CHAPTER 14

Area and Perimeter

In this chapter, we often have to deal with the problem of finding the areas of plane figures. As we know that, plane figures have only length and breadth (i.e., 2D only), hence this chapter can be featured as 2D mensuration too.

Area

It is the space enclosed within the boundary of a 2D figure. Unit sqcm, sq m (i.e., square units) etc.

Perimeter

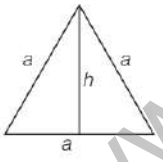
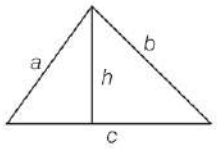
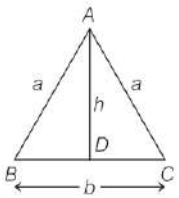
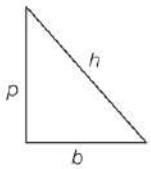
Sum of length of all the sides of 2D figure is known as its perimeter.

Unit is same as the unit of side i.e., m, cm etc.

Triangle An enclosed figure formed by three lines segment. Quadrilateral A close figure having four sides.

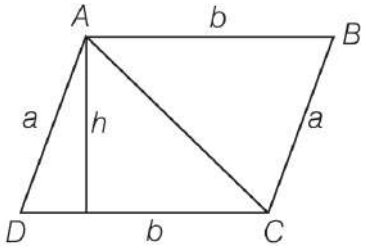
Circle A plane figure enclosed by a curve on which every point is equally distant from a fixed point.

2D Figures with their Postulates

Different Types of Triangle	Related Formulae	Different Types of Triangle	Related Formulae
• **Equilateral Triangles** Its all three sides are equal. Side = a, Height = h  	Area = $\sqrt{3}/4\ a^2$ Height, $h = \sqrt{3}/2\ a$ Perimeter = $3a$ Each angle = 60°	• **Scalene Triangle** Its all sides are unequal.  a, b, c = Unequal sides of the triangle	Area = $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$ $= \frac{1}{2} \times c \times h$ Where, $s = \frac{a+b+c}{2}$ Perimeter = $a + b + c$
• **Isosceles Triangles** Its two sides are equal. a = Equal sides b = Third unequal side h = Height  	Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$ Height, $h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2}$ $= \frac{1}{2}\sqrt{4a^2 - b^2}$ Perimeter = $a + a + b$ $= 2a + b$, $\angle ABC = \angle ACB$ $BD = DC$	• **Right Angled Triangle** One of the angle in this triangle is 90°. h = Hypotenuse, b = Base p = Perpendicular  	Area = $\frac{1}{2} \times b \times p$ Perimeter = $p + b + h$ $h^2 = p^2 + b^2$

Different Types of Quadrilaterals-

- **Parallelogram** It is a quadrilateral with opposite sides parallel and equal.



$a, b \rightarrow$ Opposite parallel sides, $h =$ Height

Related Formulae: Area $=$ Base $\times$ Height $= b \times h$

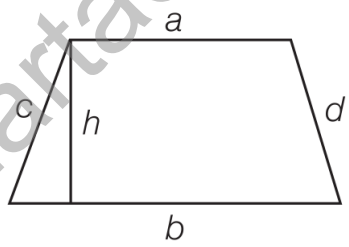
Perimeter $= 2(a + b)$

$\angle ABC = \angle ADC$

and $\angle BAD = \angle BCD$

Area $(\triangle ADC) =$ Area $(\triangle ABC)$

- **Trapezium** Quadrilateral with one of the pair of opposite sides equal is called trapezium.



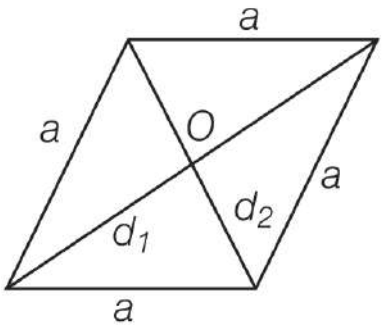
$a, b \rightarrow$ Opposite parallel sides, $h =$ Height

Related Formulae: Area $= \frac{1}{2}$(Sum of parallel sides) $\times$ Height

$$= \frac{1}{2}(a + b)h$$

Perimeter $= a + b + c + d$

- **Rhombus** It is a parallelogram whom all the four sides are equal.



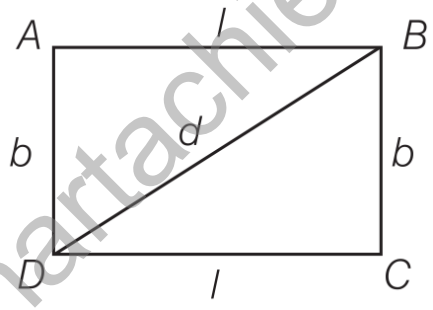
$a =$ Sides , $d_1, d_2 =$ Diagonals

Related Formulae: Area $= \frac{1}{2} \times d_1 \times d_2$

Sides $(a) = \frac{1}{2}\sqrt{{d_1}^2 + {d_2}^2}$

Perimeter $= 4a$

- **Rectangle** It is a parallelogram with equal opposite and each angle is equal to 90°.



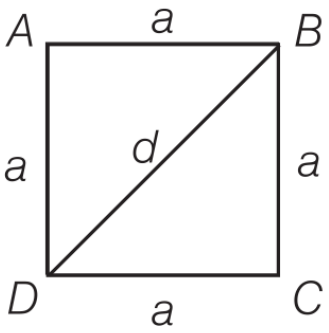
$l =$ Length, $b =$ Breadth, $d =$ Diagonal

Related Formulae: Area $= \frac{1}{2} \times d_1 \times d_2$

Sides $(a) = \frac{1}{2}\sqrt{{d_1}^2 + {d_2}^2}$

Perimeter $= 4a$

- **Square** It is a parallelogram with all 4 sides equal and each angle is equal to 90°.



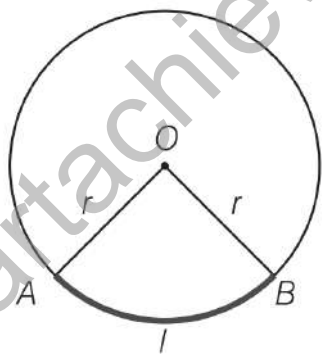
a = Sides, d = Diagonal

Related Formulae: Area $= (\text{ Side })^2 = a^2 = \frac{d^2}{2}$

Perimeter $= 4 \times$ Side $= 4a$

Diagonal $(d) = a\sqrt{2}$

- **Circle** It is a plane figure enclosed by a curve on which every point is equally distant from a fixed point called center inside the curve.



Related Formulae: Area $= \pi r^2$

Circumference (Perimeter) $= 2\pi r$

Diameter $= 2r$

Length of the arc $(l) = \frac{\pi r \theta}{180°}$

Area of sector $AOB = \frac{\pi r^2 \theta}{360°}$

$r =$ Radius

$\pi = \frac{22}{7}$

Regular Polygon in a Regular polygon, all sides and all interior angles are equal.

Each exterior angle $= \frac{360^\circ}{n}$ ($n =$ Number of sides of polygon)

Each interior angle $= 180^\circ - \frac{360^\circ}{n}$

Number of diagonals of a polygon $= \left\{\frac{n(n-1)}{2} - n\right\}$

Note-

- If the length of a rectangle is increased by $a\%$ and $b\%$ respectively, then area will be increased by $\left(a + b + \frac{ab}{100}\right)\%$.
- The area of the largest inscribed in a semicircle of radius r is equal to r^2.
- Area of a square inscribed in a circle of radius r is equal to $2r^2$.

Some Common Polygon

Number of Sides (n)	Polygon	Area
5	Pentagon	$5a^2\frac{\sqrt{3}}{4}$
6	Hexagon	$6a^2\frac{\sqrt{3}}{4}$
7	Heptagon	
8	Octagon	$2(\sqrt{2}+1)a^2$
9	Nonagon	
10	Decagon	

Solved Examples:

1. A rectangular garden is 100 m long and 60 m broad. It is surrounded by a 5 m wide road. What is the area of the road?

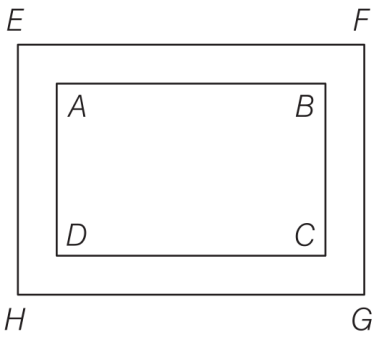
(a) 1600 m^2
(b) 1200 m^2
(c) 1000 m^2
(d) 1700 m^2

Sol. (d) Area of the rectangular garden $ABCD$

$= 100 \times 60 = 6000 \text{ m}^2$

and area of the rectangular garden with road $EFGH$

$= (100 + 2 \times 5) \times (60 + 2 \times 5) = 110 \times 70 = 7700 \text{ m}^2$



$\therefore$ Area of road $= 7700 - 6000 = 1700 \text{ m}^2$

2. If the length and breadth of a rectangle are increased by 10% and 8% respectively, then by what per cent does the area of the rectangle increase?

(a) $16\frac{2}{3}$
(b) $14\frac{2}{7}$
(c) $18\frac{4}{5}$
(d) $18\frac{2}{5}$

Sol. (c) Let the original length and breadth of the rectangle are x and y.

$\therefore$ Its original area $= xy$

Increased length $= \frac{x \times 110}{100} = \frac{11x}{10}$

and increased breadth $= y \times \frac{108}{100} = \frac{27y}{25}$

$\therefore$ Resulting area $= \frac{11x}{10} \times \frac{27y}{25} = \frac{297xy}{250}$

$\therefore$ Increase in area $= \frac{297xy}{250} - xy = \frac{47xy}{250}$

$\therefore$ Percentage increase in area $= \frac{47xy}{250} \times \frac{100}{xy} = \frac{94}{5} = 18\frac{4}{5}\%$

Alternate Method

Percentage increase in area $= \left(x + y + \frac{xy}{100}\right)$

Here, $x = 10$ and $y = 8$

$= 10 + 8 + \frac{10 \times 8}{100} = 18\frac{4}{5}\%$

3. A wire in the form of a square is cut and bent in the form of a circle. If the area of the square is 110 cm^2, what is the area of the circle?

(a) 160 cm^2
(b) 120 cm^2
(c) 140 cm^2
(d) 180 cm^2

Sol. (b) Area of the square $= 110\ \text{cm}^2$

$\therefore$ One side of the square $= \sqrt{110}$ cm

$\therefore$ Perimeter of the square $= 4 \times \sqrt{110}$ cm

But the perimeter of the square = Circumference of the circle

$\therefore$ Circumference of the circle $= 4 \times \sqrt{110}$ cm

$\therefore$ Radius of the circle $= \frac{4\sqrt{110}}{2\times\pi} = \frac{4\sqrt{110}\times 7}{2\times 22} = \frac{7\sqrt{110}}{11}$ cm

$\therefore$ Area of the circle $= \pi \times \left(\frac{7\sqrt{110}}{11}\right)^2 = \frac{22}{7} \times \frac{49\times 110}{11\times 11} = 140\ \text{cm}^2$

Alternate Method

Perimeter of the square $= 4\sqrt{110}$ cm

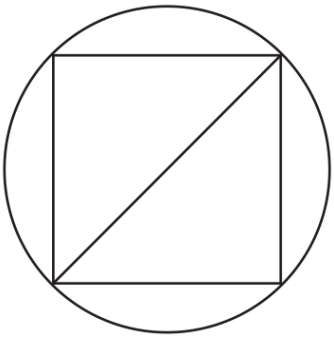
$\therefore$ Area of the circle $= \frac{(\text{Perimeter of the square})^2}{4p}$

$$= \frac{(4\sqrt{110})^2 \times 7}{4 \times 22}$$
$$= \frac{16 \times 110 \times 7}{4 \times 22} = 140\ \text{cm}^2$$

4. The radius of a circle is 11 cm. What is the area of the square inscribed in the circle?

(a) $212\ \text{cm}^2$
(b) $232\ \text{cm}^2$
(c) $242\ \text{cm}^2$
(d) $244\ \text{cm}^2$

Sol. (c)



Diagonal of the square = Diameter of the circle

$$= 2 \times 11 = 22\ \text{cm}$$

$\therefore$ Area of the square $= \frac{1}{2}(\text{ Diagonal })^2$

$$= \frac{1}{2} \times 22 \times 22 = 242\ \text{cm}^2$$

Alternate Method

Area of the square inscribed in a circle $= 2r^2$

Here, $r = 11 = 2 \times (11)^2 = 242\ \text{cm}^2$

Practice Questions

1. If the ratio of the areas of two squares is $1:4$, the ratio of their perimeters is

(a) $1:6$
(b) $1:8$
(c) $1:2$
(d) $1:4$

2. Perimeter of a triangle is 24 m and circumference of its incircle is 44 m. Find the area of the triangle.

(a) 42sqm
(b) 24sqm
(c) 48sqm
(d) 84sqm

3. The ratio of the length of the parallel sides of a trapezium is 3: 2. The shortest distance between them is 15 cm. If the area of the trapezium is 450 cm^2, then sum of lengths of the parallel sides is

(a) 15 cm
(b) 36 cm
(c) 42 cm
(d) 60 cm

4. A parallelogram has sides 15 cm and 7 cm long. The length of one of the diagonals is 20 cm. The area of then parallelogram is

(a) 42 cm^2
(c) 84 cm^2
(b) 60 cm^2
(d) 96 cm^2

5. An equilateral triangle and a regular hexagon have the same perimeter.The ratio of the area of the triangle to that of the hexagon is

(a) 3: 2
(b) 2: 3
(c) 1: 2
(d) 1: 4

6. If the edge of a cube is increased by 100%, then the surface area of the cube is increased by

(a) 100%
(b) 200%
(c) 300%
(d) 400%

7. A street of width 10 m surrounds from outside a rectangular garden whose measurement is 200 m × 180 m. The area of the path (in sqm) is

(a) 8000
(b) 7000
(c) 7500
(d) 8200

8. The length of the three sides of a Eight angle triangle are $(x-2)$cm, x cm and $(x+2)$cm, respectively. Then, the value of x is

(a) 10
(b) 8
(c) 4
(d) 0

9. The lengths of two sides of an isosceles triangle are 15 and 22, respectively. What are the possible values of perimeter?

(a) 52 or 59
(b) 52 or 60
(c) 15 or 37
(d) 37 or 29

10. A semicircular shaped window has diameter of 63 cm. Its perimeter equals to $\left(\pi = \frac{22}{7}\right)$

(a) 126 cm
(b) 162 cm
(c) 198 cm
(d) 251 cm

11. Tiling work of rectangular hall 60 m long and 40 m broad is to be completed with a square tile of 0.4 m side. If each tile cost ₹ 5, find the total cost of the tiles.

(a) ₹ 60000
(b) ₹ 65000
(c) ₹ 75000
(d) ₹ 12000

12. A horse is placed for grazing inside a square field 12 m long and is tethered to one corner by a rope 8 m long. The area it can graze is

(a) 50.18sqm
(b) 50.28sqm
(c) 50.38sqm
(d) 50.48sqm

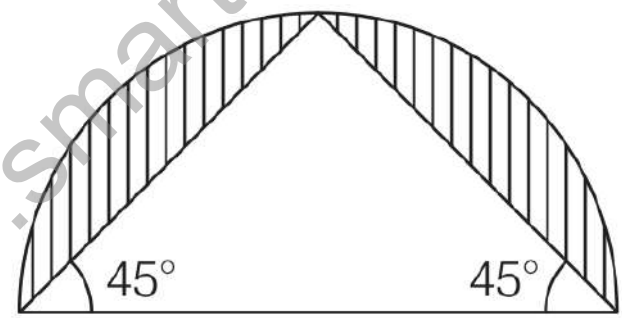
13. The diameters of two concentric circles are 8 cm and 10 cm. The area of the region between them is

(a) 2πsqcm
(b) 4πsqcm
(c) 36πsqcm
(d) 9πsqcm

14. The diagonal of square field is 50 m. The area (in m^2) of the field is

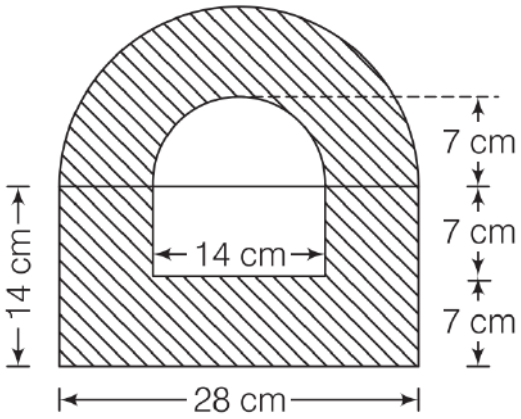
(a) 625
(b) 1250
(c) 2500
(d) 5000

15. The area of the shaded region in the given figure is



(a) $2a^2(\pi - 2)$
(b) $\frac{a^2}{2}(\pi - 2)$
(c) $a^2(\pi - 1)$
(d) $\frac{a^2}{2}(\pi - 1)$

16. The area of the shaded region shown in the given figure is



(a) $515\ cm^2$
(c) $505\ cm^2$
(b) $535\ cm^2$
(d) $525\ cm^2$

17. A square field with side 30 m is surrounded by a path of uniform width. If the area of the path is $256\ m^2$, the width of the path is

(a) 14 m
(b) 16 m
(c) 4 m
(d) 2 m

18. An equilateral triangle of side 6 cm has its corners cut off to form a regular hexagon. The area of this hexagon is

(a) $6\sqrt{3}$sqcm
(b) $3\sqrt{3}$sqcm
(c) $3\sqrt{6}$sqcm
(d) $\frac{5\sqrt{3}}{2}$sqcm

19. The area of the four walls of a room is 128sqm. The length is equal to the width and the height is 4 m. The area of the floor of the room is

(a) 32sqm
(b) 49sqm
(c) 64sqm
(d) 81sqm

20. If the radius of a circle is tripled, its perimeter will become how many times of its previous perimeter?

(a) 2
(b) $\frac{1}{3}$
(c) 9
(d) 3

21. A room is half as broad as it is high and $1\frac{1}{2}$ times as high as long. If the cost of carpeting the floor at ₹ 3 per sq m, is ₹ 144, the height (in m) of the room is

(a) 8
(b) 10
(c) 12
(d) 16

22. A rectangular plot 90 m × 50 m has two 10 m wide roads running in the middle of it, one parallel to the length and the other parallel to the breadth. Area of the plot used in roads is

(a) 1300sqm
(b) 1400sqm
(c) 1500sqm
(d) 1200sqm

ANSWERS

1.	(c)	**2.**	(d)	**3.**	(d)	**4.**	(c)	**5.**	(b)	**6.**	(c)	**7.**	(a)	**8.**	(b)	**9.**	(a)	**10.**	(b)
11.	(c)	**12.**	(b)	**13.**	(d)	**14.**	(b)	**15.**	(b)	**16.**	(d)	**17.**	(d)	**18.**	(a)	**19.**	(c)	**20.**	(d)
21.	(c)	**22.**	(a)																

Hints &Solutions

1. Ratio of perimeters

$= \sqrt{\text{Ratio of areas}}$

$= \sqrt{1} : \sqrt{4} = 1 : 2$

2. If inradius of the triangle be r, there

$$2\pi r = 44$$

$$\Rightarrow r = \frac{44}{2\pi} = 7 \text{ m}$$

Using $r = \frac{\Delta}{S}$

$$\Rightarrow \Delta = 7 \times \frac{24}{2}$$

$$= 7 \times 12 = 84 \text{ m}^2$$

3. Let the length of the parallel sides of a trapezium be $3x$ cm, $2x$ cm.

Given, Area of trapezium $= 450$

$$\frac{1}{2} \times (2x + 3x) \times 15 = 450$$

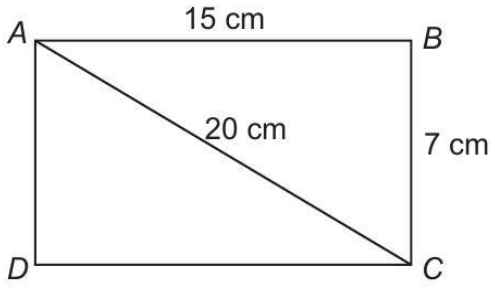
$$5x = \frac{450 \times 2}{15} = 60 \text{ cm}$$

$\therefore$ Required sum $= 60$ cm

4. In $\triangle ABC, S = \frac{15+7+20}{2} = 21$

Area of $\triangle ABC = \sqrt{21(21-15)(21-7)(21-20)}$

$= \sqrt{21 \times 6 \times 14 \times 1} = 42 \text{ cm}^2$



∴ Area of the parallelogram

$= 2 \times$ Area of $\triangle ABC$
$= 2 \times 42$
$= 84 \text{ cm}^2$

5. Let the side of an equilateral triangle is a and side of hexagon is b.

According to question,

Perimeter of equilateral triangle = Perimeter of hexagon

$$3a = 6b$$

$$\frac{a}{b} = 2$$

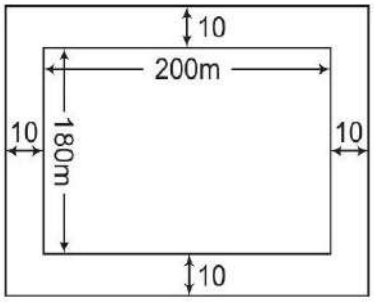
∴ Ratio of the areas $= \frac{\frac{\sqrt{3}}{4}a^2}{6\times\frac{\sqrt{3}}{4}b^2}$

$$= \frac{a^2}{6b^2} = \frac{1}{6}\left(\frac{a}{b}\right)^2$$
$$= \frac{1}{6}(2)^2 = \frac{2}{3}$$

6. The required surface area increased by

$$= \left[\left(\frac{100+x}{100}\right)^2 - 1\right] \times 100\%$$
$$= \left[\left(\frac{100+100}{100}\right)^2 - 1\right] \times 100\%$$
$$= \left[\left(\frac{200}{100}\right)^2 - 1\right] \times 100\%$$
$$= [(2)^2 - 1] \times 100\%$$
$$= (4-1) \times 100\%$$
$$= 300\%$$

7.



∴ The area of the path

$= (200 + 10 \times 2) \times (180 + 10 \times 2) - 200 \times 180$

$= 220 \times 200 - 200 \times 180$

$= 44000 - 36000$

$= 8000\ \text{m}^2$

8. In right angled triangle

(Hypotenuse $)^2 =$ (Base $)^2$

- (Perpendicular ^{2}

$\Rightarrow\ (x + 2)^2 = x^2 + (x - 2)^2$

$\Rightarrow x^2 + 4x + 4 = x^2 + x^2 - 4x + 4$

$\Rightarrow\ x^2 = 8x$

$\Rightarrow\ x = 8$

9. Perimeter of isosceles triangle $= 15 + 15 + 22$ or $15 + 22 + 22 = 52$ or 59 units

10. Perimeter of semicircular shaped window $= (\pi r + 2r)\text{cm}$

$= r(\pi + 2)\text{cm}$

$= \frac{63}{2}\left(\frac{22}{7} + 2\right)\text{cm}$

$= \frac{63}{2} \times \frac{36}{7} = 162\ \text{cm}$

11. Number of tiles $= \frac{60\times40}{0.4\times0.4} = 15000$

Total cost of the tiles $= 15000 \times 5 = ₹75000$

12. The required area $= \frac{1}{4}[\pi(8)^2]$

$$= \frac{1}{4} \times \frac{22}{7} \times 64 = 50.28\text{sqm}$$

13. The area of the region between two concentric circles

$$= \pi(R^2 - r^2) = \pi(5^2 - 4^2)$$
$$= \pi(25 - 16) = 9\pi\text{cm}^2$$

14. Area of the square field

$$= \frac{1}{2}(\text{ Diagonal })^2$$
$$= \frac{1}{2}(50)^2$$
$$= 1250\text{sqm}$$

15. If one side of the triangle is x, then in right angled triangle

$$x^2 + x^2 = 4a^2$$
$$\therefore\ x = a\sqrt{2}$$

$\therefore$ Area of a Right angle triangle

$$= \frac{1}{2} \times a\sqrt{2} \times a\sqrt{2} = a^2$$

and area of the semicircle $= \frac{1}{2}\pi a^2$

$\therefore$ Area of the shaded region

$$= \frac{1}{2}\pi a^2 - a^2$$
$$= \frac{1}{2}a^2(\pi - 2)$$

16. Area of the whole external rectangle $= 28 \times 14 = 392\ \text{cm}^2$ Area of the whole semicircle

$$= \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$
$$= 308\ \text{cm}^2$$

$\therefore$ Area of the whole external figure $= 392 + 308 = 700\ \text{cm}^2$ and area of the whole inner figure

$$= 14 \times 7 + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$
$$= 98 + 77$$
$$= 175\ \text{cm}^2$$

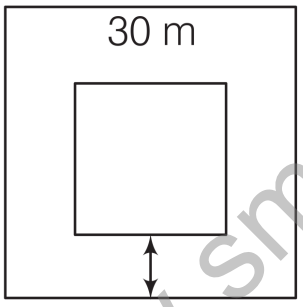
$\therefore$ Area of the shaded region

$$= 700 - 175$$
$$= 525\ \text{cm}^2$$

17. Let the breadth of the path x m.

$\therefore (30 + 2x)^2 - (30)^2 = 256$

$\Rightarrow (30 + 2x + 30)(30 + 2x - 30) = 256$



$(60 + 2x) \times 2x = 256$

$\Rightarrow x^2 + 30x - 64 = 0$

$\Rightarrow (x - 2)(x + 32) = 0$

$\therefore\ x = 2$ or -32

So, the breadth of the path be 2 m.

18. One side of an equilateral triangle $= 6$ cm $\therefore$ One side of the regular hexagon $= 2$ cm

$\therefore$Area of the hexagon

$$= \frac{\sqrt{3}}{4} \times (6)^2 - \frac{3\sqrt{3}}{4} \times (2)^2$$

$$= \frac{\sqrt{3}}{4}[36 - 12]$$

$$= \frac{\sqrt{3}}{4} \times 24$$

$$= 6\sqrt{3} \text{ cm}^2$$

19. Let the length of the room be x m.

$\because\ 128 = 2 \times 4(x + x)$

$\therefore\ x = 8$ m

So, area of the floor $= 64$ m^2

20. Let the radius of the initial circle be r.

$\therefore$ Radius of the resulting circle $= 3r$

$\therefore$ Perimeter of the initial circle $= 2\pi r$

and perimeter of the resulting circle $= 2\pi(3r) = 6\pi r$

$\therefore$ Required ratio $= \frac{6\pi r}{2\pi r} = 3$

21. Let the height of the room be x m.

$\therefore$ Length $= x \times \frac{2}{3}$

$$= \frac{2x}{3} \text{ m}$$

and Breadth $= \frac{x}{2}$ m

$\therefore$ Area of the floor $= \frac{2x}{3} \times \frac{x}{2}$

$$= \frac{x^2}{3} \text{sqm}$$

$\because\ \frac{x^2}{3} \times 3 = 144$

$\Rightarrow\ x = 12$ m

22. Area of road = Width of road × [Length of plot + Breadth of plot - Width of road]

$= 10 \times [90 + 50 - 10]$

$= 10 \times 130$

$= 1300\text{sqm}$