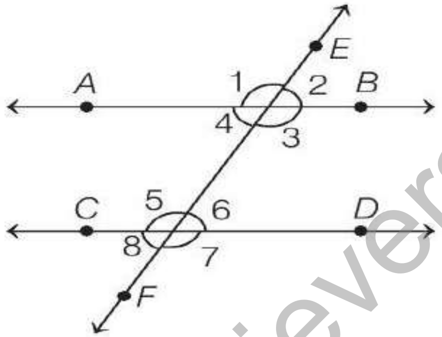
CHAPTER 17

Geometry

Plane Geometry

Plane geometry is about flat shapes like line, circle and triangle etc. These types of shapes can be easily drawn on a piece of paper.

1. Lines and Angles: Two lines in the same plane are said to be parallel, if they never meet. A line which cuts a pair of parallel line is called a transversal.



Here, two parallel lines AB and CD are cut by a transversal i.e., EF. Then,

- The corresponding angles are $\angle 1 = \angle 5, \angle 2 = \angle 6, \angle 4 = \angle 8$ and $\angle 3 = \angle 7$.
- The alternate angles are $\angle 1 = \angle 7, \angle 2 = \angle 8, \angle 3 = \angle 5$ and $\angle 4 = \angle 6$.

2. Triangles: A figure bounded by three straight lines is called a triangle. The small of all interior angles of a triangle is $180°$.

Types of Triangles: Equilateral Triangle A triangle having all sides equal is called an equilateral triangle and each angle equal to $60°$.

Scalene Triangle A triangle having all sides of different length is called a scalene triangle.

Isosceles Triangle A triangle having two sides equal is called an isosceles triangle.

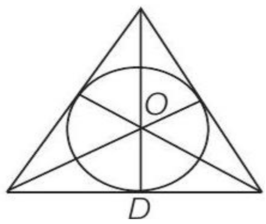
Right Angled Triangle A triangle one of whose angle measures $90°$ is called a right angled triangle.

Obtuse Angled Triangle A triangle one of whose angles lies between $90°$ and $180°$ is called an obtuse angled triangle.

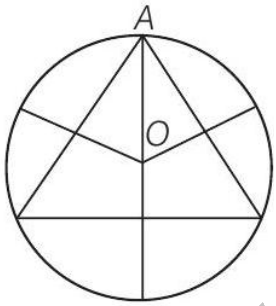
Acute Angled Triangle A triangle each of whose angle is less than $90°$ is called an acute angled triangle.

3. Circle: A circle is a set of points which are equidistant from a given point. The given point is known as the center of that circle.

$$\text{Inradius }(OD) = \frac{1}{3} \times \text{ Height } = \frac{\text{Side}}{2\sqrt{3}}$$



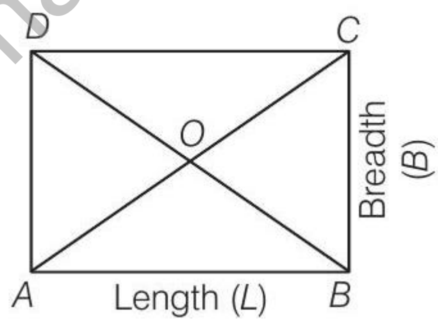
$$\text{Circumradius }(OA) = \frac{2}{3} \times \text{Height} = \frac{\text{Side}}{\sqrt{3}}$$



4. Quadrilateral: It is a plane figure bounded by four straight lines. The sum of the internal angles of a quadrilateral is equal to $360°$.

Parallelogram

A quadrilateral in which the opposite sides are equal and parallel, is called a parallelogram.



- The opposite angles are equal in magnitudes.
- The diagonals of a parallelogram are not equal in magnitudes, but they bisect each other.
- $AC \neq BD$ but $AO = OC$ and $OB = OD$.

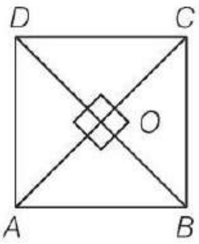
Rectangle

A parallelogram in which the adjacent sides are perpendicular to each other, is called a rectangle.

The diagonals of a rectangle are of equal magnitudes and bisect each other i.e., $AC = BD$ and $OA = OB = OC = OD$.

Square: A parallelogram in which all the sides are equal and perpendicular to each other, is called a square.

- The diagonals bisect each other at right angles and form four isosceles right angled triangles.



- The diagonals of a square are of equal mangnitudes i.e., $AC = BD$.

Polygon: Regular polygon A polygon in which all the sides are equal and also all the interior angles are equal, is called a regular polygon.

- Sum of all interior angles $= (n - 2) \times 180°$
$$= (2n - 4) \times 90°$$
- Each interior angle $= 180° -$ Exterior angle
- Each exterior angle $= \left(\frac{360°}{\text{Number of sides}}\right)$ (in degrees)
- Sum of all exterior angle $= 360°$ (always constant)
- Number of diagonals of polygon of n sides $= \frac{n(n-3)}{2}$

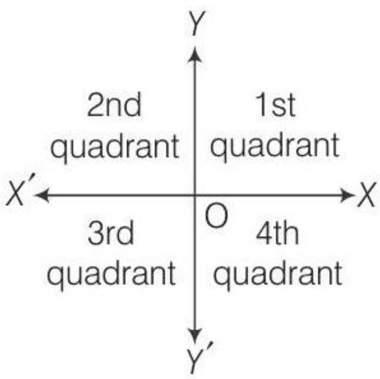
Coordinate Geometry

It is a system of geometry, where the position of points on the plane is described by using an ordered pair of numbers.

Quadrants

The X and Y-axes divide the cartesian plane into four regions referred as quadrants.

The table of sign conventions of coordinates in various quadrants is given below



Quadrant	Region	Sign of (x, y)	Example
I	XOY	$(+,+)$	$(2,3)$
II	YOX'	$(-,+)$	$(-2,4)$
III	$X'OY'$	$(-,-)$	$(-1,-2)$
IV	$Y'OX$	$(+,-)$	$(1,-3)$

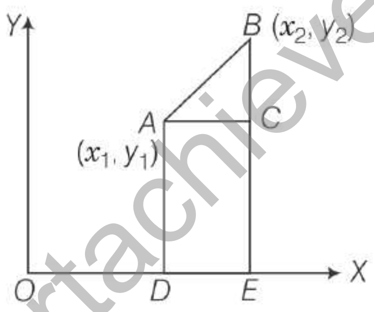
The coordinates of point O (origin) are taken as $(0,0)$.
The coordinates of any point on X-axis are of the form $(x, 0)$.

The coordinates of any point on Y-axis are of the form $(0, y)$.

Distance Formula

Distance between two points

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, then



$$\boldsymbol{AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

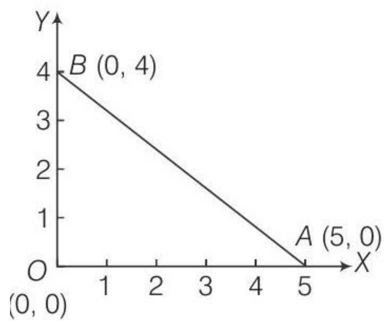
Example: The distance between $A(1,2)$ and $B(5,6)$ is

$$AB = \sqrt{(5-1)^2 + (6-2)^2} = \sqrt{4^2 + 4^2}$$
$$= \sqrt{32} = 4\sqrt{2} \text{ units}$$

Area of a Triangle

If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are three vertices of a $\triangle ABC$, then its area is given by

Area of $\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$



Example: If we have to find the area of a triangle having the vertices (0,0), (5,0) and (0,4), then

$\therefore$ Area of triangle $= \frac{1}{2}\{0(0-4)+5(4-0)+0(0-0)\}$

$$= \frac{1}{2} \times 20 = 10 \text{ sq units}$$

$\because (x_1, y_1) = (0,0), (x_2, y_2) = (5,0)$ and $(x_3, y_3) = (0,4)$

Collinearity of Three Points

Three points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, if

(i) Area of $\triangle ABC$ is 0 , i.e.

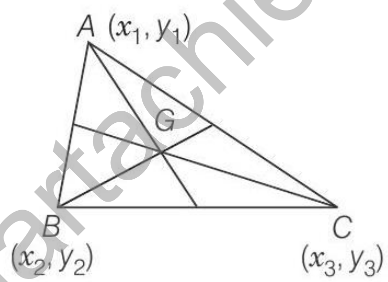
$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0.$$

(ii) Slope of $AB =$ Slope of $BC =$ Slope of AC
(iii) Distance between A and $B +$ Distance between B and $C =$ Distance between A and C

Centroid of a Triangle

Centroid is the point of intersection of all the three medians of a triangle.

If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of $\triangle ABC$, then the coordinates of its centroid are



$$\left[\frac{1}{3}(x_1+x_2+x_3), \frac{1}{3}(y_1+y_2+y_3)\right]$$

Section formulae

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points on the cartesian plane. Let point $P(x, y)$ divides the line AB in the ratio of m: n internally.

Then,

$$x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}$$

If P divides AB externally, then

$$x = \frac{mx_2-nx_1}{m-n}, y = \frac{my_2-ny_1}{m-n}$$

If P is the mid-point of AB, then

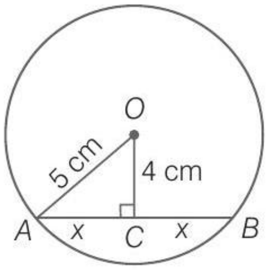
$$x = \frac{x_1+x_2}{2}, y = \frac{y_1+y_2}{2}$$

Solved Examples:

1. A chord AB is drawn in a circle with centre O and radius 5 cm. If the shortest distance between centre and chord is 4 cm, find the length of chord AB?

(a) 6 cm
(b) 5 cm
(c) 4 cm
(d) 3 cm

Sol. (a) In the adjoining figure $AO = 5$ cm (radius)



$OC = 4$ cm (shortest distance between centre and chord)

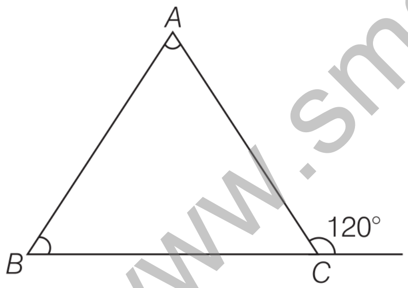
Let length of chord AB be $2x$, then $AC = x$.

In $\triangle AOB$, $AO^2 = AC^2 + OC^2$

$\Rightarrow (5)^2 = x^2 + (4)^2 \Rightarrow 25 = x^2 + 16$
$\therefore x = \sqrt{25-16} = \sqrt{9} = 3$ cm

$\therefore$ Length of chord $AB = 2x = 2 \times 3 = 6$ cm

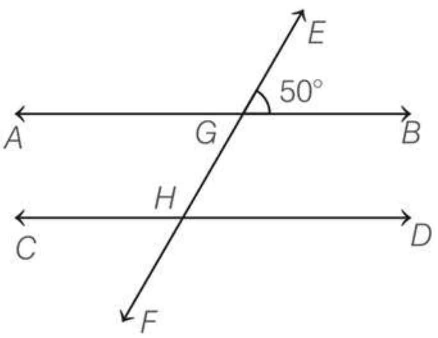
2. In the figure given, $\angle BAC : \angle ABC = 2:3$. Find the measure of $\angle ABC$.



(a) Obtuse angle
(b) Acute angle
(c) Right angle
(d) None of these

Sol. (b) Let $\angle A = 2x$ and $\angle B = 3x$
Then, $2x + 3x = 120°$
[Exterior angle is equal to the sum of the interior opposite
$\Rightarrow 5x = 120°$
$\Rightarrow x = 24°$
$\therefore \angle ABC = 3x = 3 \times 24° = 72°$

3. In the given figure, AB and CD are parallel lines.
If $\angle EGB = 50°$, find $\angle CHG$.



(a) 120°
(b) 130°
(c) 125°
(d) 140°

Sol. (b) $\angle AGH = \angle EGB$ [Vertically opposite angles]

$\angle AGH = 50°$

Now, $\angle AGH + \angle CHG = 180°$

[Interior angles on the same side of the transversal are supplementary.]

$\therefore 50° + \angle CHG = 180° \Rightarrow \angle CHG = 180° - 50° = 130°$.

4. An angle $\theta°$ is one-fourth of its supplementary angle. What is the measure of the angle $\theta°$?

(a) 36°
(b) 34°
(c) 32°
(d) 31°

Sol. (a) If the sum of two angles is 180°, the angles are said to be supplementary.

$\therefore$ The supplementary angle of $\theta°$ is $(180° - \theta°)$.

Given that, $\theta° = \frac{1}{4}(180° - \theta°)$

$\Rightarrow 4\theta° = 180° - \theta° \Rightarrow 5\theta° = 180°$

$\Rightarrow \theta° = \frac{180°}{5} = 36°$

5. The number of diagonals in a 27-sided polygon is

(a) 324
(b) 325
(c) 322
(d) 320

Sol. (a) Number of diagonals of polygon of n sides $= \frac{(n)(n-3)}{2}$

Number of diagonals of polygon of 27 sides $= \frac{27\times24}{2} = 324$

6. Find the distance between the points $A(-6,8)$ and $B(4,-8)$.

(a) 18.86 units
(b) 16.76 units
(c) 11.77 units
(d) 16.76 units

Sol. (a) Here, $A(-6,8) = A(x_1, y_1)$ and $B(4,-8) = B(x_2, y_2)$

So, $x_1 = -6, y_1 = 8,\ x_2 = 4$ and $y_2 = -8$

∴ Required distance,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\{4-(-6)\}^2 + (-8-8)^2}$$
$$= \sqrt{(4+6)^2 + (-16)^2} = \sqrt{100+256}$$
$$= \sqrt{356} \simeq 18.86 \text{ units}$$

7. Find the area of $\triangle ABC$, whose vertices are $A(8,-4), B(3,6)$ and $C(-2,4)$.

(a) 20 sq units
(b) 30 sq units
(c) 35 sq units
(d) 29 sq units

Sol. (b) Here, $A(8,-4)$, then $x_1 = 8, y_1 = -4$

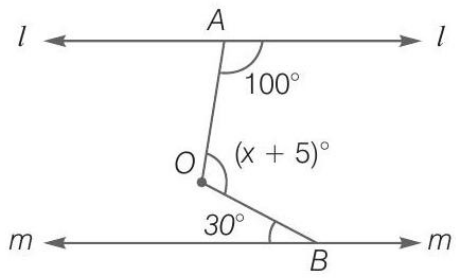
$B(3,6)$, then $x_2 = 3, y_2 = 6$
$C(-2,4)$, then $x_3 = -2, y_3 = 4$

∴ Area of $\triangle ABC$

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$
$$= \frac{1}{2}\{8(6-4) + 3(4-(-4)) + (-2)(-4-6)\}$$
$$= \frac{1}{2}\{16 + 24 + 20\} = \frac{1}{2}\times 60 = 30 \text{ sq units}$$

Practice Questions

1. In the given figure, if $l \parallel m$, then find the value of x (in degrees).

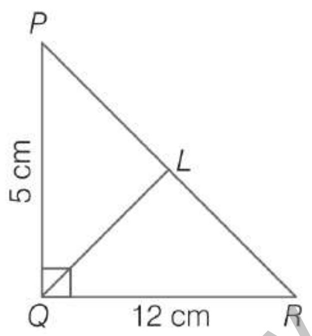


(a) $105°$
(b) $100°$
(c) $110°$
(d) $115°$

2. In a $\triangle ABC, \angle A = 90°, \angle C = 55°$ and $\overline{AD} \perp \overline{BC}$. What is the value of $\angle BAD$?

(a) $60°$
(b) $45°$
(c) $55°$
(d) $35°$

3. In the figure given below, $\angle PQR = 90°$ and QL is a median, $PQ = 5$ cm and $QR = 12$ cm. Then, QL is equal to



(a) 5 cm
(b) 5.5 cm
(c) 6 cm
(d) 6.5 cm

4. An angle which is less than $360°$ and more than $180°$, is called

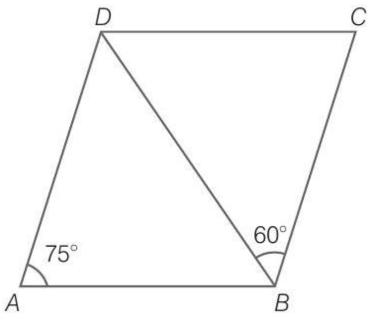
(a) a reflex angle
(b) a straight angle
(c) an acute angle
(d) an obtuse angle

5. In a $\triangle ABC, \angle A : \angle B : \angle C = 2:4:3$. The shortest side and the longest side of the triangle are respectively

(a) AC and AB
(b) BC and AC
(c) AC and BC
(d) AB and AC

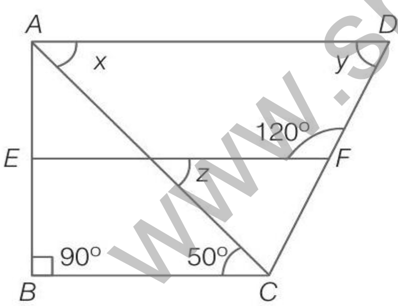
6. In the given figure, $ABCD$ is a parallelogram in which $\angle BAD = 75°$ and $\angle CBD = 60°$.

Then, $\angle BDC$ is equal to



(a) $60°$
(b) $75°$
(c) $45°$
(d) $50°$

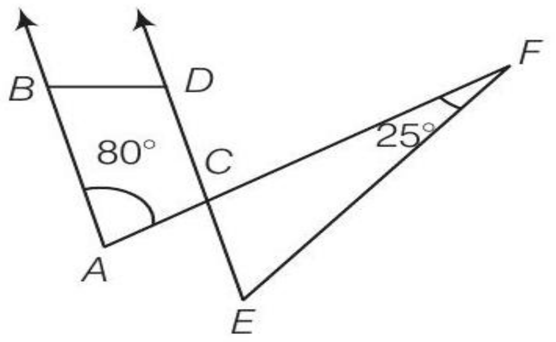
7.



In the figure given above, $ABCD$ is a trapezium. EF is parallel to AD and BC. Then, $\angle y$ is equal to

(a) $30°$
(b) $45°$
(c) $60°$
(d) $65°$

8. In the given figure, $AB \parallel CD$. If $\angle CAB = 80°$ and $\angle EFC = 25°$, then $\angle CEF$ is equal to



(a) $65°$
(b) $55°$
(c) $45°$
(d) $75°$

9. AB is the diameter of a circle with centre O and P is a point on it. If $\angle POA = 120°$, then the value of $\angle PBO$ is

(a) $30°$
(b) $50°$
(c) $60°$
(d) $40°$

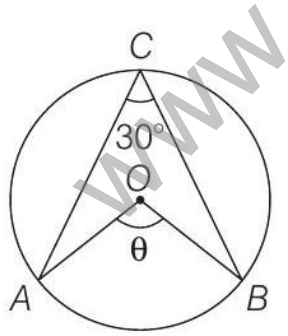
10. Three angles of a quadrilateral are $80°, 95°$ and $112°$. Its fourth angle is

(a) $78°$
(b) $73°$
(c) $85°$
(d) $100°$

11. What is the value of θ ?

Information

(I)



(II) $0 < \theta < 90°$

(a) Neither I nor II is sufficient

(b) Either I or II is sufficient

(c) Only I is sufficient

(d) Only II is sufficient

12. Find the area of the triangle formed with the three straight lines represented by

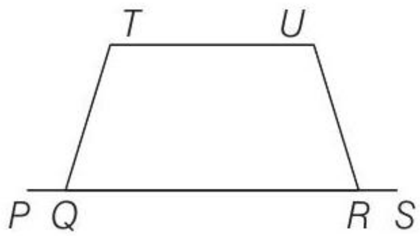
(i) $x + y = 0$; (ii) $3x = 5y$; and (iii) $y = 3x - 12$

(a) 15 units
(b) 20 units
(c) 12 units
(d) 16 units

13. Find the ratio in which line $3x + 2y = 17$ divides the line segment joined by points $(2,5)$ and $(5,2)$.

(a) $1:3$
(b) $1:2$
(c) $2:5$
(b) $3:4$

14.



In the given diagram, $TU||PS$ and points Q and R lie on PS. Also,

$\angle PQT = x°, \angle RQT = (x - 50)°$ and $\angle TUR = (x + 25)°$

What is the measure of $\angle URS$?

(a) $130°$
(b) $140°$
(c) $135°$
(d) $115°$

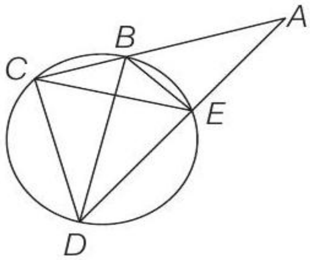
15. The ratio of the measures of the interior angles of a regular octagon to that of a regular dodecagon is

(a) $8:12$
(b) $12:8$
(c) $9:10$
() $4:5$

16. If one of the acute angles of a right-angled triangle is $55°$, what is the measure of the other acute angle?

(a) $35°$
(b) $40°$
(c) $30°$
(d) 25

17.



In the figure given above, $\angle BAE = 30°, \angle ABE = 80°$ and $\angle DBE = 50°$. What is measure of $\angle BCE$?

(a) $20°$
(b) $10°$
(c) $25°$
(d) $5°$

18. $\triangle XYZ$ is similar to $\triangle PQR$. If ratio of perimeter of $\triangle XYZ$ and perimeter of $\triangle PQR$ is $4:9$ and if $PQ = 27$ cm, then what is the length of XY (in cm)?

(a) 9
(b) 12
(c) 16
(d) 15

19. G is the centroid of the equilateral $\triangle ABC$. If $AB = 10$ cm, then length of AG is

(a) $\frac{5\sqrt{3}}{3}$ cm
(b) $\frac{10\sqrt{3}}{3}$ cm
(c) $5\sqrt{3}$ cm
(d) $10\sqrt{3}$ cm

20. If the angles of a triangle are in the ratio of $1: 2 : 3$, then find the value of the largest angle.

(a) $30°$
(b) $60°$
(c) $90°$
(d) $120°$

21. An angle is $10°$ more than one-third of its complement. Find the greater angle.

(a) $30°$
(b) $60°$
(c) $45°$
(d) $75°$

22. If the distance between the points $(x, 0)$ and $(-7,0)$ is 10 units, then the possible values of x are

(a) 3 and 17
(b) -3 and 17
(c) 3 and -17
(d) -3 and -17

23. The distance between the points $(4, -8)$ and $(k, 0)$ is 10. Find k.

(a) $k = 6$ or -2
(b) $k = 10$ or -2
(c) $k = 10$ or -4
(d) $k = 6$ or -4

24. Coordinates of a point is $(0,1)$ and ordinate of an another point is -3. If distance between both the points is 5, then abscissa of second point is

(a) 3
(b) -3
(c) ± 3
(d) 1

25. What is the reflection of the point $(6, -3)$ in the line $y = 2$?

(a) $(-2, -3)$
(b) $(6,7)$
(c) $(-6,7)$
(d) $(-2,3)$

26. If the point (x, y) is equidistant from points $(7,1)$ and $(3,5)$, then find $(x - y)$.

(a) 2
(b) 4
(c) 6
(d) 8

27. The vertices of a triangle are $A(4,4)$, $B(3, -2)$ and $C(-3,16)$. The area of the triangle is

(a) 30sq units
(b) 36 sq units
(c) 27sq units
(d) 40 sq units

28. Two vertices of an equilateral triangle are origin and $(4,0)$ What is the area of the triangle?

(a) 4 sq units
(b) $\sqrt{3}$ sq units
(c) $4\sqrt{3}$ sq units
(d) $2\sqrt{3}$ sq units

29. If the graph of the equation $2x + 3y = 6$ form a triangle with coordinates axes, then the area of triangle will be

(a) 2 sq units
(b) 3 sq units
(c) 6 sq units
(d) 1 sq unit

30. If the points $A(1, -1), B(5,2)$ and $C(k, 5)$ are collinear, then k equals

(a) 2
(b) 4
(c) 6
(d) 9

31. If two vertices of a triangle are $(5,4)$ and $(-2,4)$ and centroid is $(5,6)$, then third vertex is

(a) $(12,10)$
(b) $(10,12)$
(c) $(-10,12)$
(d) $(12, -10)$

32. A point C divides the line AB, where $A(1,3)$ and $B(2,7)$, in the ratio of $3: 4$. The coordinates of C are

(a) $\left(\frac{5}{3}, 5\right)$
(b) $(-2, -9)$
(c) $\left(\frac{3}{5}, 5\right)$
(d) $\left(\frac{10}{7}, \frac{33}{7}\right)$

33. Point $A(4,2)$ divides segment BC in the ratio $2: 5$. Coordinates of B are $(2,6)$ and C are $(9, y)$.What is the value of y?

(a) 8
(b) -8
(c) 6
(d) - 6

34. In what ratio, the line made by joining the points $A(-4, -3)$ and $B(5,2)$ intersects X-axis?

(a) $3: 2$
(b) $2: 3$
(c) $-3: 2$
(d) $-2: 3$

35. The area of the triangle with vertices at $(a, b + c), (b, c + a)$ and $(c, a + b)$ is

(a) a − b − c
(b) $ab + bc + ca$
(c) 0
(d) $a + b + c$

ANSWERS

1. (a)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (b)
11. (c)	**12.**(c)	**13.**(b)	**14.(b)**	**15.**(c)	**16.**(a)	**17.**(a)	**18.** (b)	**19.**(b)	**20.** (c)
21. (b)	**22.**(c)	**23.**(b)	**24.**(c)	**25.**(b)	**26.**(a)	**27.**(c)	**28.**(c)	**29.**(b)	**30.**(d)
31. (a)	**32.** (d)	**33.**(b)	**34.**(a)	**35.**(c)					

Hints & Solutions

2. In a right-angled triangle △ABC with ∠A = 90° and ∠C = 55°, and with (AD)¯being perpendicular to (BC)¯, we can use the fact that in a right triangle, the sum of the two acute angles is always 90°.

So, ∠B + ∠C = 90°.

Given that ∠C = 55°, we can find ∠B:

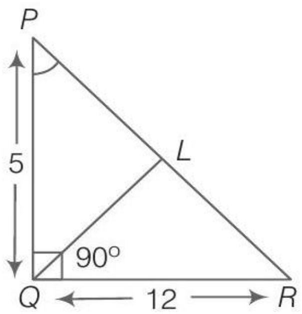
∠B + 55° = 90° ∠B = 90° - 55° ∠B = 35°

Now, since (AD)¯⊥(BC)¯, ∠BAD is complementary to ∠B. Therefore,

∠BAD = 90° - ∠B ∠BAD = 90° - 35° ∠BAD = 55°

3. Given that, $PQ = 5$ cm,

$QR = 12$ cm and QL is a median.



$\therefore\ PL = LR = \dfrac{PR}{2}$

In $\triangle PQR,\ (PR)^2 = (PQ)^2 + (QR)^2$

[by Pythagoras theorem]

$= (5)^2 + (12)^2$

$= 25 + 144 = 169 = (13)^2$

$\Rightarrow\ PR^2 = (13)^2 \Rightarrow PR = 13$

Now, by theorem, if L is the mid-point of the hypotenuse PR of a right angled $\triangle PQR$, then

$QL = \frac{1}{2}PR = \frac{1}{2}(13) = 6.5$ cm

4. An angle which is less than $360°$ and more than $180°$, is called a reflex angle.

5. Let $\angle A = 2x$

$\angle B = 4x$ and $\angle C = 3x$

We know, $\angle A + \angle B + \angle C = 180°$

$\therefore\ 2x + 4x + 3x = 180°$

$\Rightarrow\ 9x = 180° \Rightarrow x = 20°$

Now, $\angle A = 40°, \angle B = 80°$

and $\angle C = 60°$

Hence, the shortest side of triangle = side opposite to the smallest angle $= BC$ and the longest side of triangle = side opposite to the longest angle $= AC$.

6. $\angle C = A = 75°$

[opposite angles of parallelogram] In $\triangle BCD$,

$\angle CBD + \angle BCD + \angle BDC = 180°$

$\Rightarrow\ 60° + 75° + \angle BDC = 180°$

$\Rightarrow\ 135° + \angle BDC = 180°$

$\angle BDC = 45°$

8. Let $\angle CEF = x°$

Now, $AB \parallel CD$ and AF is a transversal.

$\therefore\ \angle DCF = \angle CAB = 80°$ [corresponding angles]

In $\triangle CEF$, side EC has been produced to D.

$\Rightarrow \quad x + 25 = 80°$
$\Rightarrow \quad x = 55°$

10. Let the fourth angle be $x°$.

Then, $80 + 95 + 112 + x = 360$

$\Rightarrow\ 287 + x = 360$

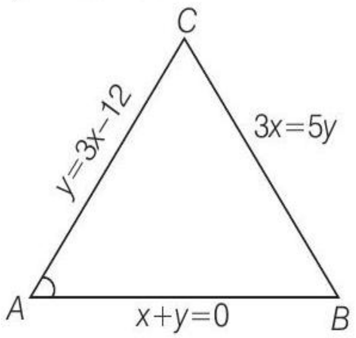
$\Rightarrow\ x = (360 - 287) = 73°$

11. From Statement (1)

$$\theta = 90 + \frac{\angle C}{2} = 90 + \frac{30}{2} = 105°$$

Hence, only statement (1) is sufficient.

12. Given equation $x + y = 0; 3x = 5y$, and $y = 3x - 12$



Coordinate of Vertex $A(3, -3) = (x_1 y_1)$

Coordinate of vertex $B(0,0) = (x_2, y_2)$

Coordinate of vertex $C(5,3) = (x_3, y_3)$

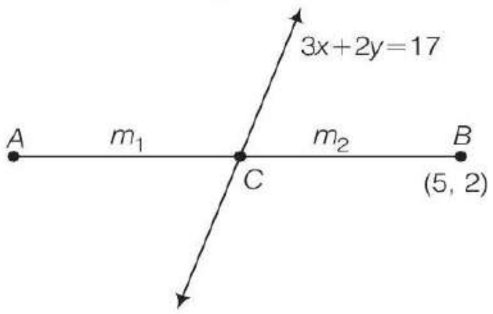
Area of $\triangle ABC = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$$= \frac{1}{2}\{3(0 - 3) + 0(3 + 3) + 5(-3 - 0)\}$$

$= \frac{1}{2}\{-9 - 15\} = \frac{1}{2} \times (-24) = 12$ units

13. Let the given lives divides $m_1 : m_2$ at point C

∴ Coordinate of point C



$$x = \frac{m_1 \times 5 + m_2 \times 2}{m_1 + m_2},$$
$$y = \frac{m_1 \times 2 + m_2 \times 5}{m_1 + m_2}$$

This point satisfying the given equation

$$3\left(\frac{5m_1 + 2m_2}{m_1 + m_2}\right) + 2\left(\frac{2m_1 + 5m_2}{m_1 + m_2}\right) = 17$$
$$\Rightarrow 15m_1 + 6m_2 + 4m_1 + 10m_2$$
$$= 17m_1 + 17m_2$$
$$\Rightarrow 19m_1 - 17m_1 = 17m_1 - 16m_2$$

14. Given $\angle PQT = x°, \angle RQT = (x - 50)°$,

$\angle TUR = (x + 25)°$

$$\angle PQT + \angle RQT = 180°$$
$$\Rightarrow \quad x + x - 50 = 180$$
$$\Rightarrow \quad 2x = 230$$
$$\Rightarrow \quad x = 115°$$
$$\because \quad \angle TUR + \angle URQ = 180°$$
$$\Rightarrow \quad (115 + 25) + \angle URQ = 180°$$
$$\Rightarrow \quad \angle URQ = 180° - 140 = 40°$$
$$\therefore \quad \angle URS = 180° - \angle URQ$$
$$= 180° - 40° = 140°$$

15. ∴ Required ratio $= \frac{\frac{(8-2)\times 180°}{8}}{\frac{(12-2)\times 180°}{12}}$

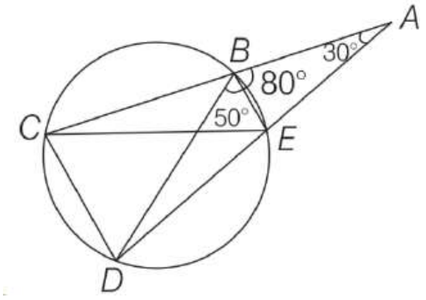
$\left[\because \text{interior angle} = \frac{(n-2)\times 180°}{n}\right]$

$$= \frac{180 \times 6 \times 12}{10 \times 8 \times 180} = \frac{9}{10}$$

16. $\therefore$ Other acute angle $= 180° - 90° - 55°$

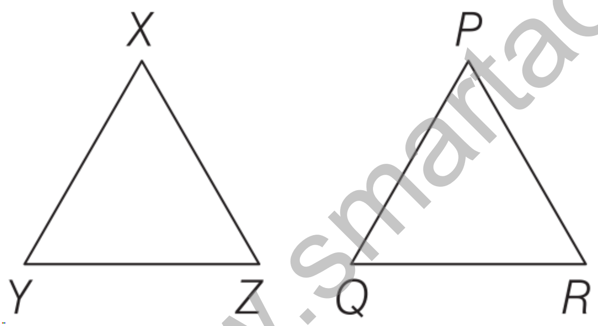
$$= 180° - 145°$$
$$= 35°$$

17. Given,



$$\angle AEB = 180° - 80° - 30° = 70°$$
$$\Rightarrow \angle EBC = 180° - 80° - 50 = 50°$$
$$= \angle CED$$
$$\angle AEB = 180° - 80° - 30° = 70°$$
$$\therefore \angle CEB = 180° - 70° - 50° = 60°$$
$$\therefore \angle BCE = 180° - 100° - 60° = 20°$$

18. Given, $\triangle XYZ - \triangle PQR$



$$\therefore \quad \frac{XY}{PQ} = \frac{YZ}{QR} = \frac{XZ}{PR} = K \text{ (say)}$$

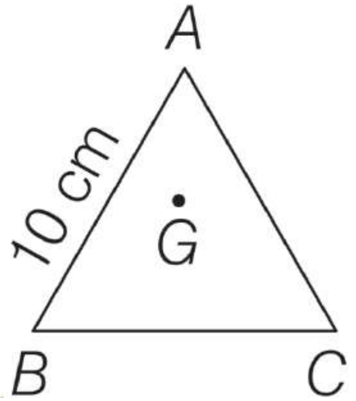
According to the question, $\frac{XY+YZ+XZ}{PQ+QR+PR} = \frac{4}{9}$

$$\Rightarrow \quad \frac{K(PQ+QR+PR)}{PQ+QR+PR} = \frac{4}{9} \Rightarrow k = \frac{4}{9}$$

or $\quad \frac{XY}{PQ} = \frac{4}{9} \Rightarrow XY = \frac{4}{9} \times 27$

$\therefore \quad XY = 12$ cm

19. In equilateral triangle, Altitude = Median



So, length of altitude, $AD = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3}$

Now, $AG = \frac{2}{3} \times AD = \frac{2 \times 5\sqrt{3}}{3} = \frac{10\sqrt{3}}{3}$ cm

20. According to the questions, largest angle $= \frac{180 \times 3}{(1+2+3)} = \frac{180}{6} \times 3 = 90°$

21. Let the other angle be $x°$.

According to question,

$$x = 10° + \frac{1}{3}(90° - x)$$
$$\Rightarrow \quad x = 10° + 30° - \frac{1}{3}x$$
$$\Rightarrow \quad x + \frac{1}{3} \times = 40°$$
$$\Rightarrow \quad \frac{4x}{3} = 40° \Rightarrow x = 30°$$

∴ The greater angle is $90° - 30° = 60°$.

22. Given, distance between the points $(x, 0)$ and $(-7,0) = 10$ units

Here, $x_1 = x, y_1 = 0, x_2 = -7$ and $y_2 = 0$

∴ Required distance

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\Rightarrow \sqrt{(-7 - x)^2 + (0 - 0)^2} = 10$$
$$\Rightarrow \pm (x + 7) = 10$$
$$\text{If } x + 7 = 10, \text{then } x = 3$$
$$\text{If } -(x + 7) = 10, \text{then } x = -17$$

23. Here, $(k-4)^2+(0+8)^2=(10)^2$

[$\because$ distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$]

$$\Rightarrow k^2+16-8k+64=100$$
$$\Rightarrow k^2-8k-20=0$$
$$\Rightarrow k^2-10k+2k-20=0$$
$$\Rightarrow k(k-10)+2(k-10)=0$$
$$\Rightarrow (k+2)(k-10)=0$$
$$\Rightarrow k=-2, k=10$$

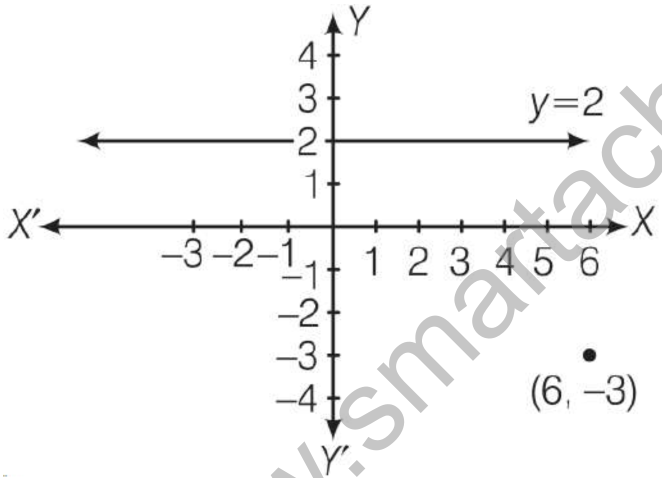
Hence, the value of k is 10 or -2 .

24. Let the abscissa be x.

Then, $(x-0)^2+(-3-1)^2=5^2$

$$\Rightarrow x^2+16=25$$
$$\Rightarrow x^2=9$$
$$\Rightarrow x=\pm 3$$

25. Here, the point $(6,-3)$ is five units away from line $y=2$.



So, its reflection point will also be 5 units away from the line $y=2$.

$\therefore$ Required point $=(6,7)$

26. Let $P(x,y)$ be equidistant from points $A(7,1)$ and $B(3,5)$.

Then, $AP=BP$, so $AP^2=BP^2$

$\therefore (x-7)^2+(y-1)^2=(x-3)^2+(y-5)^2$

$\Rightarrow x^2-14x+49+y^2-2y+1$

$=x^2-6x+9+y^2-10y+25$

$\Rightarrow 8x-8y=16$

$\therefore$ After solving, $x-y=2$

27. Let $x_1 = 4, x_2 = 3, x_3 = -3,$

$y_1 = 4, y_2 = -2$ and $y_3 = 16$

∴ Area of triangle

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

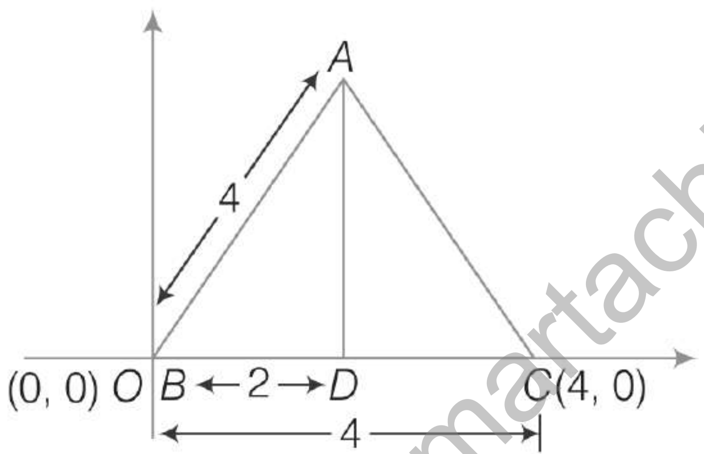
$$= \frac{1}{2}[4(-2 - 16) + 3(16 - 4) + + (-3)\{4 - (-2)\}]$$

[neglecting negative sign]

28. Since, triangle is equilateral.

$\therefore\ AB = BC = CA = 4$

$\Rightarrow BD = \frac{4}{2} = 2$



In $\triangle ADC$,

$AD^2 = 4^2 - 2^2 = 16 - 4 = 12$

$\Rightarrow\ AD = 2\sqrt{3}$

∴ Area of $\triangle ABC = \frac{1}{2} \times BC \times AD$

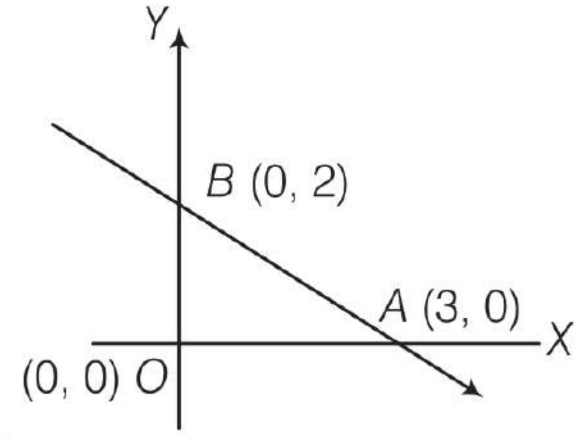
$$= \frac{1}{2} \times 4 \times 2\sqrt{3}$$
$$= 4\sqrt{3} \text{ sq units}$$

29. $\because\ 2x + 3y = 6$

$\Rightarrow\ \dfrac{2x}{6} + \dfrac{3y}{6} = 1 \Rightarrow \dfrac{x}{3} + \dfrac{y}{2} = 1$

Comparing with equation of line $\frac{x}{a} + \frac{y}{b} = 1$, we get

Intercept at X-axis $= 3$ and intercept at Y-axis $= 2$



$\therefore$ Area of $\triangle\ OAB = \frac{1}{2} \times 3 \times 2 = 3$sq units

30. Given, $x_1 = 1, x_2 = 5, x_3 = k,$

$y_1 = -1, y_2 = 2$ and $y_3 = 5.$

Since, A, B and C are collinear.

$\therefore$ Area of triangle $= 0$

$\Rightarrow \{x_1(y_2 - y_3) + x_2(y_3 - y_1)$

$+x_3(y_1 - y_2)\} = 0$

$\Rightarrow \{1(2-5) + 5(5-(-1)$

$+ k(-1-2)\} = 0$

$\Rightarrow \{-3 + 30 - 3k\} = 0$

$\Rightarrow 3k = 27$

$\therefore k = 9$

31. Let the third vertex be (x, y).

$\therefore$ Coordinates of centroid

$= \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

Given, $x_1 = x, x_2 = 5, x_3 = -2, y_1 = y, y_2 = 4, y_3 = 4$ and centroid $= (5,6)$

$\therefore\ 5 = \dfrac{x + 5 - 2}{3}$

and $\quad 6 = \frac{y+4+4}{3}$

$\Rightarrow \quad x = 12$ and $y = 10$

32. Given, $m = 3, n = 4, x_1 = 1, x_2 = 2, y_1 = 3$ and $y_2 = 7$

$\therefore$ Coordinates of C

$$= \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$
$$= \left(\frac{3 \times 2 + 4 \times 1}{3 + 4}, \frac{3 \times 7 + 4 \times 3}{3 + 4}\right)$$
$$= \left(\frac{10}{7}, \frac{33}{7}\right)$$

33. Using section formula, i.e. if a line is divided by a point in certain ratio $(m: n)$, then coordinates of point (x, y)

$$x = \frac{mx_2 + nx_1}{m + n}, y = \frac{my_2 + ny_1}{m + n}$$
$$\therefore 2 = \frac{2y + 5(6)}{2 + 5}$$
$$\Rightarrow 2y = -30 + 14 \Rightarrow 2y = -16$$
$$\Rightarrow y = -8$$

34. We know that, y-coordinate is zero on X-axis.

Given, $y_1 = -3, y_2 = 2$

$$\therefore \quad y = \frac{my_2 + ny_1}{m+n}$$
$$\Rightarrow \quad 0 = \frac{m(2)+n(-3)}{m+n}$$
$$\Rightarrow \; 2m - 3n = 0$$
$$\Rightarrow \; \frac{m}{n} = \frac{3}{2}$$

35. Here,

$$x_1 = a, y_1 = b + c$$
$$x_2 = b, y_2 = c + a$$
$$x_3 = c, y_3 = a + b$$

Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1)$

$$= \frac{1}{2}[a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a)]$$
$$= \frac{1}{2}[a \times (c - b) + b(a - c) + c(b - a)]$$
$$= \frac{1}{2}[ac - ab + ba - bc + cb - ca] = 0$$