Date: 18/10/2021

Q1. Find $A - B$, if

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}.$$

Q2. Find X, if

$$X + \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$$

Q3. For a 2×2 matrix, $A = [a_{ij}]$ whose elements are given by

$a_{ij} = \frac{i}{j}$, write the value of a_{12}.

Q4. Construct a 2×2 matrix whose element a_{ij} is given by $a_{ij} = i + 2j$.

Q5. Construct *a* 2×3 matrix whose elements are given by

$$a_{ij} = 2i - 3j$$

Q6. Construct a 2×3 matrix whose elements are given by

$$a_{ij} = i + j$$

Q7. Find x and y, if

$$\begin{bmatrix} x + 2y & 3y \\ 4x & 2 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 2 \end{bmatrix}$$

Q8. Find the values of x and y, if

$$\begin{bmatrix} x + 2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix},$$

Q9. Find the values of x and y, if

$$\begin{bmatrix} x + 3y & y \\ 7 - x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix},$$

Q10. Find AB, if

$$A = \begin{bmatrix} 0 & -3 \\ 0 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 6 \\ 0 & 0 \end{bmatrix}.$$

Q11. Find the value of y, if

$$\begin{bmatrix} y + 2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$$

Q12. Find the value of a, if

$$\begin{bmatrix} a - b & 2a + c \\ 2a - b & 3c + d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Q13. Find the matrix A. If

$$\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}.$$

Q14. Find the value of y. If

$$\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}.$$

Q15. If A = diag (1 –1 2) and B = diag (2 3 –1), find $3A + 4B$.

Q16. If A = diag (1 – 1 2) and B = diag (2 3 – 1), find $A + B$.

Q17. Find X, if

$$Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{and} \quad 2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}.$$

Q18. Find $3A - 2B$, if

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 6 \\ 5 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 & 5 \\ 5 & 3 & 2 \\ 0 & 4 & 7 \end{bmatrix}.$$

Q19. Write the order of AB and BA. If

$$A = [2\ 6\ 3\ 5] \quad \text{and} \quad B = \begin{bmatrix} 1 \\ 5 \\ 4 \\ 3 \end{bmatrix}$$

Q20. Write whether both AB and BA exist. If they exist then write the order. If

$$A = \begin{bmatrix} 5 & 4 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 5 & 1 \\ 6 & 8 & 4 \end{bmatrix}$$

Q21. If a matrix has 18 elements, what are the possible order it can have? What, if it has 5 elements?

Q22. If A is a matrix of order 3×4 and B is a matrix of order 4×3, find order of matrix (AB).

Q23. If a matrix has 24 elements, what are the possible orders it can have ? What, if it has 13 elements ?

Q24. If A is a square matrix and satisfies the relation $A^2 + A - I = 0$ then find A^{-1}.

Q25. Find the values of x and y when

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Q26. Find the matrix A. If

$$3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

Q27. Write the value of k. If

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$$

Q28. Find x from the matrix equation

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}.$$

Q29. Construct a 2×2 matrix whose elements are given by

$$a_{ij} = \begin{cases} i - j, i \geq j \\ i + j, i < j \end{cases}$$

Q30. Find x and y, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

Q31. Find the values of x and y, if

$$2\begin{bmatrix} -1 & 2 \\ 3 & x \end{bmatrix} + \begin{bmatrix} y & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 7 & 13 \end{bmatrix}$$

Q32. Construct a 2×2 matrix whose elements a_{ij} are given by :

$$a_{ij} = \frac{|-3i + j|}{2}$$

Q33. Construct a 2×3 matrix $A = [a_{ij}]$ whose elements are given by

$$a_{ij} = \frac{i - j}{i + j}.$$

Q34. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by

$$a_{ij} = \frac{(i + 2j)^2}{2}.$$

Q35. Construct a 2×2 matrix $A = [a_{ij}]$, whose elements are

$$a_{ij} = \frac{(i + j)^2}{2}$$

Q36. Find the value of y, if

$$\begin{bmatrix} x - y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$$

Q37. Find the value of x, if

$$\begin{bmatrix} 2x - y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}.$$

Q38. Find the value of x, if

$$\begin{bmatrix} 15 & x + y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x - y & 3 \end{bmatrix},$$

Q39. Find the value of *x*, if

$$\begin{bmatrix} x+y & 1 \\ 2y & 5 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix}$$

Q40. Find the value of *x*, if

$$\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix},$$

Q41. Find the value of *y*, if

$$\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}.$$

Q42. Find the value of *y*, if

$$\begin{bmatrix} 2x & 1 \\ 5 & x+2y \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 0 \end{bmatrix}$$

Q43. Find the value of *x*, if

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Q44. Find matrices *X* and *Y*, if

$$X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \quad \text{and} \quad X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

Q45. Find *X* and *Y*, if

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Q46. Find the value of *A* + *B* and *A* – *B*, if

$$A = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 \end{bmatrix}.$$

Q47. Find 3*A* – 2*B*, if

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 4 \\ 7 & 2 \end{bmatrix}$$

Q48. Find the matrix X such that 2*A* + *B* + *X* = 0, where

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

Q49. Find a matrix *A*, if

$$A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}.$$

Q50. Find x, y, z, t, if

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Q51. Find a matrix A such that $2A - 3B + 5C = 0$, where

$$B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

Q52. Find matrix X of order 3×2 such that $2A + 3X = 5B$. If

$$A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}.$$

Q53. Find the value of $x + y$ from the following equation:

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 1 & 7 \end{bmatrix}.$$

Q54. From the following matrix equation, find the value of x.

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$

Q55. Find x, y, z, t, if

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}.$$

Q56. Solve the matrix equation

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

Q57. Find the values of x and y from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

Q58. Find the values of x, y and z from the following equation:

$$\begin{bmatrix} x+y & 2 \\ 5+z & x-y \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 9 & 4 \end{bmatrix}$$

Q59. Write the value of x. If

$$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Q60. A matrix has 12 elements. What are the possible orders it can have?

Q61. If a martrix has 8 elements, what are the possible orders it can have? what if it has 5 elements?

Q62. Write the order of product matrix

$$\begin{bmatrix}1\\2\\3\end{bmatrix}[2\ \ 3\ \ 4].$$

Q63. Find x, y, z if the product of two matrices of orders $(x + 1) \times (2y + 3)$ and $(2z - 3) \times (3x - z)$ has order 5×6.

Q64. Write the value of λ. If matrix

$$A = \begin{bmatrix}3 & -3\\-3 & 3\end{bmatrix} \text{ and } A^2 = \lambda A, \text{ then}$$

Q65. Write the value of p. If matrix

$$A = \begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix} \text{ and } A^2 = pA.$$

Q66. Find A^2. If

$$A = \begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$$

Q67. Write the value of k. If matrix

$$A = \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} \text{ and } A^2 = kA.$$

Q68. Find the values of k, a and b. If

$$A = \begin{bmatrix}0 & 2\\3 & -4\end{bmatrix} \text{ and } kA = \begin{bmatrix}0 & 6a\\3b & -16\end{bmatrix}$$

Q69. Find value of x. If

$$\begin{bmatrix}3 & 4\\2 & x\end{bmatrix}\begin{bmatrix}x\\1\end{bmatrix} = \begin{bmatrix}19\\15\end{bmatrix}$$

Q70. Simplify:

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}$$

Q71. Find the values of α for which $A^2 = B$. If

$$A = \begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix} \text{ and } B = \begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}.$$

Q72. Prove that the product of matrices

$\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\\cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\\cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$ is the null matrix, when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

Q73. Find a matrix D such that $CD - AB = 0$. If

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}.$$

Q74. Find a and b. If

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \quad \text{and} \quad (A + B)^2 = A^2 + B^2$$

Q75. Prove that $(A + B)^2 \neq A^2 + 2AB + B^2$. If

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}.$$

Q76. If A, B, C are three matrices such that $A = [x \quad y \quad z]$

$$B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}, \quad C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \text{find } A(BC).$$

Q77. Show that $F(x)\,F(y) = F(x + y)$. If

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Q78. Find x and y such that $(xI + yA)^2 = A$. If

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Q79. Let

$A = \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix}$ and I be the identity matrix of order 2. show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

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S1. Given that, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$\therefore \quad A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

S2. Here $X + \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$

$$\therefore \quad X = \begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}..$$

S3. Here element a_{12} denotes the element of first row corresponding to second column.

Given that for a 2×2 matrix

$$A = [a_{ij}] \qquad a_{ij} = \frac{i}{j}$$

To find a_{12}, put $i = 1$ and $j = 2$, we get

$$a_{12} = \frac{1}{2}$$

S4. Here $a_{11} = 3 \quad a_{12} = 1 + 2 \times 2 = 1 + 4 = 5$

$a_{21} = 2 + 2 \times 1 = 2 + 2 = 4 \qquad a_{22} = 2 + 2 \times 2 = 2 + 4 = 6$

The required matrix is $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

S5. Here $a_{ij} = 2i - 3j$

$\therefore$ $a_{11} = 2 \times 1 - 3 \times 1 = -1$; $a_{12} = 2 \times 1 - 3 \times 2 = -4$; $a_{13} = 2 \times 1 - 3 \times 3 = -7$

$a_{21} = 2 \times 2 - 3 \times 1 = 1$; $a_{22} = 2 \times 2 - 3 \times 2 = -2$; $a_{23} = 2 \times 2 - 3 \times 3 = -5$

$$\therefore \quad A = \begin{bmatrix} -1 & -4 & -7 \\ 1 & -2 & -5 \end{bmatrix}_{2\times 3}$$

S6. Let $[A]_{2\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}$

Here $a_{ij} = i + j$

$\therefore$ $a_{11} = 1 + 1 = 2;\quad a_{12} = 1 + 2 = 3;\quad a_{13} = 1 + 3 = 4;$

$a_{21} = 2 + 1 = 3;\quad a_{22} = 2 + 2 = 4;\quad a_{23} = 2 + 3 = 5.$

$\therefore$ $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}_{2\times 3}$

S7. Given matrix equation is

$$\begin{bmatrix} x+2y & 3y \\ 4x & 2 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 2 \end{bmatrix}$$

Equating the corresponding elements, we get

$3y = -3 \quad \Rightarrow \quad y = \frac{-3}{3} = -1$

and $4x = 8 \quad \Rightarrow \quad x = \frac{8}{4} = 2$

$\therefore$ $x = 2$ and $y = -1$

S8. Here $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$

$-y = 3$ and $3x = 6$

$y = -3$ and $x = 2$

S9. Here $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$

$\Rightarrow$ $y = -1$ and $7 - x = 0$

$\Rightarrow$ $y = -1$ and $x = 7$.

S10. Here $A = \begin{bmatrix} 0 & -3 \\ 0 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 0 & 0 \end{bmatrix}$

$\therefore$ $AB = \begin{bmatrix} 0 & -3 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

S11. Given, $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$

Equating the corresponding elements, we have

$y + 2x = 7$ … (i)

and $\qquad -x = -2 \qquad \Rightarrow \qquad x = 2$

Substituting $x = 2$ in Eq. (i), we have

$$y + 4 = 7 \qquad \Rightarrow \qquad y = 3$$

S12. Two matrices are equal, if its corresponding elements are equal

$$a - b = -1 \qquad \dots \text{(i)}$$

$$2a - b = 0 \qquad \dots \text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$-a = -1$$

$$a = 1$$

S13. Given equation can be rewritten as

$$A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

Two matrices can be subtracted only when their order are same.

$$A = \begin{bmatrix} 9-1 & -1-2 & 4+1 \\ -2-0 & 1-4 & 3-9 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

S14. Given, $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$

On comparing, we have

$$x = 3 \quad \text{and} \quad x - y = 1$$

$$\Rightarrow \qquad y = x - 1 = 3 - 1 = 2$$

S15. We have,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\therefore \qquad 3A + 4B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \text{diag}\,(11 \quad 9 \quad 2)$$

S16. We have,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\therefore \qquad A + B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{diag } (3 \quad 2 \quad 1)$$

S17. We have,

$$Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{and} \quad 2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\therefore \qquad 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow \qquad 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow \qquad X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

S18. We have,

$$3A - 2B = 3A + (-2)\,B$$

$$\Rightarrow \qquad = \begin{bmatrix} 6 & 9 & 12 \\ 0 & 12 & 18 \\ 15 & 24 & 27 \end{bmatrix} + \begin{bmatrix} -6 & 0 & -10 \\ -10 & -6 & -4 \\ 0 & -8 & -14 \end{bmatrix} = \begin{bmatrix} 0 & 9 & 2 \\ -10 & 6 & 14 \\ 15 & 16 & 13 \end{bmatrix}$$

S19. Here order of matrix A is 1×4 and order of matrix B is 4×1.

Now number of columns in A = number of rows in B. So AB exists and order of AB is 1×1.

Also number of columns in B = number of rows in A. So BA exists and order of BA is 4×4.

S20. Since order of matrix A is 2×2 and order of matrix B is 2×3.

Now number of columns in A = number of rows in B. So AB exists and order of AB is 2×3.

Also number of columns in $B \neq$ number of rows in A. So BA does not exist.

S21. The possible orders of the matrix when it has 18 elements are 1×18; 2×9; 3×6; 6×3; 9×2; 18×1.

The possible order of the matrix when it has 5 elements are 1×5, 5×1.

S22. Order of matrix AB = 3×3

[$\because$ If a matrix A has order $x \times y$ and B has order $y \times z$, then matrix AB has order $x \times z$]

S23. The possible order of the matrix when it has 24 elements are 1×24; 2×12; 3×8; 4×6; 6×4; 8×3; 12×2; 24×1.

The possible orders of the matrix when it has 13 elements are 1×13, 13×1.

S24. Here $A^2 + A - I = 0$

Pre multiplication by A^{-1} on both sides

$$A^{-1}(A^2 + A - \text{I}) = A^{-1} \times 0$$

$\Rightarrow$ $A^{-1} A^2 + A^{-1} A - A^{-1} = 0$

$\Rightarrow$ $A + I - A^{-1} = 0$

$\Rightarrow$ $A^{-1} = A + I.$

S25. $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\Rightarrow$ $2x - 3y = 1$ and $x + y = 3.$

Now solving $2x - 3y = 1$ and $x + y = 3,$

we get, $x = 2$ and $y = 1.$

S26. Here, $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow 3A = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + B$

$$= \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix} = 3\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$\Rightarrow$ $3A = 3\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow$ $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

S27. Given that, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 3+4 & 1+10 \\ 9+8 & 3+20 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix} \Rightarrow k = 17$

S28. Firstly we calculate the multiplication of matrix in LHS and then equate the corresponding elements on both sides.

Given matrix equation is $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

$\Rightarrow \qquad \begin{bmatrix} x+6 \\ 4x+10 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

Equating the corresponding elements, we get

$x + 6 = 5$

$\Rightarrow \qquad x = -1.$

S29. Let $[A]_{2\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$

Here $a_{ij} = \begin{cases} i-j, i \ge j \\ i+j, i < j \end{cases}$

$\therefore \qquad a_{11} = 1 - 1 = 0; \qquad a_{12} = 1 + 2 = 3$

$a_{21} = 2 - 1 = 1; \qquad a_{22} = 2 - 2 = 0$

$\therefore \qquad A = \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}_{2\times2}$

S30. Given matrix equation is

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \qquad \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Equating the corresponding elements, we get

$2 + y = 5 \qquad \Rightarrow \qquad y = 3$

and $\qquad 2x + 2 = 8$

$\therefore \qquad 2x = 6$

$\therefore \qquad x = 3$

and $\qquad y = 3$

S31. $2\begin{bmatrix} -1 & 2 \\ 3 & x \end{bmatrix} + \begin{bmatrix} y & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 7 & 13 \end{bmatrix}$

$\Rightarrow \qquad \begin{bmatrix} -2 & 4 \\ 6 & 2x \end{bmatrix} + \begin{bmatrix} y & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 7 & 13 \end{bmatrix}$

$\Rightarrow \qquad \begin{bmatrix} -2+y & 2 \\ 7 & 2x+5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 7 & 13 \end{bmatrix}$

$\Rightarrow \qquad -2 + y = 1 \quad \text{and} \quad 2x + 5 = 13$

$\Rightarrow \qquad y = 1 + 2 \quad \text{and} \quad 2x = 13 - 5$

$\Rightarrow \qquad y = 3 \quad \text{and} \quad x = 4.$

S32. Let $A = [a_{ij}]$ be a 2×2 matrix that

$$a_{ij} = \frac{|-3i + j|}{2} \text{ .Then,}$$

$$a_{11} = \frac{|-3+1|}{2} = 1, a_{12} = \frac{|-3+2|}{2} = \frac{1}{2}$$

$$a_{21} = \frac{|-6+1|}{2} = \frac{5}{2}, a_{22} = \frac{|-6+2|}{2} = 2$$

$$\therefore \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 5/2 & 2 \end{bmatrix}$$

S33. We have $\quad a_{ij} = \frac{i-j}{i+j}, 1 \le i \le 2 \text{ and } 1 \le j \le 3.$

Therefore, $\quad a_{11} = 0,\ a_{12} = \frac{1}{3},\ a_{13} = -\frac{1}{2},\ a_{21} = \frac{1}{3},\ a_{22} = 0 \text{ and } a_{23} = -\frac{1}{5}.$

So the required matrix is $A = \begin{bmatrix} 0 & \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{3} & 0 & -\frac{1}{5} \end{bmatrix}$

S34. Here $a_{ij} = \frac{(i+2j)^2}{2}, 1 \le i \le 2$ and $1 \le j \le 2$. Therefore,

$$a_{11} = \frac{(1+2\times1)^2}{2} = \frac{9}{2},\ a_{12} = \frac{(1+2\times2)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2\times1)^2}{2} = 8 \quad \text{and} \quad a_{22} = \frac{(2+2\times2)^2}{2} = 18$$

So the required matrix is $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

S35. Let $\quad [A]_{2\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$

Here $\quad a_{ij} = \frac{(i+j)^2}{2}$

$\therefore \quad a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2; \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}; \quad a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

$\therefore \quad A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}_{2\times 2}$

S36. Given that $\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$

Equating the corresponding elements, we get

$$x - y = 2 \quad \text{... (i)}$$

and $\quad x = 3 \quad$ ... (ii)

Substituting $x = 3$ in Eq. (i), we get

$$3 - y = 2$$

$\Rightarrow \quad -y = -1$

or $\quad y = 1$

S37. Given matrix equation is

$$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$$

Equating the corresponding elements, we get

$$2x - y = 6 \quad \text{... (i)}$$

and $\quad y = -2$

Substituting $y = -2$ in Eq. (i), we get

$$2x + 2 = 6$$

$$2x = 4$$

or $\quad x = 2$

S38. Given, $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$

Equating the corresponding elements, we have

$$x + y = 8 \quad \text{... (i)}$$

$$y = 3$$

Substituting $y = 3$ in Eq. (i), we have

$$x + 3 = 8$$

$$\Rightarrow \quad x = 5$$

S39. Given matrix equation is $\begin{bmatrix} x+y & 1 \\ 2y & 5 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix}$

Equating the corresponding elements, we get

$$x + y = 7 \quad \text{... (i)}$$

and $$2y = 4 \quad \text{... (ii)}$$

From Eq. (ii), we get

$$y = \frac{4}{2} = 2$$

Putting the value of y in Eq. (i), we get

$$x + 2 = 7$$

$$\therefore \quad x = 5$$

S40. Given matrix equation is $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$

Equating the corresponding elements, we get

$$2x + y = 6 \quad \text{... (i)}$$

and $$3y = 0 \quad \text{... (ii)}$$

From Eq. (ii), we get

$$y = \frac{0}{3} = 0$$

Putting $y = 0$ in Eq. (i), we get

$$2x = 6$$

$$\therefore \quad x = 3$$

S41. Given matrix equation is $\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$

Equating the corresponding elements, we get

$$3y - x = 5 \quad \text{... (i)}$$

and $$-2x = -2 \quad \text{... (ii)}$$

From Eq. (ii), we get $x = \frac{-2}{2} = 1$

Putting $x = 1$ in Eq. (i), we get

$$3y - 1 = 5$$

or $$3y = 6$$

$$y = \frac{6}{3}$$

$\therefore$ $$y = 2$$

S42. Given that, $\begin{bmatrix} 2x & 1 \\ 5 & x+2y \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 0 \end{bmatrix}$

Equating the corresponding elements, we get

$$2x = 4 \quad \text{....(i)}$$

and $$x + 2y = 0 \quad \text{.... (ii)}$$

From Eq. (i), we get

$$x = \frac{4}{2} = 2$$

Putting $x = 2$ in Eq. (ii), we get

$$2 + 2y = 0$$

$\Rightarrow$ $2y = -2$ or $y = \frac{-2}{2}$

$\therefore$ $y = -1$

S43. Given matrix equation is

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Equating the corresponding elements, we get

$$3x + y = 1 \quad \text{... (i)}$$

and $$-y = 2 \quad \text{... (ii)}$$

From Eq. (ii), we get $y = -2$

Putting y = – 2 in Eq. (i), we get

$$3x - 2 = 1$$

$$3x = 3$$

$\therefore$ $$x = 1$$

S44. We have, $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$$\Rightarrow \quad (X + Y) + (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{2}\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

and, $(X + Y) - (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$$\Rightarrow \quad 2Y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$Y = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow \quad Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

S45. We have,

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \text{and} \quad X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\therefore \quad (X + Y) + (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow \quad 2X = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix}$$

$$\Rightarrow \quad 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

and, $(X + Y) - (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

$$\Rightarrow \quad 2Y = \begin{bmatrix} 7-3 & 0+0 \\ 2+0 & 5-3 \end{bmatrix}$$

$\Rightarrow \quad 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} \Rightarrow Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Thus, $\quad X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

S46. Clearly, A and B both are matrices of the same order 2×3. So, $A + B$ and $A - B$ both are defined.

Now, $\quad A + B = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 1 \\ -2 & 7 & 3 \end{bmatrix}$

$\Rightarrow \quad A + B = \begin{bmatrix} 2+0 & 3+5 & -5+1 \\ 1-2 & 2+7 & -1+3 \end{bmatrix} = \begin{bmatrix} 2 & 8 & -4 \\ -1 & 9 & 2 \end{bmatrix}$

$\Rightarrow \quad A - B = A + (-B) = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -5 & -1 \\ 2 & -7 & -3 \end{bmatrix}$

$= \begin{bmatrix} 2+0 & 3-5 & -5-1 \\ 1+2 & 2-7 & -1-3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -6 \\ 3 & -5 & -4 \end{bmatrix}$

S47. We have,

$3A = \begin{bmatrix} 6 & -3 \\ 9 & 3 \end{bmatrix}$ and $(-2)\,B = \begin{bmatrix} -2 & -8 \\ -14 & -4 \end{bmatrix}$

$\therefore \quad 3A - 2B = 3A + (-2)\,B = \begin{bmatrix} 6 & -3 \\ 9 & 3 \end{bmatrix} + \begin{bmatrix} -2 & -8 \\ -14 & -4 \end{bmatrix}$

$\Rightarrow \quad 3A - 2B = \begin{bmatrix} 6+(-2) & -3+(-8) \\ 9+(-14) & 3+(-4) \end{bmatrix} = \begin{bmatrix} 4 & -11 \\ -5 & -1 \end{bmatrix}$

S48. We have,

$2A + B + X = 0$

$\Rightarrow \quad X = -2A - B$

$\Rightarrow \quad X = -2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

$\Rightarrow \quad X = \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ -1 & -5 \end{bmatrix}$

$\Rightarrow \quad X = \begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix}$

$$\Rightarrow \qquad X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}.$$

S49. Let $B = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$. Then, the given matrix equation is $A + B = C$.

Now, $\quad A + B = C$

$\Rightarrow \quad (A + B) + (-B) = C + (-B)$

$\Rightarrow \quad A + (B + (-B)) = C + (-B)$

$\Rightarrow \quad A + O = C - B$

$\Rightarrow \quad A = C - B.$

$$= \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -6-3 \\ -3+1 & 8-4 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix}$$

S50. The given matrix equation can be written as

$$\begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$\Rightarrow \quad 2x + 3 = 9. \quad 2z - 3 = 15, \quad 2y = 12 \quad$ and $\quad 2t + 6 = 18$

$\Rightarrow \quad x = 3, \quad z = 9, \quad y = 6 \quad$ and $\quad t = 6.$

S51. We have,

$2A - 3B + 5C = 0$

$\Rightarrow \quad 2A = 3B - 5C$

$$\Rightarrow \quad 2A = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$\Rightarrow \quad 2A = \begin{bmatrix} -6-10 & 6+0 & 0+10 \\ 9-35 & 3-5 & 12-30 \end{bmatrix}$$

$$\Rightarrow \quad 2A = \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$\Rightarrow \quad A = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$\Rightarrow \qquad A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}.$$

S52. We have,

$$A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

and, $\qquad 2A + 3X = 5B.$

$$\Rightarrow \qquad 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} + 3X = 5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix} + 3X = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix}$$

$$\Rightarrow \qquad 3X = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} - \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix} = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow \qquad X = \begin{bmatrix} -2 & -10/3 \\ 4 & 14/3 \\ -31/3 & -7/3 \end{bmatrix}$$

S53. The given equation can be written as

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 1 & 7 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 1 & 7 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 1 & 7 \end{bmatrix}$$

On comparing, we have

$$2 + y = 3 \quad \Rightarrow \quad y = 1$$
$$2x + 2 = 7 \quad \Rightarrow \quad x = 5/2$$

Thus, $\qquad x + y = 5/2 + 1 = 7/2$

S54. If two matrices are equal then their corresponding elements are equal.

Given matrix equation is $\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$

Equating the corresponding elements, we get

$$x + y = 3 \quad \dots \text{(i)}$$

and $$3y = 6 \quad \dots \text{(ii)}$$

From Eq. (ii), we get

$$y = \frac{6}{3} = 2$$

Putting $y = 2$ in Eq. (i), we get

$$x + 2 = 3$$

$\therefore$ $$x = 1$$

S55.

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & x+y+6 \\ -1+z+t & 2t+3 \end{bmatrix}$$

$$\Rightarrow 3x = x + 4,\ 3y = x + y + 6,\ 3z = -1 + z + t,\ 3t = 2t + 3$$

$$\Rightarrow 2x = 4,\ x - 2y + 6 = 0,\ 2z - t + 1 = 0,\ t = 3$$

$$\Rightarrow x = 2,\ y = 4,\ z = 1,\ t = 3.$$

S56. We have, $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - \begin{bmatrix} 3x \\ 6y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2 - 3x \\ y^2 - 6y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

$$\Rightarrow x^2 - 3x = -2 \text{ and } y^2 - 6y = 9$$

$$\Rightarrow x^2 - 3x + 2 = 0 \text{ and } y^2 - 6y - 9 = 0$$

$$\Rightarrow (x - 1)(x - 2) = 0 \text{ and } y = \frac{6 \pm \sqrt{36 + 36}}{2}$$

$$\Rightarrow x = 1, 2 \text{ and } y = 3 \pm 3\sqrt{2}.$$

S57. We have,

$$\Rightarrow 2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x+3 & 14 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \quad 2x + 3 = 7,\ 2y - 4 = 14$$

$$\Rightarrow \quad x = 2,\ y = 9$$

S58. Here $\begin{bmatrix} x+y & 2 \\ 5+z & x-y \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 9 & 4 \end{bmatrix}$

$$\therefore \quad x + y = 6,\ x - y = 4 \quad \text{and} \quad 5 + z = 9$$

$$\Rightarrow \quad x + y + x - y = 6 + 4 \quad \text{and} \quad z = 9 - 5$$

$$\Rightarrow \quad 2x = 10 \quad \text{and} \quad z = 4$$

$$\Rightarrow \quad x = 5$$

Putting value of x in $x + y = 6$, we get

$$5 + y = 6$$

$$\Rightarrow \quad y = 6 - 5 = 1$$

Thus $x = 5$, $y = 1$ and $z = 4$.

S59. We have, $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

On comparing, we have

$$2x - y = 10,\ \ 3x + y = 5$$

On solving, we get

$$x = 3.$$

S60. We know that if a matrix is of order $m \times n$, then it has mn elements. Therefore, to find all possible orders of matrix with 12 elements, we will have to find all ordered pairs (a, b) such that a and b are factors of 12.

Clearly, all possible ordered pairs of this type are

(1, 12), (12, 1), (3, 4), (4, 3), (2, 6), (6, 2)

Hence possible order of the matrix are:

1×12, 12×1, 3×4, 4×3, 2×6 and 6×2.

S61. We know that if a matrix is of order $m \times n$, then it has mn elements.Therefore, to find all possible orders of matrix with 8 elements, we will have to find all ordered pairs (a, b) such that a and b are factors of 8. Clealry, all possible ordered pairs of this type are (1, 8), (8, 1), (2, 4), (4, 2).

Hence, possible orders of the matrix are 1×8, 8×1, 2×4, 4×2,

If a matrix has 5 elements, then its possible orders are 1×5 and 5×1.

S62. Use the fact that if a matrix A has order $m \times n$ and other matrix B has order $n \times z$, then the matrix AB has order $m \times z$ that means if number of columns of matrix A must be same as number of rows of matrix B then matrix multiplication AB is possible.

Let $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

and $B = [2 \ \ 3 \ \ 4]$

order of matrix $A = 3 \times 1$

and order of matrix $B = 1 \times 3$

$\therefore$ order of matrix $AB = 3 \times 3$.

S63. For the product to be defined, the number of columns in first matrix should be equal to the number of rows of the second.

$\Rightarrow \quad 2y + 3 = 2z - 3$

$\Rightarrow \quad 2(z - y) = 6$

$\Rightarrow \quad z - y = 3$ ...(i)

Also, the product has same number of rows as the first matrix.

$\Rightarrow \quad x + 1 = 5$

$\Rightarrow \quad x = 4$ ..(ii)

The product has same number of columns as the second matrix.

$\Rightarrow \quad 3x - z = 6$

$\Rightarrow \quad 3 \times 4 - z = 6$ [using (ii)]

$\Rightarrow \quad 12 - z = 6$

$\Rightarrow \quad z = 6$ ...(iii)

Using Eq. (iii) and Eq. (i),

$6 - y = 3$

$\Rightarrow \quad y = 3$

Thus, $x = 4$, $y = 3$, $z = 6$.

S64.
Given matrix, $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ ... (i)

Also, $A^2 = \lambda A$ ... (ii)

Now, $A^2 = A \cdot A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 9+9 & -9-9 \\ -9-9 & 9+9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 6\cdot 3 & -6\cdot 3 \\ -6\cdot 3 & 6\cdot 3 \end{bmatrix} = 6\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$

$\Rightarrow$ $\lambda A = 6A$ [From Eqs. (i) and (ii)]

$\therefore$ $\lambda = 6$

S65. Given that,

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \quad \text{... (i)}$$

and $A^2 = pA$ ... (ii)

Now, we have

$$A^2 = A \times A$$

$$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$= 4\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$\Rightarrow$ $A^2 = 4 \times A$ [$\because$ From Eq. (i)]

On comparing with Eq. (ii), we get

$$p = 4$$

S66. Here $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Then $A^2 = A \cdot A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \sin\theta\cos\theta + \cos\theta\sin\theta \\ -\sin\theta\cos\theta - \sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

S67. Given that, $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$

Now, $A^2 = A \cdot A$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$\Rightarrow$ $A^2 = 2A$

On comparing with $A^2 = kA$, we get

$$k = 2$$

S68. Here, $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 6a \\ 3b & -16 \end{bmatrix}$

Now $kA = k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

$\therefore$ $\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 6a \\ 3b & -16 \end{bmatrix}$

$\therefore$ $-4k = -16,\ 2k = 6a$ and $3k = 3b$

$\therefore$ $k = 4,\ 6a = 2 \times 4$ and $b = k$

$$a = \frac{8}{6} = \frac{4}{3} \text{ and } b = 4$$

Thus $k = 4,\ a = \frac{4}{3}$ and $b = 4$.

S69. Given matrix equation is $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} 3x+4 \\ 2x+x \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$$

Equating the corresponding elements, we get

$$3x + 4 = 19$$

and $\quad 2x + x = 15$

$\Rightarrow \quad 3x = 15$

and $\quad 3x = 15$

or $\quad x = 5.$

S70. Firstly we multiply $\cos\theta$ and $\sin\theta$ inside the each element of the matrix and then using the property of matrix addition we get

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$= I$ = unit matrix

S71. We have,

$$A^2 = B$$

$$\Rightarrow \quad \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \alpha^2+0 & 0+0 \\ \alpha+1 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\Rightarrow \quad \alpha^2 = 1$ and $\alpha + 1 = 5$

$\Rightarrow \quad \alpha = \pm 1$ and $\alpha = 4$, which is not possible.

Hence, there is no value of α for which $A^2 = B$ is true.

S72. We have, $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\cos\phi\sin\theta\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos^2\phi\cos\theta\sin\theta + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\phi-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \left[\because \theta-\phi = (2n+1)\frac{\pi}{2}, n \in Z \therefore \cos(\theta-\phi) = \cos(2n+1)\frac{\pi}{2} = 0\right]$$

S73. Let $D = \begin{bmatrix} a & b \\ x & y \end{bmatrix}$. Then,

$$CD - AB = 0$$

$$\Rightarrow \quad CD = AB$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ x & y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a+5x & 2b+5y \\ 3a+8x & 3b+8y \end{bmatrix} = \begin{bmatrix} 10-7 & 4-4 \\ 15+28 & 6+16 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a+5x & 2b+5y \\ 3a+8x & 3b+8y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$$

$\Rightarrow \quad 2a + 5x = 3,\ 3a + 8x = 43,\ 2b + 5y = 0$ and $3b + 8y = 22$

Solving $2a + 5x = 3$ and $3a + 8x = 43$

$$a = -191 \text{ and } x = 77.$$

Solving $2b + 5y = 0$ and $3b + 8y = 22$, we get

$$b = -110 \text{ and } y = 44.$$

$$\therefore \quad D = \begin{bmatrix} a & b \\ x & y \end{bmatrix} = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}.$$

S74. We have,

$$(A + B)^2 = A^2 + B^2$$

$\Rightarrow \quad (A + B)(A + B) = A^2 + B^2$ [By distributive law]

$\Rightarrow \quad A^2 + BA + AB + B^2 = A^2 + B^2$ [By distributive law]]

$\Rightarrow \quad BA + AB = O$

$$\Rightarrow \quad \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} + \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a-b+2 & -a+1 \\ 2a-2 & -b+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad 2a - b + 2 = 0, - a + 1 = 0, 2a - 2 = 0$ and $- b + 4 = 0$

$\Rightarrow \quad a = 1, b = 4$

S75. We have $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

$$\therefore \quad A^2 = AA = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 7 & 2 \end{bmatrix}$$

$$2AB = \begin{bmatrix} 2 & 2 \\ 14 & 4 \end{bmatrix}$$

$$B^2 = BB = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix}$$

$$\Rightarrow \quad (A + B)^2 = \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 18 & 9 \end{bmatrix} \quad \ldots \text{(i)}$$

Also, $A^2 + 2AB + B^2 = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 14 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 24 & 12 \end{bmatrix}$ ... (ii)

From (i) and (ii), we obtain that

$$(A + B)^2 \neq A^2 + 2AB + B^2.$$

S76. Since the product of matrices is associative, therefore we can find ABC either by finding $(AB)C$ or by finding $A(BC)$. Let us find $A(BC)$.

Since B is a 3×3 matrix and C is 3×1 matrix. Therefore, BC is of order 3×1.

Now, $$BC = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$$

Clearly, A is of order 1×3 and BC is of order 3×1. Therefore, $A(BC)$ is of order 1×1.

Now, $$A(BC) = \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$$

$$\Rightarrow \quad A(BC) = [\,x(ax + hy + gz) + y(hx + by + fz) + z(gx + fy + cz)\,]$$

$$\Rightarrow \quad A(BC) = [ax^2 + 2\,hxy + by^2 + cz^2 + 2\,fyz + 2\,gzx\,]$$

S77.

We have, $$F(x).\ F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad F(x).\ F(y) = \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\sin y\cos x - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad F(x).\ F(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y)$$

S78.

We have, $$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\therefore \quad xI + yA = x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + y\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad xI + yA = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} 0 & y \\ -y & 0 \end{bmatrix}$$

$$\Rightarrow \quad xI + yA = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$

Now, $(xI + yA)^2 = A$

$$\Rightarrow \quad \begin{bmatrix} x & y \\ -y & x \end{bmatrix}\begin{bmatrix} x & y \\ -y & x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x^2 - y^2 & 2xy \\ -2xy & x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$\Rightarrow \qquad x^2 - y^2 = 0$ and $2xy = 1$

$\Rightarrow \qquad x = \pm y$ and $2xy = 1$

Now two cases arise.

Case I: When $x = y$ and $2xy = 1$

In this case, we have

$$x = y \text{ and } 2xy = 1 \Rightarrow 2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \quad \left(x = \frac{1}{\sqrt{2}} \text{ and } y = \frac{1}{\sqrt{2}}\right) \text{ or } \left(x = -\frac{1}{\sqrt{2}} \text{ and } y = -\frac{1}{\sqrt{2}}\right)$$

Case II: When $x = -y$ and $2xy = 1$

In this case, we have

$$x = -y \text{ and } 2xy = 1 \Rightarrow 2x^2 = -1 \Rightarrow x = \pm\frac{i}{\sqrt{2}}$$

$$\therefore \quad \left(x = \frac{i}{\sqrt{2}} \text{ and } y = \frac{-i}{\sqrt{2}}\right) \text{ or } \left(x = -\frac{i}{\sqrt{2}} \text{ and } y = \frac{i}{\sqrt{2}}\right)$$

S79.

We have,
$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \quad \text{... (i)}$$

and,
$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}.$$

$$\therefore \quad (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 0 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 0 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & -\dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1-t^2}{1+t^2} & -\frac{2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}, \quad \text{where } t = \tan\frac{\alpha}{2}$$

$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = I + A \qquad \text{... [from Eq. (i)]}$$

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Q1. Verify that $(A - A')$ is a skew symmetric matrix. For the matrix

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix},$$

Q2. Show that the matrix

$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Q3. Show that the matrix

$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

Q4. If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or skew symmetric matrix.

Q5. Is matrix

$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ symmetric or skew symmetric?

Q6. Verify that $(A + A')$ is a symmetric matrix. For the matrix

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

Q7. For what value of x, is the matrix

$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetrix matrix?

Q8. Prove that a matrix which is both symmetric as well as skew-symmetric is a null matrix.

Q9. Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

Q10. Show that the matrix $B'AB$ symmetric or skew symmetric according as A is symmetric or skew symmetric.

Q11. Express the matrix

$A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Q12. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric. If

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}.$$

Q13. Express the following matrix as the sum of a symmetric and a skew symmetric matrix; $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$.

Q14. Express the following matrix as the sum of a symmetric and a skew symmetric matrix; $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

Q15. Express the following matrix as a sum of a symmetric and a skew-symmetric matrix and verify your result $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$.

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S1.

$$A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\therefore$ $$(A - A')' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$$

This shows that $(A - A')$ is skew symmetric matrix.

S2.

Here $$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$\therefore$ $$A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

This shows that A is skew symmetric matrix.

S3.

Here $$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

$\therefore$ $$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

This shows that A is symmetric matrix.

S4.

$$(ABA')' = (A')'(AB)'$$
$$= AB'A'$$
$$= A(-B)A' \qquad [\because B' = -B]$$
$$= -ABA'$$

Thus ABA' is a skew symmetric matrix.

S5.

Here $$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\therefore \quad A' = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = -A$$

$$A' = -A$$

Thus A is skew-symmetric.

S6.

Here $\quad A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$$\therefore \quad A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$\therefore \quad A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

Now $\quad (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = (A + A')$

This shows that $(A + A')$ is symmetric matrix.

S7. If A is matrix, then condition for skew-symmetric matrix is $A = -A^T$, where A^T is transpose of matrix A.

Given that, $\quad A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$

We know that,

if A is a skew-symmetric matrix, then

$$A = -A^T \qquad \text{... (i)}$$

From Eq. (i)

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

$\because$ Two matrix are equal, if its corresponding elements as well as order are equal.

$\Rightarrow \quad x = 2.$

S8. Let $A = [a_{ij}]$ a matrix which is both symmetric and skew-symmetric.

Now,

$A = [a_{ij}]$ is a symmetric matrix

$\Rightarrow \quad a_{ij} = a_{ji}$ for all i, j ... (i)

Also,

$A = [a_{ij}]$ is a skew-symmetric matrix.

$\therefore \quad a_{ij} = -a_{ji}$ for all i, j

$\Rightarrow \quad a_{ji} = -a_{ij}$ for all i, j ... (ii)

From (i) and (ii), we have

$a_{ij} = -a_{ij}$ for all i, j

$\Rightarrow \quad 2a_{ij} = 0$ for all i, j

$\Rightarrow \quad a_{ij} = 0$ for all i, j

$\Rightarrow \quad A = [a_{ij}]$ is a null matrix.

S9. Let A can be square matrix. Then,

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = P + Q \quad \text{(say), where}$$

$$P = \frac{1}{2}(A + A^T) \quad \text{and} \quad Q = \frac{1}{2}(A - A^T).$$

Now, $P^T = \left(\frac{1}{2}(A + A^T)\right)^T = \frac{1}{2}(A + A^T)^T \quad [\because (kA)^T = kA^T]$

$\Rightarrow \quad P^T = \frac{1}{2}(A^T + (A^T)^T) \quad [\because (A + B)^T = A^T + B^T]$

$\Rightarrow \quad P^T = \frac{1}{2}(A^T + A) \quad [\because (A^T)^T = A]$

$\Rightarrow \quad P^T = \frac{1}{2}(A + A^T) = P$ [By comm. of matrix add.]

$\therefore \quad P$ is a symmetric matrix.

Also, $Q^T = \left(\frac{1}{2}(A - A^T)\right)^T = \frac{1}{2}(A - A^T)^T = \frac{1}{2}(A^T - (A^T)^T)$

$\Rightarrow \quad Q^T = \frac{1}{2}(A^T - A) = -\frac{1}{2}(A - A^T) = -Q$

$\therefore \quad Q$ is a skew-symmetric.

Thus, $A = P + Q$, where P is symmetric matrix and Q is a skew-symmetric matrix. Hence A is expressible as the sum of a symmetric and a skew-symmetric matrix.

S10. Let A be a symmetric matrix

$$\therefore \quad A' = A \quad \text{... (i)}$$

Let $C = B'AB$

$$\therefore \quad C' = (B'AB)' = (AB)'\ (B')' = B'A'B \quad [\because (AB)' = B'A' \text{ and } (A')' = A]$$

$$= B'AB \quad \text{[Using (i)]}$$

$$= C$$

which show that $B'AB$ is a symmetric matrix.

Let A be a skew symmetric matrix

$$\therefore \quad A' = -A \quad \text{... (ii)}$$

Let $C = B'AB$

$$\therefore \quad C' = (B'AB)' = (AB)'\ (B')' = B'A'B \quad [\because (AB)' = B'A' \text{ and } (A')' = A]$$

$$= B'(-A)B = -B'AB \quad \text{[Using Eq. (ii)}$$

$$= -C$$

which shows that $B'AB$ is a skew symmetric matrix.

S11. We have,

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

So, $$A + A^T = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix}$$

and, $$A - A^T = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

Let $$P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix}$$

and, $$Q = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$$

Then, $$P^T = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix}^T = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} = P$$

and, $$Q^T = \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & -1/2 \\ -1 & 0 & 1/2 \\ 1/2 & -1/2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix} = -Q$$

Thus, P is symmetric and Q is skew-symmetric.

Also, $$P + Q = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} = A$$

Thus, we have expressed A as the sum of a symmetric and a skew-symmetric matrix.

S12. Here,

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

$\therefore$ $$A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}.$$

Now $$A + A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

and $$A - A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

Let $$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix} = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix}$$

$$P' = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} = P$$

So P is a symmetric matrix

Also $$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & 1 & -\frac{5}{2} \\ -1 & 0 & \frac{3}{2} \\ \frac{5}{2} & -\frac{3}{2} & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{bmatrix} = -Q$$

So Q is a skew symmetric matrix.

Also $$P + Q = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} = A$$

Thus, we have expressed A as the sum of a symmetric and a skew-symmetric matrix.

S13. Let ,

$$A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\therefore \quad A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Now, $A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

and $A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$

$\therefore$ $P' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} = P$

So P is a symmetric matrix.

Also $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$

$\therefore$ $Q' = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -\frac{5}{2} & \frac{3}{-2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = -Q$

So Q is a skew symmetric matrix.

Also $P + Q = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A.$

S14. Let,

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$\therefore$
$$A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Now,
$$A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

and
$$A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let
$$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$\therefore$
$$P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$$

So P is a symmetric matrix.

Let
$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$
$$Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$$

So Q is a skew symmetric matrix.

Also
$$P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A.$$

S15. Write the given matrix A as $A = P + Q$

where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$

Also, verify that P is a symmetric matrix and Q is a skew-symmetric matrix

Let $$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Let us introduce two matrices P and Q, such that

$$P = \frac{1}{2}(A + A')$$

and $$Q = \frac{1}{2}(A - A')$$

We will show that $A = (P + Q)$

First we find the matrices P and Q and check whether they are symmetric and skew-symmetric matrices.

So, $$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \quad P = \frac{1}{2}\left\{\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right\}$$

$$\therefore \quad P = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

Now, $$P' = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \text{P}$$

Since, $P' = P$

$\therefore$ P is a symmetric matrix.

Now, $$Q = \frac{1}{2}(A - A')$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

Now, $$Q' = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= -Q$$

Since, $Q' = -Q$

$\therefore$ Q is a skew-symmetric matrix.

Now,

$$P + Q = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & -4 & -8 \\ 6 & -4 & -10 \\ -2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$\therefore$ We have proved that $P + Q = A$

Hence proved.

Date: 18/10/2021

Q1. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}.$$

Q2. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}.$$

Q3. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}.$$

Q4. Using elementary transformations find the inverse of the following matrix:

$$\begin{bmatrix} 4 & 7 \\ 3 & 6 \end{bmatrix}.$$

Q5. Using elementary transformations, find inverse of

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}.$$

Q6. Using Elementary Row Transformation (*ERT*), find inverse of matrix:

$$A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}.$$

Q7. Using elementary transformation, find inverse of matrix:

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}.$$

Q8. Using elementary transformation, find inverse of matrix:

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}.$$

Q9. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}.$$

Q10. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}.$$

Q11. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

Q12. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}.$$

Q13. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}.$$

Q14. Using elementary transformations, find the inverse of the matrix:

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}.$$

Q15. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$$

Q16. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Q17. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

Q18. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

Q19. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

Q20. Using elementary transformation, find the inverse of the following matrix :

$$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix}.$$

Q21. Using elementary transformation, find the inverse of the following matrix :

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

Q22. Using elementary transformation, find the inverse of the following matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 3 \end{bmatrix}$$

Q23. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Q24. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Q25. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Q26. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Q27. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

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S1. Let,

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Now $A = IA$

$$\Rightarrow \quad \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 2R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$$

Thus $A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$ $[\because A^{-1}A = I]$

S2. Let,

$$A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Now $A = IA$

$$\Rightarrow \quad \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 7 & -2 \end{bmatrix} A$$

Applying $R_2 \to - R_2$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -7 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A$$

Thus $\qquad A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} \qquad [\because \ A^{-1}A = I]$

S3. Let,

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Now $\qquad A = IA$

$$\Rightarrow \qquad \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_2 \to \frac{1}{5} R_2$

$$\Rightarrow \qquad \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}. \qquad [\because \; A^{-1}A = I]$$

S4. Let,

$$A = \begin{bmatrix} 4 & 7 \\ 3 & 6 \end{bmatrix}$$

Now $$A = IA$$

$$\therefore \quad \begin{bmatrix} 4 & 7 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 1 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$$

Operating $R_2 \to \frac{1}{3} R_2$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & \frac{4}{3} \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{7}{3} \\ -1 & \frac{4}{3} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} 2 & -\frac{7}{3} \\ -1 & \frac{4}{3} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 6 & -7 \\ -3 & 4 \end{bmatrix}. \qquad [\because A^{-1}A = I]$$

S5. Let,

$$A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

Now $A = IA$

$$\Rightarrow \quad \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

Operating $R_2 \to \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

Operating $R_1 \to R_1 + 4R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \qquad [\because A^{-1}A = I]$$

S6. Given matrix is

$$A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$$

Let $A = IA$

$$\Rightarrow \quad \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \to R_1 - R_2$, we get

$$\therefore \quad \begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \to R_2 - 5R_1$, we get

$$\therefore \qquad \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} \cdot A$$

Now, apply $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} \cdot A$$

Now, apply $R_2 \to (-1) \cdot R_2$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} \cdot A$$

Hence, $A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$. $[\because \; A^{-1}A = I]$

S7. Given that matrix

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$$

Let $A = IA$

$$\Rightarrow \qquad \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Now, apply $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} A$$

Now, apply $R_2 \to R_2 - R_1$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$. $[\because \; A^{-1}A = I]$

S8. Given that matrix

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Now $\qquad A = IA$

$$\Rightarrow \quad \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$$

Now, apply $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A$$

Applying $R_2 \to (-1) \,.\, R_2$, we get

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

Hence, $\qquad A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \qquad [\because A^{-1}A = I]$

S9. Let,

$$A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

Now $\qquad A = IA$

$$\Rightarrow \quad \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 3R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A$$

Thus $\quad A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$. $\quad [\because \ A^{-1}A = I]$

S10. Let,

$$A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Now $\quad A = IA$

$$\Rightarrow \quad \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$$

Thus $\quad A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$. $\quad [\because \ A^{-1}A = I]$

S11. Let,

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Now $\quad A = IA$

$$\Rightarrow \quad \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}. \qquad [\because \ A^{-1} A = I]$$

S12. Let,

$$A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

Now $$A = IA$$

$$\Rightarrow \quad \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{6} R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 2R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A$$

Since R_2 has all elements zero.

Thus inverse of matrix A does not exist.

S13. Let $$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

Now $$A = IA$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & \frac{1}{2} \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 4R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -2 & 1 \end{bmatrix} A$$

Since R_2 has all elements zero.

Thus inverse of matrix A does not exist.

S14. Let $\quad A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

Now $\quad A = IA$

$$\Rightarrow \quad \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow -R_2$

$$\Rightarrow \quad \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 3R_1$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix} A$$

Applying $R_2 \to \frac{1}{2}R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}.$$ $[\because \quad A^{-1}A = I]$

S15.

Let $$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$$

Now $$A = IA$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ -2 & 2 & 1 \end{bmatrix} A$$

Operating $R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ 1 & -4 & 1 \end{bmatrix} A$$

Operating $R_3 \to -\frac{1}{7} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ -\frac{1}{7} & \frac{4}{7} & -\frac{1}{7} \end{bmatrix} A$$

Operating $R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{5}{7} & \frac{6}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{4}{7} & -\frac{1}{7} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{5}{7} & \frac{6}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{4}{7} & -\frac{1}{7} \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 & -7 & 0 \\ -5 & 6 & 2 \\ -1 & 4 & -1 \end{bmatrix} \qquad [\because A^{-1}A = I]$$

S16.

Let $$A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Now $$A = IA$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 2 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & -4 & -1 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & -4 & -1 \\ 0 & 10 & 5 \\ 0 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & -2 \end{bmatrix} A$$

Applying $R_2 \to \frac{1}{10} R_2$

$$\Rightarrow \begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ 1 & 2 & -2 \end{bmatrix} A$$

Applying $R_3 \to R_3 - 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{5}{2} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ 1 & \frac{1}{2} & -1 \end{bmatrix} A$$

Applying $R_3 \to \frac{2}{5} R_3$

$$\Rightarrow \begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$$

Applying $R_1 \to R_1 + 4R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - \frac{1}{2} R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix} \qquad [\because \ A^{-1}A = I]$$

S17.

Let $$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

Now $$A = IA$$

$$\therefore \quad \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to \frac{1}{3}R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -\frac{5}{3} \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & \frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Operating $R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & \frac{10}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{2}{3} & 1 \end{bmatrix} A$$

Operating $R_3 \to \frac{3}{10}R_3$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{10} & -\frac{1}{5} & \frac{3}{10} \end{bmatrix} A$$

Operating $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{10} & -\frac{1}{5} & \frac{3}{10} \end{bmatrix} A$$

Operating $R_1 \to R_1 + \frac{2}{3} R_3$ and $R_2 \to R_2 + \frac{5}{3} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{10} & -\frac{1}{5} & \frac{3}{10} \end{bmatrix} A$$

Thus $A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{10} & -\frac{1}{5} & \frac{3}{10} \end{bmatrix}$ $[\because A^{-1} A = I]$

S18.

Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Now $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 8R_3$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \to \frac{1}{25} R_3$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & -3 & -24 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_1 \to R_1 + 65R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_2 \to R_2 - 21R_3$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} \qquad [\because A^{-1}A = I]$$

S19. The given matrix is $A = IA$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 2R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_3$ and $R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -3 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ $[\because \quad A^{-1}A = I]$

S20.

Let $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix}$

Now $A = IA$

$$\therefore \quad \begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & -1 & 1 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & -2 \\ 0 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

Operating $R_2 \to \frac{1}{7} R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -\frac{2}{7} \\ 0 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -\frac{3}{7} & \frac{1}{7} & \frac{3}{7} \\ -1 & 0 & 2 \end{bmatrix} A$$

Operating $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 7R_2$

$$\begin{bmatrix} 1 & 0 & \frac{5}{7} \\ 0 & 1 & -\frac{2}{7} \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} & \frac{1}{7} & -\frac{4}{7} \\ -\frac{3}{7} & \frac{1}{7} & \frac{3}{7} \\ 2 & -1 & -1 \end{bmatrix} A$$

Operating $R_3 \to \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 0 & \frac{5}{7} \\ 0 & 1 & -\frac{2}{7} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} & \frac{1}{7} & -\frac{4}{7} \\ -\frac{3}{7} & \frac{1}{7} & \frac{3}{7} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \end{bmatrix} A$$

Operating $R_1 \to R_1 - \frac{5}{7}R_3$ and $R_2 \to R_2 + \frac{2}{7}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{21} & \frac{8}{21} & -\frac{7}{21} \\ -\frac{5}{21} & \frac{1}{21} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} \frac{2}{21} & \frac{8}{21} & -\frac{7}{21} \\ -\frac{5}{21} & \frac{1}{21} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \end{bmatrix} = \frac{1}{21}\begin{bmatrix} 2 & 8 & -7 \\ -5 & 1 & 7 \\ 14 & -7 & -7 \end{bmatrix} \qquad [\because A^{-1}A = I]$$

S21.

Let $$A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

Now $A = IA$

$\therefore$ $$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operating $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 3 & -2 & 7 \\ 4 & 0 & 2 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & -1 & 3 \\ 4 & 0 & 2 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

Operating $R_2 \to R_2 - 4R_1$ and $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -10 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 4 & 1 & -4 \\ 3 & 0 & -2 \end{bmatrix} A$$

Operating $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -2 \\ 0 & 4 & -10 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -2 \\ 4 & 1 & -4 \end{bmatrix} A$$

Operating $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 4R_2$

Operating $R_3 \to -\frac{1}{2} R_3$ $= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 0 & -2 \\ -8 & 1 & 4 \end{bmatrix} A$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 0 & -2 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_3$ and $R_2 \to R_2 + 2R_3$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix} A$$

Thus $A^{-1} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$ $[\because \quad A^{-1}A = I]$

S22.

Let $$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 3 \end{bmatrix}$$

Now $$A = IA$$

$$\therefore \quad \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -1 \\ 0 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Operating $R_2 \to -\frac{1}{3} R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{3} \\ 0 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & -\frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Operating $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{2}{3} & -\frac{1}{3} & 0 \\ 1 & -1 & 1 \end{bmatrix} A$$

Operating $R_1 \to R_1 - \frac{2}{9} R_3$, $\quad R_2 \to R_2 - \frac{1}{9}$ $\quad$ and $\quad R_3 \to \frac{1}{3} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{9} & \frac{5}{9} & -\frac{2}{9} \\ \frac{5}{9} & -\frac{2}{9} & -\frac{1}{9} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} \frac{1}{9} & \frac{5}{9} & -\frac{2}{9} \\ \frac{5}{9} & -\frac{2}{9} & -\frac{1}{9} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 1 & 5 & -2 \\ 5 & -2 & -1 \\ 9 & -9 & 9 \end{bmatrix} \qquad [\because A^{-1} A = I]$$

S23.

Let $$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Now $$A = IA$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to 3R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 6 & 15 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_3$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix} A$$

Applying $R_3 \to \frac{1}{3} R_3$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -12 & 5 & -5 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

Thus $$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}. \qquad [\because \; A^{-1}A = I]$$

S24. Given matrix is $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$ we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Now, applying $R_2 \to \dfrac{R_2}{9}$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -\frac{7}{9} \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Now, applying $R_3 \to R_3 + 5R_2$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -\frac{7}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{1}{3} & \frac{5}{9} & 1 \end{bmatrix} A$$

Applying $R_3 \to 9R_3$, We get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -\frac{7}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -3 & 5 & 9 \end{bmatrix} A$$

Now, applying $R_1 \to R_1 - 3R_2$, we get

$$\begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{7}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -3 & 5 & 9 \end{bmatrix} A$$

Again, applying $R_1 \to R_1 - \frac{1}{3}R_3$ and $R_2 \to R_2 + \frac{7}{9}R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$ $[\because \quad A^{-1}A = I]$

S25.

Given matrix is $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \quad \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + R_1$, we get

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Now, applying $R_2 \to \frac{1}{5} R_2$, we get

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{2}{5} \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Now, applying $R_3 \to R_3 + 2R_2$, we get

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{2}{5} & 1 \end{bmatrix} A$$

Now, applying $R_3 \to 5R_3$, we get

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{2}{5} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 2 & 5 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 2R_3$ and $R_2 \to R_2 + \frac{2}{5} R_3$,

We get

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 10 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$$

Now, applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $[\because A^{-1}A = I]$

S26.

The given matrix is $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{R_1}{3}$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to \frac{R_2}{3}$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - 4R_2$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ \frac{8}{9} & -\frac{4}{3} & 1 \end{bmatrix} A$$

Applying $R_3 \to 9R_3$,

We get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 8 & -12 & 9 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{3} R_3$

and $R_2 \to R_2 - \frac{2}{9} R_3$,

We get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$ $[\because A^{-1}A = I]$

S27.

The given matrix is $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

Let $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By performing $R_2 \to R_2 + R_1,\ R_3 \to R_3 + 3R_1$

$$\Rightarrow \quad \begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

By applying $R_1 \to (-1)\, R_1$

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

By applying $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} A$$

Now, applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 4R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{bmatrix} A$$

By applying $R_2 \to (-1)\, R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Now, applying $R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{bmatrix} A$$

By applying $R_3 \to (-1)\, R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$\Rightarrow \qquad A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \qquad [\because A^{-1}A = I]$$

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Q1. Find $A + A'$. if

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Q2. Find $A + A'$, where A' = transpose of A. If

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$$

Q3. If matrix $A = [1 \;\; 2 \;\; 3]$, write AA'.

Q4. Find $A^T - B^T$. If

$$A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Q5. Find x, $0 < x < \frac{\pi}{2}$, when $A + A' = I$. If

$$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

Q6. For the following matrices A and B, verify that

$$(AB)' = B'A' \;; \quad A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \quad B = [-1 \;\; 2 \;\; 1]$$

Q7. For the matrices A and B, verify that $(AB)' = B'A'$, where

$$A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad B = [1 \;\; 5 \;\; 7]$$

Q8. Find the values of θ satisfying the equation $A^T + A = I_2$. If

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},$$

Q9. For the matrices A and B, verify that $(AB)^T = B^T A^T$. If

$$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad B = [-2 \;\; -1 \;\; -4].$$

Q10. Find the values of a and b. If

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$$ is a matrix satisfying $AA^T = 9I_3$,

Q11. Find x if

$$\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=0$$

Q12. Find x, if

$$\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0.$$

Q13. Find the values of x, y, z if the matrix

$A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfy the equation $A^TA = I_3$.

Q14. Find A. If

$$\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix},$$

Q15. Find the value of x such that

$$\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$$

Q16. Find the matrix X so that

$$X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}.$$

Q17. Find value of $A^2 - 3A + 2I$. If

$$A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}.$$

Q18. Find k such that $A^2 = kA - 2I_2$. If

$$A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$$

Q19. Prove that $A^2 - 5A + 7I_2 = 0$. If

$$A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

Q20. Find k so that $A^2 = 8A + kI$. if

$$A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix} \quad \text{and} \quad I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Q21. Show that A is a root of the polynomial $f(x)$. If

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}; \qquad f(x) = x^3 - 6x^2 + 7x + 2.$$

Q22. Verify that $A^2 - 4A - 5I = 0$. If

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$

Q23. Show that $(aI + bA)^n = a^nI + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in N$.

If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

Q24. Prove that

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer.

Q25. Prove that $A^2 - 5A + 7I = 0$ use this to find A^4. If

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

Q26. Find the numbers a and b such that $A^2 + aA + bI = 0$, where the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}.$$

Q27. Prove that, if

$$A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

(i) $A_\alpha \cdot A_\beta = A_{\alpha+\beta}$ (ii) $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$ for every positive integer n.

Q28. Verify that $A^3 - A^2 - 3A - I = 0$. If

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}.$$

Q29. If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all $n \in N$.

Q30. Prove that

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N. \text{ If } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Q31. If a is a non-zero real or complex number. Use the principle of mathematical induction to prove that

If $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$, then $A^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix}$ for every positive integer n.

Q32. A trust fund has Rs. 30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 30000 among the two types of bonds. If the trust fund must obtain an annual total interest of

(i) Rs. 1800 (ii) Rs. 2000

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S1. Firstly we find the transpose of matrix A and then add the corresponding elements of both matrices A and A'.

Given that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\therefore$ $A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

So, $A + A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

S2. Given that matrix $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$

$\therefore$ $A' = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

So, $A + A' = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$

S3. Matrix multiplication is possible when the number of columns of first matrix is equal to the number of rows of second matrix. First we transpose the matrix A of order 1 × 3 then order of transpose matrix is 3 × 1 that matrix multiplication is possible and get resultant matrix of order 1 × 1.

Given matrix is $A = [1 \;\; 2 \;\; 3]$

$\therefore$ $A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Now, $AA' = [1 \;\; 2 \;\; 3] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$= [(1 \times 1) + (2 \times 2) + (3 \times 3)]$

$= [1 + 4 + 9]$

$= [14]$

S4. Given, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Transpose of B is $B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}.$$

S5. Here $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$\therefore$ $A' = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

Now $A + A' = I$

$\therefore$ $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} + \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $2\cos x = 1 \Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow x = \frac{\pi}{3}$.

S6. Given that, $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = [-1 \ 2 \ 1]$

To verify $(AB)' = B'A'$

LHS = $(AB)'$

Now, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times 1} [-1 \ 2 \ 1]_{1\times 3}$

$$\therefore \qquad AB = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}_{3 \times 3}$$

$$\therefore \qquad (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}_{3 \times 3} \qquad \ldots \text{(i)}$$

$$\text{RHS} = B'A'$$

Now, $$B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}_{3 \times 1} \quad \text{and} \quad A' = [1 \quad -4 \quad 3]_{1\times 3}$$

$$\therefore \qquad B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}_{3 \times 1} [1 \quad -4 \quad 3]_{1\times 3}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}_{3 \times 3} \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$(AB)' = B'A'$$

$\Rightarrow$ LHS = RHS **Hence verified.**

S7.

Here $$A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \text{ and } B = [1 \; 5 \; 7]$$

$$\therefore \qquad AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} . [1 \quad 5 \quad 7] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}' = [0 \quad 1 \quad 2]$$

$$B' = [1 \quad 5 \quad 7]' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} [0 \ \ 1 \ \ 2] = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Thus $(AB)' = B'A'$. Hence Verified

S8. We have,

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\therefore$ $$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Now, $$A^T + A = I_2$$

$\Rightarrow$ $$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$2\cos\theta = 1$$

$\Rightarrow$ $$\cos\theta = \frac{1}{2}$$

$\Rightarrow$ $$\cos\theta = \cos\frac{\pi}{3}$$

$\Rightarrow$ $$\theta = 2n\pi \pm \frac{\pi}{3}, \quad n \in Z.$$

S9.

We have, $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [-2 \quad -1 \quad -4]$

$\therefore$ $$AB = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} [-2 \quad -1 \quad -4]$$

$\Rightarrow$ $$AB = \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$$

$$\Rightarrow \qquad (AB)^T = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix} \qquad \dots \text{(i)}$$

Also, $$B^TA^T = [-2 \quad -1 \quad -4]^T \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}^T = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} [-1 \quad 2 \quad 3] = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix} \qquad \dots \text{(ii)}$$

From (i) and (ii), we observe that

$(AB)^T = B^TA^T$. Hence verified

S10. We have,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$$

$$\Rightarrow \qquad A^T = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

$$\therefore \qquad AA^T = 9I_3$$

$$\Rightarrow \qquad \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 9 & 0 & a+2b+4 \\ 0 & 9 & 2a+2-2b \\ a+2b+4 & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow \qquad a + 2b + 4 = 0, \quad 2a + 2 - 2b = 0 \quad$ and $\quad a^2 + 4 + b^2 = 9$

$\Rightarrow \qquad a + 2b + 4 = 0, \quad a - b + 1 = 0 \quad$ and $\quad a^2 + b^2 = 5$

Solving $\quad a + 2b + 4 = 0 \quad a - b + 1 = 0,$

we get $\quad a = -2 \quad$ and $\quad b = -1.$

S11.

Here $$[1 \; 2 \; 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \qquad [1+4+1 \quad 2+0+0 \quad 0+2+2] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \quad [6\ \ 2\ \ 4]\begin{bmatrix}0\\2\\x\end{bmatrix} = 0$$

$$\Rightarrow \quad 0 + 4 + 4x = 0$$

$$\Rightarrow \quad 4x = -4 \quad \Rightarrow \quad x = -1.$$

S12. Here,

$$\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad [x-0-2 \qquad 0-10-0 \qquad 2x-5-3]\begin{bmatrix}x\\4\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad [x-2\ \ -10\ \ 2x-8]\begin{bmatrix}x\\4\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad [x(x-2)-10 \times 4 + 1(2x-8)] = [0]$$

$$\Rightarrow \quad x^2 - 2x - 40 + 2x - 8 = 0$$

$$\Rightarrow \quad x^2 - 48 = 0$$

$$\Rightarrow \quad x^2 = 48$$

$$\Rightarrow \quad x = \pm\sqrt{48}$$

$$\Rightarrow \quad x = \pm 4\sqrt{3}.$$

S13. We have,

$$A = \begin{bmatrix}0 & 2y & z\\x & y & -z\\x & -y & z\end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix}0 & x & x\\2y & y & -y\\z & -z & z\end{bmatrix}$$

$$\therefore \quad A^T A = I_3$$

$$\Rightarrow \quad \begin{bmatrix}0 & x & x\\2y & y & -y\\z & -z & z\end{bmatrix}\begin{bmatrix}0 & 2y & z\\x & y & -z\\x & -y & z\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}2x^2 & 0 & 0\\0 & 6y^2 & 0\\0 & 0 & 3z^2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \qquad 2x^2 = 1, \quad 6y^2 = 1, \quad 3z^2 = 1$$

$$\Rightarrow \qquad x = \pm\frac{1}{\sqrt{2}}, \quad y = \pm\frac{1}{\sqrt{6}}, \quad z = \pm\frac{1}{\sqrt{3}}$$

S14. Since the product matrix is a 3×3 matrix and the premultiplier of A is a 3×2 matrix. Therefore, A is 2×3 matrix.

Let $$A = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}.$$

Then, the given equation becomes

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x-a & 2y-b & 2z-c \\ x & y & z \\ -3x+4a & -3y+4b & -3z+4c \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad 2x - a = -1, \quad x = 1, \quad -3x + 4a = 9, \quad 2y - b = -8, \quad y = -2,$$
$$-3y + 4b = 22, \quad 2z - c = -10, \quad z = -5, \quad -3z + 4c = 15$$

$$\Rightarrow \quad x = 1, \quad a = 3, \quad y = -2, \quad b = 4, \quad z = -5 \quad \text{and} \quad c = 0.$$

$$A = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}.$$

S15. We have,

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix} = 0$$

$$\Rightarrow \quad 7 + 2x + 12x + x^2 + 21 + 2x = 0$$

$$\Rightarrow \quad x^2 + 16x + 28 = 0$$

$$\Rightarrow \quad (x + 14)(x + 2) = 0 \quad \Rightarrow \quad x = -2 \quad \text{or} \quad -14.$$

S16. Let X have the order $m \times n$.

If its multiplication with a matrix of order 2×3 is defined.

$$n = 2.$$

Also, if the product of a matrix of order $m \times 2$ and another of order 2×3 is a matrix of order 2×3,

$$m = 2.$$

$\therefore$ X has the order 2×2.

Then $$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x_1+4x_2 & 2x_1+5x_2 & 3x_1+6x_2 \\ x_3+4x_4 & 2x_3+5x_4 & 3x_3+6x_4 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \quad x_1 + 4x_2 = -7 \quad \text{and} \quad 2x_1 + 5x_2 = -8$$

Solving these two equations, we get

$$x_1 = 1 \quad \text{and} \quad x_2 = -2$$

Also $x_3 + 4x_4 = 2$ and $2x_3 + 5x_4 = 4$

Solving these two equations, we get

$$x_3 = 2 \quad \text{and} \quad x_4 = 0$$

Thus $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

S17. Given that,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

We have to find the value of $A^2 - 3A + 2I$.

Now, $$A^2 = A \cdot A$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Now, $$3A = 3\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix}$$

and $$2I = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore \quad A^2 - 3A + 2I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{bmatrix}$$

Hence, $$A^2 - 3A + 2I = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{bmatrix}.$$

S18. We have,

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

It is given that $A^2 = kA - 2I_2$

$$\therefore \quad \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$$\Rightarrow \quad 3k - 2 = 1, \quad 4k = 4, \quad -2k = -2 \quad \text{and} \quad -2k - 2 = -4$$

$$\Rightarrow \quad k = 1.$$

S19. We have, $$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$-5A = \begin{bmatrix} (-5).3 & (-5).1 \\ (-5).(-1) & (-5).2 \end{bmatrix} = \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}$$

$$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$\therefore \quad A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow \quad A^2 - 5A + 7I_2 = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Hence proved

S20. We have

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$\therefore \quad A^2 = AA = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

and, $\quad 8A + kI = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$

$\therefore \quad A^2 = 8A + kI$

$$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

$\Rightarrow \quad 1 = 8 + k \quad \text{and} \quad 56 + k = 49 \quad \Rightarrow \quad k = -7.$

S21. We have,

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \quad \text{and} \quad f(x) = x^3 - 6x^2 + 7x + 2$$

$$f(A) = A^3 - 6A^2 + 7A + 2I_3$$

Now, $\quad A^2 = AA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$

$$A^3 = A^2A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$f(A) = A^3 - 6A^2 + 7A + 2I_3$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence, A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

S22.

Here $$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$\therefore$ $$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now $$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Thus $A^2 - 4A - 5I = 0$.

S23. Here

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

We shall prove the result by principle of mathematical Induction

$$P(n) : (aI + bA)^n = a^n I + na^{n-1} bA$$

Let $n = 1$

$$P(1) : (aI + bA)^1 = a^1 I + 1 \cdot a^{1-1} bA$$

$$(aI + bA) = aI + bA$$

which is true for $n = 1$.

Suppose it is true for $n = k$.

$\therefore$ $$P(k) : (aI + bA)^k = a^k I + k \cdot a^{k-1} bA \quad \text{... (i)}$$

Let $n = k + 1$

$\therefore$ $$P(k+1): (aI + bA)^{k+1} = a^{k+1} I + (k+1) \cdot a^k bA$$

Now $\quad$ L.H.S. $= (aI + bA)^{k+1} = (aI + bA)^k \cdot (aI + bA)$

$= (a^k I + ka^{k-1}\, bA)\,(aI + bA)$ $\quad$ [$\because$ by using (i)]

$= a^{k+1}\, I + ka^k\, IbA + a^k\, IbA + ka^{k-1}b^2A^2$

$= a^{k+1}\, I + (k+1)a^k\, bA$ $\quad$ [$\because A^2 = 0$]

$=$ R.H.S.

The result is true for $n = k + 1$ whenever it is true for $n = k$. So by principle of mathematical Induction it is true for all $n \in N$.

S24. Here,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

We shall prove the result by principle of mathematical induction

$$P(n) : A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

Let $n = 1$

$$P(1) : A^1 = \begin{bmatrix} 1+2\times 1 & -4\times 1 \\ 1 & 1-2\times 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

which is true for $n = 1$.

Suppose it is true for $n = k$

$$\therefore \quad P(k) : A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \quad ..(i)$$

Let $n = k + 1$

$$\therefore \quad P(k+1) : A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

Now L.H.S. $= A^{k+1} = A^k \cdot A^1$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \quad \text{[Using (i)]}$$

$$= \begin{bmatrix} 3(1+2k)-4k & -4(1+2k)+4k \\ 3k+1(1-2k) & -4k-1(1-2k) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$=$ R.H.S.

The result is true for $n = k + 1$ whenever it is true for $n = k$. So by principle of mathematical induction it is true for all positive integers.

S25. We have,

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = AA = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \quad A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow \quad A^2 - 5A + 7I = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\Rightarrow \quad A^2 = 5A - 7I$$

$$\therefore \quad A^4 = A^2A^2 = (5A - 7I)(5A - 7I) = 5A(5A - 7I) - 7I(5A - 7I)$$

$$= 25A^2 - 35A - 35A + 49II$$

$$= 25A^2 - 35A - 35A + 49I$$

$$= 25A^2 - 70A + 49I$$

$$= 25(5A - 7I) - 70A + 49I$$

$$= 125A - 175I - 70A + 49I$$

$$= 55A - 126I$$

$$= 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} + \begin{bmatrix} -126 & 0 \\ 0 & -126 \end{bmatrix} = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}.$$

S26. Here

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\therefore \quad A^2 = A \cdot A$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

$$\therefore \quad A^2 + aA + bI = 0$$

$$\Rightarrow \quad \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad 4 + a = 0$ and $3 + a + b = 0$

$\Rightarrow \quad a = -4$ and $3 - 4 + b = 0$

$b = 1$

Thus $a = -4$ and $b = 1$.

S27. (i) We have,

$$A_\alpha \,.\, A_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$\Rightarrow \quad A_\alpha \,.\, A_\beta = \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{bmatrix}$$

$$\Rightarrow \quad A_\alpha \,.\, A_\beta = \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = A_{\alpha+\beta}$$

(ii) We shall prove the result by mathematical induction on n.

Step 1: When $n = 1$, by the definition of integral powers of a matrix, we have

$$(A_\alpha)^1 = A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos(1.\alpha) & \sin(1.\alpha) \\ -\sin(1.\alpha) & \cos(1.\alpha) \end{bmatrix}$$

So, the result is true for $n = 1$.

Step 2: Let the result be true for $n = m$. Then,

$$(A_\alpha)^m = \begin{bmatrix} \cos m\alpha & \sin m\alpha \\ -\sin m\alpha & \cos m\alpha \end{bmatrix} \quad \ldots \text{(i)}$$

Now we will show that the result is true for $n = m + 1$ *i.e.*,

$$(A_\alpha)^{m+1} = \begin{bmatrix} \cos(m+1)\alpha & \sin(m+1)\alpha \\ -\sin(m+1)\alpha & \cos(m+1)\alpha \end{bmatrix}$$

By the definition of integral powers of a square matrix, we have

$$(A_\alpha)^{m+1} = (A_\alpha)^m \,.\, A_\alpha$$

$$\Rightarrow \quad (A_\alpha)^{m+1} = \begin{bmatrix} \cos m\alpha & \sin m\alpha \\ -\sin m\alpha & \cos m\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \quad \text{[By assumption Eq. (i)]}$$

$$\Rightarrow \quad (A_\alpha)^{m+1} = \begin{bmatrix} \cos m\alpha\cos\alpha - \sin m\alpha\sin\alpha & \cos m\alpha\sin\alpha + \sin m\alpha\cos\alpha \\ -\sin m\alpha\cos\alpha - \cos m\alpha\sin\alpha & -\sin m\alpha\sin\alpha + \cos m\alpha\cos\alpha \end{bmatrix}$$

$$\Rightarrow \quad (A_\alpha)^{m+1} = \begin{bmatrix} \cos(m\alpha+\alpha) & \sin(m\alpha+\alpha) \\ -\sin(m\alpha+\alpha) & \cos(m\alpha+\alpha) \end{bmatrix} = \begin{bmatrix} \cos(m+1)\alpha & \sin(m+1)\alpha \\ -\sin(m+1)\alpha & \cos(m+1)\alpha \end{bmatrix}$$

This shows that the result is true for $n = m + 1$, whenever it is true for $n = m$.

Hence, by the principle of mathematical induction the result is valid for every positive integer n.

S28.

Let $$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}.$$

Now $$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0-6 & 0+0-8 & -2+0-2 \\ -2+2+6 & 0+1+8 & 4-2+2 \\ 3-8+3 & 0-4+4 & -6+8+1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5+16-12 & 0+8-16 & 10-16-4 \\ 6-18+12 & 0-9+16 & -12+18+4 \\ -2+0+9 & 0+0+12 & 4+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$$

Now $$A^3 - A^2 - 3A - I = \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & -6 \\ -6 & -3 & 6 \\ 9 & 12 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+5-3-1 & -8+8-0-0 & -10+4+6-0 \\ 0-6+6-0 & 7-9+3-1 & 10-4-6-0 \\ 7+2-9-0 & 12-0-12-0 & 7-3-3-1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Thus $A^3 - A^2 - 3A - I = 0$.

S29. Here A and B are square matrices of the same order such that $AB = BA$.

We shall prove the result by principle of mathematical induction.

$$P(n) : AB^n = B^n A$$

Let $n = 1$

$$P(1) : AB^1 = B^1 A$$

which is true for $n = 1$

Suppose it is true for $n = k$

$\therefore$ $$P(k) : AB^k = B^k A$$

Let $n = k + 1$

$\therefore$ $$P(k + 1) : AB^{k+1} = B^{k+1} A.$$

Now $$\text{L.H.S.} = AB^{k+1} = A(BB^k) = AB(B^k) = (BA)B^k = B(AB^k)$$
$$= B(B^k A) = (BB^k)A = B^{k+1} A = \text{R.H.S.} \qquad [\because AB^k = B^k A]$$

The result is true for $n = k + 1$ whenever it is true for $n = k$. So by principle of mathematical induction it is true for all $n \in N$.

Also $$P(n) : (AB)^n = A^n B^n$$

Let $n = 1$

$$P(1) : AB^1 = A^1 B^1$$

which is true for $n = 1$.

Suppose it is true for $n = k$.

$\therefore$ $$P(k) : (AB)^k = A^k B^k$$

Let $n = k + 1$

$\therefore$ $$P(k + 1) : (AB)^{k+1} = A^{k+1} B^{k+1}$$

Now $$\text{L.H.S.} = (AB)^{k+1} = (AB)^k (AB) \qquad [\because (AB)^k = A^K B^K, \text{ and } AB = BA]$$
$$= (A^k B^k)(BA) = A^k(B^k B)A = A^k(B^{k+1} A)$$
$$= A^k(AB^{k+1}) = (A^k A)B^{k+1} = A^{k+1} B^{k+1}$$
$$= \text{R.H.S.}$$

This shows that the result is true for $n = k + 1$ whenever it is true for $n = k$. So by the principle of mathematical induction it is true for all $n \in N$.

S30. Here,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

We shall prove the result by principle of mathematical induction.

$$P(n) : A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$

Let $n = 1$

$$P(1) : A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

which is true for $n = 1$

Suppose it is true for $n = k$

$$\therefore \quad P(k) : A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \quad \text{... (i)}$$

Let $n = k + 1$

$$\therefore \quad P(k+1) : A^{k+1} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Now L.H.S. $= A^{k+1} = A^k \, . \, A^1$

$$= \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{[Using Eq. (i)]}$$

$$= \begin{bmatrix} 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \end{bmatrix} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

= R.H.S.

The result is true for $n = k + 1$ whenever it is true for $n = k$. So by principle of mathematical induction it is true for all $n \in N$.

S31. We have,

Step 1: When $n = 1$, by the definition of integral powers of a matrix, we have

$$A^1 = A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^1 & 1(a^{1-1}) \\ 0 & a^1 \end{bmatrix}$$

So, the result is true for $n = 1$.

Step 2: Let the result be true for $n = m$. Then

$$A^m = \begin{bmatrix} a^m & ma^{m-1} \\ 0 & a^m \end{bmatrix} \quad \text{... (i)}$$

Now we will show that the result is true for $n = m + 1$ *i.e.*,

$$A^{m+1} = \begin{bmatrix} a^{m+1} & (m+1)a^m \\ 0 & a^{m+1} \end{bmatrix}$$

By the definition of integral powers of a square matrix, we have

$$A^{m+1} = A^m . A$$

$$\Rightarrow \quad A^{m+1} = \begin{bmatrix} a^m & ma^{m-1} \\ 0 & a^m \end{bmatrix} \cdot \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} \quad \text{[By assumption eq. (i)]}$$

$$\Rightarrow \quad A^{m+1} = \begin{bmatrix} a^m . a + 0 . ma^{m-1} & a^m . 1 + ma^{m-1} . a^1 \\ a . 0 + 0 . a^m & 0.1 + a^m . a \end{bmatrix}$$

$$\Rightarrow \quad A^{m+1} = \begin{bmatrix} a^{m+1} & a^m + ma^m \\ 0 & a^{m+1} \end{bmatrix} = \begin{bmatrix} a^{m+1} & (m+1)a^m \\ 0 & a^{m+1} \end{bmatrix}$$

This shows that the result is true for $n = m + 1$, whenever it is true for $n = m$.

Hence, by the principle of mathematical induction the result is true for every positive integer n.

S32. Let Rs. x be invested in first bond and Rs. y be invested in second bond. Let A be the investment matrix and B the interest per rupee matrix. Then,

$$A = [x \quad y] \quad \text{and} \quad B = \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}$$

$$\text{Total annual interest} = AB = [x \quad y] \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = \left[\frac{5x}{100} + \frac{7y}{100}\right]$$

Also, $x + y = 30000$. … (i)

(i) If total interest is Rs. 1800. Then

$$\frac{5x}{100} + \frac{7y}{100} = 1800$$

$\Rightarrow$ $5x + 7y = 180000$ … (ii)

Solving (i) and (ii), we get : $x = y = 15000$.

(ii) If total interest is Rs. 2000. Then,

$$\frac{5x}{100} + \frac{7y}{100} = 2000$$

$\Rightarrow$ $5x + 7y = 200000$ … (iii)

Solving (i) and (ii), we get

$$x = 5000 \quad \text{and} \quad y = 25000.$$