Date: 18/10/2021

Q1. Evaluate

$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right\}$

Q2. Find the principal values of :

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Q3. Evaluate :

$\operatorname{cosec}^{-1}\left[\operatorname{cosec}\left(-\frac{\pi}{4}\right)\right]$

Q4. Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$.

Q5. Find the principal value of $\operatorname{cosec}^{-1}(2)$.

Q6. What is the principal value of $\tan^{-1}(-1)$?.

Q7. Find principal value of

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Q8. What is the domain of the function $\sin^{-1} x$.

Q9. Evaluate

$\tan\left(\frac{\pi}{3} - \cos^{-1}\frac{1}{2}\right)$

Q10. Evaluate

$\cos\left(\frac{\pi}{2} - \sin^{-1}\frac{\sqrt{3}}{2}\right)$

Q11. Evaluate

$\sin\left\{\frac{\pi}{6} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$

Q12. Evaluate

$\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

Q13. Evaluate

$\tan\left(\operatorname{cosec}^{-1}\frac{13}{5}\right)$

Q14. Evaluate

$$\cot\left(\operatorname{cosec}^{-1}\frac{17}{8}\right)$$

Q15. Evaluate

$$\cos\left(\operatorname{cosec}^{-1}\frac{13}{12}\right)$$

Q16. Evaluate

$$\sec\left(\tan^{-1}\left(\frac{8}{15}\right)\right)$$

Q17. Evaluate

$$\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$$

Q18. If $\tan^{-1}(\sqrt{3}) + \cot^{-1} x = \frac{\pi}{2}$, find x.

Q19. Evaluate

$$\operatorname{cosec}\left(\cos^{-1}\left(-\frac{12}{13}\right)\right)$$

Q20. Evaluate

$$\sin\left(\cos^{-1}\frac{4}{5}\right)$$

Q21. Evaluate

$$\tan\left(\cos^{-1}\frac{8}{17}\right)$$

Q22. Evaluate

$$\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right)$$

Q23. What is the principal value of $\sec^{-1}(-2)$?

Q24. Using principal values, write the value of

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right).$$

Q25. Write the principal value of

$$\sin^{-1}\left(-\frac{1}{2}\right).$$

Q26. Write the principal value of

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right).$$

Q27. Evaluate :

$$\sin^{-1}\left(-\frac{1}{2}\right)+2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right).$$

Q28. Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$.

Q29. Find the principal value of

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right).$$

Q30. Find the principal value of

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right).$$

Q31. Find the principal value of

$$\cos^{-1}\left(-\frac{1}{2}\right).$$

Q32. Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$.

Q33. Find principal value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

Q34. Write the value of

$$\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right).$$

Q35. Write the principal value of $\tan^{-1}(\sqrt{3})-\cot^{-1}(-\sqrt{3})$.

Q36. Write the principal value of

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)$$

Q37. Write the value of

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right).$$

Q38. Write the value of

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right).$$

Q39. Using principal values, evaluate

$$\tan^{-1}(1)+\sin^{-1}\left(-\frac{1}{2}\right).$$

Q40. Using principal values, write the value of

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right).$$

Q41. Find the principal values of

$$\cos^{-1}\left(\cos\frac{25\pi}{6}\right).$$

Q42. Using principal values, find the value of :

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

Q43. Find the principal value of

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)+\cot^{-1}\left(\cot\frac{7\pi}{6}\right).$$

Q44. Find the principal value of

$$\sin^{-1}\left(\sin\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{\pi}{6}\right).$$

Q45. Find the principal value of

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{\pi}{6}\right)+\tan^{-1}\left(\tan\frac{7\pi}{6}\right).$$

Q46. Find the principal value of

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right].$$

Q47. Find the principal value of

$$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right].$$

Q48. Evaluate

$$\tan\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$$

Q49. Write the value of

$$\tan\left(2\tan^{-1}\frac{1}{5}\right).$$

Q50. Evaluate :

$$\sin\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right].$$

Q51. Evaluate

$$\cos\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{4}\right)$$

Q52. Find the value of the following

$$\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right],\quad |x|<1,\ y>0\ \text{and}\ x,y<1.$$

Q53. Write the value of

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right].$$

Q54. Evaluate

$$\tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right)$$

Q55. Prove that :

$$\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$$

Q56. Prove that :

$$\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

Q57. Write the function in the simplest form:

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}}, |x|>1.$$

Q58. Prove that: $\sec^2(\tan^{-1}2) + \mathrm{cosec}^2(\cot^{-1}3) = 15$

Q59. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$

Q60. Evaluate:

$\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$, where $0 \le \cos^{-1}x \le \pi$ and $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$.

Q61. Solve the following equation

$$\cos(\tan^{-1}x)=\sin\left(\cot^{-1}\frac{3}{4}\right).$$

Q62. Solve for x,

$$2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x),\ x\ne\frac{\pi}{2}.$$

Q63. Solve for x :

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\,x)$$

Q64. Find the value of

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right).$$

Q65. Prove that

$$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}.$$

Q66. Prove that

$$2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$$

Q67. Prove that

$$\sin^{-1}\left(\frac{63}{65}\right)=\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right).$$

Q68. Prove that

$$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right).$$

Q69. Prove that

$$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right).$$

Q70. Prove that

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right).$$

Q71. Prove that

$$2\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}.$$

Q72. Prove that

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\tan^{-1}\frac{4}{3}.$$

Q73. Prove that

$$2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}.$$

Q74. Prove that

$$2\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{4}\right)=\tan^{-1}\left(\frac{32}{43}\right).$$

Q75. Prove that

$$\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}.$$

Q76. Prove that

$$\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}.$$

Q77. Prove that :

$$\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}.$$

Q78. Prove that :

$$\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}.$$

Q79. Prove that :

$$2\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{24}{7}.$$

Q80. Prove that :

$$4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=\frac{\pi}{4}.$$

Q81. Prove that

$$\cos\left[\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right]=\frac{33}{65}.$$

Q82. Show that

$$\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}.$$

Q83. Show that

$$\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)=x-\tan^{-1}\left(\frac{4}{3}\right).$$

Q84. Prove that,

$$\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}=\frac{\pi}{4}.$$

Q85. Prove that :

$$2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}.$$

Q86. Write the following function in the simplest form:

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x},\ x\neq 0$$

Q87. Prove that

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$$

Q88. Prove that

$$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right).$$

Q89. Prove that

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2};$$

Q90. Show that

$$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}.$$

Q91. Evaluate

$$\tan\left\{2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right\}$$

Q92. Prove that

$$\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\leq x\leq 1$$

Q93. Prove that

$$\cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right)=2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right).$$

Q94. Prove that

$$\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$$

Q95. Prove that :

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$$

Q96. Prove that :

$$\tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\}=\frac{\pi}{4}-\frac{x}{2}, \text{if } \pi<x<\frac{3\pi}{2}$$

Q97. Prove that,

$$\tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\}=\frac{\pi}{4}+\frac{x}{2}, 0<x<\frac{\pi}{2}$$

Q98. Prove that

$$\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

Q99. Prove that

$$\cos^{-1}x=2\sin^{-1}\sqrt{\frac{1-x}{2}}=2\cos^{-1}\sqrt{\frac{1+x}{2}}$$

Q100 If : $y=\cot^{-1}(\sqrt{\cos x})-\tan^{-1}(\sqrt{\cos x})$,

prove that $\sin y=\tan^2\frac{x}{2}$

Q101 If : $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ then prove that $x=\frac{a-b}{1+ab}$.

Q102 Prove that

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right)$$

Q103 If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$

Prove that $x^4+y^4+z^4+4x^2y^2z^2=2(x^2y^2+y^2z^2+z^2x^2)$

Q104 If $y=\sec\left\{\cot^{-1}\left\{\sin\left(\tan^{-1}\left(\operatorname{cosec}\left(\cos^{-1}a\right)\right)\right)\right\}\right\}$ Prove that $y^2=3-a^2$

Q105 If $x=\operatorname{cosec}\left\{\tan^{-1}\left\{\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1}a\right)\right)\right)\right\}\right\}$ Prove that $x^2=3-a^2$

Q106 If $\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$. Prove that $x=\frac{a+b}{1-ab}$.

Q107 If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$, prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$.

Q108 Prove that :

$$\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$$

Q109 If: $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$, then prove that $x^2 = \sin 2\alpha$.

Q110 Prove that :

$$\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)=(\alpha+\beta)\left(\alpha^2+\beta^2\right).$$

Q111 Prove that :

$$\tan^{-1}\left[\frac{3\sin 2\theta}{5+3\cos 2\theta}\right]+\tan^{-1}\left(\frac{1}{4}\tan\theta\right)=\theta.$$

Q112 Prove that

$$\tan^{-1}\frac{2ab}{a^2-b^2}+\tan^{-1}\frac{2xy}{x^2-y^2}=\tan^{-1}\frac{2\alpha\beta}{\alpha^2-\beta^2}$$

where $\alpha = ax - by$ and $\beta = ay + bx$

Q113 If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\alpha$ then prove that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$

Q114 Solve for x,

$$\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{8}{79}\right), x>0.$$

Q115 Solve for x,

$$\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}.$$

Q116 Solve for x,

$$\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}.$$

Q117 Solve for x,

$$\tan^{-1}\left(\frac{1+x}{1-x}\right)=\frac{\pi}{4}+\tan^{-1}x, 0<x<1$$

Q118 Prove that :

$$\tan^{-1}\left\{\frac{\cos 2\alpha\sec 2\beta+\cos 2\beta\sec 2\alpha}{2}\right\}=\tan^{-1}\left\{\tan^2(\alpha+\beta)\tan^2(\alpha-\beta)\right\}+\tan^{-1}1.$$

Q119 If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.

Q120 Solve following equation for x,

$$\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x; x>0.$$

Q121 Solve for x,

$$\cos(2\sin^{-1}x) = \frac{1}{9}, x > 0$$

Q122 Solve for x

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, \ -1 < x < 1$$

Q123 Solve for x,

$$\cos^{-1}x + \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{6}.$$

Q124 Solve for x,

$$\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}; \quad 0 < x < \sqrt{6}$$

Q125 Solve for x :

$$\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2} = \frac{\pi}{3}.$$

Q126 Solve for x :

$$\cot^{-1}x - \cot^{-1}(x+2) = \frac{\pi}{12}, \text{ where } x > 0$$

Q127 Solve for x :

$$\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$$

Q128 Solve for x :

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$$

Q129 Solve for x :

$$\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1}x$$

Q130 Solve for x :

$$\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$$

Q131 Solve for x :

$$3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

Q132 Solve for x :

$$\tan(\cos^{-1}x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$$

Q133 Solve for x :

$$\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}] = 0$$

Date: 18/10/2021

S1. We have $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right\}$

$$= \sin\left\{\frac{\pi}{3} + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}$$

$$= \sin\left\{\frac{\pi}{3} + \frac{\pi}{3}\right\} = \sin\frac{2\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

S2. We know that the principal value branch of : $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right] = \frac{\pi}{6}$$

S3. Here $\text{cosec}^{-1}\left[\text{cosec}\left(-\frac{\pi}{4}\right)\right] = \text{cosec}^{-1}\left[-\text{cosec}\frac{\pi}{4}\right]$

$$= -\text{cosec}^{-1}\left[\text{cosec}\frac{\pi}{4}\right] = -\frac{\pi}{4}$$

S4. Let $\cot^{-1}(\sqrt{3}) = \theta$

$\Rightarrow$ $\cot\theta = \sqrt{3}$

We know that the range of principal value branch of $\cot^{-1}x$ is $(0, \pi)$

$\therefore$ $\cot\theta = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$

$\Rightarrow$ $\theta = \frac{\pi}{6} \in (0, \pi)$

Thus principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

S5. Let $\text{cosec}^{-1}(2) = \theta$

$\Rightarrow$ $\text{cosec}\,\theta = 2$

Now we know that the range of principal value branch of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\therefore \quad \text{cosec}\,\theta = 2 = \text{cosec}\,\frac{\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Thus principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

S6. We know that, principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \quad \tan^{-1}(-1) = \tan^{-1}\left(-\tan\frac{\pi}{4}\right) \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$

$= \tan^{-1}\tan\left(-\frac{\pi}{4}\right) \qquad [\because \tan(-\theta) = -\tan\theta]$

$= -\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \quad \tan^{-1}(-1) = -\frac{\pi}{4}$

S7. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$= \frac{\pi}{6} \in [0, \pi]$

{As principal value branch of $\cos^{-1} x$ is $[0, \pi]$}

$\therefore \quad \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

S8. The domain of $\sin^{-1} x$ is "$-1 \le x \le 1$".

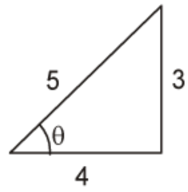
S9. $\tan\left(\frac{\pi}{3} - \cos^{-1}\frac{1}{2}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{3}\right) = \tan 0° = 0$

S10. $\cos\left(\frac{\pi}{2} - \sin^{-1}\frac{\sqrt{3}}{2}\right) = \sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

S11. $\sin\left\{\frac{\pi}{6} - \sin^{-1}\left(-\frac{1}{2}\right)\right\} = \sin\left\{\frac{\pi}{6} + \sin^{-1}\left(\frac{1}{2}\right)\right\} = \sin\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

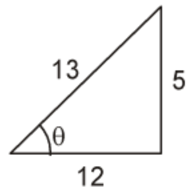
S12. $\cos\left(\tan^{-1}\frac{3}{4}\right)$



Let $\tan^{-1}\left(\frac{3}{4}\right) = \theta \Rightarrow \tan\theta = \frac{3}{4}$

$\therefore \quad \cos\left(\tan^{-1}\frac{3}{4}\right) = \cos\theta = \frac{4}{5}$

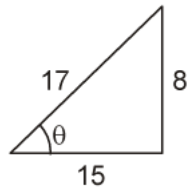
S13. $\tan\left(\operatorname{cosec}^{-1}\frac{13}{5}\right)$



Let $\operatorname{cosec}^{-1}\frac{13}{5} = \theta \Rightarrow \operatorname{cosec}\theta = \frac{13}{5}$

$\therefore \quad \tan\left(\operatorname{cosec}^{-1}\frac{13}{5}\right) = \tan\theta = \frac{5}{12}$

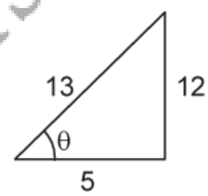
S14. $\cot\left(\operatorname{cosec}^{-1}\frac{17}{8}\right)$



Let $\operatorname{cosec}^{-1}\frac{17}{8} = \theta \Rightarrow \operatorname{cosec}\theta = \frac{17}{8}$

$\therefore \quad \cot\left(\operatorname{cosec}^{-1}\frac{17}{8}\right) = \cot\theta = \frac{15}{8}$

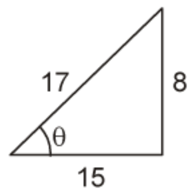
S15. $\cos\left(\operatorname{cosec}^{-1}\frac{13}{12}\right)$



Let $\operatorname{cosec}^{-1}\frac{13}{12} = \theta \Rightarrow \operatorname{cosec}\theta = \frac{13}{12}$

$\therefore \quad \cos\left(\operatorname{cosec}^{-1}\frac{13}{12}\right) = \cos\theta = \frac{5}{13}$

S16. $\sec\left(\tan^{-1}\left(\frac{8}{15}\right)\right)$

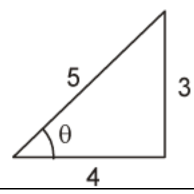


Let $\tan^{-1}\frac{8}{15} = \theta \Rightarrow \tan\theta = \frac{8}{15}$

$\therefore \quad \sec\left(\tan^{-1}\frac{8}{15}\right) = \sec\theta = \frac{17}{15}$

S17. $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right) = \cos\left(-\sin^{-1}\frac{3}{5}\right)$

$\Rightarrow \quad = \cos\left(\sin^{-1}\frac{3}{5}\right)$



Let $\sin^{-1}\dfrac{3}{5} = \theta \;\Rightarrow\; \sin\theta = \dfrac{3}{5}$

$\therefore \quad \cos\left(\sin^{-1}\dfrac{3}{5}\right) = \cos\theta = \dfrac{4}{5}$

S18. Given that, $\tan^{-1}\sqrt{3} + \cot^{-1}x = \dfrac{\pi}{2}$

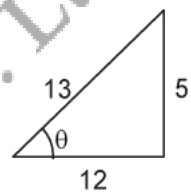
$\Rightarrow \quad \tan^{-1}\sqrt{3} = \dfrac{\pi}{2} - \cot^{-1}x$

$\Rightarrow \quad \tan^{-1}\sqrt{3} = \tan^{-1}x \qquad \left[\because \tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2}\right]$

$\therefore \quad x = \sqrt{3}$.

S19. $\text{cosec}\left(\cos^{-1}\left(-\dfrac{12}{13}\right)\right) = \text{cosec}\left(\pi - \cos^{-1}\dfrac{12}{13}\right)$

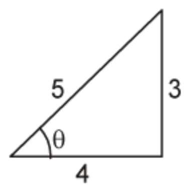
$= \text{cosec}\left(\cos^{-1}\dfrac{12}{13}\right)$



Let $\cos^{-1}\dfrac{12}{13} = \theta \;\Rightarrow\; \cos\theta = \dfrac{12}{13}$

$\therefore \quad \text{cosec}\left(\cos^{-1}\dfrac{12}{13}\right) = \text{cosec}\,\theta = \dfrac{13}{5}$

S20. $\sin\left(\cos^{-1}\dfrac{4}{5}\right)$

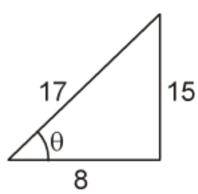


Let $\cos^{-1}\dfrac{4}{5} = \theta \;\Rightarrow\; \cos\theta = \dfrac{4}{5}$

$\therefore \quad \sin\left(\cos^{-1}\dfrac{4}{5}\right) = \sin\theta = \dfrac{3}{5}$

S21. $\tan\left(\cos^{-1}\dfrac{8}{17}\right)$

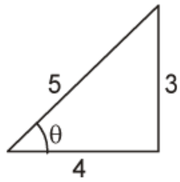
Let $\cos^{-1}\dfrac{8}{17} = \theta \;\Rightarrow\; \cos\theta = \dfrac{8}{17}$



$\therefore \quad \tan\left(\cos^{-1}\dfrac{8}{17}\right) = \tan\theta = \dfrac{15}{8}$

S22. $\cos\left(\sin^{-1}\left(\dfrac{3}{5}\right)\right)$

Let $\quad \sin^{-1}\frac{3}{5} = \theta \quad \Rightarrow \quad \sin\theta = \frac{3}{5}$



$\therefore \quad \cos\left(\sin^{-1}\frac{3}{5}\right) = \cos\theta = \frac{4}{5}$

S23. We know that, principal value branch of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore \quad \sec^{-1}(-2) = \sec^{-1}\left(-\sec\frac{\pi}{3}\right) \neq \frac{-\pi}{3}$

as $\quad \frac{-\pi}{3} \notin [0, \pi] - \left\{\frac{-\pi}{3}\right\}$

$= \sec^{-1}\left[\sec\left(\pi - \frac{\pi}{3}\right)\right] \qquad [\because \sec(\pi - \theta) = -\sec\theta]$

$= \sec^{-1}\left(\sec\frac{2\pi}{3}\right)$

$= \frac{2\pi}{3} \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore \quad \sec^{-1}(-2) = \frac{2\pi}{3}.$

S24. We know that, the principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore \quad \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{3}\right)$

$= \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right] \qquad \left[\begin{array}{l}\because \sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} \\ \text{As } \sin(-\theta) = -\sin\theta\end{array}\right]$

$= -\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \quad \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}.$

S25. We know that, the principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore \quad \sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right] \qquad \left[\because \sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} \text{ as } \sin(-\theta) = -\sin\theta\right]$$

$$= -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \quad \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

S26. As principal value branch of $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

$$\therefore \quad \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) \qquad \left[\because \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right]$$

$$= \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \quad \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

S27.

$$\sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) + 2\left[\pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$\left[\because \sin^{-1}(-\theta) = -\sin^{-1}\theta \text{ and } \cos^{-1}(-\theta) = \pi - \cos^{-1}\theta\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{6}\right) + 2\left[\pi - \cos^{-1}\left(\cos\frac{\pi}{6}\right)\right]$$

$$= -\frac{\pi}{6} + 2\left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{6} + 2 \times \frac{5\pi}{6} = -\frac{\pi}{6} + \frac{5\pi}{3}$$

$$= \frac{-\pi + 10\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$$

S28. Let $\quad \text{cosec}^{-1}(-\sqrt{2}) = \theta$

$$\Rightarrow \quad \text{cosec}\,\theta = -\sqrt{2}$$

We know that the range of principal value branch of $\text{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

$$\therefore \quad \text{cosec}\,\theta = -\sqrt{2} = -\text{cosec}\left(\frac{\pi}{4}\right) = \text{cosec}\left(-\frac{\pi}{4}\right)$$

$\Rightarrow \qquad \theta = -\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Thus principal value of $\text{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

S29. Let $\qquad \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \theta$

$\Rightarrow \qquad \cos\theta = -\frac{1}{\sqrt{2}}$

We know that the range of principal value branch of $\cos^{-1}x$ is $[0, \pi]$

$\therefore \qquad \cos\theta = -\frac{1}{\sqrt{2}} = -\cos\frac{\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

$\Rightarrow \qquad \theta = \frac{3\pi}{4} \in [0, \pi]$

Thus principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

S30. Let $\qquad \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \theta$

$\Rightarrow \qquad \sec\theta = \frac{2}{\sqrt{3}}$

We know that the range of principal value branch of $\sec^{-1}x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore \qquad \sec\theta = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$

$\Rightarrow \qquad \theta = \frac{\pi}{6} \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

Thus principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

S31. Let $\qquad \cos^{-1}\left(-\frac{1}{2}\right) = \theta$

$\Rightarrow \qquad \cos\theta = -\frac{1}{2}$

We know that the range of principal value of $\cos^{-1}x$ is $[0, \pi]$

$\therefore \qquad \cos\theta = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\Rightarrow \qquad \theta = \frac{2\pi}{3} \in [0, \pi]$

Thus principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

S32. Let $\tan^{-1}\left(-\sqrt{3}\right) = \theta$

$\Rightarrow \qquad \tan\theta = -\sqrt{3}$

We know that the range of principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \qquad \tan\theta = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$

$\Rightarrow \qquad \theta = -\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Thus principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

S33. We know that, principal value branch for $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and that of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

So, $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \tan^{-1}\tan\frac{\pi}{3} - \sec^{-1}\sec\frac{2\pi}{3} \qquad \left[\because \tan\frac{\pi}{3} = \sqrt{3} \text{ and } \sec\frac{2\pi}{3} = -2\right]$

$= \frac{\pi}{3} - \frac{2\pi}{3} = \frac{-\pi}{3}$

S34. Firstly, we check the given angle is in principal value. If it is so, then use the identity $\sin^{-1}(\sin\theta) = \theta$ and $\cos(\cos^{-1}\theta) = \theta$

$$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \cos^{-1}\cos\frac{\pi}{3} - 2\sin^{-1}\sin\left(-\frac{\pi}{6}\right)$$

$\left[\because \text{ Principal value branch for } \cos^{-1}x \text{ is } [0, \pi] \text{ and that of } \sin^{-1}x \text{ is } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$$= \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

S35. $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$ [$\because$ Principal value of $\cot^{-1} x$ is$]0, \pi[$

$= \tan^{-1}\sqrt{3} - (\pi - \cot^{-1}\sqrt{3}) \qquad \therefore \cot^{-1}(-x) = \pi - \cot^{-1} x$

$$= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$$

$$= (\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}) - \pi$$

$$= \frac{\pi}{2} - \pi = -\frac{\pi}{2} \qquad \left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$$

S36. Given that, $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \cos^{-1}\left(-\cos\frac{\pi}{3}\right)$

$$= \frac{\pi}{4} + \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \frac{\pi}{4} + \cos^{-1}\left[\cos\frac{2\pi}{3}\right]$$

$$= \frac{\pi}{4} + \frac{2\pi}{3}$$

$$= \frac{3\pi + 8\pi}{12} = \frac{11\pi}{12}$$

S37. We know that, the principal value branch of $\cos^{-1}\theta$ is $[0, \pi]$.

$$\therefore \quad \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{5\pi}{6}\right)\right] \qquad \left[\because \frac{7\pi}{6} \neq [0, \pi]\right]$$

$$= \cos^{-1}\left[\cos\frac{5\pi}{6}\right] \qquad [\because \cos(2\pi - \theta) = \cos\theta]$$

$$= \frac{5\pi}{6} \in [0, \pi]$$

$$\therefore \quad \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}.$$

S38. Principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ The principal value of

$$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right) \qquad [\because \tan(\pi - \theta) = -\tan\theta]$$

$$= \tan^{-1}\tan\left(-\frac{\pi}{4}\right) \qquad [\because -\tan\theta = \tan(-\theta)]$$

$$= -\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \quad \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = -\frac{\pi}{4}$$

S39. We need to find the value of $\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$.

Principal value branch for $\tan^{-1} x$ *is* $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and that of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$$\therefore \quad \tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \sin^{-1}\left(-\sin\frac{\pi}{6}\right) \qquad \left[\because \tan\frac{\pi}{4} = 1 \text{ and } \sin\frac{\pi}{6} = \frac{1}{2}\right]$$

$$= \tan^{-1}\tan\frac{\pi}{4} + \sin^{-1}\sin\left(-\frac{\pi}{6}\right) \qquad \left[\begin{array}{l}\because \sin(-\theta) = -\sin\theta \\ \text{So, } \sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6}\end{array}\right]$$

$$= \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}.$$

S40. We know that, principal value of $\cos^{-1}\left(\frac{1}{2}\right)$ is $\frac{\pi}{3}$ as $\cos^{-1} x \in [0, \pi]$ and $\sin^{-1}\frac{1}{2}$ is $\frac{\pi}{6}$ as $\sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$$\therefore \quad \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$

$$= \cos^{-1}\cos\frac{\pi}{3} + 2\sin^{-1}\sin\frac{\pi}{6}$$

$$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

S41. We know that the range of principal value branch of $\cos^{-1}x$ is $[0, \pi]$

$$\cos^{-1}\left(\cos\frac{25\pi}{6}\right) = \cos^{-1}\left[\cos\left(4\pi + \frac{\pi}{6}\right)\right]$$

Now, $\frac{\pi}{6} \in [0, \pi]$

$$\therefore \quad \cos^{-1}\left(\cos\frac{25\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] = \frac{\pi}{6}$$

S42. As principal value branch of $\cos^{-1} x$ is $[0, \pi]$

$$\therefore \quad \cos^{-1}\left(\cos\frac{13\pi}{6}\right) \neq \frac{13\pi}{6}, \ \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$$

$$= \cos^{-1}\cos\frac{\pi}{6} \qquad [\because \cos(2\pi + \theta) = \cos\theta]$$

$$= \frac{\pi}{6} \in [0, \pi]$$

$$\therefore \quad \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \frac{\pi}{6}$$

S43. We know that the principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\cot^{-1}x$ is $(0, \pi)$.

$\therefore$ Principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cot^{-1}\left(\cot\frac{7\pi}{6}\right)$

$$= \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\pi + \frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}.$$

S44. We know that the principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}x$ is $[0, \pi]$

$\therefore$ Principal value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$$= \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{6}\right)\right] + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}.$$

S45. We know that the principal value branch of $\text{cosec}^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ and $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore$ Principal value of $\text{cosec}^{-1}\left(\text{cosec}\,\frac{\pi}{6}\right) + \tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$$= \text{cosec}^{-1}\left(\text{cosec}\,\frac{\pi}{6}\right) + \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right]$$

$$= \frac{\pi}{6} + \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}.$$

S46. We know that the principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}x$ is $[0, \pi]$

$\therefore$ Principal value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

$$= \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{4}\right)\right] + \cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right] \qquad [\because \cos(-\theta) = \cos\theta]$$

$$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) + \cos^{-1}\left(\cos\frac{\pi}{3}\right)$$

$$= \frac{\pi}{4} + \frac{\pi}{3} = \frac{3\pi + 4\pi}{12} = \frac{7\pi}{12}$$

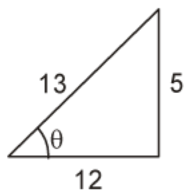
S47. We know that principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \quad \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$$

$$= \tan^{-1}\left[2 \times \frac{1}{2}\right] = \tan^{-1}(1) = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Thus principal value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ is $\frac{\pi}{4}$.

S48. $\tan\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$



Let $\quad \sin^{-1}\frac{5}{13} = \theta \;\Rightarrow\; \sin\theta = \frac{5}{13}$

$\therefore \qquad \tan\left(\sin^{-1}\frac{5}{13}\right) = \tan\theta = \frac{5}{12}$

S49. Given that, $\tan\left(2\tan^{-1}\frac{1}{5}\right)$

$$= \tan\left(\tan^{-1}\left[\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right]\right) \qquad \left[\because 2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right)\right]$$

$$= \tan\left(\tan^{-1}\left[\frac{2\times 5}{24}\right]\right)$$

$$= \tan\left(\tan^{-1}\left[\frac{5}{12}\right]\right) = \frac{5}{12}$$

S50. $$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \sin\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] \qquad \left[\because \sin^{-1}(-\theta) = -\sin^{-1}\theta\right]$$

$$= \sin\left[\frac{\pi}{2} + \sin^{-1}\left(\sin\frac{\pi}{3}\right)\right] = \sin\left[\frac{\pi}{2} + \frac{\pi}{3}\right]$$

$$= \sin\frac{5\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

S51. $\cos\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{4}\right)$

$$\Rightarrow \qquad \cos\left(\pi - \cos^{-1}\frac{\sqrt{3}}{2} + \frac{\pi}{4}\right)$$

$$\Rightarrow \qquad \cos\left(\pi - \frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\pi + \frac{\pi}{12}\right) = -\cos\frac{\pi}{12} = -\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$= -\left[\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right]$$

$$= -\left[\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right] = -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$$

S52. $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$

$$= \tan\left[\frac{1}{2}(2\tan^{-1}x) + \frac{1}{2}(2\tan^{-1}y)\right]$$

$$\left[\because \quad 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}\right]$$

$$= \tan(\tan^{-1}x + \tan^{-1}y)$$

$$= \tan\left(\tan^{-1}\frac{x+y}{1-xy}\right) \qquad \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \frac{x+y}{1-xy} \qquad [\because \tan(\tan^{-1}\theta) = \theta]$$

S53. $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$$= \tan^{-1}\left[2\sin\left(2\cdot\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\sin\frac{\pi}{3}\right] = \tan^{-1}\left(2\cdot\frac{\sqrt{3}}{2}\right)$$

$$= \tan^{-1}(\sqrt{3}) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$$

S54. $\tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right)$

Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right) = \theta \Rightarrow \cos\theta = \left(\frac{\sqrt{5}}{3}\right)$

$\therefore \quad \tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right) = \tan\frac{\theta}{2}$

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{1-\sqrt{5}/3}{1+\sqrt{5}/3}} = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$$

$$= \frac{3-\sqrt{5}}{2}$$

S55. We have,

$$\cos(\tan^{-1}x) = \cos\left\{\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right\} = \frac{1}{\sqrt{1+x^2}} \qquad \left[\because\ \tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$$

$\therefore$ $$\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$$

$$= \sin\left\{\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right\}$$

$$= \sin\left\{\sin^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right\} = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}} = \sqrt{\frac{x^2+1}{x^2+2}}$$

S56. L.H.S.

$$= \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$$

$$= \frac{9}{4}\cos^{-1}\frac{1}{3}$$

$$= \frac{9}{4}\sin^{-1}\sqrt{1-\frac{1}{9}}$$

$$= \frac{9}{4}\sin^{-1}\sqrt{\frac{8}{9}}$$

$$= \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}.$$

S57. Here,

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}},\ |x| > 1$$

Let $x = \sec\theta$, then

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}} = \tan^{-1}\frac{1}{\sqrt{\sec^2\theta - 1}} = \tan^{-1}\left(\frac{1}{\tan\theta}\right) = \tan^{-1}(\cot\theta)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\theta\right)\right] = \frac{\pi}{2}-\theta = \frac{\pi}{2}-\sec^{-1}x \qquad [\because \theta = \sec^{-1}x]$$

Thus,

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}} = \frac{\pi}{2}-\sec^{-1}x$$

S58. We have, $\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3)$

$$= \{\sec(\tan^{-1} 2)\}^2 + \{\text{cosec}\ (\cot^{-1} 3)\}^2$$

$$= \left\{\sec\left(\tan^{-1}\frac{2}{1}\right)\right\}^2 + \left\{\text{cosec}\left(\cot^{-1}\frac{3}{1}\right)\right\}^2$$

$$= \left\{\sec(\sec^{-1}\sqrt{5})\right\}^2 + \left\{\text{cosec}(\text{cosec}^{-1}\sqrt{10})\right\}^2 = (\sqrt{5})^2 + (\sqrt{10})^2 = 15$$

S59. $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi,$

$\Rightarrow \quad \cos^{-1} x + \cos^{-1} y = \pi - \cos^{-1} z$

$\Rightarrow \quad \cos^{-1} x + \cos^{-1} y = \cos^{-1}(-z) \qquad [\because \cos^{-1}(-z) = \pi - \cos^{-1} z]$

$\Rightarrow \quad \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\} = \cos^{-1}(-z)$

$\Rightarrow \quad xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$\Rightarrow \quad (xy + z)^2 = (1 - x^2)(1 - y^2)$

$\Rightarrow \quad x^2y^2 + z^2 + 2xyz = 1 - x^2 - y^2 + x^2y^2$

$\Rightarrow \quad x^2 + y^2 + z^2 + 2xyz = 1$

S60. We have,

$$\cos(2\cos^{-1}x + \sin^{-1}x) = \cos(\cos^{-1}x + \cos^{-1}x + \sin^{-1}x)$$

$$= \cos\left(\cos^{-1}x + \frac{\pi}{2}\right) = -\sin(\cos^{-1}x)$$

$$= -\sin\left(\sin^{-1}\sqrt{1-x^2}\right) = -\sqrt{1-x^2}$$

$$= -\sqrt{1-\frac{1}{25}} = -\sqrt{\frac{24}{25}}$$

S61. Given that,

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$\Rightarrow \quad \sin\left\{\frac{\pi}{2} - \tan^{-1} x\right\} = \sin\left(\cot^{-1}\frac{3}{4}\right) \qquad \left[\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right]$

$\Rightarrow \quad \frac{\pi}{2} - \tan^{-1} x = \cot^{-1}\frac{3}{4}$

$\Rightarrow \quad \tan^{-1} x + \cot^{-1}\frac{3}{4} = \frac{\pi}{2}$

This is only possible when $x = \frac{3}{4}$ $\qquad \left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in R\right]$

S62. To solve,

$$2 \tan^{-1} (\sin x) = \tan^{-1} (2 \sec x)$$

$\Rightarrow \quad \tan^{-1}\left(\frac{2\sin x}{1-\sin^2 x}\right) = \tan^{-1}\left(\frac{2}{\cos x}\right) \qquad \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), \sec x = \frac{1}{\cos x}\right]$

$\Rightarrow \quad \frac{2\sin x}{\cos^2 x} = \frac{2}{\cos x}$

$\Rightarrow \quad \tan x = 1$

$$x = \tan^{-1} (1) = \frac{\pi}{4}$$

S63. We have,

$$2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$$

$\Rightarrow \quad \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1} (2\operatorname{cosec} x)$

$\Rightarrow \quad \frac{2\cos x}{\sin^2 x} = 2 \operatorname{cosec} x$

$\Rightarrow \quad \cos x = \sin x \;\Rightarrow\; \tan x = 1 \;\Rightarrow\; x = \frac{\pi}{4}$

S64. Let $\quad \tan^{-1}(1) = \theta_1$

$\Rightarrow \quad \tan\theta_1 = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \qquad \theta_1 = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Let $\qquad \cos^{-1}\left(-\frac{1}{2}\right) = \theta_2$

$\Rightarrow \qquad \cos\theta_2 = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right)$

$= \qquad \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\Rightarrow \qquad \theta_2 = \frac{2\pi}{3} \in [0, \pi]$

Let $\qquad \sin^{-1}\left(-\frac{1}{2}\right) = \theta_3$

$\Rightarrow \qquad \sin\theta_3 = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow \qquad \theta_3 = -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \quad \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) = \theta_1 + \theta_2 + \theta_3 = \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}.$$

S65. To prove

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$$

$\therefore \qquad \text{L.H.S.} = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

$$= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{10}}\right) + \tan^{-1}\frac{1}{8}$$

[$\because$ Using $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$]

$$= \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{72}}\right)$$

$$[\because \text{ Using } \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1]$$

$$= \tan^{-1}\left(\frac{56+9}{72-7}\right) = \tan^{-1}\left(\frac{65}{65}\right)$$

$$= \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$= \frac{\pi}{4}$ = R.H.S. **Hence proved.**

S66.

$$\text{L.H.S.} = 2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right) + \tan^{-1}\frac{1}{7} \qquad \left[\because 2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{1}{\frac{3}{4}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left[\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right] \qquad \left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right) = \tan^{-1}\frac{31}{17} = \text{R.H.S.}$$

$\therefore$ **L.H.S. = R.H.S.**

Hence proved.

S67. To prove,

$$\sin^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$$

$$\text{R.H.S.} = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$$

Let $\sin^{-1}\frac{5}{13} = x \Rightarrow \sin x = \frac{5}{13}$

$$\cos^{-1}\frac{3}{5} = y \Rightarrow \cos y = \frac{3}{5}$$

Also,

$$\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$$

$$\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

We know that,

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$= \frac{5}{13}\times\frac{3}{5} + \frac{12}{13}\times\frac{4}{5} = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

$\Rightarrow$ $x + y = \sin^{-1}\left(\frac{63}{65}\right)$

$\Rightarrow$ $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{63}{65}\right)$

Hence proved.

S68. To prove,

$$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$$

Let $\cos^{-1}\left(\frac{4}{5}\right) = x$ ...(*i*)

and $\cos^{-1}\left(\frac{12}{13}\right) = y$ ...(ii)

$\Rightarrow$ $\cos x = \frac{4}{5}$ and $\cos y = \frac{12}{13}$

We know that,

$$\sin^2 x = 1 - \cos^2 x = 1-\frac{16}{25}=\frac{9}{25}$$

$\therefore$ $\sin x = \frac{3}{5}$

and $\sin^2 y = 1 - \cos^2 y = 1-\frac{144}{169}=\frac{25}{169}$

$\therefore$ $\sin y = \frac{5}{13}$

Now, we know that

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$\Rightarrow$ $\cos(x + y) = \left(\frac{4}{5}\times\frac{12}{13}\right)-\left(\frac{3}{5}\times\frac{5}{13}\right) = \frac{48}{65}-\frac{15}{65}$

$\therefore$ $\cos(x + y) = \frac{33}{65}$

$\Rightarrow$ $x + y = \cos^{-1}\frac{33}{65}$

$\Rightarrow$ $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$ $\left[\because \text{From Eqs. } (i) \text{ and } (ii),\ x=\cos^{-1}\frac{4}{5} \text{ and } y=\cos^{-1}\frac{12}{13}\right]$

$\therefore$ L.H.S. = R.H.S.

Hence proved.

S69. To prove,

$$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{36}{85}\right)$$

... (i)

Let $\sin^{-1}\left(\frac{8}{17}\right) = x$

and $\sin^{-1}\left(\frac{3}{5}\right) = y$ ... (ii)

$\Rightarrow$ $\sin x = \frac{8}{17}, \sin y = \frac{3}{5}$

$\because$ $\cos^2 x = 1 - \sin^2 x$

$= 1-\frac{64}{289}=\frac{225}{289}$

$\Rightarrow$ $\cos x = \frac{15}{17}$

Also,

$\because$ $\cos^2 y = 1 - \sin^2 y = 1-\frac{9}{25}=\frac{16}{25}$

$\Rightarrow$ $\cos y = \frac{4}{5}$

Now, we know that

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$\therefore$ $\cos(x + y) = \left(\frac{15}{17}\times\frac{4}{5}\right)-\left(\frac{8}{17}\times\frac{3}{5}\right)$ Using eq. (i) and (ii)

$\Rightarrow$ $\cos(x + y) = \frac{60}{85}-\frac{24}{85}=\frac{36}{85}$

$\Rightarrow$ $x + y = \cos^{-1}\left(\frac{36}{85}\right)$

$\Rightarrow$ $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5} = \cos^{-1}\frac{36}{85}$ Hence proved.

S70. Let

$\cos^{-1}\frac{12}{13}=x$ and $\sin^{-1}\frac{3}{5}=y$, so

$$\cos x = \frac{12}{13} \text{ and } \sin y = \frac{3}{5}$$

$\therefore$ $$\sin^2 x = 1 - \cos^2 x = 1 - \frac{144}{169} = \frac{25}{169} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

and $$\cos^2 y = 1 - \sin^2 y = 1 - \frac{9}{25} = \frac{16}{25}$$

$\Rightarrow$ $$\sin x = \frac{5}{13} \text{ and } \cos y = \frac{4}{5}$$

Now, $\sin(x + y) = \sin x \cos y + \cos x \sin y$

$\Rightarrow$ $$\sin(x + y) = \left(\frac{5}{13} \times \frac{4}{5}\right) + \left(\frac{12}{13} \times \frac{3}{5}\right)$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \frac{56}{65}$$

$\therefore$ $$\sin(x + y) = \frac{56}{65}$$

$\Rightarrow$ $$x + y = \sin^{-1}\frac{56}{65}$$

$\therefore$ $$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

Hence proved.

S71. $$\text{L.H.S.} = 2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right) \qquad \left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1 - x^2}\right]$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \times \frac{17}{31}}\right) \qquad \left[\because \tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{24 \times 31 - 17 \times 7}{7 \times 31 + 24 \times 17}\right)$$

$$= \tan^{-1}\left(\frac{744 - 119}{217 + 408}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1} \tan \frac{\pi}{4} \qquad \left[\because 1 = \tan \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

= R.H.S.

$\because$ L.H.S. = R.H.S. **Hence proved.**

S72. To prove

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \left(\frac{1}{2}\right) \tan^{-1}\left(\frac{4}{3}\right) \qquad ..(i)$$

Above equation may be written as

$$2\left[\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)\right] = \tan^{-1}\left(\frac{4}{3}\right) \qquad ...(ii)$$

Now, we prove Eq. (ii) as it is equivalent to Eq. (i).

$$\text{L.H.S.} = 2\left[\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)\right]$$

$$= 2\left[\tan^{-1}\left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}}\right)\right] \qquad \left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= 2\tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right) = 2\tan^{-1}\left(\frac{17}{34}\right)$$

$$= 2\tan^{-1}\left(\frac{1}{2}\right)$$

$$= \tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right) \qquad \left[\because\ 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) = \tan^{-1}\left(\frac{4}{3}\right) = \text{R.H.S.}$$

$\therefore$ **L.H.S. = R.H.S. Hence proved.**

S73. L.H.S. $= 2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$$= \tan^{-1}\left(\frac{\frac{2}{3}}{1-\frac{1}{9}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) \qquad \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right) = \tan^{-1}(1)$$

$$= \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4} = \text{R.H.S.}$$ **L.H.S. = R.H.S.** **Hence proved.**

S74. Here,

$$\text{L.H.S.} = 2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left[\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right] + \tan^{-1}\left(\frac{1}{4}\right) \qquad \left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\left[\frac{\frac{2}{5}}{1-\frac{1}{25}}\right] + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left[\frac{2}{5}\times\frac{25}{24}\right] + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left[\frac{5}{12}\right] + \tan^{-1}\left(\frac{1}{4}\right)$$

$$= \tan^{-1}\left[\frac{\frac{5}{12}+\frac{1}{4}}{1-\frac{5}{12}\times\frac{1}{4}}\right] \qquad \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\left[\frac{\frac{20+12}{48}}{\frac{48-5}{48}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{32}{48}}{\frac{43}{48}}\right]$$

$$= \tan^{-1}\left[\frac{32}{48}\times\frac{48}{43}\right]$$

$$= \tan^{-1}\left[\frac{32}{43}\right] = \textbf{R.H.S.}$$

S75.

$$\text{L.H.S.} = \sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)$$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{16}{25}}\right]+\sin^{-1}\left(\frac{16}{65}\right)$$

$$\left[\because \sin^{-1}x+\sin^{-1}y=\sin^{-1}\left\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\right\}\right]$$

$$= \sin^{-1}\left[\frac{4}{5}\times\sqrt{\frac{144}{169}}+\frac{5}{13}\times\sqrt{\frac{9}{25}}\right]+\sin^{-1}\left(\frac{16}{65}\right)$$

$$= \sin^{-1}\left[\left(\frac{4}{5}\times\frac{12}{13}\right)+\left(\frac{5}{13}\times\frac{3}{5}\right)\right]+\sin^{-1}\left(\frac{16}{65}\right)$$

$$= \sin^{-1}\frac{63}{65}+\sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{63}{65}\sqrt{1-\left(\frac{16}{65}\right)^2}+\frac{16}{65}\sqrt{1-\left(\frac{63}{65}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{63}{65}\sqrt{\frac{4225-256}{4225}}+\frac{16}{65}\sqrt{\frac{4225-3969}{4225}}\right]$$

$$= \sin^{-1}\left[\frac{63}{65}\times\sqrt{\frac{3969}{4225}}+\frac{16}{65}\times\sqrt{\frac{256}{4225}}\right]$$

$$= \sin^{-1}\left(\frac{63}{65}\times\frac{63}{65}+\frac{16}{65}\times\frac{16}{65}\right)$$

$$= \sin^{-1}\left(\frac{3969+256}{4225}\right)$$

$$= \sin^{-1}\left(\frac{4225}{4225}\right) = \sin^{-1}(1) = \frac{\pi}{2} = \textbf{R.H.S.}$$

S76. L.H.S. = $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}$

Let $x = \sin^{-1}\frac{3}{5}$ and $y=\sin^{-1}\frac{8}{17}$

Then, $\sin x = \frac{3}{5}$ and $\sin y = \frac{8}{17}$

$\therefore$ $\cos x = \sqrt{1-\sin^2 x}=\sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}}=\frac{4}{5}$

and $\cos y = \sqrt{1-\sin^2 y}=\sqrt{1-\frac{64}{289}} = \sqrt{\frac{225}{289}}=\frac{15}{17}$

Now, using the identity

$\cos(x + y) = \cos x \cos y - \sin x \sin y$, we get

$\cos(x + y) = \left(\frac{4}{5}\times\frac{15}{17}\right)-\left(\frac{3}{5}\times\frac{8}{17}\right)$

or $\cos(x + y) = \frac{60}{85}-\frac{24}{85}=\frac{36}{85}$ $\Rightarrow x + y = \cos^{-1}\frac{36}{85}$

$\Rightarrow$ $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17} = \cos^{-1}\frac{36}{85}$ $\left[\because x=\sin^{-1}\frac{3}{5}\text{ and } y=\sin^{-1}\frac{8}{17}\right]$

L.H.S = R.H.S.

Hence proved.

S77. R.H.S.

$= \sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

$= \sin^{-1}\frac{5}{13}+\sin^{-1}\sqrt{1-\left(\frac{3}{5}\right)^2}$ $\left[\because \cos^{-1}x=\sin^{-1}\sqrt{1-x^2}\right]$

$= \sin^{-1}\frac{5}{13}+\sin^{-1}\frac{4}{5}$

$= \sin^{-1}\left[\frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}+\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}\right]$ $\left[\because \sin^{-1}x+\sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]\right]$

$$= \sin^{-1}\left[\frac{5}{13}\times\frac{3}{5}+\frac{4}{5}\times\frac{12}{13}\right] = \sin^{-1}\left[\frac{3}{13}+\frac{48}{65}\right] = \sin^{-1}\left[\frac{63}{65}\right]$$

$$= \tan^{-1}\left[\frac{\frac{63}{65}}{\sqrt{1-\left(\frac{63}{65}\right)^2}}\right] \qquad \left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$$

$$\tan^{-1}\left[\frac{\frac{63}{65}}{\frac{16}{65}}\right] = \tan^{-1}\frac{63}{16} = \textbf{L.H.S.}$$

S78.

$$\text{L.H.S.} = \sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}$$

$$= \sin^{-1}\left[\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}+\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2}\right] \qquad \left[\because \sin^{-1}x+\sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)\right]$$

$$= \sin^{-1}\left[\frac{8}{17}\times\frac{4}{5}+\frac{3}{5}\times\frac{15}{17}\right] = \sin^{-1}\left[\frac{32}{85}+\frac{45}{85}\right] = \sin^{-1}\left[\frac{77}{85}\right]$$

$$= \tan^{-1}\left[\frac{\frac{77}{85}}{\sqrt{1-\left(\frac{77}{85}\right)^2}}\right] \qquad \left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{77}{85}}{\frac{36}{85}}\right] = \tan^{-1}\frac{77}{36} = \textbf{R. H.S.}$$

S79.

$$\text{L.H.S.} = 2\sin^{-1}\frac{3}{5}$$

$$= \sin^{-1}\left[2\times\frac{3}{5}\sqrt{1-\left(\frac{3}{5}\right)^2}\right] \qquad \left[\because 2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)\right]$$

$$= \sin^{-1}\left[2\times\frac{3}{5}\times\frac{4}{5}\right] = \sin^{-1}\left(\frac{24}{25}\right)$$

$$= \tan^{-1}\left[\frac{\frac{24}{25}}{\sqrt{1-\left(\frac{24}{25}\right)^2}}\right] \qquad \left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{24}{25}}{\sqrt{1-\frac{576}{625}}}\right] = \tan^{-1}\left[\frac{24}{25}\times\frac{25}{7}\right]$$

$$= \tan^{-1}\left[\frac{24}{7}\right] = \textbf{R.H.S.}$$

S80. We have,

$$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$$

$$= 2\left\{2\tan^{-1}\frac{1}{5}\right\} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$$

$$= 2\left\{\tan^{-1}\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right\} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} \qquad \left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}, \text{ if } |x|<1\right]$$

$$= 2\tan^{-1}\frac{5}{12} - \left\{\tan^{-1}\frac{1}{70} - \tan^{-1}\frac{1}{99}\right\}$$

$$= \tan^{-1}\left\{\frac{2\times\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2}\right\} - \tan^{-1}\left\{\frac{\frac{1}{70}-\frac{1}{99}}{1+\frac{1}{70}\times\frac{1}{99}}\right\}$$

$$= \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{29}{6931} = \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\left\{\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\times\frac{1}{239}}\right\} = \left[\frac{\frac{28680-119}{28441}}{\frac{28441+120}{28441}}\right] = \tan^{-1}\left[\frac{28561}{28561}\right] = \tan^{-1}1 = \frac{\pi}{4}.$$

S81. L.H.S.

$$= \cos\left[\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right]$$

$$= \cos\left[\sin^{-1}\left\{\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}\right\}\right]$$

$$\left[\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]\right]$$

$$= \cos\left[\sin^{-1}\left\{\frac{3}{5}\sqrt{\frac{169-25}{169}} + \frac{5}{13}\sqrt{\frac{25-9}{25}}\right\}\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{3}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{4}{5}\right)\right] = \cos\left[\sin^{-1}\left(\frac{36}{65} + \frac{20}{65}\right)\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{56}{65}\right)\right] = \cos\left[\cos^{-1}\left(\sqrt{1-\left(\frac{56}{65}\right)^2}\right)\right]$$

$$\left[\because \sin^{-1}x = \cos^{-1}\sqrt{1-x^2}\right]$$

$$= \cos\left[\cos^{-1}\left(\sqrt{\frac{4225-3136}{4225}}\right)\right]$$

$$= \cos\left[\cos^{-1}\left(\sqrt{\frac{1089}{4225}}\right)\right]$$

$$= \cos\left[\cos^{-1}\left(\frac{33}{65}\right)\right] = \frac{33}{65} = \textbf{R.H.S.}$$

S82. Here

$$\text{L.H.S.} = \sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17}$$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2} - \frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right] \quad \left[\begin{array}{l}\because \sin^{-1}x - \sin^{-1}y = \\ \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]\end{array}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\frac{64}{289}} - \frac{8}{17}\sqrt{1-\frac{9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{\frac{289-64}{289}} - \frac{8}{17}\sqrt{\frac{25-9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\times\frac{15}{17} - \frac{8}{17}\times\frac{4}{5}\right] = \sin^{-1}\left[\frac{45}{85}-\frac{32}{85}\right] = \sin^{-1}\left[\frac{13}{85}\right]$$

$$= \cos^{-1}\left[\sqrt{1-\left(\frac{13}{85}\right)^2}\right] \qquad \left[\because \sin^{-1}x = \cos^{-1}\sqrt{1-x^2}\right]$$

$$= \cos^{-1}\left[\sqrt{1-\frac{169}{7225}}\right] = \cos^{-1}\left[\sqrt{\frac{7225-169}{7225}}\right] = \cos^{-1}\left[\sqrt{\frac{7056}{7225}}\right]$$

$$= \cos^{-1}\left(\frac{84}{85}\right) = \text{R.H.S.}$$

S83. Here,

$$\text{L.H.S.} = \cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$$

Put $\frac{3}{5} = \cos\theta$

Then $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$

$\Rightarrow$ $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

Now $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right) = \cos^{-1}(\cos\theta\cos x + \sin\theta\sin x)$

$$= \cos^{-1}[\cos(x - \theta)] = x - \theta$$

$$= x - \tan^{-1}\left(\frac{4}{3}\right) \qquad \left[\because \theta = \tan^{-1}\left(\frac{4}{3}\right)\right]$$

= R.H.S.

Thus $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right) = x - \tan^{-1}\left(\frac{4}{3}\right)$

S84. L.H.S.

$$= \left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5}\right] - \tan^{-1}\frac{8}{19} \qquad \left[\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{9}{20}}\right) - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left(\frac{\frac{27}{20}}{\frac{11}{20}}\right) - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left(\frac{27}{11}\right) - \tan^{-1}\left(\frac{8}{19}\right) \qquad \left[\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= \tan^{-1}\left[\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\times\frac{8}{19}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{513-88}{209}}{\frac{209+216}{209}}\right]$$

$$= \tan^{-1}\left(\frac{425}{209}\times\frac{209}{425}\right) = \tan^{-1}(1)$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4}\right) = \frac{\pi}{4} = \text{R.H.S.}$$

$\because$ L.H.S. = R.H.S. **Hence proved**

S85. $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}$

$$= 2\left\{\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right\}+\sec^{-1}\frac{5\sqrt{2}}{7}$$

$$= 2\tan^{-1}\left\{\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}}\right\}+\tan^{-1}\sqrt{\left(\frac{5\sqrt{2}}{7}\right)^2-1} \qquad \left[\because \sec^{-1}x=\tan^{-1}\sqrt{x^2-1}\right]$$

$$= 2\tan^{-1}\frac{13}{39}+\tan^{-1}\frac{1}{7} = 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left\{\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right\}+\tan^{-1}\frac{1}{7} \qquad \left[\because 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2},\text{ if } |x|<1\right]$$

$$= \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7} = \tan^{-1}\left\{\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right\} = \tan^{-1}\left[\frac{\frac{21+4}{28}}{\frac{28-3}{28}}\right]=\tan^{-1}\left[\frac{25}{25}\right] = \tan^{-1}1$$

$$= \frac{\pi}{4}.$$

S86. Here,

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x},\ x\neq 0$$

Let $\qquad x = \tan\theta$, then

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right] = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right] \quad \left[\because \cos 2\theta = 1-2\sin^2\theta,\ \sin 2\theta = 2\sin\theta\cos\theta\right]$$

$$= \tan^{-1}\left[\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right] = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x \qquad [\because \theta = \tan^{-1} x]$$

Thus $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \frac{1}{2}\tan^{-1} x$.

S87. From L.H.S

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$$

$$= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1+\frac{x}{y}\cdot\frac{x-y}{x+y}}\right] \qquad \left[\because \tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+x\cdot y}\right)\right]$$

$$= \tan^{-1}\left[\frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}\right]$$

$$= \tan^{-1}\left(\frac{x^2 + \cancel{xy} - \cancel{xy} + y^2}{\cancel{xy} + y^2 + x^2 - \cancel{xy}}\right)$$

$$= \tan^{-1}\left(\frac{x^2+y^2}{y^2+x^2}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S}$$

L.H.S = R.H.S Hence proved

S88. L.H.S.

$$= \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\right] \qquad \left[\begin{array}{l}\because \cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \\ \text{and } \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\end{array}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right] \qquad [\because a^2-b^2=(a-b)(a+b)]$$

Dividing the numerator and denominator by cos $\frac{x}{2}$, we get

$$= \tan^{-1}\left[\frac{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}}-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}}+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}\right]$$

$$= \tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}\cdot\tan\frac{x}{2}}\right] \qquad \left[\begin{array}{l}\because 1=\tan\frac{\pi}{4}\text{ and}\\ 1\cdot\tan\frac{x}{2}=\tan\frac{\pi}{4}.\tan\frac{x}{2}\end{array}\right]$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{\pi}{4}-\frac{x}{2} \qquad \left[\because \frac{\tan A-\tan B}{1+\tan A\tan B}=\tan(A-B)\right]$$

= R.H.S.

$\therefore$ L.H.S. = R.H.S. **Hence proved**

S89. Using the relation

$$1+\sin x = \cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$= \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$$

$$1-\sin x = \cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$= \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2$$

$\therefore$ L.H.S. = $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$$

$$= \cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$$

$$= \cot^{-1}\left[\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right] = \cot^{-1}\cot\frac{x}{2}$$

$$= \frac{x}{2}.$$

$\therefore$ L.H.S. = R.H.S. **Hence proved**

S90. L.H.S.

$$= \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) \qquad \text{...(i)}$$

Let $\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right) = \theta$ ...(ii)

$\Rightarrow$ $\sin^{-1}\left(\frac{3}{4}\right) = 2\theta$

$\Rightarrow$ $\sin 2\theta = \frac{3}{4}$

$\Rightarrow$ $\dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{3}{4}$ $\left[\because \sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}\right]$

$\Rightarrow$ $8\tan\theta = 3 + 3\tan^2\theta$

$\Rightarrow$ **$3\tan^2\theta - 8\tan\theta + 3 = \theta$**

Now, by **Sridharacharya rule**

$$\tan\theta = \frac{8 \pm \sqrt{64-36}}{2\times 3} = \frac{8 \pm \sqrt{28}}{6}$$

$$\Rightarrow \qquad \tan\theta = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\left(\frac{4 \pm \sqrt{7}}{3}\right) \qquad \text{[From Eq. (ii)]}$$

$$\Rightarrow \qquad \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{4 \pm \sqrt{7}}{3}\right)$$

Taking (–)ve sign,

$$\Rightarrow \qquad \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{4 - \sqrt{7}}{3}\right)$$

$$\Rightarrow \qquad \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \tan\left\{\tan^{-1}\left(\frac{4 - \sqrt{7}}{3}\right)\right\}$$

$$\Rightarrow \qquad \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4 - \sqrt{7}}{3}$$

L.H.S. = R.H.S. **Hence proved.**

S91. $\tan\left\{2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right\}$

$$= \tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right) - \tan^{-1}1\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{2/5}{24/25}\right) - \tan^{-1}1\right\}$$

$$= \tan\left\{\tan^{-1}\frac{5}{12} - \tan^{-1}1\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right)\right\} = \tan\left\{\tan^{-1}\left(\frac{-7/12}{17/12}\right)\right\} = \frac{-7}{17}.$$

S92. Put $x = \cos 2\theta$, so that

$2\theta = \cos^{-1} x$ and $\theta = \frac{1}{2}\cos^{-1} x$

$$\therefore \quad \text{L.H.S.} = \tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right] \qquad \left[\because \begin{array}{l} 1+\cos 2A = 2\cos^2 A \\ 1-\cos 2A = 2\sin^2 A \end{array}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right]$$

$$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

Dividing numerator and denominator by $\cos\theta$, we get

$$\text{L.H.S.} = \tan^{-1}\left(\frac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}\right) = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\theta\tan\frac{\pi}{4}}\right] \qquad \left[\because 1 = \tan\frac{\pi}{4} \text{ and } 1\cdot\tan\theta = \tan\frac{\pi}{4}\cdot\tan\theta\right]$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right) \qquad \left[\because \tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\cdot\tan B}\right]$$

$$= \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

L.H.S. = R.H.S. **Hence proved.**

S93. We have,

$$\text{R.H.S.} = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$$

$$\Rightarrow \quad = \cos^{-1}\left\{\frac{1-\tan^2\frac{\alpha}{2}\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\alpha}{2}\tan^2\frac{\beta}{2}}\right\} \qquad \left[\therefore 2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$= \cos^{-1}\left\{\frac{\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2} - \sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}}{\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2} + \sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}}\right\}$$

$$= \cos^{-1}\left\{\frac{(1+\cos\alpha)(1+\cos\beta) - (1-\cos\alpha)(1-\cos\beta)}{(1+\cos\alpha)(1+\cos\beta) + (1-\cos\alpha)(1-\cos\beta)}\right\}$$

$$= \cos^{-1}\left[\frac{1+\cos\beta + \cos\alpha + \cos\alpha\cos\beta - 1 + \cos\beta + \cos\alpha - \cos\alpha\cos\beta}{1+\cos\alpha + \cos\beta + \cos\alpha\cos\beta + 1 - \cos\alpha - \cos\beta + \cos\alpha\cos\beta}\right]$$

$$= \cos^{-1}\left(\frac{\cos\alpha + \cos\beta}{1+\cos\alpha\cos\beta}\right) = \textbf{L.H.S.}$$

S94. Put $\sqrt{x} = \tan\theta$

$$\Rightarrow \quad \theta = \tan^{-1}\sqrt{x}$$

$$\text{R.H.S.} = \frac{1}{2}\cos^{-1}\left[\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right]$$

On substituting $\sqrt{x} = \tan\theta$, we get

$$\text{R.H.S.} = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{1}{2}\cos^{-1}\cos 2\theta \qquad \left[\because \frac{1-\tan^2 A}{1+\tan^2 A} = \cos 2A\right]$$

$$= \frac{1}{2}(2\theta)$$

$$= \theta \qquad [\because \theta = \tan^{-1}\sqrt{x}]$$

$= \tan^{-1}\sqrt{x}$ = L.H.S. **Hence proved.**

S95. Let

$$\cos^{-1}\left(\frac{a}{b}\right) = \theta. \text{ Then, } \cos\theta = \frac{a}{b}$$

Now, $\quad \text{LHS} = \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$\Rightarrow \quad \text{LHS} = \dfrac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}+\dfrac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$

$\Rightarrow \quad \text{LHS} = \dfrac{(1+\tan\frac{\theta}{2})^2 + (1-\tan\frac{\theta}{2})^2}{1-\tan^2\frac{\theta}{2}}$

$\Rightarrow \quad \text{LHS} = 2\left(\dfrac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right) = \dfrac{2}{\cos\theta} = \dfrac{2b}{a} = \text{RHS.}$

S96. We have,

$$\tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\frac{x}{2}}+\sqrt{2\sin^2\frac{x}{2}}}{\sqrt{2\cos^2\frac{x}{2}}-\sqrt{2\sin^2\frac{x}{2}}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2}\left|\cos\frac{x}{2}\right|+\sqrt{2}\left|\sin\frac{x}{2}\right|}{\sqrt{2}\left|\cos\frac{x}{2}\right|-\sqrt{2}\left|\sin\frac{x}{2}\right|}\right\}$$

$$= \tan^{-1}\left\{\frac{-\cos\frac{x}{2}+\sin\frac{x}{2}}{-\cos\frac{x}{2}-\sin\frac{x}{2}}\right\} \qquad \left[\begin{array}{l}\because \pi < x < \frac{3\pi}{2} \therefore \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \\ \therefore \left|\cos\frac{x}{2}\right| = -\cos\frac{x}{2}, \left|\sin\frac{x}{2}\right| = \sin\frac{x}{2}\end{array}\right]$$

$$= \tan^{-1}\left\{\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right\} = \tan^{-1}\left\{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}$$

$$= \frac{\pi}{4} - \frac{x}{2}.$$

S97. We have,

$$\tan^{-1}\left\{\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\frac{x}{2}} + \sqrt{2\sin^2\frac{x}{2}}}{\sqrt{2\cos^2\frac{x}{2}} - \sqrt{2\sin^2\frac{x}{2}}}\right\} \qquad \left\{\because 0 < \frac{x}{2} < \frac{\pi}{4} \therefore \cos\frac{x}{2} > 0, \sin\frac{x}{2} > 0\right\}$$

$$= \tan^{-1}\left\{\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right\}$$

$$= \tan^{-1}\left\{\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right\} \qquad \left\{\because 0 < x < \frac{\pi}{2} \ \therefore \frac{x}{4} < \frac{\pi}{4} + \frac{x}{2} \ < \frac{\pi}{2}\right\}$$

$$= \frac{\pi}{4} + \frac{x}{2}.$$

S98. L.H.S.

$$= \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$= \tan^{-1}\left[\frac{x + \frac{2x}{1-x^2}}{1 - x\left(\frac{2x}{1-x^2}\right)}\right] \qquad \left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left[\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right]$$

$$= \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$= \text{R.H.S.}$ **Hence proved.**

Alternate Method

We need to prove that

$$\tan^{-1} x + \tan^{-1}\left(\frac{2x}{1 - x^2}\right) = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

Let $\tan^{-1} x = \theta$

Then, $x = \tan\theta$

$$\text{R.H.S.} = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$$= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right) \qquad [\because x = \tan\theta]$$

$$= \tan^{-1}\tan 3\theta \qquad \left[\because \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right]$$

$$= 3\theta = 3\tan^{-1} x \qquad [\because \theta = \tan^{-1} x]$$

$$= \tan^{-1} x + 2\tan^{-1} x$$

$$= \tan^{-1} x + \tan^{-1}\frac{2x}{1 - x^2} \qquad \left[\because 2\tan^{-1} x = \tan^{-1}\frac{2x}{1 - x^2}\right]$$

$= \text{L.H.S.}$

Hence proved.

S99. Let $\cos^{-1} x = y$. Then, $x = \cos y$.

$$\therefore \quad 2\sin^{-1}\sqrt{\frac{1 - x}{2}} = 2\sin^{-1}\left\{\sqrt{\frac{1 - \cos y}{2}}\right\}$$

$$= 2\sin^{-1}\sqrt{\frac{2\sin^2\frac{y}{2}}{2}}$$

$$= 2\sin^{-1}\left\{\sin\frac{y}{2}\right\}$$

$$= 2\left(\frac{y}{2}\right) = y = \cos^{-1} x$$

and, $2\cos^{-1}\sqrt{\frac{1+x}{2}}$

$$= 2\cos^{-1}\sqrt{\frac{1+\cos y}{2}} = 2\cos^{-1}\sqrt{\frac{2\cos^2\frac{y}{2}}{2}}$$

$$= 2\cos^{-1}\left(\cos\frac{y}{2}\right) = 2\cdot\left(\frac{y}{2}\right) = y = \cos^{-1} x$$

Hence, $\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}.$

S100. We have,

$$y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$$

$\Rightarrow$ $$y = \frac{\pi}{2} - \tan^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$$

$\Rightarrow$ $$y = \frac{\pi}{2} - 2\tan^{-1}(\sqrt{\cos x})$$

$\Rightarrow$ $$y = \frac{\pi}{2} - \cos^{-1}\left\{\frac{1-(\sqrt{\cos x})^2}{1+(\sqrt{\cos x})^2}\right\}$$ $\left[\because 2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$

$\Rightarrow$ $$y = \frac{\pi}{2} - \cos^{-1}\left\{\frac{1-\cos x}{1+\cos x}\right\}$$

$\Rightarrow$ $$y = \frac{\pi}{2} - \cos^{-1}\left(\tan^2\frac{x}{2}\right)$$

$\Rightarrow$ $$\cos^{-1}\left(\tan^2\frac{x}{2}\right) = \frac{\pi}{2} - y$$

$\Rightarrow$ $$\tan^2\frac{x}{2} = \cos\left(\frac{\pi}{2} - y\right)$$

$\Rightarrow$ $$\tan^2\frac{x}{2} = \sin y.$$ Proved.

S101. We have

$$\text{L.H.S.} = \sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)$$

Put $a = \tan\alpha$ and $b = \tan\beta$

$$= \sin^{-1}\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right) - \cos^{-1}\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)$$

$$= \sin^{-1}(\sin 2\alpha) - \cos^{-1}(\cos 2\beta)$$

$$= 2\alpha - 2\beta = 2(\alpha - \beta)$$

$$= 2(\tan^{-1}a - \tan^{-1}b)$$

$$= 2\tan^{-1}\left(\frac{a-b}{1+ab}\right)$$

Now $\text{R.H.S} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put $x = \tan\theta$

$$= \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}\tan 2\theta = 2\theta$$

$$= 2\tan^{-1}x$$

As L.H.S. = **R.H.S.**

$$\Rightarrow \quad 2\tan^{-1}\left(\frac{a-b}{1+ab}\right) = 2\tan^{-1}x$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1}x$$

$$\therefore \quad x = \frac{a-b}{1+ab}$$

S102. **Using:** $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, we get

$$\text{L.H.S.} = 2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right) = \cos^{-1}\left\{\frac{1-\frac{a-b}{a+b}\tan^2\frac{\theta}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{\theta}{2}}\right\}$$

$$\Rightarrow \quad = \cos^{-1}\left\{\frac{a(1-\tan^2\frac{\theta}{2}) + b(1+\tan^2\frac{\theta}{2})}{a(1+\tan^2\frac{\theta}{2}) + b(1-\tan^2\frac{\theta}{2})}\right\}$$

$$\Rightarrow \quad = \cos^{-1}\left\{\frac{a\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)+b}{a+b\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)}\right\}$$

$$= \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right\}$$

= **R.H.S**

S103. We have,

$$\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$$

$$\Rightarrow \quad \sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$$

$$\Rightarrow \quad \cos(\sin^{-1} x + \sin^{-1} y) = \cos(\pi - \sin^{-1} z)$$

$$\Rightarrow \quad \cos(\sin^{-1} x)\cos(\sin^{-1} y) - \sin(\sin^{-1} x)\sin(\sin^{-1} y) = -\cos(\sin^{-1} z)$$

$$\Rightarrow \quad \sqrt{1-x^2}\sqrt{1-y^2} - xy = -\sqrt{1-z^2} \qquad \left[\begin{array}{l}\because\ \cos(\sin^{-1} x) \\ = \cos(\cos^{-1}\sqrt{1-x^2}) \\ = \sqrt{1-x^2}\end{array}\right]$$

$$\Rightarrow \quad \sqrt{(1-x^2)(1-y^2)} = xy - \sqrt{1-z^2}$$

$$\Rightarrow \quad 1 - x^2 - y^2 + x^2y^2 = x^2y^2 + 1 - z^2 - 2xy\sqrt{1-z^2}$$

[On squaruing both sides]

$$\Rightarrow \quad x^2 + y^2 - z^2 = 2xy\sqrt{1-z^2}$$

$$\Rightarrow \quad (x^2 + y^2 - z^2)^2 = 4x^2y^2(1 - z^2)$$

[Squaring again both sides]

$$\Rightarrow \quad x^4 + y^4 + z^4 - 2x^2 z^2 - 2y^2 z^2 + 2x^2 y^2 = 4x^2 y^2 - 4x^2 y^2 z^2$$

$$\Rightarrow \quad x^4 + y^4 + z^4 + 4x^2 y^2 z^2 = 2(x^2 y^2 + y^2 z^2 + z^2 x^2)$$

S104. $y = \sec\left[\cot^{-1}\left\{\sin\left(\tan^{-1}\left(\operatorname{cosec}\left(\cos^{-1} a\right)\right)\right)\right\}\right]$

$$= \sec\left[\cot^{-1}\left\{\sin\left(\tan^{-1}\left(\operatorname{cosec}\left(\operatorname{cosec}^{-1}\frac{1}{\sqrt{1-a^2}}\right)\right)\right)\right\}\right] \quad \left[\because \cos^{-1} a = \operatorname{cosec}^{-1}\frac{1}{\sqrt{1-a^2}}\right]$$

$$= \sec\left[\cot^{-1}\left\{\sin\left(\tan^{-1}\frac{1}{\sqrt{1-a^2}}\right)\right\}\right]$$

$$= \sec\left[\cot^{-1}\left\{\sin\left(\sin^{-1}\frac{1}{\sqrt{2-a^2}}\right)\right\}\right] \quad \left[\because \tan^{-1}\frac{1}{\sqrt{1-a^2}} = \sin^{-1}\frac{1}{\sqrt{2-a^2}}\right]$$

$$= \sec\left(\cot^{-1}\frac{1}{\sqrt{2-a^2}}\right) = \sec\left(\sec^{-1}\sqrt{3-a^2}\right) = \sqrt{3-a^2}$$

$\therefore \quad y^2 = 3 - a^2.$

S105. $x = \operatorname{cosec}\left[\tan^{-1}\left\{\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1} a\right)\right)\right)\right\}\right]$

$$= \operatorname{cosec}\left[\tan^{-1}\left\{\cos\left(\cot^{-1}\left(\sec\left(\sec^{-1}\frac{1}{\sqrt{1-a^2}}\right)\right)\right)\right\}\right] \quad \left[\because \sin^{-1} a = \sec^{-1}\frac{1}{\sqrt{1-a^2}}\right]$$

$$= \operatorname{cosec}\left[\tan^{-1}\left\{\cos\left(\cot^{-1}\frac{1}{\sqrt{1-a^2}}\right)\right\}\right] \quad \left[\because \cot^{-1}\frac{1}{\sqrt{1-a^2}} = \cos^{-1}\frac{1}{\sqrt{2-a^2}}\right]$$

$$= \operatorname{cosec}\left[\tan^{-1}\left\{\cos\left(\cos^{-1}\frac{1}{\sqrt{2-a^2}}\right)\right\}\right]$$

$$= \operatorname{cosec}\left(\tan^{-1}\frac{1}{\sqrt{2-a^2}}\right) = \operatorname{cosec}\left(\operatorname{cosec}^{-1}\sqrt{3-a^2}\right) = \sqrt{3-a^2}$$

$\therefore \quad x^2 = 3 - a^2$

S106. L.H.S.

$$= \sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2}$$

Put $a = \tan\alpha,\ b = \tan\beta$

$$= \sin^{-1}\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right) + \sin^{-1}\left(\frac{2\tan\beta}{1+\tan^2\beta}\right)$$

$$= \sin^{-1}\sin 2\alpha + \sin^{-1}\sin 2\beta$$

$$= 2\alpha + 2\beta = 2(\alpha + \beta)$$

$$= 2(\tan^{-1}a + \tan^{-1}b)$$

$$= 2\tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

R.H.S. $= 2\tan^{-1}x$

As L.H.S. = R.H.S.

$$2\tan^{-1}\left(\frac{a+b}{1-ab}\right) = 2\tan^{-1}x$$

$$\tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1}x$$

$$\Rightarrow \quad x = \frac{a+b}{1-ab}$$

S107. we have,

$$\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$$

$$\Rightarrow \quad \cos^{-1}\left\{\frac{xy}{ab} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right\} = \alpha$$

$$\Rightarrow \quad \frac{xy}{ab} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}} = \cos\alpha$$

$$\Rightarrow \quad \frac{xy}{ab} - \cos\alpha = \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$$

$$\Rightarrow \quad \left(\frac{xy}{ab} - \cos\alpha\right)^2 = \left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$$

$$\Rightarrow \quad \frac{x^2y^2}{a^2b^2} - \frac{2xy}{ab}\cos\alpha + \cos^2\alpha = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2}$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = 1 - \cos^2\alpha$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha$$

S108. We have,

$$\sin(\cot^{-1} x) = \sin\left\{\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right\} = \frac{1}{\sqrt{1+x^2}}$$

$$\therefore \quad \cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$$

$$= \cos\left\{\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right\}$$

$$= \cos\left\{\cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right\} = \sqrt{\frac{1+x^2}{2+x^2}} = \sqrt{\frac{x^2+1}{x^2+2}}$$

S109. We have,

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\} = \alpha$$

$$\Rightarrow \quad \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} = \tan\alpha$$

$$\Rightarrow \quad -\frac{\sqrt{1+x^2}}{\sqrt{1-x^2}} = \frac{\tan\alpha+1}{\tan\alpha-1} \qquad \left\{\begin{array}{l}\text{using componendo dividendo} \\ \text{if } \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a+b}{a-b}=\frac{c+d}{c-d}\end{array}\right\}$$

$$\Rightarrow \quad \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}} = \frac{1-\tan\alpha}{1+\tan\alpha}$$

$$\Rightarrow \quad \sqrt{\frac{1-x^2}{1+x^2}} = \frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$$

$\Rightarrow \qquad \dfrac{1-x^2}{1+x^2} = \left(\dfrac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha}\right)^2$

$\Rightarrow \qquad \dfrac{1-x^2}{1+x^2} = \dfrac{1-\sin 2\alpha}{1+\sin 2\alpha}$

$\Rightarrow \qquad x^2 = \sin 2\alpha \qquad \left\{\begin{array}{l}\text{using componendo dividendo} \\ \text{if } \dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}\end{array}\right\}$

S110. We have,

$$= \frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right) + \frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)$$

$$= \frac{\alpha^3}{2\sin^2\theta} + \frac{\beta^3}{2\cos^2\phi}$$

where $\theta = \dfrac{1}{2}\tan^{-1}\dfrac{\alpha}{\beta}$ and $\phi = \dfrac{1}{2}\tan^{-1}\dfrac{\beta}{\alpha}$

$$= \frac{\alpha^3}{1-\cos 2\theta} + \frac{\beta^3}{1+\cos 2\phi}$$

$$= \frac{\alpha^3}{1-\cos\left(\tan^{-1}\dfrac{\alpha}{\beta}\right)} + \frac{\beta^3}{1+\cos\left(\tan^{-1}\dfrac{\beta}{\alpha}\right)}$$

$$= \frac{\alpha^3}{1-\cos\left(\cos^{-1}\dfrac{\beta}{\sqrt{\alpha^2+\beta^2}}\right)} + \frac{\beta^3}{1+\cos\left(\cos^{-1}\dfrac{\alpha}{\sqrt{\alpha^2+\beta^2}}\right)}$$

$$\left[\therefore \tan^{-1}x = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$$= \frac{\alpha^3}{1-\dfrac{\beta}{\sqrt{\alpha^2+\beta^2}}} + \frac{\beta^3}{1+\dfrac{\alpha}{\sqrt{\alpha^2+\beta^2}}}$$

$$= \left\{\frac{\alpha^3}{\sqrt{\alpha^2+\beta^2}-\beta} + \frac{\beta^3}{\sqrt{\alpha^2+\beta^2}+\alpha}\right\}\sqrt{\alpha^2+\beta^2}$$

$$= \left[\frac{\alpha^3\left\{\sqrt{\alpha^2+\beta^2}+\beta\right\}}{\alpha^2+\beta^2-\beta^2}+\frac{\beta^3\left\{\sqrt{\alpha^2+\beta^2}-\alpha\right\}}{\alpha^2+\beta^2-\alpha^2}\right]\sqrt{\alpha^2+\beta^2}$$

$$= \left\{\alpha\left(\sqrt{\alpha^2+\beta^2}+\beta\right)+\beta\left(\sqrt{\alpha^2+\beta^2}-\alpha\right)\right\}\sqrt{\alpha^2+\beta^2}$$

$$= \alpha\left(\alpha^2+\beta^2\right)+\cancel{\alpha\beta\sqrt{\alpha^2+\beta^2}}+\beta(\alpha^2+\beta^2)-\cancel{\alpha\beta\sqrt{\alpha^2+\beta^2}}$$

$$= \alpha(\alpha^2+\beta^2)+\beta(\alpha^2+\beta^2)$$

$$= (\alpha+\beta)(\alpha^2+\beta^2)$$

S111. Here,

$$\text{L.H.S.} = \tan^{-1}\left[\frac{3\sin 2\theta}{5+3\cos 2\theta}\right]+\tan^{-1}\left(\frac{1}{4}\tan\theta\right)$$

$$= \tan^{-1}\left[\frac{3\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)}{5+3\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)}\right]+\tan^{-1}\left(\frac{1}{4}\tan\theta\right)$$

$$= \tan^{-1}\left[\frac{\frac{6\tan\theta}{1+\tan^2\theta}}{\frac{5+5\tan^2\theta+3-3\tan^2\theta}{1+\tan^2\theta}}\right]+\tan^{-1}\left(\frac{1}{4}\tan\theta\right)$$

$$= \tan^{-1}\left[\frac{6\tan\theta}{8+2\tan^2\theta}\right]+\tan^{-1}\left(\frac{1}{4}\tan\theta\right)$$

$$= \tan^{-1}\left[\frac{3\tan\theta}{4+\tan^2\theta}\right]+\tan^{-1}\left(\frac{1}{4}\tan\theta\right)$$

$$= \tan^{-1}\left[\frac{\frac{3\tan\theta}{4+\tan^2\theta}+\frac{1}{4}\tan\theta}{1-\frac{3\tan\theta}{4+\tan^2\theta}\times\frac{1}{4}\tan\theta}\right] \quad \left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left[\frac{\frac{12\tan\theta+4\tan\theta+\tan^3\theta}{4(4+\tan^2\theta)}}{\frac{16+4\tan^2\theta-3\tan^2\theta}{4(4+\tan^2\theta)}}\right] = \tan^{-1}\left[\frac{16\tan\theta+\tan^3\theta}{16+\tan^2\theta}\right]$$

$$= \tan^{-1}\left[\frac{\tan\theta\,(16+\tan^2\theta)}{16+\tan^2\theta}\right] = \tan^{-1}(\tan\theta) = \theta = \text{R.H.S.}$$

S112. L.H.S.

$$= \tan^{-1}\frac{2ab}{a^2-b^2} + \tan^{-1}\frac{2xy}{x^2-y^2}$$

$$= \tan^{-1}\left(\frac{2\frac{b}{a}}{1-\frac{b^2}{a^2}}\right) + \tan^{-1}\left(\frac{2\frac{y}{x}}{1-\frac{y^2}{x^2}}\right)$$

$$= 2\tan^{-1}\left(\frac{b}{a}\right) + 2\tan^{-1}\left(\frac{y}{x}\right)$$

$$= 2\left\{\tan^{-1}\frac{b}{a} + \tan^{-1}\frac{y}{x}\right\}$$

$$= 2\tan^{-1}\left(\frac{\frac{b}{a}+\frac{y}{x}}{1-\frac{by}{ax}}\right)$$

$$= 2\tan^{-1}\left(\frac{bx+ay}{ax-by}\right)$$

$$= 2\tan^{-1}\left(\frac{\beta}{\alpha}\right) = \tan^{-1}\left(\frac{2\frac{\beta}{\alpha}}{1-\frac{\beta^2}{\alpha^2}}\right)$$

$$= \tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right) = \textbf{R.H.S.}$$

S113. $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \alpha$

$$\Rightarrow \quad \cos^{-1}\left(\frac{xy}{2\times 3} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right) = \alpha$$

$\Rightarrow \quad \frac{xy}{6} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}} = \cos\alpha$

$\Rightarrow \quad \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}} = \frac{xy}{6} - \cos\alpha$

Squaring both side

$\Rightarrow \quad \left(1-\frac{x^2}{4}\right)\left(1-\frac{y^2}{9}\right) = \left(\frac{xy}{6} - \cos\alpha\right)^2$

$\Rightarrow \quad 1-\frac{x^2}{4}-\frac{y^2}{9}+\frac{x^2y^2}{36} = \frac{x^2y^2}{36}+\cos^2\alpha - 2\ \frac{x\,y}{6}\ \cos\alpha$

$\Rightarrow \quad \frac{x^2}{4}+\frac{y^2}{9}-\frac{xy}{3}\cos\alpha = 1 - \cos^2\alpha$

$\Rightarrow \quad 9x^2 + 4y^2 - 12xy\cos\alpha = 36\sin^2\alpha$

S114. $\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\frac{8}{79}\right), x > 0$

Applying identity in given equation, we get

$\tan^{-1}\left[\frac{(x+2)+(x-2)}{1-(x+2)(x-2)}\right] = \tan^{-1}\left(\frac{8}{79}\right) \qquad \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$\Rightarrow \quad \left[\frac{2x}{1-(x^2-4)}\right] = \frac{8}{79}$

$\Rightarrow \quad \frac{2x}{5-x^2} = \frac{8}{79}$

$\Rightarrow \quad \frac{x}{5-x^2} = \frac{4}{79}$

$\Rightarrow \quad 79x = 20 - 4x^2$

$\Rightarrow \quad 4x^2 + 79x - 20 = 0$

$\Rightarrow \quad 4x^2 + 80x - x - 20 = 0$

$\Rightarrow \quad 4x(x+20) - 1(x+20) = 0$

$\Rightarrow \quad (4x-1)(x+20) = 0$

$\Rightarrow \qquad x = \frac{1}{4}$ or -20

But given that $x > 0$

$\therefore \qquad x = -20$ is rejected

$\therefore \qquad x = \frac{1}{4}$

S115. We have

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

$$\Rightarrow \qquad \tan^{-1}\left[\frac{2x+3x}{1-(2x)(3x)}\right] = \tan^{-1}(1) \qquad \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow \qquad \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \tan^{-1}(1), \quad \text{if } 6x^2 < 1$$

$$\Rightarrow \qquad \frac{5x}{1-6x^2} = 1, \quad \text{if } 6x^2 < 1$$

$$\Rightarrow \qquad 5x = 1 - 6x^2, \quad \text{if } x^2 < \frac{1}{6}$$

$$\Rightarrow \qquad 6x^2 + 5x - 1 = 0, \quad \text{if } -\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$$

$$\Rightarrow \qquad (x + 1)(6x - 1) = 0, \quad \text{if } -\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$$

$$\Rightarrow \qquad x = -1 \text{ or } \frac{1}{6}, \quad \text{if } -\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$$

$\Rightarrow$ $x = -1$ is not the root of given equation

$\therefore \qquad x = \frac{1}{6}$.

S116. The given equation is $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$. We know that, $\cot^{-1} x = \tan^{-1}\frac{1}{x}$. So, the given equation can be written as

$$\tan^{-1} x + 2\tan^{-1}\left(\frac{1}{x}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1} x + \tan^{-1}\left[\frac{2\times\frac{1}{x}}{1-\frac{1}{x^2}}\right] = \frac{2\pi}{3} \qquad \left[\because 2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right)\right]$$

$$\Rightarrow \quad \tan^{-1} x + \tan^{-1}\left[\frac{\frac{2}{x}}{\frac{x^2-1}{x^2}}\right] = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1} x + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$$

Applying $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, we get

$$\tan^{-1}\left[\frac{x+\frac{2x}{x^2-1}}{1-x.\frac{2x}{x^2-1}}\right] = \frac{2\pi}{3}$$

$$\Rightarrow \quad \frac{x^3-x+2x}{x^2-1-2x^2} = \tan\frac{2\pi}{3}$$

$$\Rightarrow \quad \frac{x^3+x}{-1-x^2} = \tan\left(\pi-\frac{\pi}{3}\right)$$

$$\Rightarrow \quad \frac{x^3+x}{-(1+x^2)} = -\tan\frac{\pi}{3} \qquad [\because \tan(\pi-\theta) = -\tan\theta]$$

$$\Rightarrow \quad \frac{x(1+x^2)}{-(1+x^2)} = -\sqrt{3}$$

or $\quad x = \sqrt{3}$

S117. The given equation is

$$\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x.$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{1+x}{1-x}\right) - \tan^{-1} x = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{1+x}{1-x}-x}{1+\left(\frac{1+x}{1-x}\right)\cdot x}\right] = \frac{\pi}{4} \qquad \left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$\Rightarrow \quad \frac{1+\cancel{x}-\cancel{x}+x^2}{1-\cancel{x}+\cancel{x}+x^2} = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{1+x^2}{1+x^2} = 1 \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad 1 = 1$$

The given equation has many solutions. Because for L.H.S. all $x \in R$. It is always one and equal to R.H.S.

S118. We have,

$$\text{R.H.S.} = \tan^{-1}\{\tan^2(\alpha+\beta)\tan^2(\alpha-\beta)\} + \tan^{-1}1$$

$$= \tan^{-1}\left\{\frac{\tan^2(\alpha+\beta)\tan^2(\alpha-\beta)+1}{1-\tan^2(\alpha+\beta)\tan^2(\alpha-\beta)}\right\}$$

$$= \tan^{-1}\left\{\frac{\sin^2(\alpha+\beta)\sin^2(\alpha-\beta)+\cos^2(\alpha+\beta)\cos^2(\alpha-\beta)}{\cos^2(\alpha+\beta)\cos^2(\alpha-\beta)-\sin^2(\alpha+\beta)\sin^2(\alpha-\beta)}\right\}$$

$$= \tan^{-1}\left\{\frac{\{2\sin(\alpha+\beta)\sin(\alpha-\beta)\}^2+\{2\cos(\alpha+\beta)\cos(\alpha-\beta)\}^2}{\{2\cos(\alpha+\beta)\cos(\alpha-\beta)\}^2-\{2\sin(\alpha+\beta)\sin(\alpha-\beta)\}^2}\right\}$$

$$= \tan^{-1}\left\{\frac{(\cos 2\beta-\cos 2\alpha)^2+(\cos 2\alpha+\cos 2\beta)^2}{(\cos 2\alpha+\cos 2\beta)^2-(\cos 2\beta-\cos 2\alpha)^2}\right\}$$

$$= \tan^{-1}\left\{\frac{\cos^2 2\beta+\cos^2 2\alpha-\cancel{2\cos 2\alpha\cos 2\beta}+\cos^2 2\alpha+\cos^2 2\beta+\cancel{2\cos 2\alpha\cos 2\beta}}{\cancel{\cos^2 2\alpha}+\cancel{\cos^2 2\beta}+2\cos 2\alpha\cos 2\beta-\cancel{\cos^2 2\beta}-\cancel{\cos^2 2\alpha}+2\cos 2\alpha\cos 2\beta}\right\}$$

$$= \tan^{-1}\left\{\frac{\cos^2 2\alpha+\cos^2 2\beta}{2\cos 2\alpha\cos 2\beta}\right\}$$

$$= \tan^{-1}\left\{\frac{\cos^2 2\alpha}{2\cos 2\alpha\cos 2\beta}+\frac{\cos^2 2\beta}{2\cos 2\alpha\cos 2\beta}\right\}$$

$$= \tan^{-1}\left\{\frac{\cos 2\alpha \sec 2\beta + \cos 2\beta \sec 2\alpha}{2}\right\} = \text{L.H.S.}$$

S119.

$$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$$

$$\Rightarrow \quad \sin\left[\sin^{-1}\frac{1}{5} + \sin^{-1}\sqrt{1-x^2}\right] = 1 \qquad \left[\because \cos^{-1} x = \sin^{-1}\sqrt{1-x^2}\right]$$

$$\Rightarrow \quad \sin\left[\sin^{-1}\left(\frac{1}{5}\sqrt{1-1+x^2} + \sqrt{1-x^2}\sqrt{1-\frac{1}{25}}\right)\right] = 1 \qquad \left[\begin{array}{l}\because \sin^{-1} x + \sin^{-1} y \\ = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]\end{array}\right]$$

$$\Rightarrow \quad \sin\left[\sin^{-1}\left(\frac{1}{5}x + \sqrt{\frac{24}{25}}\sqrt{1-x^2}\right)\right] = 1$$

$$\Rightarrow \quad \frac{1}{5}x + \sqrt{\frac{24}{25}}\sqrt{1-x^2} = 1$$

$$\Rightarrow \quad \sqrt{\frac{24}{25}}\sqrt{1-x^2} = 1 - \frac{x}{5}$$

Squaring both sides, we have $\frac{24}{25}(1-x^2) = \left(1-\frac{x}{5}\right)^2$

$$\Rightarrow \quad 24 - 24x^2 = 25\left(1 + \frac{x^2}{25} - \frac{2x}{5}\right)$$

$$\Rightarrow \quad 24 - 24x^2 = 25 + x^2 - 10x$$

$$\Rightarrow \quad 25x^2 - 10x + 1 = 0$$

$$\Rightarrow \quad 25x^2 - 5x - 5x + 1 = 0$$

$$\Rightarrow \quad (5x - 1)^2 = 0$$

$$\Rightarrow \quad 5x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1}{5}$$

Thus $x = \frac{1}{5}$ is a root of given equation.

S120. Given equation is

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x \qquad x > 0.$$

$$\Rightarrow \qquad 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}x$$

$$\Rightarrow \qquad \tan^{-1}\left[\frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2}\right] = \tan^{-1}x \qquad \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{2(1-x)(1+x)}{(1+x)^2-(1-x)^2}\right] = \tan^{-1}x \qquad [\because a^2-b^2=(a-b)(a+b)]$$

$$\Rightarrow \qquad \tan^{-1}\left(\frac{2-2x^2}{4x}\right) = \tan^{-1}x$$

$$\Rightarrow \qquad \frac{1-x^2}{2x} = x$$

$$\Rightarrow \qquad 1 - x^2 = 2x^2 \Rightarrow 3x^2 = 1$$

$$\Rightarrow \qquad x^2 = \frac{1}{3}$$

$$x = \pm\frac{1}{\sqrt{3}}$$

But given $x > 0$

$$\therefore \qquad x = \frac{1}{\sqrt{3}}$$

S121. Given equation is

$$\cos(2\sin^{-1}x) = \frac{1}{9}, x > 0. \qquad \ldots(i)$$

We put $\qquad \sin^{-1}x = y$

$$\Rightarrow \qquad x = \sin y$$

$\therefore$ Eq. (i) becomes $\cos 2y = \frac{1}{9}$

$\Rightarrow$ $1-2\sin^2 y = \frac{1}{9}$ [$\because$ $\cos 2\theta = 1 - 2\sin^2\theta$]

$\Rightarrow$ $2\sin^2 y = 1-\frac{1}{9}=\frac{8}{9}$

$\Rightarrow$ $\sin^2 y = \frac{4}{9}$

$\Rightarrow$ $x^2 = \frac{4}{9}$ [$\because$ $\sin y = x$]

$\therefore$ $x = \mp\frac{2}{3}$

But given that $x > 0$

$\therefore$ $x = \frac{2}{3}$

S122. Given equation is

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}, -1 < x < 1$$

$\therefore$ The given equation becomes

$\tan^{-1}\frac{2x}{1-x^2}+\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$ $\left[\because \cot^{-1}\left(\frac{1-x^2}{2x}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$\Rightarrow$ $2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$

$\Rightarrow$ $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{6}$

$\Rightarrow$ $\frac{2x}{1-x^2} = \tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

$\Rightarrow$ $2\sqrt{3}x = 1 - x^2$

or $x^2+2\sqrt{3}x-1 = 0$

$\Rightarrow \qquad x = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2} \qquad \left[\because \text{ We know that, } x = \dfrac{-b \pm \sqrt{D}}{2a} \text{ where, } D = b^2 - 4ac\right]$

$\therefore \qquad x = \dfrac{-2\sqrt{3} \pm 4}{2} = \dfrac{4 - 2\sqrt{3}}{2} \text{ or } \dfrac{-4 - 2\sqrt{3}}{2}$

$\Rightarrow \qquad x = 2 - \sqrt{3} \text{ or } -(2 + \sqrt{3})$

But given that $-1 < x < 1$, so $x = -(2+\sqrt{3})$ is rejected. Hence, $x = 2 - \sqrt{3}$

S123. Given that,

$$\cos^{-1} x + \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{6}$$

$\Rightarrow \qquad \cos^{-1} x = \dfrac{\pi}{6} - \sin^{-1}\dfrac{x}{2}$

$\Rightarrow \qquad \cos\ \cos^{-1} x = \cos\left(\dfrac{\pi}{6} - \sin^{-1}\dfrac{x}{2}\right)$

$\Rightarrow \qquad x = \cos\dfrac{\pi}{6}\cos\left(\sin^{-1}\dfrac{x}{2}\right) + \sin\dfrac{\pi}{6}\sin\left(\sin^{-1}\dfrac{x}{2}\right)$

$[\because \cos(x - y) = \cos x \cos y + \sin x \sin y]$

$.\Rightarrow \qquad x = \dfrac{\sqrt{3}}{2}\cos\left(\sin^{-1}\dfrac{x}{2}\right) + \dfrac{1}{2}\cdot\dfrac{x}{2}$

$\Rightarrow \qquad x = \dfrac{\sqrt{3}}{2}\cos\left(\cos^{-1}\sqrt{1 - \dfrac{x^2}{4}}\right) + \dfrac{x}{4} \qquad \left[\because \sin^{-1}\dfrac{x}{2} = \cos^{-1}\sqrt{1 - \dfrac{x^2}{4}}\right]$

$\Rightarrow \qquad x = \dfrac{\sqrt{3}}{2}\left(\sqrt{1 - \dfrac{x^2}{4}}\right) + \dfrac{x}{4}$

$\Rightarrow \qquad x - \dfrac{x}{4} = \dfrac{\sqrt{3}}{2}\left(\sqrt{1 - \dfrac{x^2}{4}}\right)$

$\Rightarrow \qquad \dfrac{3x}{4} = \dfrac{\sqrt{3}}{2}\left(\sqrt{1 - \dfrac{x^2}{4}}\right)$

On squaring both sides, we get

$$\frac{9x^2}{16} = \frac{3}{4}\left(1-\frac{x^2}{4}\right)$$

$\Rightarrow$ $$\frac{3}{4}x^2 = 1-\frac{x^2}{4}$$

$\Rightarrow$ $$\frac{3}{4}x^2+\frac{x^2}{4} = 1$$

$\Rightarrow$ $$\frac{4x^2}{4} = 1 \text{ or } x^2 = 1$$

$\Rightarrow$ $x = \pm 1$.

But $x = -1$ do not satisfied the given equation. Hence, $x = 1$

S124.

$$\tan^{-1}\left[\frac{\frac{x}{2}+\frac{x}{3}}{1-\frac{x^2}{6}}\right] = \frac{\pi}{4} \qquad \left[\because \tan^{-1}x+\tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$\Rightarrow$ $$\frac{\frac{3x+2x}{6}}{\frac{6-x^2}{6}} = \tan\frac{\pi}{4}$$

$\Rightarrow$ $$\frac{5x}{6-x^2} = 1 \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$\Rightarrow$ $5x = 6 - x^2$

$\Rightarrow$ $x^2 + 5x - 6 = 0$

$\Rightarrow$ $(x-1)(x+6) = 0$

$\Rightarrow$ $x = 1 \text{ or } -6$

But given that $0 < x < \sqrt{6} \Rightarrow x > 0$

$\therefore$ $x = -6$ is rejected

Hence, $x = 1$

S125. Here,

$$\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2} = \frac{\pi}{3}$$

Put $x = \tan\theta$

$\Rightarrow$ $\theta = \tan^{-1}x$

$$\therefore \quad \frac{1}{2}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{1}{2}\sin^{-1}(\sin 2\theta)+\frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{1}{2}\times 2\theta+\frac{1}{2}\times 2\theta = \frac{\pi}{3}$$

$$\Rightarrow \quad \theta + \theta = \frac{\pi}{3}$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{3}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

$$\Rightarrow \quad \tan^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow \quad x = \tan\frac{\pi}{6}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{3}}$$

S126. We have,

$$\tan^{-1}\frac{1}{x} - \tan^{-1}\frac{1}{x+2} = \frac{\pi}{12}$$

$$\Rightarrow \quad \tan^{-1}\left\{\frac{\frac{1}{x}-\frac{1}{x+2}}{1+\frac{1}{x(x+2)}}\right\} = \frac{\pi}{12}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2}{x^2+2x+1}\right) = \frac{\pi}{12}$$

$$\Rightarrow \quad \frac{2}{x^2+2x+1} = \tan\frac{\pi}{12}$$

$\Rightarrow \quad \dfrac{2}{x^2+2x+1} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow \quad \dfrac{2}{x^2+2x+1} = \dfrac{2}{(\sqrt{3}+1)^2} \Rightarrow (x+1)^2 = (\sqrt{3}+1)^2 \Rightarrow x = \sqrt{3}$.

S127. $\Rightarrow \quad \sin^{-1} x + \sin^{-1} 2x = \dfrac{\pi}{3}$

$\Rightarrow \quad \sin^{-1} x + \sin^{-1} 2x = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow \quad \sin^{-1}x - \sin^{-1} \dfrac{\sqrt{3}}{2} = -\sin^{-1} 2x$

$\Rightarrow \sin^{-1}\left\{x\sqrt{1-\dfrac{3}{4}} - \dfrac{\sqrt{3}}{2}\sqrt{1-x^2}\right\} = \sin^{-1}(-2x)$

$\Rightarrow \quad \dfrac{x}{2} - \dfrac{\sqrt{3}}{2}\sqrt{1-x^2} = -2x$

$x - \sqrt{3}\sqrt{1-x^2} = -4x$

$\Rightarrow \quad 5x = \sqrt{3}\sqrt{1-x^2} \Rightarrow 25x^2 = 3 - 3x^2 \Rightarrow 28x^2 = 3 \Rightarrow x = \sqrt{\dfrac{3}{28}}$

$\Rightarrow \quad x = \dfrac{\sqrt{3}}{2\sqrt{7}}$

S128. We have,

$\sin^{-1}(1-x) - 2\sin^{-1}x = \dfrac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}(1-x) = \dfrac{\pi}{2} - 2\sin^{-1} x$

$\Rightarrow \quad 1 - x = \sin(\pi/2 - 2\sin^{-1}x)$

$\Rightarrow \quad 1 - x = \cos(2\sin^{-1}x)$

$\Rightarrow \quad 1 - x = \cos\{\cos^{-1}(1-2x^2)\}$ $\qquad [\because \ 2\sin^{-1}x = \cos^{-1}(1-2x^2)]$

$\Rightarrow \quad 1 - x = (1 - 2x^2)$

$\Rightarrow \qquad x = 2x^2 \quad \Rightarrow \quad x(2x - 1) = 0 \quad \Rightarrow \quad x = 0,\ \frac{1}{2}$

For, $\qquad x = \frac{1}{2}$, we have

$$\text{LHS} = \sin^{-1}(1 - x) - 2\sin^{-1} x$$

$$= \sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{2} = -\sin^{-1}\frac{1}{2} = -\frac{\pi}{6} \neq \text{R.H.S.}$$

So, $x = 1/2$ is not a root of the given equation.

Clearly, $x = 0$ satisfies the equation.

Hence, $x = 0$ is a root of the given equation.

S129. We have,

$$\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1} x$$

$$\Rightarrow \quad \sin^{-1}\left\{\frac{3x}{5}\sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5}\sqrt{1 - \frac{9x^2}{25}}\right\} = \sin^{-1} x$$

$$\Rightarrow \quad \frac{3x}{5}\sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5}\sqrt{1 - \frac{9x^2}{25}} = x$$

$$\Rightarrow \quad 3x\sqrt{25 - 16x^2} + 4x\sqrt{25 - 9x^2} = 25x$$

$$\Rightarrow \quad 3\sqrt{25 - 16x^2} + 4\sqrt{25 - 9x^2} = 25$$

$$\Rightarrow \quad 4\sqrt{25 - 9x^2} = 25 - 3\sqrt{25 - 16x^2}$$

$$\Rightarrow \quad 16(25 - 9x^2) = 625 + 9(25 - 16x^2) - 150\sqrt{25 - 16x^2}$$

$\Rightarrow$

$$\Rightarrow \quad 150\sqrt{25 - 16x^2} = 450$$

$$\Rightarrow \quad 25 - 16x^2 = 9$$

$$\Rightarrow \quad x = \pm 1$$

Hence, $x = 0, 1, -1$ are roots of the given equation.

S130. We have,

$$\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2} = \tan^{-1}1$$

$$\Rightarrow \quad \tan^{-1}\frac{x-1}{x-2} = \tan^{-1}1 - \tan^{-1}\frac{x+1}{x+2}$$

$$\Rightarrow \quad \tan^{-1}\frac{x-1}{x-2} = \tan^{-1}\left(\frac{1-\frac{x+1}{x+2}}{1+\frac{x+1}{x+2}}\right)$$

$$\Rightarrow \quad \tan^{-1}\frac{x-1}{x-2} = \tan^{-1}\frac{x+2-x-1}{x+2+x+1}$$

$$\Rightarrow \quad \tan^{-1}\frac{x-1}{x-2} = \tan^{-1}\frac{1}{2x+3}$$

$$\Rightarrow \quad \frac{x-1}{x-2} = \frac{1}{2x+3}$$

$$\Rightarrow \quad (2x+3)(x-1) = x-2$$

$$\Rightarrow \quad 2x^2+x-3 = x-2$$

$$\Rightarrow \quad 2x^2-1 = 0$$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}$$

S131. Given equation reduces to :

$$3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

Let $\qquad x = \tan\theta$

$$\Rightarrow \quad 3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$$

$$\Rightarrow \quad 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$

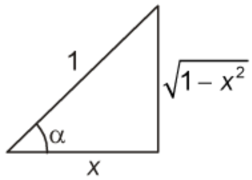
$$\Rightarrow \quad 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3}$$

$$3(2\tan^{-1}x) - 4(2\tan^{-1}x) + 2(2\tan^{-1}x) = \pi/3$$

$$\Rightarrow \quad 2\tan^{-1}x = \frac{\pi}{3} \quad \Rightarrow \quad \tan^{-1}x = \frac{\pi}{6} \quad \Rightarrow \quad x = \frac{1}{\sqrt{3}}$$

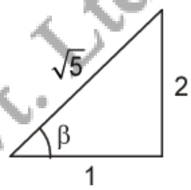
S132. We have,

$$\tan(\cos^{-1}x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$$



Let, $\cos^{-1}x = \alpha$ and $\cot^{-1}\frac{1}{2} = \beta$

$$x = \cos\alpha \qquad \frac{1}{2} = \cot\beta$$



$$\Rightarrow \quad \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow \quad \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$

$$\sqrt{5}\sqrt{1-x^2} = 2x$$

$$\Rightarrow \quad 5(1-x^2) = 4x^2$$

$$\Rightarrow \quad 5 = 9x^2$$

$$\Rightarrow \quad x^2 = \frac{5}{9}$$

$$\Rightarrow \quad x = \pm\frac{\sqrt{5}}{3}$$

S133. We have,

$$\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}] = 0$$

$$\Rightarrow \quad \sin\left[2\cos^{-1}\left\{\cot\left(\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)\right\}\right] = 0 \qquad \left[\because\ 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$\Rightarrow \quad \sin\left[2\cos^{-1}\left\{\cot\left(\cot^{-1}\left(\frac{1-x^2}{2x^2}\right)\right)\right\}\right] = 0 \qquad \left[\because \cot^{-1}x = \tan^{-1}\frac{1}{x}\right]$$

$$\Rightarrow \quad \sin\left[2\cos^{-1}\left(\frac{1-x^2}{2x}\right)\right] = 0$$

$$\Rightarrow \quad \sin\left[\sin^{-1}\left\{2\left(\frac{1-x^2}{2x}\right)\sqrt{1-\left(\frac{1-x^2}{x}\right)^2}\right\}\right] = 0 \qquad [\because 2\cos^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})]$$

$$\Rightarrow \quad \left(\frac{1-x^2}{x}\right)\sqrt{1-\left(\frac{1-x^2}{2x}\right)^2} = 0$$

$$\Rightarrow \quad \frac{1-x^2}{x} = 0 \text{ or, } \sqrt{1-\left(\frac{1-x^2}{2x}\right)^2} = 0$$

$$\Rightarrow \quad 1 - x^2 = 0, \quad \text{or} \quad \left(\frac{1-x^2}{2x}\right)^2 = 1$$

$$\Rightarrow \quad x = \pm 1 \text{ or, } (1 - x^2)^2 = 4x^2$$

Now, $(1 - x^2)^2 = 4x^2$

$$\Rightarrow \quad (1 - x^2)^2 - (2x)^2 = 0$$

$$\Rightarrow \quad (1 - x^2 - 2x)(1 - x^2 + 2x) = 0$$

$$\Rightarrow \quad 1 - x^2 - 2x = 0 \text{ or, } 1 - x^2 + 2x = 0$$

$$\Rightarrow \quad x^2 + 2x - 1 = 0 \text{ or, } x^2 - 2x - 1 = 0$$

$$\Rightarrow \quad x = -1 \pm \sqrt{2} \text{ or } x = 1 \pm \sqrt{2}$$

Hence, $x = \pm 1, -1 \pm \sqrt{2}, 1 \pm \sqrt{2}$ are the roots of the given equation.