

# Complex Numbers

## Single Correct Answer Type

1. If  $z_1, z_2, z_3$  and  $z_4$  be the consecutive vertices of a square, then  $z_1^2 + z_2^2 + z_3^2 + z_4^2$  equals
- (a)  $z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$   
 (b)  $z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4$   
 (c) 0 (d) None of the above

Key. A

Sol. We know that,  $\frac{z_2 - z_1}{z_4 - z_1} = \frac{|z_2 - z_1|}{|z_4 - z_1|} e^{i\pi/2} = i$  (as  $|z_2 - z_1| = |z_4 - z_1|$ )

$\Rightarrow (z_4 - z_1)^2 + (z_2 - z_1)^2 = 0$

similarly  $\frac{z_4 - z_3}{z_2 - z_3} = i$

$\Rightarrow (z_4 - z_3)^2 + (z_2 - z_3)^2 = 0$

On adding Eqs (i) and (ii), we get

$2(z_1^2 + z_2^2 + z_3^2 + z_4^2 - z_1z_2 - z_4z_1 - z_4z_3 - z_2z_3) = 0$   
 $z_1^2 + z_2^2 + z_3^2 + z_4^2 = z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$

2. If  $z_1, z_2$  and  $z_3$  are the vertices of an isosceles right angled triangle, right angled at the vertex  $z_2$ , then  $(z_1 - z_2)^2 + (z_3 - z_2)^2$  equals
- (a) 0 (b)  $(z_1 - z_3)^2$  (c)  $\frac{z_1 + z_3}{2}$  (d) None of these

Key. A

Sol. we know that  $\frac{z_3 - z_2}{z_1 - z_2} = \frac{|z_3 - z_2|}{|z_1 - z_2|} e^{-i\pi/2}$

$\Rightarrow z_3 - z_2 = -i(z_1 - z_2)$

$\Rightarrow (z_3 - z_2)^2 + (z_1 - z_2)^2 = 0$

3. Let  $C = \cos \frac{2p}{7} + \cos \frac{4p}{7} + \cos \frac{8p}{7}$  and  $S = \sin \frac{2p}{7} + \sin \frac{4p}{7} + \sin \frac{8p}{7}$ , then

- (a)  $C = \frac{1}{2}$  (b)  $S = \frac{1}{2}$   
 (c)  $C = \frac{\sqrt{7}}{2}$  (d)  $S = \frac{\sqrt{7}}{2}$

Key. D

Sol.  $C + iS = e^{iq} + e^{i(2q)} + e^{i(4q)}$ , where  $q = \frac{2p}{7}$

i.e  $C + iS = a + a^2 + a^4$ , where  $a = e^{iq}$  .....(i)

so,  $C - iS = \bar{a} + (\bar{a}^2) + (\bar{a}^4) = a^6 + a^5 + a^3$  .....(ii)

$$\sin ce \bar{a} = \frac{a\bar{a}}{a} = \frac{1}{a} = \frac{a^7}{a} = a^6 \text{ etc.}$$

From Eqs (i) and (ii), we get

$$2C = a + a^2 + a^3 + a^4 + a^5 + a^6 = \frac{a(a^6 - 1)}{a - 1}$$

$$\text{P } 2C = \frac{1 - a}{a - 1} (\because a^7 = 1)$$

$$\text{P } C = -\frac{1}{2}$$

$$\text{Again } (C + iS)(C - iS) = (a + a^2 + a^4)(a^6 + a^5 + a^3)$$

$$\text{P } C^2 + S^2 = 1 + a^6 + a^4 + a + 1 + a^5 + a^3 + a^2 + 1 \quad (a^7 = 1)$$

$$\text{P } \frac{1}{2} + S^2 = 2 + (1 + a + a^2 + a^3 + a^4 + a^5 + a^6)$$

$$\text{P } S^2 = 2 + 0 - \frac{1}{4} = \frac{7}{4}$$

$$\text{P } S = \frac{\sqrt{7}}{2}$$

4. The point of intersection of the curves  $\arg(z - 3i) = \frac{3p}{4}$  and  $\arg(2z + 1 - 2i) = \frac{p}{4}$  is

- (a)  $\frac{1}{4}(3 + 9i)$  (b)  $\frac{1}{4}(3 - 9i)$  (c)  $\frac{1}{2}(3 + 2i)$  (d) None of these.

Key. D

Sol. Clearly the two eqns represent two rays which are not intersecting. Hence no point of intersection.

5. If  $z_1, z_2, z_3$  are non-zero complex numbers representing the points A, B, C such that

$$\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3} \text{ . Then}$$

- (a) A, B, C are collinear.  
 (b) Circle passes through points A, B, C has centre at origin O  
 (c) Circle passes through A, B, C passes through origin.  
 (d) None of these.

Key. C

$$\text{Sol. } \frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3} \text{ P } \arg \frac{z_2 - z_1}{z_3 - z_1} = \arg \frac{z_2}{z_3} = \arg \frac{z_2}{z_3} \pm p$$

$$\text{P } \arg \frac{z_2 - z_1}{z_3 - z_1} = \arg \frac{z_2 - 0}{z_3 - 0} \pm p$$

$$\text{P } \arg \frac{z_2 - z_1}{z_3 - z_1} = \arg \frac{z_2 - 0}{z_3 - 0} \pm p$$

Sum of angles at A and origin is  $\pm p$ . Hence points O, B, A, C are concyclic.

6. If  $|2z - 4 - 2i| = |z| \sin\left(\frac{\pi}{4} - \arg z\right)$ , then locus of z is/an

- (a) Ellipse      (b) Circle      (c) Parabola      (d) Pair of straight line

Key. A

Sol. Let  $z = x + iy = r(\cos q + i \sin q)$ , then the equation is

$$|2(x-2) + 2i(y-1)| = \frac{1}{\sqrt{2}} \Rightarrow \sqrt{4(x-2)^2 + 4(y-1)^2} = \frac{1}{\sqrt{2}} \Rightarrow \sqrt{(x-2)^2 + (y-1)^2} = \frac{1}{2}$$

$$\Rightarrow \sqrt{(x-2)^2 + (y-1)^2} = \frac{1}{2}$$

It is an ellipse with focus at (2,1) and directrix  $x - y = 0$  and eccentricity =  $\frac{1}{\sqrt{2}}$ .

7. If  $|z - 3i| = 3$ , (where  $i = \sqrt{-1}$ ) and  $\arg z \in (0, \pi/2)$ , then  $\cot(\arg(z)) - \frac{6}{z}$  is equal to  
 a) 0                                      b) -i                                      c) i                                      d) none of these

Key. C

Sol. Conceptual

8. If the imaginary part of the expression  $\frac{z-1}{e^{iq}} + \frac{e^{iq}}{z-1}$  be zero, then the locus of z can be  
 (a) a straight line parallel to x-axis.  
 (b) a parabola  
 (c) a circle of radius 1  
 (d) none of these.

Key. C

Sol. Conceptual

9. If  $\cos a + \cos b + \cos g = 0 = \sin a + \sin b + \sin g$  then  $\frac{\sin 3a + \sin 3b + \sin 3g}{\sin(a + b + g)}$  is equal to  
 (a) 0      (b) 1      (c) 2      (d) 3

Key. D

Sol. Let  $a = e^{ia}, b = e^{ib}, c = e^{ig}$  clearly  $a + b + c = 0 \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

$$a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow \sin 3a + \sin 3b + \sin 3g = 3\sin(a + b + g).$$

10. If  $z$  is a complex number satisfying  $z^4 + z^3 + 2z^2 + z + 1 = 0$ , then the set of possible values of  $|z|$  is  
 (a) {1, 2}                                      (b) {1}                                      (c) {1, 2, 3}                                      (d) {1, 2, 3, 4}

Key. B

Sol. The given equation is  $(z^2 + z + 1)(z^2 + 1) = 0$ .

$$z = \pm i, w, w^2, \quad w \text{ being an imaginary cube root of unity. Thus } |z| = 1.$$

11. Let A, B and C represents the complex numbers  $z_1, z_2$  and  $z_3$  in the Argand plane. If circumcentre of the triangle ABC is at the origin, then the complex number corresponding to orthocentre is  
 (a)  $\frac{1}{4}(z_1 + z_2 + z_3)$       (b)  $\frac{1}{3}(z_1 + z_2 + z_3)$       (c)  $\frac{1}{2}(z_1 + z_2 + z_3)$       (d)  $z_1 + z_2 + z_3$

Key. D

Sol. Centroid of  $DABC$  is at  $\frac{z_1 + z_2 + z_3}{3}$ .

Orthocentre divides Centroid and circumcentre in 2:3 externally.

12. If  $z = x + iy$  then the equation  $\left| \frac{2z - i}{z + 1} \right| = m$  does not represent a circle when

- (a)  $m = \frac{1}{2}$                       (b)  $m = 1$                       (c)  $m = 2$                       (d)  $m = 3$

Key. C

Sol. The given equation is  $\left| \frac{z - \frac{i}{2}}{z + 1} \right| = \frac{m}{2}$ , which does not represent a circle when  $\frac{m}{2} = 1$ .

13.  $\alpha, \beta, \gamma$  are the roots of  $x^3 - 3x^2 + 3x + 7 = 0$  ( $\omega$  is cube root of unity) then  $\left( \frac{\alpha - 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1} \right)$  is

- (A)  $\frac{3}{\omega}$                       (B)  $\omega^2$                       (C)  $2\omega^2$                       (D)  $3\omega$

Key. A

Sol. We have  $x^3 - 3x^2 + 3x + 7 = 0$

$$\Rightarrow (x - 1)^3 + 8 = 0$$

$$\Rightarrow \left( \frac{x - 1}{-2} \right)^3 = 1$$

$$\Rightarrow \left( \frac{x - 1}{-2} \right) = 1, \omega, \omega^2$$

$$\Rightarrow x = -1; 1 - 2\omega; 1 - 2\omega^2$$

$$\therefore \alpha = -1, \beta = 1 - 2\omega; \gamma = 1 - 2\omega^2$$

$$\therefore \text{required expression} = 3\omega^2.$$

14. The complex number  $3 + 4i$  is rotated about origin by an angle of  $\frac{\pi}{4}$  and then stretched 2-times. The complex number corresponding to new position is

- (a)  $\sqrt{2}(-3 + 4i)$  (b)  $\sqrt{2}(-1 + 7i)$                       (c)  $\sqrt{2}(3 - 4i)$                       (d)  $\sqrt{2}(-1 - 7i)$

Key. B

Sol. The new complex number is  $2(3 + 4i)e^{i\pi/4} = \sqrt{2}(-1 + 7i)$ .

15. If  $(a + ib)^5 = \alpha + i\beta$  then  $(b + ia)^5$  is equal to

- (A)  $\beta - i\alpha$                       (B)  $\beta + i\alpha$                       (C)  $\alpha - \beta$                       (D)  $-\alpha - i\beta$

Key. B

Sol.  $(a + ib)^5 = \alpha + i\beta$

Taking complex conjugate

$$(a - ib)^5 = \alpha - i\beta$$

$$(-i^2 a - ib)^5 = \alpha - i\beta$$

$$(-i)^5 (b + ai)^5 = \alpha - i\beta$$

$$(b + ai)^5 = -\frac{\alpha}{i} + \beta$$

$$= \alpha i + \beta$$

16. The complex number  $a + i$ ,  $a - i$ ,  $1 + ai$ ,  $1 - ai$  where  $a \in \mathbb{R}$  taken in that order on the Argand plane represent the vertices of a parallelogram if  
 (A)  $a = 1$  (B)  $a = -1$  (C)  $a = 0$  (D) none of these

Key. B

Sol. Diagonals of Parallelogram intersects at midpoint

$$\text{Solution : } \frac{a+1}{2} = \frac{a+1}{2}$$

$$\frac{-1-a}{2} = \frac{a+1}{2}$$

$$2a = -2$$

$$a = -1$$

17. If  $(1 + i) (1 + 2i) (1 + 3i) \dots (1 + ni) = \alpha + i\beta$ , then  $2 \cdot 5 \cdot 10 \dots (1+n^2)$  is equal to (where  $\alpha, \beta, n \in \mathbb{R}$ )  
 (A)  $\alpha - i\beta$  (B)  $\alpha^2 - \beta^2$  (C)  $\alpha^2 + \beta^2$  (D) none of these

Key. C

Sol.  $(1 + i) (1 + 2i) (1 + 3i) \dots (1 + ni) = \alpha + i\beta$   
 $\alpha - i\beta = (1 - i) (1 - 2i) (1 - 3i) \dots (1 - ni)$   
 $\alpha^2 + \beta^2 = 2 \cdot 5 \cdot 10 \dots (1+n^2)$

18. If  $z = (1 + \sqrt{3}i)^{10} + (1 - \sqrt{3}i)^{10}$ , then  $\arg z$  is  
 (A)  $\frac{\pi}{2}$  (B)  $\pi$  (C)  $\frac{\pi}{4}$  (D) none of these

Key. B

Sol.  $z = a + \bar{a}$   
 $=$  always real  
 $\Rightarrow \arg z = 0$  or  $\pi$ .

19. If  $\alpha, \beta, \gamma$  are the cube roots of  $p (< 0)$ , then  $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$  for any  $x, y, z$  is equal to (where  $\omega$  is complex cube root of unity)  
 (A) 1 (B) 0 (C)  $\omega^2$  (D) 3

Key. C

Sol.  $x = -p$   
 $x^{1/3} = p^{1/3} (-1)^{1/3}$   
 $\alpha = -p^{1/3}$      $\beta = -p^{1/3}\omega$      $\gamma = -p^{1/3}\omega^2$

$$= \frac{1}{w} \frac{xw + yw^2 + z}{xw + yw^2 + z} = w^2$$

20. If the complex numbers  $z_1, z_2, z_3$  satisfying  $3z_1 = 5z_2 - 2z_3$ , then  $z_1, z_2$  and  $z_3$  lie in a  
 (A) circle (B) parabola (C) line (D) hyperbola

Key. C

Sol.  $3z_1 = 5z_2 - 2z_3$

$$z_1 = \frac{5z_2 - 2z_3}{5 - 2}$$

$\Rightarrow z_1$  divides line joining  $z_2$  and  $z_3$  externally in ratio 5 : 2

$\Rightarrow z_1, z_2, z_3$  are collinear.

21. If  $z_1$  and  $z_2$  are two complex numbers satisfying the equation  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$ , then  $\frac{z_1}{z_2}$  is a

number which is

- (A) Positive real (B) Negative real  
 (C) Zero or purely imaginary (D) None of these

Key. C

Sol.  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$

$$\left| \frac{\frac{z_1}{z_2} + 1}{\frac{z_1}{z_2} - 1} \right| = 1$$

$$\left| \frac{z_1}{z_2} + 1 \right| = \left| \frac{z_1}{z_2} - 1 \right|$$

$$\left| \frac{z_1}{z_2} + 1 \right| = \left| \frac{z_1}{z_2} - 1 \right|$$

$\Rightarrow \frac{z_1}{z_2} = 0$  or purely imaginary

22. If  $|z_1| = 2, |z_2| = 3, |z_3| = 4$  and  $|z_1 + z_2 + z_3| = 5$  then  $|4z_2z_3 + 9z_3z_1 + 16z_1z_2| =$

- a) 20 b) 24 c) 48 d) 120

Key. D

Sol.  $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$   
 $= |z_1 \bar{z}_1 \bar{z}_2 \bar{z}_3 + \bar{z}_2 \bar{z}_2 \bar{z}_3 \bar{z}_1 + \bar{z}_3 \bar{z}_3 \bar{z}_1 \bar{z}_2|$   
 $= ||z_1||z_2||z_3||z_1 + z_2 + z_3| = 120$

23. The value of  $\sin \left[ \log_e \left\{ \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)^z \right\} \right]$  is, where  $z$  satisfies the equation  $|z - 2i| = 1$

and has least modulus

(a) 1

(b) 0

(c) -1

(d)  $\frac{1}{2}$ .

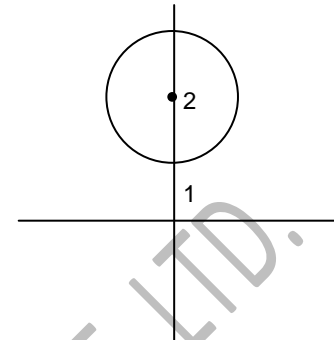
Key. C

Sol.

$$A = \sin \left[ \log \left\{ \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)^z \right\} \right]$$

$$= \sin \left[ \log e^{z\pi/2i} \right]$$

$$= \sin \left( \frac{z\pi}{2} i \right)$$



Again  $|z - 2i| = 1$  is a circle centered at  $(0, 2)$  with radius 1.

Therefore a point on circle of least modulus is  $z = i$ .

$\therefore$  By equation  $\theta$

$$A = \sin \left( -\frac{\pi}{2} \right)$$

$$= -1$$

24. If  $c^2 + s^2 = 1$ , then  $\frac{1+c+is}{1+c-is}$  is equal to

- (a)  $c - is$  (b)  $c + is$  (c)  $s + ic$  (d)  $s - ic$ .

Key. B

Sol.

$$\frac{1+c+is}{1+c-is} = \frac{(1+c)+is}{(1+c)-is} \times \frac{(1+c)+is}{(1+c)+is}$$

$$= \frac{((1+c)^2 - s^2) + i(s(1+c) + s(1+c))}{(1+c)^2 + s^2}$$

$$= \frac{(1+c^2 + 2c - s^2) + i(2s(1+c))}{(1+c)^2 + s^2}$$

$$= \frac{2c(c+1) + i2s(c+1)}{2+2c}$$

$$= c + is$$

25. If  $\omega \neq 1$  be a cube root of unity and  $(1 + \omega)^7 = l + m\omega$ , then the value of  $l + m =$

(a) 0

(b) 1

(c) 2

(d) -1

Key. C

Sol.  $\omega$  is cube root of unity

$$\therefore 1 + \omega + \omega^2 = 0$$

$$\Rightarrow 1 + \omega = -\omega^2$$

$$\text{Now if } (1 + \omega)^7 = l + m\omega$$

$$\Rightarrow (-\omega^2)^7 = l + m\omega$$

$$\Rightarrow -\omega^{14} = l + m\omega$$

$$\Rightarrow -\omega^{12} \cdot \omega^2 = l + m\omega$$

$$\Rightarrow -(\omega^3)^4 \omega^2 = l + m\omega$$

$$\Rightarrow -\omega^2 = l + m\omega$$

$$\Rightarrow 1 + \omega = l + m\omega$$

can comparison  $l = 1, m = 1$

26. One vertex of an equilateral triangle is at the origin and the other two vertices are, roots of

$$2z^2 + 2z + k = 0, \text{ then } k \text{ is}$$

- (A) 1                      (B)  $\frac{1}{3}$                       (C)  $\frac{2}{3}$                       (D)  $\frac{1}{2}$ .

Key. C

Sol.  $2z^2 + 2z + k = 0$

$$z = \frac{-2 \pm \sqrt{4 - 8k}}{4}$$

Since 'z' is a complex number

$4 - 8k$  will be negative

$$\Rightarrow k > \frac{1}{2}$$

$$(0, 0), \left(\frac{-1}{2}, \frac{\sqrt{2k-1}}{2}\right) \left(\frac{-1}{2}, -\frac{\sqrt{2k-1}}{2}\right)$$

Since triangle is equilateral

$$\therefore \frac{1}{4}(2k - 1) + \frac{1}{4} = (2k - 1)$$

$$\Rightarrow k = 2/3.$$

27. If  $\log_{\tan 30^\circ} \left( \frac{2|z|^2 + 2|z| - 3}{|z| + 1} \right) < -2$  then

- a)  $|z| < \frac{3}{2}$                       b)  $|z| > \frac{3}{2}$                       c)  $|z| > 2$                       d)  $|z| < 2$

Key. C

Sol.  $\log_{\tan 30^\circ} \left( \frac{2|z|^2 + 2|z| - 3}{|z| + 1} \right) < -2$

$$\Rightarrow \frac{2|z|^2 + 2|z| - 3}{|z| + 1} > 3$$

$$\Rightarrow ((|z| - 2)(2|z| + 3)) > 0$$

$$\Rightarrow |z| > 2$$



28. The number of common roots of the equations  $x^3 + 2x^2 + 2x + 1 = 0$  and  $x^{2012} + x^{2014} + 1 = 0$  is

- (a) 1 (b) 2 (c) 3 (d) 4

Key. D

Sol.  $x^3 + 2x^2 + 2x + 1 = 0 \Rightarrow x = -1, w, w^2$

But  $x = w, w^2$  will only satisfy  $x^3 + 2x^2 + 2x + 1 = 0$  and  $x^{2012} + x^{2014} + 1 = 0$ .

29. If  $|z| = \min \{|z-1|, |z+1|\}$  then  $|z + \bar{z}| =$

- (a) 1 (b) 2 (c) 3 (d) 4

Key. A

Sol. If  $|z| = |z-1|$

Then  $|z|^2 = |z-1|^2$

$\Rightarrow z + \bar{z} = 1$

If  $|z| = |z+1|$

Then  $|z|^2 = |z+1|^2$

$\Rightarrow z + \bar{z} = 1$

$\Rightarrow |z + \bar{z}| = 1$

30. If the roots of  $z^3 + iz^2 + 2i = 0$  represent the vertices of a  $DABC$  in the argand plane then the area of the triangle is (in square units)

- A) 3 B) 1 C) 4 D) 2

Key. D

Sol.  $(z-i)(z^2 + 2iz - 2) = 0 \Rightarrow z = i, 1-i, -1-i$

Area of  $DABC = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} = 2$  square units.

31. If  $n \geq 3$  and  $1, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_{n-1}$  are  $n$  roots of unity, then value of  $\sum_{1 \leq i < j \leq n-1} \alpha_i \alpha_j$  is

- (a) 0 (b) 1 (c) -1 (d)  $(-1)^n$

Key. B

Sol.  $x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2) \dots (x-\alpha_{n-1})$

$= x^n - x^{n-1}(1 + \alpha_1 + \dots + \alpha_{n-1}) + x^{n-2} \left( \sum_{i+j} \alpha_i \alpha_j + \alpha_1 + \alpha_2 + \dots + \alpha_{n-1} \right) + \dots - 1 = 0$

$\Rightarrow \sum_{i+j} \alpha_i \alpha_j + \alpha_1 + \alpha_2 + \dots + \alpha_n = 0$

$\sum_{i+j} \alpha_i \alpha_j = 1$

32. Let  $z = \cos \theta + i \sin \theta$ . Then, the value of  $\sum_{m=1}^{15} \operatorname{Im}(z^{2m-1})$  at  $\theta = 2^\circ$  is

- (A)  $\frac{1}{2^0}$       (B)  $\frac{1}{3 \sin 2^\circ}$       (C)  $\frac{1}{2 \sin 2^\circ}$       (D)  $\frac{1}{4 \sin 2^\circ}$

Key. D

Sol. Given that  $z = \cos \theta + i \sin \theta = e^{i\theta}$

$$\begin{aligned} \therefore \sum_{m=1}^{15} (z^{m-1}) &= \sum_{m=1}^{15} \operatorname{Im}(e^{i\theta})^{2m-1} \\ &= \sum_{m=1}^{15} \operatorname{Im} e^{i(2m-1)\theta} \\ &= \sin \theta + \sin 3\theta + \sin 5\theta + \dots + \sin 29\theta \\ &= \frac{\sin\left(\frac{\theta + 29\theta}{2}\right) \sin\left(\frac{15 \times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} \\ &= \frac{\sin(15\theta) \sin(15\theta)}{\sin \theta} = \frac{1}{4 \sin 2^\circ} \end{aligned}$$

33. If  $z_1$  is a root of the equation  $a_0 z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n = 3$ , where  $|a_i| < 2$  for  $i = 0, 1, \dots, n$ . Then

- (A)  $|z_1| > \frac{1}{3}$       (B)  $|z_1| < \frac{1}{4}$       (C)  $|z_1| > \frac{1}{4}$       (D)  $|z_1| < \frac{1}{3}$

Key. A

Sol.  $a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_{n-1} z + a_n = 3$

$$|3| = |a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_{n-1} z + a_n|$$

$$3 \leq |a_0| |z|^n + |a_1| |z|^{n-1} + \dots + |a_{n-1}| |z| + |a_n|$$

$$3 < 2(|z|^n + |z|^{n-1} + \dots + |z| + 1)$$

$$\frac{3}{2} < 1 + |z| + |z|^2 + \dots + |z|^n$$

$$\frac{1 - |z|^{n+1}}{1 - |z|} > \frac{3}{2}$$

$$2 - 2|z|^{n+1} < 3|z| - 1$$

$$3|z| - 1 > 0$$

$$|z| > \frac{1}{3}$$

34. If  $x = a+ib$  is a complex number such that  $x^2 = 3+4i$  and  $x^3 = 2+11i$  where  $i = \sqrt{-1}$  then  $a+b =$  \_\_\_\_\_

1. 1      2. 2      3. 3      4. 4

Key. 3







1.  $5\sqrt{2}$

2.5

3.  $\frac{5}{\sqrt{2}}$

4.  $\sqrt{2}$

Key. 1

Sol. A(5,-4) B(-3,2) subtends an angle  $\frac{\pi}{4}$  at C(z) on the circle hence  $\frac{\pi}{2}$  at centre

$$M \rightarrow M.dAB \therefore AM = \frac{AB}{2}$$

$$= \frac{\sqrt{64+36}}{2} = \frac{10}{2} = 5$$

$$\text{Radius} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

43.  $f(x) = 2x^3 + 2x^2 - 7x + 72$  then  $f\left(\frac{3-5i}{2}\right) = \underline{\hspace{2cm}}$

1. 1

2. 2

3. 3

4. 4

Key. 4

Sol. Let  $x = \frac{3-5i}{2}$

$$2x = 3 - 5i$$

$$(2x-3)^2 = 5i$$

$$4x^2 - 12x + 9 = 25i^2$$

$$\Rightarrow 2x^2 - 12x + 34 = 0 \Rightarrow 2x^2 - 6x + 17 = 0$$

$$2x^2 - 6x + 17 \cdot 2x^3 + 2x^2 = 7x + 72(x+4)$$

$$2x^3 - 6x^2 + 17x$$

---


$$8x^2 - 24x + 72$$

$$8x^2 - 24x + 68$$

---


$$4$$

44. If  $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$  then  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$

1.  $\frac{1}{2}$

2.  $\frac{3}{2}$

3. 4

4. 1

Key. 2

Sol. Let  $x = cis \alpha$      $y = cis \beta$      $z = cis \gamma$



C) circle  $x^2 + y^2 = 1$   
 joining  $(-1, 0)$  to  $(1, 0)$

D) line  $x = 0$  or on a line segment

Key: D

Hint:  $|z - |z + 1||^2 = |z + |z - 1||^2$   
 $\Rightarrow (z + \bar{z})(|z + 1| + |z - 1| - 2) = 0$   
 $\Rightarrow z$  lies on y-axis or  
 $Z$  lies on line segment joining the points  $(-1, 0)$  and  $(1, 0)$

48. If  $Z_1, Z_2$  are two complex numbers such that  $|Z_1| = 1, |Z_2| = 1$  then the maximum value of  $|Z_1 + Z_2| + |Z_1 - Z_2|$  is

- a) 2                                      b)  $2\sqrt{2}$                                       c) 4                                      d) none of these

Key: B

$Z_1 = \cos \alpha + i \sin \alpha, \quad Z_2 = \cos \beta + i \sin \beta$   
 $|Z_1 + Z_2| + |Z_1 - Z_2| = \sqrt{2 + 2 \cos(\alpha - \beta)} + \sqrt{2 - 2 \cos(\alpha - \beta)}$

Hint: let  $\alpha - \beta = \theta$

$2 \cos \frac{\theta}{2} + 2 \sin \frac{\theta}{2} = 2\sqrt{2} \sin \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$

49. If  $|z - 2| = \min \{|z - 1|, |z - 5|\}$ , where  $Z$  is a complex number then

- (A)  $\text{Re}(z) = \frac{3}{2}$  only                                      (B)  $\text{Re}(z) = \frac{7}{2}$  only  
 (C)  $\text{Re}(z) \in \left\{ \frac{3}{2}, \frac{7}{2} \right\}$                                       (D)  $\text{Re}(z) \in \left\{ \frac{1}{2}, \frac{3}{2} \right\}$

Key: C

Hint: draw the locus  $Z$  in argand plane.

$\text{Re}(z) \in \left\{ \frac{3}{2}, \frac{7}{2} \right\}$

50. If  $Z$  is a non-real complex number, then the minimum value of  $\frac{\text{Im} Z^5}{\text{Im}^5 Z}$  is

- (A) -1                                      (B) -2  
 (C) -4                                      (D) -5

Key: C

Hint: Let  $Z = a + ib, b \neq 0$  where  $\text{Im} Z = b$

$Z^5 = (a + ib)^5 = a^5 + {}^5C_1 a^4 bi + {}^5C_2 a^3 b^2 i^2 + {}^5C_3 a^2 b^3 i^3 + {}^5C_4 a i^4 + i^5 b^5$

$\text{Im} Z^5 = 5a^4 b - 10a^2 b^3 + b^5$

$y = \frac{\text{Im} Z^5}{\text{Im}^5 Z} = 5 \left( \frac{a}{b} \right)^4 - 10 \left( \frac{1}{b} \right)^2 + 1$



Let  $\left(\frac{a}{b}\right)^2 = x$  (say),  $x \in \mathbb{R}^+$

$$y = 5x^2 - 10x + 1 = 5[x^2 - 2x] + 1 = 5[(x-1)^2] - 4$$

Hence  $y_{\min} = -4$ .

51. Let  $z_r$  ( $1 \leq r \leq 4$ ) be complex numbers such that  $|z_r| = \sqrt{r+1}$  and

$$|30z_1 + 20z_2 + 15z_3 + 12z_4| = k |z_1z_2z_3 + z_2z_3z_4 + z_3z_4z_1 + z_4z_1z_2|$$

Then the value of k equals

- (A)  $|z_1z_2z_3|$  (B)  $|z_2z_3z_4|$   
 (C)  $|z_4z_1z_2|$  (D) None of these

Key: C

Hint: We have  $\left|\frac{z_1}{2} + \frac{z_2}{3} + \frac{z_3}{4} + \frac{z_4}{5}\right| = \frac{k}{60} |z_1z_2z_3z_4| \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4}\right|$

Now,  $z_1\bar{z}_1 = 2, z_2\bar{z}_2 = 3, z_3\bar{z}_3 = 4$  and  $z_4\bar{z}_4 = 5$

So,  $k = \frac{60}{|z_1z_2z_3z_4|} = \frac{60}{\sqrt{2}\sqrt{3}\sqrt{4}\sqrt{5}} = \sqrt{30} = |z_4z_1z_2|$

Note for objective take  $z_1 = \sqrt{2}; z_2 = \sqrt{3}; z_3 = 2; z_4 = \sqrt{5}$

52. If P(z) and A(z<sub>1</sub>) two be variable points such that  $zz_1 = |z|^2$  and  $|z - \bar{z}| + |z_1 + \bar{z}_1| = 10$  then area enclosed by the curve formed by them

- (A)  $25\pi$  (B)  $20\pi$   
 (C) 50 (D) 100

Key: C

53. A particle starts to travel from a point P on the curve  $C_1 : |z - 3 - 4i| = 5$ , where  $|z|$  is maximum. From P, the particle moves through an angle  $\tan^{-1} \frac{3}{4}$  in anticlock wise direction on  $|z - 3 - 4i| = 5$  and reaches at point Q. From Q, it comes down parallel to imaginary axis by 2 units and reaches at point R. Complex number corresponding to point R in the Argand plane is

- (A) (3+5i) (B) (3+7i) (C) (3+8i) (D) (3+9i)

Key: B

Hint:  $|z - 3 - 4i| = 5$

$$\Rightarrow (x-3)^2 + (y-4)^2 = 25$$

R is (3,7)

54. If  $|z_1| = 2$ ,  $|z_2| = 3$ ,  $|z_3| = 4$  and  $|2z_1 + 3z_2 + 4z_3| = 4$ , then absolute value of  $8z_2z_3 + 27z_3z_1 + 64z_1z_2$  equals
- (a) 24 (b) 48 (c) 72 (d) 96

Key: D

Hint:  $|8z_2z_3 + 27z_3z_1 + 64z_1z_2| =$

$$|z_1||z_2||z_3| \left| \frac{8}{z_1} + \frac{27}{z_2} + \frac{64}{z_3} \right| = (2)(3)(4) \left| \frac{8\bar{z}_1}{|z_1|^2} + \frac{27\bar{z}_2}{|z_2|^2} + \frac{64\bar{z}_3}{|z_3|^2} \right|$$

$$= 24|2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3| = 24|2z_1 + 3z_2 + 4z_3|$$

$$= (24)(4) = 96$$

55. If the ratio  $\frac{1-z}{1+z}$  is purely imaginary, then
- (a)  $0 < |z| < 1$  (b)  $|z| = 1$   
 (c)  $|z| > 1$  (d) bounds for  $|z|$  can not be decided

Key: b

Hint:  $0 = \frac{1-z}{1+z} + \frac{1-\bar{z}}{1+\bar{z}} = \frac{(1-z)(1+\bar{z}) + (1-\bar{z})(1+z)}{(1+z)(1+\bar{z})} = \frac{2(1-|z|^2)}{|1+z|^2} \Rightarrow |z| = 1$

56. If P and Q are represented by the numbers  $z_1$  and  $z_2$  such that  $\left| \frac{1}{z_2} + \frac{1}{z_1} \right| = \left| \frac{1}{z_2} - \frac{1}{z_1} \right|$ , then the circumcentre of  $\Delta OPQ$ , (where O is the origin) is
- (A)  $\frac{z_1 - z_2}{2}$  (B)  $\frac{z_1 + z_2}{2}$   
 (C)  $\frac{z_1 + z_2}{3}$  (D)  $z_1 + z_2$

Key : B

Sol :  $\left| \frac{1}{z_2} + \frac{1}{z_1} \right| = \left| \frac{1}{z_2} - \frac{1}{z_1} \right|$

$$\Rightarrow |z_1 + z_2| = |z_1 - z_2|$$

$$\Rightarrow z_1\bar{z}_2 + z_2\bar{z}_1 = 0$$

$$\Rightarrow \frac{z_1}{z_2} \text{ is purely imaginary}$$

$$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \pm \frac{\pi}{2}$$

$$\Rightarrow \angle POQ = \frac{\pi}{2}$$

Circumcentre of  $\Delta POQ$  is the mid point of PQ i.e.

57. If  $\alpha$  is non real root of  $x^7 = 1$ , then  $1 + 3\alpha + 5\alpha^2 + 7\alpha^3 + \dots + 13\alpha^6$  is equal to

- (A) 0  
 (B)  $\frac{14}{1-\alpha}$   
 (C)  $\frac{14}{\alpha-1}$   
 (D) none of these

Key: C

Hint: Let  $A = 1 + 3\alpha + 5\alpha^2 + 7\alpha^3 + \dots + 11\alpha^5 + 13\alpha^6$   
 $\alpha A = \alpha + 3\alpha^2 + 5\alpha^3 + 7\alpha^4 + \dots + 11\alpha^6 + 13\alpha^7$   
 $(1 - \alpha)A = 1 + 2\alpha + 2\alpha^2 + 2\alpha^3 + \dots + 2\alpha^6 - 13\alpha^7$   
 $= -12 + 2[\alpha + \alpha^2 + \dots + \alpha^6] = -14$   
 $A = -\frac{14}{1-\alpha}$

58. If  $z_1, z_2$  are two complex numbers satisfying the equation  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$ , then  $\frac{z_1}{z_2}$  is a number which is  
 (A) Positive real  
 (B) Negative real  
 (C) Zero  
 (D) Lying on imaginary axis

Key: D

Sol.  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1 \Rightarrow \frac{z_1 + z_2}{z_1 - z_2} = \cos \alpha + i \sin \alpha$

where  $\alpha$  is the argument of  $\frac{z_1 + z_2}{z_1 - z_2}$ . Applying componendo and dividendo, we get

$$\frac{z_1}{z_2} = \frac{1 + \cos \alpha + i \sin \alpha}{-1 + \cos \alpha + i \sin \alpha}$$

$$= \frac{2 \cos \left( \frac{\alpha}{2} \right) \left[ \cos \left( \frac{\alpha}{2} \right) + i \sin \left( \frac{\alpha}{2} \right) \right]}{2i \sin \left( \frac{\alpha}{2} \right) \left[ \cos \left( \frac{\alpha}{2} \right) + i \sin \left( \frac{\alpha}{2} \right) \right]} = -i \cot \left( \frac{\alpha}{2} \right)$$

Purely imaginary in nature

59. If  $z_1, z_2$  and  $z_3$  are the vertices of  $\Delta ABC$ , which is not right angled triangle taken in anti-clockwise direction and  $z_0$  is the circumcentre, then

$$\left( \frac{z_0 - z_1}{z_0 - z_2} \right) \frac{\sin 2A}{\sin 2B} + \left( \frac{z_0 - z_3}{z_0 - z_2} \right) \frac{\sin 2C}{\sin 2B}$$

is equal to

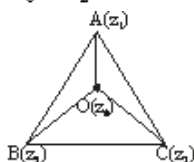
- A) 0                      B) 1                      C) -1                      D) 2

Key: C

Sol. Taking rotation at 'O'

$$\frac{z_0 - z_1}{z_0 - z_2} = \cos 2C - i \sin 2C$$

$$\frac{z_0 - z_3}{z_0 - z_2} = \cos 2A + i \sin 2A$$



$$\begin{aligned} \text{Now } & \left( \frac{z_0 - z_1}{z_0 - z_2} \right) \frac{\sin 2A}{\sin 2B} + \left( \frac{z_0 - z_3}{z_0 - z_2} \right) \frac{\sin 2C}{\sin 2B} \\ &= \frac{\sin 2A \cos 2C - i \sin 2A \sin 2C + \cos 2A \sin 2C + i \sin 2A \sin 2C}{\sin 2B} \\ &= \frac{\sin(2A + 2C)}{\sin 2B} = -1 \end{aligned}$$

60. If a complex number 'z' lies in the interior or on the boundary of a circle of radius 3 and centre at (-4, 0), then the greatest and least values of |z+1| are respectively
- A) 5, 0                      B) 6, 1                      C) 6, 0                      D) 5, 1

Key. C

Sol. It is given that  $|z+4| \leq 3$

Hence the greatest value of  $|z+1|$  is 6

Since the least value of the modulus of a complex number is zero, therefore

$$|z+1| = 0 \Rightarrow z = -1 \Rightarrow |z+4| = |-1+4| = 3$$

$$\Rightarrow |z+4| \leq 3 \text{ is satisfied by } z = -1$$

Therefore the least value of  $|z+1|$  is 0

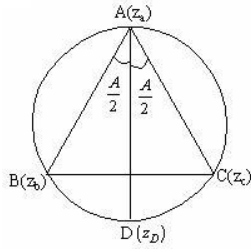
61.  $A(z_a), B(z_b), C(z_c)$  be the vertices of  $\Delta ABC$  taken in anticlockwise direction whose circum-circle is  $|z|=r$

If the internal angular bisector of angle A meets the circum circle again at  $D(z_D)$  then

- A)  $z_D = z_a z_c$                       B)  $z_D^2 = z_b z_c$                       C)  $z_D = \frac{z_b z_c}{z_a}$                       D)  $z_D = \frac{-z_b z_c}{z_a}$

Key. B

Sol. Let D represents the complex number  $z = (z_D)$



$$\angle BAD = \angle CAD = A/2$$

$$\frac{z - z_a}{|z - z_a|} = \frac{z_b - z_a}{|z_b - z_a|} e^{iA/2}$$

$$\frac{(z - z_a)^2}{|z - z_a|^2} = \frac{(z_b - z_a)^2}{|z_b - z_a|^2} e^{iA} \quad \text{----- (1)}$$

$$\text{Similarly } \frac{(z_c - z_a)^2}{|z_c - z_a|^2} = \frac{z - z_a}{|z - z_a|} e^{iA} \quad \text{----- (2)}$$

From (1) & (2)

$$\frac{z - z_a}{z - z_a} = \frac{z_b - z_a}{z_b - z_a} e^{iA}, \quad \frac{z_c - z_a}{z_c - z_a} = \frac{z - z_a}{z - z_a} e^{iA} \Rightarrow \frac{z - z_a}{z - z_a} \times \frac{z_c - z_a}{z_c - z_a} = \frac{z_b - z_a}{z_b - z_a} \frac{z - z_a}{z - z_a}$$

$$\Rightarrow \left( \frac{z - z_a}{z - z_a} \right)^2 = \frac{z_b - z_a}{z_b - z_a} \cdot \frac{z_c - z_a}{z_c - z_a}$$

$$\Rightarrow \left( \frac{\frac{z - z_a}{r^2}}{\frac{z - z_a}{r^2}} \right)^2 = \left( \frac{\frac{z_b - z_a}{r^2}}{\frac{z_b - z_a}{r^2}} \right) \left( \frac{\frac{z_c - z_a}{r^2}}{\frac{z_c - z_a}{r^2}} \right) \left( \because z_a, z_b, z_c \text{ and } z \text{ lie on } |z| = r \right)$$

$$\left( \because |z_a| = |z_b| = |z_c| = r \right)$$

$$\Rightarrow (z z_a)^2 = (z_a z_b)(z_a z_c) \Rightarrow z^2 = z_b z_c \Rightarrow z_D^2 = z_b z_c$$

62.

The least positive integer 'n' for which  $\left( \frac{1+i}{1-i} \right)^n = \frac{2}{\pi} \sin^{-1} \left( \frac{1+x^2}{2x} \right)$ , where  $x > 0$  and  $i = \sqrt{-1}$  is

A) 2

B) 4

C) 8

D) 12

Key. B

$$\because -1 \leq \frac{1+x^2}{2x} \leq 1$$

Sol.

$$\Rightarrow \left| \frac{1+x^2}{2x} \right| \leq 1 \Rightarrow \frac{1+x^2}{2|x|} \leq 1 \Rightarrow \frac{1+|x^2|}{2|x|} - 1 \leq 0 \Rightarrow \frac{(|x|-1)^2}{|x|} \leq 0$$

$$\because |x| > 0, \therefore (|x|-1)^2 \leq 0 \Rightarrow (|x|-1)^2 = 0$$

$$\therefore |x| = 1 \Rightarrow x = \pm 1$$

$$\therefore x = 1 (\because x > 0)$$

$$\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi} \cdot \sin^{-1}(1) = \frac{2}{\pi} \cdot \frac{\pi}{2} = 1 \Rightarrow \left(\frac{(1+i)^2}{2}\right)^n = 1$$

$$(i)^n = 1$$

$$\Rightarrow (-1)^{n/2} = (-1)^2, (-1)^4, (-1)^6, \dots \Rightarrow \frac{n}{2} = 2$$

$$\therefore n = 4 \text{ (least positive value)}$$

63. If 'a' is a complex number such that  $|a|=1$ , then the values of 'a', so that equation  $az^2 + z + 1 = 0$  has one purely imaginary root is

A)  $a = \cos \theta + i \sin \theta, \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$       B)  $a = \sin \theta + i \cos \theta, \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

C)  $a = \cos \theta + i \sin \theta, \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right)$       D)  $a = \sin \theta + i \cos \theta, \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right)$

Key. A

Sol.  $az^2 + z + 1 = 0 \dots (i)$

Taking conjugate of both sides,  $\overline{az^2 + z + 1} = \bar{0} \Rightarrow \bar{a}(\bar{z})^2 + \bar{z} + \bar{1} = 0$

$\bar{a}z^2 - z + 1 = 0$  (since  $\bar{z} = -z$  as 'z' is purely imaginary) .....(ii)

Eliminating 'z' from both the equations, we get  $(\bar{a}-a)^2 + 2(a+\bar{a}) = 0$

Let  $a = \cos \theta + i \sin \theta$  (since  $|a|=1$ ) so that  $(-2i \sin \theta)^2 + 2(2 \cos \theta) = 0$

$$\Rightarrow \cos \theta = \frac{-1 \pm \sqrt{1+4}}{2}$$

Only feasible value of  $\cos \theta$  is  $\frac{\sqrt{5}-1}{2}$

Hence  $a = \cos \theta + i \sin \theta$ , where  $\theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

64. The value of  $\sum_{k=1}^{10} \left( \sin \frac{2k\pi}{11} - i \cos \frac{2k\pi}{11} \right)$  is

- A) -1                      B) 0                      C) -i                      D) i

Key. D

Sol. 
$$= -i \left( \cos \frac{2k\pi}{11} + i \sin \frac{2k\pi}{11} \right) = -i \left( e^{i \frac{2\pi}{11}} \right)^k$$

Let  $e^{i \frac{2\pi}{11}} = z$

$$\begin{aligned} \therefore \sum_{k=1}^{10} \left( \sin \frac{2k\pi}{11} - i \cos \frac{2k\pi}{11} \right) &= -i \sum_{k=1}^{10} z^k \\ &= -i \left[ z + z^2 + z^3 + \dots + z^{10} \right] \\ &= -i \left[ \frac{z(z^{10} - 1)}{z - 1} \right] = -i \left[ \frac{z^{11} - z}{z - 1} \right] = i \end{aligned}$$

65. If 'z' lies on the circle  $|z - 2i| = 2\sqrt{2}$ , then the value of  $\arg \left( \frac{z-2}{z+2} \right)$  is equal to

- A)  $\frac{\pi}{3}$                       B)  $\frac{\pi}{4}$                       C)  $\frac{\pi}{6}$                       D)  $\frac{\pi}{2}$

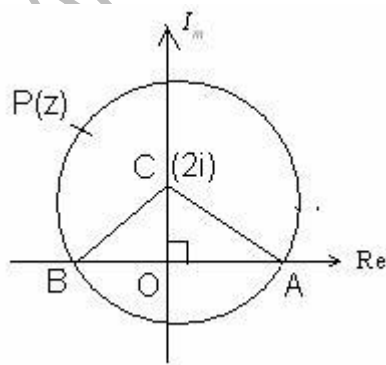
Key. B

Sol.  $CA = CB = 2\sqrt{2} \cdot OC = 2 \Rightarrow OA = OB = 2$   
 $\Rightarrow A = 2 + 0i, B = -2 + 0i$

Clearly,  $\angle BCA = \frac{\pi}{2}$

$\Rightarrow \angle BPA = \frac{\pi}{4}$

$\Rightarrow \arg \left( \frac{z-2}{z+2} \right) = \frac{\pi}{4}$



66. If 'z' is a complex number such that equation  $|z - a^2| + |z - 2a| = 3$  always represents an

ellipse, then range of  $a (\in \mathbb{R}^+)$  is

- A)  $(1, \sqrt{2})$       B)  $[1, \sqrt{3}]$       C)  $(-3, 1)$       D)  $(0, 3)$

Key. D

Sol.  $|a^2 - 2a| < 3$

$$\Rightarrow -3 < a^2 - 2a < 3 \Rightarrow -3 + 1 < a^2 - 2a + 1 < 3 + 1 \Rightarrow -2 < (a-1)^2 < 4$$

$$\therefore 0 \leq (a-1)^2 < 4 \Rightarrow -2 < a-1 < 2 \text{ or } -1 < a < 3$$

But  $a \in \mathbb{R}^+$

$$\therefore 0 < a < 3 \Rightarrow a \in (0, 3)$$

67.  $\omega$  is a non real complex cube root of unity and  $a, b \in \mathbb{R}$ . If  $\omega, \omega^2$  are roots of

$$\frac{1}{a+x} + \frac{1}{b+x} = \frac{3}{x} \text{ then } a, b \text{ are roots of}$$

- a)  $3x^2 - 6x + 2 = 0$       b)  $6x^2 - 3x + 2 = 0$   
 c)  $2x^2 - 3x + 6 = 0$       d)  $6x^2 - 2x + 3 = 0$

Key. B

Sol. The given equation simplifies  $x^2 + 2x(a+b) + 3ab = 0$ , whose roots are given table  $\omega, \omega^2$

$$\text{Hence } a+b = \frac{1}{2}, ab = \frac{1}{3}. \text{ So } a, b \text{ are roots of } x^2 - x\left(\frac{1}{2}\right) + \frac{1}{3} = 0$$

68. If  $z$  is a complex number such that  $|z-1|=1$  then  $\arg\left(\frac{1}{z} - \frac{1}{2}\right)$  may be

- a)  $\frac{\pi}{6}$       b)  $-\frac{\pi}{2}$       c)  $\frac{\pi}{4}$       d)  $-\frac{\pi}{4}$

Key. B

Sol. Since  $|z-1|=1 \Rightarrow z-1 = cis\theta \Rightarrow z = (1 + \cos\theta) + i \sin\theta = 2 \cos\frac{\theta}{2} cis\frac{\theta}{2}$

$$\therefore \frac{1}{z} - \frac{1}{2} = \frac{cis\left(-\frac{\theta}{2}\right)}{2 \cos\frac{\theta}{2}} - \frac{1}{2} = -\frac{i}{2} \tan\frac{\theta}{2} \text{ which is purely imaginary}$$

69.  $\theta \in [0, 2\pi]$  and  $z_1, z_2, z_3$  are three complex numbers such that they are collinear and  $(1 + |\sin\theta|)z_1 + (|\cos\theta| - 1)z_2 - \sqrt{2}z_3 = 0$ . If at least one of the complex numbers  $z_1, z_2, z_3$  is non-zero then number of possible values of  $\theta$  is



a) Infinite

b) 4

c) 2

d) 8

Key. B

Sol. If  $z_1, z_2, z_3$  are collinear and  $az_1 + bz_2 + cz_3 = 0$  then  $a+b+c=0$ . Hence

$$1 + |\sin \theta| + |\cos \theta| - 1 - \sqrt{2} = 0 \Rightarrow |\sin \theta| + |\cos \theta| = \sqrt{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

70. Let  $A(z_1), B(z_2), C(z_3)$  be the vertices of a triangle oriented in anti clock wise direction.

If  $BC : CA : AB = 2 : \sqrt{2} : \sqrt{3} + 1$ , then the imaginary part of  $\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4$  is

A) 0

B)  $-7 + 2\sqrt{6}$

C)  $7 - 2\sqrt{6}$

D) cannot be determined

Key. A

Sol.  $\cos A = +\frac{1}{\sqrt{2}} \Rightarrow A = \frac{\pi}{4}$

$$\therefore \frac{z_3 - z_1}{z_2 - z_1} = \left| \frac{z_3 - z_1}{z_2 - z_1} \right| \text{cis}(\pi/4)$$

$$\Rightarrow \left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4 = \left(\frac{\sqrt{2}}{\sqrt{3} + 1}\right)^4 e^{i\pi} \Rightarrow \left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4 = -\left(\frac{\sqrt{3} - 1}{2}\right)^4$$

71. A, B, C are vertices of a triangle inscribed in the circle  $|z|=1$ . Altitude from A meets the circumcircle again at D. If D, B, C represents the complex number  $z_1, z_2, z_3$  respectively then the complex number representing the reflection of D in the line BC, is

A)  $\frac{z_1 z_2 + z_1 z_3 + z_2 z_3}{z_1}$

B)  $\frac{z_1 z_2 + z_2 z_3 + z_1 z_3}{z_1 z_2 z_3}$

C)  $\frac{z_1 z_2 + z_1 z_3 - z_2 z_3}{z_1}$

D)  $\frac{z_1 z_2 + z_1 z_3 - z_2 z_3}{z_1 z_2 z_3}$

Key. C

Sol. image of D w.r.t sides of triangle is orthocenter

72. A point P representing the complex number  $z$  moves in the Argand plane so that it lies always in the region defined by  $|z-1| \leq |z-i|$  and  $|z-2-2i| \leq 1$ . If P describes the boundary of this

region then the value of  $|z|$  when the  $\arg(z)$  has least value, is

- A)  $\sqrt{5}$                       B) 7                      C)  $\sqrt{7}$                       D) 5

Key. C

Sol.  $|z| = OR = \sqrt{8-1} = \sqrt{7}$

73. Let  $P(z)$  be a variable point in the complex plane such that  $|z| = \min\{|z-1|, |z+1|\}$  then the value of  $(z + \bar{z})$  is

- A) 1 if  $\operatorname{Re}(z) > 0$                       B) 1 if  $\operatorname{Re} z < 0$                       C) 0 if  $\operatorname{Re} z > 0$                       D) 0 if  $\operatorname{Re} z < 0$

Key. A

Sol. Let  $|z| = |z-1|$  if  $\operatorname{Re} Z > 0$

$$\Rightarrow \text{lies line } z = \frac{1}{2}$$

$$\Rightarrow z + \bar{z} = \frac{1}{2} + \frac{1}{2} = 1$$

74. If  $z$  is a complex number satisfying  $|z|^2 + 2(z + \bar{z}) + 3i(z - \bar{z}) + 4 = 0, i = \sqrt{-1}$ , then the complex number  $z + 3 + 2i$  will lie on a circle with

- A) centre  $1 - 5i$ , radius 4                      B) centre  $1 + 5i$ , radius 4  
C) centre  $1 + 5i$ , radius 3                      D) centre  $1 - 5i$ , radius 3

Key. C

Sol. Given  $|z + (2 - 3i)| = 3$ , Let  $w = (z + 3 + 2i) = z + 2 - 3i + 1 + 5i$

$$\Rightarrow |w - (1 + 5i)| = |z + 2 - 3i| = 3.$$

75. The value of  $i \log_e(x-i) + i^2 \pi + i^3 \log_e(x+i) + i^4 (2 \tan^{-1} x), x > 0, i = \sqrt{-1}$  is

- A) 0                      B) 1                      C) 2                      D) 3

Key. A

Sol. Let  $i \log \frac{x-i}{x+i} - \pi + 2 \tan^{-1} x = k$

$$\Rightarrow \log \left( \frac{x+i}{x-i} \right) = (k + \pi - 2 \tan^{-1} x) i = i\theta$$

$$\Rightarrow \frac{x+i}{x-i} = e^{i\theta} \Rightarrow x = \cot \frac{\theta}{2} \Rightarrow \theta = 2 \cot^{-1} x$$

$$\therefore k + \pi - 2 \tan^{-1} x = 2 \cot^{-1} x \Rightarrow k = 0$$

76. If  $\left| \frac{z_1}{z_2} \right| = 1$  and  $\arg(z_1 z_2) = 0$ , then

- A)  $z_1 = z_2$                       B)  $|z_2|^2 = z_1 z_2$                       C)  $z_1 z_2 = 1$                       D)  $z_1 z_2 = 2$

Key. B

Sol.  $\left| \frac{z_1}{z_2} \right| = 1 \Rightarrow |z_1| = |z_2| = r_1$  as  $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$

$$\arg(z_1 z_2) = 0 \Rightarrow z_2 = r_1(\cos(-\theta_1) + i \sin(-\theta_1))$$

$$\Rightarrow z_2 = \bar{z}_1 \Rightarrow \bar{z}_2 = z_1 \Rightarrow |z_2|^2 = z_1 \cdot z_2$$

77. Let  $z$  be a complex number and  $a_k, b_k (k=1,2,3)$  are real numbers then the value of

$$\begin{vmatrix} a_1 z + b_1 \bar{z} & a_2 z + b_2 \bar{z} & a_3 z + b_3 \bar{z} \\ b_1 z + a_1 \bar{z} & b_2 z + a_2 \bar{z} & b_3 z + a_3 \bar{z} \\ b_1 z + a_1 & b_2 z + a_2 & b_3 z + a_3 \end{vmatrix} =$$

- A)  $(a_1 a_2 a_3 + b_1 b_2 b_3) |z|^2$                       B)  $(a_1 a_2 a_3 - b_1 b_2 b_3) |z|^2$   
 C)  $a_1^2 - a_2^2$                       D)  $|z|^2$

Key. C

Sol.  $\begin{vmatrix} z & \bar{z} & 1 \\ \bar{z} & z & 1 \\ 1 & z & \bar{z} \end{vmatrix} \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$

78. If  $z_1, z_2, z_3$  are the vertices of an equilateral triangle inscribed in the circle  $|z| = 1$  then area of region common to given triangle and another triangle having vertices  $-z_1, -z_2, -z_3$ , is

- A)  $\frac{\sqrt{3}}{2}$                       B)  $\frac{\sqrt{3}}{4}$                       C)  $\frac{7\sqrt{3}}{4}$                       D)  $\frac{5\sqrt{3}}{4}$

Key. A

Sol. Area of common region

$$= \text{Area of } \triangle ABC - 3 \text{ Area of } \triangle A'B'C'$$

$$= 3 \frac{\sqrt{3}}{4} - 3 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}$$

79. If  $az^2 + bz + 1 = 0$ ,  $a, b, z \in C$  and  $|a| = \frac{1}{2}$ , have a root  $\alpha$  such that  $|\alpha| = 1$  then  $|a\bar{b} - b| =$

A)  $\frac{1}{4}$

B)  $\frac{1}{2}$

C)  $\frac{5}{4}$

D)  $\frac{3}{4}$

Key. D

Sol.  $a\alpha^2 + b\alpha + 1 = 0$

$$\bar{a}\bar{\alpha}^{-2} + \bar{b}\bar{\alpha} + 1 = 0$$

$$\Rightarrow \alpha^2 + \bar{b}\alpha + \bar{a} = 0$$

$$\frac{\alpha^2}{\bar{a}\bar{b} - \bar{b}} = \frac{\alpha}{1 - |\alpha|^2} = \frac{1}{\bar{a}\bar{b} - b} \Rightarrow |a\bar{b} - b| = 1 - |\alpha|^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

80. If  $|z-1| \leq 2$  &  $|wz-1-w^2| = a$  (where 'w' is a cube root of unity) then complete set of values of a is

a)  $0 \leq a \leq 2$

b)  $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$

c)  $\frac{\sqrt{3}}{2} - \frac{1}{2} \leq a \leq \frac{1}{2} + \frac{\sqrt{3}}{2}$

d)  $0 \leq a \leq 4$

Key. D

Sol.  $|wt-1-w^2| = a$

$$\Rightarrow |w||z+1| = a$$

$$\Rightarrow |z-1| + 2 \geq a$$

81. If z is a complex number having least absolute value and  $|z-2+2i| = 1$  then z =

a)  $\left(2 - \frac{1}{\sqrt{2}}\right)(1-i)$

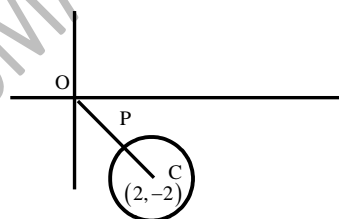
b)  $\left(2 - \frac{1}{\sqrt{2}}\right)(1+i)$

c)  $\left(2 + \frac{1}{\sqrt{2}}\right)(1-i)$

d)  $\left(2 + \frac{1}{\sqrt{2}}\right)(1+i)$

Key. A

Sol.  $OP = OC - CP$



$$= 2\sqrt{2} - 1$$

$$\therefore (0,0) (2,-2)$$

$$2\sqrt{2} - 1 : 1$$





$$z_1 + z_2 = (-a) \text{ \& } z_1 z_2 = b$$

$$-2\cos\left(\frac{\phi_1 - \phi_2}{2}\right) \left[ \cos\frac{\phi_1 + \phi_2}{2} + i \sin\frac{\phi_1 + \phi_2}{2} \right] = a$$

$$\Rightarrow \arg(a) = \frac{\phi_1 + \phi_2}{2}$$

$$\arg b = \phi_1 + \phi_2$$

$$\therefore \arg b = 2 \arg a$$

$$\text{Also } |z_1 z_2| = |b| = 1 \text{ and } |a| \leq 2$$

89. If  $|z - i| = 1$  and  $\arg(z) = \theta$  where  $\theta \in \left(0, \frac{\pi}{2}\right)$ , then  $\cot \theta - \frac{2}{z}$  equals

a)  $2i$

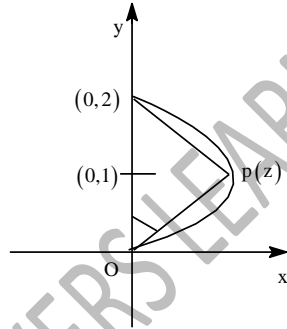
b)  $3i$

c)  $i$

d)  $-i$

Key. C

Sol.  $\angle \text{AOP} = \frac{\pi}{2} - \theta$



$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta = \frac{|z|}{2}$$

$$\text{Also } \frac{2i}{z} = \frac{OA}{OP} (\sin \theta + i \cos \theta) = \frac{2}{|z|} (\sin \theta + i \cos \theta)$$

$$= 1 + i \cot \theta$$

$$\frac{2}{z} = -i + \cot \theta$$

$$\Rightarrow \cot \theta - \frac{2}{z} = i$$

90. Let  $z = \frac{z_1 - z_2}{z_1 z_2 - 1}$ ,  $z_1 \neq \frac{1}{z_2}$ ,  $0 < |z_2| < 1$ . If  $|z| \leq 1$  then

a)  $|z_1| > 1$

b)  $|z_1| \leq 1$

c)  $2 < |z_1| < 3$

d)  $2 < |z_1| < 8$

Key. B

Sol.  $\bar{z}z - 1 = \left(\frac{z_1 - z_2}{z_1 z_2 - 1}\right) \left(\frac{\bar{z}_1 - \bar{z}_2}{\bar{z}_1 \bar{z}_2 - 1}\right) - 1$



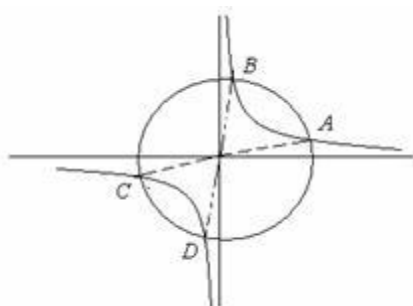


D)  $\arg z_1 + \arg z_2 + \arg z_3 + \arg z_4 = 2k\pi, k = 0, 1 \text{ or } -1$

Key. B

Sol.  $z^2 = \left(\frac{\bar{z}}{z}\right)^2 + 4i \Rightarrow \left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right) = 1 \text{ or } xy = 1 \text{ (where } z = x + iy \text{)}$

The circle  $x^2 + y^2 = 4$  intersects the rectangular hyperbola in four points, which are symmetrical about the origin in parts.



94. If  $a_1, a_2, \dots, a_n$  are real numbers with  $a_n \neq 0$  and  $\cos \alpha + i \sin \alpha$  is a root of  $z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_{n-1} z + a_n = 0$ , then the sum  $a_1 \cos \alpha + a_2 \cos 2\alpha + a_3 \cos 3\alpha + \dots + a_n \cos n\alpha$  is

- A) 0                                      B) 1                                      C) -1                                      D)  $\frac{1}{2}$

Key. C

Sol.  $\cos \alpha + i \sin \alpha$  is a root of  $a_n \left(\frac{1}{z}\right)^n + a_{n-1} \left(\frac{1}{z}\right)^{n-1} + \dots + a_2 \left(\frac{1}{z}\right)^2 + a_1 \left(\frac{1}{z}\right) + 1 = 0$ .  
Equating real parts on both sides,  
 $a_n \cos n\alpha + a_{n-1} \cos (n-1)\alpha + \dots + a_1 \cos \alpha + 1 = 0$

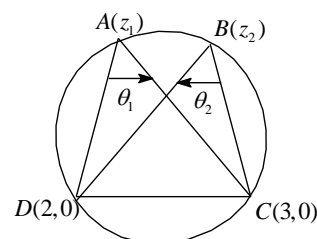
95. If  $\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right) = k$ , then points  $A(z_1), B(z_2), C(3,0)$  and  $D(2,0)$  (taken in clockwise sense) will

- A) lie on a circle only for  $k > 0$                                       B) lie on a circle only for  $k < 0$   
C) lie on a circle  $\forall k \in R$                                       D) be vertices of a square  $\forall k \in (0,1)$

Key: A

Sol :  $\arg \left(\frac{3-z_1}{2-z_1}\right) + \arg \left(\frac{2-z_2}{3-z_2}\right)$   
 $= \arg \left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right)$

If  $k > 0$ , its argument will be zero



So,  $\theta_1$  &  $\theta_2$  are equal in magnitude but opposite sign.

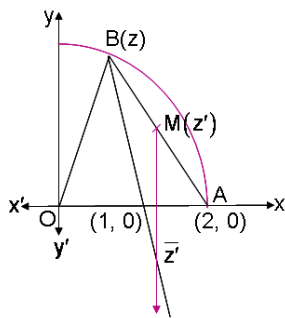
So DC subtends equal angle at A & B. So, points are concyclic if  $k > 0$

96. If A(2,0) and B(z) are two points on the circle  $|z| = 2$ . M(z') is the point on AB. If the point  $\bar{z}'$  lies on the median of the triangle OAB where O is origin then  $\arg(z')$  is

- a)  $\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$       b)  $\tan^{-1}(\sqrt{15})$       c)  $\tan^{-1}\left(\frac{5}{\sqrt{15}}\right)$       d)  $\frac{\pi}{2}$

Key: A

Sol: M(z') is mid-point of AB, so  $z' = \frac{z+2}{2}$



$$\Rightarrow \bar{z}' = \frac{\bar{z}+2}{2}$$

$$\Rightarrow z, 1, \frac{\bar{z}}{2}+1 \text{ are collinear}$$

$$\Rightarrow \begin{vmatrix} z & \bar{z} & 1 \\ \frac{\bar{z}}{2}+1 & \frac{z}{2}+1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow z\left(\frac{z}{2}+1-1\right) - \bar{z}\left(\frac{\bar{z}}{2}+1-1\right) + 1\left(\frac{\bar{z}}{2} - \frac{z}{2}\right) = 0$$

$$\Rightarrow \frac{z^2}{2} - \frac{\bar{z}^2}{2} + \frac{(\bar{z}-z)}{2} = 0$$

$$\Rightarrow (z-\bar{z})(z+\bar{z}-1) = 0$$

$$\Rightarrow z-\bar{z} = 0 \text{ or } (z+\bar{z}-1) = 0$$

$$\Rightarrow z+\bar{z} = 1 \text{ or } \operatorname{Re}(z) = \frac{1}{2}$$

$$|z| = 2 \Rightarrow \frac{1}{4} + \operatorname{Im}(z)^2 = 4$$

$$\Rightarrow \operatorname{Im}(z) = \frac{\sqrt{15}}{2}$$

$$1. \quad z = \frac{1}{2} + \frac{i\sqrt{15}}{2}$$

$$2. \quad \arg(z') = \tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$$

97. If the tangents at  $z_1, z_2$  on the circle  $|z - z_o| = r$  intersect at

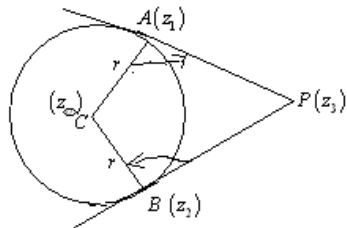
$z_3$ , then  $\frac{(z_3 - z_1)(z_o - z_2)}{(z_o - z_1)(z_3 - z_2)}$  equals

- a) 1                                      b) -1                                      c) i                                      d) -i

Key: B

Hint:  $\frac{z_3 - z_1}{z_o - z_1} = \left(\frac{PA}{AC}\right) i$  and  $\frac{z_o - z_2}{z_3 - z_2} = \left(\frac{BC}{BP}\right) (i)$

$$\frac{(z_3 - z_1)(z_o - z_2)}{(z_o - z_1)(z_3 - z_2)} = \left(\frac{PA}{AC} \times \frac{BC}{PB}\right) (-1) = -1$$



98. If  $Z$  is a complex number then the number of complex numbers satisfying the equation  $Z^{2009} = \bar{Z}$  is

- A) 3                                      B) 2009                                      C) 2010                                      D) 2011

Key: D

Sol.  $Z^{2009} = \bar{Z} \Rightarrow |Z| = 0$  or  $|Z| = 1$

99. If  $a_1, a_2, \dots, a_n$  are real numbers with

$a_n \neq 0$  and  $\cos \alpha + i \sin \alpha$  is a root of  $z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_{n-1} z + a_n = 0$ , then the

sum  $a_1 \cos \alpha + a_2 \cos 2\alpha + a_3 \cos 3\alpha + \dots + a_n \cos n\alpha$  is

- a) 0                                      b) 1                                      c) -1                                      d) 1/2

Key: C

Sol.  $\cos \alpha + i \sin \alpha$  is a root of  $a_n \left(\frac{1}{z}\right)^n + a_{n-1} \left(\frac{1}{z}\right)^{n-1} + \dots + a_2 \left(\frac{1}{z}\right)^2 + a_1 \left(\frac{1}{z}\right) + 1 = 0$ . Equating real parts on both sides,  $a_n \cos n\alpha + a_{n-1} \cos(n-1)\alpha + \dots + a_1 \cos \alpha + 1 = 0$ .







$$\frac{\pi}{2} + 2 \arg w = \pi$$

$$2 \arg w = \frac{\pi}{2}$$

$$\arg w = \frac{\pi}{4} \Rightarrow \arg(z) = \frac{3\pi}{4}$$

111. If A ( $Z_1$ ) B( $Z_2$ ) C( $Z_3$ ) are vertices of a triangle such that

$$Z_3 = \left( \frac{Z_2 - iZ_1}{1 - i} \right) \text{ and } |Z_1| = 3, |Z_2| = 4 \text{ and } |Z_2 + iZ_1| = |Z_1| + |Z_2| \text{ then area of triangle ABC is}$$

1.  $\frac{5}{2}$

2. 0

3.  $\frac{25}{2}$

4.  $\frac{25}{4}$

Key. 4

Sol.  $|z_2 + iz_1| = |z_1| + |z_2| \Rightarrow z_2, iz_1, 0$  are collinear.

$$\therefore \arg(iz_1) = \arg z_2$$

$$\Rightarrow \arg i + \arg z_1 = \arg z_2$$

$$\Rightarrow \arg z_2 - \arg z_1 = \frac{\pi}{2}$$

$$z_3 = \frac{z_2 - iz_1}{1 - i}$$

$$(1 - i)z_3 = z_2 - iz_1$$

$$z_3 - z_2 = i(z_3 - z_1)$$

$$\frac{z_3 - z_2}{z_3 - z_1} = i \Rightarrow \arg\left(\frac{z_3 - z_2}{z_3 - z_1}\right) = \frac{\pi}{2} \text{ and } |z_3 - z_2| = |z_3 - z_1|$$

$$\therefore AB=BC, \therefore AB^2 = AC^2 + BC^2$$

$$25 = 2AC^2$$

$$\Rightarrow AC = \frac{5}{\sqrt{2}}$$

$$\text{Required area} = \frac{1}{2} \times \frac{5}{\sqrt{2}} \times \frac{5}{\sqrt{2}} = \frac{25}{4} \text{ sq. units}$$

112. The radius of the circle given by  $\arg\left(\frac{Z - 5 + 4i}{Z + 3 - 2i}\right) = \frac{\pi}{4}$

1.  $5\sqrt{2}$

2. 5

3.  $\frac{5}{\sqrt{2}}$

4.  $\sqrt{2}$

Key. 1

Sol. A(5,-4) B(-3,2) subtends an angle  $\frac{\pi}{4}$  at C(z) on the circle hence  $\frac{\pi}{2}$  at centre

$$M \rightarrow M.dAB \therefore AM = \frac{AB}{2}$$

$$= \frac{\sqrt{64+36}}{2} = 5$$

$$\text{Radius} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

113.  $f(x) = 2x^3 + 2x^2 - 7x + 72$  then  $f\left(\frac{3-5i}{2}\right) = \underline{\hspace{2cm}}$

1. 1

2. 2

3. 3

4. 4

Key. 4

Sol. Let  $x = \frac{3-5i}{2}$

$$2x = 3 - 5i$$

$$(2x - 3)^2 = 5i$$

$$4x^2 - 12x + 9 = 25i^2$$

$$\Rightarrow 2x^2 - 12x + 34 = 0 \Rightarrow 2x^2 - 6x + 17 = 0$$

$$2x^2 - 6x + 17) 2x^3 + 2x^2 = 7x + 72(x + 4$$

$$2x^3 - 6x^2 + 17x$$

---


$$8x^2 - 24x + 72$$

$$8x^2 - 24x + 68$$

---

4

114. If  $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$  then  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$

\_\_\_\_\_

1.  $\frac{1}{2}$

2.  $\frac{3}{2}$

3. 4

4. 1

Key. 2

Sol. Let  $x = cis \alpha$      $y = cis \beta$      $z = cis \gamma$

$$\text{Clearly } x + y + z = 0, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

$$x^2 + y^2 + z^2 = (x + y + z)^2 - 2xyz \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = 0$$

$$= cis 2\alpha + cis 2\beta + cis 2\gamma = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$$





Key. A

Sol.  $a_0z^n + a_1z^{n-1} + a_2z^{n-2} + \dots + a_{n-1}z + a_n = 3$   
 $|3| = |a_0z^n + a_1z^{n-1} + a_2z^{n-2} + \dots + a_{n-1}z + a_n|$   
 $3 \leq |a_0||z|^n + |a_1||z|^{n-1} + \dots + a_{n-1}|z| + |a_n|$   
 $3 < 2(|z|^n + |z|^{n-1} + \dots + |z| + 1)$   
 $\frac{3}{2} < 1 + |z| + |z|^2 + \dots + |z|^n$   
 $\frac{1 - |z|^{n+1}}{1 - |z|} > \frac{3}{2}$   
 $2 - 2|z|^{n+1} < 3|z| - 1$   
 $3|z| - 1 > 0$   
 $|z| > \frac{1}{3}$

118. If  $n \geq 3$  and  $1, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_{n-1}$  are  $n$  roots of unity, then value of  $\sum_{1 \leq i < j \leq n-1} \alpha_i \alpha_j$  is  
 (a) 0 (b) 1 (c) -1 (d)  $(-1)^n$

Key. B

Sol.  $x^n - 1 = (x - 1)(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1})$   
 $= x^n - x^{n-1}(1 + \alpha_1 + \dots + \alpha_{n-1}) + x^{n-2} \left( \sum_{i+j} \alpha_i \alpha_j + \alpha_1 + \alpha_2 + \dots + \alpha_{n-1} \right) + \dots - 1 = 0$   
 $\Rightarrow \sum_{i+j} \alpha_i \alpha_j + \alpha_1 + \alpha_2 + \dots + \alpha_{n-1} = 0$   
 $\sum_{i+j} \alpha_i \alpha_j = 1$

119. If the equation  $z^2 + z + \alpha = 0$  has a purely imaginary root and  $\alpha$  lies on the circle  $|z| = 1$  then the imaginary part of that root, is (are)

- (A)  $\pm\sqrt{2}$  (B) 0  
 (C)  $\pm\sqrt{2 - \sqrt{2}}$  (D)  $\pm\sqrt{\frac{\sqrt{5} - 1}{2}}$

Key. D

Sol. Let  $z = i\beta$  ( $\beta \in \mathbb{R}$ ) be a root, then  
 $-\beta^2 + i\beta + \alpha = 0 \Rightarrow \alpha = \beta^2 - i\beta$   
 Now as  $|\alpha| = 1$   
 $\Rightarrow \beta^4 + \beta^2 = 1 \Rightarrow \beta^2 = \frac{-1 + \sqrt{5}}{2}$

120. Let  $z(\alpha, \beta) = \cos\alpha + e^{i\beta} \sin\alpha$  ( $\alpha, \beta \in \mathbb{R}, i = \sqrt{-1}$ ) then the exhaustive set of values of modulus of  $z(\theta, 2\theta)$ , as  $\theta$  varies, is

- (A)  $[0, 1]$  (B)  $[0, \sqrt{2}]$   
 (C)  $[1, 2]$  (D)  $[\sqrt{2}, 2]$

Key. B

Sol.  $|z| \theta, 2\theta = |\cos\theta + e^{i2\theta} \sin\theta|$   
 $= |\cos\theta + \sin\theta \cos 2\theta + i \sin\theta \sin 2\theta| = \sqrt{(\cos\theta + \sin\theta \cos 2\theta)^2 + \sin^2 \theta \cos^2 \theta}$   
 $= \sqrt{1 + \sin 4\theta} \in [0, \sqrt{2}]$

121. If  $|z| = 1$  and  $z \neq \pm 1$  then one of the possible values of  $\arg(z) - \arg(z + 1) - \arg(z - 1)$ , is  
 (A)  $-\pi/6$  (B)  $\pi/3$   
 (C)  $-\pi/2$  (D)  $\pi/4$

Key. C

Sol.  $\arg(z) - \arg(z + 1) - \arg(z - 1) = \arg\left(\frac{z}{z^2 - 1}\right) = \arg\left(\frac{z}{z^2 - z\bar{z}}\right)$   
 $= \arg\left(\frac{1}{z - \bar{z}}\right) = \arg(\text{purely imaginary no.})$

122. If  $z_1, z_2, z_3$  are three distinct complex numbers and  $a, b, c$  are three positive real numbers such that

$$\frac{a}{|z_2 - z_3|} = \frac{b}{|z_3 - z_1|} = \frac{c}{|z_1 - z_2|} \text{ then } \frac{a^2}{z_2 - z_3} + \frac{b^2}{z_3 - z_1} + \frac{c^2}{z_1 - z_2} \text{ is}$$

- a)  $3abc$  b)  $(abc)^3$  c)  $a + b + c$  d)  $0$

Key. D

Sol.  $\frac{a}{|z_2 - z_3|} = \lambda \Rightarrow \frac{a^2}{z_2 - z_3} = \lambda^2 (\bar{z}_2 - \bar{z}_3)$  etc

123. If  $|z_1| = 2, |z_2| = 3, |z_3| = 4$  and  $|2z_1 + 3z_2 + 4z_3| = 4$  then the absolute value of  $8z_2z_3 + 27z_3z_1 + 64z_1z_2$  equals  
 (A) 24 (B) 48 (C) 72 (D) 96

Key. D

SOL.  $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$   
 $= |z_1z_2z_3| \left| \frac{8}{z_1} + \frac{27}{z_2} + \frac{64}{z_3} \right|$   
 $= 24 |2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3|$   
 $= 24 \times 4 = 96$

124. If  $\omega$  is a cube root of unity, then  $\omega + \omega^{\left(\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \dots + \infty\right)}$  is  
 (A)  $i$  (B)  $i^2$   
 (C)  $0$  (D) none of these

Key. B

Sol.  $\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \dots + \infty = \frac{1}{2} \left( 1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \dots + \infty \right)$

$$= \frac{1}{2} \left( \left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots \infty \right) = \frac{1}{2} \left( \frac{1}{1 - \frac{3}{4}} \right) = \frac{1}{2} \cdot \frac{1}{\frac{1}{4}} = 2$$

So expression =  $\omega + \omega^2 = -1 = i^2$ .

125. Let P be a point on the circumcircle of the triangle whose vertices A, B, C (P, A, B, C are in order) are represented by the complex numbers  $\omega^2, 2i\omega$  and  $-4$  ( $\omega$  is a non real cube root of unity) respectively such that  $PA \cdot BC = PC \cdot AB$  then the complex number associated with the mid-point of PB is

- (A)  $\omega - 1$  (B) 0  
 (C)  $-i$  (D)  $\omega - \omega^2$

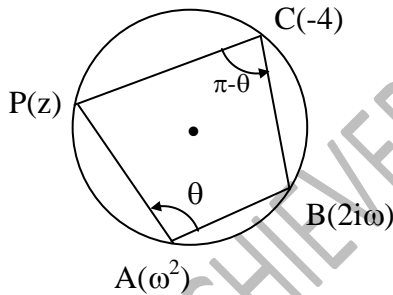
Key. B

Sol. Applying rotation formula at A and C,

$$\frac{z - \omega^2}{2i\omega - \omega^2} = \frac{PA}{AB} e^{i\theta}, \quad \frac{2i\omega + 4}{z + 4} = \frac{BC}{PC} e^{i(\pi - \theta)}$$

Multiplying we get,  $\frac{z - \omega^2}{2i\omega - \omega^2} \times \frac{2i\omega + 4}{z + 4} = -1$

$\Rightarrow z = -2i\omega$



126. The complex numbers satisfying  $(3Z+1)(4Z+1)(6Z+1)(12Z+1) = 2$  is

- a)  $\frac{\sqrt{33}-5}{4}$       b)  $\frac{\sqrt{33}+5}{24}$       c)  $\frac{-i\sqrt{23}-5}{24}$       d)  $\frac{-i\sqrt{23}+5}{24}$

Key. C

Sol. Given equation can be written as  $(144z^2 + 60z + 4)(144z^2 + 60z + 6) = 48 \Rightarrow t(t+2) = 48$

Where  $t = 144z^2 + 60z + 4$

$\therefore t = 6$  or  $-8$  hence  $z = \frac{-5 \pm \sqrt{33}}{24}, \frac{-5 \pm i\sqrt{23}}{24}$

127. If  $|z| = 1$  and  $z' = \frac{1+z^2}{z}$ , then

- (A)  $z'$  lie on a line not passing through origin  
 (B)  $|z'| = \sqrt{2}$   
 (C)  $\text{Re}(z') = 0$



$$= 1 + 3\alpha + 5\alpha^2 + \dots + 13\alpha^6 \text{ (AGP)}$$

$$= \frac{-14}{1-\alpha} = \frac{-14}{1 - \operatorname{cis} \frac{2K\pi}{7}}$$

$$= -7 \left[ 1 + i \cot \frac{K\pi}{7} \right]$$

132. All the complex numbers  $z$  that satisfy the equation  $z^{10} = (1-z)^{10}$  lie on

- a)  $x = \frac{1}{2}$                       b)  $x = -\frac{1}{2}$                       c)  $y = \frac{1}{2}$                       d)  $y = -\frac{1}{2}$

Key. A

$$\frac{z^{10}}{(1-z)^{10}} = 1 \Rightarrow \frac{z}{1-z} = 1^{1/10} = \operatorname{cis} \frac{2K\pi}{10} \text{ (} K = 0 \text{ to } 9)$$

Sol.

$$\Rightarrow z = \frac{\operatorname{cis} \frac{2K\pi}{10}}{1 + \operatorname{cis} \frac{2K\pi}{10}} = \frac{1}{2} + \frac{i}{2} \tan \frac{K\pi}{10}$$

133. If  $z$  is a complex number such that  $|z-1|=1$  then  $\arg \left( \frac{1}{z} - \frac{1}{2} \right)$  may be

- a)  $\frac{\pi}{6}$                       b)  $-\frac{\pi}{2}$                       c)  $\frac{\pi}{4}$                       d)  $-\frac{\pi}{4}$

Key. B

Sol. Since  $|z-1|=1 \Rightarrow z-1 = \operatorname{cis} \theta \Rightarrow z = (1 + \cos \theta) + i \sin \theta = 2 \cos \frac{\theta}{2} \operatorname{cis} \frac{\theta}{2}$

$$\therefore \frac{1}{z} - \frac{1}{2} = \frac{\operatorname{cis} \left( -\frac{\theta}{2} \right)}{2 \cos \frac{\theta}{2}} - \frac{1}{2} = -\frac{i}{2} \tan \frac{\theta}{2} \text{ which is purely imaginary}$$

134.  $\theta \in [0, 2\pi]$  and  $z_1, z_2, z_3$  are three complex numbers such that they are collinear and

$(1 + |\sin \theta|)z_1 + (|\cos \theta| - 1)z_2 - \sqrt{2}z_3 = 0$ . If at least one of the complex numbers  $z_1, z_2, z_3$  is non-zero then number of possible values of  $\theta$  is

- a) Infinite                      b) 4                      c) 2                      d) 8

Key. B

Sol. If  $z_1, z_2, z_3$  are collinear and  $az_1 + bz_2 + cz_3 = 0$  then  $a+b+c=0$ . Hence

$$1 + |\sin \theta| + |\cos \theta| - 1 - \sqrt{2} = 0 \Rightarrow |\sin \theta| + |\cos \theta| = \sqrt{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

135. If  $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_{n-1}$  be the  $n, n^{\text{th}}$  roots of unity, then value of  $\sum_{i=0}^{n-1} \frac{\alpha_i}{(3-\alpha_i)}$  is equal to

- A)  $\frac{n}{3^n - 1}$       B)  $\frac{n-1}{3^n - 1}$       C)  $\frac{n+1}{3^n - 1}$       D)  $\frac{n+2}{3^n - 1}$

Key. A

Sol. Let  $P = \sum_{i=0}^{n-1} \frac{\alpha_i}{3 - \alpha_i} = -\sum_{i=0}^{n-1} \frac{(3 - \alpha_i) - 3}{(3 - \alpha_i)} = 3 \sum_{i=0}^{n-1} \frac{1}{3 - \alpha_i} - \sum_{i=0}^{n-1} 1 \dots (i)$

$$Z^n - 1 = \prod_{i=0}^{n-1} (Z - \alpha_i) \log(Z^n - 1) = \sum_{i=0}^{n-1} \ln(Z - \alpha_i)$$

Diff. both sides w.r.t Z

$$\frac{nZ^{n-1}}{Z^n - 1} = \sum_{i=0}^{n-1} \frac{1}{z - \alpha_i} \text{ Put } Z = 3$$

$$\Rightarrow \frac{n3^{n-1}}{3^n - 1} = \sum_{i=0}^{n-1} \frac{1}{3 - \alpha_i}$$

$$P = \frac{3n3^{n-1}}{3^n - 1} - n = \frac{n3^n}{3^n - 1} - n = \frac{n}{3^n - 1}$$

136. Let  $|Z_1 - 1| = 1, |Z_2 + 4| = 2$  then maximum value of  $|Z_1 - Z_2|$  is  
 A) 8      B) 5      C) 4      D) 2

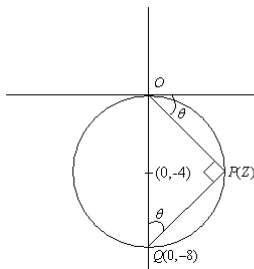
Key. A

Sol. Max. distance between two curves lies along their common normal.

137. The complex number Z has argument  $\theta, -\frac{\pi}{2} < \theta < 0$  and  $|Z + 4i| = 4$ , then  $\cot \theta + \frac{8}{Z} =$   
 A)  $1 - i$       B)  $1 + i$       C)  $i$       D)  $-i$

Key. C

Sol.



Applying rotation at P  $\frac{Z + 8i}{Z} = -i \cot \theta$

138. Let  $|Z - (1 + i)| < 2$  then  $|iZ + 1 + 2i|$   
 A)  $< 7$       B)  $< 9$       C)  $< 5$       D)  $< 10$

Key. C

Sol.  $|iZ + 1 + 2i| = |i(z - (1 + i)) + 3i| \leq |Z - (1 + i)| + 3 < 2 + 3$

139. If  $x^6 = 2x^3 - 1$  and x is not real then  $\sum_{r=1}^{50} (x^r + x^{2r})^3 =$   
 A) 0      B) 256      C) 76      D) 94

Key. D

Sol.  $x = \omega, \omega^2$

$$\omega^r + \omega^{2r} = \begin{cases} 2, & \text{if } r \text{ is a multiple of } 3 \\ -1, & \text{if } r \text{ is not of a multiple of } 3 \end{cases}$$

140. If  $A(z_1), B(z_2), C(z_3)$  be the vertices of triangle ABC in which  $\angle ABC = \frac{\pi}{4}$  and  $\frac{AB}{BC} = \sqrt{2}$

then  $z_2$  is equal to

- (a)  $z_3 + i(z_1 + z_3)$       (b)  $z_3 - i(z_1 - z_3)$       (c)  $z_3 + i(z_1 - z_3)$       (d)  $z_3 - i(z_2 - z_1)$

Key. B

Sol.  $\frac{z_1 - z_2}{z_3 - z_2} = \sqrt{2}e^{i\frac{\pi}{4}}$

141. If  $|z_1| = 2, |z_2| = 3, |z_3| = 4$  and  $|z_1 + z_2 + z_3| = 5$  then  $|4z_2z_3 + 9z_3z_1 + 16z_1z_2| =$

- a) 20      b) 24      c) 48      d) 120

Key. D

Sol.  $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$   
 $= |z_1\bar{z}_1z_2z_3 + \bar{z}_2z_2z_3z_1 + \bar{z}_3z_3z_1z_2|$   
 $= ||z_1||z_2||z_3||z_1 + z_2 + z_3| = 120$

142. If  $\log_{\tan 30^\circ} \left( \frac{2|z|^2 + 2|z| - 3}{|z| + 1} \right) < -2$  then

- a)  $|z| < \frac{3}{2}$       b)  $|z| > \frac{3}{2}$       c)  $|z| > 2$       d)  $|z| < 2$

Key. C

Sol.  $\log_{\tan 30^\circ} \left( \frac{2|z|^2 + 2|z| - 3}{|z| + 1} \right) < -2$   
 $\Rightarrow \frac{2|z|^2 + 2|z| - 3}{|z| + 1} > 3$   
 $\Rightarrow ((|z| - 2)(2|z| + 3)) > 0$   
 $\Rightarrow |z| > 2$

143.  $z_1$  and  $z_2$  be two complex numbers with  $\alpha$  and  $\beta$  as their principal arguments, such that

$\alpha + \beta > \pi$ , then principal  $\text{Arg}(z_1z_2)$  is

- a)  $\alpha + \beta + \pi$       b)  $\alpha + \beta - \pi$       c)  $\alpha + \beta - 2\pi$       d)  $\alpha + \beta$

Key. C



Sol. Take  $z_1 = i, z_2 = \omega$  Arg  $z_1 = \frac{\pi}{2}, \text{Arg } z_2 = \frac{2\pi}{3}$

$$\text{Arg } z_1 + \text{Arg } z_2 = \frac{7\pi}{6} \text{ should be equivalent to } \frac{7\pi}{6} - 2\pi$$

144. If the square root of  $\frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{2i} \left( \frac{x}{y} + \frac{y}{x} \right) + \frac{31}{16}$  is  $\pm \left( \frac{x}{y} + \frac{y}{x} - \frac{i}{m} \right)$  then m is

- a) 2    b) 3    c) 4    d) 5

Key. C

Sol.  $\left( \frac{x}{y} + \frac{y}{x} - \frac{i}{m} \right)^2 = \frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{2i} \left( \frac{x}{y} + \frac{y}{x} \right) + \frac{31}{16}$

L.H.S =

$$\begin{aligned} &= \left( \frac{x}{y} + \frac{y}{x} \right)^2 - \frac{2i}{m} \left( \frac{x}{y} + \frac{y}{x} \right) - \frac{1}{m^2} \\ &= \frac{x^2}{y^2} + \frac{y^2}{x^2} + 2 + \frac{4}{m} \cdot \frac{1}{2i} \left( \frac{x}{y} + \frac{y}{x} \right) - \frac{1}{m^2} \\ m &= 4 \end{aligned}$$

145. If  $\left| \frac{z_1 - 2z_2}{2 - z_1 z_2} \right| = 1$  and  $|z_2| \neq 1$ , then value of  $|z_1| =$

- a) 2    b) 1    c) 4    d) 5

Key. A

Sol. Conceptual

146. If  $A_1(z_1), A_2(\overline{z_1})$  are the adjacent vertices of a regular polygon. If  $\frac{\text{Im}(\overline{z_1})}{\text{Re}(z_1)} = 1 - \sqrt{2}$  then

- number of sides of the polygon is equal to  
a) 6    b) 8    c) 16    d) 12

Key. B

Sol. Clearly origin is the centre of the polygon  
Let  $z_1 = re^{i\theta}$   
 $\overline{z_1} = re^{-i\theta}$   
 $\text{Re}(z) = r\cos\theta$   
 $\text{Im}(\overline{z_1}) = -r\sin\theta$   
 $\Rightarrow -\frac{\sin\theta}{\cos\theta} = 1 - \sqrt{2} \Rightarrow \tan(\theta) = \sqrt{2} - 1$   
 $\Rightarrow \theta = \frac{\pi}{8}$  if 'n' be the no. of sides then  $\theta = \frac{\pi}{n}$   
 $\Rightarrow n = 8$

147. If exactly one root of  $z^2 + az + b = 0$  where  $a, b \in C$  is purely imaginary, then

a)  $(\bar{b} - b)^2 = -(a\bar{b} + \bar{a}b)(a + \bar{a})$

b)  $(\bar{b} - b)^2 = -(a\bar{b} + \bar{a}b)(a - \bar{a})$

c)  $(\bar{b} - b)^2 = -(a\bar{b} - \bar{a}b)(a + \bar{a})$

d)  $(\bar{b} - b)^2 = -(a\bar{b} - \bar{a}b)(a - \bar{a})$

Key. A

Sol.  $z^2 + az + b = 0$   
Let  $z_0$  is the purely imaginary root of the equation

Then  $z_0^2 + az_0 + b = 0$

$\Rightarrow z_0 + \bar{z}_0 = 0$

$\Rightarrow \bar{z}_0 = -z_0$

We have  $\overline{z_0^2 + az_0 + b} = 0 \Rightarrow \bar{z}_0^2 + \bar{a}\bar{z}_0 + \bar{b} = 0$

Now  $\bar{z}_0^2 + \bar{a}\bar{z}_0 + \bar{b} = 0$  and  $z_0^2 - \bar{a}z_0 + \bar{b} = 0$

We should have a common root. Find common root.

148.  $z_1$  and  $z_2$  are the roots of  $z^2 - az + b = 0$ , where  $|z_1| = |z_2| = 1$  and  $a, b$  are non-zero complex numbers, then

a)  $\text{Arg}(a) = 2 \text{Arg}(b)$

b)  $2 \text{Arg}(a) = \text{Arg}(b)$

c)  $\text{Arg}(a) = \text{Arg}(b)$

d) none of these

Key. B

Sol.  $z_1 + z_2 = a$        $z_1 z_2 = b$

Since  $|z_1| = |z_2| = 1$

$\Rightarrow \text{Arg}(a) = \frac{1}{2} [\text{Arg}(z_1) + \text{Arg}(z_2)]$

Also  $\text{Arg}(b) = \text{Arg}(z_1 z_2)$

$\therefore \text{Arg}(a) = \frac{1}{2} (\text{Arg}(b)) \Rightarrow 2 \text{Arg}(a) = \text{Arg}(b)$

149. If  $|z - 2 + 2i| = 1$ , then the least value of  $|z|$  is

a)  $\sqrt{8} + 1$

b)  $\sqrt{6} + 1$

c)  $\sqrt{6} - 1$

d)  $\sqrt{8} - 1$

Key. D

Sol.  $|z - 2 + 2i| = 1 \Rightarrow z - 2 + 2i = \cos\theta + i\sin\theta$

$z = (2 + \cos\theta) + i(\sin\theta - 2)$

$|z| = \sqrt{4 + 4\cos\theta + \cos^2\theta + 4 - 4\sin\theta + \sin^2\theta}$

$= \sqrt{9 + 4(\cos\theta - \sin\theta)}$

$$= \sqrt{9 + 4\sqrt{2} \cos\left(\theta + \frac{\pi}{4}\right)}$$

$$|z| \text{ is least if } \cos\left(\theta + \frac{\pi}{4}\right) = -1 = \sqrt{9 - 4\sqrt{2}}$$

$$= \sqrt{9 - 2\sqrt{8}} = \sqrt{8} - 1$$

150. If the imaginary part of  $\frac{2z+1}{iz+1}$  is -4, then the locus of the point representing  $z$  in the complex plane is

- 1) a straight line                      2) a parabola                      3) a circle                      4) an ellipse

Key. 3

Sol. Let  $z = x + iy$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$$

$$= \frac{(2x+1)+2iy}{(1-y)+ix}$$

$$= \frac{[(2x+1)+2iy][(1-y)-ix]}{(1-y)^2 + x^2}$$

since  $\text{Im}\left(\frac{2z+1}{iz+1}\right) = -4$ , we get  $\frac{2y(1-y) - x(2x+1)}{x^2 + (1-y)^2} = -4$

$$\Rightarrow 2x^2 + 2y^2 + x - 6y + 4 = 0$$

Which represents a circle.

151. If  $Z_K = \cos\frac{K\pi}{10} + i\sin\frac{K\pi}{10}$  then  $z_1 z_2 z_3 z_4$  is equal to

- 1) -1                      2) 1                      3) -2                      4) 2

Key. 1

Sol. Let  $z_K = w^K$  where  $w = \cos\frac{\pi}{10} + i\sin\frac{\pi}{10}$

$$\therefore z_1 z_2 z_3 z_4 = w \cdot w^2 \cdot w^3 \cdot w^4$$

$$= w^{10}$$

$$= \cos\frac{10\pi}{10} + i\sin\frac{10\pi}{10} \text{ (Q Demoviere's theorem)}$$

$$= \cos\pi + i\sin\pi$$

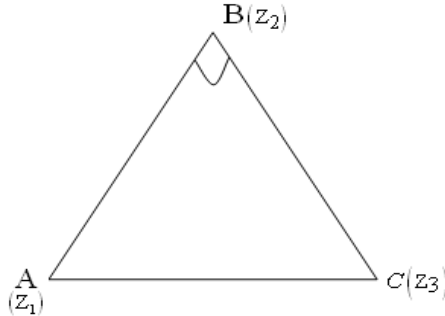
$$= -1$$

152. If  $z_1, z_2, z_3$  are the vertices of an isosceles triangle, right angled at the vertex  $z_2$ , then value of  $(z_1 - z_2)^2 + (z_2 - z_3)^2$  is

- 1) -1                                  2) 0                                  3)  $(z_1 - z_3)^2$                                   4) None of these

Key. 2

Sol. Since  $A(Z_1), B(Z_2), C(Z_3)$  is an Isosceles right angled triangle with right angle at B  
 $BA = BC$  and  $\angle ABC = 90^\circ$



$$\Rightarrow |Z_1 - Z_2| = |Z_3 - Z_2| \text{ and } \arg\left(\frac{z_3 - z_2}{z_1 - z_2}\right) = \pi/2$$

$$\therefore \frac{z_3 - z_2}{z_1 - z_2} = \frac{|z_3 - z_2|}{|z_1 - z_2|} \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = i$$

$$(z_3 - z_2)^2 = -(z_1 - z_2)^2$$

$$\Rightarrow (z_1 - z_2)^2 + (z_2 - z_3)^2 = 0$$

153. If  $a, b, c, p, q, r$  are three non-zero complex numbers such that

$$\frac{p}{a} + \frac{q}{b} + \frac{r}{c} = 1 + i \text{ and } \frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 0 \text{ then value of } \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} \text{ is}$$

- 1) 0                                  2) -1                                  3) 2i                                  4) -2i

Key. 3

Sol. We have  $(1+i)^2 = \left(\frac{p}{a} + \frac{q}{b} + \frac{r}{c}\right)^2$

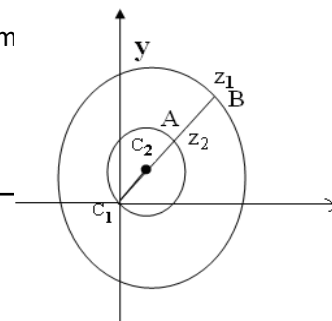
$$1 - 1 + 2i = \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} + 2\left(\frac{qr}{bc} + \frac{rp}{ca} + \frac{pq}{ab}\right)$$

$$2i = \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} + \frac{2abc}{pqr} \left(\frac{a}{p} + \frac{q}{b} + \frac{r}{c}\right)$$

$$= \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} + \frac{2abc}{pqr} (0)$$

$$= \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2}$$

154. For all complex numbers  $z_1, z_2$  satisfying  $|z_1| = 12$  and  $|z_2 - 3 - 4i| = 5$ , the minimum of  $|z_1 - z_2|$  is



1) 0

2) 2

3) 7

4) 17

Key. 2

Sol.  $|z_1| = 12 \Rightarrow z_1$  lies on circle with centre  $c_1$  at origin and radius 12

$|z_2 - 3 - 4i| = 5 \Rightarrow z_2$  lies on the circle with centre  $c_2 (3 + 4i)$  and radius 5.

$\therefore |z_1 - z_2|$  will be minimum.

If  $z_1$  and  $z_2$  lies on the line joining  $c_1$  and  $c_2$  i.e on the line  $z = 3 + 4i$

Minimum value of  $|z_1 - z_2| = AB$

$$= c_1B - c_1A$$

$$12 - 10 = 2$$

155. If  $z_1, z_2, z_3$  are complex numbers such that  $|z_1| = |z_2| = |z_3| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1$  then

$|z_1 + z_2 + z_3|$  is

1) equal to 1

2) less than 1

3) greater than 3

4) equal to 3

Key. 1

Sol. Q  $|z_1| = |z_2| = |z_3| = 1$  we get  $z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$

$$\therefore 1 = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| \Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \bar{z}_3 = \frac{1}{z_3}$$

$$= |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$$

$$= |z_1 + z_2 + z_3|$$

156. The complex numbers  $z_1, z_2$  and  $z_3$  satisfying  $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$  are the vertices of a

triangle which is

1) of area  $\sqrt{3}$

2) right angled and isosceles

3) equilateral

4) obtuse-angled and isosceles

Key. 3

Sol.  $\left| \frac{z_1 - z_3}{z_2 - z_3} \right| = \left| \frac{1 - i\sqrt{3}}{2} \right| \Rightarrow \frac{|z_1 - z_3|}{|z_2 - z_3|} = \sqrt{\frac{1 + 3}{4 + 4}} = 1$

$$\Rightarrow |z_1 - z_3| = |z_2 - z_3|$$

Again  $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$

$$\frac{z_1 - z_3}{z_2 - z_3} - 1 = \frac{1 - i\sqrt{3}}{2} - 1$$

$$\frac{z_1 - z_2}{z_2 - z_3} = \frac{-1 - i\sqrt{3}}{2}$$

$$\left| \frac{z_1 - z_2}{z_2 - z_3} \right| = \left| \frac{-1 - i\sqrt{3}}{2} \right|$$

$$\frac{|z_1 - z_2|}{|z_2 - z_3|} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\Rightarrow |z_1 - z_2| = |z_2 - z_3|$$

∴  $z_1, z_2$  and  $z_3$  are the vertices of an equilateral triangle.

157. The vertices B and D of a parallelogram are  $1 - 2i$  and  $4 + 2i$  respectively. If the diagonals are at right angles and  $|AC| = 2|BD|$ , then the complex number representing A is

- 1)  $\frac{3}{2}i + \frac{1}{2}$                       2)  $3i - 4$                       3)  $3i - \frac{3}{2}$                       4)  $\frac{5}{2}$

Key. 3

Sol. Let affix of A be Z.

M = Mid point of BD

$$= \left( \frac{5}{2}, 0 \right)$$

$$\angle AMB = 90^\circ$$

$$\text{Arg} \left( \frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}} \right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}} = \left| \frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}} \right| \text{cis} \frac{\pi}{2}$$

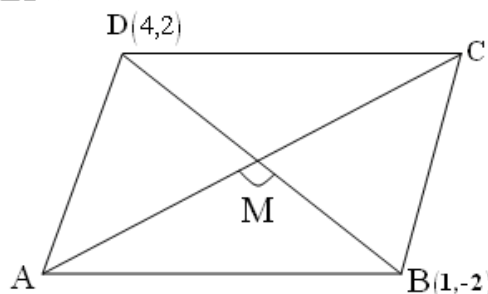
$$= \frac{|BM|}{|AM|} i$$

$$= \frac{\frac{|BD|}{2}}{\frac{|AC|}{2}} i$$

$$= \frac{|BD|}{|AC|} i$$

$$= \frac{1}{2} i \quad (\text{Q } |AC| = 2|BD|)$$

$$\therefore \left( \frac{-3}{2} - 2i \right) \frac{2}{i} = z - \frac{5}{2}$$



$$\Rightarrow z = \frac{5}{2} - \frac{3}{i} - c_1$$

$$= \frac{-3}{2} + 3i$$

Hence option( 3 )

158. For all  $z$  satisfying  $|z+1-i|=1$  we have

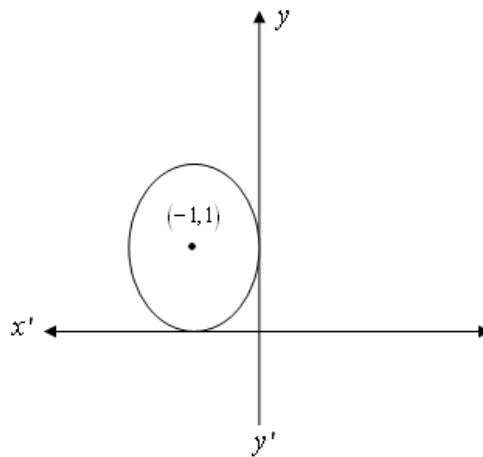
- 1)  $\frac{\pi}{2} \leq \text{Arg}z \leq \pi$       2)  $-\pi \leq \text{Arg}z \leq \frac{-\pi}{2}$       3)  $-\pi < \text{Arg} z \leq \frac{\pi}{2}$       4) None of

these

Key. 1

Sol.  $|z+1-i|=1 \Rightarrow |z-(-1)+i|=1$

$\therefore$  Locus of  $z$  is a circle whose centre is  $(-1,1)$  and radius=1



The circle touches both the axes in the second quadrant

All points on this circle lie in the region +ve Y-axis corresponding to  $\text{Arg} z = \frac{\pi}{2}$

and -ve X-axis corresponds to  $\text{Arg}z = \pi$

$$\therefore \frac{\pi}{2} \leq \text{Arg} Z \leq \pi$$

Hence option ( 1 )

159. The equation  $|z-4i|+|z+4i|=10$  represents

- 1) a circle      2) an ellipse      3) a line segment      4) None of these

Key. 2

Sol.  $p(z)$ ,  $A(4i)$ ,  $B(-4i)$  then

$$|AB| = |-4i - 4i|$$

$$= |-8i|$$

$$= 8$$

Now  $|z-4i|+|z+4i|=10$

$$\Rightarrow |PA| + |PB| = 10$$

$$> |AB|$$

∴ Locus of P is an ellipse  
Hence option is (2).

160. If  $z^5 = (z-1)^5$  then the roots are represented in the argand plane by the points that are

- |                                |                  |
|--------------------------------|------------------|
| 1) Collinear                   | 2) Concylic      |
| 3) Vertices of a parallelogram | 4) None of these |

Key. 1

Sol. Let Z be a complex number satisfying

$$Z^5 = (Z-1)^5$$

$$\Rightarrow |Z^5| = |(Z-1)^5|$$

$$\Rightarrow |Z|^5 = |Z-1|^5$$

$$\Rightarrow |Z| = |Z-1|$$

Thus Z lies on the perpendicular bisector of the segment joining the  
Origin and A(1+i 0) i.e Z lies on  $\text{Re } Z = \frac{1}{2}$

Hence option (1)

161. Let  $|z - 5 + 12i| \leq 1$  and the least and greatest values of  $|z|$  are  $m$  and  $n$  and if  $l$  be the least positive value of  $\frac{x^2 + 24x + 1}{x}$  ( $x > 0$ ), then  $l$  is

- |                     |           |         |         |
|---------------------|-----------|---------|---------|
| (A) $\frac{m+n}{2}$ | (B) $m+n$ | (C) $m$ | (D) $n$ |
|---------------------|-----------|---------|---------|

Key. 2

Sol.  $r^{\text{th}}$  term of given expression

$$= r(r+1-w)(r+1-w^2)$$

$$= (r+1-1)(r+1-w)(r+1-w^2)$$

$$= (r+1)^3 - 1 \left[ \begin{array}{l} \therefore \text{Let } r+1 = x \\ (x-1)(x-w)(x-w^2) = x^3 - 1 \end{array} \right]$$

$$\therefore \text{ Given expression value} = \sum_{r=1}^{n-1} r(r+1-w)(r+1-w^2)$$

$$= \sum_{r=1}^n (r+1)^3 - 1$$

$$= 2^3 + 3^3 + \dots + n^3 - (n-1)$$

$$= (1^3 + 2^3 + 3^3 + \dots + n^3) - n$$





164. The complex numbers  $\sin x + i \cos 2x$  and  $\cos x - i \sin 2x$  are conjugate to each other for

- 1)  $x = n\pi$  2)  $x = 0$   
 3)  $x = \left(n + \frac{1}{2}\right)\pi$  4) no value of  $x$

Key. 4

Sol. Let  $Z_1 = \sin x + i \cos 2x$ ,  $Z_2 = \cos x - i \sin 2x$

$$\overline{Z_1} = Z_2$$

$$\sin x - i \cos 2x = \cos x - i \sin 2x$$

$$\Rightarrow \sin x = \cos x \text{ and } \cos 2x = \sin 2x$$

$$\Rightarrow \tan x = 1 \text{ and } \tan 2x = 1$$

$$\Rightarrow x = \frac{\pi}{4} \text{ and } x = \frac{\pi}{8} \text{ which is not possible. Hence there is no value of } x$$

Hence option (4).

165. Suppose  $z_1, z_2, z_3$  are the vertices of an equilateral triangle inscribed in the circle  $|z| = 2$ . If

$z_1 = 1 + i\sqrt{3}$  then  $z_2$  may be

- 1)  $1 + i\sqrt{3}, 2$  2)  $3 - i\sqrt{2}, 4$  3)  $1 - i\sqrt{3}, -2$  4)  $2 - i\sqrt{3}, 1$

Key. 3

Sol. Let  $Z_1, Z_2, Z_3$  are the vertices A, B, C of equilateral triangle ABC inscribed in a circle

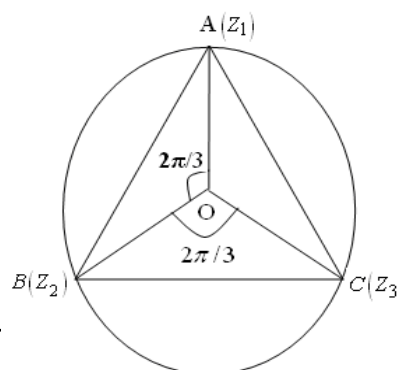
$|Z| = 2$  with centre  $(0, 0)$  and radius 2.

Given  $Z_1 = 1 + i\sqrt{3}$

Rotating  $OA$  about  $O$  by an angle  $\frac{2\pi}{3}$  we have

$$\frac{Z - 0}{1 + i\sqrt{3} - 0} = \frac{|Z - 0|}{|1 + i\sqrt{3} - 0|} e^{\pm i \frac{2\pi}{3}}$$

$$Z = (1 + i\sqrt{3}) \left( \cos \frac{2\pi}{3} \pm i \sin \frac{2\pi}{3} \right)$$



$$\begin{aligned}
 &= (1+i\sqrt{3})\left(\frac{-1 \pm i\sqrt{3}}{2}\right) \\
 &= -\frac{(1+i\sqrt{3})(1-i\sqrt{3})}{2} \quad (\text{or}) \quad \frac{-(1+i\sqrt{3})(1+i\sqrt{3})}{2} \\
 &= \frac{-(1+3)}{2} \quad \text{or} \quad \frac{-(1-3+2i\sqrt{3})}{2} \\
 &\quad -2 \quad \text{or} \quad 1-i\sqrt{3}
 \end{aligned}$$

Hence option (3)

166. If  $a = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$  then the quadratic equation whose roots are

$$\alpha = a + a^2 + a^4 \quad \text{and} \quad \beta = a^3 + a^5 + a^6 \text{ is}$$

- |                      |                       |
|----------------------|-----------------------|
| 1) $x^2 + x + 2 = 0$ | 2) $x^2 - 5x + 7 = 0$ |
| 3) $x^2 - x + 2 = 0$ | 4) $x^2 + x - 2 = 0$  |

Key. 1

Sol.  $a = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$

$$a^7 = \left(\cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}\right)^7$$

$$= \cos 2\pi + i \sin 2\pi$$

$$= 1 \dots\dots\dots(1)$$

$$\text{Sum of roots} = \alpha + \beta$$

$$= a + a^2 + a^4 + a^3 + a^5 + a^6$$

$$= a + a^2 + a^3 + a^4 + a^5 + a^6$$

$$= \frac{a(1-a^6)}{1-a}$$

$$= \frac{a-a^7}{1-a}$$

$$= \frac{a-1}{1-a}$$

$$= -1 \dots\dots\dots(2)$$

Product of roots =  $\alpha\beta$

$$= (a + a^2 + a^4)(a^3 + a^5 + a^6)$$

$$= a^4 + a^5 + a^7 + a^6 + a^7 + a^9 + a^7 + a^8 + a^{10}$$

$$= a^4 + a^5 + 1 + a^6 + 1 + a^2 + 1 + a + a^3 \quad (\text{Q from (1)})$$

$$= 3 + a + a^2 + a^3 + a^4 + a^5 + a^6$$

$$= 3 + (-1) \quad (\text{Q from (2)})$$

$$= 3 - 1 = 2$$

Required equation is  $x^2 - x(-1) + 2 = 0$

$$x^2 + x + 2 = 0$$

Hence option (1)

167. Let  $z$  and  $w$  be two non zero complex numbers such that  $|z| = |w|$  and  $\arg z + \arg w = \pi$ .

Then  $z$

- 1)  $w$                       2)  $\bar{w}$                       3)  $-\bar{w}$                       4)  $2w$

Key. 3

Sol. Let  $\arg w = \theta$

$$\therefore \text{Arg } z = \pi - \theta$$

$$w = |w|(\cos \theta + i \sin \theta) \text{ and } z = |z|[\cos(\pi - \theta) + i \sin(\pi - \theta)]$$

$$= |w|(-\cos \theta + i \sin \theta)$$

$$= -|w|(\cos \theta - i \sin \theta)$$

$$= -\bar{w}$$

Hence option (3)

168. If  $|z_1 - 1| \leq 1, |z_2 - 2| \leq 2, |z_3 - 3| \leq 3$  then the greatest value of  $|z_1 + z_2 + z_3|$  is

- 1) 6                      2) 7                      3) 9                      4) 12

Key. 4

Sol.  $|z_1 + z_2 + z_3| = |(z_1 - 1) + (z_2 - 2) + (z_3 - 3) + 6|$

$$\leq |z_1 - 1| + |z_2 - 2| + |z_3 - 3| + 6$$

$$\leq 1 + 2 + 3 + 6$$

$$\leq 12$$

Greatest value of  $|z_1 + z_2 + z_3| = 12$

Hence option (4)

169. The greatest and least value of  $|z_1 + z_2|$  if  $z_1 = 24 + 7i$  and  $|z_2| = 6$

1) 31, 19

2) 25, 6

3) 31, 6

4) 19, 6

Key. 1

Sol.  $|z_1 + z_2| \leq |z_1| + |z_2|$

$$= |24 + 7i| + 6$$

$$= \sqrt{(24)^2 + 7^2} + 6 = 25 + 6 = 31$$

Also  $|z_1 + z_2| = |z_1 - (-z_2)|$

$$\geq ||z_1| - |z_2|| = |25 - 6| = 19$$

∴ Least value = 19, Greatest value = 31

Hence option (1)

170. If  $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$  then Modulus and argument of  $(1 + \cos 2\alpha) + i \sin 2\alpha$  is

1)  $-2 \sin \alpha, \frac{\pi}{6}$

2)  $-2 \cos \alpha, \alpha - \pi$

3)  $-2 \sin \alpha, \alpha - \pi$

4) None of

these

Key. 2

Sol. Let  $z = (1 + \cos 2\alpha) + i \sin 2\alpha$

$$= 2 \cos^2 \alpha + 2i \sin \alpha \cos \alpha$$

$$= 2 \cos \alpha [\cos \alpha + i \sin \alpha]$$

$$= -2 \cos \alpha [-\cos \alpha - i \sin \alpha]$$

$$= -2 \cos \alpha [\cos(\alpha - \pi) + i \sin(\alpha - \pi)] \quad \left[ \text{Q } \frac{\pi}{2} < \alpha < \frac{3\pi}{2} \right]$$

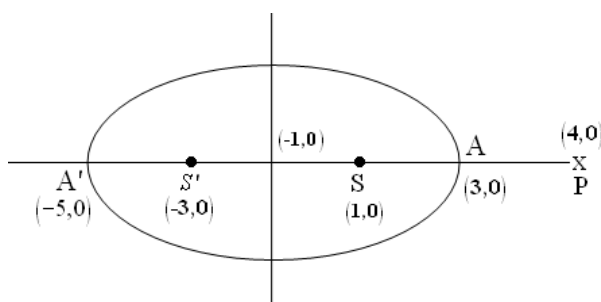
$$\therefore |z| = -2 \cos \alpha \text{ and } \arg z = \alpha - \pi$$

Hence option (2)

171. If  $|z-1| + |z+3| \leq 8$  then the minimum and maximum values of  $|z-4|$  respectively is

- 1) 1, 9    2) 10, 15    3) 16, 22    4) 13, 18

Key. 1



Sol.

Given  $|z-1| + |z+3| \leq 8$

$\therefore z$  lies inside or on the ellipse whose foci are (1, 0) and (-3, 0) and vertices are (-5, 0) and (3, 0). Clearly the minimum and maximum values of  $|z-4|$  are 1 and 9 respectively representing the distances PA and PA'.

$$\therefore 1 \leq |z-4| \leq 9$$

Hence option (1)

172. If  $|z-25i| \leq 15$  then |maximum argz – minimum argz| equals

- 1)  $2 \cos^{-1} \frac{3}{5}$     2)  $2 \cos^{-1} \frac{4}{5}$   
 3)  $\frac{\pi}{2} + \cos^{-1} \frac{3}{5}$     4)  $\sin^{-1} \frac{3}{5} - \cos^{-1} \frac{3}{5}$

Key. 2

Sol. If  $|z-25i| \leq 15$  then  $z$  lies either in the interior and or on the boundary of the circle with centre at  $C(0, 25)$  and radius equal to 15.

The least argument is for point A and greatest argument is for point B from right

$$\Delta OAC, \cos\left(\frac{\pi}{2} - \theta\right) = \frac{OA}{OC} = \frac{20}{25} = \frac{4}{5}$$

$$\frac{\pi}{2} - \theta = \cos^{-1}\left(\frac{4}{5}\right)$$

Now for  $|z - 25i| \leq 15$

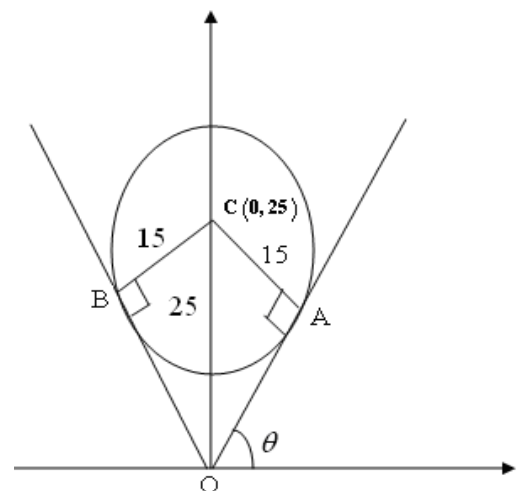
$$|\text{Maximum argz} - \text{Minimum argz}| = |\text{ArgB} - \text{ArgA}|$$

$$= \angle BOA$$

$$= \angle BOX - \angle AOX = \frac{\pi}{2} + \frac{\pi}{2} - \theta - \theta$$

$$= \pi - 2\theta$$

$$= 2 \cos^{-1} \frac{4}{5}$$



Hence option (2)

173. Let  $z_1$  and  $z_2$  be two non-zero complex numbers such that  $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$  then the origin and

points represented by  $z_1$  and  $z_2$

1) lie on a straight line      2) form a right triangle

3) form an equilateral triangle

4) None of these

Key. 3

Sol. Let  $\frac{z_1}{z_2} = z$  then  $z + \frac{1}{z} = 1$

$$\Rightarrow z^2 - z + 1 = 0$$

$$\Rightarrow z = \frac{1 \pm i\sqrt{3}}{2}$$

$$\therefore \frac{z_1}{z_2} = \frac{1 \pm i\sqrt{3}}{2}$$

If  $z_1$  and  $z_2$  are represented by A and B respectively and O be the origin, then

$$\frac{OA}{OB} = \frac{|z_1|}{|z_2|} = \left| \frac{1 \pm \sqrt{3}i}{2} \right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\Rightarrow OA = OB$$

$$\text{Also, } \frac{AB}{OB} = \frac{|z_2 - z_1|}{|z_2|} = \left| 1 - \frac{z_1}{z_2} \right|$$

$$= \left| 1 - \left( \frac{1}{2} \pm \frac{\sqrt{3}}{2} i \right) \right|$$

$$\left| \frac{1}{2} \mp \frac{\sqrt{3}}{2} i \right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\Rightarrow AB = OB$$

$$\text{Thus } OA = OB = AB$$

$\therefore \Delta AOB$  is an equilateral triangle.

Hence option (3)

174. If the equation,  $z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$ , where  $a_1, a_2, a_3, a_4$  are real coefficients

different from zero has a pure imaginary root then the expression  $\frac{a_3}{a_1 a_2} + \frac{a_1 a_4}{a_2 a_3}$  has the

value equal to:

- (A) 0                      (B) 1                      (C) -2                      (D) 2

Key. 2

Sol. we know that

$$\frac{1}{x-1} + \frac{1}{x-w} + \frac{1}{x-w^2} + \dots + \frac{1}{x-w^{n-1}} = \frac{n(x^{n-1})}{2^n - 1}$$

Put  $x = 2$  we get

$$\frac{1}{2-1} + \frac{1}{2-w} + \frac{1}{2-w^2} + \dots + \frac{1}{2-w^{n-1}} = \frac{n \cdot 2^{n-1}}{2^n - 1}$$

$$\frac{1}{2-w} + \frac{1}{2-w^2} + \dots + \frac{1}{2-w^{n-1}} = \frac{n \cdot 2^{n-1}}{2^n - 1} = \frac{n \cdot 2^{n-1} - 2^n + 1}{2^n - 1} = \frac{2^n(n-2) + 2}{2(2^n - 1)}$$

Hence option (4)

175. If  $\alpha, \beta$  be the roots of the equation  $u^2 - 2u + 2 = 0$  & if  $\cot \theta = x + 1$ , then

$$\frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta}$$
 is equal to



(A)  $\frac{\sin n\theta}{\sin^n \theta}$

(B)  $\frac{\cos n\theta}{\cos^n \theta}$

(C)  $\frac{\sin n\theta}{\cos^n \theta}$

(D)  $\frac{\cos n\theta}{\sin^n \theta}$

Key. 1

$$S = 1 + 3\alpha + 5\alpha^{2\alpha^n} + (2n-3)^{n-2} 4(2n-1)\alpha^{n-1}$$

$$\alpha S = \alpha + 3\alpha^2 + \dots + (2n-3)\alpha^{n-1} + (2n-1)\alpha^n$$

Sol. Let  $S(1-\alpha) = 1 + 2\alpha + 2\alpha^2 + \dots + 2\alpha^{n-1} - (2n-1)\alpha^n$

$$= 1 + 2\alpha(1 + \alpha + \alpha^2 + \dots + \alpha^{n-2}) - (2n-1)\alpha^n$$

$$= 1 + \frac{2\alpha(1-\alpha^{n-1})}{1-\alpha} - (2n-1)\alpha^n$$

$$= 1 + \frac{2(\alpha - \alpha^n)}{1-\alpha} - (2n-1)\alpha^n$$

$$= 1 + \frac{2(\alpha-1)}{1-\alpha} - (2n-1) \quad (\text{Q } \alpha^n = 1)$$

$$= 1 - 2 - 2n + 1 = -2n$$

$$S = \frac{-2n}{1-\alpha} = \frac{2n}{\alpha-1}$$

Hence option (4)

176. If  $z = (\lambda + 3) - i\sqrt{5 - \lambda^2}$  then the locus of  $z$  is

- 1) ellipse                      2) semi circle                      3) parabola                      4) straight line

Key. 2

Sol. Let  $z = x + iy$  then  $x = \lambda + 3, y = -\sqrt{5 - \lambda^2}$

$$\Rightarrow (x-3)^2 = \lambda^2 \text{ and } y^2 = 5 - \lambda^2$$

(1)                      (2)

From (1) and (2)  $(x-3)^2 = 5 - y^2$

$$\Rightarrow (x-3)^2 + y^2 = 5$$

Clearly it is a semi circle as  $y < 0$ . Hence part of the circle lies below the x-axis.  
Hence option (2)

177. If  $x^2 + x + 1 = 0$  then the value of  $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \dots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$  is

- 1) 27                                      2) 72                                      3) 45                                      4) 54

Key. 4

Sol.  $x^2 + x + 1 = 0 \Rightarrow x = w \text{ or } w^2$

Let  $x = w$  then  $x + \frac{1}{x} = w + \frac{1}{w} = w + w^2 = -1$

$x^2 + \frac{1}{x^2} = w^2 + \frac{1}{w^2} = w^2 + w = -1$

$x^3 + \frac{1}{x^3} = w^3 + \frac{1}{w^3} = 1 + 1 = 2$

$x^4 + \frac{1}{x^4} = w^4 + \frac{1}{w^4} = w + \frac{1}{w} = -1 \text{ etc.}$

$\therefore \left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \dots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$

$= 18 + 9(2)^2 = 54$

Hence option (4)

178. If centre of a regular hexagon is at origin and one of the vertices on Argand diagram is  $1 + 2i$ , then its perimeter is

- 1)  $2\sqrt{5}$                                       2)  $6\sqrt{2}$                                       3)  $4\sqrt{5}$                                       4)  $6\sqrt{5}$

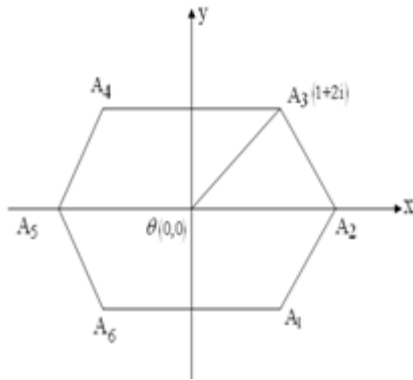
Key. 4

Sol. Let the vertices be  $z_1, z_2, z_3, z_4, z_5, z_6$  w.r.t centre O at origin  $|z_3| = \sqrt{5}$

Now  $\Delta O A_2 A_3$  is equilateral  $\Rightarrow OA_2 = OA_3 = A_2 A_3 = \sqrt{5}$

Perimeter =  $6\sqrt{5}$

Hence option (4)

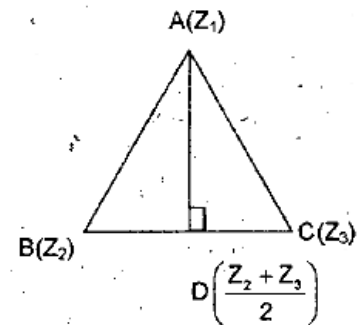


179. Let  $A(Z_1)$ ,  $B(Z_2)$ ,  $C(Z_3)$  be the vertices of an equilateral triangle  $ABC$ , then the value of  $\arg\left(\frac{Z_2 + Z_3 - 2Z_1}{Z_3 - Z_2}\right)$  is equal to
- a)  $\frac{\pi}{3}$       b)  $\frac{\pi}{4}$       c)  $\frac{\pi}{2}$       d)  $\frac{\pi}{6}$

Ans. c

$$\arg\left(\frac{Z_2 + Z_3 - 2Z_1}{Z_3 - Z_2}\right) = \arg 2 \left\{ \frac{\left(\frac{Z_2 + Z_3 - Z_1}{2}\right)}{Z_3 - Z_2} \right\}$$

$$= \arg \left\{ \frac{\left(\frac{Z_2 + Z_3 - Z_1}{2}\right)}{Z_3 - Z_2} \right\} = \frac{\pi}{2}$$



Clearly  $AD \perp BC$

180. If  $|z - 1 - i| = 1$ , then the locus of a point represented by the complex number  $5(z - i) - 6$  is
- a) circle with centre  $(1, 0)$  and radius 3      b) circle with centre  $(-1, 0)$  and radius 5
- c) line passing through origin      d) line passing through  $(-1, 0)$

Ans. b

Let  $w = 5(z - i) - 6$

$$\Rightarrow |w + 1| = 5|z - 1 - i| = 5$$

181. Let  $z$  be a complex number satisfying  $|z^2 + 2z \cos \alpha| \leq 1$ , ( $\alpha \in R$ ) then maximum value  $\cot|z|$  must be
- a)  $\sqrt{2} + 1$       b)  $\sqrt{3} - 1$       c)  $\sqrt{3} + 1$       d)  $\sqrt{6}$

Ans. c

$$|z^2 + 2z \cos \alpha| \leq 1 \Rightarrow |z||z + 2 \cos \alpha| \leq 1$$

$$\Rightarrow |z + 2 \cos \alpha| \leq |z| + |2 \cos \alpha|$$

$$\Rightarrow |z|^2 (|z| + |2 \cos \alpha|)^2 \leq 1$$

$$\Rightarrow |z| \in [0, \sqrt{3}+1]$$

182.  $Z_1$  and  $Z_2$  are the roots of  $Z^2 - aZ + b = 0$  where  $|Z_1| = |Z_2| = 1$  and  $a, b \in C$ , then  
 a)  $\arg(a) = \arg(b)$       b)  $\arg(a) = 2 \arg(b)$       c)  $2\arg(a) = \arg(b)$       d) none of these

Ans. c  
 $Z_1 + Z_2 = a, Z_1 Z_2 = b$  and  $|Z_1| = |Z_2| = 1$

$$\therefore \arg(a) = \frac{1}{2} \{ \arg(Z_2) + \arg(Z_1) \} \text{ and } \arg(b) = \arg(Z_1 Z_2) = \arg(Z_1) + \arg(Z_2)$$

$$\therefore 2 \arg(a) = \arg(b)$$

183. If  $|z-1|=1$  and  $\arg z = \theta (z \neq 0)$  and  $0 < \theta < \pi/2$ , then  $1 - \frac{2}{z}$  is equal to

- a)  $\tan \theta$       b)  $i \tan \theta$       c)  $\tan \frac{\theta}{2}$       d)  $i \tan \frac{\theta}{2}$

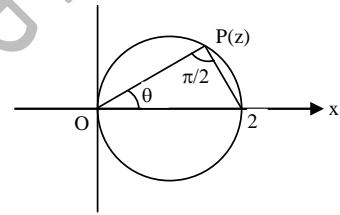
Ans. b

$$\arg\left(\frac{2-z}{0-z}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{z-2}{z} = \frac{AP}{OP} i$$

$$\tan \theta = \frac{AP}{QP}$$

$$\text{then } \frac{z-2}{z} = i \tan \theta$$



184. If complex number  $z$  satisfies  $|z-6i| = \text{Im}(z)$ , then range of  $(\arg z - \arg \bar{z})$  will be

- a)  $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$       b)  $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$       c)  $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$       d)  $\left[\frac{3\pi}{4}, \frac{5\pi}{3}\right]$

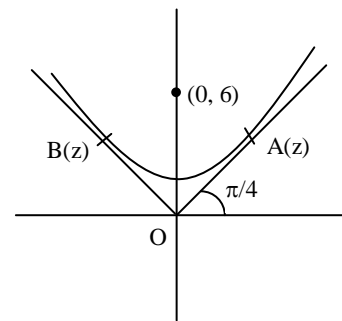
Ans. a

Clearly  $z$  lies on a parabola focus at  $(0, 6)$  and  $x$ -axis as directrix as  $\arg \bar{z} = -\arg z$ . Point of contact of the tangent drawn from origin to the parabola will correspond to the maximum and minimum argument.

$$(\arg z)_{\min} = \frac{\pi}{4}$$

$$(\arg z)_{\max} = \frac{3\pi}{4}$$

$$\frac{\pi}{2} \leq 2 \arg z \leq \frac{3\pi}{2}$$



185. If  $z_1, z_2, z_3$  are three distinct complex numbers and  $a, b, c$  are three positive real numbers

such that  $\frac{a}{|z_2 - z_3|} = \frac{b}{|z_3 - z_1|} = \frac{c}{|z_1 - z_2|}$  then the value of  $\frac{a^2}{z_2 - z_3} + \frac{b^2}{z_3 - z_1} + \frac{c^2}{z_1 - z_2}$  is

- a) 0      b) 1      c) 2      d) 3

Ans. a

$$\left(\frac{a^2}{z_2 - z_3}\right) + \left(\frac{b^2}{z_3 - z_1}\right) + \left(\frac{c^2}{z_1 - z_2}\right) = \lambda(\overline{z_2 - z_3} + \overline{z_3 - z_1} + \overline{z_1 - z_2}) = 0$$

186. If  $|z - 1| + |z + 3| \leq 8$  then the range of values of  $|z - 4|$  is  
 a) [0, 7]                      b) [1, 8]                      c) [1, 9]                      d) [2, 5]

Ans. c

$z$  lies inside or on the ellipse with foci (1, 0) and (-3, 0). Hence minimum and maximum values of  $|z - 4|$  are 1 and 9.

187. If  $x_1, x_2, x_3, \dots, x_n$  are the roots of  $x^n + ax + b = 0$ , then the value of  $(x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \dots (x_1 - x_n)$  is  
 a)  $nx_1^n + a$                       b)  $nx_1^{n-1} + a$                       c)  $nx_1 + a^{n-1}$                       d)  $nx_1 + a^n$

Ans. b

$$x^n + ax + b = 0 = (x - x_1)(x - x_2)(x - x_3) \dots (x - x_n)$$

$$\Rightarrow (x - x_2)(x - x_3) \dots (x - x_n) = \frac{x^n + ax + b}{x - x_1}$$

$$\therefore (x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \dots (x_1 - x_n) = \lim_{x \rightarrow x_1} \left( \frac{x^n + ax + b}{x - x_1} \right) = nx_1^{n-1} + a$$

188. Triangle ABC, A( $z_1$ ), B( $z_2$ ) and C( $z_3$ ) is inscribed in the circle  $|z| = 5$ . If H( $z_H$ ) be the orthocentre of triangle ABC, then  $Z_H$  is equal to  
 (A)  $\frac{2}{3}(z_1 + z_2 + z_3)$                       (B)  $\frac{4}{3}(z_1 + z_2 + z_3)$   
 (C)  $(z_1 + z_2 + z_3)$                       (D)  $3(z_1 + z_2 + z_3)$

Key. C

Sol. Circumcentre of triangle ABC is origin. Let G( $z_G$ ) be it's centroid, then

$$z_G = \frac{1}{3}(z_1 + z_2 + z_3) \text{ the points O (0), G}(z_G), \text{H}(z_H) \text{ are collinear and } OG : GH = 1 : 2$$

$$z_G = \frac{2 \times 0 + 1 \times z_H}{3} = z_H = 3z_G = z_1 + z_2 + z_3$$

189. If tangents drawn to circle  $|z| = 2$  at A( $z_1$ ) and B( $z_2$ ) meet at P( $z_P$ ), then

- (A)  $z_P = \left(\frac{z_1 + z_2}{2}\right)$                       (B)  $z_P = \frac{2(z_1 + z_2)}{\sqrt{z_1 z_2}}$   
 (C)  $z_P = \frac{2z_1 z_2}{z_1 + z_2}$                       (D)  $z_P^2 = z_2 z_1$

Key. C

Sol. Equation of tangent at A( $z_1$ ) is

$$\frac{z}{z_1} + \frac{\bar{z}}{\bar{z}_1} = 2 \Rightarrow \frac{z}{z_1} + \frac{\bar{z}}{4} = 2$$

$$\Rightarrow \frac{z}{z_1^2} + \frac{\bar{z}}{4} = \frac{2}{z_1}$$

Equation of tangent at B ( $z_2$ ) is

$$\frac{z}{z_2^2} + \frac{\bar{z}}{4} = \frac{2}{z_2}$$

$$\Rightarrow z \left( \frac{1}{z_1^2} - \frac{1}{z_2^2} \right) = 2 \left( \frac{1}{z_1} - \frac{1}{z_2} \right)$$

$$\Rightarrow z = \frac{2z_1z_2}{z_1 + z_2}$$

190. If  $t^2 + t + 1 = 0$ , then the value of  $\left(t + \frac{1}{t}\right)^2 + \left(t^2 + \frac{1}{t^2}\right)^2 + \dots + \left(t^{27} + \frac{1}{t^{27}}\right)^2$  is

- (A) 27  
(C) 45

- (B) 72  
(D) 54

Key. D

Sol.  $s = \left(t + \frac{1}{t}\right)^2 + \left(t^2 + \frac{1}{t^2}\right)^2 + \dots + \left(t^{27} + \frac{1}{t^{27}}\right)^2$

Let  $t = \omega$  then

$$S = \{(-1)^2 + (-1)^2 + \dots 18 \text{ terms}\} + \{(2)^2 + \dots 9 \text{ terms}\}$$

$$= 18 + 9 \times 4 = 18 + 36 = 54$$

191. Let  $n$  is of the form of  $3P$  where  $P$  is an odd integer then,  ${}^nC_0 + {}^nC_3 + {}^nC_6 + {}^nC_9 + \dots + {}^nC_n$  equals

(A)  $\frac{1}{3}(2^n - 2)$

(B)  $\frac{2}{3}(2^n - 2)$

(C)  $\frac{1}{3}(2^{n-1} - 2)$

(D)  $\frac{2}{3}(2^n + 2)$

Key. A

Sol.  $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$

$$(1+\omega)^n = C_0 + C_1\omega + C_2\omega^2 + \dots + C_n\omega^n$$

$$(1+\omega^2)^n = C_0 + C_1\omega^2 + C_2\omega^4 + \dots + C_n\omega^{2n}$$

$$2^n = C_0 + C_1 + C_2 + \dots + C_n$$

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$$2^n + (-\omega)^n + (-\omega^2)^n = 3C_0 + 3C_3 + \dots + 3^nC_n$$

$$C_0 + C_3 + C_6 + \dots + C_n = \frac{1}{3} [2^n + (-1)^n \omega^n + (-1)^n \omega^{2n}]$$

$$= \frac{1}{3} [2^n + (-1)^{3P} \omega^{3P} + (-1)^{3P} \omega^{6P}]$$

$$= \frac{1}{3} [2^n - 1 - 1] = \frac{1}{3} [2^n - 2]$$

192. Let  $z_1$  and  $z_2$  be two complex numbers with  $\alpha$  and  $\beta$  as their principal arguments, such that  $\alpha + \beta > \pi$ , then principal  $\arg(z_1z_2)$  is given by

- (A)  $\alpha + \beta + \pi$  (B)  $\alpha + \beta - \pi$   
 (C)  $\alpha + \beta - 2\pi$  (D)  $\alpha + \beta$

Key. C

Sol.  $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2m\pi, m \in \mathbb{I}$

$= \alpha + \beta - 2\pi$  which should be equivalent to negative angle  $\frac{7\pi}{6} - 2\pi$

193. Let  $z$  and  $\omega$  be two complex numbers, such that  $|z|^2 \omega - |\omega|^2 z = z - \omega$  and  $z \neq \omega$ , then

- (A)  $z = \bar{\omega}$  (B)  $z\bar{\omega} = 1$   
 (C)  $\bar{z}\omega = 2$  (D)  $z\bar{\omega} = 2$

Key. B

Sol.  $|z|^2 \omega - |\omega|^2 z = z - \omega$

$\Rightarrow \omega[1 + |z|^2] = z[1 + |\omega|^2]$

$\omega\bar{\omega} = \frac{[1 + |z|^2]}{[1 + |\omega|^2]} = z\bar{z} = |\omega|^2 \left\{ \frac{1 + |z|^2}{1 + |\omega|^2} \right\}$

$\Rightarrow z\bar{\omega}$  is real number and therefore

$z\bar{\omega} = \omega\bar{z} \dots\dots (1)$

$|z|^2 \omega - |\omega|^2 z = z - \omega$

$z\bar{z}\bar{\omega} - \omega\bar{\omega}\bar{z} - z + \omega = 0$

$z(\bar{z}\bar{\omega} - 1) - \omega(\bar{\omega}\bar{z} - 1) = 0 \dots\dots (2)$

From (1) and (2)

$z(z\bar{\omega} - 1) - \omega(\omega\bar{z} - 1) = 0$

$(z\bar{\omega} - 1)(z - \omega) = 0$

$z\bar{\omega} = 1 = \bar{z}\omega$  Since  $z \neq \omega$

194. The point of intersection of the curves  $\arg(z - 3i) = \frac{3\pi}{4}$  and  $\arg(2z + 1 - 2i) = \frac{\pi}{4}$  is

- (A)  $\frac{3}{4} + i\frac{9}{4}$  (B)  $1 + 3i$   
 (C)  $1 + i$  (D) no solution

Key. D

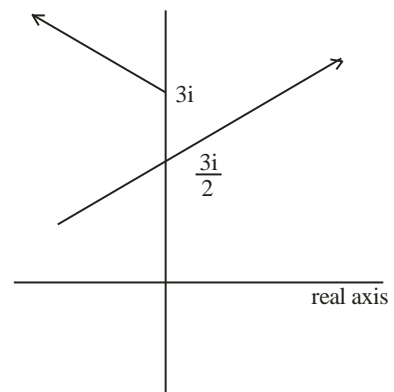
Sol.

$\arg(z - 3i) = \frac{3\pi}{4} \dots\dots (1)$

$\arg(2z + 1 - 2i) = \frac{\pi}{4}$

$\Rightarrow \arg\left(z + \frac{1}{2} - i\right) + \arg(2) = \frac{\pi}{4}$

$\Rightarrow \arg\left[z - \left(-\frac{1}{2}\right) + i\right] = \frac{\pi}{4} \dots\dots (2)$



No point of intersection of (1) and (2)

195. Let  $\left| \frac{z_1 - 2z_2}{2 - z_1\bar{z}_2} \right| = 1$  and  $|z_2| \neq 1$  where  $z_1$  and  $z_2$  are complex numbers.

The value of  $|z_1|$  is

- (A) 1 (B) 2  
 (C) cannot be obtained (D) none of these

Key. B

Sol.  $|z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$  i.e.,

$$\begin{aligned} (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) &= (2 - z_1\bar{z}_2)(2 - \bar{z}_1z_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1z_2) \\ \Rightarrow z_1\bar{z}_1 - 2z_2\bar{z}_1 - 2z_1\bar{z}_2 + 4z_2\bar{z}_2 &= 4 - 2z_1\bar{z}_2 - 2\bar{z}_1z_2 + |z_1|^2|z_2|^2 \\ \Rightarrow (|z_1|^2 - 4)(|z_2|^2 - 1) &= 0 \\ \text{Since } |z_2| \neq 1, \\ \Rightarrow |z_1|^2 = 4 \Rightarrow |z_1| &= 2 \end{aligned}$$

196. Points  $A(z_1)$ ,  $B(z_2)$  and  $C(z_3)$  form a triangle with centroid  $z_0$ . If triangles  $XCB$ ,  $CYA$  and  $BAZ$  similar to triangle  $ABC$  are outwardly drawn on the sides of  $\Delta ABC$ , then centroid of  $\Delta XYZ$  is

- (A)  $3z_0$  (B)  $-z_0$   
 (C)  $z_0$  (D)  $-2z_0$

Key. C

Sol.

The quadrilateral  $abxc$  is a parallelogram. if  $z$  is the affix of  $x$ ,

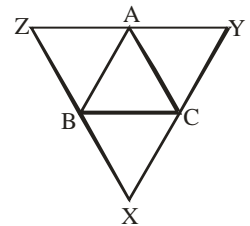
$$\frac{1}{2}(z_1 + z) = \frac{1}{2}(z_2 + z_3)$$

$$z = z_2 + z_3 - z_1$$

similarly affix of  $y$  is  $z_1 + z_3 - z_2$  and that of  $z$  is  $z_1 + z_2 - z_3$

centroid of  $\Delta XYZ$  is

$$\begin{aligned} \frac{1}{3}(z_2 + z_3 - z_1 + z_1 + z_3 - z_2 + z_1 + z_2 - z_3) \\ = \frac{1}{3}(z_1 + z_2 + z_3) = z_0 \end{aligned}$$



197. The roots of  $1 + z + z^3 + z^4 = 0$  are represented by the vertices of

- (A) a square (B) an equilateral triangle  
 (C) a rhombus (D) a rectangle

Key. B

SOL. The given equation is  $(1 + z)(1 + z^3) = 0$  the distinct roots being  $-1, -\omega, -\omega^2$  which if be represented by points  $a, b$  and  $c$  in that order

$$ab = |1 - \omega| = |\omega| |\omega^2 - 1| = |\omega^2 - 1|$$

$$bc = |\omega - \omega^2| = |\omega^2| |\omega^2 - 1| = |\omega^2 - 1|$$

$$ca = |\omega^2 - 1|$$



**THE THREE POINTS REPRESENT THE VERTICES OF AN EQUILATERAL TRIANGLE.**

198. If  $\omega$  is any complex number such that  $z\omega = |z|^2$  and  $|z - \bar{z}| + |\omega + \bar{\omega}| = 4$ , then as  $\omega$  varies, then the area bounded by the locus of  $z$  is  
 (A) 4 sq. Units (B) 8 sq. Units  
 (C) 16 sq. Units (D) 12 sq. Units

Key. B

Sol.  $z\omega = |z|^2 \Rightarrow \omega = \bar{z}$   
 $|z + \bar{z}| + |z - \bar{z}| = 4$   
 $|x| + |y| = 2$

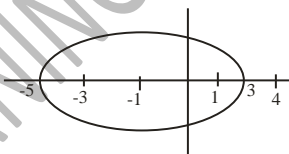
Which is a square  $\therefore$  Area = 8 sq. Units

199. If  $|z - 1| + |z + 3| \leq 8$ , then the range of values of  $|z - 4|$  is,  
 (A) (0, 8) (B) [1, 9]  
 (C) [0, 8] (D) [5, 9]

Key. B

Sol.

$z$  lies inside or on the ellipse. Clearly the minimum distance of  $z$  from the given point 4 is 1 and maximum distance is 9



200. The reflection of the complex number  $\frac{6+10i}{(1+i)^2}$  in the straight line  $i\bar{z} = z$ , is  
 (A)  $-3 + 5i$  (B)  $-3 - 5i$   
 (C)  $3 - 5i$  (D)  $3 + 5i$

Key. A

Sol.  $\frac{6+10i}{(1+i)^2} = \frac{6+10i}{2i} = 5 - 3i$   
 Put  $z = x + iy$   
 $i(x - iy) = x + iy$   
 $ix + y = x + iy$   
 $\Rightarrow (x - y) - i(x - y) = 0$   
 $\Rightarrow x - y = 0$

Reflection is  $(-3 + 5i)$

201. If  $z_1, z_2$  and  $z_3$  be the vertices of  $\Delta ABC$ , taken in anti-clock wise direction and  $z_0$  be the circumcentre, then  $\left(\frac{z_0 - z_1}{z_0 - z_2}\right) \frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right) \frac{\sin 2C}{\sin 2B}$  is equal to

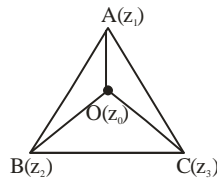
- (A) 0 (B) 1  
 (C) -1 (D) 2

Key. C

Sol. Taking rotation at O

$$\frac{z_0 - z_1}{z_0 - z_2} = \cos 2C - i \sin 2C$$

$$\frac{z_0 - z_3}{z_0 - z_2} = \cos 2A + i \sin 2A$$



Now 
$$\left( \frac{z_0 - z_1}{z_0 - z_2} \right) \frac{\sin 2A}{\sin 2B} + \left( \frac{z_0 - z_3}{z_0 - z_2} \right) \frac{\sin 2C}{\sin 2B}$$

$$= \frac{\sin 2A \cos 2C - i \sin 2A \sin 2C + \cos 2A \sin 2C + i \sin 2A \sin 2C}{\sin 2B}$$

$$= \frac{\sin(2A + 2C)}{\sin 2B} = -1$$

202. If  $z = \cos \alpha + i \sin \alpha$ ,  $0 < \alpha < \pi/6$  then the argument of  $\frac{z^4 - 1}{z^3 + 1}$  is

(A)  $\frac{\pi}{2} + \frac{\alpha}{2}$

(B)  $\frac{\pi}{2} - \frac{\alpha}{2}$

(C)  $\frac{3\alpha}{2}$

(D)  $2\alpha - \frac{\pi}{2}$

Key. A

Sol. 
$$\arg \left( \frac{z^4 - 1}{z^3 + 1} \right) = \arg(z^4 - 1) - \arg(z^3 + 1) = \left( 2\alpha - \frac{\pi}{2} + \pi \right) - \frac{3\alpha}{2} = \frac{\pi + \alpha}{2}$$

203. If  $|z| = 1$  and  $z' = \frac{1+z^2}{z}$ , then

(A)  $z'$  lie on a line not passing through origin (B)  $|z'| = \sqrt{2}$

(C)  $\text{Re}(z') = 0$

(D)  $\text{Im}(z') = 0$

Key. D

Sol. Conceptual

204. The number of complex numbers  $z$  satisfying  $|z + \bar{z}| + |z - \bar{z}| = 4$  and  $|z + 2i| + |z - 2i| = 4$  is/are

(A) 0

(B) 1

(C) 2

(D) 4

Key. C

SOL.  $\therefore |x| + |y| = 2$  .....(i)

$|z + 2i| + |z - 2i| = 4$  .....(ii)

eq. (i) represent square & (ii) represent line segment solution are  $z = \pm 2i$ .

205. If  $z_1, z_2$  are complex numbers such that  $z_1^3 - 3z_1z_2^2 = 2$  and  $3z_1^2z_2 - z_2^3 = 11$  then  $|z_1^2 + z_2^2| =$

A) 3

B) 4

C) 5

D) 6

Key. C

Sol.  $z_1^3 - 3z_1z_2^2 + 3iz_1^2z_2 - iz_2^3 = 2 + 11i \Rightarrow (z_1 + iz_2)^3 = 2 + 11i$

Similarly  $(z_1 - iz_2)^3 = 2 - 11i$

$|z_1^2 + z_2^2| = |(z_1 + iz_2)(z_1 - iz_2)| = |(2 + 11i)^{1/3}(2 - 11i)^{1/3}| = 5$

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