22. Five bells first begin to toll together and then at intervals of 3, 5, 7, 8 and 10 s. Find after what interval they will again toll together. How many times does they toll together in one hour?
- (a) 14 min, 3 times
 - (b) 12 min, 4 times
 - (c) 14 min, 4 times
 - (d) 12 min, 3 times
23. The LCM of $(16 - x^2)$ and $(x^2 + x - 6)$ is
- (a) $(x - 3)(x + 3)(4 - x^2)$
 - (b) $4(4 - x^2)(x + 3)$
 - (c) $(4 - x^2)(x - 3)$
 - (d) $(16 - x^2)(x - 2)(x + 3)$
24. The LCM of two numbers is $(a + b)$ and their HCF is $P(a - b)$. If one of the numbers is P , then the other number is
- (a) $\frac{Pa}{b}$
 - (b) Pab
 - (c) $a^2 - b^2$
 - (d) $\frac{(a+b)}{P(a-b)}$

ANSWERS

1. (d)	2. (c)	3. (b)	4. (c)	5. (c)	6. (b)	7. (a)	8. (a)	9. (a)	10. (c)
11.(d)	12.(b)	13.(b)	14.(a)	15.(a)	16.(d)	17.(c)	18. (d)	19.(d)	20. (a)
21.(c)	22.(c)	23.(d)	24.(c)						

Hints & Solutions

1. Required HCF

$$= \text{HCF of } \frac{4}{3}, \frac{10}{9} \text{ and } \frac{16}{15} = \frac{\text{HCF of (4,10 and 16)}}{\text{LCM of (3,9 and 15)}} = \frac{2}{45}$$

2. Let the two numbers be $2x, 3x$ respectively.

$$\text{LCM of numbers} = 48$$

$$2 \times 3x = 48$$

$$x = \frac{48}{6} = 8$$

$$\therefore \text{Sum of two numbers} = 2 \times 8 + 3 \times 8$$

$$= 16 + 24 = 40$$

3. Smallest number of 5 digits = 10000

When 10000 is divided by 41, remainder = 37

$$\text{Hence, required number} = 10000 + (41 - 37) = 10004$$

$$4. 99 = 3 \times 3 \times 11$$

$$101 = 101$$

$$176 = 2 \times 2 \times 2 \times 2 \times 11$$

$$182 = 2 \times 7 \times 13$$

Hence, 176 has the highest divisors.

5. Minimum distance each should walk

= LCM of 80 cm, 85 cm, 90 cm

5	80, 85, 90
2	16, 17, 18
	8, 17, 9

Minimum distance = $5 \times 2 \times 8 \times 17 \times 9 = 12240$ cm

6. Required time = LCM of 42, 56 and 635

2	42, 56, 63
3	21, 28, 63
3	7, 28, 21
7	7, 28, 7
4	1, 4, 1
	1, 1, 1

\therefore Required time = $2 \times 3 \times 3 \times 7 \times 4 = 504$ s

8. The LCM of 22, 54, 108, 135

2	22, 54, 108, 135
3	11, 27, 54, 135
9	11, 9, 18, 45
	11, 1, 2, 5

= $2 \times 3 \times 9 \times 11 \times 5 \times 2 = 5940$

9. HCF of $\frac{5}{6}, \frac{10}{18}, \frac{25}{36}$

$$= \frac{\text{HCF of } 5, 10, 25}{\text{LCM of } 6, 18, 36} = \frac{5}{36}$$

10. The LCM of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$

$$= \frac{\text{LCM of } 1, 5, 2, 4}{\text{HCF of } 3, 6, 9, 27}$$

$$= \frac{20}{3}$$

11. LCM of 5, 15, 20 = 60
HCF of 5, 15, 20 = 5

The required ratio = $\frac{60}{5} = 12:1$

12. Product of 1st × 2nd number = HCF × LCM

⇒ $540 \times 2\text{nd number} = 18 \times 3780$

⇒ $2\text{nd number} = \frac{18 \times 3780}{540} = 126$

13. Side of each square tiles = HCF of 1517 cm and 902 cm = 41 cm

Required numbers of tiles = $\frac{1517 \times 902}{41 \times 41} = 814$

14. Required number = HCF of (1356 – 12),
(1868 – 12), (2764 – 12)

= HCF of (1344, 1856, 2752)

1856) 2752(1

1856

896) 1856(2

1792

64) 896(14

64

256

256

x

HCF of 2752 and 1856 is 64 .

Now, HCF of 64 and LCM 1344

$$\begin{array}{r} 64) 1344(21 \\ \underline{128} \\ 64 \\ \underline{64} \\ \times \\ \text{HCF} = 64 \end{array}$$

15. We know that LCM of the given numbers is always divisible by their HCF. So, 11 is the HCF

16. Let the numbers be x and y . Then, $x - y = 2$

$$\begin{aligned} \Rightarrow y &= x - 2 \\ \text{LCM} \times \text{HCF} &= x \times y = 24 \\ \Rightarrow x(x - 2) &= 24 \\ \Rightarrow x^2 - 2x - 24 &= 0 \\ \Rightarrow (x - 6)(x + 4) &= 0 \\ \Rightarrow x &= 6 \end{aligned}$$

The numbers are 6 and 4 respectively.

17. HCF = Highest common factor = 3^5

So, 3^5 the highest common factor in $3^8, 3^5, 3^9, 3^{14}$.

18. LCM - Least Common Factor So, 4^{12} is least common factor in $4^5, 4^{-81}, 4^{12}$ and 4.

19. ₹ 1.20 = 120 paise

₹ 3.40 = 340 paise

₹ 4.80 = 480 paise

So, HCF of 120, 340, 480

$$= 20 \times 6, 20 \times 17, 20 \times 24$$

HCF = 20 paise (highest common factor)

20. Since, the HCF of two numbers is 21, hence let the numbers be $21x$ and $21y$ respectively.

We know that product of two numbers = HCF \times LCM

$$21x \times 21y = 21 \times 4641$$

$$\therefore xy = 221$$

$$= (13 \times 17)$$

because x and y are co-prime.

Therefore, the numbers are $(21 \times 13, 21 \times 17) \Rightarrow (273, 357)$

21. Required number

$$= \text{HCF of } (132 - 62), (237 - 132) \text{ and } (237 - 62)$$

$$= \text{HCF of } 70, 105 \text{ and } 175 = 35$$

22. Required time interval

$$= \text{LCM of } 3, 5, 7, 8 \text{ and } 10$$

$$= 840 \text{ s}$$

$$= 14 \text{ min}$$

Number of times they will toll together in one hour = $\frac{60}{14} = 4$ times.

(ignoring the fraction part)

23. $(16 - x^2) = (4 - x)(4 + x)$

$$(x^2 + x - 6) = (x + 3)(x - 2) \text{ LCM}$$

of $(16 - x^2)$ and $(x^2 + x - 6)$

$$= (16 - x^2)(x - 2)(x + 3)$$