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Q1. A die is thrown three times. Find $P(E/F)$ where
E : 4 appears on the third toss, F : 6 and 5 appear respectively on first two tosses.

Q2. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$ find

(i) $P(A \cap B)$ (ii) $P(A/B)$ (iii) $P(B/A)$

Q3. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$, find :

(i) $P(A \cap B)$ (ii) $P(A/B)$ (iii) $P(A \cup B)$

Q4. A coin is tossed three times, Find $P(E/F)$ in each case where

(i) E : head on third toss, F : heads on first two tosses

(ii) E : at least two heads, F : at most two heads

(iii) E : at most two tails, F : at least one tail.

Q5. Two coins are tossed once, Find $P(E/F)$ in each case where

(i) E : tail appears on one coin, F : one coin shows head

(ii) E : no tail appears, F : no head appears

Q6. An instructor has a question bank consisting of 300 easy true/false questions, 200 difficult true/false questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Q7. 12 cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number.

Q8. A family has 2 children. Find the probability that both are boys, if it is known that

(i) at least one of children is a boy. (ii) elder child is a boy.

Q9. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl (ii) at least one is a girl?

Q10. Consider the experiment of throwing a die. If a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event, 'the coin shows a tail' given that 'at least one die shows a 3'.

Q11. Given that two numbers appearing on throwing two die are different. Find the probability of the event 'the sum of numbers on the die is 4'.

Q12. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that the student reads neither Hindi nor English newspaper.

(b) If the student reads Hindi newspaper, find the probability that the same student reads English newspaper also.

(c) If the student reads English newspaper, find the probability that the same student reads Hindi newspaper also.

Q13. A fair die is rolled. Consider events $E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$. Find

(i) $P(E/F)$ and $P(F/E)$ (ii) $P(E/G)$ and $P(G/E)$ (iii) $P[(E \cup F)/G]$ and $P[(E \cap F)/G]$

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S1. A die is tossed three times then $n(S) = 216$

$$E = \{(1, 1, 4), (1, 2, 4), (1, 3, 4), (1, 4, 4) (1, 5, 4), (1, 6, 4), (2, 1, 4), (2, 2, 4), (2, 3, 4), (2, 4, 4), (2, 5, 4), (2, 6, 4), (3, 1, 4), (3, 2, 4), (3, 3, 4), (3, 4, 4), (3, 5, 4), (3, 6, 4), (4, 1, 4), (4, 2, 4), (4, 3, 4), (4, 4, 4), (4, 5, 4), (4, 6, 4), (5, 1, 4), (5, 2, 4), (5, 3, 4), (5, 4, 4), (5, 5, 4), (5, 6, 4), (6, 1, 4) (6, 2, 4) (6, 3, 4), (6, 4, 4), (6, 5, 4), (6, 6, 4)\}$$

$$F = \{6, 5, 1), (6, 5, 2) (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)\}$$

$$E \cap F = \{6, 5, 4\}$$

$\therefore$ $$P(E) = \frac{36}{216} = \frac{1}{6}, \text{ P (F)} = \frac{6}{216} = \frac{1}{36} \text{ and } P(E \cap F) = \frac{1}{216}$$

$\therefore$ $$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{1}{36}} = \frac{1}{216} \times \frac{36}{1} = \frac{1}{6}$$

S2. Here $$P(A) = \frac{6}{11}, \; P(B) = \frac{5}{11} \text{ and } P(A \cup B) = \frac{7}{11}$$

(i) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\therefore$ $$\frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B) \Rightarrow P(A \cap B) = 1 - \frac{7}{11} = \frac{4}{11}$$

(ii) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{11} \times \frac{11}{5} = \frac{4}{5}$$

(iii) $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{4}{11} \times \frac{11}{6} = \frac{2}{3}$$

S3. Here $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$

(i) Now $$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$\therefore \quad 0.4 = \dfrac{P(A \cap B)}{0.8} \Rightarrow P(A \cap B) = 0.4 \times 0.8 = 0.32$

(ii) $\quad P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.32}{0.5} = 0.64$

(iii) $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.5 - 0.32 = 0.98$

S4. When a coin is tossed three times then

(i) $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E = \{HHH, HTH, THH, TTH\}$

$F = \{HHH, HHT\}$

$E \cap F = \{HHH\}$

$\therefore \quad P(E) = \dfrac{4}{8} = \dfrac{1}{2}, P(F) = \dfrac{2}{8} = \dfrac{1}{4}$ and $P(E \cap F) = \dfrac{1}{8}$

$\therefore \quad P(E/F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{1}{8}}{\frac{1}{4}} = \dfrac{1}{8} \times \dfrac{4}{1} = \dfrac{1}{2}$

(ii) $E = \{HHT, HTH, THH, HHH\}$

$F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E \cap F = \{HHT, HTH, THH\}$

$\therefore \quad P(E) = \dfrac{4}{8} = \dfrac{1}{2}, P(F) = \dfrac{7}{8}$ and $P(E \cap F) = \dfrac{3}{8}$

$\therefore \quad P(E/F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{3}{8}}{\frac{7}{8}} = \dfrac{3}{8} \times \dfrac{8}{7} = \dfrac{3}{7}$

(iii) $E = \{HTT, THT, TTH, HHT, HTH, THH, HHH\}$

$F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E \cap F = \{HTT, THT, TTH, HHT, HTH, THH\}$

$\therefore \quad P(E) = \dfrac{7}{8}, P(F) = \dfrac{7}{8}$ and $P(E \cap F) = \dfrac{6}{8} = \dfrac{3}{4}$

$$\therefore \qquad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$$

S5. When two coins are tossed then

(i) $S = \{HH, HT, TH, TT\}$

$E = \{HT, TH\}$

$F = \{HT, TH\}$

$E \cap F = \{HT, TH\}$

$$\therefore \qquad P(E) = \frac{2}{4} = \frac{1}{2}, P(F) = \frac{2}{4} = \frac{1}{2} \text{ and } P(E \cap F) = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \qquad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1/2}{1/2} = 1$$

(ii) $E = \{HH\}$

$F = \{TT\}$

$E \cap F = \phi$

$$\therefore \qquad P(E) = \frac{1}{4}, \text{P(F)} = \frac{1}{4} \text{ and } P(E \cap F) = \frac{0}{4} = 0$$

$$\therefore \qquad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{1/4} = 0.$$

S6. Here total questions = 300 + 200 + 500 + 400 = 1400

Let A be the event that selected question is an easy question.

$$\therefore \qquad P(A) = \frac{300+500}{1400} = \frac{800}{1400} = \frac{4}{7}$$

Let B be the event that selected question is a multiple choice question.

$$P(B) = \frac{500+400}{1400} = \frac{900}{1400} = \frac{9}{14}$$

Now $A \cap B$ is the event so that the selected question is easy and multiple choice question.

$$\therefore \qquad P(A \cap B) = \frac{500}{1400} = \frac{5}{14}$$

$$\therefore \qquad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{5}{14}}{\frac{9}{14}} = \frac{5}{9}.$$

S7. Let us define the events as

A : Number on the drawn card is more than 3.

B : Number on the card is an even number

Here, total sample space, $n(S) = 12$

$\therefore \quad A = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and $B = \{2, 4, 6, 8, 10, 12\}$

So, $\quad A \cap B = \{4, 6, 8, 10, 12\}$

$\Rightarrow \quad P(A) = \frac{9}{12} = \frac{3}{4}$ $\quad$ [$\because$ Total cases = 12]

and $\quad P(B) = \frac{6}{12} = \frac{1}{2}$ and $P(A \cap B) = \frac{5}{12}$

We have, to find $P\left(\frac{B}{A}\right)$.

By using conditional probability, we have

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}; P(A) \neq 0$$

$$= \frac{P(A \cap B)}{P(A)} = \frac{\frac{5}{12}}{\frac{3}{4}} = \frac{5}{12} \times \frac{4}{3} = \frac{5}{9}$$

Hence, required probability = $\frac{5}{9}$.

S8. Let B represents elder child which is a boy and b represents younger child which is also a boy. And G represents elder child which is a girl and g represents younger child which is also a girl. The sample space of the above is given as $S = \{Bb, Bg, Gg, Gb\}$

$\therefore \quad n(S) = 4$

Let us define event A : both children are boys, then $A = \{Bb\}$

$\Rightarrow \quad n(A) = 1$

(i) Let B : at least one of the children is a boy and A = both children are boys.

We have to find $P\left(\frac{A}{B}\right)$, we know that

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}; P(B) \neq 0 \quad \text{... (i)}$$

Now, $\quad A \cap B = \{Bb\}$

$\Rightarrow \quad P(A \cap B) = \frac{1}{4}$ $\quad$ ... (ii) and $B = \{Bb, Bg, Gb\}$

$\Rightarrow$ $n(B) = 3$

$\Rightarrow$ $P(B) = \frac{3}{4}$ ... (iii)

$\therefore$ From Eqs. (i), (ii) and (iii), we get

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

$\therefore$ Required probability = $\frac{1}{3}$

(ii) Let B : The elder child is a boy and A : Both children are boys.

We have to find $P\left(\frac{A}{B}\right)$.

Now, $B = \{Bb, Bg\}$

$\Rightarrow$ $P(B) = \frac{2}{4} = \frac{1}{2}$ ... (iv)

and $A \cap B = \{Bb\}$

$\Rightarrow$ $P(A \cap B) = \frac{1}{4}$ ... (v)

$\therefore$ From Eqs. (i), (iv) and (v), we get

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

$\therefore$ Required probability = $\frac{1}{2}$

S9. Here $S = \{BB, BG, GB, GG\}$

(i) Let $A = \{GG\}$

$B = \{BG, GG\}$

$\therefore$ $A \cap B = \{GG\}$

$\therefore$ $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{4} = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$

$\therefore$ $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

(ii) Let $A = \{GG\}$

$B = \{BG, GB, GG\}$

$(A \cap B) = \{GG\}$

$\therefore \quad P(A) = \frac{1}{4}, P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{4}$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$$

S10. Here, $S = \{(3, 1), (3, 2), (3, 3)\ (3, 4), (3, 5), (3, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\ (1, H)\ (1, T), (2, H), (2, T), (4, H), (4, T), (5, H), (5, T)\}$

Let A be the event of getting a tail on coin.

$A = \{(1, T), (2, T)\ (4, T), (5, T)\}$

Let B be the event of getting 3 on at least one die.

$B = \{(3, 1), (3, 2), (3, 3)\ (3, 4)\ (3, 5)\ (3, 6), (6, 3)\}$

$A \cap B = \phi$

$\therefore \quad P(A) = \frac{4}{20} = \frac{1}{5}, P(B) = \frac{7}{20}$ and $(A \cap B) = \frac{0}{20} = 0$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{\frac{7}{20}} = 0$$

S11. Here two die are tossed then $n(S) = 36$

Let A be the event of getting the sum of numbers on the die is 4

$\therefore \quad A = \{(1, 3), (2, 2)\ (3, 1)\}$

Let B be the event of getting two different numbers on two dice.

$\therefore \quad B = [\{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1)\ (3, 2)\ (3, 4)\ (3, 5), (3, 6)\ (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4)\ (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)\}]$

$\therefore \quad A \cap B = \{(1, 3), (3, 1)\}$

Now $P(A) = \frac{3}{36} = \frac{1}{12}, P(B) = \frac{30}{36} = \frac{5}{6}$ and $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{5}{6}} = \frac{1}{18} \times \frac{6}{5} = \frac{1}{15}$$

S12. Let A be the event that a student reads Hindi newspaper and B be the event that a student reads English newspaper.

$$\therefore \quad P(A) = \frac{60}{100} = 0.6, \quad P(B) = \frac{40}{100} = 0.4$$

and $$P(A \cap B) = \frac{20}{100} = 0.2$$

(a) Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.6 + 0.4 - 0.2 = 0.8$$

Probability that student reads neither Hindi nor English newspaper

$$= 1 - P(A \cup B) = 1 - 0.8 = 0.2 = \frac{1}{5}$$

(b) $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$$

(c) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}.$$

S13. Here $E = \{1, 3, 5\}, \quad F = \{2, 3\} \quad \text{and} \quad G = \{2, 3, 4, 5\}$

$\therefore \quad E \cap F = \{3\}, \ E \cap G = \{3, 5\}, \ (E \cup F) \cap G = \{2, 3, 5\}$

and $(E \cap F) \cap G = \{3\}$

$$\therefore \quad P(E) = \frac{3}{6} = \frac{1}{2}, \quad P(F) = \frac{2}{6} = \frac{1}{3}, \quad P(G) = \frac{4}{6} = \frac{2}{3}$$

$$P(E \cap F) = \frac{1}{6}, \quad P(E \cap G) = \frac{2}{6} = \frac{1}{3}, \quad P[(E \cup F) \cap G] = \frac{3}{6} = \frac{1}{2}$$

and $$P[(E \cap F) \cap G] = \frac{1}{6}$$

(i) $$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2}$$

and $$P(F/E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$$

(ii) $$P(E/G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

and $P(G/E) = \frac{P(E \cap G)}{P(E)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$

(iii) $P(E \cup F)/G] = \frac{P[(E \cup F) \cap G]}{P(G)} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

and $P[(E \cap F)/G] = \frac{P[(E \cap F) \cap G]}{P(G)} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}$

Date: 19/10/2021

Q1. A bag contains 15 tickets numbered from 1 to 15. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

Q2. A card is drawn from a well shuffled deck of 52 cards and then a second card is drawn. Find the probability that the first card is a spade and the second card is a club if the first card is not replaced.

Q3. Two balls are drawn from an urn containing 2 white 3 red and 4 black balls one by one without replacement. What is the probability that at least one ball is red?

Q4. What is the probability of drawing in succession an ace, king and queen of clubs from a deck of cards under the assumption that the cards are not replaced after each draw?

Q5. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Q6. A box of orange is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good; the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

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S1. Let A and B be the events of drawings an even number ticket in the first and second drawn respectively.

In the first draw, there are 7 even numbers out of 15 numbers.

$$\therefore \qquad P(A) = \frac{7}{15}$$

After first draw, there are 14 tickets left.

In the second drawn, one even number ticket is drawn out of 14 tickets.

$$\therefore \qquad P(B/A) = \frac{6}{14} = \frac{3}{7}$$

$$\therefore \qquad P(A \cap B) = P(A) \cdot P(B/A) = \frac{7}{15} \times \frac{3}{7} = \frac{1}{5}$$

S2. Let A be the event of drawing a spade card in the first drawn and B be the event of drawing a club card in the second drawn. In the first draw, one spade card is drawn out of 13 spade cards.

$$P(A) = \frac{13}{52} = \frac{1}{4}$$

After the first draw, there are 51 cards left.

In the second draw, one club card is drawn out of 13 club cards.

$$\therefore \qquad P(B/A) = \frac{13}{51}$$

$$\therefore \qquad P(A \cap B) = P(A) \cdot P(B/A) = \frac{1}{4} \times \frac{13}{51} = \frac{13}{204}.$$

S3. Total balls = 2 + 3 + 4 = 9

Let A and B be the events that no red ball is drawn in first drawn and second drawn, respectively.

$\therefore$ Probability that at least one red ball is drawn = $1 - P(A \cap B)$

In the first draw

$$P(A) = \frac{6}{9} = \frac{2}{3}$$

After first draw, there are 8 balls left.

In the second draw

$$P(B/A) = \frac{5}{8}$$

$$\therefore \quad 1 - P(A \cap B) = 1 - P(A) \cdot P(B/A) = 1 - \frac{2}{3} \times \frac{5}{8} = 1 - \frac{5}{12} = \frac{7}{12}$$

S4. Let A, B and C be the events of drawings an ace of clubs first draw, a king of clubs in the second draw and a queen of clubs in the third draw respectively.

In the first draw, one ace of clubs is drawn out of 52 cards

$$\therefore \quad P(A) = \frac{1}{52}$$

After first drawn, there are 51 cards left.

In the second draw, one king of clubs is drawn out of 51 cards

$$\therefore \quad P(B/A) = \frac{1}{51}$$

After second draw, there are 50 cards left.

In the third draw, one queen of clubs is drawn out of 50 cards

$$\therefore \quad P(\text{C/AB}) = \frac{1}{50}$$

$$\therefore \quad P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/AB) = \frac{1}{52} \times \frac{1}{51} \times \frac{1}{50} = \frac{1}{132600}.$$

S5. Let A and B be the events of drawing a black card in first draw and second draw respectively.

In the first draw, there are 26 black cards out of 52 cards

$$\therefore \quad P(A) = \frac{26}{52} = \frac{1}{2}$$

After first draw, there are 51 cards left.

In the second draw, there are 25 black cards out of 51 cards

$$\therefore \quad P(B/A) = \frac{25}{51}$$

$$\therefore \quad P(A \cap B) = P(A) \cdot P(B/A) = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

S6. The box contain 15 oranges out of which 12 are good and 3 are bad.

In the first draw, one orange is drawn out of 12 good oranges.

$$\therefore \quad P(A) = \frac{12}{15}$$

After first draw, there are 14 oranges left.

In the second draw, one orange is drawn out of 11 good ornages.

$$\therefore \quad P(B/A) = \frac{11}{14}$$

After second draw, there are 13 oranges left.

In the third draw, one orange is drawn out of 10 good oranges.

$$\therefore \qquad P(C/AB) = \frac{10}{13}$$

$$\therefore \qquad P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/AB) = \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}.$$

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Q1. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $(A \cap B) = \frac{1}{8}$, find P (not A and not B).

Q2. If events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P (not A or not B) $= \frac{1}{4}$. State whether A and B are independent?

Q3. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'the number is even' and B be the event 'the number is red'. Are A and B independent?

Q4. Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$ Find
(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A / B)$ (iv) $P(B / A)$

Q5. Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find
(i) P(A and B) (ii) P(A and not B) (iii) P(A or B) (iv) P(neither A nor B)

Q6. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are (i) mutually exclusive (ii) independent.

Q7. A die is tossed thrice. Find the probability of getting an odd number at least once.

Q8. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red (ii) first ball is black and second is red
(iii) one of them is black and other is red

Q9. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time.

Q10. A speak truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

Q11. Probabilities of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$, respectively. If both try to solve problem independently, find the probability that
(i) problem is solved, (ii) exactly one of them solves the problem.

Q12. A speaks truth 75% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? Do you think that statement of B is true?

Q13. P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact?

Do you think, when they agree, means both are speaking truth?

Q14. In a hockey match, both teams *A* and *B* scored same number of goals upto the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternatively and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team *A* was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

Q15. A fair coin and an unbiased die are tossed. Let *A* be the event 'head appears on the coin' and *B* be the event '3 on the die'. Check whether *A* and *B* are independent events or not.

Q16. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events *E* and *F* independent?

(i) *E* : the card drawn is a spade, *F* : the card drawn in an ace.

(ii) *E* : the card drawn is black, *F* : the card drawn is a king.

(iii) *E* : the card drawn is a king or queen, *F* : the card drawn is a queen or jack.

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S1. Here $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$

$P(\text{not } A \text{ and not } B) = P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

$P(\text{not } A \text{ and not } B) = 1 - \frac{5}{8} = \frac{3}{8}.$

S2. Here $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$

Now $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

$\therefore$ $\frac{1}{4} = 1 - P(A \cap B) \Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$

Now $P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$

$\therefore$ $P(A \cap B) \neq P(A) \times P(B)$

Thus A and B are not independent.

S3. Let A be the event when the number on the die is even.

There are 3 even numbers out of 6 numbers

$\therefore$ $P(A) = \frac{3}{6} = \frac{1}{2}$

Let B be the event when the number is red.

There are 3 numbers marked red out of 6 numbers

$\therefore$ $P(B) = \frac{3}{6} = \frac{1}{2}$

Now $A \cap B$ is the event when numbers is even and marked red.

There is only one number which is even and marked red

$\therefore$ $P(A \cap B) = \frac{1}{6}$

Now $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\therefore \quad P(A \cap B) \neq P(A) \times P(B)$

Thus A and B are not independent events.

S4. $P(A) = 0.3$ and $P(B) = 0.4$

(i) Since A and B are independent

$\therefore \quad P(A \cap B) = P(A) \times P(B)$

$= 0.3 \times 0.4 = 0.12$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.12$

$= 0.7 - 0.12 = 0.58$

(iii) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.4} = 0.3$

(iv) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.12}{0.3} = 0.4$

S5. Here $P(A) = 0.3$ and $P(B) = 0.6$

(i) Since A and B are independent events

$\therefore \quad P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$

$= 0.3 \times 0.6 = 0.18$

(ii) $P(A \text{ and not } B) = P(A \cap \bar{B}) = P(A) \times P(\bar{B})$

$= P(A) \times [1 - P(B)] = 0.3 \times [1 - 0.6]$

$= 0.3 \times 0.4 = 0.12$

(iii) $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18 = 0.9 - 0.18 = 0.72$

(iv) $P(\text{neither } A \text{ nor } B) = P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$

$= [1 - P(A)] [1 - P(B)]$

$= (1 - 0.3)(1 - 0.6) = 0.7 \times 0.4 = 0.28.$

S6. Here,

$$P(A) = \frac{1}{2}, P(A \cup B) = \frac{3}{5} \text{ and } P(B) = p.$$

(i) A and B are mutually exclusive

$$\Rightarrow \quad P(A \cap B) = 0$$

Now $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore \quad \frac{3}{5} = \frac{1}{2} + p - 0 \Rightarrow p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$$

(ii) A and B are independent

$\therefore \quad P(A \cap B) = P(A) \times P(B)$

Now $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$$

$$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2} \times p$$

$$\Rightarrow \quad \frac{3}{5} - \frac{1}{2} = p\left(1 - \frac{1}{2}\right) \Rightarrow \frac{1}{10} = p \times \frac{1}{2} \Rightarrow p = \frac{1}{5}.$$

S7. When a die is thrown, there are 3 odd numbers on the die out of 6 numbers.

$\therefore$ Probability of getting odd number = $\frac{3}{6} = \frac{1}{2}$

Probability of getting even number = $1 - \frac{1}{2} = \frac{1}{2}$

Now probability of getting no odd number when the die is tossed thrice

= Probability of getting even number when the die is tossed thrice

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$\therefore$ Probability of getting an odd number at least once when the die is tossed thrice

$$= 1 - \frac{1}{8} = \frac{7}{8}.$$

S8. Total number of balls =10 + 8 = 18

(i) Probability of getting both red balls

$$= \frac{8}{18} \times \frac{8}{18} = \frac{16}{81}.$$

(ii) Probability of getting black ball first and then red ball

$$= \frac{10}{18} \times \frac{8}{18} = \frac{20}{81}.$$

(iii) Probability of getting one black and other red ball

$$= \frac{10}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18} = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}.$$

S9. Given, Probability of student *A* coming in time,

$$P(A) = \frac{3}{7}$$

$\Rightarrow$ Probability of student *A* not coming in time,

$$P(\overline{A}) = 1 - \frac{3}{7} = \frac{4}{7}$$

Probability of student *B* coming in time,

$$P(B) = \frac{5}{7}$$

$\Rightarrow$ Probability of student of *B* not coming in time,

$$P(\overline{B}) = 1 - \frac{5}{7} = \frac{2}{7}$$

$\therefore$ Required Probability

$$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$$

$$= P(A) \cdot P(\overline{B}) + P(\overline{A}) \cdot P(B)$$

$$= \frac{3}{7} \cdot \frac{2}{7} + \frac{4}{7} \cdot \frac{5}{7}$$

$$= \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$$

S10. Given,

$$P(A_T) = \frac{60}{100}, \quad P(\overline{A}_T) = 1 - \frac{60}{100} = \frac{40}{100}$$

$$P(B_T) = \frac{90}{100}, \quad P(\overline{B}_T) = 1 - \frac{90}{100} = \frac{10}{100}$$

P (*A* and *B* are contradict to each other)

$$= P(A_T) \cdot P(\overline{B}_T) + P(\overline{A}_T) \cdot P(B_T)$$

$$= \frac{60}{100} \times \frac{10}{100} + \frac{40}{100} \times \frac{90}{100}$$

$$= \frac{600 + 3600}{10000} = \frac{4200}{10000} = \frac{42}{100}$$

Percentage of (*A* and *B* are contradict to each other) = 42%

Yes, the statement of *B* will carry more weights as he speaks truth in more number of cases than *A*.

S11. Let

$P(A)$ = Probability that A solves the problems

$P(B)$ = Probability that B solves the problems

$P(\overline{A})$ = Probability that A does not solve the problems

$P(\overline{B})$ = Probability that B does not solve the problems

According to the question, given that

$$P(A) = \frac{1}{2}$$

$\Rightarrow$ $$P(\overline{A}) = 1 - \frac{1}{2} = \frac{1}{2} \qquad \left[\because P(A) + P(\overline{A}) = 1\right]$$

and $$P(B) = \frac{1}{3}$$

$\Rightarrow$ $$P(\overline{B}) = 1 - \frac{1}{3} = \frac{2}{3}$$

(i) P (problems is solved) $= P(A)\cdot P(\overline{B}) + P(\overline{A})\cdot P(B) + P(A)\cdot P(B)$

$$= \left(\frac{1}{2}\times\frac{2}{3}\right) + \left(\frac{1}{2}\times\frac{1}{3}\right) + \left(\frac{1}{2}\times\frac{1}{3}\right)$$

$$= \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

$\therefore$ Probability that the problem is solved $= \frac{2}{3}$

Alternate Method:

P (problem is solved) = 1 – P (none of them solve problem)

$$= 1 - P(\overline{A})P(\overline{B}) = 1 - \left(\frac{1}{2}\times\frac{2}{3}\right) \quad \left[\because P(\overline{A}) = \frac{1}{2} \text{ and } P(\overline{B}) = \frac{2}{3}\right]$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

(ii) P (exactly one of them solves the problem)

$$= P(A)\cdot P(\overline{B}) + P(\overline{A})\cdot P(B)$$

$$= \left(\frac{1}{2}\times\frac{2}{3}\right) + \left(\frac{1}{2}\times\frac{1}{3}\right)$$

$$= \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}.$$

S12. Given,

$$P(A_T) = \frac{75}{100}, \quad P(\overline{A}_T) = 1 - \frac{75}{100} = \frac{25}{100}$$

$$P(B_T) = \frac{90}{100}, \quad P(\overline{B}_T) = 1 - \frac{90}{100} = \frac{10}{100}$$

$P(A$ and B are contradict to each other)

$$= P(A_T \cap \overline{B}_T) + (\overline{A}_T \cap B_T)$$

$$= P(A_T) \cdot P(\overline{B}_T) + P(\overline{A}_T) \cdot P(B_T)$$

$$= \frac{75}{100} \times \frac{10}{100} + \frac{25}{100} \times \frac{90}{100}$$

$$= \frac{750 + 2250}{10000} = \frac{3000}{10000} = \frac{3}{10}$$

$\therefore$ Percentage of $P(A$ and B are contradict to each other)

$$= \frac{3}{10} \times 100 = 30\%$$

We think that statement of B is false.

S13. Given,

$$P(P_T) = \frac{70}{100}, \quad P(\overline{P}_T) = 1 - \frac{70}{100} = \frac{30}{100}$$

$$P(Q_T) = \frac{80}{100}, \quad P(\overline{Q}_T) = 1 - \frac{80}{100} = \frac{20}{100}$$

P (A and B are agree to each other)

$$= P(P_T \cap Q_T) + P(\overline{P}_T \cap \overline{Q}_T)$$

$$= P(P_T) \cdot P(Q_T) + P(\overline{P}_T) \cdot P(\overline{Q}_T)$$

$$= \frac{70}{100} \cdot \frac{80}{100} + \frac{30}{100} \cdot \frac{20}{100}$$

$$= \frac{5600}{10000} + \frac{600}{10000}$$

$$= \frac{6200}{10000} = \frac{62}{100}$$

$\therefore$ Percentage of A and B are agree to each other

$$= \frac{62}{100} \times 100 = 62\%$$

No, agree does not mean they are speaking truth.

S14. Let, E_1 = Event of A getting six and E_2 = Event of B getting six

In throwing a die,

Total number of sample spaces $n(S) = 6$

$$\therefore \qquad P(E_1) = P(E_2) = \frac{1}{6}$$

$\therefore$ Probability of not getting a six $P(\overline{E}_1) = P(\overline{E}_2) = 1 - \frac{1}{6} = \frac{5}{6}$

Since, the referee gives first chance to captain A for throwing a die.

$$\therefore \qquad \text{Probability of } A \text{ winning} = \left[P(E_1) + P(\overline{E}_1 \cap \overline{E}_2 \cap E_1) + \ldots\right]$$

$$= P(E_1) + P(\overline{E}_1) \cdot P(\overline{E}_2) \cdot P(E_1) + \ldots$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \ldots\right]$$

$$= \frac{1}{6}\left(\frac{1}{1 - \frac{25}{36}}\right) \qquad \left[\therefore \text{Sum of infinie GP, } i.e., S_\infty = \frac{a}{1-r}\right]$$

$$= \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

$$\therefore \qquad \text{Probability of } B \text{ winning} = P(\overline{E}_1 \cap E_2) + P(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_1 \cap E_2) + \ldots$$

$$= P(\overline{E}_1) \cdot P(E_2) + P(\overline{E}_1) P(\overline{E}_2) P(\overline{E}_1) P(E_2) + \ldots$$

$$= \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots$$

$$= \frac{5}{36}\left[1 + \left(\frac{5}{6}\right)^2 + \ldots\right]$$

$$= \frac{5}{36}\left(\frac{1}{1 - \frac{25}{36}}\right) = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$$

Here, we see that $P(A) > P(B)$.

Hence, team A has more chance of winning a game.

As the referee first give a chance to team A, hence it is not a fair decision.

S15. Let A be the event when head appears on the coin.

$$\therefore \qquad P(A) = \frac{1}{2}$$

Let B be the event when 3 appears on the die.

$\therefore \qquad P(B) = \frac{1}{6}$

The sample space for 'a coin is tossed and a die is tossed' is

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

Now $A \cap B$ is 'head appears on the coin and 3 appears on the die'

$\therefore \qquad A \cap B = \{H3\}$

$\therefore \qquad P(A \cap B) = \frac{1}{12}$

Now $\qquad P(A \cap B) = P(A) \times P(B) = \frac{1}{12}$

which shows that A and B are independent events.

S16. (i) E : the card drawn is a spade

$\therefore \qquad P(E) = \frac{13}{52} = \frac{1}{4}$

F : the card drawn in an ace

$\therefore \qquad P(F) = \frac{4}{52} = \frac{1}{13}$

$E \cap F$: the card drawn in an ace of spade

$\therefore \qquad P(E \cap F) = \frac{1}{52}$

Now $\quad P(E) \times P(F) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} = P(E \cap F)$

Thus events E and F are independent.

(ii) E : the card drawn is black

$\therefore \qquad P(E) = \frac{26}{52} = \frac{1}{2}$

F : the card drawn is a king

$\therefore \qquad P(F) = \frac{4}{52} = \frac{1}{13}$

$E \cap F$: the card drawn is a king of black colour

$\therefore \qquad P(E \cap F) = \frac{2}{52} = \frac{1}{26}$

Now $\quad P(E) \times P(F) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} = P(E \cap F)$

Thus events E and F are independent.

(iii) E : the card drawn is a king or queen

$\therefore \quad P(E) = \frac{8}{52} = \frac{2}{13}$

F : the card drawn is a queen or jack

$\therefore \quad P(F) = \frac{8}{52} = \frac{2}{13}$

$E \cap F$: the card drawn is a queen

$\therefore \quad P(E \cap F) = \frac{4}{52} = \frac{1}{13}$

Now $\quad P(E) \times P(F) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169}$

$\therefore \quad P(E \cap F) \neq P(E) \times P(F)$

Thus events E and F are not independent.

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Q1. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

Q2. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin?

Q3. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the sum of the numbers obtained is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4 .... 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8?

Q4. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A fair die is cast, if the face 1 or 3 turns up, a ball is taken from the first bag, and if any other face turns up a ball is chosen from the second bag. Find the probability of choosing a black ball.

Q5. A contents of three bags I, II and III are as follows:

Bag I: 1 white, 2 black and 3 red balls,

Bag II: 2 white, 1 black and 1 red ball, and

Bag III: 4 white, 5 black and 3 red balls.

A bag is chosen at random and two balls are drawn. What is the probability that the balls are white and red?

Q6. There are two bags. The first bag contains 5 white and 3 black balls the second bag contains 3 white and 5 black balls. Two balls are drawn at random from the first bag and are put into the second bag without noticing their colours. Then two balls are drawn from the second bag. Find the probability that the balls are white and black.

Q7. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball is transferred form first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

Q8. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. *A* ball is drawn from the first bag and without noticing its colours is put in the second bag. A ball is then drawn from the second bag. Find the probability that the ball drawn is blue in colour.

Q9. A bag contains 4 red and 3 back balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag, one ball is drawn. Find the probability that the ball drawn is red.

Q10. Find the probability of drawing a one-rupee coin from a purse with two compartment one of which contains 3 fifty-paise coins and 2one-rupee coins other contains 2 fifty-paise coins and 3one-rupee coins.

Q11. In a bolt factory, machines *A*, *B* and *C* manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. What is the probability that the bolt drawn is defective?

Date: 19/10/2021

S1. Let A be the event that the construction job will be completed on time, E_1 be the event that there will be a strike and E_2 be the event that there will be no strike.

We have,

$$P(E_1) = 0.65,\ P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$$

$$P(A/E_1) = 0.32 \quad \text{and} \quad P(A/E_2) = 0.80$$

By total probability theorem, we have

$$\begin{aligned} \text{Required probability} &= P(A) \\ &= P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) \\ &= 0.65 \times 0.32 + 0.35 \times 0.80 \\ &= 0.208 + 0.28 = 0.488 \end{aligned}$$

S2. Consider the following events:

$$E_1 = \text{Selecting first purse,}$$

$$E_2 = \text{selecting second purse,}$$

$$A = \text{coin drawn is silver coin}$$

We have, $\quad P(E_1) = P(E_2) = \frac{1}{2}, \quad P(A/E_1) = \frac{2}{6}, \quad P(A/E_2) = \frac{4}{7}$

Required probability $= P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$

$$= \frac{1}{2} \times \frac{2}{6} + \frac{1}{2} \times \frac{4}{7} = \frac{19}{42}.$$

S3. Let E_1 = the coin shows a head, E_2 = the coin shows a tail,

A = the noted number is 7or 8.

Then, $\quad P(E_1) = 1/2,\ P(E_2) = 1/2, \quad P(A/E_1) = 11/36 \quad \text{and} \quad P(A/E_2) = 2/11$

Required probability $\quad P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{11} = \frac{193}{792}.$$

S4. Let E_1, E_2 and A be the events defined as follows:

E_1 = The die shows 1 or 3

E_2 = The die shows 2, 4, 5 or 6

and, A = The ball drawn in black

We have, $P(E_1) = \frac{2}{6} = \frac{1}{3}$, $P(E_2) = \frac{4}{6} = \frac{2}{3}$

If E_1 occurs, then the first bag is chosen and the probability of drawing a black ball from it is $\frac{3}{7}$

$\therefore$ $P(A/E_1) = \frac{3}{7}$

If E_2 occurs, then the second bag is chosen and the probability of drawing a black ball from it is $\frac{4}{7}$

$\therefore$ $P(A/E_2) = \frac{4}{7}$

By the law of total probability, we have

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) = \frac{1}{3}\times\frac{3}{7}+\frac{2}{3}\times\frac{4}{7} = \frac{11}{21}.$$

S5. Let E_1 = bag I is selected, E_2= bag II is selected, E_3 = bag III is selected

and, A = two balls drawn from the chosen bag are white and red

Then, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$$P(A/E_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{3}{15} = \frac{1}{5},$$

$$P(A/E_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2}{6} = \frac{1}{3} \quad \text{and} \quad P(A/E_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{12}{66} = \frac{2}{11}$$

$\therefore$ Required probability

$$= P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$$

$$= \frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11} = \frac{109}{495}.$$

S6. A white and a black ball can be drawn from the second bag in the following mutually exclusive ways:

(i) By transferring 2 black balls from first bag to the second bag and then drawing a white and a black ball from it.

(ii) By transferring 2 white balls from first bag to the second bag and then drawing a white and a black ball from it.

(iii) By transferring one white and one black ball from first bag to the second bag and then drawing a white and a back ball from it.

Let E_1, E_2, E_3 and A be the events as defined below :

E_1 = two black balls are drawn from the first bag,

E_2 = two white balls are drawn from the first bag,

E_3 = one white and one black ball is drawn from the first bag,

A = two balls drawn from the second bag are white and black.

We have, $P(E_1) = \frac{^3C_2}{^8C_2} = \frac{3}{28}$, $P(E_2) = \frac{^5C_2}{^8C_2} = \frac{5}{14}$ and $P(E_3) = \frac{^3C_1 \times ^5C_1}{^8C_2} = \frac{15}{28}$.

If E_1 has already occurred, that is, if two black balls have been transferred from the first bag to the second bag, then the second bag will contain 3 white and 7 black balls, therefore the probability of drawing a white and a black ball from the second bag is $\frac{^3C_1 \times ^7C_1}{^{10}C_2}$

$$\therefore \quad P(A/E_1) = \frac{^3C_1 \times ^7C_1}{^{10}C_2} = \frac{7}{15}$$

Similarly, we have

$$P(A/E_2) = \frac{^5C_1 \times ^5C_1}{^{10}C_2} = \frac{5}{9} \quad \text{and} \quad P(A/E_3) = \frac{^4C_1 \times ^6C_1}{^{10}C_2} = \frac{8}{15}$$

By the law of total probability, we have

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$$

$$= \frac{3}{28} \times \frac{7}{15} + \frac{5}{14} \times \frac{5}{9} + \frac{15}{28} \times \frac{8}{15} = \frac{673}{1260}.$$

S7. A white ball can be drawn form the second bag in two mutually exclusive ways:

(i) By transferring a white ball from first bag to the second bag and then drawing a white ball from it.

(ii) By transferring a black ball from first bag to the second bag and then drawing a white ball from it.

Let E_1, E_2 and A be the events defined as follows:

E_1 = a ball drawn from the first bag is white

E_2 = a ball drawn from the first bag is black

A = a white ball is drawn from the second bag

Since the first bag contains 4 white and 5 black balls, we have

$$P(E_1) = \frac{4}{9} \text{ and } P(E_2) = \frac{5}{9}$$

If E_1 has already occurred, that is a white ball has already been transferred from first bag to the second bag, then the second bag contains 7 white and 7 black balls.

So, $P(A/E_1) = \frac{7}{14}$

If E_2 has already occurred, that is a black ball has been transferred from first bag to the second bag, then the second bag contains 6 white and 8 black balls. So,

$$P(A/E_2) = \frac{6}{14}$$

By the law of total probability, we have

P (Getting a white ball)

$$= P(A) = P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) = \frac{4}{9}\times\frac{7}{14}+\frac{5}{9}\times\frac{6}{14}$$

$$= \frac{58}{126} = \frac{29}{63}.$$

S8. A blue colour ball can be drawn from the second bag in the following mutually exclusive ways :

(i) By transferring a blue ball from first bag to the second bag and then drawing a blue ball from the second bag.

(ii) By transferring a red ball from first bag to the second bag and then drawing a blue ball from the second bag.

Let E_1, E_2 and A be the events defined as follows :

E_1 = ball drawn from first bag is blue

E_2 = ball drawn from first bag is red

A = ball drawn from the second bag is blue

Since first bag contains 6 red and 5 blue balls,

we have

$$P(E_1) = \frac{5}{11} \text{ and } P(E_2) = \frac{6}{11}$$

If E_1 has already occurred, that is, if a blue ball is transferred from the first bag to the second bag, then the second bag contains 5 red and 9 blue balls, therefore the probability of drawing a blue balls from the second bag is $\frac{9}{14}$

$$\therefore \qquad P(A/E_1) = \frac{9}{14}$$

Similarly, we have $P(A/E_2) = \frac{8}{14}$

By the law of total probability, we have

$$P(A) = P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) = \frac{5}{11}\times\frac{9}{14}+\frac{6}{11}\times\frac{8}{14} = \frac{93}{154}.$$

S9. A red ball can be drawn in two mutually exclusive ways.

(i) Selecting bag I and then drawing a red ball from it.

(ii) Selecting bag II and then drawing a red ball from it.

Let E_1, E_2 and A denote the events defined as follows:

$$E_1 = \text{Selecting bag I,}$$
$$E_2 = \text{Selecting bag II,}$$
$$A = \text{Drawing a red ball}$$

Since one of the two bags is selected randomly

$$\therefore \quad P(E_1) = \frac{1}{2} \text{ and } P(E_2) = \frac{1}{2}$$

Now $P(A/E_1)$ = Probability of drawing a red ball when the first bag has been chosen.

$$= \frac{4}{7} \qquad \left[\because \text{First bag contains 4 red and 3 black balls}\right]$$

and, $P(A/E_2)$ = Probability of drawing a red when the second bag has been selected

$$= \frac{2}{6} \qquad [\because \text{Second bag contains 2 red and 4 black balls}]$$

Using the low of total probability, we have

$$P(\text{red ball}) = P(A) = P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)$$
$$= \frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{2}{6} = \frac{19}{42}.$$

S10. A one rupee coin can be drawn in two mutually exclusive ways:

(i) Selecting compartment I and then drawing a rupee coin from it.

(ii) Selecting compartment II and then drawing a rupee coin from it.

Let E_1, E_2 and A be the events defined as follows:

E_1 = the first compartment of the purse is chosen,

E_2 = the second compartment of the purse is chosen,

A = a rupee coin is drawn from the purse.

Since one of the two compartment is chosen randomly, we have

$$P(E_1) = \frac{1}{2} = P(E_2)$$

Also,

$P(A/E_1)$ = Probability drawing a rupee coin given that the first compartment of the purse is chosen.

$= \dfrac{2}{5}$ $\left[\because \begin{array}{l}\text{First compatment contains 3 fifty}\\ \text{paise coins and 2 one rupee coins}\end{array}\right]$

and $P(A/E_2)$ = Probability of drawing a rupee coin given that second compartment of purse is chosen

$= \dfrac{3}{5}$ [$\because$ second compartments contain 2 fifty paise coins and 3 one rupee coins]

By the law of total probability, we have

P(Drawing a one rupee coin]

$$= P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) = \frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{3}{5} = \frac{1}{2}.$$

S11. Let E_1, E_2, E_3 and A be the events defined as follows:

E_1 = the bolts is manufactured by machine A;

E_2 = the bolts is manufactured by machine B;

E_3 = the bolt is manufactured by machine C,

and, A = the bolt is defective. Then,

$$P(E_1) = \frac{25}{100} = \frac{1}{4},\quad P(E_2) = \frac{35}{100},\quad P(E_3) = \frac{40}{100}$$

$P(A/E_1)$ = Probability that the bolt drawn is defective given the condition that it is manufactured by machine A

= 5/100

Similarly, we have

$$P(A/E_2) = \frac{4}{100} \text{ and } P(A/E_3) = \frac{2}{100}$$

Using the law of total probability, we have

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$$

$$= \frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{40}{100}\times\frac{2}{100} = 0.0345.$$

Date: 19/10/2021

Q1. A is known to speak the truth 3 times out of 5 times. He throws a die and reports that it is 1. Find the probability that it is actually 1.

Q2. Two groups are competing for the post of board of directors of a corporation. The probabilities that the first and second group wins are 0.6 and 0.4, respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product was introduced by second group.

Q3. An insurance company insured 2000 scooters and 3000 motor cycles. The probability of an accident involving a scooter is 0.01 and that of a motor cycle is 0.02. An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motor cycle.

Q4. A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is a number greater than 4. Find the probability that it is actually a number greater than 4.

Q5. A speaks the truth 8 times out of 10 times. A die is tossed. He reports that it was 5. What is the probability that it was a actually 5?

Q6. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Q7. A bag A contains 1 white and 6 red balls, bag B contains 4 white and 3 red balls. One ball is drawn at random from one of the chosen bags and is found to be white, Find the probability that it was drawn from bag A.

Q8. Two bags contain 4 red and 4 black, 2 red and 6 black balls. One ball is drawn from one of the bags and found to be red. Find the probability that it was drawn from the second bag

Q9. A bag contains 4 red and 4 black balls. Other bag contains 2 red and 6 black balls. One of the bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that ball is drawn from first bag?

Q10. Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at the random. What is the probability of this person being male? Assume that there are equal number of males and females.

Q11. A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter has come from (i) CALCUTTA (ii) TATANAGAR?

Q12. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year result report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual exams. At the end of year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hosteller?

Q13. Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she throw 1, 2, 3 or 4 with the die.

Q14. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods, is more beneficial for the patient.

Q15. Suppose that the reliability of a HIV test is specified as follows:

Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV –ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV?

Q16. The compressors used in refrigerators are manufactured by three different factories at Pune, Nasik and Nagpur. It is known that the Pune factory produces twice as many compressors as Nasik are, which produces the same number as the Nagpur are (during the same period). Experience also shows that 0.2% of the compressors produced at Pune as well as at Nasik are defective and so are 0.4% of those produced at Nagpur. A quality controller chooses a compressor and finds it a defective one. What is the probability that it was produced at Nasik factory?

Q17. A company has two plants to manufacture T.V.'s. The first plant manufactures 70% of the T.V.'s and the rest are manufactured by the other plant. 80% of T.V's manufactured by first are rated to standard quality; while that of the second plant only 70% are of standard quality. If a T.V. chosen at random is found to be of standard quality, find the probability that it was produced by the first plant.

Q18. A factory has two machines *A* and *B*. Past record shows that machine *A* produced 60% of items of output and machine *B* produced 40% of items. Further, 2% of items, produced by machine *A* and 1% produced by machine *B* were defective. All the items are put into a stockpile and then one item is chosen at random from this and this is found to be defective. What is the probability that it was produced by machine *B*?

Q19. In a bolt factory machines, *A*, *B* and *C* manufacture 25%, 35% and 40% of total production, respectively. Out of their total output 5%, 4% and 2% are defective bolts. A bolt is drawn at random and is found to be defective. What is the probability that it is manufactured by machine *B*?

Q20. In a bulb factory machines *A*, *B* and *C* manufacture 60%, 30% and 10% bulbs, respectively. 1%, 2% and 3% of bulbs produced, respectively by *A*, *B* and *C* are found to be defective. A bulb is picked up at random from total production and found to be defective. Find the probability that this bulb was produced by machine *A*.

Q21. A company has two plants to manufacture motorcycles. 70% motorcycles are manufactured at the first plant, while 30% are manufactured at the second plant. At first plant, 80% motorcyles are rated of the standard quality while at the second plant, 90% are rated of standard quality. A motorcycle, randomly picked up, and is found to be of standard quality. Find the probability that it has come out from the second plant.

Q22. A manufacturer has three machine operators *A*, *B* and *C*. The first operator *A* produces 1% defective items, whereas the other two operators *B* and *C* produce 5% and 7% defective items respectively. *A* is on the job for 50% of the time, *B* is on the job for 30% of the time and *C* is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by *A*?

Q23. A factory has three machines *X*, *Y* and *Z*, prdoucing 1,000, 2,000 and 3,000 bolts per day respectively. The machine *X* produces 1% defective bolts, *Y* produces 1.5% and *Z* produces 2% defective bolts. At the end of a day, a bolt is drawn at random and is found to be defective. What is the probability that this defective bolt has been produced by machine *X*?

Q24. A company has two plants to manufacture motor cycles. Plant 1 manufactures 70% of motor cycles and Plant 2 manufactures 30%. At plant 1,80% of the motor cycles are rated of standard quality and at plant 2, 90% of the motor cycles are rated of standard quality. A motor cycle is chosen at random and is found to be of standard quality. Find the probability that it has come from Plant 1.

Q25. A doctor is to visit a patient. From the past experience, it is known that the probability that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probability that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives he is late. What is the probability that he came by train?

Q26. An insurance company insured 2000 scooter drivers, 4000 car drivers, 6000 truck drivers. The probability of their meeting with an accident are 0.01, 0.03 and 0.15, respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

Q27. A doctor is to visit a patient. From the past experience it is known that the probabilities of doctor coming by train, bus, scooter and taxi are $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}$ and $\frac{2}{5}$, respectively. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter, respectively but by taxi he will not be late. When he arrives, is late; what is the probability that he came by bus?

Q28. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Q29. Two bags *A* and *B* contain 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag *A*, two balls are drawn at random and then transferred to bag *B*. A ball is drawn from bag *B* and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black?

Q30. There are three coins. One is a two headed coin (having head on both faces), other is a biased coin that comes up heads 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, and it shows head. What is the probability that it was the two headed coin.

Q31. There are three coins. One is a two tailed coin (having tail on both faces), another is a biased coin that comes up heads 60% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, and it shows tail. What is the probability that it is a two tailed coin?

Q32. There are three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and take out a coin. If the coin is of gold, what is the probability that the other coin in box is also of gold?

Q33. In a class 5% of boys and 10% of girls have an *IQ* of more than 150. In the class 60% are boys and rest girls. If a student is selected at random and found to have an *IQ* of more than 150, find probability that the student is a boy.

Q34. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both clubs. Find the probability of the lost card being of club?

Q35. A card from a pack of 52 cards is missing. From the remaining cards of the pack, two cards are drawn and are found to be heart. Find the probability of the missing card to be a heart.

Q36. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that the copies the answer is 1/6. The probability that his answer is correct, given that he copied it, is 1/8. Find the probability that he knew the answer to the question, given that he correctly answered it.

Q37. Coloured balls are distributed in three bags as shown in the following table

Bag	Colour of Ball		
	Black	White	Red
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and then two balls are randomly drawn from selected bag. They happen to be black and red. What is the probability that they come from bag I?

Q38. Three bags contain balls as shown in the table

Bag	White Balls	Black Balls	Red Balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from bag III.

Q39. Bag I contains 3 red and 4 black balls and bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II.

Q40. A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white?

Q41. Three urns *A*, *B* and *C* contain 6 red and 4 white; 2 red and 6 white and 1 red and 5 white balls respectively. An urn is chosen random and a ball is drawn. If the ball drawn is found to be red, find the probability that the ball was drawn from the urn *A*.

Date: 19/10/2021

S1. Let E_1, E_2 and A events be as below :

E_1 = 1 occurs, E_2 = 1 does not occur

and A = man reports that it is 1

We have : $P(E_1) = \frac{1}{6}, P(E_2) = \frac{5}{6}$

Now $P(A/E_1)$ = Probability that the man reports that it is 1, given that 1 has occurred on the die

= Probability that the man speaks truth = $\frac{3}{5}$

and $P(A/E_2)$ = Probability that the man reports that it is 1, given that 1 has not occurred on the die

= Probability that the man does not speak truth = $1-\frac{3}{5} = \frac{2}{5}$

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} = \frac{\left(\frac{1}{6}\right)\left(\frac{3}{5}\right)}{\left(\frac{1}{6}\right)\left(\frac{3}{5}\right)+\left(\frac{5}{6}\right)\left(\frac{2}{5}\right)}$$

$$= \frac{3}{3+10} = \frac{3}{13}$$

S2. Let us define the events as

E_1 : First group wins

E_2 : Second group wins

A : The new product is launched

To find : $P\left(\frac{E_2}{A}\right)$

Now, by Baye's theorem, we have

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \quad \ldots(i)$$

Now, $P(E_1)$ = Probability that first group wins = 0.6

$P(E_2)$ = Probability that second group wins = 0.4

and $P\left(\frac{A}{E_1}\right)$ = Probability that first group wins and it launches the new product = 0.7

$P\left(\frac{A}{E_2}\right)$ = Probability that second group wins and it launches the new product = 0.3

Putting above values in Eq. (i), we get

$$P\left(\frac{E_2}{A}\right) = \frac{0.4 \times 0.3}{(0.6 \times 0.7) + (0.4 \times 0.3)}$$

$$= \frac{0.12}{0.42 + 0.12}$$

$$= \frac{0.12}{0.54}$$

$$= \frac{12}{54} = \frac{2}{9}$$

Hence, the required probability = $\frac{2}{9}$.

S3. Let E_1, E_2 and A events be as below :

E_1 : Insurance of scooter

E_2 : Insurance of a motor cycle

A : Accident of vehicle

We have : $P(E_1) = \frac{2000}{5000} = \frac{2}{5}$, $P(E_2) = \frac{3000}{5000} = \frac{3}{5}$

and $P(A/E_1)$= 0.01, $P(A/E_2)$ = 0.02

By Baye's Theorem,

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{3}{5}(0.02)}{\frac{2}{5}(0.01) + \frac{3}{5}(0.02)} = \frac{0.06}{0.02 + 0.06} = \frac{0.06}{0.08} = \frac{6}{8} = \frac{3}{4}$$

S4. Let us define the events as

E_1 : A number greater than 4 appears on the die

E_2 : A number greater than 4 does not appear on the die

A : Man reports that it is a number greater than 4.

To find : $P\left(\frac{E_1}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \quad \text{...(i)}$$

Now, $P(E_1)$ = Probability of getting a number greater than 4 = $\frac{2}{6} = \frac{1}{3}$

[$\because$ There are only two numbers greater than 4 *i.e.,* 5 and 6]

$P(E_2)$ = Probability that the number greater than 4 does not appear = $1 - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$

$P\left(\frac{A}{E_1}\right)$ = Probability that the man reports it is a number greater than 4 on the die and it is so *i.e.,* he is speaking truth = $\frac{3}{5}$ [Given that he speaks truth 3 out of 5 times]

$P\left(\frac{A}{E_2}\right)$ = Probability that man reports it is a number greater than 4 on the die but it is not so *i.e.,* he is not speaking truth = $1 - \frac{3}{5} = \frac{2}{5}$

Putting above values in eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3}\times\frac{3}{5}}{\left(\frac{1}{3}\times\frac{3}{5}\right)+\left(\frac{2}{3}\times\frac{2}{5}\right)}$$

$$= \frac{\frac{3}{5}}{\frac{3}{5}+\frac{4}{5}} = \frac{\frac{3}{5}}{\frac{7}{5}} = \frac{3}{7}$$

Hence, the required probability = $\frac{3}{7}$.

S5. Let E_1 and E_2 be the events when A speaks the truth or not respectively.

$\therefore \quad P(E_1) = \frac{8}{10} = \frac{4}{5}, \quad P(E_2) = 1 - \frac{4}{5} = \frac{1}{5}$

Let A be the event when there is 5.

$\therefore \quad P(A/E_1) = \frac{1}{6}$ and $P(A/E_2) = \frac{5}{6}$

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} = \frac{\left(\frac{4}{5}\right)\left(\frac{1}{6}\right)}{\left(\frac{4}{5}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{5}\right)\left(\frac{5}{6}\right)} = \frac{4}{4+5} = \frac{4}{9}$$

S6. Let E_1 and E_2 be the events when lost card is a diamond and not a diamond respectively.

$\therefore \quad P(E_1) = \frac{13}{52} = \frac{1}{4}$

and $\quad P(E_2) = \frac{39}{52} = \frac{3}{4}$

Let A be the event :

"two cards drawn from the remaining pack are diamonds"

$\therefore \quad P(A/E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12 \times 11}{51 \times 50}$

and $\quad P(A/E_2) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13 \times 12}{51 \times 50}$

By Bayes' Theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{12 \times 11}{51 \times 50}\right)}{\left(\frac{1}{4}\right)\left(\frac{12 \times 11}{51 \times 50}\right) + \left(\frac{3}{4}\right)\left(\frac{13 \times 12}{51 \times 50}\right)}$$

$$= \frac{12 \times 11}{12 \times 11 + 3 \times 13 \times 12} = \frac{132}{132 + 468} = \frac{132}{600} = \frac{11}{50}$$

S7. Let the events, E_1, E_2 and A be as below :

E_1 : Ball drawn from bag A

E_2 : Ball drawn from bag B

and S = Ball drawn is white

We have to find $P(E_1/A)$

Since both the bags are equally likely to be selected,

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Also, $P(A/E_1) = \frac{1}{7}$ and $P(A/E_2) = \frac{4}{7}$

By Baye's Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{7}\right)} = \frac{1}{1+4} = \frac{1}{5}.$$

S8. Let the events E_1, E_2 and A be as below:

E_1 : Ball drawn from bag I

E_2 : Ball drawn from bag II

A : Ball drawn is red.

We have to find $P(E_2/A)$

Since both the bags are equally likely to be selected,

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}$$

Also $P(A/E_1) = \frac{4}{8} = \frac{1}{2}$ and $P(A/E_2) = \frac{2}{8} = \frac{1}{4}$

By Bayes' Theorem,

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\left(\frac{1}{4}\right)}{\frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{4}\right)} = \frac{\frac{1}{8}}{\frac{1}{8}(2+1)} = \frac{1}{3}.$$

S9. Let us define the events as

E_1 : Bag I is selected

E_2 : Bag II is selected

A : The drawn ball is a red ball

To find : $P\left(\frac{E_1}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \quad \text{... (i)}$$

Now, $P(E_1) = P(E_2) = \frac{1}{2}$

and $P\left(\frac{A}{E_1}\right)$ = Probability that the red ball is drawn from bag I = $\frac{4}{8} = \frac{1}{2}$

[$\because$ Bag I contains 4 red and 4 black balls.]

$P\left(\frac{A}{E_2}\right)$ = Probability that the red ball is drawn from bag II = $\frac{2}{8} = \frac{1}{4}$

[$\because$ Bag II contains 2 red and 6 black balls]

Putting above values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2}\times\frac{1}{2}}{\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{2+1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}}$$

$$= \frac{1}{2}\times\frac{4}{3} = \frac{2}{3}$$

Hence, required probability = $\frac{2}{3}$.

S10. Let us define events

E_1 : Person selected is a male

E_2 : Person selected is a female

A : Person selected has grey hair

To find : $P\left(\frac{E_1}{A}\right)$. *i.e.*, probability of selecting person is male.

Now, by Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \qquad \ldots(i)$$

Now, we find the various probabilities as follows

$P(E_1) = P$ (person selected is a male) = $\frac{1}{2}$

and $P(E_2) = P$ (person selected is a female) = $\frac{1}{2}$

[$\because$ Given that, there are equal number of males and females]

Now, $P\left(\frac{A}{E_1}\right)$ = Probability of selecting a male person who has grey hair = $\frac{5}{100}$ [Given]

and $P\left(\frac{A}{E_2}\right)$ = Probability of selecting a female person who has grey hair = $\frac{0.25}{100}$ [Given]

Putting all above probabilities values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2}\times\frac{5}{100}}{\left(\frac{1}{2}\times\frac{5}{100}\right)+\left(\frac{1}{2}\times\frac{0.25}{100}\right)}$$

$$= \frac{\frac{5}{200}}{\frac{5}{200}+\frac{0.25}{200}} = \frac{5}{5+0.25}$$

$$= \frac{5}{5.25} = \frac{500}{525}$$

$$= \frac{100}{105} = \frac{20}{21}$$

$\therefore$ Required probability = $\frac{20}{21}$.

S11. Let E_1 be the event that the letter came from CALCUTTA and E_2 be the event that the letter came from TATANAGAR. Let A denote the event that two consecutive letters visible on the envelope are TA.

Since the letters have come either from CALCUTTA or TATANAGAR. Therefore,

$$P(E_1) = \frac{1}{2} = P(E_2)$$

If E_1 has occurred, then it means that the letter came from CALCUTTA. In the word CALCUTTA there are 8 letters in which TA occurs in the end. Considering TA as one letter there are seven letters out of which one can be in 7 ways. Therefore,

$$P(A/E_1) = \frac{1}{7}$$

If E_2 has occurred, then the letter came from TATANAGAR. In the word TATANAGAR there are 9 letters in which TA occurs twice. Considering one of the two TA's as one letter there are 8 letters. Therefore,

$$P(A/E_2) = \frac{2}{8}$$

By Baye's theorem, we have

(i) $$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} = \frac{\frac{1}{2}\times\frac{1}{7}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{2}{8}} = \frac{4}{11}$$

(ii) $$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} = \frac{\frac{1}{2}\times\frac{2}{8}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{2}{8}} = \frac{7}{11}$$

S12. Let us define the events as

E_1 : Students reside in a hostel.

E_2 : Students are day scholars.

A : Students get A grade

To find : $P\left(\frac{E_1}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \quad \text{... (i)}$$

Now, given that

$$P(E_1) = \text{Probability that students reside in a hostel} = \frac{60}{100}$$

and $$P(E_2) = \text{Probability that students do not reside in a hostel} = \frac{40}{100}$$

Also, $$P\left(\frac{A}{E_1}\right) = \text{Probability that students who are hosteller get } A \text{ grade} = \frac{30}{100}$$

and $$P\left(\frac{A}{E_2}\right) = \text{Probability that students who are day scholars get } A \text{ grade} = \frac{20}{100}$$

Putting all above values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{60}{100} \times \frac{30}{100}}{\left(\frac{60}{100} \times \frac{30}{100}\right) + \left(\frac{40}{100} \times \frac{20}{100}\right)}$$

$$= \frac{1800}{1800 + 800} = \frac{1800}{2600} = \frac{18}{26}$$

$\therefore$ $$P\left(\frac{E_1}{A}\right) = \frac{9}{13}.$$

S13. Let us define events

E_1 : Girl gets 5 or 6 on a die

E_2 : Girl gets 1, 2, 3 or 4 on die

A : She gets exactly one head

To find : $P\left(\frac{E_2}{A}\right)$ by Baye's theorem, we have

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)} \quad \text{...(i)}$$

Now, $P(E_1)$ = Probability of getting 5 or 6 on a die = $\frac{2}{6} = \frac{1}{3}$

$P(E_2)$ = Probability of getting 1, 2, 3, or 4 on a die = $\frac{4}{6} = \frac{2}{3}$

$P\left(\frac{A}{E_1}\right)$ = Probability of girl getting exactly one head when she gets 5 or 6

i.e., when she throws coin thrice = $\frac{3}{8}$

[$\because$ {*HHH*, *TTT*, *HHT*, *HTH*, *THH*, *TTH*, *THT*, *TTH*}]

$P\left(\frac{A}{E_2}\right)$ = Probability of girl getting exactly one head when she gets 1, 2, 3, or 4

i.e., when she throws coin once = $\frac{1}{2}$

Putting all above values in eq. (i), we get

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{2}{3}\times\frac{1}{2}}{\left(\frac{1}{3}\times\frac{3}{8}\right)+\left(\frac{2}{3}\times\frac{1}{2}\right)}$$

$$= \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{3+8}{24}} = \frac{\frac{1}{3}}{\frac{11}{24}}$$

$$= \frac{1}{3}\times\frac{24}{11} = \frac{8}{11}$$

S14. Let E_1 : The patient follows meditation and yoga.

E_2 : The patient uses drug, then E_1 and E_2 are mutually exclusive

and $P(E_1) = P(E_2) = \frac{1}{2}$

Let E : The selected patient suffers a heart attack.

then $P\left(\frac{E}{E_1}\right) = \frac{40}{100}\left(1-\frac{30}{100}\right) = \frac{28}{100}$

and $P\left(\frac{E}{E_2}\right) = \frac{40}{100}\left(1-\frac{25}{100}\right) = \frac{30}{100}$

$\therefore$ P (patient who suffers heart attack follows meditation and yoga).

$$P\left(\frac{E_1}{E}\right) = \frac{P\left(\frac{E}{E_1}\right)P(E_1)}{P\left(\frac{E}{E_1}\right)P(E_1)+P\left(\frac{E}{E_2}\right)P(E_2)}$$ [Using Baye's theorem]

$$= \frac{\frac{28}{100} \times \frac{1}{2}}{\frac{28}{100} \times \frac{1}{2} + \frac{30}{100} \times \frac{1}{2}}$$

$$= \frac{28}{58} = \frac{14}{29}$$

Yoga course is more beneficial for the patient than a drug.

S15. Consider the following events :

E_1 = The person selected is actually having HIV

E_2 = The person selected is not having HIV

A = The person's HIV test is diagnosed as +ve

We have,

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001,\ P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$$

$P(A/E_1)$ = Probability that the person tested as HIV +ve given that he/she is actually having HIV.

$$\Rightarrow \quad P(A/E_1) = \frac{90}{100} = 0.9$$

and,

$\Rightarrow$ $P(A/E_2)$ = Probability that the person tested as HIV +ve given that he/she is actually not having HIV

$$\Rightarrow \quad P(A/E_2) = \frac{1}{100} = 0.01$$

Now,

Required probability

$$= P(E_1/A)$$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01} = \frac{90}{1089}$$

S16. Let the compressors manufactured by factories at Pune, Nasik and Nagpur be in the ratio 2 : 1 : 1.

$$P(E_1) = \frac{2}{4} = \frac{1}{2}, \quad P(E_2) = P(E_3) = \frac{1}{4},$$

where E_1, E_2, E_3 are events for Pune, Nasik and Nagpur respectively.

$$P(D/E_1) = \frac{0.2}{100}, \quad P(D/E_2) = \frac{0.2}{100} \quad \text{and} \quad P(D/E_3) = \frac{0.4}{100},$$

where $D \equiv$ Defective Compressor.

By Bayes's Theorem,

$$P(E_2/D) = \frac{P(E_2)P(D/E_2)}{P(E_1)P(D/E_1) + P(E_2)P(D/E_2) + P(E_3)P(D/E_3)}$$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{0.2}{100}\right)}{\left(\frac{1}{2}\right)\left(\frac{0.2}{100}\right) + \frac{1}{4}\left(\frac{0.2}{100}\right) + \frac{1}{4}\left(\frac{0.4}{100}\right)} = \frac{0.2}{2(0.2) + 0.2 + 0.4} = \frac{0.2}{1} = \frac{1}{5}.$$

S17. Let us define the events :

E_1 : T.V. from 1st plant

E_2 : T.V. from 2nd plant

A : T.V. of standard quality

We have : $P(E_1) = \frac{70}{100} = \frac{7}{10}, \; P(E_2) = \frac{30}{100} = \frac{3}{10},$

Also, $P(A/E_1) = \frac{80}{100} = \frac{4}{5}$ and $P(A/E_2) = \frac{70}{100} = \frac{7}{10}$

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{7}{10}\right)\left(\frac{4}{5}\right)}{\frac{7}{10}\left(\frac{4}{5}\right) + \frac{3}{10}\left(\frac{7}{10}\right)} = \frac{28}{50} \times \frac{100}{56 + 21} = \frac{56}{77} = \frac{8}{11}$$

S18. Let us define the events

E_1 : Items produced by machine A

E_2 : Items produced by machine B

A : Item is defective

To find : $P\left(\frac{E_2}{A}\right)$

Now, by Baye's theorem, we have

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)} \qquad \ldots\text{(i)}$$

Now, $P(E_1)$ = Probability that item is produced by machine $A = \frac{60}{100}$

$P(E_2)$ = Probability that item is produced by machine $B = \frac{40}{100}$

$P\left(\frac{A}{E_1}\right)$ = Probability that defective item is produced by machine $A = \frac{2}{100}$

$P\left(\frac{A}{E_2}\right)$ = Probability that defective item is produced by machine $B = \frac{1}{100}$

Putting above values in Eq. (i), we get

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{40}{100} \times \frac{1}{100}}{\left(\frac{60}{100} \times \frac{2}{100}\right) + \left(\frac{40}{100} \times \frac{1}{100}\right)}$$

$$= \frac{40}{120 + 40} = \frac{40}{160} = \frac{1}{4}$$

$\therefore$ Required probability $= \frac{1}{4}$.

S19. Let us define the events as

E_1 : Bolt is manufactured by machine A.

E_2 : Bolt is manufactured by machine B.

E_3 : Bolt is manufactured by machine C.

A : Bolt drawn is defective.

To find : $$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{\left[P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)\right]} \quad \text{...(i)}$$

Now, $P(E_1)$ = Probability that bolt is manufactured by machine $A = \frac{25}{100}$

$P(E_2)$ = Probability that bolt is manufactured by machine $B = \frac{35}{100}$

$P(E_3)$ = Probability that bolt is manufactured by machine $C = \frac{40}{100}$

Now, $P\left(\frac{A}{E_1}\right)$ = Probability that machine A produces defective bolt $= \frac{5}{100}$

$P\left(\frac{A}{E_2}\right)$ = Probability that machine B produces defective bolt = $\frac{4}{100}$

$P\left(\frac{A}{E_3}\right)$ = Probability that machine C produces defective bolt = $\frac{2}{100}$

Putting above values in Eq. (i), we get

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{35}{100}\times\frac{4}{100}}{\left[\left(\frac{25}{100}\times\frac{5}{100}\right)+\left(\frac{35}{100}\times\frac{4}{100}\right)+\left(\frac{40}{100}\times\frac{2}{100}\right)\right]}$$

$$= \frac{140}{125+140+80} = \frac{140}{345} = \frac{28}{69}$$

Hence, required probability = $\frac{28}{69}$.

S20. Let us define the events as

E_1 : Bulb is manufactured by machine A.

E_2 : Bulb is manufactured by machine B.

E_3 : Bulb is manufactured by machine C.

A : Bulb drawn is defective.

To find : $P\left(\frac{E_1}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{\left[P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)\right]} \quad \text{... (i)}$$

Now, given that $P(E_1)$ = 60% = $\frac{60}{100}$, $P(E_2)$ = 30% = $\frac{30}{100}$, $P(E_3)$ = 10% = $\frac{10}{100}$

and $P\left(\frac{A}{E_1}\right)$ = Probability that defective bulb is manufactured by machine A = 1% = $\frac{1}{100}$ [Given]

$P\left(\frac{A}{E_2}\right)$ = Probability that defective bulb is manufactured by machine B = 2% = $\frac{2}{100}$ [Given]

$P\left(\frac{A}{E_3}\right)$ = Probability that defective bulb is manufactured by machine C = 3% = $\frac{3}{100}$ [Given]

Putting all above values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{60}{100}\times\frac{1}{100}}{\left[\left(\frac{60}{100}\times\frac{1}{100}\right)+\left(\frac{30}{100}\times\frac{2}{100}\right)+\left(\frac{10}{100}\times\frac{3}{100}\right)\right]}$$

$$= \frac{60}{60+60+30} = \frac{60}{150} = \frac{2}{5}$$

Hence, required probability = $\frac{2}{5}$.

S21. Let us define the events as

E_1 : Motorcycle manufactured in plant I

E_2 : Motorcycle manufactured in plant II

A : Motorcycle of standard quality.

To find : $P\left(\frac{E_2}{A}\right)$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \quad \dots (i)$$

$$P(E_1) = \frac{70}{100} = \frac{7}{10}, \quad P\left(\frac{A}{E_1}\right) = \frac{80}{100} = \frac{8}{10}$$

$$P(E_2) = \frac{30}{100} = \frac{3}{10}, \quad P\left(\frac{A}{E_2}\right) = \frac{90}{100} = \frac{9}{10}$$

Substituting the above values in eq. (i), we have

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{3}{10}\cdot\frac{9}{10}}{\frac{7}{10}\times\frac{8}{10}+\frac{3}{10}\times\frac{9}{10}} = \frac{27}{56+27} = \frac{27}{83}$$

S22. Let the events be :

E_1 : Item is from machine A

E_2 : Item is from machine B

E_3 : Item is from machine C

and A : Item is defective

$\therefore$ $P(E_1) = \frac{50}{100}$, $P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{20}{100}$

Also $P(A/E_1) = \frac{1}{100}$, $P(A/E_2) = \frac{5}{100}$ and $P(A/E_3) = \frac{7}{100}$

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$

$$= \frac{50}{50 + 150 + 140} = \frac{50}{340} = \frac{5}{34}$$

S23. Let X, Y, Z and D be event be as given below :

X : bolt is manufactured by machine X and

Y : bolt is manufactured by machine Y

Z : bolt is manufactured by machine Z

D : bolt drawn is defective

We have : $P(X) = \frac{1000}{6000} = \frac{1}{6}$, $P(Y) = \frac{2000}{6000} = \frac{1}{3}$ and $P(Z) = \frac{3000}{6000} = \frac{1}{2}$,

$$P(D/X) = \frac{1}{100}, \quad P(D/Y) = \frac{3}{200}, \quad P(D/Z) = \frac{2}{100},$$

where D denotes defective bolt.

By Bayes' Theorem,

$$P(X/D) = \frac{P(X)P(D/X)}{P(X)P(D/X) + P(Y)P(D/Y) + P(Z)P(D/Z)}$$

$$= \frac{\frac{1}{6}\left(\frac{1}{100}\right)}{\frac{1}{6}\left(\frac{1}{100}\right) + \frac{1}{3}\left(\frac{3}{200}\right) + \frac{1}{2}\left(\frac{2}{100}\right)} = \frac{1}{1+3+6} = \frac{1}{10}$$

S24. Let the events E_1, E_2 and A be as below:

E_1 : Plant 1 is chosen

E_2 : Plant 2 is chosen

and A : Motor cycle is of standard quality

We have : $P(E_1) = \frac{70}{100}$, $P(E_2) = \frac{30}{100}$, $P(A/E_1) = \frac{80}{100}$ and $P(A/E_2) = \frac{90}{100}$

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)+\left(\frac{30}{100}\right)\left(\frac{90}{100}\right)} = \frac{5600}{5600+2700} = \frac{56}{83}$$

S25. Let E_1, E_2, E_3, E_4 and A events be as below :

E_1 : Doctor travels by train

E_2 : Doctor travels by bus

E_3 : Doctor travels by scooter

E_4 : Doctor travels by other means of transport

and A : Doctor visits the patient late

We have : $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{1}{10}$, $P(E_4) = \frac{2}{5}$

and $P(A/E_1) = \frac{1}{4}$, $P(A/E_2) = \frac{1}{3}$, $P(A/E_3) = \frac{1}{12}$, $P(A/E_4) = 0$

By Bayes' Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)+P(E_4)P(A/E_4)}$$

$$= \frac{\left(\frac{3}{10}\right)\left(\frac{1}{4}\right)}{\left(\frac{3}{10}\right)\left(\frac{1}{4}\right)+\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{10}\right)\left(\frac{1}{12}\right)+\left(\frac{2}{5}\right)(0)} = \frac{3}{40}\times\frac{120}{9+8+1} = \frac{9}{18} = \frac{1}{2}$$

S26. Let us define the events as

E_1 : Insured person is a scooter driver. E_2 : Insured person is a car driver.

E_3 : Insured person is a truck driver.

A : Insured person meets with an accident

To find : $P\left(\frac{E_1}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{\left[P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)\right]} \quad \dots \text{(i)}$$

Total insurance = 12000

Now, $P(E_1)$ = Probability that the insured person is a scooter driver

$$= \frac{2000}{12000} = \frac{1}{6}$$

$P(E_2)$ = Probability that the insured person is a car driver

$$= \frac{4000}{12000} = \frac{1}{3}$$

$P(E_3)$ = Probability that the insured person is a truck driver

$$= \frac{6000}{12000} = \frac{1}{2}$$

Now, $P\left(\frac{A}{E_1}\right)$ = Probability that scooter driver meets with an accident = 0.01 [Given]

$P\left(\frac{A}{E_2}\right)$ = Probability that car driver meets with an accident = 0.03 [Given]

$P\left(\frac{A}{E_3}\right)$ = Probability that truck driver meets with an accident = 0.15 [Given]

Putting all above values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{6}\times 0.01}{\left[\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.15\right)\right]}$$

$$= \frac{0.00167}{0.00167+0.01+0.075} = \frac{0.00167}{0.08667} = 0.019$$

Hence, required probability = 0.019

S27. Let us define the events

E_1 : Doctor comes by train. E_2 : Doctor comes by bus.

E_3 : Doctor comes by scooter. E_4 : Doctor comes by taxi.

A : Doctor arrives late.

To find : $P\left(\frac{E_2}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,.\,P\left(\frac{A}{E_2}\right)}{\left[P(E_1)\,.\,P\left(\frac{A}{E_1}\right)+P(E_2)\,.\,P\left(\frac{A}{E_2}\right)+P(E_3)\,.\,P\left(\frac{A}{E_3}\right)+P(E_4)\,.\,P\left(\frac{A}{E_4}\right)\right]} \quad \text{... (i)}$$

Now, given that

Now, $P(E_1) = \frac{1}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{3}{10}$ and $P(E_4) = \frac{2}{5}$

Now, $P\left(\frac{A}{E_1}\right)$ = Probability that doctor arrives late when he comes by train = $\frac{1}{4}$ [Given]

$P\left(\frac{A}{E_2}\right)$ = Probability that doctor arrives late when he comes by bus = $\frac{1}{3}$ [Given]

$P\left(\frac{A}{E_3}\right)$ = Probability that doctor arrives late when he comes by scooter = $\frac{1}{12}$ [Given]

$P\left(\frac{A}{E_4}\right)$ = Probability that doctor arrives late when he comes by taxi = 0

[$\because$ Given that, when the doctor comes by taxi he will not late]

Putting above values in eq. (i), we get

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{5}\times\frac{1}{3}}{\left[\left(\frac{1}{10}\times\frac{1}{4}\right)+\left(\frac{1}{5}\times\frac{1}{3}\right)+\left(\frac{3}{10}\times\frac{1}{12}\right)+\left(\frac{2}{5}\times 0\right)\right]}$$

$$= \frac{\frac{1}{15}}{\frac{1}{40}+\frac{1}{15}+\frac{3}{120}} = \frac{\frac{1}{15}}{\frac{3+8+3}{120}} = \frac{\frac{1}{15}}{\frac{14}{120}} = \frac{1}{15}\times\frac{120}{14} = \frac{4}{7}$$

Hence, required probability = $\frac{4}{7}$.

S28. Let us define the events

E_1 : Six appears on a die.

E_2 : Six does not appear on the die.

A : Man reports that it is a six.

To find : $P\left(\frac{E_1}{A}\right)$ *i.e.*, man reports actually a six

Now by Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \quad \text{... (i)}$$

Now, we find the various probabilities as follows

$P(E_1) = P$ (Six appears on a die) = $\frac{1}{6}$

$P(E_2)$ = P (Six does not appear on a die) = $1 - \frac{1}{6} = \frac{5}{6}$

$P\left(\frac{A}{E_1}\right)$ = Probability that six appear on a die and man also reports that it is a six, *i.e.,* he is speaking the truth = $\frac{3}{4}$.

[$\because$ Given that, man speaks truth 3 out of 4 times, so the probability of speaking truth is $\frac{3}{4}$]

$P\left(\frac{A}{E_2}\right)$ = Probability that six does not appears on the die but man reports it is a six *i.e.*, he is not speaking the truth = $1 - \frac{3}{4} = \frac{1}{4}$

Putting all above probabilities in eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\left(\frac{1}{6}\times\frac{3}{4}\right)}{\left(\frac{1}{6}\times\frac{3}{4}\right)+\left(\frac{5}{6}\times\frac{1}{4}\right)} = \frac{\frac{3}{4}}{\frac{3}{4}+\frac{5}{4}} = \frac{3}{8}$$

S29. Let the events be as below :

E_1 : 2 white balls are transferred from A to B

E_2 : 2 black balls are transferred from A to B

E_3 : 1 white and 1 black balls are transferred from A to B

and A = 1 black ball is drawn from B

Now $$P(E_1) = \frac{^4C_2}{^7C_2} = \frac{4\times3}{7\times6} = \frac{2}{7}$$

$$P(E_2) = \frac{^3C_2}{^7C_2} = \frac{3\times2}{7\times6} = \frac{1}{7}$$

$$P(E_3) = \frac{^4C_1\times{}^3C_1}{^7C_2} = \frac{4\times3}{\frac{7\times6}{1\times2}} = \frac{4\times3}{7\times3} = \frac{4}{7}$$

and $P(A/E_1) = \frac{2}{6} = \frac{1}{3}$, $P(A/E_2) = \frac{4}{6} = \frac{2}{3}$

and $P(A/E_3) = \frac{3}{6} = \frac{1}{2}$

By Baye's Theorem,

$$P(E_3/A) = \frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{4}{7}\times\frac{1}{2}}{\frac{2}{7}\times\frac{1}{3}+\frac{1}{7}\times\frac{2}{3}+\frac{4}{7}\times\frac{1}{2}} = \frac{2}{7}\times\frac{42}{4+4+12}$$

$$= \frac{2\times 6}{20} = \frac{12}{20} = \frac{3}{5}.$$

S30. Let us define the events

E_1 : A two headed coin is selected.

E_2 : A biased coin is selected.

E_3 : Unbiased coin is selected.

A : Head comes up.

To find : $P\left(\frac{E_1}{A}\right)$

Now, by Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{\left[P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)\right]} \quad ...(i)$$

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

and $P\left(\frac{A}{E_1}\right)$ = Probability that head comes up on a two headed coin = 1

$P\left(\frac{A}{E_2}\right)$ = Probability that head comes up on a biased coin = 75% = $\frac{75}{100}$

and $P\left(\frac{A}{E_3}\right)$ = Probability that head comes up on an unbiased coin = $\frac{1}{2}$

Putting above values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right)+\left(\frac{75}{100}\times\frac{1}{3}\right)+\left(\frac{1}{3}\times\frac{1}{2}\right)}$$

$$= \frac{1}{1 + \frac{75}{100} + \frac{1}{2}} = \frac{1}{\frac{100 + 75 + 50}{100}}$$

$$= \frac{100}{225} = \frac{4}{9}$$

Hence, required probability = $\frac{4}{9}$.

S31. Let us define the events as

E_1 : A two tailed coin is selected.

E_2 : Biased coin is selected.

E_3 : An unbiased coin is selected.

A : A two tailed coin shows tail.

To find : $P\left(\frac{E_1}{A}\right)$

Now, by Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) . P\left(\frac{A}{E_1}\right)}{\left[P(E_1) . P\left(\frac{A}{E_1}\right) + P(E_2) . P\left(\frac{A}{E_2}\right) + P(E_3) . P\left(\frac{A}{E_3}\right)\right]} \quad \text{...(i)}$$

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

and $P\left(\frac{A}{E_1}\right)$ = Probability that tail is shown by two tailed coin = 1

$P\left(\frac{A}{E_2}\right)$ = Probability that a biased coin shows a tail = 40% = $\frac{40}{100}$

[$\because$ Given that, the biased coin shows heads 60% of the times.
So, it will show tails 40% of the times.]

and $P\left(\frac{A}{E_3}\right)$ = Probability that unbiased coins shows a tail = $\frac{1}{2}$

Putting above values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3} \times 1}{\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times \frac{40}{100}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)}$$

$$= \frac{1}{1 + \frac{40}{100} + \frac{1}{2}} = \frac{1}{\frac{100 + 40 + 50}{100}}$$

$$= \frac{100}{190} = \frac{10}{19}$$

Hence, required probability = $\frac{10}{19}$.

S32. Let us define the events

E_1 : Box I is selected

E_2 : Box II is selected

E_3 : Box III is selected

A : The drawn coin is also a gold coin

To find : $$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{\left[P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)\right]} \quad ...(i)$$

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$P\left(\frac{A}{E_1}\right)$ = Probability that a gold coin is drawn from box I = $\frac{2}{2}$ = 1

[∵ Box I contains both gold coins]

$P\left(\frac{A}{E_2}\right)$ = Probability that a gold coin is drawn from box II = 0

[∵ Box II has both silver coins]

$P\left(\frac{A}{E_3}\right)$ = Probability that a gold coin is drawn from box III = $\frac{1}{2}$

[∵ Box III contains 1 gold and 1 silver coins]

Putting above values in eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3} \times 1}{\left(\frac{1}{3} \times 1\right) + \left(\frac{1}{3} \times 0\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)}$$

$$= \frac{1}{1+0+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

Hence, required probability = $\frac{2}{3}$.

S33. Let us define the events as

E_1 : Boys are selected.

E_2 : Girls are selected.

A : The student has an IQ of more than 150.

To find : $P\left(\frac{E_1}{A}\right)$

Now, By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \qquad ...(i)$$

Now, $P(E_1) = 60\% = \frac{60}{100}$

and $P(E_2) = 40\% = \frac{40}{100}$

[$\because$ Given that, in the class 60% students are boys, so 40% are girls]

Also, $P\left(\frac{A}{E_1}\right)$ = Probability that a boys has an IQ of more than 150 = 5% = $\frac{5}{100}$

and $P\left(\frac{A}{E_2}\right)$ = Probability that a girls has an IQ of more than 150 = 10% = $\frac{10}{100}$

Putting above values in eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{60}{100}\times\frac{5}{100}}{\left(\frac{60}{100}\times\frac{5}{100}\right)+\left(\frac{40}{100}\times\frac{10}{100}\right)}$$

$$= \frac{300}{300+400}$$

$$= \frac{300}{700} = \frac{3}{7}$$

Hence, required probability = $\frac{3}{7}$.

S34. Let us define the events as

E_1 : The lost card is a diamond. E_2 : The lost card is a spade.

E_3 : The lost card is a club. E_4 : The lost card is a heart.

A : Two cards drawn from remaining pack are club.

To find : $$P\left(\frac{E_3}{A}\right) = \frac{P(E_3).P\left(\frac{A}{E_3}\right)}{\left[P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)+P(E_4).P\left(\frac{A}{E_4}\right)\right]} \quad \text{... (i)}$$

Now, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52} = \frac{1}{4}$

and $P\left(\frac{A}{E_1}\right)$ = Probability that two drawn cards are clubs when the lost card is a diamond

$$= \frac{^{13}C_2}{^{51}C_2}$$

Similarly, $P\left(\frac{A}{E_2}\right) = P\left(\frac{A}{E_4}\right) = \frac{^{13}C_2}{^{51}C_2}$

and $P\left(\frac{A}{E_3}\right)$ = Probability that two drawn cards are clubs when the lost card is of clubs also

$$= \frac{^{12}C_2}{^{51}C_2}$$

Putting above values in eq. (i), we get

$$P\left(\frac{E_3}{A}\right) = \frac{\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}}{\left[\left(\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}\right)+\left(\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}\right)+\left(\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}\right)+\left(\frac{1}{4}\times\frac{^{13}C_2}{^{51}C_2}\right)\right]}$$

$$= \frac{\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}}{\frac{3}{4}\cdot\frac{^{13}C_2}{^{51}C_2}+\frac{1}{4}\times\frac{^{12}C_2}{^{51}C_2}} = \frac{^{12}C_2}{3.^{13}C_2+{}^{12}C_2}$$

$$\left[\because {}^{12}C_2 = \frac{12!}{2!\,10!} = \frac{12\times 11}{2} = 66 \quad \text{and } {}^{13}C_2 = \frac{13!}{2!\,11!} = \frac{13\times 12}{2} = 78\right]$$

$$= \frac{66}{3\times 78+66} = \frac{66}{234+66} = \frac{66}{300} = \frac{11}{50}$$

Hence, required probability = $\frac{11}{50}$.

S35. Let E_1, E_2, E_3, E_4 and A be the events as defined below :

E_1 = the missing card is heart card,

E_2 = the missing card is spade card,

E_3 = the missing card is club card,

E_4 = the missing card is a diamond card, and

A = Drawing two heart cards from the remaining cards.

Then, $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{13}{52} = \frac{1}{4}$, $P(E_3) = \frac{13}{52} = \frac{1}{4}$, $P(E_4) = \frac{13}{52} = \frac{1}{4}$

$P(A/E_1)$ = Probability of drawing two heart cards given that one heart card is missing

$\Rightarrow \quad P(A/E_1) = \frac{^{12}C_2}{^{51}C_2}$

$P(A/E_2)$ = Probability of drawing two heart cards given that one spade card is missing

$\Rightarrow \quad P(A/E_2) = \frac{^{13}C_2}{^{51}C_2}$

Similarly, we have

$P(A/E_3)$ = Probability of drawing two heart cards given that one club card is missing

$= \frac{^{13}C_2}{^{51}C_2}$

and $P(A/E_4)$ = Probability of drawing two heart cards given that one diamond card is missing

$= \frac{^{13}C_2}{^{51}C_2}$

By Baye's Theorem, we have

Required probability

= $P(E_1/A)$ = Probability of the missing card to be heart

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + P(E_4)P(A/E_4)}$$

$$= \frac{\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2}}{\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2} + \frac{1}{4} \times \frac{^{13}C_2}{^{51}C_2} + \frac{1}{4} \times \frac{^{13}C_2}{^{51}C_2} + \frac{1}{4} \times \frac{^{13}C_2}{^{51}C_2}} = \frac{^{12}C_2}{^{12}C_2 + ^{13}C_2 + ^{13}C_2 + ^{13}C_2}$$

$$= \frac{66}{66 + 78 + 78 + 78} = \frac{66}{300} = \frac{11}{50}$$

S36. Let E_1, E_2, E_3 and A be the events defined as follows :

E_1 = the examinee guesses the answer,

E_2 = the examinee copies the answer,

E_3 = the examinee knows the answer,

and A = the examinee answers correctly.

We have, $P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{6}$

Since E_1, E_2, E_3 are mutually exclusive and exhaustive events.

$\therefore$ $P(E_1) + P(E_2) + P(E_3) = 1$

$\Rightarrow$ $P(E_3) = 1 - \{P(E_1) + P(E_2)\} = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$

If E_1 has already occurred, then the examinee guesses. Since there are four choices out of which only one is correct, therefore the probability that he answers correctly given that he has made a guess is 1/4 *i.e.*, $P(A/E_1) = 1/4$.

It is given that $P(A/E_2) = 1/8$, and

$P(A/E_3)$ = Probability that the answer correctly given that he knew the answer

$\Rightarrow$ $P(A/E_3) = 1$

By Baye's Theorem, we have

Required probability

$$= P(E_3/A)$$

$$= \frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1} = \frac{24}{29}.$$

S37. Let us define the events as

E_1 : Bag I is selected.

E_2 : Bag II is selected.

E_3 : Bag III is selected.

A : Two drawn balls are found to be black and red.

To find : $P\left(\frac{E_1}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{\left[P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)\right]} \quad \text{... (i)}$$

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

and $P\left(\frac{A}{E_1}\right)$ = Probability that red and black balls are drawn from bag I

$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$ [$\because$ Bag I contains total of 6 balls including 1 black and 3 red balls]

$= \frac{1\times 3}{\frac{6!}{2!\,4!}} = \frac{1\times 3\times 2\times 4!}{6!} = \frac{1\times 3\times 2}{6\times 5}$

$\therefore \quad P\left(\frac{A}{E_1}\right) = \frac{1}{5}$

and $P\left(\frac{A}{E_2}\right)$ = Probability that red and black balls are drawn from bag II

$= \frac{{}^2C_1 \times {}^1C_1}{{}^7C_2}$

[$\because$ Bag II contains total of 7 balls including 2 black balls and 1 red ball]

$= \frac{2\times 1}{\frac{7!}{2!5!}} = \frac{2\times 1\times 2!\times 5!}{7!}$

$= \frac{2\times 2}{7\times 6} = \frac{2}{21}$

$P\left(\frac{A}{E_3}\right)$ = Probability that red and black balls are drawn from bag III

$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$

[$\because$ Bag III contains total of 12 balls including 4 black balls and 3 red balls]

$= \frac{4\times 3}{\frac{12!}{2!\,10!}} = \frac{4\times 3\times 2!\times 10!}{12!}$

$$= \frac{4 \times 3 \times 2}{12 \times 11} = \frac{2}{11}$$

Putting above values in Eq. (i), we get

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3} \times \frac{1}{5}}{\left(\frac{1}{3} \times \frac{1}{5}\right) + \left(\frac{1}{3} \times \frac{2}{21}\right) + \left(\frac{1}{3} \times \frac{2}{11}\right)}$$

$$= \frac{\frac{1}{5}}{\left(\frac{1}{5} + \frac{2}{21} + \frac{2}{11}\right)}$$

$$= \frac{\frac{1}{5}}{\frac{231 + 110 + 210}{1155}}$$

$$= \frac{1}{5} \times \frac{1155}{551} = \frac{231}{551}.$$

S38. Let us define the events as

E_1 : Bag I is selected.

E_2 : Bag II is selected.

E_3 : Bag III is selected.

A : Two balls are drawn which are found to be white and red.

To find : $P\left(\frac{E_3}{A}\right)$

By Baye's theorem, we have

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3) . P\left(\frac{A}{E_3}\right)}{\left[P(E_1) . P\left(\frac{A}{E_1}\right) + P(E_2) . P\left(\frac{A}{E_2}\right) + P(E_3) . P\left(\frac{A}{E_3}\right)\right]} \quad \ldots(i)$$

Now, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P\left(\frac{A}{E_1}\right)$ = Probability that white and red balls are drawn from bag I

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} \quad [\because \text{ Bag I contains total 6 balls including 1 white and 3 red balls}]$$

$$= \frac{1 \times 3}{\frac{6!}{2!4!}} = \frac{1 \times 3 \times 2! \times 4!}{6!} = \frac{1 \times 3 \times 2}{6 \times 5}$$

$$\therefore \quad P\left(\frac{A}{E_1}\right) = \frac{1}{5}$$

Now, $P\left(\frac{A}{E_2}\right)$ = Probability that white and red balls are drawn from bag II

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} \quad [\because \text{ Bag II contains total 4 balls including 2 white and 1 red balls}]$$

$$= \frac{2 \times 1}{\frac{4!}{2!\,2!}} = \frac{2 \times 1 \times 2! \times 2!}{4!}$$

$$= \frac{2 \times 1 \times 2}{4 \times 3}$$

$$\therefore \quad P\left(\frac{A}{E_2}\right) = \frac{1}{3}$$

and $P\left(\frac{A}{E_3}\right)$ = Probability that the white and red balls are drawn from bag III

$$= \frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} \quad [\because \text{ Bag III contains total 9 balls including 4 white and 2 red balls}]$$

$$= \frac{4 \times 2}{\frac{9!}{2!7!}} = \frac{4 \times 2 \times 2! \times 7!}{9!}$$

$$= \frac{4 \times 2 \times 2}{9 \times 8}$$

$$\therefore \quad P\left(\frac{A}{E_3}\right) = \frac{2}{9}$$

Putting all above values in Eq. (i), we get

$$P\left(\frac{E_3}{A}\right) = \frac{\frac{1}{3} \times \frac{2}{9}}{\left(\frac{1}{3} \times \frac{1}{5}\right) + \left(\frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{2}{9}\right)}$$

$$= \frac{\frac{2}{9}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{9}} = \frac{\frac{2}{9}}{\frac{27+45+30}{135}}$$

$$= \frac{2}{9} \times \frac{135}{102} = \frac{15}{51} = \frac{5}{17}.$$

Hence, required probability = $\frac{5}{17}$.

S39. Let us define the events

E_1 : Bag I is selected.

E_2 : Bag II is selected.

A : Ball drawn is found to be red.

To find : $P\left(\frac{E_2}{A}\right)$ *i.e.,* Probability that the red ball is drawn from bag II.

By Baye's theorem, we have

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)} \qquad \ldots(i)$$

Now, we find the various probabilities as follows

$$P(E_1) = P(E_2) = \frac{1}{2}$$

[$\because$ There are two bag, I and II and probability of their selection is 50% *i.e.*, 1/2]

Also, $P\left(\frac{A}{E_1}\right)$ = Probability of drawing a red ball from bag I = $\frac{3}{3+4}$

[$\because$ Bag I contains 3 red and 4 black balls]

$\therefore$ $P\left(\frac{A}{E_1}\right) = \frac{3}{7}$

and $P\left(\frac{A}{E_2}\right)$ = Probability of drawing a red ball from bag II = $\frac{5}{5+6}$

[$\because$ Bag II contains 5 red and 6 black balls]

$\therefore$ $P\left(\frac{A}{E_2}\right) = \frac{5}{11}$

Putting all above probabilities in Eq. (i), we get

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{2}\times\frac{5}{11}}{\left(\frac{1}{2}\times\frac{3}{7}\right)+\left(\frac{1}{2}\times\frac{5}{11}\right)} = \frac{\frac{5}{22}}{\frac{3}{14}+\frac{5}{22}} = \frac{\frac{5}{22}}{\frac{33+35}{154}} = \frac{5}{22}\times\frac{154}{68}$$

$\therefore$ $$P\left(\frac{E_2}{A}\right) = \frac{35}{68}$$

Hence, required probability = $\frac{35}{68}$.

S40. Let the events E_1, E_2 and A be as below :

E_1 : Bag contains 2 white and 2 non-white balls

E_2 : Bag contains 3 white and 1 non-white ball

E_3 : Bag contains 4 white balls

and A : Drawing 2 white balls

Now $$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A/E_1) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{\frac{4\times 3}{1\times 2}} = \frac{1}{6}$$

$$P(A/E_2) = \frac{{}^3C_2}{{}^4C_2} = \frac{3}{\frac{4\times 3}{1\times 2}} = \frac{3}{6} = \frac{1}{2}$$

and $$P(A/E_3) = \frac{{}^4C_2}{{}^4C_2} = 1$$

By Bayes' Theorem,

$$P(E_3/A) = \frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\left(\frac{1}{3}\right)(1)}{\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)(1)}$$

$$= \frac{1/3}{\frac{1}{3}\left(\frac{1}{6}+\frac{1}{2}+1\right)} = \frac{1}{\frac{1}{6}+\frac{1}{2}+1}$$

$$= \frac{6}{1+3+6} = \frac{6}{10} = \frac{3}{5}.$$

S41. Let the events E, E_2, E_3 and A be as below:

E_1 : Ball drawn from urn A

E_2 : Ball drawn from urn B

E_3 : Ball drawn from urn C

and A : Ball drawn is red

We have to find $P(E_1/A)$.

Since the three urns are equally likely to be selected,

$$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Also $P(A/E_1) = \frac{6}{10}$, $P(A/E_2) = \frac{2}{8}$ and $P(A/E_3) = \frac{1}{6}$

By Baye's Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{6}{10}\right)}{\left(\frac{1}{3}\right)\left(\frac{6}{10}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{8}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{6}\right)}$$

$$= \frac{\frac{6}{10}}{\frac{6}{10} + \frac{2}{8} + \frac{1}{6}} = \frac{6}{10} \times \frac{120}{72 + 30 + 20} = \frac{72}{122} = \frac{36}{61}$$

Date: 19/10/2021

Q1. A random variable X has the following probability distribution values of X,

X	:	0	1	2	3	4	5	6	7
$P(X)$	:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find each of the following:

(i) k (ii) $P(X < 6)$ (iii) $P(X \geq 6)$ (iv) $P(0 < X < 5)$

Q2. The random variable X has a probability distribution $P(x)$ to the following form, where k is some number:

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the value of k (ii) Find $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$

Q3. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the value x has the following form, where k is some unknown constant.

$$P(X = x) \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k (ii) What is the probability that you study at least two hours?

(iii) Exactly two hours? (iv) At most two hours?

Q4. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. find

(i) the probability distribution of X (ii) $P(X \geq 0)$

(iii) $P(X < 1)$ (iv) $P(0 < X < 2)$

Q5. Two cards are drawn without replacement from a well-shuffled deck of 52 cards. Determine the probability distribution of the number of face cards (*i.e.*, Jack, Queen, King and Ace).

Q6. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of kings.

Q7. An urn contains 4 white and 6 red balls. Four balls are drawn at random from the urn. Find the probability distribution of the number of white balls.

Q8. An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in a random draw of three balls.

Q9. A coin is biased so that the head is 3 times as likely to occurs as tail. If coin is tossed twice, find the probability distribution for the number of tails.

Q10. A coin is tossed until a head appears or the tail appears 4 times in succession. Find the probability distribution of the number of tosses.

Q11. Find the probability distribution of X for the number of heads in two tosses of a coin?

Q12. An unbiased die is thrown twice. Find the probability distribution of the number of sixes.

Q13. A die is loaded in such a way that an even number is twice likely to occur as an odd number. If the die is tossed twice, find the probability distribution of the random variable X representing the perfect squares in the two tosses.

Q14. A random variable X can take all non-negative integral values and the probability that X takes the value r is proportional to α^r $(0 < \alpha < 1)$. Find $P(X = 0)$.

Q15. A random variable X has following probability distributions

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k; (ii) $P(X < 3)$; (iii) $P(X > 6)$; (iv) $(0 < X < 3)$.

Q16. Find the probability distribution of number of doublets in three tosses of a pair of dice.

Q17. Two cards are drawn successively with replacement, from a well shuffled deck of 52 cards. Find the probability distribution of number of aces.

Q18. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of number of defective bulbs?

Q19. Four bad oranges are mixed accidently with16 good oranges. Find the probability distribution of the number of bad oranges in a draw of two oranges.

Q20. We take 8 identical slips of paper, write the number 0 on one of them, the number 1 on three of the slips, the number 2 on three of the slips and the number 3 on one of the slips. These slips are folded, put in a box and thoroughly mixed. One slip is drawn at random from the box. If X is the random variable denoting the number written on the drawn slip, find the probability distribution of X.

Date: 19/10/2021

S1. (i) Since the sum of all the probabilities in a probability distribution is always unity. Therefore,

$$P(X = 0) + P(X = 1) + ... + P(X = 7) = 1$$

$\Rightarrow$ $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow$ $10k^2 + 9k - 1 = 0$

$\Rightarrow$ $(10k - 1)(k + 1) = 0$

$\Rightarrow$ $10k - 1 = 0 \quad [\because k \geq 0 \therefore k + 1 \neq 0]$

$\Rightarrow$ $k = \dfrac{1}{10}$

(ii) $P(X < 6) = P(X= 0) + P(X= 1) + P(X= 2) + P(X= 3) + P(X= 4) + P(X= 5)$

$P(X < 6) = 0 + k + 2k + 2k + 3k + k^2$

$\Rightarrow$ $P(X < 6) = k^2 + 8k$

$\Rightarrow$ $P(X < 6) = \left(\dfrac{1}{10}\right)^2 + \dfrac{8}{10}$ $[\because k = 1/10]$

$\Rightarrow$ $P(X < 6) = \dfrac{81}{100}$

(iii) $P(X \geq 6) = P(X = 6) + P(X = 7)$

$\Rightarrow$ $P(X \geq 6) = 2k^2 + 7k^2 + k$

$\Rightarrow$ $P(X \geq 6) = 9k^2 + k$

$\Rightarrow$ $P(X \geq 6) = \dfrac{9}{100} + \dfrac{1}{10}$ $[\because k = 1/10]$

$\Rightarrow$ $P(X \geq 6) = \dfrac{19}{100}$

ALITER:

$$P(X \geq 6) = 1 - P(X < 6) = 1 - \frac{81}{100} = \frac{19}{100}$$

(iv) $P(0 < X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$\Rightarrow$ $P(0 < X < 5) = k + 2k + 2k + 3k$

$\Rightarrow$ $P(0 < X < 5) = 8k$

$\Rightarrow$ $P(0 < X < 5) = \dfrac{8}{10}$ $[\because k = 1/10]$

$\Rightarrow$ $P(0 < X < 5) = \dfrac{4}{5}$

S2. (i) The Probability distribution of X is

X :	0	1	2
$P(X)$:	k	$2k$	$3k$

The given distribution of probability will be a probability distribution, if

$$P(X = 0) + P(X = 1) + P(X = 2) = 1$$

$$\Rightarrow \quad k + 2k + 3k = 1 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$$

(ii) $P(X < 2) = P(X = 0) + P(X = 1) = k + 2k = 3k = \frac{3}{6} = \frac{1}{2}$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = k + 2k + 3k = 6k = 1$

$P(X \geq 2) = 1 - P(X < 2) = 1 - \frac{1}{2} = \frac{1}{2}$.

S3. The probability distribution of X is

X:	0	1	2	3	4
$P(X)$:	0.1	k	$2k$	$2k$	k

(i) The given distribution is a probability distribution.

$\therefore \quad P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$

$\Rightarrow \quad 0.1 + k + 2k + 2k + k = 1$

$6k = 0.9 \Rightarrow k = 0.15$

(ii) Required probability $= P(X \geq 2)$

$= P(X = 2) + P(X = 3) + P(X = 4)$

$= 2k + 2k + k = 5k = 5 \times 0.15 = 0.75$

(iii) Required probability $= P(X = 2) = 2k = 2 \times 0.15 = 0.3$

(iv) Required probability $= P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.1 + k + 2k = 0.1 + 3k = 0.1 + 3 \times 0.15 = 0.55$

S4. (i) Clearly, X can assume values 0, 1, 2, 3 such that

$$P(X = 0) = \text{Probability of getting no defective item} = \frac{{}^{7}C_4}{{}^{10}C_4} = \frac{1}{6}$$

$$P(X = 1) = \text{Probability of getting one defective item} = \frac{{}^{3}C_1 \times {}^{7}C_3}{{}^{10}C_4} = \frac{1}{2}$$

$$P(X = 2) = \text{Probability of getting two defective items} = \frac{{}^{3}C_2 \times {}^{7}C_2}{{}^{10}C_4} = \frac{3}{10}$$

and, $P(X = 3) =$ Probability of getting three defective items $= \frac{{}^3C_3 \times {}^7C_1}{{}^{10}C_4} = \frac{1}{30}$

Hence, the probability distribution of X is

X:	0	1	2	3
$P(X)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

(ii) $P(X \geq 0) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = \frac{1}{6} + \frac{1}{2} + \frac{3}{10} + \frac{1}{30} = 1$

(iii) $P(X < 1) = P(X = 0) = \frac{1}{6}$

(iv) $P(0 < X < 2) = P(X = 1) = \frac{1}{2}$

S5. Let X denote the number of face cards in two draws. Then, X can take values 0, 1, 2.

Let F_i denote the event of getting a face card in i^{th} draw.

Then,

$P(X = 0) =$ Probability of getting no face card

$\Rightarrow \quad P(X = 0) = P(\bar{F}_1 \cap \bar{F}_2)$

$\Rightarrow \quad P(X = 0) = P(\bar{F}_1)\, P(\bar{F}_2 / \bar{F}_1)$ [By multiplication theorem]

$\Rightarrow \quad P(X = 0) = \frac{36}{52} \times \frac{35}{51} = \frac{105}{221}$

$P(X = 1) =$ Probability of getting one face card and one other card

$\Rightarrow \quad P(X = 1) = P(F_1 \cap \bar{F}_2) + P(\bar{F}_1 \cap F_2)$ [By addition theorem]

$\Rightarrow \quad P(X = 1) = P(F_1)\, P(\bar{F}_2 / F_1) + P(\bar{F}_1)\, P(F_2 / \bar{F}_1)$

$\Rightarrow \quad P(X = 1) = \frac{16}{52} \times \frac{36}{51} + \frac{36}{52} \times \frac{16}{51} = \frac{96}{221}$

$P(X = 2) =$ Probability of getting both face cards

$P(X = 2) = P(F_1 \cap F_2)$

$\Rightarrow \quad P(X = 2) = P(F_1)\, P(F_2 / F_1)$

$\Rightarrow \quad P(X = 2) = \frac{16}{52} \times \frac{15}{51} = \frac{20}{221}$

Hence, the required probability distribution is

X:	0	1	2
$P(X)$:	$\frac{105}{221}$	$\frac{96}{221}$	$\frac{20}{221}$

S6. Let X denote the number of kings. Then, X can take values 0, 1, or 2.

Let S_i denote the event of getting a king in the i^{th} draw and F_i denote the event of not getting a king in the i^{th} draw. Then,

$$P(X = 0) = \text{Probability of not getting a king in the two draws}$$

$$= P(\text{not a king in } 1^{st} \text{ draw and a king in second draw})$$

$$P(F_1 \cap F_2) = P(F_1)\,P(F_2) \qquad [\because F_1 \text{ and } F_2 \text{ are independent}]$$

$$= \frac{48}{52} \times \frac{48}{52} = \frac{144}{169},$$

$$P(X = 1) = \text{Probability of getting one king in the two draws}$$

$$P((S_1 \cap F_2) \text{ or } (F_1 \cap S_2)) = P(S_1 \cap F_2) + P(F_1 \cap S_2)$$

$$= P(S_1)\,P(F_2) + P(F_1)\,P(S_2) \qquad [\text{By multiplication theorem}]$$

$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{24}{169},$$

and, $$P(X = 2) = \text{Probability of getting kings in both the draws}$$

$$= P(S_1 \cap S_2) = P(S_1)\,P(S_2) \qquad [\text{By multiplication theorem}]$$

$$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

Thus, the probability distribution of X is

X:	0	1	2
$P(X)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

S7. Let X denote the number of white balls drawn from the urn. Since there are 4 white balls therefore X can take values, 0, 1, 2, 3 and 4

Now, $$P(X = 0) = \text{Probability of getting no white ball}$$

$$\Rightarrow \quad P(X = 0) = \text{Probability that 4 balls drawn are red} = \frac{{}^6C_4}{{}^{10}C_4} = \frac{1}{14},$$

$$\Rightarrow \quad P(X = 1) = \text{Probability of getting one white ball} = \frac{{}^4C_1 \times {}^6C_3}{{}^{10}C_4} = \frac{8}{21}'$$

$$P(X = 2) = \text{Probability of getting two white balls} = \frac{{}^4C_2 \times {}^6C_2}{{}^{10}C_4} = \frac{6}{14}'$$

$$P(X = 3) = \text{Probability of getting three white balls} = \frac{^4C_3 \times ^6C_1}{^{10}C_4} = \frac{4}{35'}$$

and' $$P(x = 4) = \text{Probability of getting 4 white balls} = \frac{^4C_4}{^{10}C_4} = \frac{1}{210}$$

Thus, the probability distribution of X is given by

X:	0	1	2	3	4
$P(X)$:	$\frac{1}{14}$	$\frac{8}{21}$	$\frac{6}{14}$	$\frac{4}{35}$	$\frac{1}{210}$

S8. When three balls are drawn, there may be all red, 2 red, 1 red or no red ball at all. Thus, if X denotes the number of red balls in a random draw of three balls. Then, X can take values 0, 1, 2, 3.

Now, $$P(X = 0) = P(\text{Getting no red ball})$$

$$\Rightarrow \quad P(X = 0) = P(\text{Getting three white balls})$$

$$\Rightarrow \quad P(X = 0) = \frac{^4C_3}{^7C_3} = \frac{4 \times 3 \times 2}{7 \times 6 \times 5} = \frac{4}{35}$$

$$P(X = 1) = P(\text{Getting one red and two white balls}) = \frac{^3C_1 \times ^4C_2}{^7C_3} = \frac{18}{35}$$

$$P(X = 2) = P(\text{Getting two red and one white ball}) = \frac{^3C_2 \times ^4C_1}{^7C_3} = \frac{12}{35}$$

$$P(X = 3) = P(\text{Getting three red balls}) = \frac{^3C_3}{^7C_3} = \frac{1}{35}$$

Thus, the probability distribution of the number of red balls is given by

X:	0	1	2	3
$P(X)$:	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

S9. Let p be the probability of getting a tail in a single toss of a coin. Then, probability of getting a head = $3p$.

Since" getting head" and getting tail" are mutually exclusive and exhaustive events in a single toss of a coin.

$$\therefore \quad P(H) + P(T) = 1$$

$$\Rightarrow \quad 3p + p = 1 \Rightarrow p = \frac{1}{4}$$

$$\Rightarrow \quad P(H) = \frac{3}{4} \text{ and } P(T) = \frac{1}{4}$$

Let X denote the number of tails in two tosses of a coin. Then, X can take values 0, 1, 2.

Now,

$P(X = 0) =$ Probability of getting no tail

$\Rightarrow \quad P(X = 0) =$ Probability of getting both heads

$\Rightarrow \quad P(X = 0) = P(HH)$

$\Rightarrow \quad P(X = 0) = P(H) \times P(H)$ [$\because$ two trials are independent]

$\Rightarrow \quad P(X = 0) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

$P(X = 1) =$ Probability of getting one tail and one head.

$\Rightarrow \quad P(X = 1) = P(H)\,P(T) + P(T)\,P(H)$

$\Rightarrow \quad P(X = 1) = \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} = \frac{3}{8}$

$P(X = 2) =$ Probability of getting both tails

$\Rightarrow \quad P(X = 2) = P(TT)$

$\Rightarrow \quad P(X = 2) = P(T)\,P(T) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Hence, the probability distribution of X is

X:	0	1	2
$P(X)$:	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

S10. Let S be the sample space associated the with the given random experiment.

Then,

$$S = \{H, TH, TTH, TTTH, TTTT\}$$

Let X denote the number of tosses. Then, X can take values 1, 2, 3 and 4

Now,

$$P(X = 1) = P(H) = \frac{1}{2},$$

$$P(X = 2) = P(TH) = P(T)\,P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(X = 3) = P(TTH) = P(T)\,P(T)\,P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8},$$

and, $\quad P(X = 4) = P(TTTH \text{ or } TTTT) = P(TTTH) + P(TTTT)$

$\Rightarrow \quad P(X = 4) = P(T)\,P(T)\,P(T)\,P(H) + P(T)\,P(T)\,P(T)\,P(T)$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

Thus, the probability distribution of X is given by

X:	1	2	3	4
$P(X)$:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

S11. When two coins are tossed, there may be 1 head, 2 heads or no head at all.

Thus, the possible values of X are 0, 1, 2.

Now, $P(X = 0) = P$ (getting no head) $= P(TT) = \frac{1}{4}$

$P(X = 1) = P$ (getting one head)

$= P(HT \text{ or } TH) = \frac{2}{4} = \frac{1}{2}$

$P(X = 2) = P$ (getting two heads) $= P(HH) = \frac{1}{4}$

Thus, the required probability distribution of X is given by$X$:

X:	0	1	2
$P(X)$:	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

S12. Let X denote the number of times six occurs i.e. the number of sixes. Since the die is thrown twice, X can take values 0, 1 and 2.

Let S_i denote the event that a six occurs on the die in i^{th} throw and F_i denote the event that the six does not occur in the i^{th} throw. Then,

$P(X = 0) =$ Probability of not getting six in both the throw

$= P(F_1 \text{ and } F_2) = P(F_1 \cap F_2)$

$= P(F_1)\, P(F_2)$ $\quad [\because F_1, F_2$ are independent events$]$

$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(X = 1) =$ Probability of getting one six in two throws

$= P[(F_1 \text{ and } S_2) \text{ or } (S_1 \text{ and } F_2)]$

$= P(F_1)\, P(S_2) + P(S_1)\, P(F_2)$ $\quad \left[\begin{array}{r}\text{By multiplication theorem} \\ \text{for indep. events}\end{array}\right]$

$= \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} = \frac{10}{36} = \frac{5}{18}$

and, $P(X = 2)$ = Probability of getting sixes in both the throws

$= P(S_1 \cap S_2)$

$= P(S_1)\,P(S_2)$ [By Multiplication Theorem]

$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Thus, the probability distribution of X is

X:	0	1	2
$P(X)$:	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

S13. Let P be the probability of getting an odd number in a single throw of a die. Then, probability of getting an even number is $2p$.

We have, $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$\Rightarrow$ $p + 2p + p + 2p + p + 2p = 1$ [$\because$ Sum of the probability = 1]

$\Rightarrow$ $9p = 1 \Rightarrow p = \frac{1}{9}$

Now, Probability of getting a perfect square *i.e.*, 1 or 4 in a single throw of a die

$= P(1) + P(4) = p + 2p = 3p = \frac{3}{9} = \frac{1}{3}$

Since X denotes the number of perfect square in two tosses. Then, X can take values 0, 1, 2 such that

$P(X = 0)$ = Probability of not getting perfect squares in both the tosses

$\Rightarrow$ $P(X = 0) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$\Rightarrow$ $P(X = 1)$ = Probability of getting perfect squares in one of the two tosses

$\Rightarrow$ $P(X = 1) = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$

$\Rightarrow$ $P(X = 2)$ = Probability of getting perfect squares in both tosses

$\Rightarrow$ $P(X = 2) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Hence, the probability distribution of X is

X:	0	1	2
$P(X)$:	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

S14. We have, $P(X = r) \propto \alpha^r \Rightarrow P(X = r) = \lambda\,\alpha^r$, $r = 0, 1, 2, \ldots$

Since sum of the all the probabilities in a probability distribution is 1.

$\therefore$ $P(X = 0) + P(X = 1) + P(X = 2) + \ldots = 1$

$\Rightarrow \quad \lambda\alpha^0 + \lambda\alpha^1 + \lambda\alpha^2 + ...) = 1 \quad [\because P(X = r) = \lambda\alpha^r \text{ (given)}]$

$\Rightarrow \quad \lambda(1 + \alpha + \alpha^2 + \alpha^3 + ...) = 1$

$\Rightarrow \quad \lambda \cdot \left(\frac{1}{1-\alpha}\right) = 1$

$\Rightarrow \quad \lambda = 1 - \alpha$

$\therefore \quad P(X = r) = (1 - \alpha)\,\alpha^r,\ r = 0, 1, 2, ...$

Hence $\quad P(X = 0) = (1 - \alpha)\,\alpha^0 = (1 - \alpha).$

S15. We know that, total probability of an experiment = 1

(i) $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow \quad 10k^2 + 9k - 1 = 0$

$\Rightarrow \quad 10k^2 + 10k - k - 1 = 0$

$\Rightarrow \quad 10k(k + 1) - 1(k + 1) = 0$

$\Rightarrow \quad (10k - 1)(k + 1) = 0$

$\therefore \quad k = \frac{1}{10} \text{ or } -1 \quad [\because k > 0 \Rightarrow k+1 \neq 0]$

$\Rightarrow \quad k = \frac{1}{10}$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k = 3k$

$= 3\left(\frac{1}{10}\right) \quad \left[\because k = \frac{1}{10}\right]$

$= \frac{3}{10}$

(iii) $P(X > 6) = P(X = 7) = 7k^2 + k$

$= 7\left(\frac{1}{10}\right)^2 + \frac{1}{10} \quad \left[\because k = \frac{1}{10}\right]$

$= \frac{7}{100} + \frac{1}{10} = \frac{7+10}{100} = \frac{17}{100}$

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$= k + 2k = 3k$

$= 3\left(\frac{1}{10}\right) \quad \left[\because k = \frac{1}{10}\right]$

$= \frac{3}{10}.$

S16. We know that, when a pair of dice is thrown, then total number of outcomes = 36.

Also, probability of getting of doublet in one throw = $\frac{6}{36} = \frac{1}{6}$

[$\because$ Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]

$\therefore$ Probability of not getting a doublet = $1 - \frac{1}{6} = \frac{5}{6}$

Let X = Number of doublets is three tosses of pair of dice

So, X can take values 0, 1, 2 and 3.

Now, $P(X = 0) = P$(not getting a doublet)

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$P(X = 1) = P$(getting a doublet once only)

$= P$(getting a doublet in 1st throw) + P(getting a doublet in 2nd throw) + P(getting a doublet in 3rd throw)

$$= \left(\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$$

$$= \frac{25}{216} + \frac{25}{216} + \frac{25}{216} = \frac{75}{216}$$

$P(X = 2) = P$(getting a doublet in two times)

$= P$(doublet in 1st and 2nd throw) + P(doublet in 2nd and 3rd throw) + P(doublet in 1st and 3rd throw)

$$= \left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$$

$$= \frac{5}{216} + \frac{5}{216} + \frac{5}{216} = \frac{15}{216}$$

$P(X = 3) = P$(getting a doublet in all the three throw)

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$\therefore$ The required probability distribution is

X	0	1	2	3
***P*(*X*)**	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

S17. Let X = Number of aces

Since, two cards are drawn, so X can take values 0, 1 and 2.

Now, probability of getting an ace = $\frac{4}{52} = \frac{1}{13}$

And probability of not getting an ace = $1 - \frac{1}{13} = \frac{12}{13}$

$$P(X = 0) = P(\text{not getting an aces})$$

$$= \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$$

$$P(X = 1) = P(\text{getting one ace card})$$

$$= \frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13} = \frac{24}{169}$$

$$P(X = 2) = P(\text{getting two ace card})$$

$$= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$

∴ The required probability distribution is

X	0	1	2
***P*(*X*)**	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

S18. Here, given that total number of bulbs = 10

Number of defective bulbs = 3

Number of bulbs which are not defective = 7

Now, let X be the number of defective bulbs drawn. We take $X = 0, 1$ and 2. As 2 bulbs are drawn from a lot of a 10 bulbs.

Now, we find the respective probabilities at X = 0, 1 or 2.

∴ $P(X = 0)$ = Probability that no defective bulb is drawn

$$= \frac{{}^3C_0 \times {}^7C_2}{{}^{10}C_2} = \frac{1 \times \frac{7!}{2!5!}}{\frac{10!}{2!8!}}$$

$$= \frac{7!}{2!5!} \times \frac{2!8!}{10!} = \frac{7 \times 6}{10 \times 9}$$

$$= \frac{42}{90} = \frac{14}{30} = \frac{7}{15}$$

$\therefore \qquad P(X=0) = \frac{7}{15}$

Now, $P(X = 1)$ = Probability that one defective bulb is drawn *i.e.*, the other drawn bulb is non-defective

$$= \frac{^3C_1 \times {^7C_1}}{^{10}C_2} = \frac{3 \times 7}{\frac{10!}{2!8!}} \qquad \left[\because {^nC_1} = n\right]$$

$$= \frac{3 \times 7 \times 8! \times 2!}{10!} = \frac{3 \times 7 \times 2}{10 \times 9} = \frac{14}{30} = \frac{7}{15}$$

Now, $P(X = 2)$ = Probability that both the bulbs drawn are defective.

$$= \frac{^3C_2}{^{10}C_2} = \frac{3}{\frac{10!}{2! \times 8!}} \qquad \left[\because {^nC_{n-1}} = n\right]$$

$$= \frac{3 \times 2! \times 8!}{10!} = \frac{3 \times 2}{10 \times 9}$$

$$= \frac{6}{90} = \frac{2}{30} = \frac{1}{15}$$

Hence, the required probability distribution is

X	0	1	2
***P*(*X*)**	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

S19. Let X denote the number of bad oranges in draw of 2 oranges drawn from group of 16 good oranges and 4 bad oranges. Since there are 4 bad oranges in the group, therefore, X can take values 0, 1 and 2.

Now, $\qquad P(X = 0)$ = Probability of getting no bad orange = $\frac{^{16}C_2}{^{20}C_2} = \frac{12}{19}$,

$P(X = 1)$ = Probability of getting one bad orange = $\frac{^4C_1 \times {^{16}C_1}}{^{20}C_2} = \frac{32}{95}$,

and, $\qquad P(X = 2)$ = Probability of getting two bad orange = $\frac{^4C_2}{^{20}C_2} = \frac{3}{95}$

Thus, the probability distribution of X is given by

X:	0	1	2
$P(X)$:	$\frac{12}{19}$	$\frac{32}{95}$	$\frac{3}{95}$

S20. Clearly, X takes values 0, 1, 2, 3 such that

$$P(X = 0) = \text{ Probability of getting a slip marked } 0 = \frac{1}{8}$$

$$P(X = 1) = \text{ Probability of getting a slip marked } 1 = \frac{3}{8}$$

$$P(X = 2) = \text{ Probability of getting a slip marked } 2 = \frac{3}{8}$$

$$P(X = 3) = \text{ Probability of getting a slip marked } 3 = \frac{1}{8}$$

Hence, the probability distribution of X is

X:	0	1	2	3
$P(X)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Date: 19/10/2021

Q1. In a game, a person is paid Rs. 5 if he gets all heads or all tails when three coins are tossed, and he will pay Rs. 3 if either one or two heads show. What can he expect to win on the average per game?

Q2. In a meeting 70% of the members favour a certain proposal, 30% being opposed. A member is selected at random and let $X = 0$ if he opposed, and $X = 1$ if he in favour. Find $E(X)$ and Var (X).

Q3. A random variable X has the following probability distribution:

X	:	–2	–1	0	1	2	3
$P(X)$	:	0.1	k	0.2	$2k$	0.3	k

(i) Find the value of k. (ii) Calculate the mean of X. (iii) Calculate the variance of X.

Q4. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One students is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of random variable X? Find mean, variance and standard deviation of X.

Q5. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find $E(X)$ and P(X) of random variable X.

Q6. Find the mean, variance and standard deviation of the number of heads in a simultaneous toss of three coins.

Q7. A die is tossed twice. A" success" is getting an odd number" on a random toss. Find the variance of the number of success.

Q8. Find the probability distribution of the number of sixes in three tosses of a die. Find also the mean and variance of the distribution.

Q9. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as getting a number greater than 4. Also, find the mean and variance of the distribution.

Q10. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation and variance of X.

Q11. A coin weighted so that $P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$ is tossed three times. Let X be the random variable which denotes the longer string of heads which occurs. Find the probability distribution, mean and variance of X.

Q12. A fair coin is tossed until a head or five tails occur. If X denotes the number of tosses of the coin, find the mean of X.

Q13. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the items in the sample are drawn one by one without replacement, find

(i) The probability distribution of X (ii) Mean of X (iii) Variance of X

Q14. There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean and variance.

Q15. In a game a man wins a rupee for a six and loose a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/looses.

Q16. Find the mean of number of heads in three tosses of a coin.

Q17. Two cards are drawn simultaneously (without replacement) from a well shuffled deck of 52 cards. Find the mean and variance of number of red cards.

Q18. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled deck of 52 cards. Find the mean, variance and standard deviation of number of aces.

Date: 19/10/2021

S1. Let X be the amount received by the person. Then, X can take values 5 and3 such that

$$P(X = 5) = \text{Probability of getting all heads or all tails when three coins are tossed}$$

$$\Rightarrow \quad P(X = 5) = \frac{2}{8} = \frac{1}{4}$$

$$P(X = -3) = \text{Probability of getting one or two heads}$$

$$\Rightarrow \quad P(X = -3) = \frac{6}{8} = \frac{3}{4}$$

$\therefore$ Expected amount of win, on the average, per game

$$= \bar{X} = \Sigma p_i x_i = 5 \times \frac{1}{4} - 3 \times \frac{3}{4} = -1$$

Thus, the person will, on the average, lose Rs 1 per toss of the coins.

S2. The probability distribution of X is

X	:	0	1
$P(X)$	:	$\frac{30}{100}$	$\frac{70}{100}$

$$\therefore \quad E(X) = \frac{30}{100} \times 0 + \frac{70}{100} \times 1 = \frac{7}{10}$$

and, $$E(X^2) = \frac{30}{100} \times 0^2 + \frac{70}{100} \times 1^2 = \frac{7}{10}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{7}{10} - \frac{49}{100} = \frac{21}{100}$$

Hence, $$E(X) = \frac{7}{10} \text{ and Var}(X) = \frac{21}{100}$$

S3. Since sum of the probabilities in a frequency distribution is always unity

$$\therefore \quad 0.1 + k + 0.2 + 2k + 0.3 + k = 1$$

$$\Rightarrow \quad 0.6 + 4k = 1 \Rightarrow 4k = 0.4 \Rightarrow k = 0.1$$

Computation of mean and variance

x_i	p_i	$p_i x_i$	$p_i x_i^2$
– 2	0.1	– 0.2	0.4
– 1	0.1	– 0.1	0.1
0	0.2	0	0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.1	0.3	0.9
		$\Sigma p_i x_i = 0.8$	and $\Sigma p_i x_i^2 = 2.8$

Hence

$\therefore$ Mean = 0.8

and, Variance = $\Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = 2.8 - (0.8)^2 = 2.8 - 0.64 = 2.16$

S4. We observe that X takes values 14, 15, 16, 17, 18, 19, 20 and 21 such that

$$P(X = 14) = \frac{2}{15},\ P(X = 15) = \frac{1}{15},\ P(X = 16) = \frac{2}{15},\ P(X = 17) = \frac{3}{15}$$

$$P(X = 18) = \frac{1}{15},\ P(X = 19) = \frac{2}{15},\ P(X = 20) = \frac{3}{15},\ P(X = 21) = \frac{1}{15}$$

So, the probability distribution of X is as given below :

X	:	14	15	16	17	18	19	20	21
$P(X)$	:	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Computation of mean and variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
14	$\frac{2}{15}$	$\frac{28}{15}$	$\frac{392}{15}$
15	$\frac{1}{15}$	$\frac{15}{15}$	$\frac{225}{15}$
16	$\frac{2}{15}$	$\frac{32}{15}$	$\frac{512}{15}$
17	$\frac{3}{15}$	$\frac{51}{15}$	$\frac{867}{15}$
18	$\frac{1}{15}$	$\frac{18}{15}$	$\frac{324}{15}$
19	$\frac{2}{15}$	$\frac{38}{15}$	$\frac{722}{15}$
20	$\frac{3}{15}$	$\frac{60}{15}$	$\frac{1200}{15}$
21	$\frac{1}{15}$	$\frac{21}{15}$	$\frac{441}{15}$
		$\Sigma p_i x_i = \frac{263}{15}$	$\Sigma p_i x_i^2 = \frac{4683}{15}$

We have, $\Sigma p_i x_i = \frac{263}{15}$ and $\Sigma p_i x_i^2 = \frac{4683}{15}$

$\therefore$ Mean $= \Sigma p_i x_i = \frac{263}{15} = 17.53$

Variance $= \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$

$\Rightarrow$ Variance $= \frac{4683}{15} - \left(\frac{263}{15}\right)^2 = \frac{70245 - 69169}{225} = \frac{1076}{225}$

$\therefore$ Standard Deviation $= \sqrt{\text{Variance}} = \sqrt{\frac{1076}{225}} = \frac{\sqrt{1076}}{15} = \frac{32.80}{15} = 2.186$

S5. We observe that X can take values 2, 3, 4, 5, 6 such that

$P(X = 2) =$ Probability that the larger of two numbers is 2

$\Rightarrow$ $P(X = 2) =$ Probability of getting 1 in first selection and 2 in second selection or getting 2 in first selection and 1 in second selection

$\Rightarrow$ $P(X = 2) = \frac{1}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{1}{5} = \frac{2}{30} = \frac{1}{15}$

$P(X = 3) =$ Probability that the larger of two numbers is 3

$\Rightarrow$ $P(X = 3) =$ Probability of getting a number less than 3 in first selection and 3 in second selection or getting 3 in first selection and a number less than 3 in second selection.

$\Rightarrow$ $P(X = 3) = \frac{2}{6}\times\frac{1}{5}+\frac{1}{6}\times\frac{2}{5} = \frac{4}{30} = \frac{2}{15}$

$\Rightarrow$ ($X =$ 4) Probability that the larger of two numbers is 4

$$P(X = 4) = \frac{3}{6}\times\frac{1}{5}+\frac{1}{6}\times\frac{3}{5} = \frac{6}{30} = \frac{1}{5}$$

$$P(X = 5) = \frac{4}{6}\times\frac{1}{5}+\frac{1}{6}\times\frac{4}{5} = \frac{8}{30} = \frac{4}{15}$$

$$P(X = 6) = \frac{5}{6}\times\frac{1}{5}+\frac{1}{6}\times\frac{5}{5} = \frac{10}{30} = \frac{1}{3}$$

Thus, the probability distribution of X is

X	:	2	3	4	5	6
$P(X)$	:	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

$\therefore$ $E(X) = \frac{1}{15}\times 2+\frac{2}{15}\times 3+\frac{1}{5}\times 4+\frac{4}{15}\times 5+\frac{1}{3}\times 6$

$\Rightarrow$ $E(X) = \frac{70}{15} = \frac{14}{3}$.

S6. Let X denote the number of heads in a simultaneous toss of three coins.

Then X can take values 0, 1, 2, 3.

Now, $P\,(X = 0) = P\,(TTT) = \frac{1}{8}$

$$P\,(X = 1) = \text{P}\,(HTT \text{ or } TTH \text{ or } THT) = \frac{3}{8}$$

$$P\,(X = 2) = \text{P}\,(HHT \text{ or } THH \text{ or } HTH) = \frac{3}{8}$$

and, $P\,(X = 3) = \text{P}\,(HHH) = \frac{1}{8}$

Thus, the probability distribution of X is given by

X	:	0	1	2	3
$P(X)$	:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Computation of variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\Sigma p_i x_i = \frac{3}{2}$	$\Sigma p_i x_i^2 = 3$

We have,

$$\Sigma p_i x_i = \frac{3}{2} \text{ and } \Sigma p_i x_i^2 = 3$$

$\therefore$ $\bar{X}$ = Mean = $\Sigma p_i x_i = \frac{3}{2}$

and, $\text{Var}(X) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$

$\therefore$ Standard deviation = $\sqrt{\text{Var}(X)} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} = 0.866$

Hence, Mean = $\frac{3}{2}$, Variance = $\frac{3}{4}$ and Standard deviation = 0.866

S7. Let X denote the number of successes in two tosses of a die. Then, X can take values, 0, 1, 2. Let S_i and F_i denote the success and failure respectively in i^{th} toss. Then,

$P(S_i)$ = Probability of getting an odd number in i^{th} toss = $\frac{3}{6} = \frac{1}{2}$

$\Rightarrow$ $P(F_i) = \left(1 - \frac{1}{2}\right) = \frac{1}{2}$

Now, $P(X = 0)$ = Probability of getting no success in two tosses of a die

$\Rightarrow$ $(X = 0) = P(F_1 \cap F_2)$

$\Rightarrow$ $P(X = 0) = P(F_1)\, P(F_2)$ [By Multiplication Theorem]

$\Rightarrow$ $P(X = 0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\left[\because P(F_1) = P(F_2) = \frac{1}{2}\right]$

$P(X = 1)$ = Probability of getting one success in two tosses of a die

$\Rightarrow \quad P(X=1) = P\left((S_1 \cap F_2) \cup (F_1 \cap S_2)\right)$

$\Rightarrow \quad (X=1) = P(S_1 \cap F_2) + P(F_1 \cap S_2)$

$\Rightarrow \quad P(X=1) = P(S_1)\,P(F_2) + P(F_1)\,P(S_2) = \frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2} = \frac{1}{2}$

and, $\quad P(X=2) =$ Probability of getting two successes in two tosses of a die

$\Rightarrow \quad P(X=2) = P(S_1 \cap S_2) = P(S_1)\,P(S_2) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

Thus, the probability distribution of X i.e. the number of successes in two tosses of a die, is given by

X	:	0	1	2
$P(X)$	:	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Computation of variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{1}{2}$	1
		$\Sigma p_i x_i = 1$	$\Sigma p_i x_i^2 = \frac{3}{2}$

We have, $\quad \Sigma p_i x_i = 1$ and $\Sigma p_i x_i^2 = \frac{3}{2}$

$\therefore \quad \text{Var}(X) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{3}{2} - 1 = \frac{1}{2}$

S8. Let X denote the number of sixes in three tosses of a die. Then, X can take values 0, 1, 2,3.

Let S_i denote the event of getting a six in i^{th} toss, $i = 1, 2, 3$

Then,

$$P(S_i) = \frac{1}{6} \text{ and } P(\bar{S}_i) = \frac{5}{6}; i = 1, 2, 3$$

Now, $\quad P(X=0) = P(\bar{S}_1 \cap \bar{S}_2 \cap \bar{S}_3)$

$\Rightarrow \quad P(X=0) = P(\bar{S}_1)P(\bar{S}_2)P(\bar{S}_3)$ $\quad [\because \bar{S}_1, \bar{S}_2, \bar{S}_3$ are independent events$]$

$\Rightarrow \quad P(X=0) = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{125}{216}$

$P(X=1) = P(S_1 \cap \bar{S}_2 \cap \bar{S}_3) \cup (\bar{S}_1 \cap S_2 \cap \bar{S}_3) \cup (\bar{S}_1 \cap \bar{S}_2 \cap S_3)$

$\Rightarrow \quad P(X=1) = P(S_1 \cap \bar{S}_2 \cap \bar{S}_3) + P(\bar{S}_1 \cap S_2 \cap \bar{S}_3) + P(\bar{S}_1 \cap \bar{S}_2 \cap S_3)$

$\Rightarrow \quad P(X=1) = P(S_1)P(\bar{S}_2)P(\bar{S}_3) + P(\bar{S}_1)P(S_2)P(\bar{S}_3) + P(\bar{S}_1)P(\bar{S}_2)P(S_3)$

$\Rightarrow \quad P(X=1) = \frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}+\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{75}{216}$

$\Rightarrow \quad P(X=2) = P(S_1 \cap S_2 \cap \bar{S}_3) \cap (\bar{S}_1 \cap S_2 \cap S_3) \cap (S_1 \cap \bar{S}_2 \cap S_3)$

$\Rightarrow \quad P(X=2) = P(S_1 \cap S_2 \cap \bar{S}_3) + P(\bar{S}_1 \cap S_2 \cap S_3) + P(S_1 \cap \bar{S}_2 \cap S_3)$

$\Rightarrow \quad P(X=2) = P(S_1)\,P(S_2)\,P(\bar{S}_3) + P(\bar{S}_1)P(S_2)P(S_3) + P(S_1)P(\bar{S}_2)P(S_3)$

$\Rightarrow \quad P(X=2) = \frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{15}{216}$

and, $\quad P(X=3) = P(S_1 \cap S_2 \cap S_3) = P(S_1)\,P(S_2)\,P(S_3) = \frac{1}{6}\times\frac{1}{6}\times\frac{1}{6} = \frac{1}{216}$

Thus, the probability distribution of X given by

X	:	0	1	2	3
$P(X)$	:	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

Computation of mean and variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{125}{216}$	0	0
1	$\frac{75}{216}$	$\frac{75}{216}$	$\frac{75}{216}$
2	$\frac{15}{216}$	$\frac{30}{216}$	$\frac{60}{216}$
3	$\frac{1}{216}$	$\frac{3}{216}$	$\frac{9}{216}$
		$\Sigma p_i x_i = \frac{108}{216} = \frac{1}{2}$	$\Sigma p_i x_i^2 = \frac{144}{216}$

We have,

$$\Sigma p_i x_i = \frac{108}{216} = \frac{1}{2} \text{ and } \Sigma p_i x_i^2 = \frac{144}{216}$$

$\therefore \quad \text{Mean} = \bar{X} = \Sigma p_i x_i = \frac{1}{2}$

and $\quad \text{Var}(X) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{144}{216} - \left(\frac{1}{2}\right)^2 = \frac{90}{216} = \frac{5}{12}$

Hence, $\quad \text{Mean} = \frac{1}{2}$ and $\text{Variance} = \frac{5}{12}$

S9. Let X denote the number of success in two tosses of a die. Then, X can take values 0, 1, 2.

Let S_i = Getting a success in i^{th} toss and, F_i = Getting a failure i^{th} toss

Then, $\quad P(S_1)$ = Probability of getting a number greater than 4 in first toss

$\Rightarrow \quad P(S_1) = \frac{2}{6} = \frac{1}{3}$

Also, $\quad P(S_2) = \frac{1}{3}$

$\therefore \quad P(F_1) = P(F_2) = \frac{2}{3}$

Now, $\quad P(X = 0)$ = Probability of getting no success in two tosses of a die

$\Rightarrow \quad P(X = 0) = P(F_1 \cap F_2)$

$\Rightarrow \quad P(X = 0) = P(F_1) \times P(F_2)$ [By multiplication theorem]

$\Rightarrow \quad P(X = 0) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(X = 1)$ = Probability of getting one success in two tosses of a die

$\Rightarrow \quad (X = 1) = \{(S_1 \cap F_2) \cup (F_1 \cap S_2)\}$

$\Rightarrow \quad P(X = 1) = P(S_1 \cap F_2) + P(F_1 \cap S_2)$

$\Rightarrow \quad P(X = 1) = P(S_1)\, P(F_2) + P(F_1)\, P(S_2)$

$\Rightarrow \quad P(X = 1) = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$

and, $\quad P(X = 2)$ = Probability of getting two successes in two tosses of a die

$\Rightarrow \quad P(X = 2) = P(S_1 \cap S_2) = P(S_1)\, P(S_2) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Thus, the probability distribution of X is given by

X	:	0	1	2
$P(X)$	:	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Computation of mean and variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{4}{9}$	0	0
1	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{4}{9}$
2	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{4}{9}$
		$\Sigma p_i x_i = \frac{6}{9}$	$\Sigma p_i x_i^2 = \frac{8}{9}$

We have,

$$\Sigma p_i x_i = \frac{6}{9} = \frac{2}{3} \quad \text{and} \quad \Sigma p_i x_i^{\,2} = \frac{8}{9}$$

$$\bar{X} = \text{Mean} = \Sigma p_i x_i = \frac{2}{3}$$

and $$\text{Var}(X) = \Sigma p_i x_1^2 - (\Sigma p_i x_i)^2 = \frac{8}{9} - \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Hence, $$\text{Mean} = \frac{2}{3} \quad \text{and} \quad \text{Variance} = \frac{4}{9}$$

S10. Clearly, X can take values 0, 1and 2.

We have,

$$P(X = 0) = \text{Probability of not getting six on any dice} = \frac{25}{36}$$

$$P(X = 1) = \text{Probability of getting one six} = \frac{10}{36}$$

$$P(X = 2) = \text{Probability of getting two sixes} = \frac{1}{36}$$

Thus, the probability distribution of X is given by

X	:	0	1	2
$P(X)$	:	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Computation of mean and variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{25}{36}$	0	0
1	$\frac{10}{36}$	$\frac{10}{36}$	$\frac{10}{36}$
2	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$
		$\Sigma p_i x_i = \frac{12}{36}$	$\Sigma p_i x_i^2 = \frac{14}{36}$

We have,

$$\Sigma p_i\, x_i = \frac{12}{36} = \frac{1}{3} \quad \text{and} \quad \Sigma p_i x_i^2 = \frac{7}{18}$$

$\therefore$ $$E(X) = \Sigma p_i\, x_i = \frac{1}{3}$$

and, $$\text{Var}(X) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{7}{18} - \frac{1}{9} = \frac{5}{18}$$

Hence, $$E(X) = \frac{1}{3} \quad \text{and} \quad \text{Var}(X) = \frac{5}{18}$$

S11. The random variable X is defined on the sample space S given by

$$S = \{TTT, HTT, THT, TTH, THH, HTH, HHT, HHH\}$$

Note that the string of heads means the sequence of consecutive heads.

Since X dentoes the longest string of heads. Therefore,

$$X(TTT) = 0, X(THT) = 1, X(HTT) = 1, X(TTH) = 1, X(HTH) = 1,$$

$$X(HHT) = 2, X(THH) = 2 \text{ and } X(HHH) = 3$$

Now, $$P(X = 0) = P(TTT) = P(T)\,P(T)\,P(T) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$$

$$P(X = 1) = P(THT \cup HTT \cup TTH \cup HTH)$$

$\Rightarrow$ $$P(X = 1) = P(THT) + P(HTT) + P(TTH) + P(HTH)$$

$\Rightarrow$ $$P(X = 1) = P(T)\,P(H)\,P(T) + P(H)\,P(T)\,P(T) + P(T)\,P(T)\,P(H) + P(H)\,P(T)\,P(H)$$

$\Rightarrow \quad P(X = 1) = \frac{1}{4}\times\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}+\frac{3}{4}\times\frac{1}{4}\times\frac{3}{4}$

$= \frac{3}{64}+\frac{3}{64}+\frac{3}{64}+\frac{9}{64} = \frac{18}{64}$

$P(X = 2) = P(THH \cup HHT)$

$\Rightarrow \quad P(X = 2) = P(THH) + P(HHT)$

$\Rightarrow \quad P(X = 2) = P(T)\,P(H)\,P(H) + P(H)\,P(H)\,P(T)$

$\Rightarrow \quad P(X = 2) = 2\left(\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}\right) = \frac{18}{64}$

and $\quad P(X = 3) = P(HHH)$

$\Rightarrow \quad P(X = 3) = P(H)\,P(H)\,P(H) = \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$

So, the probability distribution is as follows:

X	:	0	1	2	3
$P(X)$	:	$\frac{1}{64}$	$\frac{18}{64}$	$\frac{18}{64}$	$\frac{27}{64}$

Computation of variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{64}$	0	0
1	$\frac{18}{64}$	$\frac{18}{64}$	$\frac{18}{64}$
2	$\frac{18}{64}$	$\frac{36}{64}$	$\frac{72}{64}$
3	$\frac{27}{64}$	$\frac{81}{64}$	$\frac{243}{64}$
		$\Sigma p_i x_i = \frac{135}{64}$	$\Sigma p_i x_i^2 = \frac{333}{64}$

We have, $\quad \Sigma p_i x_i = \frac{135}{64}$ and $\Sigma p_i x_i^2 = \frac{333}{64}$

$\therefore \quad \text{Mean} = \Sigma p_i x_i = \frac{135}{64} = 2.1$

and, Variance = $\Sigma p_i x_i^2 - (\text{Mean})^2 = \frac{333}{64} - (2.1)^2 = 5.2 - 4.41 = 0.79$

Hence, Mean = 2.1 and Variance = 0.79.

S12. The sample space related to the given random experiment is given by

$$S = \{H, TH, TTH, TTTH, TTTTH, TTTTT\}$$

Clearly, X assumes values 1, 2, 3, 4, 5 such that

$$P(X = 1) = P(H) = \frac{1}{2}$$

$$P(X = 2) = P(TH) = P(T)\,P(H) = \frac{1}{4}$$

$$P(X = 3) = P(TTH) = P(T)\,P(T)\,P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(X = 4) = P(TTTH) = P(T)\,P(T)\,P(T)\,P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

and, $P(X = 5) = P(TTTTH \cup TTTTT)$

$\Rightarrow$ $P(X = 5) = P(TTTTH) + P(TTTTT)$

$\Rightarrow$ $P(X = 5) = P(T)\,P(T)\,P(T)\,P(T)\,P(H) + P(T)\,P(T)\,P(T)\,P(T)\,P(T)$

$\Rightarrow$ $P(X = 5) = \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5 = \frac{1}{16}$

So, the probability distribution of X is given by

X	:	1	2	3	4	5
$P(X)$	:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

Now,

$$\text{Mean} = \Sigma p_i x_i = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{16} \times 4 + \frac{1}{16} \times 5 = \frac{31}{16} = 1.9$$

S13. (i) Clearly, X can assume values 0, 1, 2, 3, such that

$$P(X = 0) = \frac{{}^7C_4}{{}^{10}C_4} = \frac{1}{6}, \quad P(X) = 1 = \frac{{}^3C_1 \times {}^7C_3}{{}^{10}C_4} = \frac{1}{2}$$

$$P(X = 2) = \frac{{}^3C_2 \times {}^7C_1}{{}^{10}C_4} = \frac{1}{10}, \text{ and } P(X = 3) = \frac{{}^3C_3 \times {}^7C_1}{{}^{10}C_4} = \frac{1}{30}$$

So, the probability distribution of X is given below.

X :	0	1	2	3
$P(X)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{30}$

Computation of variance

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{4}{10}$
3	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{3}{10}$
		$\Sigma p_i x_i = \frac{8}{10}$	$\Sigma p_i x_i^2 = \frac{12}{10}$

(ii) We have, $\Sigma p_i x_i = \frac{8}{10}$

$\therefore \quad \bar{X} = \text{Mean} = \Sigma p_i x_i = \frac{8}{10}$

(iii) Now, $\text{Var}(X) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{12}{10} - \frac{64}{100} = \frac{56}{100}$

Hence, Mean $= \frac{8}{10}$ and Variance $= \frac{56}{100}$

S14. Clearly, X can take values from 3 to 9.

We have,

$P(X = 3)$ = Probability of getting 3 as the sum

$\Rightarrow$ $P(X = 3) = P${Getting 1 in first draw and 2 in second drawn) or (Getting 2 in first draw and 1 in second drawn)}

$$P(X = 3) = \frac{1}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{4} = \frac{1}{10}$$

$P(X = 4) = P${(Getting 1 in first drawn and 3 in second drawn) or (Getting 3 in first drawn and 1 in second drawn)}

$$\Rightarrow \quad P(X = 4) = \frac{1}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{4} = \frac{1}{10}$$

Similarly

$$P(X=5) = \frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4} = \frac{1}{5}$$

$$P(X=6) = \frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4} = \frac{1}{5}$$

$$P(X=7) = \frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4} = \frac{1}{5}$$

$$P(X=8) = \frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4} = \frac{1}{10}$$

$$P(X=9) = \frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4} = \frac{1}{10}$$

Thus, the probability distribution of X is as given below :

X :	3	4	5	6	7	8	9
$P(X)$:	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

Computation of means and variance

	x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
	3	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{9}{10}$
	4	$\frac{1}{10}$	$\frac{4}{10}$	$\frac{16}{10}$
	5	$\frac{1}{5}$	$\frac{5}{5}$	$\frac{25}{5}$
	6	$\frac{1}{5}$	$\frac{6}{5}$	$\frac{36}{5}$
	7	$\frac{1}{5}$	$\frac{7}{5}$	$\frac{49}{5}$
	8	$\frac{1}{10}$	$\frac{8}{10}$	$\frac{64}{10}$
	9	$\frac{1}{10}$	$\frac{9}{10}$	$\frac{81}{10}$
Total		$\Sigma p_i = 1$	$\Sigma p_i x_i = \frac{60}{10} = 6$	$\Sigma p_i x_i^2 = \frac{390}{10} = 39$

We have, $\Sigma p_i x_i = 6$ and $\Sigma p_i x_i^2 = 39$

$\therefore$ $\bar{X}$ = Mean = $\Sigma p_i x_i = 6$

and, $\text{Var}(X) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = 39 - 6^2 = 3$

S15. The man may get six in the first throw and then the quits the game. He may get a number other than six in the first throw and in the second throw he may get six and quits the game. In the first two throws he gets a number other than six and in third throw he may get a six. He may not get six in any one of three throws.

Let X be the amount the wins/looses. Then, X can take values 1, 0, –1, – 3 such that

$$P(X = 1) = P\text{ (Getting six in first throw)} = \frac{1}{6}$$

$$P(X = 0) = P\text{ (Getting an other number in first throw and six in second throw)}$$

$$P(X = 0) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$$

$P(X = -1) = P$ (Getting numbers other than 6 in first two throws and a six in third throw)

$$\Rightarrow \quad P(X = -1) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$P(X = -3) = P$ (Getting a number other than six in first three throws)

$$\Rightarrow \quad (X = -3) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

Thus, the probability distribution of X is as given below

X	:	1	0	– 1	– 3
$P(X)$	:	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{216}$	$\frac{125}{216}$

$$\therefore \quad E(X) = 1 \times \frac{1}{6} + 0 \times \frac{5}{36} + (-1) \times \frac{25}{216} + (-3) \times \frac{125}{216}$$

$$\Rightarrow \quad E(X) = \frac{36 + 0 - 25 - 375}{216} = -\frac{364}{216} = -\frac{91}{54}$$

S16. Let X = Number of heads when a coin is tossed three times

Sample space of experiment is

$$S = \{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT\}$$

X can take values 0, 1, 2 and 3.

Now, $P(X = 0) = P$ (no head occurs) = $\frac{1}{8}$

$P(X = 1) = P$ (one head occurs) = $\frac{3}{8}$

$P(X = 2) = P$ (two head occurs) = $\frac{3}{8}$

$P(X = 3) = P$ (three head occurs) = $\frac{1}{8}$

$\therefore$ The probability distribution table is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Now, we find mean as follows

X	**P(X)**	$X_i P_i$
0	$\frac{1}{8}$	0
1	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$
		$\frac{12}{8}$

$\therefore$ Mean = $\Sigma X_i P_i$

$$= \frac{12}{8} = \frac{3}{2}$$

S17. First we find the probability distribution table. Let X be the number of red cards. Then, X can take values 0, 1 and 2.

Now, $P(X = 0) = P$ (having no red card)

$$\frac{^{26}C_2}{^{52}C_2} = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

$P(X = 1) = P$ (having one red card)

$$= \text{Probability of getting one red card and other black card}$$

$$= \frac{^{26}C_2 \times {}^{26}C_1}{^{52}C_1} = \frac{26 \times 26 \times 2}{52 \times 51} = \frac{26}{51}$$

$P(X = 2) = P$ (having two red cards)

$$\frac{^{26}C_2}{^{52}C_2} = \frac{26}{52} \times \frac{25}{51} = \frac{25}{102}$$

$\therefore$ The probability distribution of number of red cards is

X	0	1	2
$P(X)$	$\frac{25}{102}$	$\frac{26}{51}$	$\frac{25}{102}$

Now, we know that mean $= \Sigma X \cdot P(X)$

and variance $= \Sigma X^2 \cdot P(X) - (\Sigma X \cdot P(X))^2$

X	$P(X)$	$X \cdot P(X)$	$X^2 \cdot P(X)$
0	$\frac{25}{102}$	0	0
1	$\frac{26}{51}$	$\frac{26}{51}$	$\frac{26}{51}$
2	$\frac{25}{102}$	$\frac{25}{51}$	$\frac{50}{51}$

$\therefore$ Mean $= \Sigma X \cdot P(X)$

$$= 0 + \frac{26}{51} + \frac{25}{51} = \frac{51}{51} = 1$$

Variance $= \Sigma X^2 P(X) - (\Sigma X \cdot P(X))^2$

$$= \frac{76}{51} - (1)^2 = \frac{76}{51} - 1 = \frac{76 - 51}{51} = \frac{25}{51}$$

S18. Given that, total number of cards = 52

Also, total ace cards = 4 and other cards = 52 – 4 = 48

Let X denotes the number of ace cards drawn. We take values of X as X = 0, 1 and 2.

$\therefore$ $P(X = 0)$ = Probability that no ace card is drawn *i.e.*, both the cards drawn are non-aces

$$= \frac{^{4}C_0 \times {}^{48}C_2}{^{52}C_2} = \frac{\frac{48 \times 47}{2}}{\frac{52 \times 51}{2}}$$

$$= \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$P(X = 1)$ = Probability that one ace card is drawn *i.e.*, the other drawn card will be non-ace card.

$$= \frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{4 \times 48}{\frac{52 \times 51}{2}}$$

$$= \frac{4 \times 48 \times 2}{52 \times 51} = \frac{32}{221}$$

$P(X = 2)$ = Probability that both drawn cards are aces

$$= \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{\frac{4!}{2!\,2!}}{\frac{52!}{2!\,50!}} = 4 \times 3 \times \frac{50!}{52!}$$

$$= \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

$\therefore$ The probability distribution table is

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Now, we find mean, variance and standard deviation, we know that mean of a probability distribution is given as

mean = $\Sigma x_i \times p_i$ ... (i)

and its variance is given by, variance = $\sum x_i^2 p_i - \left(\sum x_i \cdot p_i\right)^2$ ... (ii)

$\therefore$ We have

X	$P(X)$	$x_i \cdot p_i$	$x_i^2 p_i$
0	$\frac{188}{221}$	0	0
1	$\frac{32}{221}$	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$	$\frac{4}{221}$
Total		$\frac{34}{221}$	$\frac{36}{221}$

$\therefore$ We have, $\Sigma x_i p_i = \dfrac{34}{221}$ and $\Sigma x_i^2 p_i = \dfrac{36}{221}$

Putting above values in Eqs. (i) and (ii), we get

$$\text{mean} = \frac{34}{221} = 0.1538$$

and $$\text{variance} = \frac{36}{221} - \left(\frac{34}{221}\right)^2$$ [$\because$ From Eq. (ii)]

$$= \frac{36}{221} - \frac{1156}{48841} = \frac{7956 - 1156}{48841}$$

$$= \frac{6800}{48841} = 0.139$$

Also, standard deviation = $\sqrt{\text{variance}}$

$$= \sqrt{\frac{6800}{48841}} = \frac{82.46}{221} = 0.37$$

Hence, mean = 0.154, variance = 0.139 and standard deviation = 0.37.

Date: 19/10/2021

Q1. In a hurdles race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Q2. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Q3. In a 20-question true-false examination suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Q4. Ten eggs are drawn successively, with replacement, from a lot containing 10% defective eggs. Find the probability that there is at least one defective eggs.

Q5. A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of (i) 5 successes (ii) at least 5 successes (iii) at most 5 successes (iv) at least one success (v) no success.

Q6. Five dice are thrown simultaneously. If the occurrence of an even number in a single dice is considered a success find the probability of at most 3 successes.

Q7. Three cards are drawn successively with replacement from a well shuffled deck of 52 cards. If getting a card of spade is a success, find the probability distribution of number of successes.

Q8. A pair of dice is thrown 4 times. If getting a doublet is a successes, find probability distribution of number of successes.

Q9. A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of

(i) no success? (ii) 6 successes?

(iii) at least 6 successes? (iv) at most 6 successes?

Q10. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is just one step away from the starting point.

Q11. An urn contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one with replacement, what is the probability that

(i) all are white? (ii) only 3 are white?

(iii) none is white? (iv) at least three are white?

Q12. A coin is tossed 5 times. What is the probability of getting at least 3 heads?

Q13. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Q14. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining third six in the sixth throw of die.

Q15. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right handed ?

Q16. An urn contains 25 balls of which 10 balls bear a mark '*A*' and the remaining 15 balls bear a mark '*B*'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

(i) all will bear '*A*' mark (ii) not more than 2 will bear '*B*' mark

(iii) the number of balls with '*A*' mark and '*B*' mark will be equal

(iv) at least one ball will bear '*B*' mark

Q17. The probability of a man hitting a target is 1/4. How many times must he fire so that the probability of his hitting the target at least once is greater than 2/3 ?

Q18. How many dice must be thrown so that there is a better than even chance of obtaining a six?

Q19. Find the probability distribution of the number of heads when a coin is tossed three times.

Q20. A bag contains 3 red and 4 black balls. One ball is drawn and then replaced in the bag and the process is repeated. Every time if the ball drawn is red we say that the draw has resulted in success. Let *X* be the number of successes in 3 draws. Assuming that at each draw each ball is equally likely to be selected, find the probability distribution of *X*.

Q21. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

(*i*) at least once (*ii*) exactly once (*iii*) at least twice?

Q22. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%.

Q23. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must the shooter fire so that the probability of hitting the target at least once is more than 0.99?

Q24. How many times must a man toss a fair coin, so that the probability of having atleast one head is more than 80%?

Q25. The probability of a man hitting a target is 1/2. How many times must he fire so that the probability of hitting the target at least once is more than 90%.

Q26. Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six.

Date: 19/10/2021

S1. Let X denote the number of hurdles knocked down by the player. Then, X follows binomial distribution with $n = 10$, $p = 1 - \frac{5}{6} = \frac{1}{6}$ and $q = \frac{5}{6}$.

$$\therefore \quad P(X = r) = {}^{10}C_r\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{10-r};\ r = 0, 1, 2, \ldots, 10$$

Required probability = $P(X < 2)$

$$= P(X = 0) + P(X = 1)$$

$$= \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^9$$

$$= \left(\frac{5}{6}\right)^9 \left\{\frac{5}{6} + \frac{10}{6}\right\} = \frac{5^{10}}{2 \times 6^9}$$

S2. Let X denote the number of defective bulbs in a sample of 4 bulbs drawn successively with replacement. Then, X follow binomial distribution with parameters $n = 4$, $p = \frac{6}{30} = \frac{1}{5}$ and $q = 1 - \frac{1}{5} = \frac{4}{5}$ such that

$$\text{P}(X = r) = {}^4C_r\left(\frac{1}{5}\right)^r\left(\frac{4}{5}\right)^{4-r};\quad r = 0, 1, 2, 3, 4.$$

The probability distribution of X is

X:	0	1	2	3	4
$P(X)$:	$\left(\frac{4}{5}\right)^4$	$\left(\frac{4}{5}\right)^4$	$6 \times \frac{1}{25} \times \frac{16}{25} = \frac{96}{625}$	$4 \times \frac{1}{125} \times \frac{4}{5} = \frac{16}{625}$	$\left(\frac{1}{5}\right)^4$

S3. Here,

$$n = 20, \quad p = \frac{1}{2}, \quad q = \frac{1}{2}$$

$$\therefore \quad P(X = r) = {}^{20}C_r\left(\frac{1}{2}\right)^{20}$$

Required probability = $P(X \geq 12) = \sum_{r=12}^{20} {}^{20}C_r\left(\frac{1}{2}\right)^{20}$

S4. Let X denote the number of defective eggs in a sample of 10 eggs drawn successively with replacement. Then, X follows binomial distribution with parameters n = 10, $p = \frac{10}{100} = \frac{1}{10}$ and $q = \frac{9}{10}$.

$\therefore$ $$P(X = r) = {}^{10}C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{10-r}, r = 0, 1, 2, ..., 10$$

Required probability = $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} = 1 - \frac{9^{10}}{10^{10}}$$

S5. Let p denotes the probability of getting an odd number in a single throw of the die. Then,

$$p = \frac{3}{6} = \frac{1}{2} \quad \text{and} \quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Let X denote the number of successes in 6 trials. Then, X is a binomial variate with parameter $n = 6$, $p = \frac{1}{2}$.

The probability of r successes is given by

$$P(X = r) = {}^6C_r \left(\frac{1}{2}\right)^{6-r} \left(\frac{1}{2}\right)^r, \quad \text{where} \quad r = 0, 1, 2, ..., 6$$

or $$P(X = r) = {}^6C_r \left(\frac{1}{2}\right)^6, \quad \text{where} \quad r = 0, 1, 2, ..., 6 \quad ... \text{(i)}$$

(i) Probability of 5 successes = $P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$ [Using (i)]

(ii) Probability of at least 5 successes = $P(X \geq 5)$

$$= P(X = 5) + P(X = 6)$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^6 + {}^6C_6 \left(\frac{1}{2}\right)^6 \quad \text{[Using (i)]}$$

$$= (6 + 1)\frac{1}{64} = \frac{7}{64}$$

(iii) Probability of at most 5 successes = $P(X \leq 5) = 1 - P(X > 5)$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 \quad \text{[Using (i)]}$$

$$= 1 - \frac{1}{64} = \frac{63}{64}$$

(iv) Probability of at least one success = $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^6C_0 \left(\frac{1}{2}\right)^6 \quad \text{[Using (i)]}$$

$$= 1 - \frac{1}{64} = \frac{63}{64}$$

(v) Probability of no success = $P(X = 0) = {}^6C_0 \left(\frac{1}{2}\right)^6$ [Using (i)]

$$= \frac{1}{64}.$$

S6. Let X denote the number of successes in 5 throws of a die and let p be the probability of getting an even number in a single throw of a die. Then,

$$p = \frac{3}{6} = \frac{1}{2} \quad \text{and} \quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

The probability of r successes in five throws of die is given by

$$P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} = {}^5C_r \left(\frac{1}{2}\right)^5, \quad \text{where } r = 0, 1, 2, ..., 5$$

$\therefore$ Required probability = $P(X \leq 3) = 1 - P(X > 3)$

$\Rightarrow$ Required probability = $1 - \{P(X = 4) + P(X = 5)\}$

$\Rightarrow$ Required probability = $1 - \left\{ {}^5C_4 \left(\frac{1}{2}\right)^5 + {}^5C_5 \left(\frac{1}{2}\right)^5 \right\}$

$\Rightarrow$ Required probability = $1 - \left(\frac{5}{32} + \frac{1}{32}\right) = \frac{26}{32} = \frac{13}{16}$

S7. Now, we know that the probability of getting spade card = $\frac{13}{52} = \frac{1}{4}$

i.e., $p = \frac{1}{4}$

and probability of not getting a spade card

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

i.e., $q = \frac{3}{4}$

We know that, probability of X success by using the binomial distribution is given by

$$P(X) = {}^nC_x \, p^x q^{n-x}$$

Using above formula, we calculate

$$P(X = 0), \quad P(X = 1), \quad P(X = 2) \quad \text{and} \quad P(X = 3)$$

We have, $n = 3$, $p = \frac{1}{4}$, $q = \frac{3}{4}$

$\therefore$ $$P(X = 0) = {}^3C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$P(X = 1) = {}^3C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^2 = 3 \times \frac{1}{4} \times \frac{9}{16} = \frac{27}{64}$$

$$P(X = 2) = {}^3C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right) = 3 \times \frac{1}{16} \times \frac{3}{4} = \frac{9}{64}$$

$$P(X = 3) = {}^3C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0 = 1 \times \frac{1}{64} \times 1 = \frac{1}{64}$$

$\therefore$ Required probability distribution is

X	0	1	2	3
$P(X)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

S8. We know, that the probability of getting doublet $= \frac{6}{36} = \frac{1}{6}$

[$\because$ Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]

So, $$p = \frac{6}{36} = \frac{1}{6}$$

i.e., $$p = \frac{1}{6}$$

Probability of not getting a doublet $= 1 - \frac{1}{6} = \frac{5}{6}$

i.e., $$q = \frac{5}{6}$$

Here, we have $$n = 4, \quad p = \frac{1}{6}, \quad q = \frac{5}{6}$$

Let X denotes the number of doublets, X can take values 0, 1, 2, 3 and 4, we know that by binomial distribution : $P(X) = {}^nC_x \, p^x \, q^{n-x}$.

Using above formula, we find $P(X = 0)$, $P(X = 1)$, $P(X = 2)$, $P(X = 3)$ and $P(X = 4)$.

$\therefore \qquad P(X=0) = {}^4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4 = 1\times1\times\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$

$P(X=1) = {}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 = 4\times\frac{1}{6}\times\frac{125}{216} = \frac{500}{1296}$

$P(X=2) = {}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{4!}{2!\,2!}\times\frac{1}{36}\times\frac{25}{36} = \frac{6\times25}{36\times36} = \frac{150}{1296}$

$P(X=3) = {}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 = 4\times\frac{1}{216}\times\frac{5}{6} = \frac{20}{1296}$

$P(X=4) = {}^4C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = 1\times\left(\frac{1}{6}\right)^4\times1 = \frac{1}{1296}$

$\therefore$ The required probability distribution is

X	0	1	2	3	4
***P*(*X*)**	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

or

X	0	1	2	3	4
***P*(*X*)**	$\frac{625}{1296}$	$\frac{125}{324}$	$\frac{75}{648}$	$\frac{5}{324}$	$\frac{1}{1296}$

S9. Let p denotes the probability of getting a total of 7 in a single throw of a pair of dice Then,

$p = \frac{6}{36} = \frac{1}{6}$ [$\because$ The sum can be 7 in any one of the ways: (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)]

$\therefore \qquad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Let X denote the number of successes in 7 throws of a pair of dice. The X is a binomial variate with parameters $n = 7$ and $p = 1/6$ such that

Now $\qquad P(X = r) = {}^7C_r\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{7-r}, \quad r = 0, 1, 2, ..., 7 \qquad$ ...(i)

(i) Probability of no success $= P(X=0) = {}^7C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{7-0} = \left(\frac{5}{6}\right)^7$ [Using (i)]

(ii) Probability of 6 successes $= P(X = 6) = {}^7C_6 \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right)^{7-6}$ [Using (i)]

$= 35\left(\frac{1}{6}\right)^7$

(iii) Probability of at least 6 successes $= P(X \geq 6)$

$= P(X = 6) + P(X = 7)$

$= {}^7C_6 \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right)^{7-6} + {}^7C_7 \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right)^0$ [Using (i)]

$= 7 \cdot \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^7 = \left(\frac{1}{6}\right)^6 \left(\frac{35}{6} + \frac{1}{6}\right) = \left(\frac{1}{6}\right)^5$

(iv) Probability of at most 6 successes $= P(X \leq 6)$

$= 1 - P(X > 6)$

$= 1 - P(X = 7)$

$= 1 - {}^7C_7 \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right)^0$

$= 1 - \left(\frac{1}{6}\right)^7$

S10. Let p denote the probability that the man takes a step forward. Then, $p = 0.4$

$\therefore \quad q = 1 - p = 1 - 0.4 = 0.6$

Let X denote the number of steps taken in the forward direction. Since the steps are independent of each other, therefore X is a binomial variate with parameters $n = 11$ and $p = 0.4$ such that

$$P(X = r) = {}^{11}C_r (0.4)^r (0.6)^{11-r};\ r = 0, 1, 2, \ldots, 11 \quad \ldots \text{(i)}$$

Since the man is one step away from the initial point, he is either one step forward or one step backward from the initial point at the end of eleven steps. If he is one step forward, then he must have taken six steps forward and five steps backward and if he is one step backward, then he must have taken five steps forward and six steps backward. Thus, either $X = 6$ or $X = 5$.

$\therefore$ Required probability $= P[(X = 5)$ or $(X = 6)]$

$\Rightarrow$ Required probability $= P(X = 5) + P(X = 6)$

$\Rightarrow$ Required probability $= {}^{11}C_5 (0.4)^5 (0.6)^{11-5} + {}^{11}C_6 (0.4)^6 (0.6)^{11-6}$ [Using (i)]

$\Rightarrow$ Required probability $= {}^{11}C_5 (0.4)^5 (0.6)^5 [0.6 + 0.4]$ [$\because {}^{11}C_5 = {}^{11}C_6$]

$\Rightarrow$ Required probability $= 462 (0.4)^5 (0.6)^5 = 462 (0.24)^5$

S11. Let p denote the probability of drawing a white ball from an urn containing 5 white, 7 red and 8 black balls. Then,

$$p = \frac{{}^5C_1}{{}^{20}C_1} = \frac{5}{20} = \frac{1}{4}$$

So, $$q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

Let X denote the number of white balls in 4 draws with replacement. Then, X is a binomial variate with parameter $n = 4$ and $p = \frac{1}{4}$ such that

$$P(X = r) = \text{Probability that } r \text{ balls are white} = {}^4C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{4-r};$$

$$r = 0, 1, 2, 3, 4 \qquad \text{... (i)}$$

Now,

(i) Probability that all are white $= P(X = 4)$

$$= {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{4-4} = \left(\frac{1}{4}\right)^4 \qquad \text{[Using (i)]}$$

(ii) Probability that only 3 are white $= P(X = 3) = {}^4C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{4-3}$ [Using (i)]

$$= 3\left(\frac{1}{4}\right)^3$$

(iii) Probability that none is white $= P(X = 0) = {}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4$ [Using (i)]

$$= \left(\frac{3}{4}\right)^4$$

(iv) Probability that at least three are white

$$= P(X \geq 3)$$
$$= P(X = 3) + P(X = 4)$$
$$= {}^4C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{4-3} + {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^0 \qquad \text{[Using (i)]}$$
$$= 13\left(\frac{1}{4}\right)^4$$

S12. Let p denote the probability of getting head in a single toss of a coin. Then,

$$p = \frac{1}{2} \quad \text{and} \quad \text{so, } q = \frac{1}{2}$$

Let X denote the number of heads in 5 tosses of a coin. Then, X is a binomial variate with parameters $n = 5$, $p = \frac{1}{2}$ such that

$$P(X = r) = {}^5C_r\left(\frac{1}{2}\right)^{5-r}\left(\frac{1}{2}\right)^{r} = {}^5C_r\left(\frac{1}{2}\right)^{5}, \quad \text{where } r = 0, 1, 2, ..., 5 \qquad ... (i)$$

Now,

Probability of at least 3 heads

$$= P(X \geq 3)$$
$$= P(X = 3) + P(X = 4) + P(X = 5)$$
$$= {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$
$$= \{{}^5C_3 + {}^5C_4 + {}^5C_5\}\left(\frac{1}{2}\right)^5 \qquad \text{[Using } (i)\text{]}$$
$$= (10 + 5 + 1) \times \frac{1}{32} = \frac{1}{2}$$

S13. We know that, probability of getting correct answer on a multiple choice exam with three possible answers = $\frac{1}{3}$

$\therefore$ $$p = \frac{1}{3}$$

$$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Also, total number of question = 5

$\therefore$ $n = 5$

So, we get $n = 5$, $p = \frac{1}{3}$, $q = \frac{2}{3}$

$\therefore$ Required probability = P (getting four or more correct answers)

$= P(4) + P(5)$

By binomial distribution, we know that

$$P(x) = {}^nC_x p^x q^{n-x}$$

$\therefore$ $$P(4) = {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1$$

$$= 5 \times \frac{1}{81} \times \frac{2}{3} = \frac{10}{243}$$

and $$P(5) = {}^5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0$$

$$= 1 \times \frac{1}{243} \times 1 = \frac{1}{243}$$

$\therefore$ Required probability = $P(4) + P(5)$

$$= \frac{10}{243} + \frac{1}{243} = \frac{11}{243}$$

S14. First, we find the probability of 2 sixes in first five throws by using binomial distribution for this, we have $n = 5$, $p = \frac{1}{6}$, $q = 1 - \frac{1}{6} = \frac{5}{6}$ and $x = 2$.

We know that by binomial distribution, we have

$$P(x) = {}^nC_x p^x q^{n-x}$$

$\therefore$ $$P(2) = {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

$$= \frac{5!}{2!3!} \times \frac{1}{36} \times \frac{125}{216}$$

$$= \frac{5 \times 4}{2} \times \frac{1}{36} \times \frac{125}{216}$$

$$= \frac{10 \times 125}{36 \times 216} = \frac{625}{3888}$$

Now, P (obtaining third six in the sixth throw)

= P (2 sixes in first five throws) × P (third six in sixth throw)

$$= \frac{625}{3888} \times \frac{1}{6} \qquad \left[\because P \text{ (getting six on a die)} = \frac{1}{6}\right]$$

$\therefore$ Required probability = $\frac{625}{23328}$

S15. We have,

p = probability that a person is right-handed = $\frac{90}{100} = \frac{9}{10}$

$\therefore$ $q = 1 - p = \frac{1}{10}$

Let X denote the number of right-handed persons in a sample of 10 persons. Then,

X follows binomial distribution with parameters $n = 10$, $p = \frac{9}{10}$ and $q = \frac{1}{10}$.

$$\therefore \qquad P(X = r) = {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}, \; r = 0, 1, 2, \ldots, 10$$

Required probability = $P(X \le 6)$

$$= 1 - P(X > 6)$$

$$= 1 - [P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)]$$

$$= 1 - \sum_{r=7}^{10} P(X = r)$$

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

S16. Let p denote the probability of drawing a ball which bears mark 'A'. Then

$$p = \frac{10}{25} = \frac{2}{5}$$

Let X denote the number of balls which bear mark 'A' in 6 draws. Then, X is binomial variate with parameters $n = 6$ and $p = \frac{2}{5}$.

Also $\qquad q = 1 - p = 1 - \frac{2}{5} = \frac{3}{5}$

Now $\qquad P(X = r)$ = Probability of getting r balls bearing mark 'A'

$$= {}^6C_r \left(\frac{2}{5}\right)^r \left(\frac{3}{5}\right)^{6-r}, \quad r = 0, 1, 2, \ldots, 6 \qquad \ldots \text{(i)}$$

(i) Probability that all balls bear 'A' mark = $P(X = 6)$

$$= {}^6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^{6-6} \qquad \text{[Using (i)]}$$

$$= \left(\frac{2}{5}\right)^6$$

(ii) Not more than 2 balls will bear 'B' mark means that there can be either no ball or one ball or two balls of 'B' mark. This implies that there can be either 6 or 5 or 4 balls of 'A' mark.

$\therefore$ Required probability

$$= P(X \ge 4)$$

$$= P(X = 4) + P(X = 5) + P(X = 6)$$

$$= {}^6C_4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + {}^6C_5\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right) + {}^6C_6\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0$$

$$= 7\left(\frac{2}{5}\right)^4$$

ALITER: Let p denote the probability that a ball drawn mark '*B*'. Then,

$$p = \frac{15}{25} = \frac{3}{5}$$

Let Y denote the number of balls which bear mark '*B*' in 6 draws. Then, Y is a binomial variate with parameters $n = 6$ and $p = 3/5$ such that

$$P(Y = r) = {}^6C_r\left(\frac{3}{5}\right)^r\left(\frac{2}{5}\right)^{6-r}, \quad r = 0, 1, 2, ..., 6 \qquad ...\text{(ii)}$$

Required probability

$$= P(Y \le 2)$$

$$= P(Y = 0) + P(Y = 1) + P(Y = 2)$$

$$= {}^6C_0\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^6 + {}^6C_1\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^5 + {}^6C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^4 \qquad \text{[Using (ii)]}$$

$$= 7\left(\frac{2}{5}\right)^4$$

(iii) Required probability

$$= P(X = 3) = {}^6C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^{6-3} \qquad \text{[Using (i)]}$$

$$= 20\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^3$$

(iv) Probability that at least one ball will bear '*B*' mark

$$= P(Y \ge 1)$$

$$= 1 - P(Y = 0) = 1 - {}^6C_0\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^6 \qquad \text{[Using (ii)]}$$

$$= 1 - \left(\frac{2}{5}\right)^6$$

S17. Suppose the man fires n times and let X denotes the number of times he hits the target. Then,

$$P(X = r) = {}^nC_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{n-r}, \quad r = 0, 1, 2, ..., n$$

Now, $$P(X \geq 1) > \frac{2}{3}$$

$\Rightarrow$ $$1 - P(X = 0) > \frac{2}{3}$$

$\Rightarrow$ $$1 - {}^nC_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n > \frac{2}{3}$$

$\Rightarrow$ $$1 - \left(\frac{3}{4}\right)^n > \frac{2}{3} \quad \Rightarrow \quad \left(\frac{3}{4}\right)^n < \frac{1}{3}$$

Clearly, $$\frac{3}{4} \nless \frac{1}{3'}\left(\frac{3}{4}\right)^2 \nless \frac{1}{3'}\left(\frac{3}{4}\right)^3 \nless \frac{1}{3}, \quad \text{but} \quad \left(\frac{3}{4}\right)^4 = \frac{81}{256} < \frac{1}{3}$$

$\therefore$ $$\left(\frac{3}{4}\right)^n < \frac{1}{3} \quad \Rightarrow \quad n = 4, 5, 6, \ldots$$

Hence, the man must fire at least 4 times.

S18. Let n dice be thrown, and let X denote the number of sixes. Then,

$$P(X = r) = {}^nC_r\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^r, \; r = 0, 1, 2, \ldots, n,$$

We have to find the smallest value of n for which $P(X = 0)$ is less than $\frac{1}{2}$.

Now, $$P(X = 0) < \frac{1}{2} \quad \Rightarrow \quad \left(\frac{5}{6}\right)^n < \frac{1}{2}$$

Clearly, $$\frac{5}{6} \nless \frac{1}{2'}\left(\frac{5}{6}\right)^2 \nless \frac{1}{2'}\left(\frac{5}{6}\right)^3 \nless \frac{1}{2'} \text{ but } \left(\frac{5}{6}\right)^4 = \frac{625}{1296} < \frac{1}{2}$$

$\therefore$ $$P(X = 0) < \frac{1}{2} \quad \Rightarrow \quad \left(\frac{5}{6}\right)^n < \frac{1}{2} \quad \Rightarrow \quad n = 4, 5, \ldots$$

Thus, at least 4 dice must be thrown.

S19. Let X denote the number of heads obtained in three tosses of a coin, then X can take values 0, 1, 2, and 3.

Let p denote the probability of obtaining a head in a single toss of a coin. Then,

$$p = \frac{1}{2} \quad \text{and} \quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Since three trials are independent, therefore X is a binomial variate with parameters $n = 3$ and $p = 1/2$ such that

$$P(X = r) = {}^3C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{3-r} = {}^3C_r\left(\frac{1}{2}\right)^3, r = 0, 1, 2, 3.$$

X:	0	1	2	3
$P(X)$:	${}^3C_0\left(\frac{1}{2}\right)^3$	${}^3C_1\left(\frac{1}{2}\right)^3$	${}^3C_2\left(\frac{1}{2}\right)^3$	${}^3C_3\left(\frac{1}{2}\right)^3$

or,

X:	0	1	2	3
$P(X)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

S20. Let p denote the probability of success in a draw. Then,

$$p = \text{Probability of getting a red ball in a draw} = \frac{3}{7}$$

$$\therefore \qquad q = 1 - p \quad \Rightarrow \quad q = 1 - \frac{3}{7} = \frac{4}{7}$$

Since the ball drawn in each draw is replaced in the bag, therefore three trials are independent. Consequently X, the number of successes, can take values 0, 1, 2 and 3 and is a binomial variate with parameters $n = 3$, $p = 3/7$ such that

$$P(X = r) = \text{Probability of } r \text{ successes} = {}^3C_r\left(\frac{3}{7}\right)^r\left(\frac{4}{7}\right)^{3-r}, \; r = 0, 1, 2, 3.$$

Thus, the probability distribution of X is given by

X:	0	1	2	3
$P(X)$:	${}^3C_0\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^3$	${}^3C_1\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^2$	${}^3C_2\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^1$	${}^3C_3\left(\frac{3}{7}\right)^3$

or,

X:	0	1	2	3
$P(X)$:	$\left(\frac{4}{7}\right)^3$	$\frac{9}{7}\left(\frac{4}{7}\right)^2$	$\frac{12}{7}\left(\frac{3}{7}\right)^2$	$\left(\frac{3}{7}\right)^3$

S21. Here,

$$p = \frac{1}{100}, \quad n = 50, \quad q = \frac{99}{100}$$

$$\therefore \qquad P(X = r) = {}^{50}C_r\left(\frac{1}{100}\right)^r\left(\frac{99}{100}\right)^{50-r}$$

(i) Required probability = $P(X \geq 1) = 1 - P(X = 0) = 1 - \left(\frac{99}{100}\right)^{50}$

(ii) Required probability = $P(X = 1) = {}^{50}C_1 \times \frac{1}{100} \times \left(\frac{99}{100}\right)^{49}$

(iii) Required probability = $P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$

$$= 1 - \left(\frac{99}{100}\right)^{50} - {}^{50}C_1 \times \frac{1}{100}\left(\frac{99}{100}\right)^{49}$$

S22. Suppose the man tosses a fair coin n times and X denote the number of heads in n tosses. Then,

$$P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^n \qquad \left[\because p = q = \frac{1}{2}\right]$$

It is given that

$P(X \geq 1) > 0.9$

$\Rightarrow \quad 1 - P(X = 0) > 0.9$

$\Rightarrow \quad 1 - {}^nC_0 \left(\frac{1}{2}\right)^{\boldsymbol{n}} > 0.9 \Rightarrow \left(\frac{1}{2}\right)^{\boldsymbol{n}} < \frac{1}{10} \Rightarrow 2^{\boldsymbol{n}} > 10 \Rightarrow \boldsymbol{n} = 4, 5, 6, \ldots.$

Hence, the man must toss the coin at least 4 times.

S23. Let the shooter fire n times and let X denotes the number of times the shooter hits the target. Then, X follows binomial distribution with $p = \frac{3}{4}$ and $q = \frac{1}{4}$ such that

$$P(X = r) = {}^nC_r \frac{3^r}{4^n}$$

It is given that

$P(X \geq 1) > 0.99$

$\Rightarrow \quad 1 - P(X = 0) > 0.99$

$\Rightarrow \quad 1 - \frac{1}{4^n} > 0.99 \Rightarrow \frac{1}{4^n} < 0.01 \Rightarrow 4^n > \frac{1}{0.01} \Rightarrow 4^n > 100$

The least value of n satisfying this inequality is 4. Hence, the shooter must fire at least 4 times.

S24. Let man tosses the coin n times. Now, given that P(having atleast on head) > 80%.

i.e., $\quad P(X \geq 1) > \frac{80}{100}$

where, X is the number of heads.

$\Rightarrow \quad 1 - P(X = 0) > \frac{80}{100}$

$\Rightarrow \quad 1 - {}^nC_0 p^0 q^n > \frac{80}{100} \qquad$ [Using $P(x) = {}^nC_x p^x q^{n-x}$]

$\Rightarrow \quad 1 - {}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n > \frac{8}{10}$ $\quad \left[p = \text{probability of getting a head once} = \frac{1}{2} \text{ and } q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}\right]$

$\Rightarrow \quad 1 - \frac{1}{2^n} > \frac{4}{5}$

$\Rightarrow \quad \frac{1}{2^n} < 1 - \frac{4}{5}$

$\Rightarrow \quad \frac{1}{2^n} < \frac{1}{5}$

$\Rightarrow \quad 2^n > 5$ ...(i)

Inequality (i) is satisfied for $n \geq 3$.

Hence, coin must be tossed 3 or more times.

S25. Suppose he fires n times. Let X denote the number of times he hits the target in n trials. Then,

$$P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n, \ r = 0, 1, 2, ..., n$$

Now, $\quad P(X \geq 1) > \frac{90}{100}$

$\Rightarrow \quad 1 - P(X = 0) > \frac{90}{100}$

$\Rightarrow \quad P(X = 0) < 1 - \frac{90}{100}$

$\Rightarrow \quad {}^nC_0 \left(\frac{1}{2}\right)^n < \frac{1}{10} \quad \Rightarrow \quad {}^nC_0 \left(\frac{1}{2}\right)^n < \frac{1}{10}$

Clearly, $\quad \frac{1}{2} \not< \frac{1}{10}, \left(\frac{1}{2}\right)^2 \not< \frac{1}{10}, \left(\frac{1}{2}\right)^3 \not< \frac{1}{10}$, but $\left(\frac{1}{2}\right)^4 < \frac{1}{10}$

$\therefore \quad \left(\frac{1}{2}\right)^n < \frac{1}{10} \Rightarrow \quad n = 4, 5, 6, ...$

Thus, he must fire at least 4 times.

S26. Let X denote the number of dice showing five or six in a set of six dice. Then, X follows binomial distribution with $n = 6$, p = probability of getting 5 or 6 in a single throw of a die = $\frac{2}{6} = \frac{1}{3}$ and $q = \frac{2}{3}$.

$\therefore \quad P(X = r) = {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}, \ r = 0, 1, 2, 3, ..., 6.$

$\Rightarrow \quad P(X \geq 3) = 1 - P(X < 3)$

$\Rightarrow \quad P(X \geq 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

$\Rightarrow \quad P(X \geq 3) = 1 - \left[{}^6C_0 \left(\frac{2}{3}\right)^6 + {}^6C_1 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^5 + {}^6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 \right]$

$\Rightarrow \quad P(X \geq 3) = 1 - \left(\frac{2}{3}\right)^6 \left[1 + 3 + \frac{15}{4}\right] = 1 - \left(\frac{2}{3}\right)^6 \left(\frac{31}{4}\right)$

$\Rightarrow \quad P(X \geq 3) = 1 - \frac{64 \times 31}{729 \times 4} = 1 - \frac{496}{729} = \frac{233}{729}$

Thus, the frequency that at least three dice show five or six when six dice are thrown 729 times.

$$= 729 \,.\, P(X \geq 3) = 729 \times \frac{233}{729} = 233$$

Date: 19/10/2021

Q1. A die is thrown 20 times. Getting a number greater than 4 is considered a success. Find the mean and variance of the number of successes.

Q2. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quite as and when he gets a six. Find the expected value of the amount he wins/loses.

Q3. If two dice are rolled 12 times, obtain the mean and the variance of the distribution of successes, if getting a total greater than 4 is considered a success.

Q4. The sum of mean and variance of a binomial distribution is 15 and the sum of their squares is 117. Determine the distribution.

Q5. The sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution.

Q6. If the sum of the mean and variance of a binomial distribution for 5 trials is 1.8, find the distribution.

Q7. Find the binomial distribution for which the mean is 4 and variance is equal to 3.

Q8. If X follows binomial distribution with mean 4 and variance 2, find $P(|X - 4| \leq 2)$.

Q9. The mean and variance of a binomial distribution are 4 and 4/3 respectively, find $P(X \geq 1)$.

Q10. A perfect cubic die is thrown a large number of times of sets of 8. The occurrence of 5 or 6 is called a success. In what proportion of the sets would you expect 3 successes.

Date: 19/10/2021

S1. The distribution of the 'number of successes' is a binomial distribution with $n = 20$ and,

$$p = \text{Probability of getting a number greater than } 4 = \frac{2}{6} = \frac{1}{3}$$

$\therefore$ $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Now Mean = np and Variance = npq

$\Rightarrow$ Mean = $20 \times \frac{1}{3} = 6.66$ and Variance = $20 \times \frac{1}{3} \times \frac{2}{3} = 4.44$

Hence, Mean = 6.66 and Variance = 4.44.

S2. Let n denote the number of throws required to get a six and X denote the amount won/lost.

The man may get a six in the very first throw of the die or in 2nd throw or in the third throw (as he has decided to throw a die at most thrice).

Thus, we have the following probability distribution for X.

Number of throws (n) :	1	2	3	3
Amount won/lost (X) :	1	0	– 1	– 3
Probability ($P(X)$) :	$\frac{1}{6}$	$\frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$\therefore$ $E(X) = 1 \times \frac{1}{6} + 0 \times \frac{5}{36} + (-1) \times \frac{25}{216} + (-3)\frac{125}{216} = -\frac{364}{216}$

S3. Let X be a binomial variate with parameters n and p. Then,
We have,

p = Probability of getting a total greater than 4 in a single throw of a pair of dice.

$\Rightarrow$ $p = 1 - \frac{6}{36} = \frac{5}{6}$

$\Rightarrow$ $q = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$

$\therefore$ Mean = $np = \frac{5}{6} \times 12 = 10$

and, Variance = $npq = 12 \times \frac{5}{6} \times \frac{1}{6} = \frac{5}{3}$

S4. Let n and p be the parameters of the distribution. Then,

Mean = np and Variance = npq.

Now, $np + npq = 15$ [Given]

and, $n^2p^2 + n^2p^2q^2 = 117$ [Given]

$\Rightarrow$ $np(1 + q) = 15$ and $n^2p^2(1 + q^2) = 117$

$\Rightarrow$ $n^2p^2(1 + q)^2 = 225$ and $n^2p^2(1 + q^2) = 117$

$$\Rightarrow \quad \frac{n^2p^2(1+q)^2}{n^2p^2(1+q^2)} = \frac{225}{117} \quad \Rightarrow \quad \frac{(1+q)^2}{(1+q^2)} = \frac{225}{117}$$

$$\Rightarrow \quad \frac{1+q^2+2q}{1+q^2} = \frac{225}{117} \quad \Rightarrow \quad 1 + \frac{2q}{1+q^2} = \frac{225}{117}$$

$$\Rightarrow \quad \frac{2q}{1+q^2} = \frac{108}{117}$$

$$\Rightarrow \quad \frac{1+q^2}{2q} = \frac{13}{12}$$

$$\Rightarrow \quad \frac{1+q^2+2q}{1+q^2-2q} = \frac{13+12}{13-12}$$

$$\Rightarrow \quad \left(\frac{1+q}{1-q}\right)^2 = 25 \quad \Rightarrow \quad \frac{1+q}{1-q} = 5$$

$$\Rightarrow \quad 6q = 4 \quad \Rightarrow \quad q = \frac{2}{3} \quad \Rightarrow \quad p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$$

Putting $p = \frac{1}{3}$, $q = \frac{2}{3}$ in $np + npq = 15$, we get

$$\frac{n}{3} + \frac{2n}{9} = 15 \quad \Rightarrow \quad \frac{5n}{9} = 15 \quad \Rightarrow \quad n = 27$$

Thus, $n = 27$, $p = \frac{1}{3}$ and $q = \frac{2}{3}$

Hence, the distribution is given by

$$P(X = r) = {}^{27}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{27-r}, \; r = 0, 1, 2, \ldots, 27.$$

S5. Let n and p be the parameters of the given binomial distribution. Then,

Mean = np and Variance = npq, where $q = 1 - p$

Now, Mean + Variance = 24 and, Mean × Variance = 128

$\Rightarrow \quad np + npq = 24 \quad \text{and} \quad np \times npq = 128$

$\Rightarrow \quad np(1 + q) = 24 \quad \text{and} \quad n^2p^2 \times q = 128$

$\Rightarrow \quad np = \frac{24}{1+q} \quad \text{and} \quad n^2p^2 = \frac{128}{q}$

$\Rightarrow \quad \left(\frac{24}{1+q}\right)^2 = \frac{128}{q}$

$\Rightarrow \quad 576q = 128\ (1 + q)^2$

$\Rightarrow \quad 9q = 2(1 + 2q + q^2)$

$\Rightarrow \quad 2q^2 - 5q + 2 = 0$

$\Rightarrow \quad (2q - 1)\ (q - 2) = 0$

$\Rightarrow \quad q = \frac{1}{2} \qquad [\because \quad q \neq 2]$

$\therefore \quad p = 1 - q = \frac{1}{2}$

Putting $p = \frac{1}{2}$, $q = \frac{1}{2}$ in $np + npq = 24$, we get

$$\frac{n}{2} + \frac{n}{4} = 24 \quad \Rightarrow \quad n = 32$$

Let X be the binomial variate. Then, the probability distribution of X is given by

$$P(X = r) = {}^{32}C_r\left(\frac{1}{2}\right)^{32-r}\left(\frac{1}{2}\right)^r; \quad r = 0, 1, 2, ..., 32.$$

S6. Let n and p be the parameters of the distribution. Then,

Mean = np and Variance = npq

It is given that

$n = 5$ and, Mean + Variance = 1.8

$\Rightarrow \quad np + npq = 1.8$

$\Rightarrow \quad 5p + 5pq = 1.8$

$\Rightarrow \quad 5p + 5p\ (1 - p) = 1.8$

$\Rightarrow \quad 5p^2 - 10p + 1.8 = 0$

$\Rightarrow \quad p^2 - 2p + 0.36 = 0$

$\Rightarrow \quad (p - 0.2)\ (p - 1.8) = 0$

$\Rightarrow \quad p = 0.2 \qquad [\therefore \quad p \not> 1]$

Thus, we have

$$n = 5, \quad p = 0.2 \text{ and } q = 0.8$$

Therefore, if X denotes the binomial variate, then

$$P(X = r) = {}^5C_r (0.2)^r (0.8)^{5-r}, \ r = 0, 1, 2, 3, 4, 5.$$

This is the required binomial distribution.

S7. Let X be a binomial variate with parameters n and p. Then,

$$\text{Mean} = 4, \quad \text{Variance} = 3$$

$$\Rightarrow \quad np = 4 \quad \text{and} \quad npq = 3 \quad \Rightarrow \quad \frac{npq}{np} = \frac{3}{4} \quad \Rightarrow \quad q = \frac{3}{4}$$

$$\therefore \quad p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

Putting $p = \frac{1}{4}$ in $np = 4$, we get : $n = 16$

Then, we have

$$n = 16, \ p = \frac{1}{4} \quad \text{and} \quad q = \frac{3}{4}$$

Hence, the binomial distribution is given by

$$P(X = r) = {}^{16}C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{16-r}, \quad r = 0, 1, 2, ..., 16.$$

S8. Let n and p be the parameters of the binomial distribution. Then,

$$\text{Mean} = 4 \quad \text{and,} \quad \text{Variance} = 2$$

$$\Rightarrow \quad np = 4 \quad \text{and} \quad npq = 2$$

$$\Rightarrow \quad \frac{npq}{np} = \frac{2}{4} \quad \Rightarrow \quad q = \frac{1}{2} \quad \Rightarrow \quad p = \frac{1}{2}$$

Putting $p = \frac{1}{2}$ in $np = 4$, we get : $n = 8$.

Thus, X be a binomial variate with parameters $n = 8$ and $p = \frac{1}{2}$

$$\therefore \quad P(X = r) = {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} = {}^8C_r \left(\frac{1}{2}\right)^8, \quad r = 0, 1, 2, ..., 8.$$

Now

$$P(|X - 4| \le 2) = P(-2 \le X - 4 \le 2)$$

$$\Rightarrow \quad P(|X - 4| \le 2) = P(2 \le X \le 6)$$

$\Rightarrow \quad P(|X-4| \le 2) = P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$\Rightarrow \quad P(|X-4| \le 2) = {}^8C_2\left(\frac{1}{2}\right)^8 + {}^8C_3\left(\frac{1}{2}\right)^8 + {}^8C_4\left(\frac{1}{2}\right)^8 + {}^8C_5\left(\frac{1}{2}\right)^8 + {}^8C_6\left(\frac{1}{2}\right)^8$

$\Rightarrow \quad P(|X-4| \le 2) = ({}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6)\left(\frac{1}{2}\right)^8 = \frac{119}{128}$

S9. Let X be a binomial variate with parameters n and p. Then,

Mean = np and Variance = npq

$\Rightarrow \quad np = 4$ and $npq = \frac{4}{3}$ $\quad [\because$ Mean = 4, Var $(X) = \frac{4}{3}$ (Given)]

$\Rightarrow \quad \frac{npq}{np} = \frac{\frac{4}{3}}{4} \Rightarrow q = \frac{1}{3} \Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$ $\quad [\because p = 1 - q]$

Putting $p = \frac{2}{3}$ in $np = 4$, we get

$n \times \frac{2}{3} = 4 \Rightarrow n = 6.$

Thus, we have

$n = 6, \quad p = \frac{2}{3}$ and $q = \frac{1}{3}$

$\therefore \quad P(X = r) = {}^nC_r p^r q^{n-r} \Rightarrow P(X = r) = {}^6C_r\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{6-r}, \quad r = 0, 1, 2, ..., 6.$

Now, $\quad P(X \ge 1) = 1 - P(X < 1)$

$\Rightarrow \quad P(X \ge 1) = 1 - P(X = 0)$

$\Rightarrow \quad P(X \ge 1) = 1 - {}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6 = 1 - \left(\frac{1}{3}\right)^6 = 1 - \frac{1}{729} = \frac{728}{729}$

S10. Let there be n sets of 8 dice.

We have, p = Probability of getting 5 or a 6 with six faced die = $\frac{2}{6} = \frac{1}{3}$

$\therefore \quad q = 1 - \frac{1}{3} = \frac{2}{3}$

Let X denote the number of successes in one set of 8 dice. Then,

$$P(X = r) = {}^8C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{8-r}, \quad r = 0, 1, 2, ..., 8.$$

$\therefore$
$$P(X = 3) = {}^8C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^5 = \frac{1792}{6561}$$

Total number of sets in which we get 3 successes = $N \cdot P(X = 3) = \frac{1792}{6561}N$

So, percentage of 3 successes in 100 sets = $\frac{1792}{6561}N \times \frac{1}{N} \times 100 = 27.31\%$.