Date: 19/10/2021

Q1. Using integration, find the area of the region bounded by the curve $y = |x-1|$, the x-axis and the ordinates $x = -2$ and $x = 4$.

Q2. Using integration, find the area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -2$ and $x = 3$.

Q3. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.

Q4. Sketch the region $\{(x, y) : 4x^2 + 9y^2 = 36\}$ and find its area, using integration.

Q5. Using method of integration find the area bounded by the curve $|x| + |y| = 1$.

Q6. Using integration, find the area of the region bounded by the following curves, after making a rough sketch:

$$y = 1 + |x+1|, \quad x = -3, \quad x = 3, \quad y = 0.$$

Q7. Find the area bounded by the curves $y = x$ and $y = x^3$.

Q8. Draw the rough sketch of $y^2 = x + 1$ and $y^2 = -x + 1$ and determine the area enclosed by the two curves.

Q9. Find the area bounded by the parabola $y^2 = 4x$ and the straight line $x + y = 3$.

Q10. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

Q11. Draw a rough sketch and find the area of the region bounded by the two parabolas $y^2 = 8x$ and $x^2 = 8y$, by using method of integration.

Q12. Using the method of integration, find the area of the region bounded by the following lines: $5x - 2y - 10 = 0$, $x + y - 9 = 0$, $2x - 5y - 4 = 0$

Q13. Using the method of integration, find the area of the region bounded by the following lines: $3x - y - 3 = 0$, $2x + y - 12 = 0$, $x - 2y - 1 = 0$

Q14. Using integration, find the area of ΔABC, whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$.

Q15. Using integration, find the area of the triangle ABC where A is $(2, 3)$, B is $(4, 7)$ and C is $(6, 2)$.

Q16. Using integration, find the area of the triangle ABC whose vertices are $A(-1, 1)$, $B(0, 5)$ and $C(3, 2)$.

Q17. Draw the rough sketch of the curve :

$y = \sin^2 x,\ x \in \left[0, \frac{\pi}{2}\right]$. Find the area enclosed between the curve, x-axis and the line $x = \frac{\pi}{2}$.

Q18. Draw a rough sketch of the curve $y = \cos^2 x$ in $[0, \pi]$ and find the area enclosed by the curve, the lines $x = 0$, $x = \pi$ and x-axis.

Q19. Draw a rough sketch of the curves $y = \sin x$ and $y = \cos x$ as x varies from 0 to $\frac{\pi}{2}$. Find the area of the region enclosed by the curves and the y-axis.

Q20. Find the area bounded by the curve $|x| + y = 1$ and axis of x.

Q21. Find the area bounded by curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

Q22. Sketch the region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$. Also, find the area of the region, using integration.

Q23. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

Or

Find the area of the region given by
$\{(x, y) : x^2 \le y \le |x|\}$.

Q24. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above X-axis and between $x = -6$ to $x = 0$.

Q25. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the ordinates $x = 0$ and $x = ae$, where $b^2 = a^2 (1 - e^2)$ and $0 < e < 1$.

Q26. Find the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

Q27. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

Q28. AOB is the part of the ellipse $9x^2 + y^2 = 36$, in the first quadrant such that $OA = 2$ and $OB = 6$. Find the area between the arc AB and the chord AB.

Q29. Find the area of the region bounded by the curves $x = 2y - y^2$ and $y = 2 + x$.

Q30. Find the area of the region $\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$.

Q31. Find the area of the region bounded by the region enclosed by the curves

$$(x - 6)^2 + y^2 = 36$$

and

$$x^2 + y^2 = 36.$$

Q32. Find the area of the region enclosed between two circles $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$.

Q33. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

Q34. Find the area of the region enclosed by the parabola $y^2 = x$ and the line $x + y = 2$.

Q35. Find the area enclosed by the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.

Q36. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

Q37. Using integration, find the area enclosed by the parabola $y^2 = 4ax$ and the chord $y = mx$.

Q38. Find the area of the region bounded by the $y = x^2 + 1$, $y = x$, $x = 0$ and $y = 2$.

Q39. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

Q40. Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

Q41. Find the area of circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Q42. Using integration, find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

Q43. Find the area lying above the x-axis and included between the curves $x^2 + y^2 = 8x$ and $y^2 = 4x$.

Q44. Find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$, by using integration.

Q45. Find the area of the region bounded by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$

Q46. Find the area of the region $\{(x, y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$.

Q47. Find area of the region.
$\{(x, y): x^2 + y^2 \le 4, x + y \ge 2\}$.

Q48. Find the area of the region
$\{(x, y) : (x^2 + y^2) \le 1 \le x + y\}$.

Q49. Using integration, find the area of the following region
$\{(x, y) : |x - l| \le y \le \sqrt{5 - x^2}\}$.

Q50. Using integration, find the area of the following region
$\left\{(x, y) : \frac{x^2}{9} + \frac{y^2}{4} \le 1 \le \frac{x}{3} + \frac{y}{2}\right\}$

Q51. Using integration, find the area of ΔABC, the coordinates of whose vertices are $A(2, 5)$, $B(4, 7)$ and $C(6,2)$.

Q52. Using integration, find the area of ΔABC, coordinates of whose vertices are $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$.

Q53. Using method of integration, find the area of the region bounded by the lines
$2x + y = 4$; $3x - 2y = 6$; $x - 3y + 5 = 0$.

Q54. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Q55. Using method of integration find the area of region bounded by lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$

Q56. Using integration, find the area of the triangle ABC, Whose vertices are $A(3, 0)$, $B(4, 5)$ and $C(5, 1)$.

Q57. Using integration find the area of region bounded by the triangle whose vertices are $(-1, 0)$, $(1, 3)$ and $(3, 2)$.

Q58. Find the area of the triangle formed by the lines $y = x + 1$, $3y = x + 5$ and $y = -x + 7$ by method of integration.

Q59. Find the area of the region $\{(x, y) : y^2 \le 6ax \text{ and } x^2 + y^2 \le 16a^2\}$ using method of integration.

Q60. Sketch the curves and identify the region bounded by $x = \frac{1}{2}$, $x = 2$, $y = \log_e x$ and $y = 2^x$. Find the area of this region.

Q61. Show that areas under the curves $y = \sin^2 x$ and $y = \cos^2 x$ between $x = 0$ and $x = \pi$ are as 1 : 1.

Q62. Show that areas under the curves $y = \sin x$ and $y = \sin 2x$ between $x = 0$ and $x = \frac{\pi}{3}$ are as 2 : 3.

Q63. Draw a rough sketch of the curves $y = \sin x$ and $y = \cos x$ as x varies from 0 to $\frac{\pi}{2}$ and find the area of the region enclosed by them and x-axis.

Q64. Using integration, find the area of triangular region whose vertices are (1, 0), (2, 2) and (3, 1).

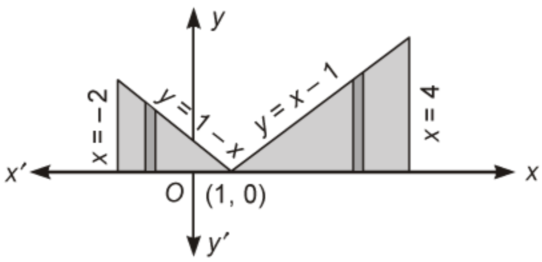
Q65. Using integration, find the area of the region bounded by the triangle whose vertices are (1, 3), (2, 5) and (3, 4).

Date: 19/10/2021

S1. The given equation to the curve is $y = |x - 1|$

$$\Rightarrow \quad y = \begin{cases} x - 1, & x - 1 \ge 0 \\ -(x - 1), & x - 1 < 0 \end{cases}$$

$$= \begin{cases} x - 1, & x \ge 1 \\ -(x - 1), & x < 1 \end{cases} \quad \text{... (i)}$$



Limits are $x = -2$ and $x = 4$

$\therefore$ The required area

$$= \int_{-2}^{4} y\,dx = \int_{-2}^{1} y\,dx + \int_{1}^{4} y\,dx$$

$$= -\int_{-2}^{1} (x-1)\,dx + \int_{1}^{4} (x-1)\,dx \quad \text{[From Eq. (i)]}$$

$$= -\left[\frac{x^2}{2} - x\right]_{-2}^{1} + \left[\frac{x^2}{2} - x\right]_{1}^{4}.$$

$$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{4}{2} + 2\right)\right] + \left[\left(\frac{16}{2} - 4\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{9}{2} + \frac{9}{2} = 9 \text{ sq unit}$$

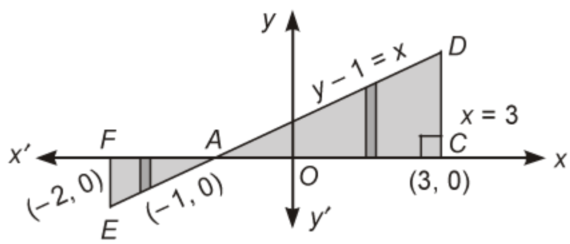
S2. $y - 1 = x$ or $y = x + 1$ is the given line DE. ... (i)

$x = -2$ is the line EF.

$x = 3$ is the line CD.

Let A be a point of intersection of (i) and x-axis.

Limits are $x = -2$ and $x = -1$ for the area AEF and the limits for the area ACD are $x = -1$ and $x = 3$.



The required area = Shaded area

$$= |\Delta AFE| + |\Delta ACD|$$

$$= \left|\int_{-2}^{-1}(x+1)\,dx\right| + \left|\int_{-1}^{3}(x+1)\,dx\right|$$

$$= \left|\left[\frac{x^2}{2}+x\right]_{-2}^{-1}\right| + \left|\left[\frac{x^2}{2}+x\right]_{-1}^{3}\right|$$

$$= \left|\left(\frac{1}{2}-1\right)-(2-2)\right| + \left|\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right|$$

$$= \frac{1}{2}+8 = 8.5 \text{ sq. units.}$$

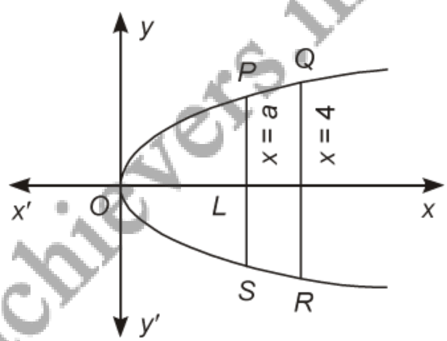
S3. Graph of the curve $x = y^2$ is a parabola as shown in the figure.

It's vertex is O and axis is x-axis.

$\therefore$ Area of the region $ORQO$

$$A_1 = 2\int_0^4 y\,dx = 2\int_0^4 \sqrt{x}\,dx = 2\times\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4 = \frac{4}{3}\left[4^{\frac{3}{2}}-0\right] = \frac{32}{3}\text{ sq. units}$$

Area of the region bounded by the curve and ordinate $x = a$ is



$$A_2 = 2\int_0^a y\,dx = 2\int_0^a \sqrt{x}\,dx = 2\cdot\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^a = \frac{4}{3}\left(a^{\frac{3}{2}}-0\right) = \frac{4}{3}\cdot a^{\frac{3}{2}}$$

Now, PS [$x = a$] divides the area of the region $ORQO$ into two equal parts

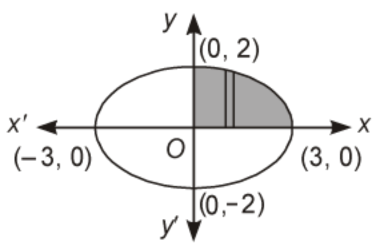
$\therefore$ Area of region $ORQO$ = 2 Area of the region $OSPO$

$\therefore$ $A_1 = 2A_2$

$$\therefore \quad \frac{32}{3} = 2\cdot\frac{4}{3}a^{\frac{3}{2}} \quad\Rightarrow\quad a^{\frac{3}{2}} = 4 \quad\Rightarrow\quad a = 4^{\frac{2}{3}}$$

S4. Region $\{(x, y) : 4x^2 + 9y^2 = 36\}$ = Region bounded

This is an ellipse, with major axis, $2a = 2\times 3 = 6$ and minor axis, $2b = 2\times 2 = 4$



Limits for the shaded area are $x = 0$ and $x = 3$.

The required area of the ellipse

$$= 4\int_0^3 y\,dx = 4\int_0^3 2\sqrt{1-\frac{x^2}{9}}\,dx \qquad \left[\because\ \frac{x^2}{9}+\frac{y^2}{4}=1 \ \Rightarrow\ \frac{y^2}{4}=1-\frac{x^2}{9} \ \Rightarrow\ y = 2\sqrt{1-\frac{x^2}{9}}\right]$$

$$= \frac{8}{3}\int_0^3 \sqrt{3^2 - x^2}\,dx = \frac{8}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 \qquad \left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$= \frac{8}{3}\left[0 + \frac{9}{2}\sin^{-1}1 - 0 - 0\right] = \frac{8}{3}\times\frac{9}{2}\times\frac{\pi}{2} = 6\pi \text{ sq. units.}$$

S5. In I Quadrant, $x > 0$, $y > 0$

$|x| = x \qquad |y| = y$

The line is $\quad x + y = 1 \quad$ ... (i)

In II Quadrant, $x < 0$, $y > 0$

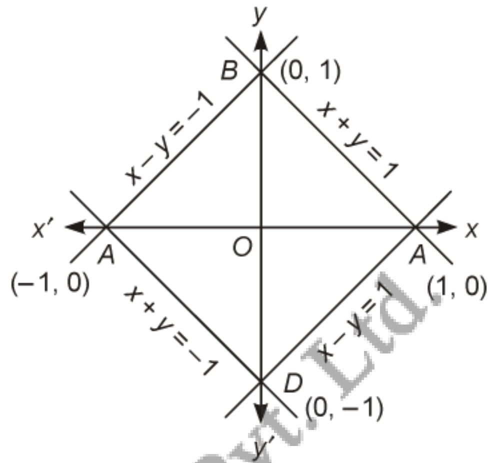
$|x| = -x \qquad |y| = y$

The line is $\quad -x + y = 1$

or $\quad x - y = -1 \quad$ ... (ii)

In III Quadrant, $x < 0$, $y < 0$

$|x| = -x \qquad |y| = -y$



The line is $\quad -x - y = 1$

or $\quad x + y = -1 \quad$ ... (iii)

In IV Quadrant, $x > 0$, $y < 0$

$|x| = x \qquad |y| = -y$

The line is $\quad x - y = 1 \quad$ ... (iv)

$|x| + |y| = 1$ represents four lines forming square $ABCD$, Area of square $ABCD$ = 4 × Area of $\Delta AOB = 4 \times \int_0^1 (1-x)\,dx$ since $x + y = 1$ is the equation of the line.

$$= 4 \times \left[x - \frac{x^2}{2}\right]_0^1$$

$$= 4\left(1 - \frac{1}{2}\right) = 4 \times \frac{1}{2} = 2 \text{ sq. unit}$$

S6. Given curve is

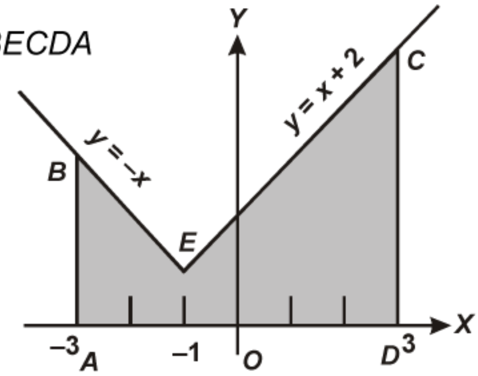
$$y = 1 + |x+1| = \begin{cases} 1 + x + 1, & \text{if } x + 1 \geq 0 \\ 1 - (x+1), & \text{if } x + 1 < 0 \end{cases}$$

or, $$y = \begin{cases} x + 2, & \text{if } x \geq -1 \\ -x, & \text{if } x < -1 \end{cases} \qquad \text{... (i)}$$

Given lines are $x = -3, \quad x = 3, \quad y = 0$ ... (ii)

The rough sketch of (i) has been shown in the figure.

$\therefore$ The required area = the area of the shaded region $ABECDA$



$$= \int_{-3}^{-1} y_{BE}\, dx + \int_{-1}^{3} y_{EC}\, dx$$

$$= \int_{-3}^{-1} (-x)\, dx + \int_{-1}^{3} (x+2)\, dx$$

$$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\frac{1}{2}(1-9) + \left[\left(\frac{9}{2}+6\right) - \left(\frac{1}{2}-2\right)\right]$$

$= 4 + 12 = 16$ sq. units.

S7. Equation of given curves are

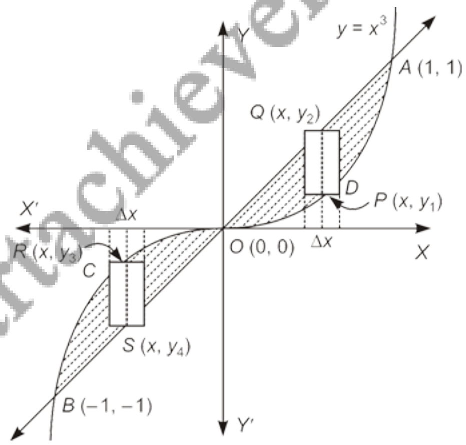
$y = x$ ... (i)

and $y = x^3$ ... (ii)

From Eq. (i) and Eq. (ii), $x = x^3$

$\Rightarrow \quad x - x^3 = 0$

$\Rightarrow \quad x(1 - x^2) = 0$



$\therefore \quad x = 0$

or $\quad x^2 = 1$

$\therefore \quad x = 0, x = 1$

or $\quad x = -1$

From Eq. (i), $\quad x = 0 \Rightarrow y = 0$

$x = 1 \Rightarrow y = 1$ and $x = -1 \Rightarrow y = -1$

Now, required area = area $OBCO$ + area $ODAO$

$$= \int_{-1}^{0} \left[y_{\text{curve(ii)}} - y_{\text{line (i)}} \right] dx + \int_{0} \left[y_{\text{line(i)}} - y_{\text{curve (ii)}} \right] dx$$

$$= \int_{-1}^{0}(x^3 - x)\,dx + \int_{0}^{1}(x - x^3)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1}$$

$$= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - \frac{1}{4}\right) - 0\right]$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ sq. units.}$$

S8. Given curves are $y^2 = x + 1$ ... (i)

and $y^2 = -x + 1$ or, $y^2 = -(x - 1)$ ... (ii)

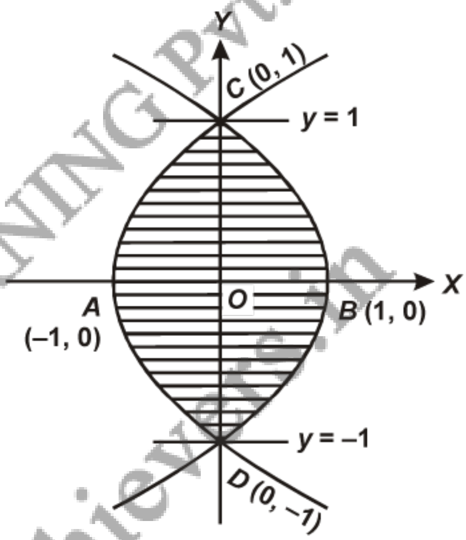
Curve (i) is the parabola having axis $y = 0$ and vertex $(-1, 0)$.

Curve (ii) is the parabola having axis $y = 0$ and vertex $(1, 0)$.

(i) – (ii) $\Rightarrow$ $2x = 0 \Rightarrow x = 0$

From (i), $x = 0 \Rightarrow y = \pm 1$

Required area = area $ACBDA$



$$= \int_{-1}^{1}\left(x_{\text{ curve }(ii)} - x_{\text{ curve}(i)}\right)dy$$

$$= \int_{-1}^{1}\left[(1 - y^2) - (y^2 - 1)\right]dy$$

$$= 2\int_{-1}^{1}(1 - y^2)dy = 2\left[y - \frac{y^3}{3}\right]_{-1}^{1}$$

$$= 2\left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] = \frac{8}{3} \text{ sq. units.}$$

S9. Given curves are $y^2 = 4x$ ... (i)

and $x + y = 3$ ... (ii)

Curve (i) is a right handed parabola whose vertex is (0, 0) and axis is $y = 0$.

Line (ii) cuts x-axis at (3, 0) and y-axis at (0, 3). Here required area $OCDAO$ is bounded by curves (i) and (ii) and abscissa at A and C.

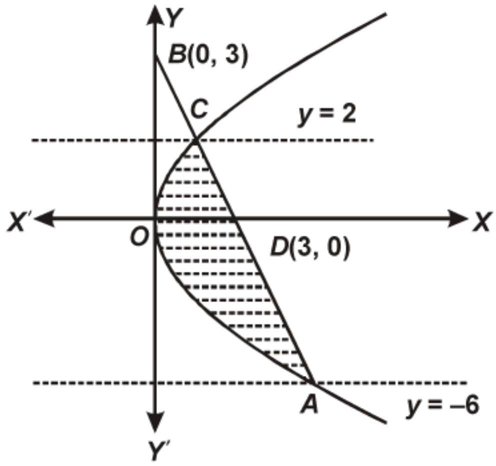
Hence we will find the values of y from equations (i) and (ii).

Putting the value of x from equation (ii) in (i), we get

$$y^2 = 4(3 - y)$$

or, $$y^2 + 4y - 12 = 0$$

$\therefore$ $$y = -6, 2$$



Required area $OCDAO = \int_{-6}^{2}(x_1 - x_2)\,dy$

$$= \int_{-6}^{1}\left(x_{line\,(i)} - x_{curve\,(ii)}\right)dy$$

$$= \int_{-6}^{2}\left[(3-y) - \frac{y^2}{4}\right]dy = \left[3y - \frac{y^2}{2} - \frac{y^3}{12}\right]_{-6}^{2}$$

$$= \left(6 - 2 - \frac{2}{3}\right) - \left(-18 - 18 + \frac{216}{12}\right) = \frac{10}{3} + 18 = \frac{64}{3} \text{ sq. units.}$$

S10. Given curve is $y = x^2 + 2$.

Limits are $x = 0$ to $x = 3$.

We have to find the area bounded by the curve $y = x^2 + 2$, lines $y = x$, $x = 3$ and $x = 0$.

Shaded region in the adjoining figure is required area.

$\therefore$ Required area = Area $OACDO$ – Area $ODBO$

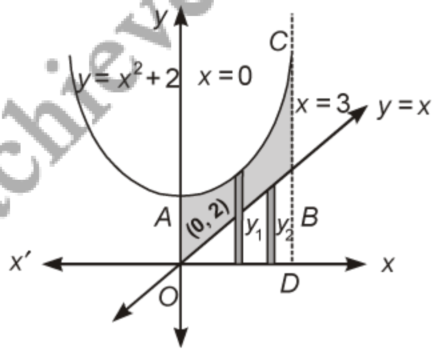
$$= \int_0^3 y_1\,dx - \int_0^3 y_2\,dx$$

$$= \int_0^3 (x^2 + 2)dx - \int_0^3 x\,dx$$

$$= \left[\frac{x^3}{3} + 2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$$

$$= \left[\left(\frac{27}{3} + 6\right) - (0)\right] - \left[\frac{9}{2} - 0\right]$$

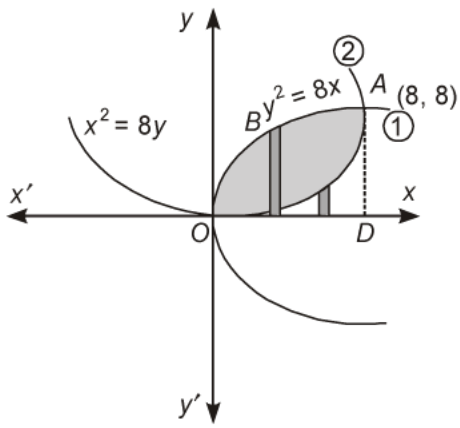
$$= 15 - \frac{9}{2} = \frac{21}{2} \text{ sq. units.}$$



S11. The given parabolas are

$$y^2 = 8x \quad \text{... (i)}$$

$$x^2 = 8y \quad \text{... (ii)}$$



Eq. (i) and (ii) meet at $O(0, 0)$ and $A(8, 8)$.

$\therefore$ Required area = Area $OBADO$ – Area $OADO$

$$= \int_0^8 (y_1 - y_2)\,dx$$

Limits are $x = 0$ and $x = 8$.

$$= \int_0^8 \left(\sqrt{8}x - \frac{x^2}{8}\right)dx$$

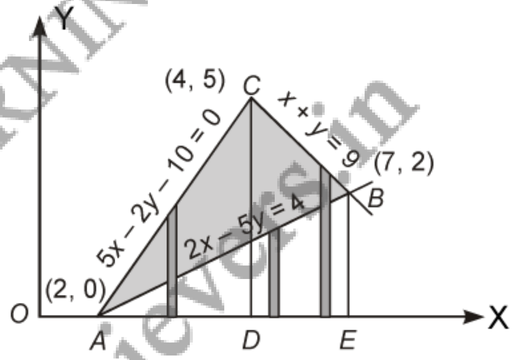
$$= \left[2\sqrt{2}\cdot\frac{x^{3/2}}{3/2} - \frac{1}{8}\frac{x^3}{3}\right]_0^8$$

$$= \frac{4\sqrt{2}}{3}16\sqrt{2} - \frac{64}{3} = \frac{64}{3} \text{ sq. units}.$$

S12. $5x - 2y - 10 = 0 \quad \text{... (i)}$

$x + y - 9 = 0 \quad \text{... (ii)}$

$2x - 5y - 4 = 0 \quad \text{... (iii)}$



Required area = Shaded portion of the figure = Area ABC

= Area ACD + Area $BCDE$ – Area AEB

$$= \int_2^4 y_1\,dx + \int_4^7 y_2\,dx - \int_2^7 y_3\,dx$$

$$= \int_2^4 \frac{5x-10}{2}dx + \int_4^7 (9-x)dx - \int_2^7 \frac{2x-4}{5}dx$$

$$= \frac{1}{2}\left[\frac{5x^2}{2} - 10x\right]_2^4 + \left[9x - \frac{x^2}{2}\right]_4^7 - \frac{1}{5}(x^2 - 4x)_2^7$$

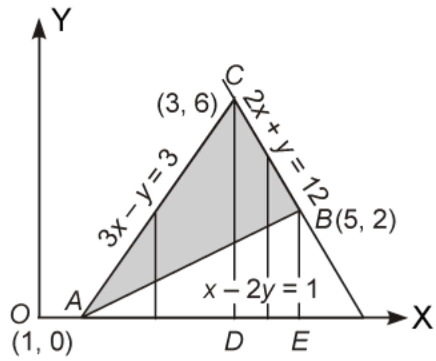
$$= \frac{1}{2}[40 - 40 - 10 + 20] + \left(63 - \frac{49}{2} - 36 + 8\right) - \frac{1}{5}(49 - 28 - 4 + 8)$$

$$= \frac{1}{2}(10) + \frac{21}{2} - 5 = \frac{21}{2} \text{ Sq. unit.}$$

S13. $3x - y = 3$ ... (i)

$2x + y = 12$ ... (ii)

$x - 2y = 1$ ... (iii)



Required area = Area of shaded portion of the figure

$$= \int_1^3 y_1\,dx + \int_3^5 y_2\,dx - \int_1^5 y_3\,dx$$

$$= \int_1^3 (3x-3)dx + \int_3^5 (12-2x)dx - \int_1^5 \frac{x-1}{2}dx$$

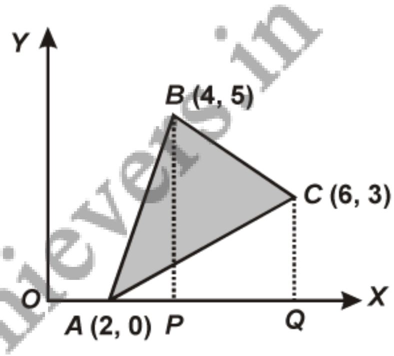
$$= \left(\frac{3x^2}{2} - 3x\right)_1^3 + [12x - x^2]_3^5 - \frac{1}{2}\left(\frac{x^2}{2} - x\right)_1^5$$

$$= \left(\frac{27}{2} - 9 - \frac{3}{2} + 3\right) + (60 - 25 - 36 + 9) - \frac{1}{2}\left(\frac{25}{2} - 5 - \frac{1}{2} + 1\right)$$

$= 6 + 8 - 4 = 10$ Sq. unit.

S14. The equation of side AB is

$$y - 0 = \frac{(5-0)}{(4-2)}(x-2) \quad \text{or} \quad y = \frac{5}{2}(x-2) \qquad \text{... (i)}$$



The equation of side BC is

$$y - 5 = \frac{(3-5)}{(6-4)}(x-4) \quad \text{or} \quad y = -x + 9 \qquad \text{... (ii)}$$

The equation of side AC is

$$y - 0 = \frac{(3-0)}{(6-2)}(x-2) \quad \text{or} \quad y = \frac{3}{4}(x-2) \qquad \text{... (iii)}$$

We draw perpendiculars BP and CQ on the X-axis.

$\therefore$ area of ΔABC = ar (ΔAPB) + ar (trapezium $BPQC$) – ar (ΔAQC)

$$= \int_2^4 y_{AB}dx + \int_4^6 y_{BC}dx - \int_2^6 y_{AC}dx$$

$$= \frac{5}{2}\int_2^4 (x-2)dx + \int_4^6 (9-x)dx - \frac{3}{4}\int_2^6 (x-2)dx$$

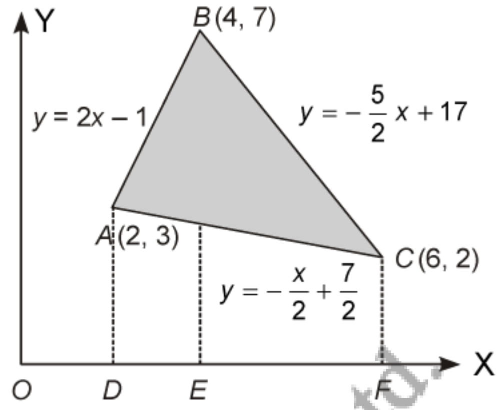
$$= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[9x - \frac{x^2}{2}\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$$

$$= \frac{5}{2}[0-(-2)] + (36-28) - \frac{3}{4}[6-(-2)]$$

$= (5 + 8 - 6)$ sq. units

$= 7$ sq. units.

S15. Equation of AB, $y - 3 = \frac{7-3}{4-2}(x-2)$



$\Rightarrow$ $y = 2x - 1$ ... (i)

Equation of BC, $y - 7 = \frac{2-7}{6-4}(x-4)$

$\Rightarrow$ $y = \frac{-5}{2}x + 17$ ... (ii)

Equation of AC, $y - 3 = \frac{2-3}{6-2}(x-2)$

$\Rightarrow$ $y = -\frac{x}{4} + \frac{7}{2}$ ... (iii)

Required area = ⏢ $ABED$ + ⏢ $BEFC$ – ⏢ $ADFC$

$$= \int_2^4 (2x-1)\,dx + \int_4^6 \left(\frac{-5}{2}x + 17\right)dx - \int_2^6 \left(-\frac{x}{4} + \frac{7}{2}\right)dx$$

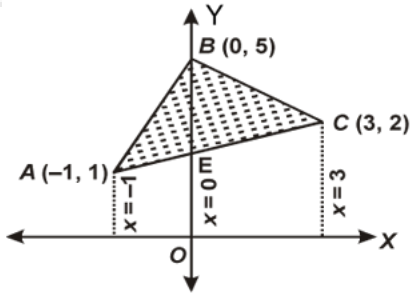
$$= [x^2 - x]_2^4 + \left[\frac{-5x^2}{4} + 17x\right]_4^6 - \left[\frac{-x^2}{8} + \frac{7x}{2}\right]_2^6.$$

$= [(16-4) - (4-2)] + [(-45 + 102) - (20 + 68)]$

$$-\left[\left(-\frac{36}{8} + 21\right) - \left(-\frac{4}{8} + \frac{14}{2}\right)\right]$$

$= (10) + (9) - (10) = 9$ sq. unit

S16. Equation of AB is



$y - 1 = \frac{5-1}{0+1}(x+1)$

or, $y - 1 = 4\,(x + 1)$

or, $y = 4x + 5$ ... (i)

Equation of BC is $y - 5 = \frac{5-2}{0-3}(x-0)$

or, $y = 5 - x$ ... (ii)

Equation of AC is $y - 1 = \frac{2-1}{3+1}(x+1)$

or $\quad 4y = x + 5 \quad$ ... (iii)

Now $\quad$ area of ΔABC = area of $ABEA$ + area $EBCE$

$$= \int_{-1}^{0}(y_{AB} - y_{AE})\,dx + \int_{0}^{3}(y_{BC} - y_{EC})\,dx$$

$$= \int_{-1}^{0}\left[4x + 5 - \frac{x+5}{4}\right]dx + \int_{0}^{3}\left[5 - x - \frac{x+5}{4}\right]dx$$

$$= \left[2x^2 + 5x\right]_{-1}^{0} - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{0}^{3}$$

$$= \left[0 - (2-5)\right] - \frac{1}{4}\left[0 - \left(\frac{1}{2} - 5\right)\right] + \left[\left(15 - \frac{9}{2}\right) - 0\right] - \frac{1}{4}\left[\left(\frac{9}{2} + 15\right) - 0\right]$$

$$= 3 - \frac{9}{8} + \frac{21}{2} - \frac{39}{8}$$

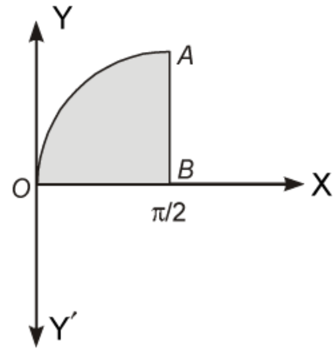
$$= 3 + \frac{21}{2} - 6$$

$$= \frac{15}{2} \text{ sq. units.}$$

S17. Some points on the $\sin^2 x$ graph are:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	0	0.25	0.49	0.64	1

By plotting points and joining them, we trace the curve.



Area bounded by curve $y = \sin^2 x$ between $x = 0$ and $x = \frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} y\,dx = \int_0^{\frac{\pi}{2}} \sin^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}(1 - \cos 2x)\,dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

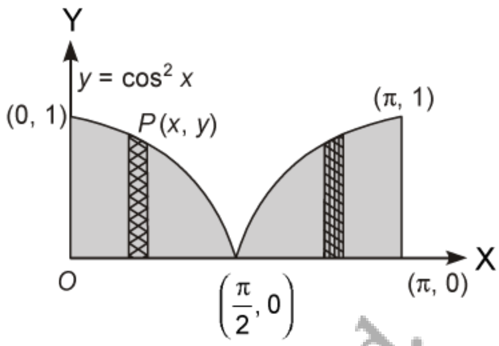
$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - (0 - 0)\right] = \frac{\pi}{4} \text{ sq. units}.$$

S18. The values of $\cos^2 x$ at different points lying between 0 to π are given below:

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos^2 x$	1	0.75	0.5	0.25	0	0.25	0.5	0.75	1

With the help of these points, we draw a graph.

The limits are $x = 0$ and $x = \pi$.



$\therefore$ Required area = $\int_0^\pi y\,dx = \int_0^\pi \cos^2 x\,dx$

[$\because$ $P(x, y)$ lies on $y = \cos^2 x$]

$$= \frac{1}{2}\int_0^\pi (1+\cos 2x)\,dx = \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^\pi$$

$$= \frac{1}{2}\left[\left(\pi + \frac{\sin 2\pi}{2}\right) - 0\right] = \frac{\pi}{2} \text{ sq. units.}$$

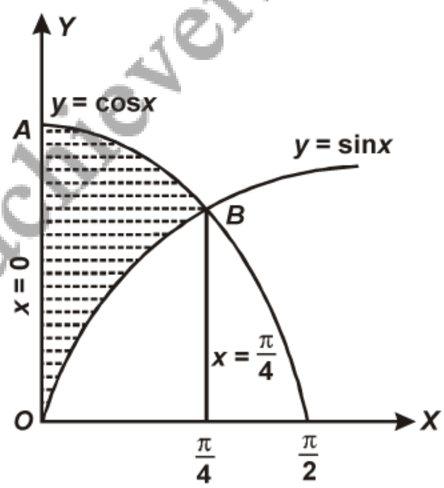
S19. Given curves are $y = \sin x$ ... (i)

and $y = \cos x$ ... (ii)

From (i) and (ii), $\sin x = \cos x$

$\Rightarrow$ $\tan x = 1$

$\Rightarrow$ $x = \frac{\pi}{4}$ $\left(\because 0 \le x \le \frac{\pi}{2}\right)$



Required area $OABO = \int_0^{\frac{\pi}{4}} (y_1 - y_2)\,dx$

$$= \int_0^{\pi/4} (y_{\text{curve}(ii)} - y_{\text{curve}(i)})\,dx$$

$$= \int_0^{\frac{\pi}{4}} \cos x\,dx - \int_0^{\frac{\pi}{4}} \sin x\,dx$$

$$= [\sin x]_0^{\frac{\pi}{4}} + [\cos x]_0^{\frac{\pi}{4}}$$

$$= \left(\sin\frac{\pi}{4} - \sin 0\right) + \left(\cos\frac{\pi}{4} - \cos 0\right)$$

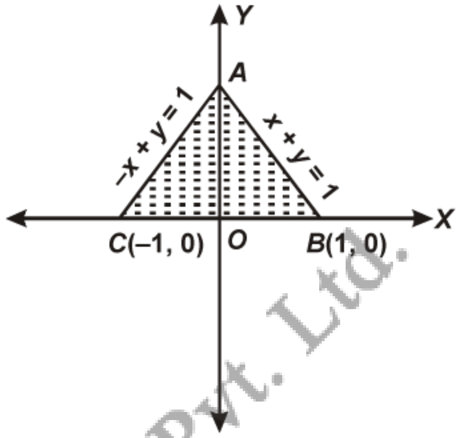
$$= \frac{1}{\sqrt{2}} - 0 + \frac{1}{\sqrt{2}} - 1 = (\sqrt{2} - 1) \text{ sq. units.}$$

S20. Given curve is $|x| + y = 1$

$\therefore$ Curve is $x + y = 1$, when $x \geq 0$ ... (i)

and $-x + y = 1$, when $x < 0$ ... (ii)

The graph of the curve is as given in the figure



$\therefore$ Required area = area $CAOC$ + area $OABO$

$$= \int_{-1}^{0} y_{AC}\, dx + \int_{0}^{1} y_{AB}\, dx$$

$$= \int_{-1}^{0} y\, dx + \int_{0}^{1} y\, dx$$

$$= \int_{-1}^{0} (x+1)\, dx + \int_{0}^{1} (1-x)\, dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-1}^{0} + \left[x - \frac{x^2}{2}\right]_{0}^{1}$$

$$= \left[0 - \left(\frac{1}{2} - 1\right)\right] + \left[\left(1 - \frac{1}{2}\right) - 0\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \text{ sq. unit}$$

Second method:

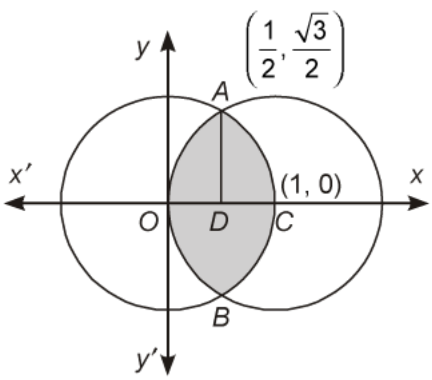
Required area = 2 area of ΔABO

$$= 2 \cdot \frac{1}{2} \cdot OB \cdot OA = 2 \cdot \frac{1}{2} \cdot 1 \cdot 1 = 1 \text{ sq. unit}$$

S21. The given circles are:

$$(x - 1)^2 + y^2 = 1 \quad \text{... (i)}$$

$$x^2 + y^2 = 1 \quad \text{... (ii)}$$



Subtracting Eq. (ii) from Eq. (i), we get

$(x - 1)^2 - x^2 = 0 \quad \Rightarrow \quad (x - 1)^2 = x^2$

$\Rightarrow \quad x^2 - 2x + 1 = x^2 \quad \Rightarrow \quad 2x = 1$

$\Rightarrow \quad x = \frac{1}{2}$

Putting $x = \frac{1}{2}$ in Eq (ii), we get

$$\frac{1}{4} + y^2 = 1 \quad \Rightarrow \quad y^2 = 1 - \frac{1}{4} = \frac{3}{4} \quad \Rightarrow \quad y = \frac{\pm\sqrt{3}}{2}$$

Thus, these intersect at $A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $B\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$.

The area of enclosed region

$= \text{Area } OBCAO = 2 \text{ Area } OACO$

$= 2\,[\text{Area } OAD + \text{Area } ACD]$

$$= 2\int_0^{\frac{1}{2}} y_1\,dx + 2\int_{\frac{1}{2}}^{1} y_2\,dx$$

$$= 2\left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx\right]$$

$$= 2\left\{\left[\frac{(x-1)\sqrt{1-(x-1)^2}}{2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + 2\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}\right\}$$

$$= \left[-\frac{1}{2}\sqrt{1-\frac{1}{4}} + \sin^{-1}\left(-\frac{1}{2}\right)\right] - [0+\sin^{-1}(-1)] + [0+\sin^{-1}1] - \left[\frac{1}{2}\sqrt{1-\frac{1}{4}} + \sin^{-1}\frac{1}{2}\right]$$

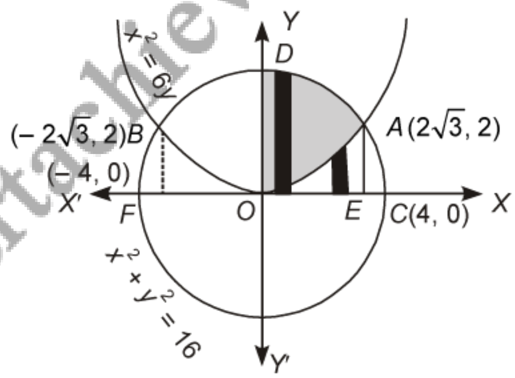
$$= \left[-\frac{\sqrt{3}}{4} + \left(-\frac{\pi}{6}\right) - \left(-\frac{\pi}{2}\right)\right] + \left(\frac{\pi}{2} - \frac{\sqrt{3}}{4} - \frac{\pi}{6}\right)$$

$$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}\ \text{sq. units}.$$

S22. Here, the curves are

$x^2 + y^2 = 16$ ... (i) (circle)

$x^2 = 6y$ ... (ii) (parabola)



Now, Eq. (i) and (ii) meet at $A(2\sqrt{3}, 2)$ and $B(-2\sqrt{3}, 2)$. Also, (i) meets x-axis at $C(4, 0)$ and $F(-4, 0)$.

Hence, the limits are $x = 0$ and $x = 2\sqrt{3}$ for the area $OADO$.

The required area = $OADBO$.

$= 2 \,.\, \text{Area } OADO$ ($\because$ by symmetry)

$= 2\,[\text{Area } OEADO - \text{Area } OEA]$

$$= 2\left[\int_0^{2\sqrt{3}} \sqrt{16-x^2}\,dx - \int_0^{2\sqrt{3}} \frac{x^2}{6}\,dx\right]$$

$$= 2\int_0^{2\sqrt{3}} \left[\sqrt{4^2-x^2} - \frac{x^2}{6}\right]dx$$

$\Rightarrow \quad x^2 + \left(\frac{x^2}{6}\right) = 16$

$\Rightarrow \quad x^4 + 36x^2 - 576 = 0$

$\Rightarrow \quad x^4 + 48x^2 - 12x^2 - 576 = 0$

$\Rightarrow \quad x^2(x^2+48) - 12(x^2+48) = 0$

$\Rightarrow \quad (x^2+48)(x^2+12) = 0$

$\Rightarrow \quad x^2 - 12 = 0 \quad \Rightarrow \quad x^2 = 12$

$$= 2\left[\frac{x\sqrt{16 - x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4} - \frac{x^3}{18}\right]_0^{2\sqrt{3}}$$

$$= 2\left[\frac{1}{2}\times 2\sqrt{3}\times 2 + 8\sin^{-1}\frac{2\sqrt{3}}{4} - \frac{24\sqrt{3}}{18}\right]$$

$$= \left[4\sqrt{3} + 16\sin^{-1}\frac{\sqrt{3}}{2} - \frac{8}{3}\sqrt{3}\right]$$

$$= \frac{16\pi + 4\sqrt{3}}{3}\text{ sq. units}.$$

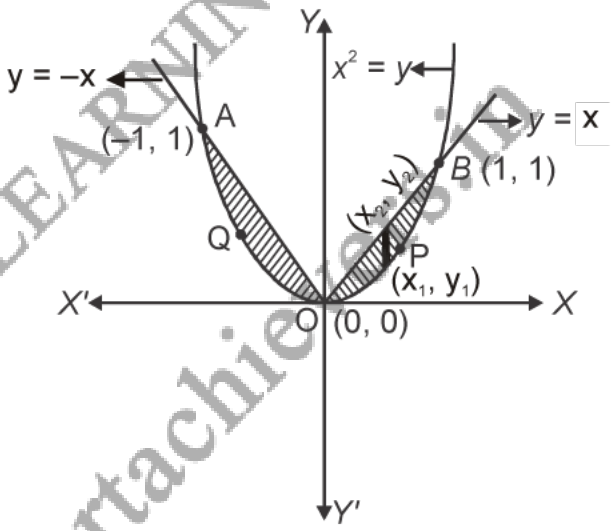
S23. Given curve are

$$x^2 = y \quad \text{... (i)}$$

and $$y = |x| \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$x^2 = |x|$$



Case I^st: when $x \le 0$

$\Rightarrow$ $$x^2 = -x$$

$\Rightarrow$ $$x(x + 1) = 0$$

$\Rightarrow$ $$x = 0, -1$$

Putting the values of x in Eq. (i), we get $y = 0, 1$

Case II^nd: when $x \ge 0$

$\Rightarrow$ $$x^2 = x$$

$\Rightarrow$ $$x(x - 1) = 0$$

$\Rightarrow$ $$x = 0, 1$$

Putting the values of x in Eq. (i), we get $y = 0, 1$

$\therefore$ Both curve cut each other at points

$$A(-1, 1) \quad O(0, 0) \quad \text{and} \quad B(1, 1)$$

Now Area of curve *OPBO*

$$= \int_0^1 (y_2 - y_1)dx = \int_0^1 (x - x^2)dx$$

As (x_1, y_1) lie on $x^2 = y$ and (x_2, y_2) lie on $y = x$

$\therefore$ $$y_1 = x^2 \quad \text{and} \quad y_2 = x$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \left[\left(\frac{1}{2} - \frac{1}{3}\right) - (0-0)\right] = \frac{3-2}{6} = \frac{1}{6}$$

Hence, required area = 2 × Area of curve *OPBO* [By symmetry]

$$= 2 \times \frac{1}{6} = \frac{1}{3} \text{ sq. units}.$$

S24. First, we sketch the graph of

$$y = |x + 3|,$$

We know that

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

$$\therefore \quad y = |x+3| \begin{cases} x+3, & \text{if } x+3 \geq 0 \\ -(x+3), & \text{if } x+3 < 0 \end{cases}$$

$$\Rightarrow \quad y = |x+3| \begin{cases} x+3, & \text{if } x \geq -3 \\ -(x+3), & \text{if } x < -3 \end{cases}$$

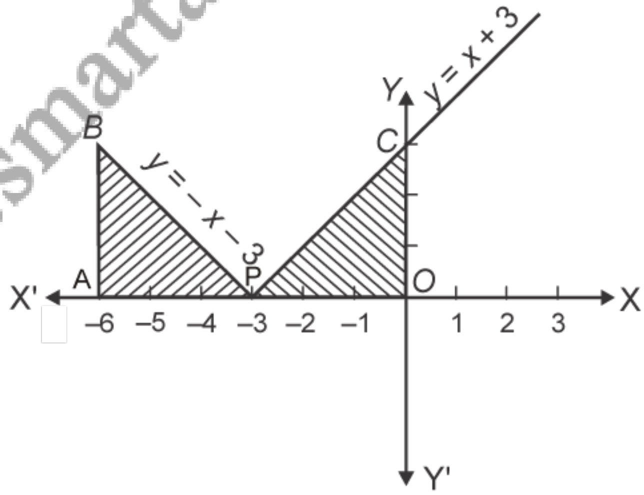
So, we have $y = x + 3$ for $x \geq -3$ and $y = -x - 3$ for $x < -3$.

Also, $y = x + 3$ is the straight line which cuts x and y axes at (– 3, 0) and (0, 3) respectively.
⇒ $y = x + 3, x \geq -3$ represents the part of line which lies on the right side of $x = -3$.

Similarly, $y = -x - 3, x < -3$ represents the part of line $y = -x - 3$ which lies as left side of $x = -3$.

A sketch of $y = |x + 3|$ is shown adjoining.

∴ Required area = Area of *ABPA* + Area of *PCOP*



$$= \int_{-6}^{-3} (-x-3)\,dx + \int_{-3}^{0} (x+3)\,dx$$

$$= \left[-\frac{x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

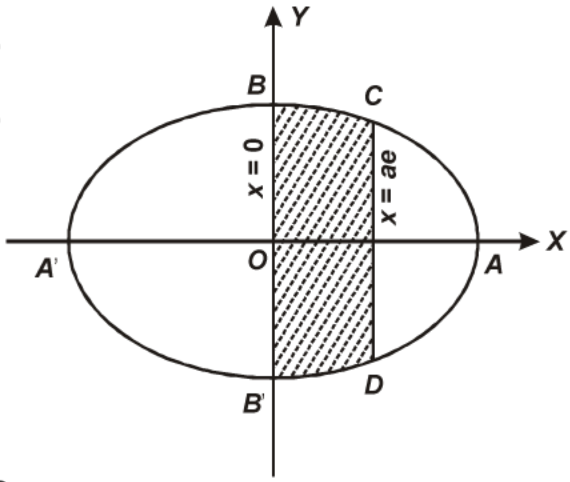
$$= \left[\left(-\frac{9}{2} + 9\right) - (-18+18)\right] + \left[0 - \left(\frac{9}{2} - 9\right)\right]$$

$$= \left(-\frac{9}{2} - \frac{9}{2}\right) + (9+9) = 18 - 9 = 9$$

Hence, required area = 9 sq units

S25. The given ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1 \quad \text{... (i)}$$



Given line is $x = ae$ ... (ii)

From (i), $\frac{y^2}{b^2} = 1-\frac{x^2}{a^2}$

$\Rightarrow$ $y = \pm\frac{b}{a}\sqrt{a^2-x^2}$ ... (iii)

The required area has been shaded in the figure.

Since the shaded portion is symmetric about the x-axis.

$\therefore$ Required area = 2 (Area of the region bounded by the given ellipse, x-axis and the lines $x = 0$ and $x = ae$)

$$= 2\int_0^{ae} y\, dx$$

$$= 2\int_0^{ae} \frac{b}{a}\sqrt{a^2-x^2}\, dx \qquad [\because\ y \geq 0 \text{ in the first quadrant}]$$

$$= 2\frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$$

$$= \frac{b}{a}\left[\left(ae\sqrt{a^2-a^2e^2}+a^2\sin^{-1}e\right)-\left(0+\frac{a^2}{2}\sin^{-1}0\right)\right]$$

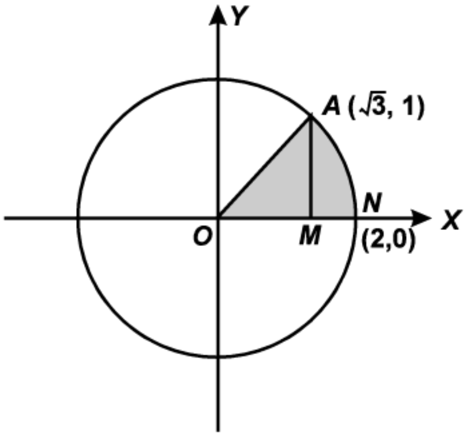
$$= ab\left(e\sqrt{1-e^2}+\sin^{-1}e\right) \text{ sq. units.}$$

S26. The given circle is $x^2 + y^2 = 4$ ... (i)

Its centre is (0, 0) and radius is 2

The given line is $x = \sqrt{3}y$ ... (ii)

From (i) and (ii), we get



$3y^2 + y^2 = 4$

$\Rightarrow$ $4y^2 = 4$

$\Rightarrow$ $y^2 = 1$

$\Rightarrow$ $y = 1, \quad -1.$

But in first quadrant $y \geq 0$

$\therefore$ $y = 1$

From Eq. (ii), when $y = 1,\ x = \sqrt{3}.1 = \sqrt{3}$

$\therefore$ The point of intersection of circle (i) and line (ii) in the first quadrant is $A\left(\sqrt{3}, 1\right)$

Now, the required area = the area of the shaded region

= area $OAMO$ + area $MANM$

$$= \int_0^{\sqrt{3}} y_{\text{line (ii)}} dx + \int_{\sqrt{3}}^{2} y_{\text{circle (i)}} dx$$

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\, dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x\sqrt{4 - x^2}}{2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{\sqrt{3}}\left(\frac{3}{2} - 0\right) + 2\sin^{-1} 1 - \frac{\sqrt{3}}{2}\ \sqrt{4-3}\ - 2\sin^{-1}\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$$

$$= \frac{\pi}{3} \text{ sq. units.}$$

S27. The equation of the curves are:

$$x^2 + y^2 = a^2 \qquad \dots \text{(i)}$$

$$x = \frac{a}{\sqrt{2}} \qquad \dots \text{(ii)}$$

Clearly, equation (i) represents a circle and (ii) is the equation of straight line parallel to y-axis at a distance of $\frac{a}{\sqrt{2}}$ units to the right of y-axis.

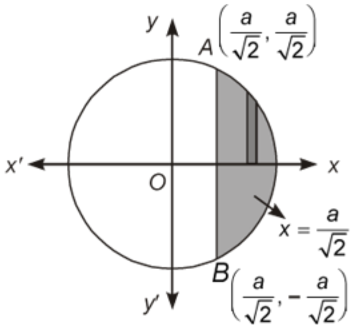
Solving (i) and (ii), we get

$$\left(\frac{a}{\sqrt{2}}\right)^2 + y^2 = a^2 \quad \Rightarrow \quad y^2 = a^2 - \frac{a^2}{2} = \frac{a^2}{2}$$

$$\Rightarrow \qquad y = \pm\frac{a}{\sqrt{2}}$$

Point of intersection of (i), and (ii) is $\left(\frac{a}{\sqrt{2}}, \pm\frac{a}{\sqrt{2}}\right)$

The smaller region bounded by these two is the shaded portion shown in the figure.



$\therefore$ Required area = Shaded region = $2\int_{a/\sqrt{2}}^{a} y\,dx$

$$= 2\int_{a/\sqrt{2}}^{a} \sqrt{a^2 - x^2}\,dx$$

$$= 2\left[\frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\sin^{-1}\frac{x}{a}\right]_{a/\sqrt{2}}^{a}$$

$$= 2\left[\frac{1}{2}a^2\sin^{-1}1 - \frac{1}{2}\frac{a}{\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} - \frac{1}{2}a^2\sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= \frac{a^2\pi}{4} - \frac{a^2}{2} = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right) \text{sq. units}.$$

S28. Given equation of the ellipse is $9x^2 + y^2 = 36$

or $\quad \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1$ ... (i)

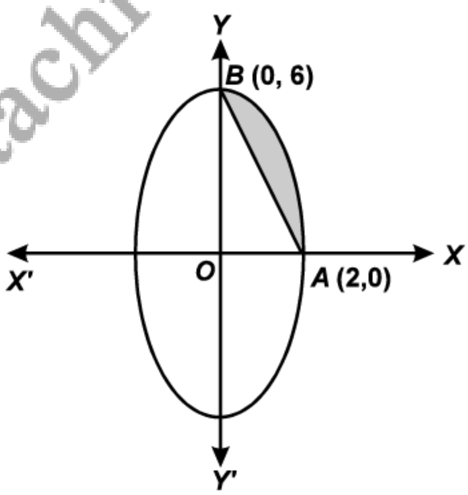
It's shape is as given in the figure. The equation of the chord AB is

$$y - 0 = \frac{6-0}{0-2}(x-2)$$

or, $\quad y = -3(x-2)$

or, $\quad y = -3x + 6$ ...(ii)

Required area = area of the shaded region



$$= \int_0^2 [y\,dx(\text{forcurve (i)}) - y\,dx(\textit{for line (ii)})]$$

$$= \int_0^2 (y_1 - y_2)\,dx$$

$$= 3\int_0^2 \sqrt{4 - x^2}\,dx - \int_0^2 (6 - 3x)\,dx$$

$$= 3\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[6x - \frac{3x^2}{2}\right]_0^2$$

$$= 3\left[\frac{2}{2}\times 0 + 2\sin^{-1}(1)\right] - \left[12 - \frac{12}{2}\right]$$

$$= 3 \times 2 \times \frac{\pi}{2} - 6$$

$$= (3\pi - 6) \text{ sq. units.}$$

S29. Given curves are $y = 2 + x$ ... (i)

and $x = 2y - y^2$ ... (ii)

Curve (i) is a straight line which cuts x-axis at $(-2, 0)$ and y-axis at $(0, 2)$.

Curve (ii) is a parabola and it's equation is quadratic in y.

Now, $x = 2y - y^2$

$\therefore$ $-x = y^2 - 2y$

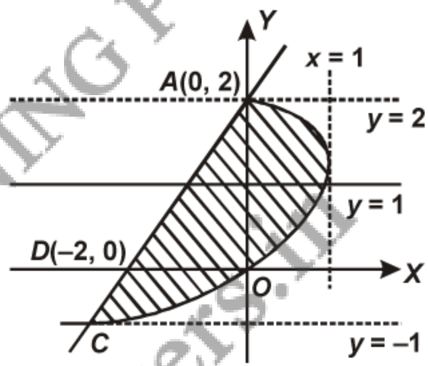
or $-x + 1 = (y - 1)^2$

or, $(y - 1)^2 = -(x - 1)$ ... (iii)

Vertex of parabola (iii) is $(1, 1)$ and its axis is $y = 1$.

From (ii), when $x = 0$, $y = 0, 2$. Therefore, curve (iii) cuts y-axis at $(0, 0)$ and $(0, 2)$. When $y = 0$, $x = 0$, therefore, curve (ii) cuts x-axis at $(0, 0)$.

Putting the value of x from equation (i) in equation (ii), we get



$$y - 2 = 2y - y^2$$

or, $y^2 - y - 2 = 0$

or, $(y - 2)(y + 1) = 0$

$\therefore$ $y = -1, 2$

Now, required area of shaded region $= \int_{-1}^{2} -(x_{curve(ii)} - x_{line(i)})dy$

$$= \int_{-1}^{2} -\left[(2y - y^2) - (y - 2)\right]dy = \int_{-1}^{2}\left(-y^2 + y + 2\right)dy$$

$$= \left[-\frac{y^3}{3} + \frac{y^2}{2} + 2y\right]_{-1}^{2} = \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= \frac{9}{2} \text{ sq. units.}$$

S30. The required region is the intersection of the following regions

$R_1 = \{(x, y) : 0 \le y \le x^2 + 1\}$;

$R_2 = \{(x, y) : 0 \le y \le x + 1\}$; and $R_3 = \{(x, y) : 0 \le x \le 2\}$

Given curves are $y = x^2 + 1$ or, $x^2 = y - 1$ ... (i)

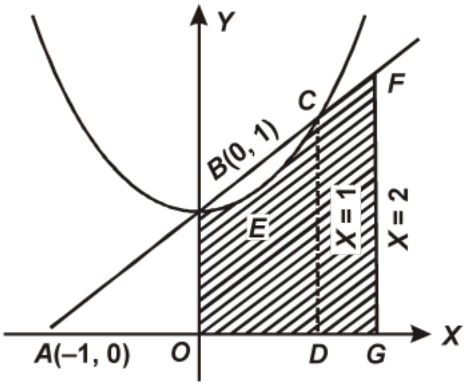
$y = x + 1$ ... (ii)

Curve (i) is a parabola having axis $x = 0$ and vertex (0, 1).

Curve (ii) is the straight line cutting x-axis at (– 1, 0) and y-axis at (0, 1).

Region R_1 is the exterior of the parabola (i) above x-axis.

Region R_2 is the region above x-axis towards that side of the line on which origin lies R_3 is the region between ordinates $x = 0$ and $x = 2$.



Curves (i) and (ii) intersect at points (0, 1) and (1, 2).

Now, required area = (area $ODCEBO$) + (area $DGFCD$)

$$= \int_0^1 y_{curve(i)} dx + \int_1^2 y_{curve(ii)} dx$$

$$= \int_0^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx$$

$$= \left[\frac{x^3}{3} + x\right]_0^1 + \left[\frac{x^2}{2} + x\right]_1^2 = \left(\frac{4}{3} + \frac{5}{2}\right) = \frac{23}{6} \text{ sq. units.}$$

S31. The given equations of the two circles are

$$(x - 6)^2 + y^2 = 36 \qquad \text{... (i)}$$

which has centre (6, 0) and radius $r = 6$

and $$x^2 + y^2 = 36 \qquad \text{... (ii)}$$

which has centre (0, 0) and radius $r = 6$.

First we sketch the required region.

Now, we find the point of intersection of the two circles. Putting $y^2 = 36 - x^2$ from Eq. (ii) in Eq. (i) we get

$$(x - 6)^2 + 36 - x^2 = 36$$

$$\Rightarrow \quad (x - 6)^2 - x^2 = 0 \qquad [\because (a^2 - b^2) = (a - b)(a + b)]$$

$$(x-6-x)(x-6+x) = 0$$

$$\Rightarrow \quad -6(2x-6) = 0$$

$$\Rightarrow \quad 2x-6 = 0$$

$$\Rightarrow \quad x = 3$$

Now, when $x = 3$, then

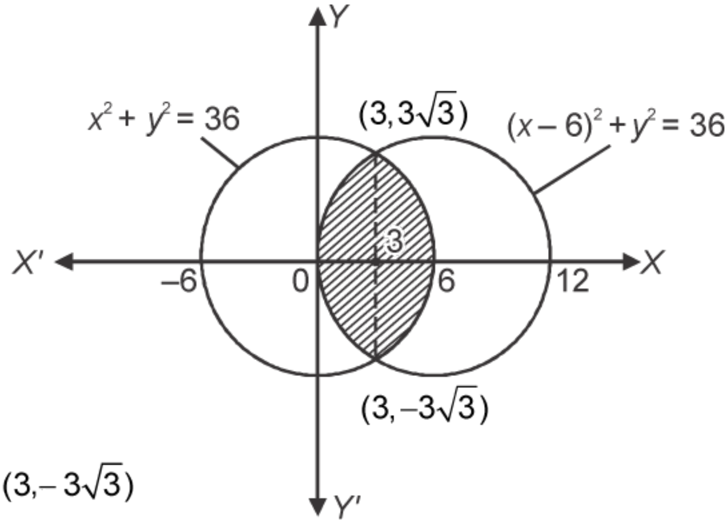
$$y^2 = 36 - x^2$$

$$= 36 - 9 = 27$$

or $$y^2 = 27$$

$$\Rightarrow \quad y = \pm 3\sqrt{3}$$

$\therefore$ Points of intersection are $(3, 3\sqrt{3})$ and $(3, -3\sqrt{3})$



Now, required area

$$= 2 \times \left[\int_0^3 y\,dx \text{ from circle (i)} + \int_3^6 y\,dx \text{ from circle (ii)}\right]$$

$$= 2 \times \left[\int_0^3 \sqrt{36-(x-6)^2}\,dx + \int_3^6 \sqrt{36-x^2}\,dx\right] \quad \left[\because \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C\right]$$

$$= 2\left\{\left[\frac{x-6}{2}\sqrt{36-(x-6)^2} + \frac{36}{2}\sin^{-1}\left(\frac{x-6}{6}\right)\right]_0^3 + \left[\frac{x}{2}\sqrt{36-x^2} + \frac{36}{2}\sin^{-1}\frac{x}{6}\right]_3^6\right\}$$

$$= 2\left\{\left[\frac{x-6}{2}\sqrt{36-(x-6)^2} + 18\sin^{-1}\left(\frac{x-6}{6}\right)\right]_0^3 + \left[\frac{x}{2}\sqrt{36-x^2} + 18\sin^{-1}\frac{x}{6}\right]_3^6\right\}$$

$$= 2\left\{\left[-\frac{3}{2}\sqrt{36-9} + 18\sin^{-1}\left(-\frac{1}{2}\right) - 18\sin^{-1}(-1)\right] + \left[18\sin^{-1}1 - \frac{3}{2}\sqrt{36-9} - 18\sin^{-1}\frac{1}{2}\right]\right\}$$

$$= 2\left\{\left[-\frac{3}{2}\sqrt{27} + 18\sin^{-1}\sin\left(-\frac{\pi}{6}\right) - 18\sin^{-1}\sin\left(-\frac{\pi}{2}\right)\right] + \left[18\sin^{-1}\sin\frac{\pi}{2} - \frac{3}{2}\sqrt{27} - 18\sin^{-1}\sin\frac{\pi}{6}\right]\right\}$$

$$\left[\because -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right), -1 = \sin\left(-\frac{\pi}{2}\right), 1 = \sin\frac{\pi}{2}, \frac{1}{2} = \sin\frac{\pi}{6}\right]$$

$$= 2\left\{\left[\frac{-3\sqrt{27}}{2} - 3\pi + 9\pi\right] + \left[9\pi - \frac{3\sqrt{27}}{2} - 3\pi\right]\right\}$$

$$= 2\left\{\left[\frac{-6\sqrt{27}}{2} + 12\pi\right] = -6\sqrt{27} + 24\pi\right\}$$

$$= -6 \times 3\sqrt{3} + 24\pi = 24\pi - 18\sqrt{3}$$

Hence, required area = $24\pi - 18\sqrt{3}$ sq. units.

S32. Given, equation of two circles are

$$x^2 + y^2 = 9 \quad \text{... (i)}$$

which has centre (0, 0) and radius $r = 3$ and

$$(x - 3)^2 + y^2 = 9 \quad \text{... (ii)}$$

which has centre (3, 0) and radius $r = 3$

Now, we find points of intersection of two circles.

Putting $y^2 = 9 - x^2$ from Eq. (i) in Eq. (ii), we get

$$(x - 3)^2 + 9 - x^2 = 9$$

$$\Rightarrow \quad (x - 3)^2 - x^2 = 0$$

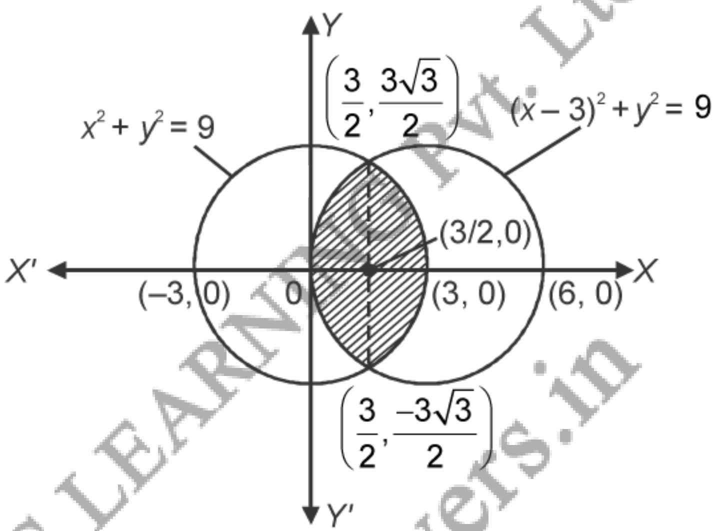
$$\Rightarrow \quad x^2 + 9 - 6x - x^2 = 0$$

$$\Rightarrow \quad 9 - 6x = 0$$

$$\Rightarrow \quad 6x = 9$$

$$\Rightarrow \quad x = \frac{9}{6}$$

$$\Rightarrow \quad x = \frac{3}{2}$$



when $x = \frac{3}{2}$, then $\quad y^2 = 9 - x^2 = 9 - \left(\frac{3}{2}\right)^2$

$$= 9 - \frac{9}{4} = \frac{27}{4}$$

$$\therefore \quad y = \pm\sqrt{\frac{27}{4}} = \pm\frac{3\sqrt{3}}{2}$$

Hence, point of intersection of the two circles are

$$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right) \quad \text{and} \quad \left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$$

Now, we sketch the required region

Now, required area

$$= 2\left[\int_0^{3/2} y\,dx \text{ from equation of circle (ii)} + \int_{3/2}^{3} y\,dx \text{ from equation of circle (i)}\right]$$

$$= 2\left[\int_0^{3/2}\sqrt{9-(x-3)^2}\,dx + \int_{3/2}^{3}\sqrt{9-x^2}\,dx\right] \quad \left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C\right]$$

$\therefore$ Required area

$$= 2\left\{\left[\frac{x-3}{2}\sqrt{9-(x-3)^2}+\frac{9}{2}\sin^{-1}\left(\frac{x-3}{3}\right)\right]_0^{3/2}+\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{3/2}^{3}\right\}$$

$$= 2\left[\frac{3/2-3}{2}\sqrt{9-\left(\frac{3}{2}-3\right)^2}+\frac{9}{2}\sin^{-1}\left(\frac{3/2-3}{3}\right)-\frac{9}{2}\sin^{-1}(-1)\right]$$

$$+2\left[0+\frac{9}{2}\sin^{-1}(1)-\frac{3}{4}\sqrt{9-\frac{9}{4}}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}.\frac{3}{2}\right)\right]$$

$$= 2\left[-\frac{3}{4}\sqrt{9-\frac{9}{4}}+\frac{9}{2}\sin^{-1}\left(-\frac{1}{2}\right)+\frac{9}{2}\sin^{-1}1\right]+2\left[\frac{9\pi}{4}-\frac{3}{4}\sqrt{\frac{27}{4}}-\frac{9}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

Now, we know that $-\frac{1}{2}=\sin\left(-\frac{\pi}{6}\right), 1=\sin\frac{\pi}{2}$ and $\frac{1}{2}=\sin\frac{\pi}{6}$

$$= 2\left[-\frac{3}{4}\sqrt{\frac{27}{4}}-\frac{9}{2}\sin^{-1}\left(\frac{1}{2}\right)+\frac{9\pi}{4}\right]+2\left[\frac{9\pi}{4}-\frac{9\sqrt{3}}{8}-\frac{9\pi}{12}\right]$$

$$= 2\left[-\frac{3}{4}\times\frac{3\sqrt{3}}{2}-\frac{9}{2}\times\frac{\pi}{6}+\frac{9\pi}{4}\right]+2\left[\frac{9\pi}{4}-\frac{9\sqrt{3}}{8}-\frac{9\pi}{12}\right]$$

$$= -\frac{9\sqrt{3}}{4}-\frac{9\pi}{6}+\frac{9\pi}{2}+9\frac{\pi}{2}-\frac{9\sqrt{3}}{4}-\frac{9\pi}{6}$$

$$= \frac{-18\sqrt{3}}{4}-\frac{18\pi}{6}+\frac{18\pi}{2}=\frac{-9\sqrt{3}}{2}-3\pi+9\pi = \left(6\pi-\frac{9\sqrt{3}}{2}\right) \text{ sq. units.}$$

S33. Firstly we draw a square formed by the lines $x = 0$, $x = 4$ and $y = 0$ and $y = 4$ after that we draw two parabola which intersect each other on the square such that the whole region divided into three parts.

Let $ABCD$ be the square whose sides are represented by following equations:

Equation of OA is $y = 0$

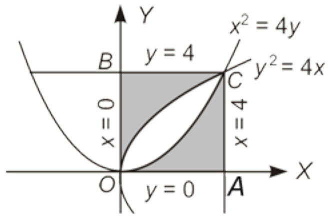
Equation of AC is $x = 4$

$\therefore$ Equation of BC is $y = 4$

Equation of BO is $x = 0$

First, we find area of the region bounded by curve $y^2 = 4x$ and $x^2 = 4y$.

Solving equation $y^2 = 4x$ and $x^2 = 4y$ simultaneously,



$$x\left(\frac{x^2}{4}\right)^2 = 4x$$

$$\frac{x^4}{16} - 4x = 0$$

$$x(x^3 - 64) = 0$$

$$x(x-4)(x^2 + 4x + 16) = 0$$

$$x = 0, 4$$

$$y = 0, 4$$

$$= \int_0^4 (y_2 - y_2)dx$$

as $(x_2\ y_2)$ lie on $y^2 = 4x$ and (x_1, y_1) lie on $x^2 = 4y$

$\therefore$ $$y_2 = 2\sqrt{x} \quad \text{and} \quad y_1 = \frac{x^2}{4}$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$$= \left[2 \cdot \frac{2}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4 = \left[\left(\frac{4}{3}\cdot(4)^{3/2} - \frac{64}{12}\right) - (0-0)\right]$$

$$= \frac{4}{3}\cdot(2^2)^{3/2} - \frac{64}{12} = \frac{4}{3}\cdot(2)^2 - \frac{64}{12}$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq units}$$

$\therefore$ Area bounded by curves $y^2 = 4x$ and $x^2 = 4y$ is $\frac{16}{3}$ sq. units ... (i)

Now, we find area bounded by curve $x^2 = 4y$ and the lines $x = 0$, $x = 4$ and X-axis

$$= \int_0^4 \frac{x^2}{4} dx = \left[\frac{x^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq. units.} \quad \text{... (ii)}$$

Similarly, we find area bounded by curve $y^2 = 4x$ and the lines, $y = 0$, $y = 4$ and Y-axis

$$= \int_0^4 x dy = \int_0^4 \frac{y^2}{4} dy$$

$$= \left[\frac{y^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq. units} \quad \text{... (iii)}$$

From Eqs. (i), (ii) and (iii), it is calculated that area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ divides the area of square in three equal parts.

S34. Given, equations of parabola and line are

$$y^2 = x \quad \text{... (i)}$$

which has vertex (0, 0) and axis along X-axis and $x + y = 2$ ... (ii)

x	2	0
y	0	2

$\therefore$ Line passes through the points (2, 0) and (0, 2).

Now, we sketch the graph of required region.

Now, we find the points of intersection of two figures. Putting $x = 2 - y$ from Eq. (ii) in Eq. (i) we get

$$y^2 = 2 - y$$

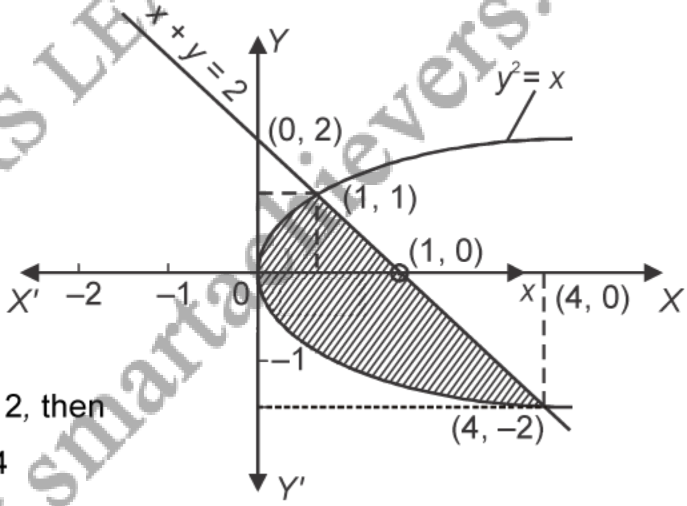
$\Rightarrow$ $y^2 + y - 2 = 0$

$\Rightarrow$ $y^2 + 2y - 1y - 2 = 0$

$\Rightarrow$ $y(y + 2) - 1(y + 2) = 0$

$\Rightarrow$ $(y - 1)(y + 2) = 0$

$\Rightarrow$ $y = 1$ or -2



when $y = 1$, then $x = 2 - y = 1$ and when $y = -2$, then

$$x = 2 - y = 2 - (-2) = 4$$

$\therefore$ Points of intersection are (1, 1) and (4, –2).

Now, Required area

$$= \int_{-2}^{1} (x\,dy \text{ from equation of line}) - (x\,dy \text{ from equation of parabola})$$

$$= \int_{-2}^{1} (2 - y - y^2)dy = \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)$$

$$= 2-\frac{5}{6}+6-\frac{8}{3}=8-\frac{5}{6}-\frac{8}{3}$$

$$= \frac{48-5-16}{6}=\frac{48-21}{6}$$

$$= \frac{27}{6}=\frac{9}{2}$$

Hence, required area $= \frac{9}{2}$ sq. units.

S35. Given, equations are $4y = 3x^2$ ... (i)

which is a parabola with vertex (0, 0) and axis along y-axis.

and $3x - 2y + 12 = 0$ ... (ii)

is a straight line.

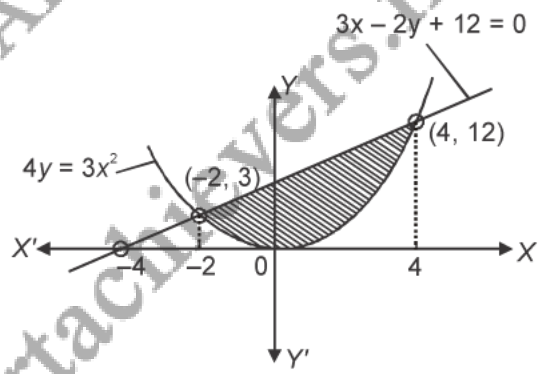
Consider the equations of line $3x - 2y + 12 = 0$

x	–4	–2
y	0	3

$\therefore$ Line passes through points (– 4, 0) and (– 2, 3).

Now, we find the point of intersection

Putting $2y = 3x + 12$ from Eq. (ii) in Eq. (i), we get



$$2(2y) = 3x^2$$

$\Rightarrow \quad 2(3x + 12) = 3x^2$

$\Rightarrow \quad 6x + 24 - 3x^2 = 0$

$\Rightarrow \quad x^2 - 2x - 8 = 0$

$\Rightarrow \quad x^2 - 4x + 2x - 8 = 0$

$\Rightarrow \quad x(x - 4) + 2(x - 4) = 0$

$\Rightarrow \quad (x + 2)(x - 4) = 0$

$\therefore \quad x = -2$ or 4

When $x = -2$,

then $2y = 3(-2) + 12 = 6$

$\Rightarrow \quad y = 3$

and when $x = 4$,

then $2y = 3(4) + 12$

$\Rightarrow \quad y = 12$

$\therefore$ Points of intersection are (– 2, 3) and (4, 12).

Now, we sketch the required region

$\therefore$ Required area $= \int_{-2}^{4} (y\,dx$ from equation of line) – ($y\,dx$ from equation of parabola)

$$= \int_{-2}^{4} \frac{3x+12}{2}\,dx - \int_{-2}^{4} \frac{3x^2}{4}\,dx$$

$$= \left[\frac{3x^2}{4} + \frac{12x}{2}\right]_{-2}^{4} - \left[\frac{3x^3}{12}\right]_{-2}^{4}$$

$$= \left[\frac{3x^2}{4} + 6x\right]_{-2}^{4} - \left[\frac{x^3}{4}\right]_{-2}^{4}$$

$$= [(12 + 24) - (3 - 12)] - [16 + 2]$$

$$= 36 + 9 - 18$$

$$= 27 \text{ sq. units}$$

S36. The equations of the given curves are

$$x^2 = 4y \qquad \text{... (i)}$$

and $$x = 4y - 2 \qquad \text{... (ii)}$$

Eq. (i) represents a parabola with vertex at the origin and axis along positive direction of y-axis.

Eq. (ii) represents a straight line which meets the coordinate axes at $(-2, 0)$ and $\left(0, \frac{1}{2}\right)$, respectively.

To find the points of intersection of the given parabola and the line, we solve Eqs. (i) and (ii) simultaneously.

i.e., $(4y - 2)^2 = 4y$

$\Rightarrow$ $16y^2 + 4 - 16y = 4y$

$\Rightarrow$ $16y^2 - 20y + 4 = 0$

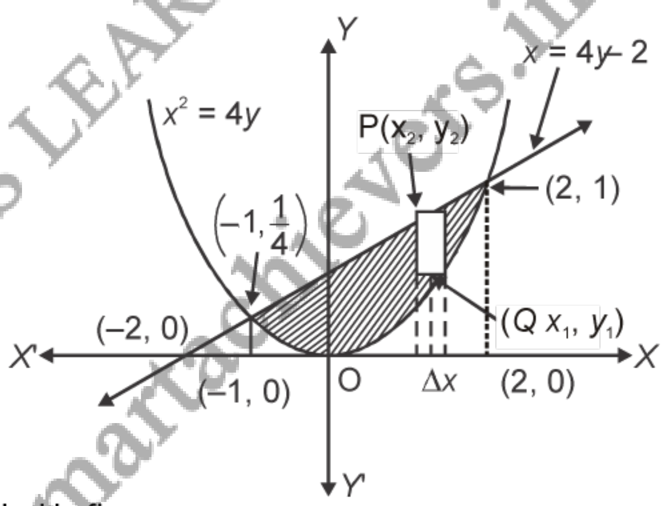
$\Rightarrow$ $4y^2 - 5y + 1 = 0$

$\Rightarrow$ $(4y - 1)(y - 1) = 0 \quad \Rightarrow \quad y = 1, \frac{1}{4}$

Putting the values of y in Eq. (i),

we get $x = -1, 2$.

$\therefore$ The points of intersection of the given parabola and the line are $(2, 1)$ and $(-1, 1/4)$.



The region whose area is to be found out is shaded in figure.

$\therefore$ Required area A is given by

$$A = \int_{-1}^{2} (y_2 - y_1)\,dx$$

$$\Rightarrow \quad A = \int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx$$

$\Rightarrow$ [$\because$ $P(x_2, y_2)$ and $Q(x_1, y_1)$ lie on Eqs. (ii) and (i), respectively $y_2 = \dfrac{x+2}{4}$ and $y_1 = \dfrac{x^2}{4}$]

$$\Rightarrow \quad A = \left[\frac{x^2}{8} + \frac{1}{2}x - \frac{x^3}{12} \right]_{-1}^{2}$$

$$\Rightarrow \quad A = \left[\frac{4}{8} + \frac{2}{2} - \frac{8}{12} \right] - \left[\frac{1}{8} - \frac{1}{2} + \frac{1}{12} \right]$$

$$\Rightarrow \quad = \frac{(48 + 96 - 64)}{96} - \frac{(12 - 48 + 8)}{96} = \frac{80 + 28}{96}$$

$$= \frac{108}{96} = \frac{9}{8} \text{ Sq. units}$$

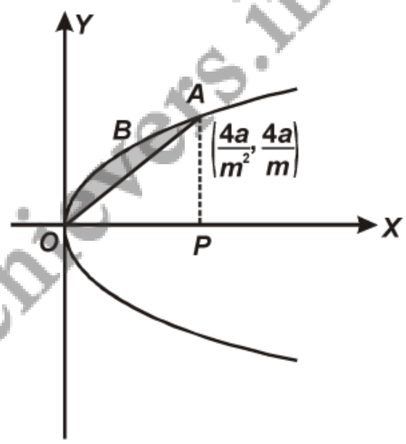
S37. The given curves are $y^2 = 4ax$ ... (i)

and $y = mx$ ... (ii)

Clearly, $y^2 = 4ax$ is a parabola, passing through the origin.

and $y = mx$ is a line passing through the origin.

Putting $y = mx$ from (ii) in (i), we get



$$m^2x^2 = 4ax$$

$$\Rightarrow \quad x(m^2x - 4a) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = \frac{4a}{m^2}$$

Now, from (ii) $x = 0$

$\Rightarrow$ $y = 0$

and $x = \dfrac{4a}{m^2}$

$\Rightarrow$ $y = \dfrac{4a}{m}$

$\therefore$ The points of intersection of the given parabola and the chord are $O(0, 0)$ and $A\left(\dfrac{4a}{m^2}, \dfrac{4a}{m}\right)$.

Draw $AP \perp OX$

Required area = (area $OBAO$)

$$= \int_0^{\frac{4a}{m^2}} (y_1 - y_2)\,dx$$

$$= \int_0^{\frac{4a}{m^2}} (y \text{ for the parabola})\, dx - \int_0^{\frac{4a}{m^2}} (y \text{ for the line})\, dx$$

$$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax}\, dx - \int_0^{\frac{4a}{m^2}} mx\, dx$$

$$= 2\sqrt{a} \cdot \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^{\frac{4a}{m^2}} - \left[\frac{mx^2}{2}\right]_0^{\frac{4a}{m^2}}$$

$$= \left[\frac{4\sqrt{a}}{3} \cdot \frac{8}{m^3} a^{\frac{3}{2}} - \frac{m}{2} \cdot \frac{16a^2}{m^4}\right]$$

$$= \left(\frac{32a^2}{3m^3} - \frac{8a^2}{m^3}\right)$$

$$= \left(\frac{8a^2}{3m^3}\right) \text{ sq. units}$$

Hence, the required area = $\left(\frac{8a^2}{3m^3}\right)$ sq. units

S38. Given curves are $y = x^2 + 1$ or $x^2 = y - 1$ ... (i)

and $y = x$ ... (ii)

Curve (i) is a parabola having axis $x = 0$ and vertex (0, 1).

Now, ΔOBD is the region bounded by the curve $y = x$, the y-axis and the abscissa $y = 0$ and $y = 2$.

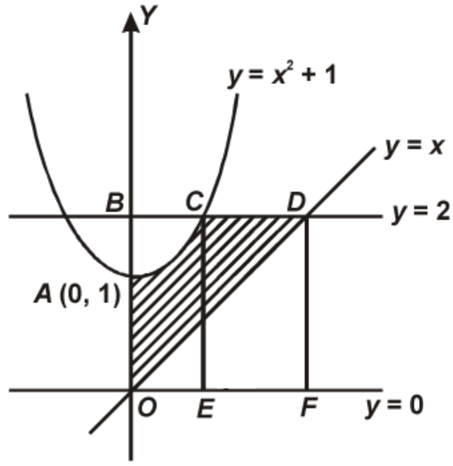
$\therefore$ Area of $\Delta OBD = \int_0^2 x_{OD}\, dy = \int_0^2 y\, dy$ [$\because$ From (ii), $x = y$]

$$= \left[\frac{y^2}{2}\right]_0^2 = \frac{4}{2} - 0 = 2 \text{ sq. units.}$$

Also the region $ABCA$ is bounded by the curve $x = \sqrt{y-1}$, the y-axis and the abscissa $y = 1$ and $y = 2$.

$\left[\because x^2+1=y \Rightarrow x=\pm\sqrt{y-1}. \text{ But } x \geq 0 \text{ for the part } AC \text{ of the parabola}\right]$

The area of the region



$$ABCA = \int_1^2 x\, dy_{curve(i)}$$

$$= \int_1^2 \sqrt{y-1}\, dy$$

$$= \left[\frac{(y-1)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2$$

$$= \frac{2}{3}\left[1^{\frac{3}{2}} - 0\right]$$

$$= \frac{2}{3} \text{ sq. units.}$$

Hence, the required shaded area = Area ΔOBD – Area $ABCA$ = $2 - \frac{2}{3} = \frac{4}{3}$ sq. units.

Second method:

Required area = area $OACEO$ + area $ECDF$ – area of ΔODF

$$= \int_0^1 y_{curve(i)}\, dx + \int_1^2 y_{CD} dx - \int_0^2 y_{OD} dx$$

$$= \int_0^1 (x^2+1) dx + \int_1^2 2\, dx - \int_0^2 x\, dx$$

$$= \left[\frac{x^3}{3} - x\right]_0^1 + 2[x]_1^2 - \left[\frac{x^2}{2}\right]_0^2$$

$$= \left(\frac{1}{3} + 1 + 2 - 2\right) \text{ sq. units} = \frac{4}{3} \text{ sq. units.}$$

S39. Given, equations of two circles are

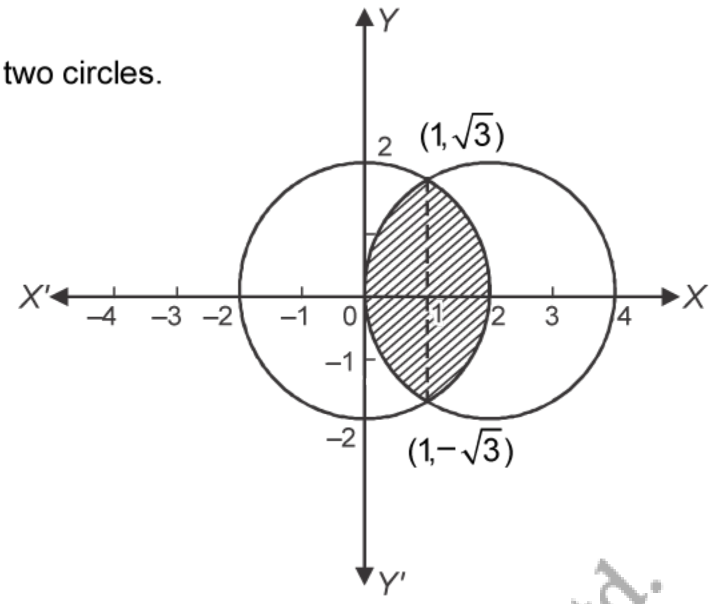
$$x^2 + y^2 = 4 \qquad \text{... (i)}$$

and $$(x-2)^2 + y^2 = 4 \qquad \text{... (ii)}$$

Comparing both Eqs. (i) and (ii) with standard form of equation of circle *i.e.*, $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) = centre and r = radius, we get

Circle with Eq. (i) has centre (0, 0) and $r = 2$ and circle with Eq. (ii) has centre (2, 0) and $r = 2$.

First, we sketch the required region.



Now, we find the points of intersection of the two circles.

Subtracting Eq. (i) from Eq. (ii), we get

$$(x-2)^2 - x^2 = 0$$

$$\Rightarrow \quad x^2 + 4 - 4x - x^2 = 0$$

$$\Rightarrow \quad 4 - 4x = 0$$

$$\Rightarrow \quad 4x = 4$$

$$\Rightarrow \quad x = 1$$

Putting $x = 1$ in Eq. (i), we get

$$1 + y^2 = 4$$

$$\Rightarrow \quad y^2 = 3$$

$$\Rightarrow \quad y = \pm\sqrt{3}$$

$\therefore$ The two circles intersect each other at the point $(1, \sqrt{3})$ and $(1, -\sqrt{3})$

Now, area of shaded region

$$= 2\left[\left\{\int_0^1 y\,dx \text{ of circle (ii) } + \int_1^2 y\,dx \text{ of circle (i)}\right\}\right]$$

$$= 2\left[\int_0^1 \sqrt{4-(x-2)^2}\,dx + \int_1^2 \sqrt{4-x^2}\,dx\right] \quad \left[\because \int \sqrt{a^2-x^2}\,dx = \frac{x^2}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$= 2\left\{\left[\frac{x-2}{2}\sqrt{4-(x-2)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1 + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_1^2\right\}$$

$$= 2\left\{\left[\frac{x-2}{2}\sqrt{4-(x-2)^2} + 2\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1 + \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_1^2\right\}$$

$$= 2\left\{\left[-\frac{1}{2}\sqrt{3} + 2\sin^{-1}\left(-\frac{1}{2}\right) - 2\sin^{-1}(-1)\right] + \left[2\sin^{-1}1 - \frac{1}{2}\sqrt{3} - 2\sin^{-1}\frac{1}{2}\right]\right\}$$

$$= 2\left[-\frac{\sqrt{3}}{2} + 2\sin^{-1}\sin\left(-\frac{\pi}{6}\right) - 2\sin^{-1}\sin\left(-\frac{\pi}{2}\right) + 2\sin^{-1}\sin\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\sin^{-1}\sin\frac{\pi}{6}\right]$$

$$= 2\left[-\frac{\sqrt{3}}{2} + 2\left(-\frac{\pi}{6}\right) - 2\left(-\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{2} - 2\left(\frac{\pi}{6}\right)\right]$$

$$= 2\left[-\frac{\sqrt{3}}{2} - \frac{\pi}{3} + \pi + \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]$$

$$= 2\left[\frac{-2\sqrt{3}}{2} - \frac{\pi}{3} + \pi + \pi - \frac{\pi}{3}\right]$$

$$= 2\left[-\sqrt{3} + 2\pi - \frac{2\pi}{3}\right] = 2\left[-\sqrt{3} + \frac{4\pi}{3}\right]$$

$$= \left[\frac{8\pi}{3} - 2\sqrt{3}\right] \text{ sq. units}$$

Hence, required area $= \left[\frac{8\pi}{3} - 2\sqrt{3}\right]$ sq. units

S40. Given equation are

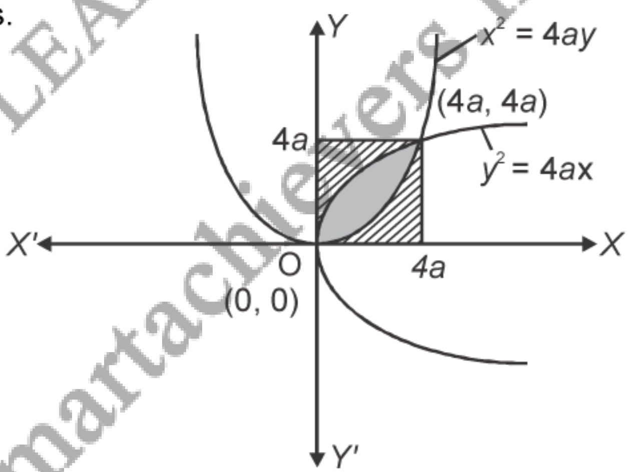
$$y^2 = 4ax \quad \text{... (i)}$$

which is parabola having vertex (0, 0) and axis along X-axis and

$$x^2 = 4ay \quad \text{... (ii)}$$

which is parabola having vertex (0, 0) and axis along Y-axis.

First, we find the point of intersection of two curves.



Putting $x = \frac{y^2}{4a}$ from Eq. (i) in Eq. (ii), we get

$$x^2 = 4ay$$

$$\Rightarrow \quad \left(\frac{y^2}{4a}\right)^2 = 4ay$$

$$\Rightarrow \quad \frac{y^4}{16a^2} - 4ay = 0$$

$$\Rightarrow \quad y\left(\frac{y^3}{16a^2} - 4a\right) = 0$$

$$\Rightarrow \quad y = 0 \text{ or } \frac{y^3}{16a^2} - 4a = 0$$

$$\Rightarrow \quad y = 0$$

or $\quad y^3 = 64a^3 = (4a)^3$

$\therefore \quad y = 0$ or $4a$

Now, when $y = 0$, then

$$x^2 = 4ay = 4a \times 0 = 0$$

$$\therefore \quad x = 0$$

and when $y = 4a$, then

$$x^2 = 4ay = 4a \times 4a = 16a^2$$

$$\therefore \quad x = \pm 4a$$

Hence, the points of intersection are (0, 0) and (4a, 4a).

Now, we sketch the graph of required region

$\therefore$ Required area = $\int_0^{4a} [y\ dx$ from equation of parabola (i)] – [$y\,dx$ from equation of parabola (ii)]

$$= \int_0^{4a} \left(2\sqrt{a} \cdot \sqrt{x} - \frac{x^2}{4a}\right) dx$$

$$\left[\because y^2 = 4ax \Rightarrow y = \sqrt{4ax} \Rightarrow y = 2\sqrt{a} \cdot \sqrt{x} \text{ and } x^2 = 4ay \Rightarrow y = \frac{x^2}{4a}\right]$$

$$= \left[2\sqrt{a} \cdot x^{3/2} \cdot \frac{2}{3} - \frac{x^3}{12a}\right]_0^{4a}$$

$$= \left[\left(\frac{4\sqrt{a}}{3} \cdot (4a)^{3/2} - \frac{64a^3}{12a}\right) - (0 - 0)\right]$$

$$= \left[\frac{4\sqrt{a}}{3} \cdot (4)^{3/2} \cdot (a)^{3/2} - \frac{16a^2}{3}\right]$$

$$= \left[\frac{4\sqrt{a}}{3} \cdot (2^2)^{3/2} \cdot ((\sqrt{a})^2)^{3/2} - \frac{16a^2}{3}\right]$$

$$= \left[\frac{32\sqrt{a}}{3} \times a\sqrt{a}\right] - \frac{16a^2}{3}$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3} = \frac{16a^2}{3}$$

Hence, the required area = $\frac{16a^2}{3}$ sq. units.

S41. The given equation of the two curves are

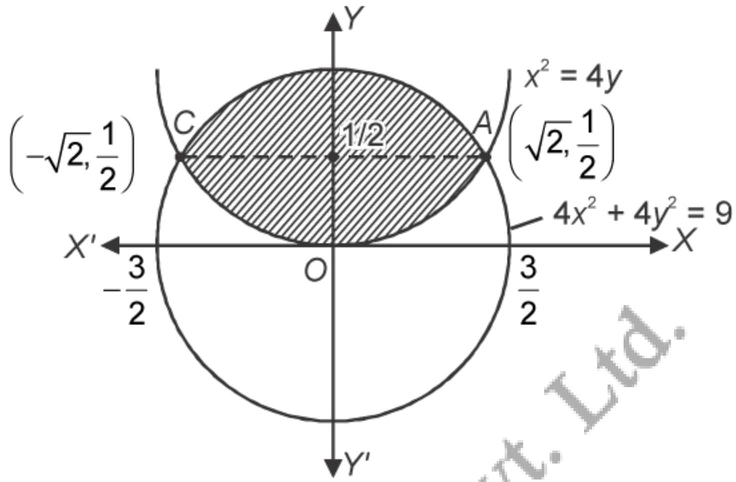
$$4x^2 + 4y^2 = 9 \text{ or } x^2 + y^2 = \frac{9}{4} \qquad \dots \text{(i)}$$

Which is a circle with centre (0, 0) and radius, $r = \frac{3}{2}$ and $x^2 = 4y$ ...(ii)

Which is a parabola with vertex (0, 0) and along Y-axis.

Next, we find the points of intersection of the two curves.

Putting $x^2 = 4y$ from Eq. (ii) in Eq. (i), we get



$$y^2 + 4y = \frac{9}{4}$$

or $\quad 4y^2 + 16y - 9 = 0$

$\Rightarrow \quad 4y^2 + 18y - 2y - 9 = 0$

$\Rightarrow \quad 2y(2y + 9) - 1(2y + 9) = 0$

$\Rightarrow \quad (2y - 1)(2y + 9) = 0$

$\Rightarrow \quad y = \frac{1}{2}$ or $-\frac{9}{2}$

Now, when $y = \frac{1}{2}$, then $x^2 = 4y = 4 \times \frac{1}{2} = 2$

$\Rightarrow \quad x = \pm\sqrt{2}$

and when $y = \frac{9}{2}$, then $x^2 = 4y = 4\left(-\frac{9}{2}\right) = -18$

$\Rightarrow \quad x^2 = -18$

which is rejected as there is no real value of x exist.

Hence the two curve meet at point $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(-\sqrt{2}, \frac{1}{2}\right)$

Now, required area of shaded region

$$= 2\left[\int_0^{1/2} x\,dy \text{ from parabola} + \int_{1/2}^{3/2} x\,dy \text{ from circle}\right]$$

$$\text{Required area} = 2\left[\int_0^{1/2} 2\sqrt{y}\,dy + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - y^2}\,dy\right]$$

[$\because$ From equation of parabola $x^2 = 4y \Rightarrow x = 2\sqrt{y}$ and from equation of circle $x^2 + y^2 = \frac{9}{4}$, we get $x = \sqrt{\frac{9}{4} - y^2}$]

$$= 2\left[\int_0^{1/2} 2\sqrt{y}\,dy + \int_{1/2}^{3/2} \sqrt{\left(\frac{3}{2}\right)^2 - y^2}\,dy\right]$$

$$= 2\left\{\left[2\cdot y^{3/2}\cdot\frac{2}{3}\right]_0^{1/2} + \left[\frac{y}{2}\sqrt{\frac{9}{4}-y^2}+\frac{9}{8}\sin^{-1}\frac{2y}{3}\right]_{1/2}^{3/2}\right\}$$

$$\left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}-\sin^{-1}\frac{x}{a}+C\right]$$

$$= 2\left\{\left[\frac{4}{3}\cdot\left(\frac{1}{2}\right)^{3/2}\right]+\left[\frac{9}{8}\sin^{-1}1-\frac{1}{4}\sqrt{\frac{8}{4}}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]\right\}$$

$$= 2\left\{\left[\frac{4}{3}\times\frac{1}{((\sqrt{2})^2)^{3/2}}\right]+\left[\frac{9}{8}\sin^{-1}\sin\frac{\pi}{2}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]\right\} \qquad \left[\because 1=\sin\frac{\pi}{2}\right]$$

$$= 2\left\{\left[\frac{4}{3}\times\frac{1}{2\sqrt{2}}\right]+\left[\frac{9}{8}\times\frac{\pi}{2}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]\right\}$$

$$= 2\left\{\left[\frac{2}{3\sqrt{2}}+\frac{9\pi}{16}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]\right\}$$

$$= 2\left[\frac{2\sqrt{2}}{6}+\frac{9\pi}{16}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{\sqrt{2}}{3}-\frac{\sqrt{2}}{4}+\frac{9\pi}{16}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{4\sqrt{2}-3\sqrt{2}}{12}+\frac{9\pi}{16}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{\sqrt{2}}{12}+\frac{9\pi}{16}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= \left(\frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\right) \text{ sq. units}$$

S42. Given, curves are $x^2 + y^2 = 16$ ... (i)

and $y^2 = 6x$ ... (ii)

$x^2 + y^2 = 16$ is a circle with its centre at (0, 0) and radius = 4 units. And $y^2 = 6x$ is a parabola with its vertex at (0, 0) and axis $y = 0$.

We have to find the area of the shaded region.

Putting the value of y^2 from (ii) in (i), we get

$$x^2 + 6x - 16 = 0$$

$\Rightarrow \quad (x + 8)(x - 2) = 0$

$\Rightarrow \quad x = -8$ or, $x = 2$

Now from (ii), $x = -8$

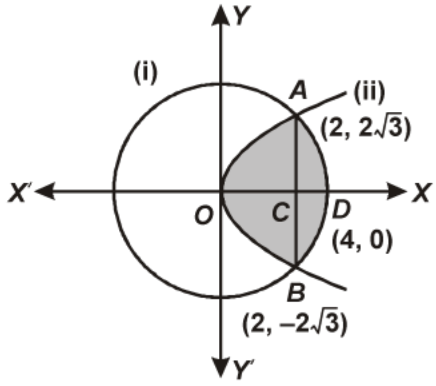
$\Rightarrow \quad y^2 = -48$

$\Rightarrow$ y is imaginary

and $x = 2$

$\Rightarrow \quad y^2 = (6 \times 2) = 12$

$\Rightarrow \quad y = \pm 2\sqrt{3}$



Thus, the points of intersection of the given curves are $A\left(2, 2\sqrt{3}\right)$ and $B\left(2, -2\sqrt{3}\right)$.

Since each of the given equations contains only even powers of y, each one is symmetric about the x-axis.

$\therefore$ Required area $OADBO$ = 2 (area $OCDAO$)

= 2 (area $OCAO$ + area $CDAC$)

$$= 2\left[\int_0^2 y_{\text{curve (ii)}}\,dx + \int_2^4 y_{\text{curve (i)}}\,dx\right]$$

$$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$$

$$= 2\left\{\left[\frac{2\sqrt{6}}{3}x^{\frac{3}{2}}\right]_0^2 + \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4\right\}$$

$$= 2\left\{\left(\frac{2\sqrt{6}}{3}\cdot 2^{\frac{3}{2}} - 0\right) + 8\sin^{-1}1 - \left(2\sqrt{3} + 8\sin^{-1}\frac{1}{2}\right)\right\}$$

$$= 2\left(\frac{8\sqrt{3}}{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}\right)$$

$$= 2\left(\frac{2\sqrt{3}}{3} + \frac{8\pi}{3}\right) = \frac{4}{3}\left(\sqrt{3} + 4\pi\right) \text{ sq. units}$$

Hence, the required area = $\frac{4}{3}\left(\sqrt{3} + 4\pi\right)$ sq. units

S43. The given curves are $x^2 + y^2 = 8x$ ... (i)

and $y^2 = 4x$ ... (ii)

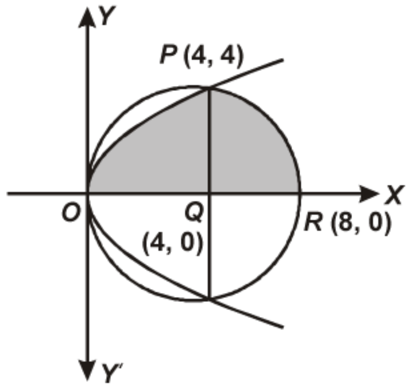
The equation (i) can be written as $(x - 4)^2 + (y - 0)^2 = 4^2$, which represents a circle with centre (4, 0) and radius 4 units. The equation (ii) *i.e.*, $y^2 = 4x$ represents a right handed parabola with vertex (0, 0) and axis $y = 0$.

A rough sketch of the curves is shown in figure solving (i) and (ii), we get

$$x^2 + 4x = 8x$$

$\Rightarrow$ $x^2 - 4x = 0$

$\Rightarrow$ $x = 0, 4$



When $x = 0$, $y = 0$ and when $x = 4$, $y = 4$ (in first quadrant). Thus, the points of intersection of the two curves are (0, 0) and (4, 4) above the x-axis. The required area has been shaded in figure.

Required area $OPRQO$ =area $OPQO$ + area $QPRQ$

$$= \int_0^4 y\,dx \text{ (for parabola)} + \int_4^8 y\,dx \text{ (for circle)}$$

$$= \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2 - (x-4)^2}\,dx$$

$\left(\because y^2 = 4x \Rightarrow y = 2\sqrt{x} \text{ and } (x-4)^2 + y^2 = 4^2 \Rightarrow y = \sqrt{4^2 - (x-4)^2} \text{ in the first quadrant}\right)$

$$= 2\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 + \int_0^4 \sqrt{4^2 - t^2}\,dt$$

`(In 2nd integral, put $x - 4 = t \Rightarrow dx = dt$, when $x = 4$, $t = 0$ and when $x = 8$, $t = 4$)

$$= \frac{4}{3}\left[4^{\frac{3}{2}} - 0\right] + \left[\frac{t\sqrt{4^2 - t^2}}{2} + \frac{4^2}{2}\sin^{-1}\frac{t}{4}\right]_0^4$$

$$= \frac{4}{3}\times 8 + (0 + 8\sin^{-1}1) - (0 + 8\sin^{-1}0)$$

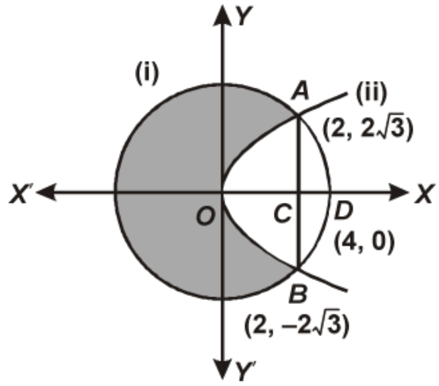
$$= \frac{32}{3} + 8\cdot\frac{\pi}{2} - 0 = \frac{4}{3}(8 + 3\pi) \text{ sq. units.}$$

S44. Given, curves are $x^2 + y^2 = 16$ ... (i)

and $y^2 = 6x$ ... (ii)

$x^2 + y^2 = 16$ is a circle with its centre at (0, 0) and radius = 4 units. And $y^2 = 6x$ is a parabola with it's vertex at (0, 0) and axis $y = 0$.

We have to find the area of the shaded region.



Putting the value of y^2 from (ii) in (i), we get

$$x^2 + 6x - 16 = 0$$

$\Rightarrow \quad (x + 8)(x - 2) = 0$

$\Rightarrow \quad x = -8 \quad \text{or,} \quad x = 2$

Now from (ii), $\quad x = -8$

$\Rightarrow \quad y^2 = -48$

$\Rightarrow$ y is imaginary

and $\quad x = 2$

$\Rightarrow \quad y^2 = (6 \times 2) = 12$

$\Rightarrow \quad y = \pm 2\sqrt{3}$

Thus, the points of intersection of the given curves are $A\left(2, 2\sqrt{3}\right)$ and $B\left(2, -2\sqrt{3}\right)$.

Since each of the given equations contains only even powers of y, each one is symmetric about the x-axis.

$$\therefore \quad \text{Area } OADBO = 2\,(\text{area } OCDAO)$$

$$= 2\,(\text{area } OCAO + \text{area } CDAC)$$

$$= 2\left[\int_0^2 y_{\text{curve (ii)}}\,dx + \int_2^4 y_{\text{curve (i)}}\,dx\right]$$

$$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$$

$$= 2\left\{\left[\frac{2\sqrt{6}}{3}x^{\frac{3}{2}}\right]_0^2 + \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4\right\}$$

$$= 2\left\{\left(\frac{2\sqrt{6}}{3}\cdot 2^{\frac{3}{2}} - 0\right) + 8\sin^{-1}1 - \left(2\sqrt{3} + 8\sin^{-1}\frac{1}{2}\right)\right\}$$

$$= 2\left(\frac{8\sqrt{3}}{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}\right)$$

$$= 2\left(\frac{2\sqrt{3}}{3} + \frac{8\pi}{3}\right) = \frac{4}{3}\left(\sqrt{3} + 4\pi\right) \text{ sq. units}$$

$$= \frac{4}{3}\left(\sqrt{3} + 4\pi\right) \text{ sq. units}$$

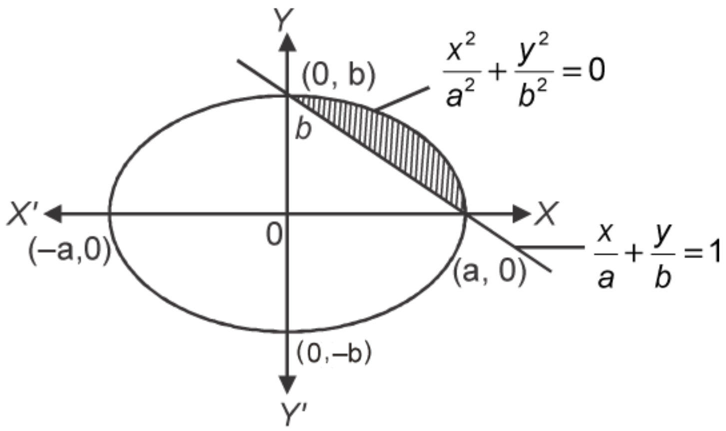
Hence $\quad$ Required area = area of the circle – area $OADBO$

$$= \pi.4^2 - \frac{4}{3}\left(\sqrt{3} + 4\pi\right) = \frac{4}{3}\left(8\pi - \sqrt{3}\right) \text{ sq. units.}$$

S45. Given equation of ellipse and the straight line are

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1 \qquad \text{... (i)}$$

and $$\frac{x}{a}+\frac{y}{b} = 1 \qquad \text{... (ii)}$$



Ellipse with Eq. (i) has vertices ($\pm a$, 0) and ($0 \pm b$) and centre (0, 0). while the line with Eq. (ii) has x-intercept a and y-intercept b.

$\Rightarrow$ Line passes through (a, 0) and (0, b).

$\therefore$ Graph of the above region is as shown in the figure.

Clearly, points of intersection are (a, 0) and (0, b).

$\therefore$ Required area = $\int_0^a$ ($y\,dx$ from equation of ellipse) – ($y\,dx$ from equation of line) ... (iii)

Now, equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\Rightarrow \quad \frac{y^2}{b^2} = 1-\frac{x^2}{a^2}=\frac{a^2-x^2}{a^2}$$

$$\Rightarrow \quad y^2 = \frac{b^2}{a^2}(a^2-x^2)$$

$$\Rightarrow \quad y = \frac{b}{a}\sqrt{a^2-x^2} \qquad \text{... (iv)}$$

and equation of line is $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{y}{b} = 1-\frac{x}{a}=\frac{a-x}{a}$$

or $$y = \frac{b}{a}(a-x) \quad \text{... (v)}$$

Hence, from Eqs. (iii), (iv) and (v), we get

$$\text{Required area} = \int_0^a \left[\frac{b}{a}\sqrt{a^2-x^2} - \frac{b}{a}(a-x)\right]dx$$

$$= \frac{b}{a}\left[\int_0^a \sqrt{a^2-x^2}\,dx - \int_0^a (a-x)\,dx\right]$$

$$\left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C\right]$$

$$\therefore \quad \text{Area} = \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - \frac{b}{a}\left[ax-\frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1}1\right]-\frac{b}{a}\left[a^2-\frac{a^2}{2}\right]$$

$$= \frac{ba^2}{2a}\sin^{-1}\sin\frac{\pi}{2}-\frac{b}{a}\left(\frac{a^2}{2}\right) \qquad \left[\because 1=\sin\frac{\pi}{2}, \therefore \sin^{-1}1=\sin^{-1}\sin\frac{\pi}{2}\right]$$

$$= \left(\frac{ba}{2}\times\frac{\pi}{2}\right)-\frac{ab}{2}$$

$$= \frac{\pi ab}{4}-\frac{ab}{2}$$

$$= \left(\frac{\pi}{4}-\frac{1}{2}\right)ab \text{ sq. units}$$

S46. Given, region is $\{(x, y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$

Above region has a parabola

$$y^2 = 4x \quad \text{... (i)}$$

which has vertex (0, 0) and axis along X-axis and a circle

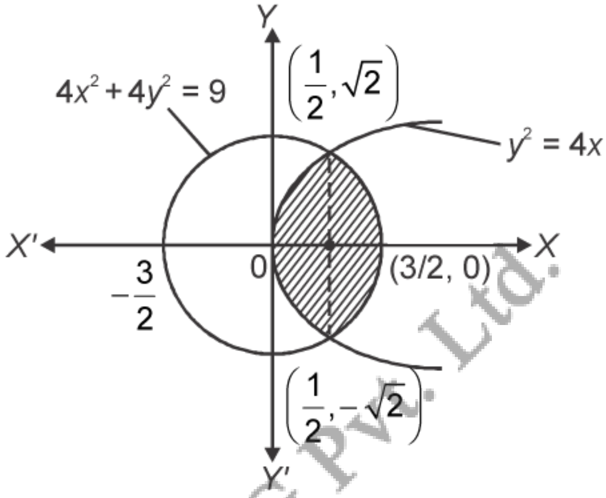
$$4x^2 + 4y^2 = 9 \quad \text{... (ii)}$$

or $$x^2 + y^2 = \frac{9}{4}$$

which is a circle with centre (0, 0) and radius, $r = \frac{3}{2}$

First, we find the point of intersection of two figures.

Putting $y^2 = 4x$ from Eq. (i) in Eq. (ii) we get



$$4x^2 + 4(4x) = 9$$

$$\Rightarrow \quad 4x^2 + 16x - 9 = 0$$

$$\Rightarrow \quad 4x^2 + 18x - 2x - 9 = 0$$

$$\Rightarrow \quad 2x(2x + 9) - 1(2x + 9) = 0$$

$$\Rightarrow \quad (2x - 1)(2x + 9) = 0$$

$$\therefore \quad x = \frac{1}{2} \text{ or } -\frac{9}{2}$$

when $x = \frac{1}{2}$, then $y^2 = \frac{9}{4} - x^2 = \frac{9}{4} - \frac{1}{4} = \frac{8}{4} = 2$

$$\therefore \quad y = \pm\sqrt{2}$$

and $x = -\frac{9}{2}$, then $y^2 = \frac{9}{4} - x^2$

$$= \frac{9}{4} - \frac{81}{4} = \frac{-72}{4} \le 0$$

so, $x = -\frac{9}{2}$, is rejected as $y^2 = -\frac{72}{4}$ is not possible.

Hence, the point of intersection are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$

Now, we sketch the required region.

$\therefore$ Required area = $2\left[\int_0^{1/2} y\,dx \text{ from equation of parabola} + \int_{1/2}^{3/2} y\,dx \text{ from equation of circle}\right]$

$$= 2\left[\int_0^{1/2} 2\sqrt{x}\,dx + \int_{1/2}^{3/2}\sqrt{\frac{9}{4} - x^2}\,dx\right] \qquad \left[\because \int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C\right]$$

$$= 4\left[x^{3/2} \cdot \frac{2}{3}\right]_0^{1/2} + 2\left[\frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\right]_{1/2}^{3/2}$$

$$= \frac{8}{3}\left[\left(\frac{1}{2}\right)^{3/2} - 0\right] + 2\left[\left(\frac{9}{8}\sin^{-1}\frac{2}{3}\cdot\frac{3}{2}\right) - \left(\frac{1}{4}\right)\sqrt{\frac{9}{4} - \frac{1}{4}} - \frac{9}{8}\sin^{-1}\frac{2}{3}\cdot\frac{1}{2}\right]$$

$$= \left[\frac{8}{3} \times \frac{1}{\{(\sqrt{2})^2\}^{3/2}}\right] + 2\left[\frac{9}{8}\sin^{-1}1 - \frac{1}{4}\sqrt{2} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= \left(\frac{8}{3} \times \frac{1}{2\sqrt{2}}\right) + 2\left[\frac{9}{8}\sin^{-1}1 - \frac{1}{4}\sqrt{2} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= \frac{4}{3\sqrt{2}} + 2\left[\frac{9}{8}\cdot\sin^{-1}\sin\frac{\pi}{2} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right] \qquad \left[\because 1 = \sin\frac{\pi}{2}\right]$$

$$= \frac{4\sqrt{2}}{3\sqrt{2} \times \sqrt{2}} + 2\left[\frac{9}{8}\cdot\frac{\pi}{2} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= \left(\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{2}\right) + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$= \left(\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{2}\right) + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$= \left(\frac{4\sqrt{2} - 3\sqrt{2}}{6}\right) + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

Hence, required area

$$= \left(\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}\right) \text{ sq. units}$$

S47. Given region is $\{(x, y): x^2 + y^2 \le 4, x + y \ge 2$

The above region has a circle whose equation is

$$x^2 + y^2 = 4 \qquad \ldots \text{(i)}$$

with centre (0, 0) and radius 2 and line whose equation is

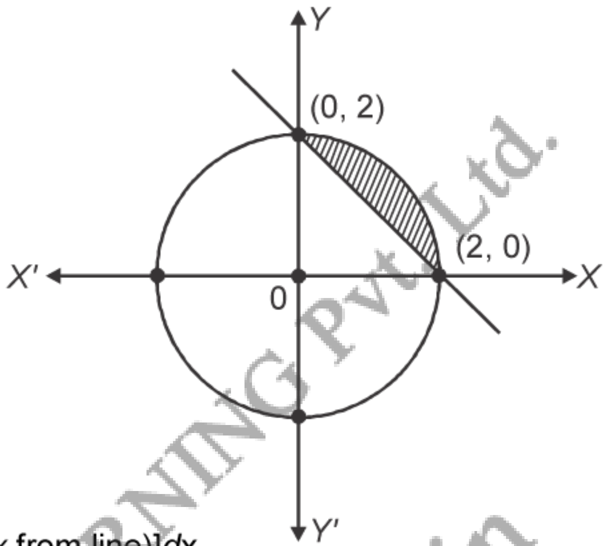
$$x + y = 2 \qquad \text{... (ii)}$$

x	0	2
y	2	0

Rough sketch for the above region is given as

Point of intersection is calculated as follows

$$x^2 + y^2 = 4 \qquad [\because\ y = 2 - x]$$



$$x^2 + (2 - x)^2 = 4$$

$\Rightarrow$ $x^2 + 4 + x^2 - 4x = 4$

$\Rightarrow$ $2x^2 - 4x = 0$

$\Rightarrow$ $2x(x - 2) = 0$

$\Rightarrow$ $x = 0$ or 2

when $x = 0$, then $y = 2 - 0 = 2$

and when $x = 2$, then $y = 2 - 2 = 0$

Points of intersection are (0, 2) and (2, 0).

Required area of region = $\int_0^2 [y dx$ from circle) – ($y dx$ from line)]dx

$$= \int_0^2 \left[\sqrt{4 - x^2} - (2 - x)\right] dx \qquad \left[\because\ x^2 + y^2 = 4, y^2 = 4 - x^2, y = \sqrt{4 - x^2}\right]$$

$$= \int_0^2 \sqrt{4 - x^2} dx - \int_0^2 (2 - x) dx$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$

$$\left[\because \int \sqrt{a^2 - x^2} dx = \left\{\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right\}\right]$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$

$$= \left[\frac{2}{2}\sqrt{4 - 4} + 2\sin^{-1}\left(\frac{2}{2}\right) - 0 - 2\sin^{-1} 0\right] - \left[4 - \frac{4}{2} - 0\right]$$

$$= (2\sin^{-1}1 - 0) - \left[4 - \frac{4}{2}\right]$$

$$= 2\sin^{-1}\sin\frac{\pi}{2} - 2 \qquad \left[\because 1 = \sin\frac{\pi}{2}\right]$$

$$= 2\cdot\frac{\pi}{2} - 2 = \pi - 2$$

$\therefore$ Area of region = $(\pi - 2)$ sq. units.

S48. The given region is $\{(x, y) : (x^2 + y^2) \le 1 \le x + y\}$

The above region has a circle whose equation is $x^2 + y^2 = 1$ and a line whose equation is $x + y = 1$.

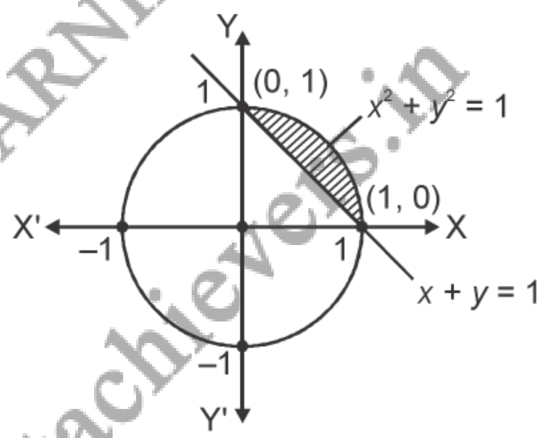
First, we sketch the region. The circle $x^2 + y^2 = 1$ has centre (0, 0) and radius 1. The line $x + y = 1$ passes through the point (0,1) and (1,0).

Shaded region is the required bounded region whose area is to be calculated. Now, we find points of intersection of the circle and the line by solving their equations. We have

$$x^2 + y^2 = 1 \qquad \text{... (i)}$$

$$x + y = 1 \qquad \text{... (ii)}$$

Putting $y = (1 - x)$ from Eq. (ii) in Eq. (i), we get



$$x^2 + (1 - x)^2 = 1$$

$\Rightarrow$ $$x^2 + 1 + x^2 - 2x = 1$$

$$2x^2 - 2x = 0$$

$$2x(x - 1) = 0$$

$\Rightarrow$ $$x = 0 \quad \text{or} \quad 1$$

where $x = 0$, $y = 1 - x = 1 - 0 = 1$

and $x = 1$, $y = 1 - x = 1 - 1 = 0$

$\therefore$ Points of intersection are (0, 1) and (1, 0).

Hence, required area = Area of shaded region = $= \int_0^1 y\,dx$ (from circle) $- \int_0^1 y\,dx$ [From line]

$$= \int_0^1 \left[\sqrt{1 - x^2} - (1 - x)\right] dx$$

$$= \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$$

$$\left[\because \int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C\right]$$

$$= \left(\frac{1}{2}\sin^{-1}1\right) - \left(1 - \frac{1}{2}\right) \qquad [\because \sin^{-1}0 = 0]$$

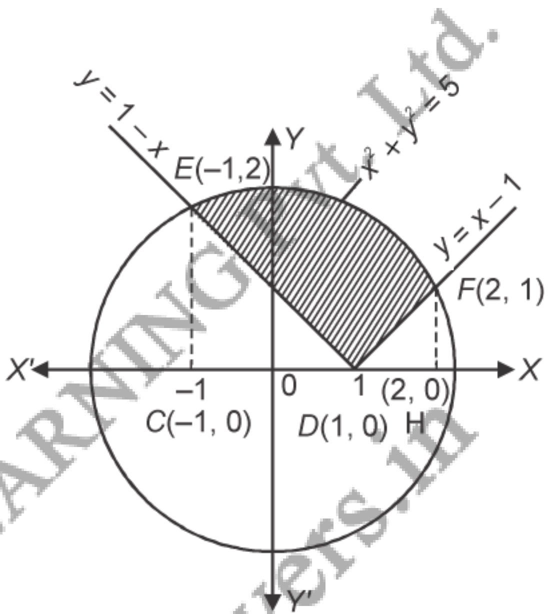
$$= \frac{1}{2}\cdot\sin^{-1}\sin\frac{\pi}{2} - \frac{1}{2} \qquad \left[\therefore \sin^{-1}1 = \sin^{-1}\sin\frac{\pi}{2}\right]$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2} = \left(\frac{\pi}{4} - \frac{1}{2}\right)\text{sq. units}$$

S49. From the given region, we observe that the two equations are $y = |x - 1|$ and $y = \sqrt{5 - x^2}$

Using definition, $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

Write $y = |x - 1| = \begin{cases} x - 1, & \text{if } x \geq 1 \\ -x + 1, & \text{if } x < 1 \end{cases}$



Sketch all the above figures and find the required area.

Given region is

$$\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$$

Above region has two equations

$$y = |x - 1| \text{ and } y = \sqrt{5 - x^2}$$

$\therefore$ $y = \begin{cases} x - 1, & \text{if } x \geq 1 \\ 1 - x, & \text{if } x < 1 \end{cases}$

Also, other equation is $y = \sqrt{5 - x^2}$

Squaring both sides, we get

$$y^2 = 5 - x^2 \text{ or } x^2 + y^2 = 5$$

It is a circle with centre (0, 0) and radius $r = \sqrt{5}$.

Now, we sketch the graph of required region.

Now, we find the point of intersection of the curves.

We have $y = 1 - x$ ... (i)

$y = x - 1$ ... (ii)

$x^2 + y^2 = 5$ ... (iii)

Points of intersection of Eqs. (i) and (iii),

Putting $y = (1 - x)$ from Eq. (i) in Eq. (iii),

We get

$$x^2 + (1 - x)^2 = 5$$

$\Rightarrow \quad x^2 + 1 + x^2 - 2x = 5$

$\Rightarrow \quad 2x^2 - 2x - 4 = 0$

$\Rightarrow \quad x^2 - x - 2 = 0$

$\Rightarrow \quad x^2 - 2x + x - 2 = 0$

$\Rightarrow \quad x(x - 2) + 1(x - 2) = 0$

$\Rightarrow \quad (x + 1)(x - 2) = 0$

$\Rightarrow \quad x = -1$ or 2 but $x = 2$ rejected, as $x < 1$

$\Rightarrow \quad 2x^2 - 2x - 4 = 0$

Now, when $x = -1$, then

$$y^2 = 5 - x^2 = 5 - 1 = 4$$

$\Rightarrow \quad y^2 = 4$

or $\quad y = \pm 2$

$\Rightarrow$ Points of intersection are $(-1, 2)$ and $(-1, -2)$.

Points of intersection of Eqs. (ii) and (iii).

Putting $y = x - 1$ from Eq. (ii) and (iii), we get

$$x^2 + (x - 1)^2 = 5$$

$\Rightarrow \quad x^2 + x^2 + 1 - 2x = 5$

$\Rightarrow \quad 2x^2 - 2x - 4 = 0$

$\Rightarrow \quad x^2 - x - 2 = 0$

$\Rightarrow \quad (x - 2)(x + 1) = 0$

$\Rightarrow \quad x = -1$ or 2 but $x = -1$ rejected as $x \geq 1$.

Hence, the two curve intersect at $(2, \pm 1)$.

Now, required area

$$= \int_{-1}^{2} y\, dx \text{ from circle} - \int_{-1}^{1} (y = 1 - x)dx - \int_{1}^{2} (y = x - 1)dx$$

$$= \int_{-1}^{2} \sqrt{5 - x^2}\, dx - \int_{-1}^{1} (1 - x)dx - \int_{1}^{2} (x - 1)dx$$

$$\left[\because \int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C\right]$$

$$= \left[\frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$$

$$= \left[\left(\frac{2}{2}\sqrt{5-4}+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right)-\left\{-\frac{1}{2}\sqrt{4}+\frac{5}{2}\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right)\right\}\right]$$

$$-\left[\left(1-\frac{1}{2}\right)-\left(-1-\frac{1}{2}\right)\right]-\left[\left(\frac{4}{2}-2\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= 1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}+\frac{1}{2}\times 2-\frac{5}{2}\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right)-\left(\frac{1}{2}+\frac{3}{2}\right)-\left(0+\frac{1}{2}\right)$$

$$= 1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}+1+\frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)-2-\frac{1}{2} \quad \left[\because \sin^{-1}(-\theta)=-\sin^{-1}\theta\right]$$

$$= -\frac{1}{2}+\frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{5}\right)$$

Hence, required area is $\left[\frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{5}\right)-\frac{1}{2}\right]$ sq. unit

S50. The given region is $\left\{(x,y): \frac{x^2}{9}+\frac{y^2}{4}\le 1\le \frac{x}{3}+\frac{y}{2}\right\}$

Above region has two figures whose equations are

$$\frac{x^2}{9}+\frac{y^2}{4} = 1 \quad \text{... (i)}$$

Which is an ellipse with vertices (± 3, 0) and (0, ± 2)

and $$\frac{x}{3}+\frac{y}{2} = 1 \quad \text{... (ii)}$$

Which is a straight line.

Comparing Eq. (ii) with intercept form of the line $\frac{x}{a}+\frac{y}{b} = 1$

where a = x intercept and b = y intercept

We get $a = 3$

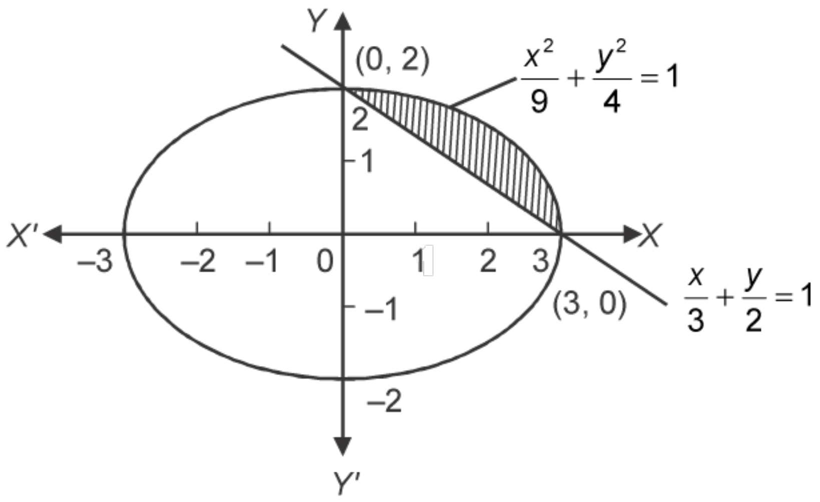
and $b = 2$

Line cuts X-axis at (3, 0) and Y-axis at (0, 2). ... (iii)

Also, vertices of ellipse are ($\pm$3, 0) and (0 $\pm$ 2) ... (iv)

Using statements (iii) and (iv), we get that ellipse and line intersect at (3, 0) and (0, 2).

Now, we sketch the required region



Now, required area = $\int_0^3$ (y dx from ellipse – (y dx from line)

$$= \int_0^3 \left[\frac{2}{3}\sqrt{9-x^2} - \left(\frac{6-2x}{3}\right)\right]dx$$

$$= \frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx - \frac{2}{3}\int_0^3 (3-x)dx$$

$$\left[\because \frac{x^2}{9}+\frac{y^2}{4}=1 \Rightarrow \frac{y^2}{4}=1-\frac{x^2}{9}=\frac{9-x^2}{9} \text{ and } y^2=\frac{4}{9}(9-x^2)\Rightarrow y=\frac{2}{3}\sqrt{9-x^2}\right.$$

$$\left.\frac{x}{3}+\frac{y}{2} = 1 \Rightarrow \frac{y}{2} = 1-\frac{x}{3} \text{ or } y=\frac{2}{3}(3-x)\right]$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - \frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

$$\left[\therefore \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$= \frac{2}{3}\left[\frac{9}{2}\sin^{-1}1-\frac{9}{2}\sin^{-1}0\right]-\frac{2}{3}\left[9-\frac{9}{2}\right]$$

$$= \frac{2}{3}\left[\frac{9}{2}\sin^{-1}\sin\frac{\pi}{2}-0\right]-\frac{2}{3}\times\frac{9}{2} \qquad \left[\because \sin^{-1}1=\sin^{-1}\sin\frac{\pi}{2}=\frac{\pi}{2}\right]$$

$$= \frac{2}{3}\left[\frac{9}{2}\times\frac{\pi}{2}\right]-3$$

$$= \frac{3\pi}{2} - 3$$

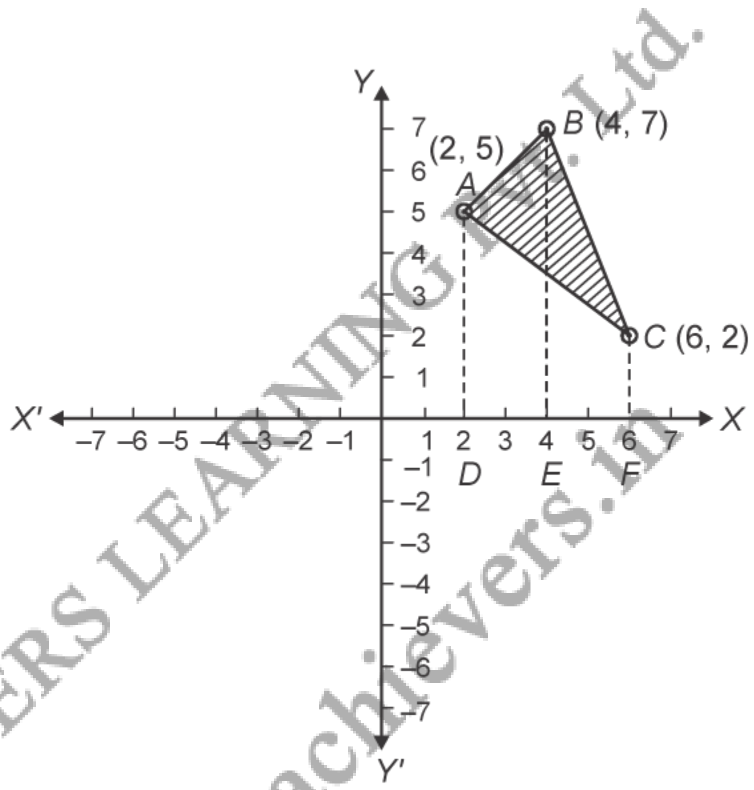
$$= 3\left(\frac{\pi}{2} - 1\right)$$

Hence, required area = $3\left(\frac{\pi}{2} - 1\right)$ sq. units

S51. The given vertices of ΔABC are $A(2, 5)$, $B(4, 7)$ and $C(6, 2)$. Plotting above points, we get the required triangular region as follows

Now, we find equations of sides AB, BC and CA We know that equation of the line passing through the points $(x_1\ y_1)$ and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



Using above result, the equations are calculated as follows

Equation of AB whose points are $A(2,5)$ and $B(4, 7)$

$$y - 5 = \frac{2}{2}(x - 2)$$

$$\Rightarrow \quad y - 5 = x - 2$$

$$\Rightarrow \quad y = x + 3 \qquad \text{... (i)}$$

Equation of BC, whose points are $B(4, 7)$ and $C(6, 2)$.

$$y - 7 = \frac{-5}{2}(x - 4)$$

$$\Rightarrow \quad y = \frac{-5x}{2} + 10 + 7$$

$$\text{or} \quad y = \frac{-5x + 34}{2} \qquad \text{... (ii)}$$

Equation of CA whose points are $C(6, 2)$ and $A(2, 5)$.

$$y - 2 = \frac{3}{-4}(x - 6)$$

$$\Rightarrow \quad y = \frac{-3x}{4} + \frac{18}{4} + 2$$

$$\Rightarrow \quad y = \frac{-3x + 18 + 8}{4}$$

$$\Rightarrow \quad y = \frac{-3x + 26}{4} \qquad \text{... (iii)}$$

Hence, the required area is given by

$$= \text{Area } ABED + \text{Area } BEFC - \text{Area } ADFC$$

$$= \int_2^4 y\,dx \text{ from Eq. } AB + \int_4^6 y dx \text{ from Eq. } BC - \int_2^6 y dx \text{ from Eq. } AC$$

$$= \int_2^4 (x+3)dx + \int_4^6 \left(\frac{-5x+34}{2}\right)dx - \int_2^6 \frac{-3x+26}{4}dx$$

$$= \left[\frac{x^2}{2}+3x\right]_2^4 + \left[\frac{34x}{2}-\frac{5x^2}{4}\right]_4^6 - \left[\frac{-3x^2}{8}+\frac{26x}{4}\right]_2^6$$

$$= [(8+12)-(2+6)] + [(102-45)-(68-20)] - \left[\left(-\frac{108}{8}+\frac{156}{4}\right)-\left(-\frac{3}{2}+\frac{26}{2}\right)\right]$$

$$= 12 + 57 - 48 + \frac{27}{2} - 39 - \frac{3}{2} + 13$$

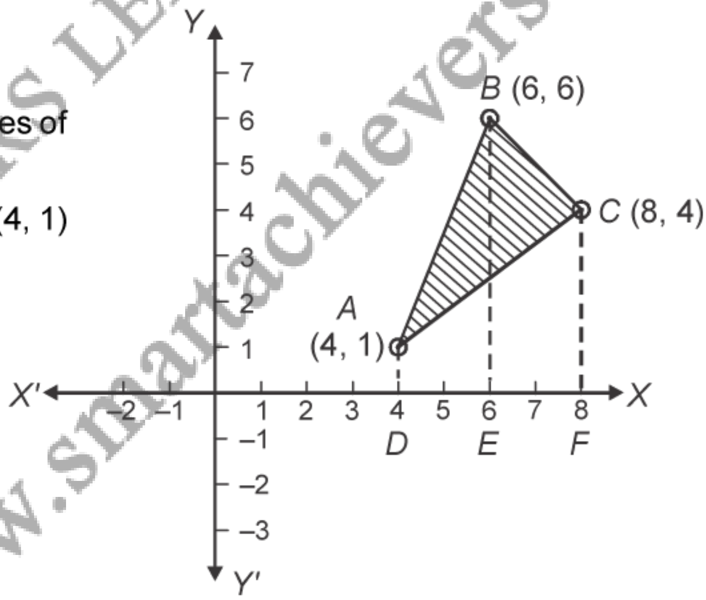
$$= -5 + 12 = 7 \text{ sq units.}$$

Hence, required area = 7 sq. units.

S52. Given, vertices of ΔABC are $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$. First we plot these vertices and get the required triangular region.

We know that equation of line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



Using above equation, we find equation of sides of ΔABC.

$\therefore$ Equation of side AB whose points are $A(4, 1)$ and $B(6, 6)$ is given below

$$y - 1 = \frac{5}{2}(x-4)$$

$$\Rightarrow \quad y = \frac{5x-20}{2} + 1$$

$$\Rightarrow \quad y = \frac{5x-18}{2} \quad \text{... (i)}$$

Equation of side BC whose points are $B(6, 6)$ and $C(8, 4)$, is

$$y - 6 = \frac{-2}{2}(x-6)$$

$$\Rightarrow \quad y - 6 = -x + 6$$

$$\Rightarrow \quad y = -x + 12 \quad \text{... (ii)}$$

and equation of side CA whose points are $C(8, 4)$ and $A(4, 1)$ is

$$y - 4 = \frac{-3}{-4}(x - 8) \quad \Rightarrow \quad y - 4 = \frac{3}{4}(x - 8)$$

$$\Rightarrow \qquad y - 4 = \frac{3x - 24}{4}$$

$$\Rightarrow \qquad y = \frac{3x - 24}{4} + 4$$

$$\therefore \qquad y = \frac{3x - 8}{4} \qquad \text{... (iii)}$$

Now required area of shaded region

$\Rightarrow$ Area of $ADEB$ + Area $BEFC$ – Area of $ADFC$

$$\Rightarrow \int_6^4 y\,dx \text{ from eq. } AB + \int_6^8 y\,dx \text{ from eq. } BC - \int_4^8 y\,dx \text{ from eq. } AC$$

$$\Rightarrow \int_4^6 \left(\frac{5x - 18}{2}\right) dx + \int_6^8 (12 - x) dx - \int_4^8 \left(\frac{3x - 8}{4}\right) dx$$

$$\Rightarrow \frac{1}{2}\int_4^6 (5x - 18)\,dx + \int_6^8 (12 - x)\,dx - \frac{1}{4}\int_4^8 (3x - 8)\,dx$$

$$\Rightarrow \left[\frac{5x^2}{4} - \frac{18x}{2}\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3x^2}{8} - \frac{8x}{4}\right]_4^8$$

$$\Rightarrow \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$$

$\Rightarrow$ [(45 – 54) – (20 – 36)] + [(96 – 32) – (72 – 18)] – [(24 – 16) – (6 – 8)]

$\Rightarrow$ [(–9 + 16) + (64 – 54) – (8 + 2)]

$\Rightarrow$ [7 + 10 – 10] = 7

Hence, required area = 7 sq. units.

S53. Given, equations of lines are

$$2x + y = 4 \qquad \text{... (i)}$$

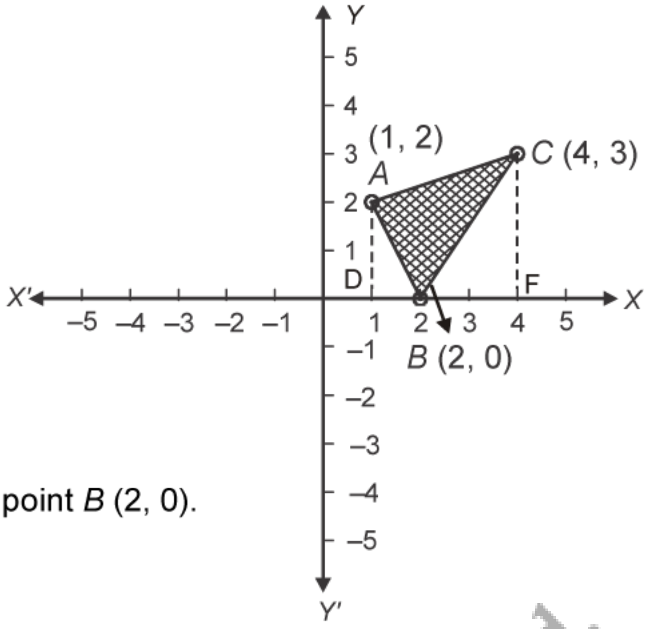
$$3x - 2y = 6 \qquad \text{... (ii)}$$

$$x - 3y = -5 \qquad \text{... (iii)}$$

First, we sketch the required triangular region. Now, the line $2x + y = 4$ passes through the points (2, 0) and (1, 2), the line $3x - 2y = 6$ passes through points (2, 0) and (4, 3) and the line $x - 3y = -5$

passes through points (1, 2) and (4, 3).

$\therefore$ Graph of triangular region is shown below.



Now, solving Eqs. (i) and (ii), we get

$$2(2x + y = 4)$$

$$\Rightarrow \quad 4x + 2y = 8$$

$$3x - 2y = 6$$

$$7x = 14$$

$$\Rightarrow \quad x = 2$$

$\therefore$ $y = 4 - 2(2) = 4 - 4 = 0$

$\therefore$ Lines $2x + y = 4$ and $3x - 2y = 6$ meet at the point B (2, 0).

Again, solving Eqs. (ii) and (iii), we get

$$3(x - 3y = -5)$$

$$\Rightarrow \quad 3x - 9y = -15$$

$$3x - 2y = 6$$

$$-7y = -21$$

$$\Rightarrow \quad y = 3$$

$$\therefore \quad x = 3(3) - 5 = 9 - 5 = 4$$

Lines $3x - 2y = 6$ and $x - 3y = -5$ meet at the point C (4, 3).

Solving Eqs. (iii) and (i), we get

$$3(2x + y = 4)$$

$$\Rightarrow \quad 6x + 3y = 12$$

$$x - 3y = -5$$

$$7x = 7$$

$$\Rightarrow \quad x = 1$$

$$\therefore \quad y = \frac{1+5}{3} = \frac{6}{3} = 2$$

$\Rightarrow \therefore$ Lines $2x + y = 4$ and $x - 3y = -5$ meet at the point A (1, 2).

Hence, required area = Area of $ADFC$ – (Area of ΔADB + Area of ΔCFB)

$$= \int_1^4 y_{AC}\,dx - \int_1^2 y_{AB}\,dx - \int_2^4 y_{BC}\,dx$$

$$= \int_1^4 \left(\frac{x+5}{3}\right)dx - \int_1^2 (4-2x)\,dx - \int_2^4 \left(\frac{3x-6}{2}\right)dx$$

$$= \frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4 - [4x - x^2]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2}-6x\right]_2^4$$

$$= \frac{1}{3}\left[\left(\frac{16}{2}+20\right)-\left(\frac{1}{2}+5\right)\right] - [(8-4)-(4-1)] - \frac{1}{2}[0-(6-12)]$$

$$= \frac{1}{3}\left(28-\frac{11}{2}\right) - (1) - \frac{1}{2}(0+6)$$

$$= \frac{1}{3}\times\frac{45}{2} - 1 - 3 = \frac{15}{2} - 4 = \frac{7}{2} \text{ sq. units}$$

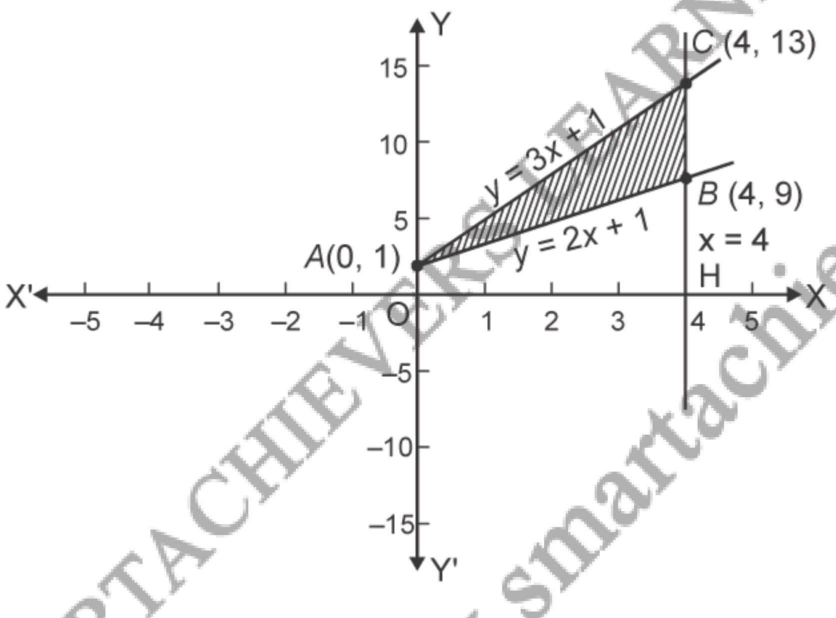
S54. We have to find the triangular region area whose equations of sides of triangle are given as

$y = 2x + 1$ ... (i)

$y = 3x + 1$ ... (ii)

$x = 4$ ... (iii)

First of all, we sketch the graph of the triangular region. The line $y = 2x + 1$ passes through points (0, 1) and (1, 3) and the line $y = 3x + 1$ passes through points (0, 1) and (1, 4). Graph of the required region is given below.



We have to find the area of ΔABC shaded above.

Now, solving Eqs. (i) and (ii), we get

$y = 2x + 1$

$\underline{y = 3x + 1}$

$0 = -x + 0$

or $x = 0 \Rightarrow y = 2(0) + 1 = 1$

$\therefore$ Lines $y = 2x + 1$ and $y = 3x + 1$ meet at the point $A(0, 1)$.

Again, solving Eqs. (ii) and (iii), we get

$$y = 3x + 1$$

$$\Rightarrow \quad y = 3(4) + 1 \qquad [\because x = 4]$$

$$\Rightarrow \quad y = 12 + 1 = 13$$

$\therefore$ Lines $y = 3x + 1$ and $x = 4$ meet at the point $C(4, 13)$.

Solving Eqs. (i) and (iii), we get

$$y = 2x + 1$$

$$\Rightarrow \quad y = 2(4) + 1 \qquad [\because x = 4]$$

$$\Rightarrow \quad y = 8 + 1 = 9$$

Lines $y = 2x + 1$ and x = 4 meet at the point 6(4,9).

Hence, required area = Area of $AOHCA -$ Area of $AOHBA$

$$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$$

$$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$$

$$= \left[\frac{3(16)}{2} + 4\right] - [16 + 4]$$

$$= 24 + 4 - 16 - 4 = 8 \text{ sq. units.}$$

S55. Firstly plot the given lines and find their point of intersection.

Given equation of lines are

$$3x - 2y = -1 \qquad \dots \text{(i)}$$

$$2x + 3y = 21 \qquad \dots \text{(ii)}$$

$$x - 5y = -9 \qquad \dots \text{(iii)}$$

Solving Eqs. (i) and (ii) as follows; we get

$$2(3x - 2y = -1)$$

$$3(2x + 3y = 21)$$

$$\Rightarrow \quad 6x - 4y = -2$$

$$\underline{6x \pm 9y = \underline{}\ 63}$$

$$-13y = -65$$

$$y = \frac{65}{13} = 5$$

Putting $y = 5$ in Eq. (i), we get

$$3x - 2y = -1$$
$$3x - 10 = -1$$

$$3x = 9 \quad \Rightarrow \quad x = \frac{9}{3} = 3$$

Lines (i) and (ii) intersect each other at point (3, 5).

Now, solving Eqs. (ii) and (iii) as follows, we get

$$1(2x + 3y = 21)$$
$$2(x - 5y = -9)$$

$$2x + 3y = 21$$
$$\underline{2x \mp 10y = \mp 18}$$
$$13y = 39$$

$$y = \frac{39}{13} = 3$$

Putting $y = 3$ in Eq. (iii), we get

$$x - 5y = -9 \quad \Rightarrow \quad x - 5(3) = -9$$
$$x - 15 = -9$$

$$x = -9 + 15 = 6$$
$$x = 6, \quad y = 3$$

So, lines (ii) and (iii) intersect each other at point (6, 3).

Now, solving Eqs. (i) and (iii), we get

$$1 \times (3x - 2y = -1)$$
$$3 \times (x - 5y = -9)$$

$$3x - 2y = -1$$
$$\underline{3x \mp 15y = \mp 27}$$
$$13y = 26$$

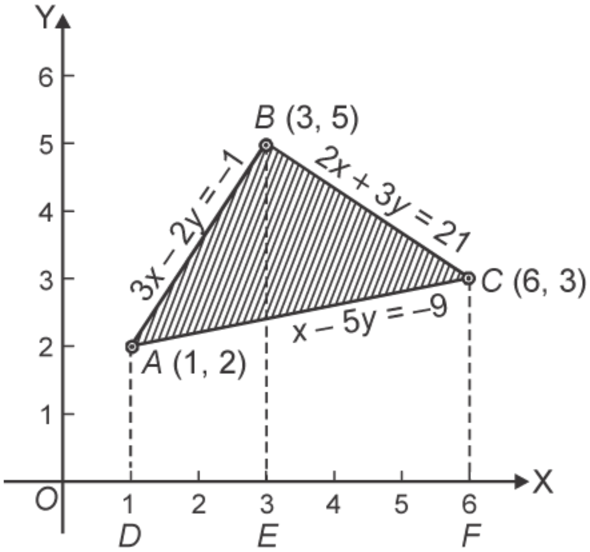
$$y = \frac{26}{13} = 2$$

Put $y = 2$ in Eq. (iii), we get

$$x - 5(2) = -9 \quad \Rightarrow \quad x - 10 = -9$$
$$x = -9 + 10 \quad \Rightarrow \quad x = 1$$

So, the lines (i) and (iii) intersect each other at point (1, 2).

Now, we draw the rough sketch of these lines as



Now, required area of triangle

$$= \text{Area of } (ABED) + \text{Area of } (BEFC) - \text{Area of } (ADFC)$$

$$= \int_1^3 y\,dx \text{ from Eq. (i)} + \int_6^3 y\,dx \text{ from Eq. (ii)} - \int_1^6 y\,dx \text{ from Eq. (iii)}$$

$$= \int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{21-2x}{3}dx - \int_1^6 \frac{x+9}{5}dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2}+x\right]_1^3 + \frac{1}{3}\left[21x-\frac{2x^2}{2}\right]_3^6 - \frac{1}{5}\left[\frac{x^2}{2}+9x\right]_1^6$$

$$= \frac{1}{2}\left[\left(\frac{27}{2}+3\right)-\left(\frac{3}{2}+1\right)\right] + \frac{1}{3}[(126-36)-(63-9)] - \frac{1}{5}\left[(18+54)-\left(\frac{1}{2}+9\right)\right]$$

$$= \frac{1}{2}\left[\frac{33}{2}-\frac{5}{2}\right] + \frac{1}{3}[90-54] - \frac{1}{5}\left[72-\frac{19}{2}\right]$$

$$= \frac{1}{2}\left[\frac{28}{2}\right] + \frac{1}{3}(36) - \frac{1}{5}\left[\frac{125}{2}\right]$$

$$= 7 + 12 - \frac{25}{2}$$

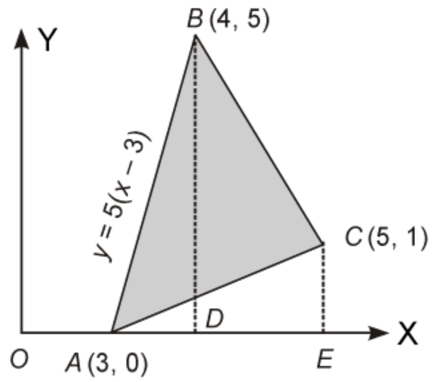
$$= \frac{14+24-25}{2}$$

$$= \frac{13}{2} \text{ Sq. units}$$

S56. Here we have a triangle ABC whose vertices are $A(3, 0)$, $B(4, 5)$, $C(5, 1)$.

Equation of the line AB passing through $A(3, 0)$ and $B(4, 5)$ is

$$y - 0 = \frac{5-0}{4-3}(x-3) \quad \Rightarrow \quad y = 5(x-3) \qquad \text{... (i)}$$



Equation of the line CB passing through $C(5, 1)$ and $B(4, 5)$ is

$$y - 1 = \frac{5-1}{4-5}(x-5)$$

$$y - 1 = -4(x-5) \quad \Rightarrow \quad y = -4x + 21 \qquad \text{... (ii)}$$

Equation of the line AC passing through $A(3, 0)$ and $C(5, 1)$ is

$$y - 0 = \frac{1-0}{5-3}(x-3) \quad \Rightarrow \quad y = \frac{1}{2}(x-3) \qquad \text{... (iii)}$$

$$\text{Area of } \Delta ABC = \int_3^4 y_1\,dx + \int_4^5 y_2\,dx - \int_3^5 y_3\,dx$$

$$= \int_3^4 (5x-15)dx + \int_4^5 (-4x+21)dx - \int_3^5 \frac{1}{2}(x-3)dx$$

$$= \left(\frac{5x^2}{2} - 15x\right)_3^4 + (-2x^2 + 21x)_4^5 - \left(\frac{x^2}{4} - \frac{3x}{2}\right)_3^5.$$

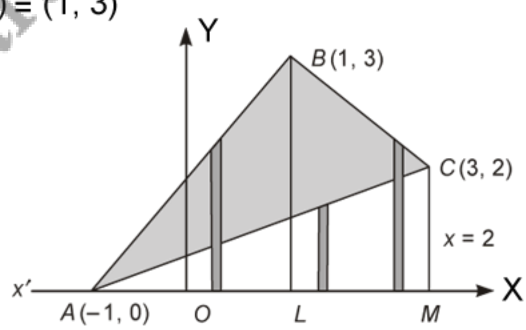
$$= \left[(40-60) - \left(\frac{45}{2} - 45\right)\right] + \left[(-50+105) - (-32+84)\right]$$

$$- \left[\left(\frac{25}{4} - \frac{15}{2}\right) - \left(\frac{9}{4} - \frac{9}{2}\right)\right]$$

$$= \left(\frac{5}{2}\right) + (3) - (1) = \frac{9}{2} \text{ sq. unit}$$

S57. Equation of the line AB where $A(x_1, y_1) = (-1, 0)$, $B(x_2, y_2) = (1, 3)$

$$y - 0 = \frac{3-0}{1+1}(x+1) \quad \Rightarrow \quad y = \frac{3}{2}(x+1)$$



Equation of line BC where

$B(x_1, y_1) = (1, 3)$ and $C(x_2, y_2) = (3, 2)$

$$y - 3 = \frac{2-3}{3-1}(x-1)$$

$$\Rightarrow \qquad y - 3 = \frac{-1}{2}(x-1) \quad \Rightarrow \quad y = -\frac{1}{2}x + \frac{7}{2}$$

Equation of line AC where $A(x_1, y_1) = (-1, 0)$ and $C(x_2, y_2) = (3, 2)$.

$$y - 0 = \frac{2-0}{3+1}(x+1) \quad \Rightarrow \quad y = \frac{1}{2}(x+1).$$

Limits: for the area of ΔALB are $x = -1$, $x = 1$.

for the area of ⏢ $BLMC$ are $x = 1$, $x = 3$.

for the area of $\Delta\ AMC$ are $x = -1$, $x = 3$

Area of ΔABC = area of ΔALB + Area of trapezium $BLMC$ – Area of $\Delta\ AMC$

$$= \int_{-1}^{1} y_{AB}\,dx + \int_{1}^{3} y_{BC}\,dx - \int_{1}^{3} y_{AC}\,dx$$

$$= \frac{3}{2}\int_{-1}^{1}(x+1)dx + \int_{1}^{3}\left(-\frac{1}{2}x+\frac{7}{2}\right)dx - \frac{1}{2}\int_{-1}^{3}(x+1)dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} - \frac{1}{2}\left[\frac{x^2}{2}-7x\right]_{1}^{3} - \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$= \frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right] - \frac{1}{2}\left[\frac{9}{2}-21-\frac{1}{2}+7\right] - \frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right]$$

$$= \frac{3}{2}[2] - \frac{1}{2}[-10] - \frac{1}{2}[8]$$

= 3 + 5 – 4 = 4 Sq. units.

S58. Required area of ΔABC = area $ADEBA$ + area $BEFCB$ – area $ADFCA$ ... (i)

Equation of AB is $y = x + 1$ ... (ii)

Equation of BC is $y = -x + 7$ ... (iii)

Equation of AC is $y = \frac{x}{3} + \frac{5}{3}$ ... (iv)

Solving (ii) and (iii),

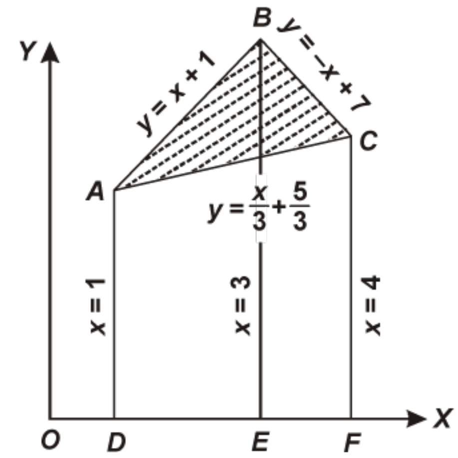
we get $x = 3$, $\therefore\ OE = 3$

Solving (iii) and (iv),

we get, $x = 4, \quad \therefore \; OF = 4$

Solving (ii) and (iv),

we get $x = 1, \quad \therefore \; OD = 1$



Now area $ADEBA = \int_1^3 y_{AB}\, dx$

$$= \int_1^3 (x+1)\, dx = \left[\frac{x^2}{2} + x\right]_1^3$$

$$= \left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} + 1\right) = 6$$

area $BEFCB = \int_3^4 y_{BC}\, dx = \int_3^4 (-x+7)\, dx$

$$= \left[-\frac{x^2}{2} + 7x\right]_3^4 = (-8 + 28) - \left(-\frac{9}{2} + 21\right) = \frac{7}{2}$$

area $ADFCA = \int_1^4 y_{AC}\, dx = \int_1^4 \left(\frac{x}{3} + \frac{5}{3}\right) dx$

$$= \left[\frac{x^2}{6} + \frac{5}{3}x\right]_1^4 = \left(\frac{16}{6} + \frac{20}{3}\right) - \left(\frac{1}{6} + \frac{5}{3}\right)$$

$$= \frac{15}{6} + \frac{15}{3} = \frac{45}{6} = \frac{15}{2}$$

From Eq. (i), area of $\Delta ABC = 6 + \frac{7}{2} - \frac{15}{2} = 2$ sq. units.

S59. Given curves are $y^2 \le 6ax$ and $x^2 + y^2 \le 16a^2$

Now, solving both equations of the given curve, we get the intersection point

$$x^2 + 6ax = 16a^2$$

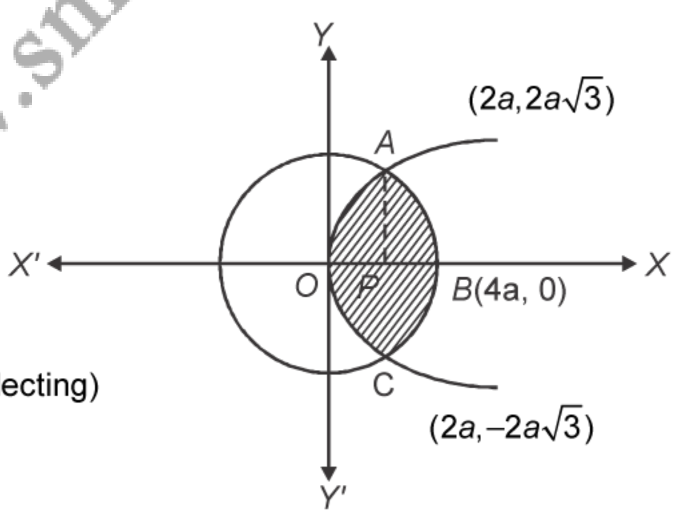
$\Rightarrow \quad x^2 + 6ax - 16a^2 = 0$

$\Rightarrow \quad x^2 + 8ax - 2ax - 16a^2 = 0$

$\Rightarrow \quad x(x + 8a) - 2a(x + 8a) = 0$

$\Rightarrow \quad (x + 8a)\,(x - 2a) = 0$

$\Rightarrow \quad x = -8a, 2a$



Put $x = -8a$ in first curve it is not satisfied (neglecting)

Put $x = 2a$ in first curve, we get $(y = \pm 2a\sqrt{3})$

$\therefore$ Required intersection points $(2a, 2a\sqrt{3})$ and $(2a, -2a\sqrt{3})$.

$\therefore$ Required area= Area of curve *OABC*

$$= 2\,[\text{Area of curve } OAP + \text{Area of curve } APB]$$

$$= 2\left[\int_0^{2a}\sqrt{6ax}\,dx + \int_{2a}^{4a}\sqrt{16a^2 - x^2}\,dx\right]$$

$$= 2\left\{\sqrt{6a}\left[\frac{2}{3}x^{3/2}\right]_0^{2a} + \left[\frac{x}{2}\sqrt{16a^2 - x^2} + \frac{16}{2}a^2\sin^{-1}\frac{x}{4a}\right]_{2a}^{4a}\right\}$$

$$= 2\left\{\sqrt{6a}\,\frac{2}{3}(2a)^{3/2} + \left(\frac{16a^2}{2}\times\frac{\pi}{2} - a\,2a\sqrt{3} - \frac{16a^2}{2}\times\frac{\pi}{6}\right)\right\}$$

$$= 2\left[\frac{\sqrt{2}}{\sqrt{3}}\cdot 2a^{1/2}\cdot 2\sqrt{2}\cdot a^{3/2} + 4a^2\pi - 2a^2\sqrt{3} - \frac{4}{3}a^2\pi\right]$$

$$= 2\left[\frac{8}{\sqrt{3}}a^2 + \frac{8a^2\pi}{3} - 2a^2\sqrt{3}\right]$$

$$= \frac{16\sqrt{3}}{3}a^2 + \frac{16a^2\pi}{3} - 4\sqrt{3}a^2$$

$$= \frac{4\sqrt{3}}{3}a^2 + \frac{16a^2\pi}{3}$$

$$= \frac{4}{\sqrt{3}}a^2 + \frac{16a^2\pi}{3}$$

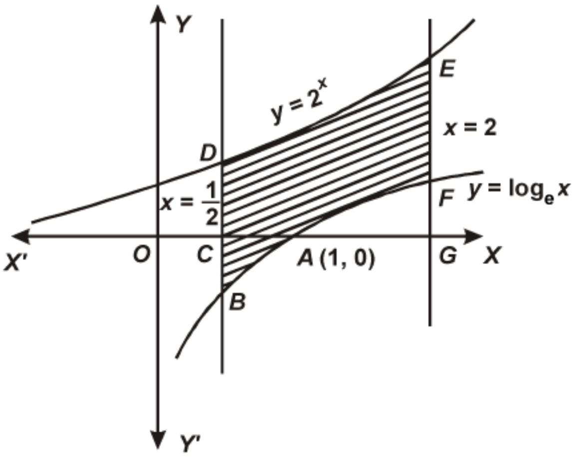
S60. Given curves are $y = \log_e x$ ... (i)

$y = 2^x$ ... (ii)

Given lines are $x = \frac{1}{2}$ and $x = 2$

Hence shaded part is the region bounded by curves (i) and (ii) and the lines $x = \frac{1}{2}$ and $x = 2$.

Required area $BCDEFAB = \int_{\frac{1}{2}}^{2}(y_2 - y_1)\,dx$



$$= \int_{\frac{1}{2}}^{2}\left[y_{\text{curve (ii)}} - y_{curve\,(i)}\right]dx$$

$$= \int_{\frac{1}{2}}^{2}\left(2^x - \log x\right)dx$$

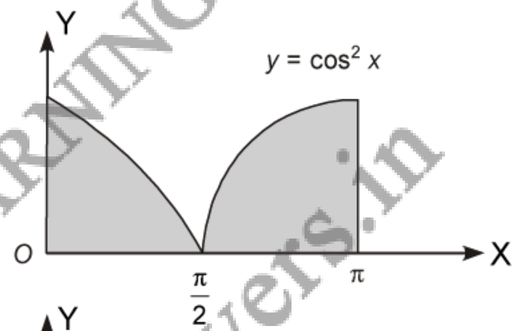
$$= \left[\frac{2^x}{\log 2} - (x\log x - x)\right]_{\frac{1}{2}}^{2}$$

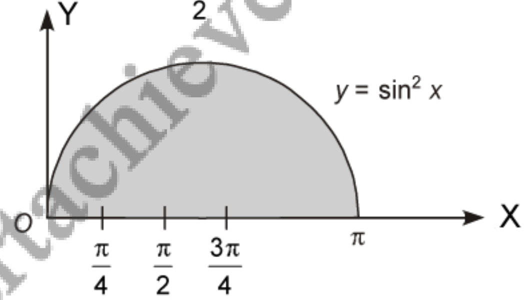
$$= \left(\frac{4}{\log 2} - 2\log 2 + 2\right) - \left(\frac{\sqrt{2}}{\log 2} - \frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\right)$$

$$= \left(\frac{4-\sqrt{2}}{\log 2} + \frac{3}{2} - \frac{5}{2}\log 2\right) \text{ sq. units.} \qquad \left[\because \log\left(\frac{1}{2}\right) = -\log 2\right]$$

S61. Here, we have

$y = \sin^2 x$ and $y = \cos^2 x$





A_1 = Area of curve $\sin^2 x$

$$= \int_0^{\pi} \sin^2 x\,dx = \frac{1}{2}\int_0^{\pi}(1-\cos 2x)\,dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \frac{1}{2}\left(\pi - \frac{\sin 2\pi}{2}\right) = \frac{\pi}{2} \qquad \text{... (i)}$$

and A_2 = Area of curve $\cos^2 x$

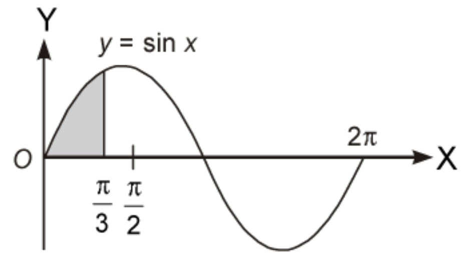
$$= \int_0^{\pi} \cos^2 x\,dx = \frac{1}{2}\int_0^{\pi}[1+\cos 2x]\,dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{2}(\pi + 0) = \frac{\pi}{2} \qquad \text{... (ii)}$$

$$\frac{A_1}{A_2} = \frac{\pi/2}{\pi/2} = \frac{1}{1}$$

$\therefore$ $A_1 : A_2 = 1 : 1$.

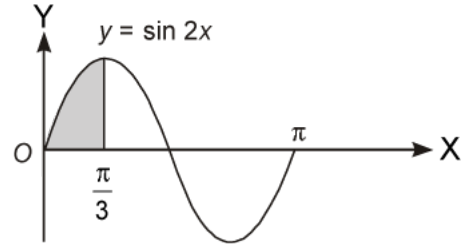
S62. Area Bounded by $x = 0$ and $x = \frac{\pi}{3}$



$$A_1 = \int_0^{\frac{\pi}{3}} \sin x \, dx = \left[-\cos x\right]_0^{\frac{\pi}{3}}$$

$$= \left[-\cos\frac{\pi}{3} + \cos 0\right]$$

$$= \left[-\frac{1}{2} + 1\right] = \frac{1}{2}$$



$$A_2 = \int_0^{\frac{\pi}{3}} \sin 2x \, dx = -\left[\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{3}}$$

$$= -\frac{1}{2}\left[\cos\frac{2\pi}{3} - \cos 0^\circ\right]$$

$$= -\frac{1}{2}\left[-\frac{1}{2} - 1\right] = \frac{3}{4}$$

$$\frac{A_1}{A_2} = \frac{1/2}{3/4} = \frac{2}{3}$$

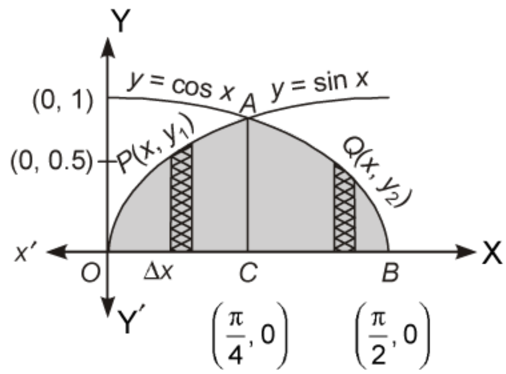
$\therefore \quad A_1 : A_2 = 2 : 3.$

S63. The values of sin x and cos x at different points lying between 0 to $\frac{\pi}{2}$ are given below:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin x	0	0.5	0.716	0.866	1
cos x	1	0.866	0.716	0.5	0

With the help of these points, we draw a graph. These two curves intersect each other at $x = \frac{\pi}{4}$. At the point of intersection, we have

$\sin x = \cos x \quad \Rightarrow \quad x = \frac{\pi}{4}$



Required area = Area *OACO* + Area *ACBA*

$$= \int_0^{\pi/4} y_1 dx + \int_{\pi/4}^{\pi/2} y_2 dx$$

$$= \int_0^{\pi/4} \sin x dx + \int_{\pi/4}^{\pi/2} \cos x dx$$

$$= \left[-\cos x\right]_0^{\pi/4} + \left[-\sin x\right]_{\pi/4}^{\pi/2}$$

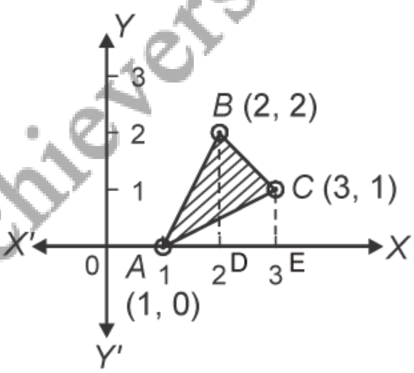
$$\left[\begin{array}{l} \because \; P(x, y_1) \text{ lies on } y = \sin x \text{ and} \\ \quad\; Q(x, y_2) \text{ lies on } y = \cos x \\ \because \; y_1 = \sin x \text{ and } y_2 = \cos x \end{array}\right.$$

$$= \left(-\frac{1}{\sqrt{2}} + 1\right) + \left(1 - \frac{1}{\sqrt{2}}\right) = 2 - \frac{2}{\sqrt{2}} = (2 - \sqrt{2}) \text{ sq. units.}$$

S64. Given vertices of triangle are (1, 0) (2, 2) and (3, 1). First, we plot these vertices to get the required triangular region.

We know that equation of the line passing through point $(x_1, y_1), (x_2, y_2)$ is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



Using above equation, we find the equation of sides of ΔABC.

$\therefore$ Equation of side *AB* whose points are *A* (1, 0) and *B* (2, 2) is

$$y - 0 = \frac{2}{1}(x - 1)$$

$\Rightarrow \qquad y = 2x - 2 \qquad \ldots \text{(i)}$

Equation of side *BC* whose points are *B* (2, 2) and *C* (3, 1) is

$$y - 2 = \frac{-1}{1}(x - 2)$$

$$y - 2 = -x + 2$$

or $\qquad y = -x + 4 \qquad \ldots \text{(ii)}$

Equation of *CA* whose points are *C* (3, 1) and *A* (1, 0) is

$$y - 1 = \frac{-1}{-2}(x - 3)$$

$$\Rightarrow \quad y - 1 = \frac{x-3}{2}$$

$$\Rightarrow \quad y = \frac{x-3}{2} + 1 = \frac{x-1}{2} \qquad \text{... (iii)}$$

$\therefore$ Required area = Area of BAD + Area of $BDEC$ – Area of AEC

$= \int_1^2 (y\,dx$ from equation of $AB + \int_2^3 (y\,dx$ from equation of $BC) - \int_1^3 (y\,dx$ from equation of $CA)$

$$= \int_1^2 (2x-2)dx + \int_2^3 (4-x)dx - \int_1^3 \frac{x-1}{2}dx$$

$$= [x^2 - 2x]_1^2 + \left[4x - \frac{x^2}{2}\right]_2^3 - \left[\frac{x^2}{4} - \frac{x}{2}\right]_1^3$$

$$= [(4-4)-(1-2)] + \left[\left(12 - \frac{9}{2}\right) - (8-2)\right] - \left[\left(\frac{9}{4} - \frac{3}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= 0 + 1 + \left[\frac{15}{2} - 6\right] - \left[\frac{3}{4} + \frac{1}{4}\right] = 1 + \frac{15}{2} - 6 - 1$$

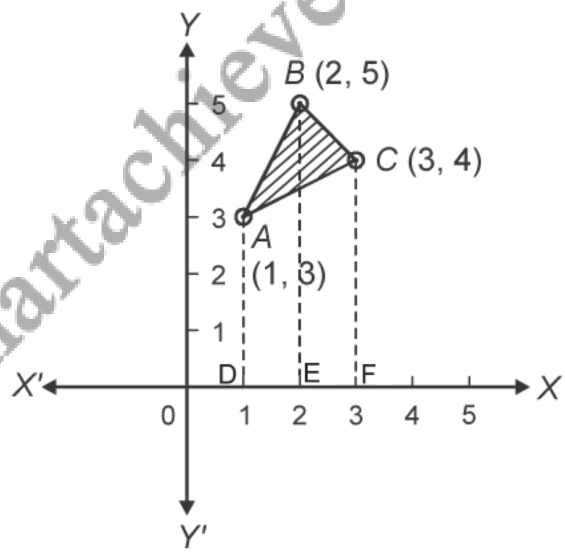
$$= \frac{15}{2} - 6 = \frac{3}{2} \text{ sq. units.}$$

S65. Given, vertices are (1, 3) (2, 5) and (3, 4).

First, we plot these vertices and get the required triangular region as follows.

We know that equation of line passing through two points, (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



Using above equation, we find equation of sides of triangle.

$\therefore$ Equation of side AB whose points are $A(1, 3)$ and $B(2, 5)$; is

$$y - 3 = \frac{2}{1}(x - 1)$$

$$\Rightarrow \quad y = 2x - 2 + 3$$

$$\text{or} \quad y = 2x + 1 \qquad \text{... (i)}$$

Equation of side BC whose points are $B(2, 5)$ and $C(3, 4)$ is

$$y - 5 = \frac{-1}{1}(x - 2)$$

$$\Rightarrow \quad y - 5 = -x + 2$$

or $\quad y = 7 - x \quad$ ... (ii)

Equation of side *CA* whose points are *C* (3, 4) and *A* (1, 3) is

$$y - 4 = \frac{-1}{-2}(x - 3)$$

$$\Rightarrow \quad y - 4 = \frac{x - 3}{2}$$

$$\Rightarrow \quad y = \frac{x - 3}{2} + 4$$

$$\Rightarrow \quad y = \frac{x + 5}{2} \quad \text{... (iii)}$$

Now, Required area = Area *ADEBA* + Area *BEFC* – Area *ADFC*

$= \int_1^2 (y\,dx$ from equation of $AB) + \int_2^3 (y\,dx$ from equation of $BC) - \int_1^3 (y\,dx$ from equation of $CA)$

$$= \int_1^2 (2x + 1)dx + \int_2^3 (7 - x)dx - \int_1^3 \frac{x + 5}{2}dx$$

$$= \left[\frac{2x^2}{2} + x\right]_1^2 + \left[7x - \frac{x^2}{2}\right]_2^3 - \left[\frac{x^2}{4} + \frac{5x}{2}\right]_1^3$$

$$= \left[x^2 + x\right]_1^2 + \left[7x - \frac{x^2}{2}\right]_2^3 - \left[\frac{x^2}{4} + \frac{5x}{2}\right]_1^3$$

$$= [(4 + 2) - (1 + 1)] + \left[\left(21 - \frac{9}{2}\right) - (14 - 2)\right] - \left[\left(\frac{9}{4} + \frac{15}{2}\right) - \left(\frac{1}{4} + \frac{5}{2}\right)\right]$$

$$= 4 + \left(9 - \frac{9}{2}\right) - (2 + 5) = 4 + \frac{9}{2} - 7$$

$$= \frac{8 + 9 - 14}{2} = \frac{3}{2} \text{ sq. units.}$$