Date: 19/10/2021

Q1. Find a normal vector to the plane $2x - y + 2z = 5$. Also, find a unit vector normal to the plane.

Q2. Find the equation of plane passing through the point $\hat{i} + \hat{j} + \hat{k}$ and parallel to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 5$.

Q3. Write the equation of the plane whose intercepts on the coordinate axes are – 4, 2 and 3.

Q4. Reduce the equation of the plane $2x + 3y - 4z = 12$ to intercept form and find its intercepts on the coordinate axes.

Q5. Find the angle between the planes $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$.

Q6. Show that the planes $2x + 6y + 6z = 7$ and $3x + 4y - 5z = 8$ are at right angles.

Q7. Find the angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$.

Q8. If the line $\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\vec{r} \cdot (3\hat{i} - 2\hat{j} + m\hat{k}) = 14$, find the value of m.

Q9. Write the distance of following plane from origin, $2x - y + 2z + 1 = 0$.

Q10. Find the vector equation of a plane which is at a distance of 8 units from the origin and which is normal to the vector $2\hat{i} + \hat{j} + 2\hat{k}$.

Q11. Find the distance of the point (2, 1, 0) from the plane $2x + y + 2z + 5 = 0$.

Q12. Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$.

Q13. Find the distance between the parallel planes $x + y - z + 4 = 0$ and $x + y - z + 5 = 0$.

Q14. Find the angle between the normal to the planes $2x - y + z = 6$ and $x + y + 2z = 7$.

Q15. Find distance of the plane $3x - 4y + 12z = 3$ from the origin.

Q16. Write the intercept cut-off by plane $2x + y - z = 5$ on x-axis.

Q17. Reduce the equation $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 5$ to normal form and hence find the length of perpendicular from the origin to the plane.

Q18. Find the equation of the plane passing through the point (1, – 1, 2) having (2, 3, 2)as direction ratios of normal to the plane.

Q19. Prove that if a plane has the intercepts a, b, c along their respective axes and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.

Q20. If the points (1, 1, λ) and (–3, 0, 1) be equidistant from the plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$, find the value of λ.

Q21. If from a point $P(a, b, c)$ perpendiculars PA and PB are drawn to yz and zx-planes, find the vector equation of the plane OAB.

Q22. Find the equation of the plane passing through the point (– 1, 2, 1) and perpendicular to the line joining the points (– 3, 1, 2) and (2, 3, 4). Find also the perpendicular distance of the origin from this plane.

Q23. A vector $\vec{n}$ of magnitude 8 units is inclined to x-axis at 45°, y-axis at 60° and an acute angle with z-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to $\vec{n}$, find its equation in vector form.

Q24. Let $\vec{n}$ be a vector of magnitude $2\sqrt{3}$ such that it makes equal acute angles with the coordinate axes. Find the vector and Cartesian forms of the equation of a plane passing through (1, –1, 2) and normal to $\vec{n}$.

Q25. A variable plane which remains at a constant distance $3p$ from the origin cut the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

Q26. Find the equation of the plane passing through the intersection of the planes $4x - y + z = 10$ and $x + y - z = 4$ and parallel to the line with direction ratios proportional to 2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane.

Q27. State whether the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is parallel to the plane $\vec{r}\cdot\vec{n} = d$. Show that the line $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$ is parallel to the plane $\vec{r}\cdot(-2\hat{i} + \hat{k}) = 5$. Also, find the distance between the line and the plane.

Q28. Show that the line whose vector equation is $\vec{r} = (2\vec{i} - 2\vec{j} + 3\vec{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ is parallel to the plane $\vec{r}\cdot(\vec{i} + 5\vec{j} + \vec{k}) = 5$. Also find the distance between them.

Q29. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and $\vec{r}\cdot\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}\cdot\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

Q30. Find the coordinates of the point, where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ intersects the plane $x - y + z - 5 = 0$. Also, find the angle between the line and the plane.

Q31. Find the coordinates of point where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ meets the plane $x + y + 4z = 6$.

Q32. Find the equation of the plane passing through the points (1, 0, – 1), (3, 2, 2) and parallel to the line $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{3}$.

Q33. Find the equation of plane that contains the point (1, – 1, 2) and is perpendicular to each of planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

Q34. Find the plane passing through (4, – 1, 2) and parallel to the lines

$$\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2} \quad \text{and} \quad \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$$

Q35. Find the vector equation of the plane through the points (2, 1, – 1) and (– 1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$.

Q36. Prove that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ are coplanar. Also, find the plane containing these two lines.

Q37. Find the equation of the plane through the line of intersection of $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-\hat{j})+4=0$ and perpendicular to $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8=0$.

Q38. Show that the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ are coplanar. Also, find the plane containing these two lines.

Q39. Find the vector equation of the plane that contains the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$. Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane thus obtained.

Q40. Find the equation of plane(s) passing through the intersection of planes $x+3y+6=0$ and $3x-y-4z=0$ and whose perpendicular distance from origin is unity.

Q41. Find the equation of plane passing through the point A (1, 2, 1) and perpendicular to the line joining points P (1, 4, 2) and Q (2, 3, 5). Also, find distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$.

Q42. Find the Cartesian equation of the plane passing through points A (0, 0, 0) and B (3, –1, 2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

Q43. Find the equation of plane passing through the point (– 1, 3, 2) and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=5$.

Q44. Find the vector equation of plane passing through the points A (2, 2, – 1), B (3, 4, 2) and C (7, 0, 6). Also, find the Cartesian equation of plane.

Q45. Find the equation of plane passing through point (1, 1, – 1) and perpendicular to planes $x+2y+3z-7=0$ and $2x-3y+4z=0$.

Q46. Find the vector and Cartesian equation of a plane containing the two lines $\vec{r}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+5\hat{k}\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$. Also, show that the line $\vec{r}=\left(2\hat{i}+5\hat{j}+2\hat{k}\right)+\lambda_1\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ lies in the plane.

Q47. Find the equation of plane passing through the point (1, 2, 1) and perpendicular to line joining points (1, 4, 2) and (2, 3, 5). Also, find the coordinates of foot of the perpendicular and the perpendicular distance of the point (4, 0, 3) from above found plane.

Q48. Find the equation of plane passing through point P(1, 1, 1) and containing the line $\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(3\hat{i}-\hat{j}-5\hat{k})$. Also, show that plane contains the line $\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(\hat{i}-2\hat{j}-5\hat{k})$.

Q49. Find the equation of plane passing through the point (– 1, – 1, 2) and perpendicular to each planes $2x+3y-3z=2$ and $5x-4y+z=6$.

Q50. Find the equation of plane determined by the points A(3, – 1, 2), B(5, 2, 4) and C(– 1, – 1, 6). Also, find distance of point P(6, 5, 9) from plane.

Q51. Find the equation of plane passing through points (3, 4, 1) and (0, 1, 0) and parallel to line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$.

Q52. Find the coordinates of the point where the line through (3, –4, –5) and (2, –3, 1) crosses the plane, passing through the point (2, 2, 1), (3, 0, 1) and (4, – 1, 0).

Q53. Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r}\cdot\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$.

Q54. Find the equation of the plane through the points (2, 1, – 1) and (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$. Also, show that the plane thus obtained contains the line $\vec{r}=\hat{i}+3\hat{j}+4\hat{k}+\lambda(3\hat{i}-2\hat{j}-5\hat{k})$.

Q55. Find the equation of plane passing through the line of intersection of planes $r\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $r\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to x-axis.

Q56. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+3\hat{j}\right)-6=0$ and $\vec{r}\cdot\left(3\hat{i}-\hat{j}-4\hat{k}\right)=0$, whose perpendicular distance from origin is unity.

Q57. Find the equation of plane passing through the line of intersection of planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

Q58. Find the coordinates of image of point (1, 3, 4) in the plane $2x - y + z + 3 = 0$.

Q59. Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$.

Q60. Find the image of the point (3, – 2, 1) in the plane $3x - y + 4z = 2$.

Q61. Find the coordinates of the foot of perpendicular and the perpendicular distance of point $P(3, 2, 1)$ from the plane $2x - y + z + 1 = 0$. Find also image of the point in the plane.

Q62. Find the length and foot of perpendicular from point $P(7, 14, 5)$ to planes $2x + 4y - z = 2$. Also find the image of point P in the plane.

Q63. Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

Q64. Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Q65. From the point $P(1, 2, 4)$ a perpendicular is drawn on the plane $2x + y - 2z + 3 = 0$. Find the equation, the length and the coordinates of foot of perpendicular.

Q66. Find the distance of point (– 2, 3, – 4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

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S1. We know that the coefficient of x, y and z respectively in the Cartesian equation of a plane determine the direction ratios of a vector normal to the plane. Therefore, direction ratios of a vector $\vec{n}$ normal to the given plane are proportional to 2, – 1, 2.

$$\therefore \quad \vec{n} = 2\hat{i} - \hat{j} + 2\hat{k}$$

Thus, a unit vector normal to the plane is given by

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{4+1+4}} = \frac{1}{3}(2\hat{i} - \hat{j} + 2\hat{k})$$

S2. The equation of a plane parallel to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 5$ is

$$\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = d$$

Since it passes through $\hat{i} + \hat{j} + \hat{k}$. Therefore,

$$(\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = d \Rightarrow 2 - 1 + 2 = d \Rightarrow d = 3.$$

Hence the equation of the required plane is $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 3$

S3. We know that the equation of a plane whose intercepts on the coordinate axes are a, b and c respectively, is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Here, $a = -4$, $b = 2$, and $c = 3$. So, the equation of the required plane is

$$\frac{x}{-4} + \frac{y}{2} + \frac{z}{3} = 1 \Rightarrow -3x + 6y + 4z = 12$$

S4. The equation of the given plane is

$$2x + 3y - 4z = 12 \Rightarrow \frac{2x}{12} + \frac{3y}{12} - \frac{4z}{12} = 1 \Rightarrow \frac{x}{6} + \frac{y}{4} + \frac{z}{-3} = 1$$

This is of the form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. So, the intercepts made by the plane with the coordinate axes are 6, 4 and -3 respectively.

S5. We know that the angle between the planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is given by

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

Here, $\vec{n}_1 = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{n}_2 = \hat{i} + \hat{j} + 2\hat{k}$

$$\therefore \quad \cos\theta = \frac{(2\hat{i} - \hat{j} + \hat{k})\cdot(\hat{i} + \hat{j} + 2\hat{k})}{|2\hat{i} - \hat{j} + \hat{k}|\cdot|\hat{i} + \hat{j} + 2\hat{k}|} = \frac{2-1+2}{\sqrt{4+1+1}\sqrt{1+1+4}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

S6. We know that the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are at right angles, if

$$\cos\theta = a_1a_2 + b_1b_2 + c_1c_2 = 0$$

We have, (2) (3) + (6) (4) + (6) (– 5) = 6 + 24 – 30 = 0

Therefore, planes $2x + 6y + 6z = 7$ and $3x + 4y - 5z = 8$ are at right angles.

S7. We know that the angle θ between the line $\vec{r} = \vec{a} + \lambda\vec{b}$ and the plane $\vec{r}\cdot\vec{n} = d$ is given by

$$\sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}$$

Here, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{n} = 2\hat{i} - \hat{j} + \hat{k}$.

$$\therefore \quad \sin\theta = \frac{(\hat{i} - \hat{j} + \hat{k})\cdot(2\hat{i} - \hat{j} + \hat{k})}{\sqrt{1^2 + (-1)^2 + 1^2}\sqrt{2^2 + (-1)^2 + 1^2}} = \frac{2+1+1}{\sqrt{3}\sqrt{6}} = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

S8. The given line is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$ and the given plane is normal to the vector $\vec{n} = 3\hat{i} - 2\hat{j} + m\hat{k}$.

If the line is parallel to the plane, then normal to the plane is perpendicular to the line

$$\therefore \quad \vec{b} \perp \vec{n}$$

$$\Rightarrow \quad \vec{b}\cdot\vec{n} = 0$$

$$\Rightarrow \quad (2\hat{i} + \hat{j} + 2\hat{k})\cdot(3\hat{i} - 2\hat{j} + m\hat{k}) = 0$$

$$\Rightarrow \quad 6 - 2 + 2m = 0 \Rightarrow m = -2$$

S9. Given equation of plane is $2x - y + 2z + 1 = 0$ and the point is (0, 0, 0).

On comparing with $Ax + By + Cz + D = 0$

We have, $A = 2, \quad B = -1, \quad C = 2, \quad D = 1$

And point $x_1 = 0, \quad y_1 = 0, \quad z_1 = 0$

$$\therefore \quad d = \left|\frac{2(0) - 1(0) + 2(0) + 1}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}\right| \qquad \left[\because d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}\right]$$

$$\Rightarrow \quad d = \left|\frac{1}{\sqrt{4+1+4}}\right|$$

$$= \left|\frac{1}{\sqrt{9}}\right| = \frac{1}{3} \text{ unit}$$

S10. Here, $d = 8$ and $\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$.

$$\therefore \quad \hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{4+1+4}} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Hence, the required equation of the plane is

$$\vec{r} \cdot \left(\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right) = 8 \quad \Rightarrow \quad \vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 24.$$

S11. We know that the distance of the point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is given by

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

So, required distance = $\dfrac{|2 \times 2 + 1 + 2 \times 0 + 5|}{\sqrt{2^2 + 1^2 + 2^2}} = \dfrac{|4 + 1 + 0 + 5|}{\sqrt{4+1+4}} = \dfrac{10}{3}$.

S12. The distance from point (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$ is $\dfrac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}$.

Given, equation of plane is

$$2x - 3y + 6z + 21 = 0$$

$\therefore$ Length of the perpendicular drawn from the origin to this plane

$$= \left|\frac{2 \cdot 0 - 3 \cdot 0 + 6 \cdot 0 + 21}{\sqrt{(2)^2 + (-3)^2 + (6)^2}}\right|$$

$$= \left|\frac{0 - 0 + 0 + 21}{\sqrt{4 + 9 + 36}}\right| = \frac{21}{\sqrt{49}} = \frac{21}{7}$$

$$= 3 \text{ units.}$$

S13. Let $P(x_1, y_1, z_1)$ be any point on $x + y - z + 4 = 0$. Then,

$$x_1 + y_1 - z_1 + 4 = 0$$

The length of the perpendicular from $P(x_1, y_1, z_1)$ to $x + y - z + 5 = 0$ is

$$\left|\frac{x_1 + y_1 - z_1 + 5}{\sqrt{1^2 + 1^2 + (-1)^2}}\right| = \frac{|-4+5|}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Therefore, the distance between the two given parallel planes is $\frac{1}{\sqrt{3}}$.

S14. Let $\vec{n}_1$ and $\vec{n}_2$ be vectors normal to the planes $2x - y + z = 6$ and $x + y + 2z = 7$.

Direction ratios of normal to $2x - y + z = 6$ are proportional to 2, –1, 1

So, $\vec{n}_1 = 2\hat{i} - \hat{j} + \hat{k}$

Direction ratios of normal to $x + y + 2z = 7$ are proportional to 1, 1, 2

So $\vec{n}_2 = \hat{i} + \hat{j} + 2\hat{k}$

Let θ be the angle between the normal's $\vec{n}_1$ and $\vec{n}_2$. Then,

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$$\Rightarrow \quad \cos\theta = \frac{2\times1 + (-1)\times1 + 1\times2}{\sqrt{2^2 + (-1)^2 + 1^2}\sqrt{1^2 + 1^2 + 2^2}} = \frac{4-1}{\sqrt{4+1+1}\sqrt{1+1+4}} = \frac{3}{\sqrt{6}\sqrt{6}}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}.$$

S15. Given equation of plane is $3x - 4y + 12z - 3 = 0$ and the point is (0, 0, 0). We know that distance of the plane $Ax + By + Cz + D = 0$ to the point (x_1, y_1, z_1) is

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Here, $x_1 = y_1 = z_1 = 0$ and $A = 3, B = -4, C = 12, D = -3$.

$$\therefore \quad d = \frac{|3(0) - 4(0) + 12(0) - 3|}{\sqrt{(3)^2 + (-4)^2 + (12)^2}}$$

$$d = \frac{|-3|}{\sqrt{9+16+144}} = \frac{3}{\sqrt{169}} = \frac{3}{13} \text{ unit}$$

S16. Firstly we convert the given equation of plane in intercept form *i.e.*, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, which cut the x-axis at $(a, 0, 0)$.

Given equation of plane is

$$2x + y - z = 5$$

Divide both sides by 5, we get

$$\frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1 \text{ or } \frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{5} + \frac{z}{(-5)} = 1$$

Comparing above equation of plane with the intercept form of equation of plane.

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

where, a = x-intercept, b = y-intercept, c = z-intercept

We get, $a = \frac{5}{2}$

i.e., Intercept cut off on x-axis = $\frac{5}{2}$.

S17. The given equation is

$$\vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 5$$

$\Rightarrow$ $\vec{r} \cdot \vec{n} = 5,$ where $\vec{n} = 3\hat{i} - 4\hat{j} + 12\hat{k}$

Since $|\vec{n}| = \sqrt{3^2 + (-4)^2 + 12^2} = 13 \neq 1$. Therefore the given equation is not in normal form. To reduce it to normal form, we divide both sides by $|\vec{n}|$ i.e.,

$$\vec{r} \cdot \frac{\vec{n}}{|\vec{n}|} = \frac{5}{|\vec{n}|}$$

$$\Rightarrow \quad \vec{r} \cdot \left(\frac{3}{13}\hat{i} - \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k}\right) = \frac{5}{13}$$

This is the normal form of the equation of given plane. The length of the perpendicular from the origin is $\frac{5}{13}$.

S18. Here the plane passes through the point having position vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and is normal to the vector $\vec{n} = 2\hat{i} + 3\hat{j} + 2\hat{k}$. So, the vector equation of the plane is

$$\therefore \quad (\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow \quad \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \quad \vec{r} \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) = (\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 3\hat{j} + 2\hat{k})$$

$$\Rightarrow \quad \vec{r} \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) = 2 - 3 + 4$$

$\Rightarrow \qquad \vec{r}\cdot(2\hat{i}+3\hat{j}+2\hat{k}) = 3$

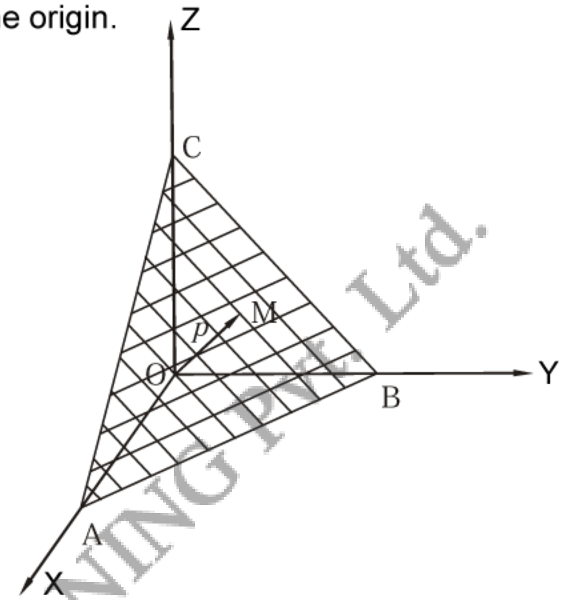
The Cartesian equation of the plane is

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+3\hat{j}+2\hat{k}) = 3$ [Putting $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$]

$\Rightarrow \qquad 2x + 3y + 2z = 3$

S19. The equation of the plane having intercepts *a*, *b* and *c* on the coordinate axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

It is given that this plane is at a distance of *p* units from the origin.



$$\therefore \qquad \left|\frac{\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = p$$

$$\Rightarrow \qquad \left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = p \Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = p^2$$

$$\Rightarrow \qquad \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}} = p^2 \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$$

Hence proved.

S20.

$$\therefore \qquad \left|\frac{(\hat{i}+\hat{j}+\lambda\hat{k})\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13}{\sqrt{9+16+144}}\right| = \left|\frac{(-3\hat{i}+0\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13}{\sqrt{9+16+144}}\right|$$

$$\Rightarrow \qquad \left|\frac{3+4-12\lambda+13}{13}\right| = \left|\frac{-9+0-12+13}{13}\right|$$

$$\Rightarrow \qquad |20-12\lambda| = 8$$

$$\Rightarrow \qquad 20-12\lambda = \pm 8$$

$$\Rightarrow \qquad 20-12\lambda = 8 \text{ or, } 20-12\lambda = -8$$

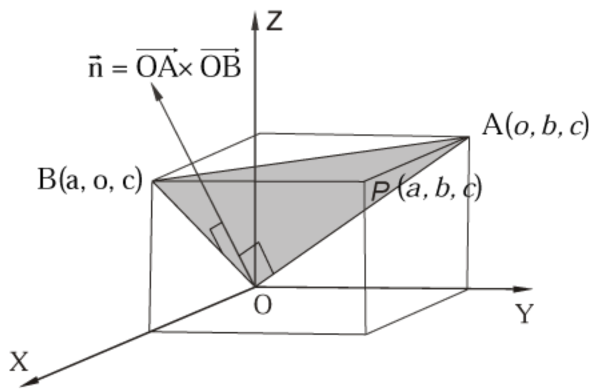
$\Rightarrow 12\lambda = 12$ or, $12\lambda = 28 \Rightarrow \lambda = 1$ or, $\lambda = \frac{7}{3}$.

S21. The coordinates *A* and *B* are (0, *b*, *c*) and (*a*, 0, *c*) respectively.

$\therefore \qquad \overrightarrow{OA} = b\hat{j}+c\hat{k}$ and $\overrightarrow{OB} = a\hat{i}+c\hat{k}$

The plane *OAB* passes through $O\,(\vec{0})$ and is perpendicular to $\vec{n}=\overrightarrow{OA}\times\overrightarrow{OB}$.

We have



$$\vec{n} = \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = bc\hat{i} + ac\hat{j} - ab\hat{k}$$

So, the equation of plane *OAB* is

$$(\vec{r} - \vec{0}) \cdot \vec{n} = 0$$

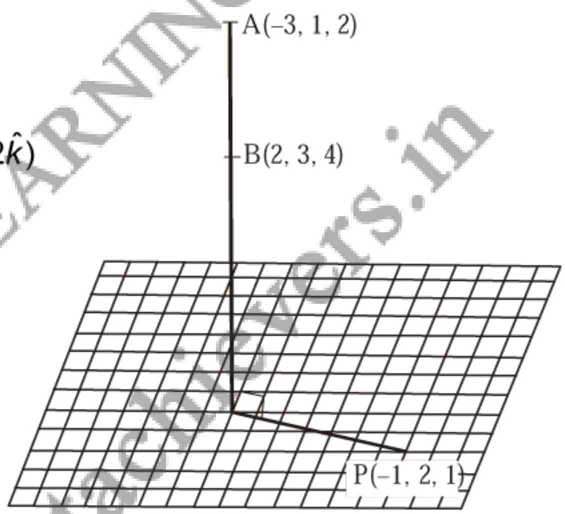
$$\Rightarrow \qquad \vec{r} \cdot \vec{n} = 0 \Rightarrow \vec{r} \cdot (bc\hat{i} + ca\hat{j} - ab\hat{k}) = 0.$$

S22. The required plane passes through the point (– 1, 2, 1) having position vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ and is perpendicular to the line joining the points *A* (– 3, 1, 2) and *B* (2, 3, 4). Therefore, a vector normal to the plane is given by

$$\vec{n} = \overrightarrow{AB} = (2\hat{i} + 3\hat{j} + 4\hat{k}) - (-3\hat{i} + \hat{j} + 2\hat{k}) = 5\hat{i} + 2\hat{j} + 2\hat{k}$$

We know that the vector equation of a plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0 \quad \Rightarrow \quad \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$



Therefore, the equation of the required plane is

$$\vec{r} \cdot (5\hat{i} + 2\hat{j} + 2\hat{k}) = (-\hat{i} + 2\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\Rightarrow \qquad \vec{r} \cdot (5\hat{i} + 2\hat{j} + 2\hat{k}) = -5 + 4 + 2$$

$$\Rightarrow \qquad \vec{r} \cdot (5\hat{i} + 2\hat{j} + 2\hat{k}) = 1.$$

To find the distance of this plane from the origin,

we reduce its equation to normal form.

We have, $|\vec{n}| = \sqrt{5^2 + 2^2 + 2^2} = \sqrt{33}$

S23. Let γ be the angle made by $\vec{n}$ with *z*-axis. The direction cosines of $\vec{n}$ are

$$l = \cos 45^\circ = \frac{1}{\sqrt{2}}, m = \cos 60^\circ = \frac{1}{2} \text{ and } n = \cos \gamma$$

$$\therefore \qquad l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \qquad \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$$

$\Rightarrow \qquad n^2 = \frac{1}{4} \Rightarrow n = \frac{1}{2} \qquad [\because \gamma \text{ is acute } \therefore n = \cos\gamma > 0]$

We have, $\quad |\vec{n}| = 8$

$\therefore \qquad \vec{n} = |\vec{n}|\,(l\hat{i} + m\hat{j} + n\hat{k})$

$$\Rightarrow \qquad \vec{n} = 8\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}\right) = 4\sqrt{2}\hat{i} + 4\hat{j} + 4\hat{k}$$

The required plane passes through the point $(\sqrt{2}, -1, 1)$ having position vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \hat{k}$.

So, its vector equation is

$$(\vec{r} - \vec{a})\cdot\vec{n} = 0$$

$$\Rightarrow \qquad \vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$$\Rightarrow \qquad \vec{r}\cdot(4\sqrt{2}\hat{i} + 4\hat{j} + 4\hat{k}) = (\sqrt{2}\hat{i} - \hat{j} + \hat{k})\cdot(4\sqrt{2}\hat{i} + 4\hat{j} + 4\hat{k})$$

$$\Rightarrow \qquad \vec{r}\cdot(4\sqrt{2}\hat{i} + 4\hat{j} + 4\hat{k}) = 8 - 4 + 4$$

$$\Rightarrow \qquad \vec{r}\cdot(4\sqrt{2}\hat{i} + 4\hat{j} + 4\hat{k}) = 8$$

$$\Rightarrow \qquad \vec{r}\cdot(\sqrt{2}\hat{i} + \hat{j} + \hat{k}) = 2.$$

S24. Let α, β and γ be the angles made by $\vec{n}$ with *ox*, *oy* and *oz* respectively. Then,

$$\alpha = \beta = \gamma \Rightarrow \cos\alpha = \cos\beta = \cos\gamma \Rightarrow l = m = n,$$

where l, m, n are direction cosines of $\vec{n}$.

But, $\quad l^2 + m^2 + n^2 = 1$

$\therefore \qquad l = m = n \Rightarrow 3l^2 = 1 \Rightarrow l = \frac{1}{\sqrt{3}} \qquad [\because \alpha \text{ is acute } \therefore \cos\alpha = l > 0]$

Thus, $\qquad \vec{n} = 2\sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) \qquad \left[\text{Using } \vec{r} = |\vec{r}|\,(l\hat{i} + m\hat{j} + n\hat{k})\right]$

$$\Rightarrow \qquad \vec{n} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

The required plane passes through a point (1, – 1, 2) having position vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and is normal to $\vec{n}$. So, its vector equation is

$$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$$\Rightarrow \qquad \vec{r}\cdot(2\hat{i} + 2\hat{j} + 2\hat{k}) = (\hat{i} - \hat{j} + 2\hat{k})\cdot(2\hat{i} + 2\hat{j} + 2\hat{k})$$

$\Rightarrow \quad \vec{r}\cdot(2\hat{i}+2\hat{j}+2\hat{k}) = 2 - 2 + 4$

$\Rightarrow \quad \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 2$

The Cartesian equation of this plane is

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = 2 \Rightarrow x + y + z = 2$

S25. Let the equation of the plane be

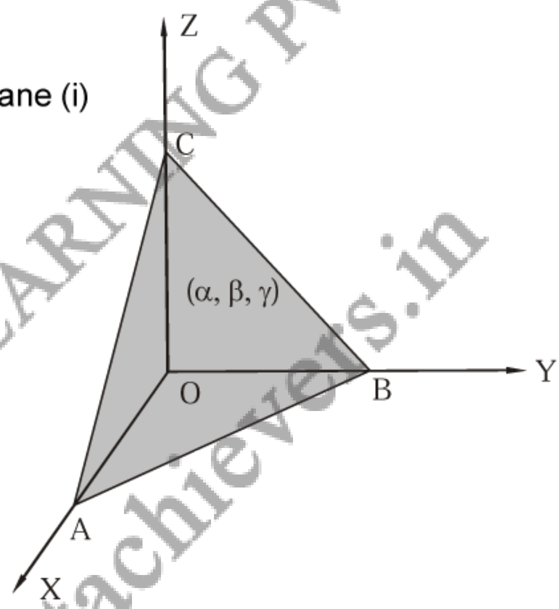
$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1 \qquad \text{... (i)}$$

where a, b, c are variables

This meets X, Y and Z axes at $A\,(a, 0, 0)$, $B\,(0, b, 0)$ and $C\,(0, 0, c)$

Let (α, β, γ) be the coordinates of the centroid of triangle ABC. Then,

$$\alpha = \frac{a+0+0}{3} = \frac{a}{3}, \beta = \frac{0+b+0}{3} = \frac{b}{3}, \gamma = \frac{0+0+c}{3} = \frac{c}{3}. \qquad \text{... (ii)}$$



The plane (i) is at a distance $3p$ from the origin.

$\therefore \quad 3p$ = Length of perpendicular from (0, 0, 0) to the plane (i)

$$\Rightarrow \quad 3p = \frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow \quad 3p = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} \Rightarrow \frac{1}{9p^2} = \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \qquad \text{... (iii)}$$

From (ii), we have

$$a = 3\alpha,\ b = 3\beta \text{ and } c = 3\gamma.$$

Substituting the values of a, b, c in (iii), we obtain

$$\frac{1}{9p^2} = \frac{1}{9\alpha^2}+\frac{1}{9\beta^2}+\frac{1}{9\gamma^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$$

So, the locus of (α, β, γ) is

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} \Rightarrow x^{-2}+y^{-2}+z^{-2} = p^{-2}.$$

S26. The equation of a plane passing through the intersection of the given planes is

$$(4x - y + z - 10) + \lambda\,(x + y - z - 4) = 0$$

$$\Rightarrow \quad x(4 + \lambda) + y(\lambda - 1) + z(1 - \lambda) - 10 - 4\lambda = 0 \qquad \text{... (i)}$$

This plane is parallel to the line with direction ratios proportional to 2, 1, 1.

$\therefore \quad 2(4 + \lambda) + 1(\lambda - 1) + (1 - \lambda) = 0 \quad \Rightarrow 2\lambda + 8 = 0 \quad \Rightarrow \lambda = -4$

Putting $\lambda = -4$ in (i), we obtain

$$-5y + 5z + 6 = 0 \qquad \text{... (ii)}$$

This is the equation of the required plane.

Now, Length of the perpendicular from (1, 1, 1) on (ii) is given by

$$d = \left|\frac{-5\times1+5\times1-6}{\sqrt{5^2+(-5)^2}}\right| = \frac{6}{5\sqrt{2}} = \frac{3\sqrt{2}}{5}.$$

S27. The line $\vec{r} = \vec{a} + \lambda\vec{b}$ is parallel to the plane $\vec{r}\cdot\vec{n} = d,$ if the normal to the plane is perpendicular to the line i.e.

$$\vec{b} \perp \vec{n} \;\Rightarrow \vec{b}\cdot\vec{n} = 0$$

Here, $\vec{a} = \hat{i} + \hat{j}, \vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{n} = -2\hat{i} + \hat{k}.$

We have, $\vec{b}\cdot\vec{n} = (2\hat{i} + \hat{j} + 4\hat{k})\cdot(-2\hat{i} + 0\hat{j} + \hat{k}) = -4 + 0 + 4 = 0$

$\Rightarrow \quad \vec{b} \perp \vec{n}$

$\Rightarrow$ Given line is parallel to the given plane.

Distance between the line and the plane

= Length of perpendicular from the point $\vec{a} = \vec{i} + \hat{j}$ to the plane $\vec{r}\cdot\vec{n} = d.$

$$= \frac{|(\vec{a}\cdot\vec{n} - d|}{|\vec{n}|} = \frac{|(\hat{i} + \hat{j})\cdot(-2\hat{i} + \hat{k}) - 5|}{\sqrt{(-2)^2 + (0)^2 + 1^2}} = \frac{|(-2 - 0 + 0) - 5|}{\sqrt{5}} = \frac{7}{\sqrt{5}}.$$

S28. The given line passes through the point having position vector $\vec{a} = 2\vec{i} - 2\vec{j} + 3\vec{k}$ and is parallel to the vector $\vec{b} = \vec{i} - \vec{j} + 4\vec{k}$.

We have, $\vec{b}\cdot\vec{n} = (\vec{i} - \hat{j} + 4\hat{k})\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 1 - 5 + 4 = 0$

So, $\vec{b}$ perpendicular to $\vec{n}.$

Hence, the given line is parallel to the given plane. The distance between the line and the parallel plane is the distance between any point on the line and the given plane. Since the line passes through the point $\vec{a} = 2\vec{i} - 2\hat{j} + 3\hat{k}.$ Therefore,

Distance between the line and the plane

= Length of perpendicular from $\vec{a} = 2\vec{i} - 2\hat{j} + 3\hat{k}$ to the given plane

$$= \frac{\left|(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5\right|}{\sqrt{1^2 + 5^2 + 1^2}} = \frac{|(2 - 10 + 3) - 5|}{\sqrt{27}} = \frac{10}{\sqrt{27}}.$$

S29. Let, the given equation of plane in Cartesian form are

$$\pi_1 = x + 2y + 3z - 4 = 0 \qquad (\because \ \vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0)$$

and $$\pi_2 = 2x + y - z + 5 = 0 \qquad (\because \ \vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0)$$

Equation of plane passing through intersection of the planes π_1 and π_2 is

$$(x + 2y + 3z - 4) + k(2x + y - z + 5) = 0$$

$$\Rightarrow \quad x(1 + 2k) + y(2 + k) + z(3 - k) - 4 + 5k = 0$$

and $$5x + 3y - 6z + 8 = 0 \qquad \left(\because \vec{r} \cdot \left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0\right)$$

If $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0 \perp$ to each other then their normal also $\perp$ to each other. Then,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\therefore \quad 5(1 + 2k) + 3(2 + k) - 6(3 - k) = 0$$

$$\Rightarrow \quad 5 + 10k + 6 + 3k - 18 + 6k = 0$$

$$\Rightarrow \quad 19k - 7 = 0$$

$$\Rightarrow \quad k = \frac{7}{19}$$

$\therefore$ Required plane will be

$$x\left(1 + \frac{14}{19}\right) + y\left(2 + \frac{7}{19}\right) + z\left(3 - \frac{7}{19}\right) - 4 + \frac{35}{19} = 0$$

$$\Rightarrow \quad x\left(\frac{33}{19}\right) + y\left(\frac{45}{19}\right) + z\left(\frac{50}{19}\right) - \frac{41}{19} = 0$$

$$\Rightarrow \quad 33x + 45y + 50z - 41 = 0.$$

S30. Given, equation of line is

$$\frac{x - 2}{3} = \frac{y + 1}{4} = \frac{z - 2}{2} = \lambda \quad \text{[Say]}$$

$$[\because \ x = 3\lambda + 2, \ y = 4\lambda - 1, \ z = 2\lambda + 2]$$

Then, $((3\lambda + 2), (4\lambda - 1), (2\lambda + 2))$ be any point on the given line.

This point lies on the plane $x - y + z - 5 = 0$

$$\therefore \quad (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) - 5 = 0$$

$$\Rightarrow \quad 3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 - 5 = 0$$

$$\lambda = 0$$

$\therefore$ Point of intersection = $(3 \times 0 + 2, 4 \times 0 - 1, 2 \times 0 + 2) = [2, -1, 2]$

Let θ be the angle between line and plane.

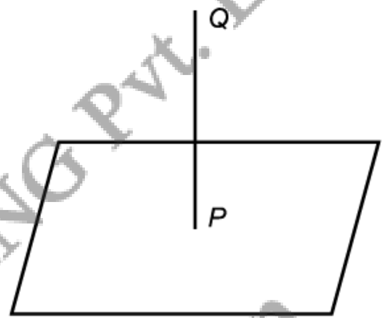
$$\therefore \quad \sin\theta = \frac{3)(1) + 4(-1) + 2(1)}{\sqrt{9+16+4}\sqrt{1+1+1}} \qquad \left[\because \sin\theta = \frac{al + bm + cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}\right]$$

$(\because a = 3, b = 4, c = 2; l = 1, m = 1, n = 1)$

$$\sin\theta = \frac{3-4+2}{\sqrt{29}\sqrt{3}} = \frac{1}{\sqrt{87}}$$

$$\theta = \sin^{-1}\frac{1}{\sqrt{87}}.$$

S31. Given equation of line is $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$.



$\therefore$ Any variable point on the given line is

$$\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4} = \lambda \text{ [Say]}$$

$$\Rightarrow \quad x = 2\lambda - 1, y = 3\lambda - 2, z = 4\lambda - 3$$

$\therefore$ Arbitrary point on the line is $P(2\lambda - 1, 3\lambda - 2, 4\lambda - 3)$

Now, as point P lies on the plane. So, it satisfies the equation of plane

$$x + y + 4z = 6$$

$$\therefore \quad (2\lambda - 1) + (3\lambda - 2) + 4(4\lambda - 3) = 6$$

$$\Rightarrow \quad 2\lambda - 1 + 3\lambda - 2 + 16\lambda - 12 = 6$$

$$\Rightarrow \quad 21\lambda - 21 = 0$$

$$\Rightarrow \quad 21\lambda = 21$$

$$\lambda = 1$$

Putting $\lambda = 1$ in point P, we get the required point

$$P(2 - 1, 3 - 2, 4 - 3) = (1, 1, 1)$$

S32. The equation of a plane passing through $(1, 0, -1)$ is

$$a(x - 1) + b(y - 0) + c(z + 1) = 0 \qquad \text{... (i)}$$

This passes through (3, 2, 2). So,

$$a(3 - 1) + b(2 - 0) + c(2 + 1) = 0 \Rightarrow 2a + 2b + 3c = 0 \qquad \text{... (ii)}$$

The plane (i) is parallel to the line $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{3}$.

Therefore, normal to the plane is perpendicular to the line

$\therefore \quad a(1) + b(-2) + c(3) = 0$ ... (iii)

Solving (ii) and (iii) by cross-multiplication, we get

$$\frac{a}{(2)(3)-(3)(-2)} = \frac{b}{(1)(3)-(2)(3)} = \frac{c}{(2)(-2)-(2)(1)}$$

$$\Rightarrow \quad \frac{a}{12} = \frac{b}{-3} = \frac{c}{-6} \Rightarrow \frac{a}{4} = \frac{b}{-1} = \frac{c}{-2} = \lambda \text{ (say)}$$

$\Rightarrow \quad a = 4\lambda,\ b = -\lambda,\ c = -2\lambda$

Substituting the values of a, b, c in (i), we obtain: $4\lambda(x-1) - \lambda y - 2\lambda(z+1) = 0$

$\Rightarrow \quad 4x - 4 - y - 2z - 2 = 0$

$\Rightarrow \quad 4x - y - 2z - 6 = 0$

This is the equation of required plane.

S33. Equation of plane passing through point (1, – 1, 2) is given by

$a(x-1) + b(y+1) + c(z-2) = 0$ ... (i)

Now, given that plane (i) is perpendicular to planes

$2x + 3y - 2z = 5$ ... (ii)

and $\quad x + 2y - 3z = 8$ ... (iii)

We know that when two planes

$a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ are perpendicular, then

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore$ We get, $\quad 2a + 3b - 2c = 0$ [From Eqs. (i) and (ii)]

and $\quad a + 2b - 3c = 0$ [From Eqs. (i) and (iii)]

or we may write $\quad 2a + 3b = 2c$ ... (iv)

$a + 2b = 3c$ ... (v)

Multiplying Eq. (v) by 2 and subtracting it from Eq. (iv), we get

$$\begin{array}{r} 2a + 3b = 2c \\ \underline{\overset{-}{}2a \overset{-}{+} 4b = \overset{-}{}6c} \\ -b = -4c \end{array}$$

$\Rightarrow \quad b = 4c$

Putting $b = 4c$ in Eq. (v), we get

$a + 8c = 3c$

$\Rightarrow \qquad a = -5c$

Now, putting $a = -5c$ and $b = 4c$ in Eq. (i), we get the required equation of planes as

$-5c(x-1) + 4c(y+1) + c(z-2) = 0$

$\Rightarrow -5(x-1) + 4(y+1) + (z-2) = 0$ [Divide both sides by c]

$\Rightarrow \qquad -5x + 5 + 4y + 4 + z - 2 = 0$

$\Rightarrow \qquad 5x - 4y - z - 7 = 0$

S34. The equation of a plane passing through (4, – 1, 2) is

$$a(x-4) + b(y+1) + c(z-2) = 0 \qquad \text{...(i)}$$

It is parallel to the lines

$$\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2} \quad \text{and} \quad \frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$$

$\therefore \qquad 3a - b + 2c = 0$

$a + 2b + 3c = 0$

Using cross-multiplication, we get

$$\frac{a}{-3-4} = \frac{b}{2-9} = \frac{c}{6+1} \text{ or, } \frac{a}{1} = \frac{b}{1} = \frac{c}{-1} = \lambda \text{ or } a = \lambda,\ b = \lambda,\ c = -\lambda$$

Substituting the values of a, b, c in (i), we obtain

$\lambda(x-4) + \lambda(y+1) - \lambda(z-2) = 0$

$(x-4) + (y+1) - (z-2) = 0 \quad \text{or} \quad x + y - z - 1 = 0$

as the equation of the required plane.

S35. The required plane passes through two points $P(2, 1, -1)$ and $Q(-1, 3, 4)$. Let $\vec{a}$ and $\vec{b}$ be the position vectors of points P and Q respectively.

Then, $\qquad \vec{a} = 2\hat{i} + \hat{j} - \hat{k} \quad \text{and} \quad \vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \qquad \overrightarrow{PQ} = \vec{b} - \vec{a} = -\hat{i} + 3\hat{j} + 4\hat{k} - (2\hat{i} + \hat{j} - \hat{k})$

$= -3\hat{i} + 2\hat{j} + 5\hat{k}$

The required plane is perpendicular to the plane $x - 2y + 4z = 10$. Let $\vec{n}_1$ be the normal vector to this plane, then $\vec{n}_1 = \hat{i} - 2\hat{j} + 4\hat{k}$.

Let n be the normal vector to the required plane. Then,

$$\vec{n} = \vec{n}_1 \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ -3 & 2 & 5 \end{vmatrix} = -18\hat{i} - 17\hat{j} - 4\hat{k}$$

The required plane passes through a point having position vector $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and is normal to the vector $\vec{n}_1 = -18\hat{i} - 17\hat{j} - 4\hat{k}$. So its vector equation is

$$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$$\Rightarrow \quad \vec{r}\cdot\left(-18\hat{i} - 17\hat{j} - 4\hat{k}\right) = \left(2\hat{i} + \hat{j} - \hat{k}\right)\cdot\left(-18\hat{i} - 17\hat{j} - 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{r}\cdot\left(-18\hat{i} - 17\hat{j} - 4\hat{k}\right) = -36 - 17 + 4$$

$$\Rightarrow \quad \vec{r}\cdot\left(18\hat{i} + 17\hat{j} + 4\hat{k}\right) = 49$$

S36. We know that the lines

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} \text{ and, } \frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Here, $x_1 = -1$, $y_1 = -3$, $z_1 = -5$, $x_2 = 2$, $y_2 = 4$, $z_2 = 6$, $l_1 = 3$, $m_1 = 5$, $n_1 = 7$, $l_2 = 1$, $m_2 = 4$, $n_2 = 7$

$$\therefore \quad \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 3 & 7 & 11 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 21 - 98 + 77 = 0$$

So, the given lines are coplanar.

The equation of the plane containing the lines is

$$\begin{vmatrix} x+1 & y+3 & z+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x + 1)(35 - 28) - (y + 3)(21 - 7) + (z + 5)(12 - 5) = 0$$

$$\Rightarrow \quad 7x + 7 - 14y - 42 + 7z + 35 = 0$$

$$\Rightarrow \quad 7x - 14y + 7z + 42 - 42 = 0$$

$$\Rightarrow \quad x - 2y + z = 0$$

S37. The equation of any plane through the line of intersection of the given planes is

$$\left[\vec{r}\cdot(2\hat{i} - 3\hat{j} + 4\hat{k}) - 1\right] + \lambda\left[\vec{r}\cdot(\hat{i} - \hat{j}) + 4\right] = 0$$

$$\Rightarrow \quad \vec{r}\cdot[(2 + \lambda)\hat{i} - (3 + \lambda)\hat{j} + 4\hat{k}] = 1 - 4\lambda \qquad \text{...(i)}$$

If plane (i) is perpendicular to $\vec{r}\cdot[(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$, then

$$[(2+\lambda)\hat{i}-(3+\lambda)\hat{j}+4\hat{k}]\cdot(2\hat{i}-\hat{j}+\hat{k}) = 0 \qquad [\text{Using } \vec{n}_1\cdot\vec{n}_2=0]$$

$$2(2+\lambda)+(3+\lambda)+4 = 0$$

$$\Rightarrow \qquad 3\lambda+11=0 \Rightarrow \lambda = -\frac{11}{3}$$

Putting $\lambda = -\frac{11}{3}$ in (i), we obtain the equation of the required plane as

$$\vec{r}\cdot\left\{\left(2-\frac{11}{3}\right)\hat{i}-\left(3-\frac{11}{3}\right)\hat{i}+4\hat{k}\right\} = 1+\frac{44}{3}$$

$$\Rightarrow \qquad \vec{r}\cdot(-5\hat{i}+2\hat{j}+12\hat{k}) = 47.$$

S38. We know that the lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are coplanar if

$$\vec{a}_1\cdot(\vec{b}_1\times\vec{b}_2) = \vec{a}_2\cdot(\vec{b}_1\times\vec{b}_2)$$

and the equation of the plane containing them is

$$\vec{r}\cdot(\vec{b}_1\times\vec{b}_2) = \vec{a}_1\cdot(\vec{b}_1\times\vec{b}_2)$$

Here, $\vec{a}_1=\hat{i}+\hat{j}-\hat{k}, \vec{b}_1=3\hat{i}-\hat{j}, \vec{a}_2=4\hat{i}+0\hat{j}-\hat{k}$ and $\vec{b}_2=2\hat{i}+0\hat{j}+3\hat{k}$.

$$\therefore \qquad \vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 0\\ 2 & 0 & 3\end{vmatrix} = -3\hat{i}-9\hat{j}+2\hat{k}$$

$$\Rightarrow \qquad \vec{a}_1\cdot(\vec{b}_1\times\vec{b}_2) = (\hat{i}+\hat{j}-\hat{k})\cdot(-3\hat{i}-9\hat{j}+2\hat{k}) = -3-9-2=-14$$

$$\text{and,} \qquad \vec{a}_2\cdot(\vec{b}_1\times\vec{b}_2) = (4\hat{i}+0\hat{j}-\hat{k})\cdot(-3\hat{i}-9\hat{j}+2\hat{k}) = -12+0-2=-14$$

$$\Rightarrow \qquad \vec{a}_1\cdot(\vec{b}_1\times\vec{b}_2) = \vec{a}_2\cdot(\vec{b}_1\times\vec{b}_2)$$

So, the given lines are coplanar.

The equation of the plane containing the given lines is

$$\vec{r}\cdot(\vec{b}_1\times\vec{b}_2) = \vec{a}_1\cdot(\vec{b}_1\times\vec{b}_2)$$

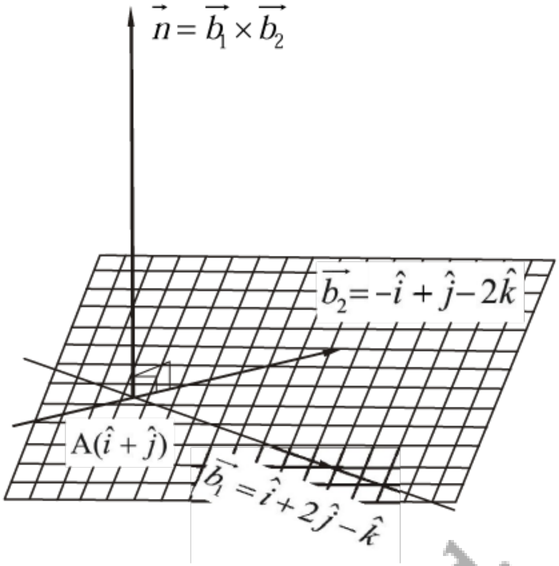
$$\Rightarrow \qquad \vec{r}\cdot(\vec{b}_1\times\vec{b}_2) = -14$$

$$\Rightarrow \qquad \vec{r}\cdot(-3\hat{i}-9\hat{j}+2\hat{k}) = -14 \qquad \left[\because \vec{b}_1\times\vec{b}_2=-3\hat{i}-9\hat{j}+2\hat{k}\right]$$

$$\Rightarrow \qquad \vec{r}\cdot(3\hat{i}+9\hat{j}-2\hat{k}) = 14$$

S39. The two given lines pass through the point having position vector $\vec{a}=\hat{i}+\hat{j}$ and are parallel to the vectors $\vec{b}_1=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}_2=-\hat{i}+\hat{j}-2\hat{k}$ respectively. Therefore, the plane containing the given lines also passes through the point with position vector $\vec{a}=\hat{i}+\hat{j}$. Since the plane contains the lines which are parallel to the vectors $\vec{b}_1$ and $\vec{b}_2$ respectively. Therefore, the plane is normal to the vector.

$$\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$



Thus, the vector equation of the required planes is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0 \text{ or, } \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \quad \vec{r} \cdot (-3\hat{j} + 3\hat{j} + 3\hat{k}) = (\hat{i} + \hat{j}) \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k})$$

$$\Rightarrow \quad \vec{r} \cdot (-3\hat{j} + 3\hat{j} + 3\hat{k}) = -3 + 3$$

$$\Rightarrow \quad \vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0$$

The length of perpendicular from P(2, 1, 4) to the above plane is given by

$$d = \left| \frac{(2\hat{i} + \hat{j} + 4\hat{k}) \cdot (-\hat{i} + \hat{j} + \hat{k})}{\sqrt{(-1)^2 + 1^2 + 1^2}} \right| = \frac{|-2 + 1 + 4|}{\sqrt{3}} = \sqrt{3}$$

S40. Eq. of plane Passing through the intersection of planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ is

$$(x + 3y + 6) + \lambda\,(3x - y - 4z) = 0$$

$$x + 3y + 6 + 3\lambda x - \lambda y - 4\lambda z = 0$$

$$\Rightarrow \quad x(1 + 3\lambda) + y(3 - \lambda) - 4\lambda z + 6 = 0 \qquad \text{... (i)}$$

which is the general form of equation of plane. Now, given that perpendicular distance of plane (i) from origin is unity.

$\therefore$ We have,

$$\left| \frac{(1 + 3\lambda)(0) + (3 - \lambda)(0) - 4\lambda(0) + 6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} \right| = 1$$

$\left[\because \text{ Distance of point } (x_1, y_1, z_1) \text{ from a plane } ax + by + cz + d = 0 \quad d = \left| \dfrac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|. \right]$

$$\Rightarrow \quad \left| \frac{6}{\sqrt{1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2}} \right| = 1$$

$$\Rightarrow \quad \frac{6}{\sqrt{26\lambda^2 + 10}} = 1$$

$$\Rightarrow \quad 6 = \sqrt{26\lambda^2 + 10}$$

Squaring both sides, we get $\quad 36 = 26\lambda^2 + 10$

$$\Rightarrow \quad 26\lambda^2 = 26$$

$\Rightarrow \qquad \lambda^2 = 1$

$\Rightarrow \qquad \lambda = \pm 1$

Now, putting $\lambda = 1$ in eq. (i), we get

$\Rightarrow \qquad 4x + 2y - 4z + 6 = 0$

$\Rightarrow \qquad 2x + y - 2z + 3 = 0 \qquad \text{... (ii)}$

and putting $\lambda = -1$ in eq. (i), we get

$\Rightarrow \qquad -2x + 4y + 4z + 6 = 0$

$\Rightarrow \qquad x - 2y - 2z - 3 = 0 \qquad \text{... (iii)}$

Eqs. (ii) and (iii) are the required equations of the plane.

S41. Equation of plane passing through the point $A(1, 2, 1)$ is given as

$$a(x-1) + b(y-2) + c(z-1) = 0 \qquad \text{... (i)}$$

[$\because$ Equation of plane passing through (x_1, y_1, z_1) having DR's a, b, c is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Now, DR's of line PQ, where $P(1, 4, 2)$ and $Q(2, 3, 5)$ are $2 - 1, 3 - 4, 5 - 2 = 1, -1, 3$.

Since, plane (i) is perpendicular to line PQ.

$\therefore$ DR's of plane (i) are $1, -1, 3$

i.e., $\qquad a = 1, \quad b = -1, \quad c = 3$

Putting values of a, b, c in Eq. (i), we get the required equation of plane as

$$1(x-1) - 1(y-2) + 3(z-1) = 0$$

$\Rightarrow \qquad x - 1 - y + 2 + 3z - 3 = 0$

or $\qquad x - y + 3z - 2 = 0 \qquad \text{... (ii)}$

To find distance of the above plane from the line

$$\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-7}{-1} \qquad \text{... (iii)}$$

Now, DR's of above line are $(2, -1, -1)$ and DR's of normal of Plane of Eq. (ii) are $(1, -1, 3)$

Now, we have $\qquad a_1a_2 + b_1b_2 + c_1c_2 = 0$

Now $\qquad 2(1) - 1(-1) - 1(3) = 2 + 1 - 3 = 0$

$\Rightarrow$ Line (iii) is parallel to plane (ii).

$\therefore$ The required distance = perpendicular distance of the point $(-3, 5, 7)$ to the plane (ii).

$\therefore$ We have,

$$d = \left|\frac{(-3)(1)+(5)(-1)+7(3)-2}{\sqrt{(1)^2+(-1)^2+(3)^2}}\right|$$

$$\left[\begin{array}{l}\text{Distance of the point } (x_1, y_1, z_1) \text{ to the plane} \\ \quad ax + by + cz + d = 0, \\ \quad d = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}\end{array}\right]$$

$$= \left|\frac{-3-5+21-2}{\sqrt{1+1+9}}\right| = \left|\frac{11}{\sqrt{11}}\right| = \sqrt{11} \text{ units.}$$

S42. Equation of plane passing through the point A (0, 0, 0) is

$$a(x-0) + b(y-0) + c(z-0) = 0$$

or $$ax + by + cz = 0 \quad \text{... (i)}$$

[Using one point form of plane $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Since, the plane (i) passes through the point B (3, –1, 2).

$\therefore$ Put $x = 3$, $y = -1$, $z = 2$ in Eq. (i), we get

$$3a - b + 2c = 0 \quad \text{... (ii)}$$

Also, the plane (i) is parallel to the line

$$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

$\therefore$ $a(1) + b(-4) + c(7) = 0$ [If plane is parallel to the line, then normal to the plane is perpendicular to the line $\therefore$ $a_1a_2 + b_1b_2 + c_1c_2 = 0$]

$$\Rightarrow \quad a - 4b + 7c = 0 \quad \text{... (iii)}$$

Now, multiplying Eq. (iii) by 3 and subtracting it from Eq. (ii), we get

$$\begin{array}{r} 3a - b + 2c = 0 \\ \underline{\overset{}{-}3a \underset{+}{-} 12b \underset{-}{+} 21c = 0} \\ 11b - 19c = 0 \end{array}$$

$$\Rightarrow \quad b = \frac{19}{11}c$$

Putting $b = \dfrac{19}{11}c$ in Eq. (ii), we get

$$3a - \frac{19}{11}c + 2c = 0$$

$$\Rightarrow \quad 3a + \frac{-19c + 22c}{11} = 0$$

$$\Rightarrow \quad 3a + \frac{3c}{11} = 0$$

$$\Rightarrow \quad 3a = -\frac{3c}{11}$$

$$\Rightarrow \quad a = -\frac{c}{11}.$$

Now, putting $a = -\dfrac{c}{11}$ and $b = \dfrac{19}{11}c$ in Eq. (i), we get the required equation of plane as

$$\frac{-c}{11}x + \frac{19c}{11}y + cz = 0$$

$$\Rightarrow \quad -\frac{x}{11} + \frac{19y}{11} + z = 0$$

$$\Rightarrow \quad -x + 19y + 11z = 0$$

$$\Rightarrow \quad x - 19y - 11z = 0$$

Note: If a line is parallel to the plane, then normal to the plane is perpendicular to the line.

S43. Let the required equation of plane passing through (– 1, 3, 2) is

$$a(x + 1) + b(y - 3) + c(z - 2) = 0 \quad \text{... (i)}$$

[$\because$ Equation of plane passing through (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Given that plane (i) is perpendicular to the planes whose equations are

$$x + 2y + 3z = 5 \quad \text{... (ii)}$$

and $$3x + 3y + z = 5 \quad \text{... (iii)}$$

$\therefore$ We have, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Using the above result first in Eqs. (i), (ii) and then in Eqs. (i) and (iii), we get

$$a + 2b + 3c = 0 \quad \text{... (iv)}$$

and $$3a + 3b + c = 0 \quad \text{... (v)}$$

Multiplying, (iv) by 3 and subtracting it from Eq. (v), we get

$$\begin{array}{r} 3a + 3b + c = 0 \\ \underline{\overset{-}{}3a \underset{-}{+} 6b \underset{-}{+} 9c = 0} \\ -3b - 8c = 0 \end{array}$$

$$\Rightarrow \quad -3b = 8c \quad \text{or} \quad b = -\frac{8c}{3}$$

Putting $b = -\dfrac{8c}{3}$ in Eq. (iv), we get

$$a + 2\left(-\frac{8c}{3}\right) + 3c = 0$$

$$\Rightarrow \qquad a - \frac{16c}{3} + 3c = 0$$

or $$a = \frac{16c}{3} - 3c = \frac{16c - 9c}{3} = \frac{7c}{3}$$

$$\therefore \qquad a = \frac{7c}{3}$$

Finally, putting $a = \frac{7c}{3}$ and $b = -\frac{8c}{3}$ in Eq. (i), we get the required equation of plane as

$$\frac{7c}{3}(x+1) - \frac{8c}{3}(y-3) + c(z-2) = 0$$

Divide both sides by c, we get

$$\frac{7}{3}(x+1) - \frac{8}{3}(y-3) + (z-2) = 0$$

$$\Rightarrow \qquad 7x + 7 - 8y + 24 + 3z - 6 = 0$$

or $$7x - 8y + 3z + 25 = 0$$

S44. First, we check whether the points are collinear or not.

Given points are $A(2, 2, -1)$, $B(3, 4, 2)$ and $C(7, 0, 6)$.

$$\therefore \qquad AB = \sqrt{(3-2)^2 + (4-2)^2 + (2+1)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{1+4+9} = \sqrt{14}$$

$$BC = \sqrt{(7-3)^2 + (0-4)^2 + (6-2)^2}$$

$$= \sqrt{16+16+16}$$

$$= \sqrt{48} = 4\sqrt{3}$$

$$\therefore \qquad CA = \sqrt{(2-7)^2 + (2-0)^2 + (-1-6)^2}$$

$$= \sqrt{25+4+49} = \sqrt{78}$$

$\because \quad AB + BC \neq CA$, so points A, B, C are not collinear.

Now, the equation of plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$\therefore$ We get,

$$\begin{vmatrix} x-2 & y-2 & z+1 \\ 3-2 & 4-2 & 2+1 \\ 7-2 & 0-2 & 6+1 \end{vmatrix} = 0$$

$\because$ $(x_1, y_1, z_1) = (2, 2, -1)$, $(x_2, y_2, z_2) = (3, 4, 2)$ and $(x_3, y_3, z_3) = (7, 0, 6)$

$$\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$$

Expanding along R_1

$\Rightarrow$ $(x-2)(14+6) - (y-2)(7-15) + (z+1)(-2-10) = 0$

$\Rightarrow$ $(x-2) \cdot 20 - (y-2)(-8) + (z+1)(-12) = 0$

$\Rightarrow$ $20x - 40 + 8y - 16 - 12z - 12 = 0$

$\Rightarrow$ $20x + 8y - 12z - 68 = 0$

Dividing both sides by 4, we get

$$5x + 2y - 3z = 17$$

is the required Cartesian equation of plane.

Also, we have to find the vector equation of plane.

We know that vector form of Cartesian equation $ax + by + cz = d$ of plane is given by $r \cdot \left(a\hat{i} + b\hat{j} + c\hat{k}\right) = d$.

$\therefore$ Required vector equation of plane is

$$\vec{r} \cdot \left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$$

S45. Let the required equation of plane passing through $(1, 1, -1)$ is

$$a(x-1) + b(y-1) + c(z+1) = 0 \qquad \text{... (i)}$$

[$\because$ Equation of plane passing through point (x_1, y_1, z_1) is given by $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Given that the plane (i) is perpendicular to the planes whose equations are

$$x + 2y + 3z - 7 = 0 \qquad \text{... (ii)}$$

and

$$2x - 3y + 4z = 0 \qquad \text{... (iii)}$$

$\therefore$ We have, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Using the above result first in Eqs. (i) and (ii) and then in Eqs. (i) and (iii), we get

$$a + 2b + 3c = 0 \quad \text{... (iv)}$$

and $$2a - 3b + 4c = 0 \quad \text{... (v)}$$

Using cross multiplication method, we get

$$\frac{a}{\begin{vmatrix} 2 & 3 \\ -3 & 4 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}}$$

$$\Rightarrow \quad \frac{a}{8+9} = \frac{-b}{4-6} = \frac{c}{-3-4}$$

$$\Rightarrow \quad \frac{a}{17} = \frac{-b}{-2} = \frac{c}{-7} = k \quad \text{[Say]}$$

$\therefore \quad a = 17k, \; b = 2k$ and $c = -7k$

Putting above values in Eq. (i), we get the required equation of planes as

$$17k(x - 1) + 2k(y - 1) - 7k(z + 1) = 0$$

$$\Rightarrow \quad 17x - 17 + 2y - 2 - 7z - 7 = 0 \quad \text{[Divide by } k\text{]}$$

$$\Rightarrow \quad 17x + 2y - 7z - 26 = 0$$

S46. Given equations of lines are

$$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \quad \text{... (i)}$$

and $$\vec{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(3\hat{i} - 2\hat{j} + 5\hat{k}) \quad \text{... (ii)}$$

Comparing eqs. (i) and (ii) with the vector equation of line which is $\vec{r} = \vec{a} + \lambda\vec{b}$, we get

$$\vec{a}_1 = 2\hat{i} + \hat{j} - 3\hat{k}, \quad \vec{b}_1 = \hat{i} + 2\hat{j} + 5\hat{k}$$

and $$\vec{a}_2 = 3\hat{i} + 3\hat{j} + 2\hat{k}, \quad \vec{b}_2 = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

Now, the required plane which contains the lines Eqs. (i) and (ii) will pass through $\vec{a}_1 = 2\hat{i} + \hat{j} - 3\hat{k}$. Also, the required plane has $\vec{b}_1$ and $\vec{b}_2$ parallel to it.

$\therefore$ The plane is normal to the vector

$$\vec{n} = \vec{b}_2 \times \vec{b}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}$$

$$= \hat{i}(10 + 10) - \hat{j}(5 - 15) + \hat{k}(-2 - 6)$$

$$= 20\hat{i} + 10\hat{j} - 8\hat{k}$$

$\therefore$ The vector equation of required plane is given by

$$(\vec{r}-\vec{a})\cdot n = 0 \text{ or } \vec{r}\cdot\vec{n} = \vec{a}_1\cdot\vec{n} \qquad [\text{Here, } \vec{a} = \vec{a}_1]$$

$$\Rightarrow \vec{r}\cdot\left(20\hat{i}+10\hat{j}-8\hat{k}\right) = \left(2\hat{i}+\hat{j}-3\hat{k}\right)\cdot\left(20\hat{i}+10\hat{j}-8\hat{k}\right)$$

or $\vec{r}\cdot\left(20\hat{i}+10\hat{j}-8\hat{k}\right) = 40 + 10 + 24 = 74$

or $\vec{r}\cdot\left(10\hat{i}+5\hat{j}-4\hat{k}\right) = 37$ [Divide by 2]

is the required equation of plane. ... (iii)

Also, its Cartesian equation is given by

$10x + 5y - 4z = 37$ [$\because$ Vector form of plane $\vec{r}\cdot\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right) = d$ can be written in its Cartesian form as $a_1x + a_2y + a_3z = d$]

Next, we have to show that the line

$$\vec{r} = \left(2\hat{i}+5\hat{j}+2\hat{k}\right)+\lambda_1\left(3\hat{i}-2\hat{j}+5\hat{k}\right) \qquad \text{... (iv)}$$

lies in the plane (iii).

Now, the above line will lie on plane (iii) when it passes through the point $\vec{a} = 2\hat{i}+5\hat{j}+2\hat{k}$ of line (iv)

$$\therefore \quad \vec{r}\cdot\left(10\hat{i}+5\hat{j}-4\hat{k}\right) = \left(2\hat{i}+5\hat{j}+2\hat{k}\right)\cdot\left(10\hat{i}+5\hat{j}-4\hat{k}\right)$$

$$= 20 + 25 - 8 = 37$$

$\therefore$ $\vec{a}$ lies on plane whose equation is given by Eq. (iii).

Hence, line (iv) lies on the plane (iii).

Hence proved.

S47. First we find equation of plane passing through point $R(1, 2, 1)$ and is perpendicular to line PQ where $P(1, 4, 2)$ and $Q(2, 3, 5)$.

DR's of the line PQ are

$$= (2 - 1, 3 - 4, 5 - 2) = (1, -1, 3)$$

Let the required equation of plane which passes through point $R\,(1, 2, 1)$ is

$$a\,(x - 1) + b(y - 2) + c(z - 1) = 0 \qquad \text{... (i)}$$

Plane (i) is perpendicular to line PQ.

$\therefore$ We have,

$1(x - 1) - 1(y - 2) + 3(z - 1) = 0$ [$\because$ Since, plane is perpendicular to the line, the DR's of normal to the plane is proportional to the DR's of a line *i.e.*, $a \propto 1$, $b \propto -1$, $c \propto 3$.]

or $x - 1 - y + 2 + 3z - 3 = 0$

or $x - y + 3z - 2 = 0$ ... (ii)

is the required equation of plane.

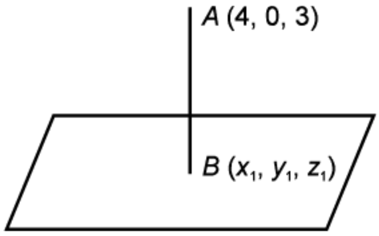
Next, we have to find the foot of perpendicular and perpendicular distance of the point (4, 0, 3) from above plane.

So, let $B(x_1, y_1, z_1)$ be the foot of perpendicular on above plane (ii). It must satisfies Eq. (ii).

$\therefore$ $x_1 - y_1 + 3z_1 - 2 = 0$ ... (iii)

Also, DR's of line *AB* normal to above plane (i) are given by

$x_1 - 4,\ y_1 - 0,\ z_1 - 3$



$\therefore$ We have, $$\frac{x_1 - 4}{1} = \frac{y_1 - 0}{-1} = \frac{z_1 - 3}{3}$$

[$\because$ DR's of line *AB* and plane (iii) are proportional]

Let, $$\frac{x_1 - 4}{1} = \frac{y_1 - 0}{-1} = \frac{z_1 - 3}{3} = \lambda \quad \text{[Say]}$$

$\Rightarrow$ $x_1 = \lambda + 4,\ y_1 = -\lambda,\ z_1 = 3\lambda + 3$ ...(iv)

Putting above values of x_1, y_1 and z_1 in Eq. (iii), we get

$$\lambda + 4 + \lambda + 9\lambda + 9 - 2 = 0$$

$\Rightarrow$ $11\lambda + 11 = 0$

$\Rightarrow$ $11\lambda = -11$ or $\lambda = -1$

Putting $\lambda = -1$ in Eq. (iv), we get the required foot of perpendicular as

$$B(x_1, y_1, z_1) = B(\lambda + 4, -\lambda, 3\lambda + 3) = B(3, 1, 0)$$

Also, perpendicular distance *AB* where *A*(4, 0, 3) and *B*(3, 1, 0) is given by using distance formula *i.e.*,

$$AB = \sqrt{(3-4)^2 + (1-0)^2 + (0-3)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{1+1+9} = \sqrt{11} \text{ units.}$$

Note: If line is perpendicular to the plane, then DR's of normal to the plane is proportional to the DR's of a line.

S48. Equation of plane passing through point *P*(1, 1, 1) is given by

$a(x - 1) + b(y - 1) + c(z - 1) = 0$ ... (i)

[$\because$ Equation of plane passing through (x_1, y_1, z_1) is given as $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Given equation of line is

$$\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} - 5\hat{k}\right) \quad \text{... (ii)}$$

DR's of the parallel vector line are 3, – 1 and –5 and the line passes through point (– 3, 1, 5).

Now, as the plane (i) contains line (ii), we get

$a(-3-1) + b(1-1) + c(5-1) = 0$

As plane contains a line, it means point of line contains a plane

$\Rightarrow \quad -4a + 4c = 0 \quad \text{... (iii)}$

Also, since DR's of plane are normal to that of line

$\therefore \quad 3a - b - 5c = 0 \quad \text{...(iv)}$

[$\because$ Plane contains line, it means DR's of plane is perpendicular to the line *i.e.*, $aa_1 + bb_1 + cc_1 = 0$]

From Eq. (iii), we get

$$-4a = -4c \quad \Rightarrow \quad a = c$$

Putting, $a = c$ in Eq. (iv), we get

$3c - b - 5c = 0$

or $\quad -b - 2c = 0$

or $\quad b = -2c$

Finally putting $a = c$ and $b = -2c$ in Eq. (i), we get the required equation of plane as

$c(x-1) - 2c(y-1) + c(z-1) = 0$

Divide both sides by c, we get

$x - 1 - 2y + 2 + z - 1 = 0$

or $\quad x - 2y + z = 0 \quad \text{... (v)}$

Next, we have to show that the above plane (v) contains the line

$$r = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} - 5\hat{k}\right) \quad \text{... (vi)}$$

Vector equation of plane (v) is

$$r \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0 \quad \text{... (vii)}$$

The plane (vii) will contains line (vi), if

$$\left(\hat{i} - 2\hat{j} - 5\hat{k}\right) \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0 \quad [\because \ b_1 \cdot b_2 = 0]$$

$\Rightarrow \quad (1)(1) - 2(-2) - 5(1) = 0$

$\Rightarrow \quad 1 + 4 - 5 = 0$

or $0 = 0$

which is true.

$\therefore$ The plane contains the given line.

S49. Let the required equation of plane passing through the point $(-1, -1, 2)$ is

$$a(x + 1) + b(y + 1) + c(z - 2) = 0 \quad \text{... (i)}$$

Given that plane (i) is perpendicular to the planes whose equations are

$$2x + 3y - 3z = 2 \quad \text{... (ii)}$$

and $$5x - 4y + z = 6 \quad \text{... (iii)}$$

We know that when two planes are perpendicular, we have

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

where, a_1, b_1, c_1 and a_2, b_2, c_2 are DR's of two planes.

Using above results in Eqs. (i) and (ii) and then in Eqs. (i) and (iii), we get

$$2a + 3b - 3c = 0 \quad \text{... (iv)}$$

and $$5a - 4b + c = 0 \quad \text{... (v)}$$

Multiplying Eq. (v) by 3 and adding it to Eq. (iv), we get

$$\begin{array}{r} 2a + 3b - 3c = 0 \\ 15a - 12b + 3c = 0 \\ \hline 17a - 9b \qquad = 0 \end{array}$$

$\Rightarrow$ $$17a = 9b$$

$\Rightarrow$ $$a = \frac{9}{17}b$$

Putting $a = \frac{9}{17}b$ in Eq. (v), we get

$$5\left(\frac{9b}{17}\right) - 4b + c = 0$$

$\Rightarrow$ $$\frac{45b - 68b}{17} + c = 0$$

$\Rightarrow$ $$c = \frac{23b}{17}$$

Putting $a = \frac{9b}{17}$ and $c = \frac{23b}{17}$ in Eq. (i), we get

$$\frac{9b}{17}(x+1) + b(y+1) + \frac{23b}{17}(z-2) = 0$$

Divide both sides by b, we get

$$\frac{9}{17}(x+1)+(y+1)+\frac{23}{17}(z-2) = 0$$

$\Rightarrow$ $9x + 9 + 17y + 17 + 23z - 46 = 0$

or $9x + 17y + 23z - 20 = 0$

is the required equation of plane.

S50. Given points are $A(3, -1, 2)$, $B(5, 2, 4)$, $C(-1, -1, 6)$.

First we check whether the points are collinear or not. Using distance formula, we have

$$AB = \sqrt{(5-3)^2+(2+1)^2+(4-2)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\right]$$

$$= \sqrt{4+9+4} = \sqrt{17}$$

$$BC = \sqrt{(-1-5)^2+(-1-2)^2+(6-4)^2} = \sqrt{36+9+4} = \sqrt{49}$$

$$CA = \sqrt{(3+1)^2+(-1+1)^2+(2-6)^2} = \sqrt{16+0+16} = \sqrt{32}$$

$\because$ $AB + BC \neq CA$

$\therefore$ Given points are non-collinear.

Now, we know that equation of plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \qquad \text{... (i)}$$

We have $(x_1, y_1, z_1) = A(3, -1, 2)$, $(x_2, y_2, z_2) = B(5, 2, 4)$ and $(x_3, y_3, z_3) = C(-1, -1, 6)$. Putting above values in Eq. (i), we get the required equation as

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 5-3 & 2+1 & 4-2 \\ -1-3 & -1+1 & 6-2 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$\Rightarrow$ $(x-3)(12-0)-(y+1)(8+8)+(z-2)(0+12) = 0$

$\Rightarrow$ $12x - 36 - 16y - 16 + 12z - 24 = 0$

$\Rightarrow$ $3x - 4y + 3z - 19 = 0$ [Divide by 4] ... (ii)

Next, we find distance of point $P(6, 5, 9)$ to the above plane (ii) by using the formula.

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Here, $A = 3$, $B = -4$, $C = 3$, $D = -19$

$$x_1 = 6,\ y_1 = 5,\ z_1 = 9$$

$$\therefore \quad d = \left|\frac{18 - 20 + 27 - 19}{\sqrt{9 + 16 + 9}}\right| = \left|\frac{6}{\sqrt{34}}\right| = \frac{6}{\sqrt{34}}$$

$$\therefore \quad \text{Distance} = \frac{6\sqrt{34}}{34} \text{ units} = \frac{3\sqrt{34}}{17} \text{ units}$$

S51. Equation of plane passing through the point (3, 4, 1) is given as

$$a(x-3) + b(y-4) + c(z-1) = 0 \qquad \text{... (i)}$$

[Using one point form $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Since, given plane (i) is also passing through point (0, 1, 0).

So, this point also satisfies equation of plane.

$$\therefore \quad a(0-3) + b(1-4) + c(0-1) = 0$$

or $\quad -3a - 3b - c = 0$

or $\quad 3a + 3b + c = 0 \qquad \text{... (ii)}$

Also, given that plane (i) is parallel to the line

$$\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$$

$$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$$

[$\because$ Line is parallel to the plane, therefore normal to the plane is perpendicular to the line.]

where, $\quad a_1 = a,\ b_1 = b,\ c_1 = c$

and $\quad a_2 = 2,\ b_2 = 7,\ c_2 = 5$

$\therefore$ We get, $\quad 2a + 7b + 5c = 0 \qquad \text{... (iii)}$

Multiplying Eq. (ii) by 2 and Eq. (iii) by 3 and subtracting, we get

$$\begin{array}{r} 6a + 6b + 2c = 0 \\ \underset{-}{}6a \underset{-}{+} 21b \underset{-}{+} 15c = 0 \\ \hline -15b - 13c = 0 \end{array}$$

$$\Rightarrow \quad -15b = 13c \quad \Rightarrow \quad b = \frac{-13}{15}c$$

Putting $b = \frac{-13c}{15}$ in Eq. (ii), we get

$$3a + 3\left(\frac{-13c}{15}\right) + c = 0,$$

$$\Rightarrow \quad 3a - \frac{13}{5}c + c = 0$$

$$\Rightarrow \quad 3a - \frac{8c}{5} = 0,$$

$$\Rightarrow \quad a = \frac{8c}{15},$$

Putting $a = \frac{8c}{15}, \quad b = -\frac{13c}{15}$

In Eq. (i), we get the required equation of plane as

$$\frac{8c}{15}(x-3) - \frac{13c}{15}(y-4) + c(z-1) = 0$$

Divide both sides by c, we get

$$\frac{8}{15}(x-3) - \frac{13}{15}(y-4) + z - 1 = 0$$

$$\Rightarrow \quad 8x - 24 - 13y + 52 + 15z - 15 = 0$$

or $$8x - 13y + 15z + 13 = 0$$

S52. Equation of the line passing through the points (3, –4, –5) and (2, –3, 1) is

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5}$$

or $$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda \quad \text{[say]}$$

$\because$ Any variable point on this line be $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ ... (i)

Now, equation of plane passes through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

Where $(x_1, y_1, z_1) = (2, 2, 1)$, $(x_2, y_2, z_2) = (3, 0, 1)$ and $(x_3, y_3, z_3) = (4 -1, 0)$

$\therefore$ Equation of plane $\begin{vmatrix} x-2 & y-2 & z-1 \\ 3-2 & 0-2 & 1-1 \\ 4-2 & -1-2 & 0-1 \end{vmatrix} = 0$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$$

$$(x-2)(2-0) - (y-2)(-1-0) + (z-1)(-3+4) = 0$$

$$2x - 4 + y - 2 + z - 1 = 0$$

$$2x + y + z - 7 = 0 \quad \text{... (ii)}$$

The variable point of line lies on plane (i)

$$\therefore \quad 2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0$$

$$-2\lambda + 6 + \lambda - 4 + 6\lambda - 5 - 7 = 0$$

$$5\lambda - 10 = 0$$

$$\lambda = 2$$

Put the value $\lambda = 2$ in Eq. (i)

$\therefore$ Point of intersection of line and plane $(1, -2, 7)$.

S53. Consider the required line be parallel to vector b given by,

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

The position vector of the point (1, 2, 3) is

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by

$$\vec{r} = a + \lambda b$$

$$\therefore \quad \vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) \quad \text{... (i)}$$

The equation of the given planes are

$$\vec{r} \cdot \left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5 \quad \text{... (ii)}$$

and

$$\vec{r} \cdot \left(3\hat{i} + \hat{j} + \hat{k}\right) = 6 \quad \text{... (iii)}$$

The line in Eq. (i) and plane in Eq. (ii) are parallel. Therefore, the normal to the plane of Eq. (ii) and the given line are perpendicular.

$$\therefore \quad \left(\hat{i} - \hat{j} + 2\hat{k}\right) \cdot \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$$

$$\Rightarrow \quad (b_1 - b_2 + 2b_3) = 0$$

$$\Rightarrow \quad (b_1 - b_2 + 2b_3) = 0 \quad \text{... (iv)}$$

Similarly,

$$\left(3\hat{i} + \hat{j} + \hat{k}\right)\cdot\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$$

$\Rightarrow$ $$3b_1 + b_2 + b_3 = 0 \quad \dots \text{(v)}$$

From Eq. (iv) and (v), we obtain

$$\frac{b_1}{(-1)\times 1 - 1\times 2} = \frac{b_2}{2\times 3 + (-1)\times 1} = \frac{b_3}{1\times 1 - 3(-1)}$$

$\Rightarrow$ $$\frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$$

Therefore, the direction ratios of $\vec{b}$ are –3, 5 and 4.

$\therefore$ $$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Substituting the value of b in eq. (i), we obtain

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

This is the equation of the required line.

S54. The equation of any plane through (2, 1, – 1) is

$$a(x-2) + b(y-1) + c(z+1) = 0 \quad \dots \text{(i)}$$

If it passes through (–1, 3, 4), then

$$a(-1-2) + b(3-1) + c(4+1) = 0$$

$\Rightarrow$ $$-3a + 2b + 5c = 0 \quad \dots \text{(ii)}$$

If plane (i) is perpendicular to the plane $x - 2y + 4z = 10$, then

$$a - 2b + 4c = 0 \quad \dots \text{(iii)}$$

Solve (ii) and (iii) by the method of cross-multiplication, we obtain

$$\frac{a}{8+10} = \frac{b}{5+12} = \frac{c}{6-2}$$

$\Rightarrow$ $$\frac{a}{18} = \frac{b}{17} = \frac{c}{4} = \lambda(\text{say})$$

$\Rightarrow$ $$a = 18\lambda,\ b = 17\lambda \text{ and } c = 4\lambda.$$

Putting $a = 18\lambda$, $b = 17\lambda$ and $c = 4\lambda$ in (i), we obtain

$$18\lambda(x-2) + 17\lambda(y-1) + 4\lambda(z+1) = 0$$

$$18x - 36 + 17y - 17 + 4z + 4 = 0$$

$\Rightarrow \qquad 18x + 17y + 4z = 49 \qquad \text{... (iv)}$

This is the required equation of the plane.

The coordinates of any point on the lines are $(3\lambda - 1, -2\lambda + 3, -5\lambda + 4)$. Substituting in (iv), we obtain

$$\text{L.H.S.} = 18(3\lambda - 1) + 17(-2\lambda + 3) + 4(-5\lambda + 4) = 49 = \text{R.H.S.}$$

So, $(3\lambda - 1, -2\lambda + 3, -5\lambda + 4)$ lies on (iv). Hence, plane in (iv) contains the given line.

Alter: The required plane passes through the points $P(2, 1, -1)$ and $Q(-1, 3, 4)$. Let $\vec{a}$ and $\vec{b}$ be the position vectors of points P and Q respectively. Then, $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$ and $\overrightarrow{PQ} = \vec{b} - \vec{a} = -3\hat{i} + 2\hat{j} + 5\hat{k}$

The required plane is perpendicular to the plane $x - 2y + 4z = 10$. Let $\vec{n_1}$ be the normal vector to this plane. Then, $\vec{n_1} = \hat{i} - 2\hat{j} + 4\hat{k}$.

Let $\vec{n}$ be the normal vector to the desired plane. Then,

$$\vec{n} = \vec{n}_1 \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ -3 & 2 & 5 \end{vmatrix} = -18\hat{i} - 17\hat{j} - 4\hat{k}$$

The required plane passes through a point having position vector $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and is normal to the vector $\vec{n} = -18\hat{i} - 17\hat{j} - 4\hat{k}$. So, its vector equation is

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k}) = (2\hat{i} + \hat{j} - \hat{k}) \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k})$$

$$\Rightarrow \vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k}) = -36 - 17 + 4$$

$$\Rightarrow \vec{r} \cdot (18\hat{i} + 17\hat{j} + 4\hat{k}) = 49 \qquad \text{... (i)}$$

The position vector of any point on the given lines is

$(3\lambda - 1)\hat{i} + (3 - 2\lambda)\hat{j} + (4 - 5\lambda)\hat{k}$

Put the point in Eq. (i)

$$\text{L.H.S} = \left((3\lambda - 1)\hat{i} + (3 - 2\lambda)\hat{j} + (4 - 5\lambda)\hat{k}\right) \cdot (18\hat{i} + 17\hat{j} + 4\hat{k})$$

$$= 18\,(3\lambda - 1) + 17\,(3 - 2\lambda) + 4\,(4 - 5\lambda)$$

$$= 54\lambda - 18 + 51 - 34\lambda + 16 - 20\lambda$$

$= 49 = \text{R.H.S.}$

Clearly, it satisfies (i). Hence, the plane in (i) contains given line.

The Cartesian equation of the plane is $18x + 17y + 4z = 49$.

S55. Given equation of planes are $r \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $r \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$

Put $r = x\hat{i} + y\hat{j} + z\hat{k}$ in above equations, It can be written in Cartesian form as

$$x + y + z - 1 = 0 \qquad \text{... (i)}$$

and $$2x + 3y - z + 4 = 0 \qquad \text{... (ii)}$$

Let the required equation of plane passing through the line of intersection of planes (i) and (ii) is

$$(x + y + z - 1) + \lambda (2x + 3y - z + 4) = 0$$

$$\Rightarrow \quad x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 - \lambda) + (-1 + 4\lambda) = 0 \qquad \text{... (iii)}$$

$\therefore$ DR's of the above planes are $1 + 2\lambda$, $1 + 3\lambda$, $1 - \lambda$.

Also, DR's of the x-axis are (1, 0, 0).

Now, since given that the above plane (iii) is parallel to the x-axis.

$\therefore$ We have, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

where, $a_1 = 1 + 2\lambda$, $b_1 = 1 + 3\lambda$, $c_1 = 1 - \lambda$

and $a_2 = 1$, $b_2 = 0$, $c_2 = 0$

$\therefore \quad 1(1 + 2\lambda) + 0\,(1 + 3\lambda) + 0\,(1 - \lambda) = 0$

$\Rightarrow \quad 1 + 2\lambda = 0$

$\Rightarrow \quad 2\lambda = -1$

$\Rightarrow \quad \lambda = -\dfrac{1}{2}$

Putting $\lambda = -\dfrac{1}{2}$ in Eq. (iii), we get the required equation of plane as

$$x\left(1 - \frac{2 \times 1}{2}\right) + y\left(1 - \frac{3}{2}\right) + z\left(1 + \frac{1}{2}\right) + \left(-1 - \frac{4}{2}\right) = 0$$

$$\Rightarrow \quad \frac{-y}{2} + \frac{3z}{2} - \frac{6}{2} = 0$$

$$\Rightarrow \quad y - 3z + 6 = 0.$$

S56. Given planes are $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$. The equation of any plane passing through the line of intersection of these planes is

$$\left[r \cdot (\hat{i} + 3\hat{j}) - 6\right] + \lambda\left[\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k})\right] = 0$$

or $\quad r \cdot \left[(3\lambda + 1)\hat{i} + (3 - \lambda)\hat{j} - 4\lambda\hat{k}\right] - 6 = 0$ ... (i)

Given that, perpendicular distance from origin on this plane is unity.

i.e., $\quad \dfrac{|(3\lambda + 1) \times 0 + (3 - \lambda) \times 0 - 4\lambda \times 0 - 6|}{\sqrt{(3\lambda + 1)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} = 1$

$\Rightarrow \quad 6 = \sqrt{9\lambda^2 + 1 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2}$

$\Rightarrow \quad 6 = \sqrt{26\lambda^2 + 10}$

Squaring on both sides, we get

$\Rightarrow \quad 26\lambda^2 + 10 = 36$

$\Rightarrow \quad 26\lambda^2 = 26$

$\Rightarrow \quad \lambda^2 = 1$

$\Rightarrow \quad \lambda = \pm 1$

$\therefore$ Required equation of planes are

$r \cdot \left(4i + 2\hat{j} - 4\hat{k}\right) - 6 = 0$, when ($\lambda$ = 1)

and $\quad r \cdot \left(-2i + 4\hat{j} - 4\hat{k}\right) - 6 = 0$, when ($\lambda$ = – 1)

S57. Given equation of planes are

$2x + y - z - 3 = 0$ ... (i)

and $\quad 5x - 3y + 4z + 9 = 0$ ... (ii)

Let the required equation of plane which passes through the line of intersection of planes (i) and (ii) is

$(2x + y - z - 3) + \lambda (5x - 3y + 4z + 9) = 0$ ... (iii)

$\Rightarrow \quad x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + (-3 + 9\lambda) = 0$ ... (iv)

Here, DR's of plane are $2 + 5\lambda$, $1 - 3\lambda$, $-1 + 4\lambda$ given that the plane (i) is parallel to the line whose

equation is $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-5}{5}$

DR's of the line are 2, 4, 5.

Since, the plane is parallel to the line.

$\therefore$ We have, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

where, $a_1 = 2 + 5\lambda$, $b_1 = 1 - 3\lambda$, $c_1 = -1 + 4\lambda$ and $a_2 = 2$, $b_2 = 4$, $c_2 = 5$

$\Rightarrow \quad 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(-1 + 4\lambda) = 0$

$\Rightarrow \quad 4 + 10\lambda + 4 - 12\lambda - 5 + 20\lambda = 0$

$\Rightarrow \quad 18\lambda + 3 = 0$

or $\quad \lambda = -\dfrac{3}{18} = -\dfrac{1}{6}$

$\therefore \quad \lambda = -\dfrac{1}{6}$

Putting $\lambda = -\dfrac{1}{6}$ in Eq. (iii), we get the required equation of plane is

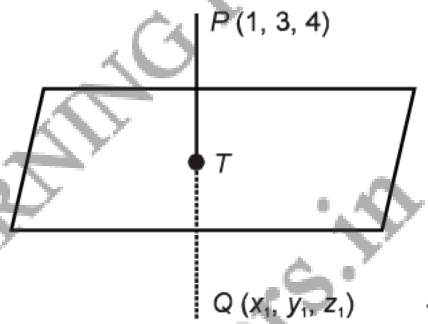
$$(2x + y - z - 3) - \frac{1}{6}(5x - 3y + 4z + 9) = 0$$

$\Rightarrow \quad 12x + 6y - 6z - 18 - 5x + 3y - 4z - 9 = 0$

$\Rightarrow \quad 7x + 9y - 10z - 27 = 0$

S58. Let $Q(x_1, y_1, z_1)$ be the image of the point $P(1, 3, 4)$ on the plane whose equation is

$2x - y + z + 3 = 0$ ... (i)



Now, the line PT is normal to the plane, so the DR's of PT are proportional to the DR's of plane which are 2, –1 and 1.

$\therefore$ Equation of line PT where $P(1, 3, 4)$ is given by

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} \quad \text{... (ii)}$$

$\left[\because \text{Equation of line passing through one point is } \dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}\right]$

Let $\quad \dfrac{x-1}{2} = \dfrac{y-3}{-1} = \dfrac{z-4}{1} = \lambda \quad$ [Say]

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3 - \lambda,\ z = 4 + \lambda$

$\therefore$ Coordinates of the point T are $(2\lambda + 1, 3 - \lambda, 4 + \lambda)$ ... (iii)

From the figure, we see that the point T lies on plane, so we put $x = 2\lambda + 1$, $y = 3 - \lambda$ and $z = 4 + \lambda$ in Eq. (i).

$\therefore \quad 2(2\lambda + 1) - (3 - \lambda) + (4 + \lambda) + 3 = 0$

$\Rightarrow \quad 4\lambda + 2 - 3 + \lambda + 4 + \lambda + 3 = 0$

$\Rightarrow \quad 6\lambda + 6 = 0$

$\Rightarrow \quad 6\lambda = -6 \quad \text{or} \quad \lambda = -1$

Putting $\lambda = -1$ in Eq. (iii), we get the point $T(-1, 4, 3)$

Since, T is the mid-point of line PQ. So, by using mid-point formula, we get

$$\left(\frac{x_1+1}{2}, \frac{y_1+3}{2}, \frac{z_1+4}{2}\right) = (-1, 4, 3) \qquad \left[\because \text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)\right]$$

$$\Rightarrow \qquad \frac{x_1+1}{2} = -1,\ \frac{y_1+3}{2} = 4,\ \frac{z_1+4}{2} = 3$$

or $\quad x_1 = -2 - 1,\ y_1 = 8 - 3,\ z_1 = 6 - 4$

or $\qquad x_1 = -3,\ y_1 = 5,\ z_1 = 2$

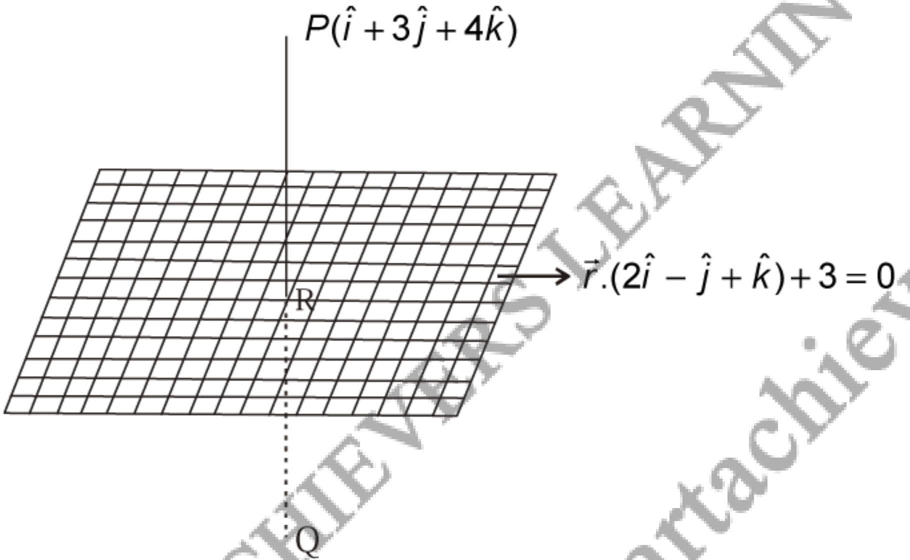
Hence, image Q of the point $P(1, 3, 4)$ is $Q(-3, 5, 2)$.

S59. Let Q be the image of the point $P(\hat{i}+3\hat{j}+4\hat{k})$ in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$. Then, PQ is normal to the plane. Since PQ passes through P and is normal to the given plane, therefore equation of line PQ is

$$\vec{r} = (\hat{i}+3\hat{j}+4\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k})$$

Since Q lies on the line PQ, so let the position vector of Q be

$$(\hat{i}+3\hat{j}+4\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k}) = (1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}.$$



Since R is the mid-point of PQ. Therefore, position vector of R is

$$\frac{[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}]+[\hat{i}+3\hat{j}+4\hat{k}]}{2}$$

$$= (\lambda+1)\hat{i}+\left(3-\frac{\lambda}{2}\right)\hat{j}+\left(4+\frac{\lambda}{2}\right)\hat{k}$$

Since R lies on the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$

$$\therefore \quad \left\{(\lambda+1)\hat{i}+\left(3-\frac{\lambda}{2}\right)\hat{j}+\left(4+\frac{\lambda}{2}\right)\hat{k}\right\}\cdot(2\hat{i}-\hat{j}+\hat{k})+3 = 0$$

$\Rightarrow \quad 2\lambda + 2 - 3 + \dfrac{\lambda}{2} + 4 + \dfrac{\lambda}{2} + 3 = 0 \Rightarrow \lambda = -2.$

Put the value λ in point Q

Thus, the position vector of Q is

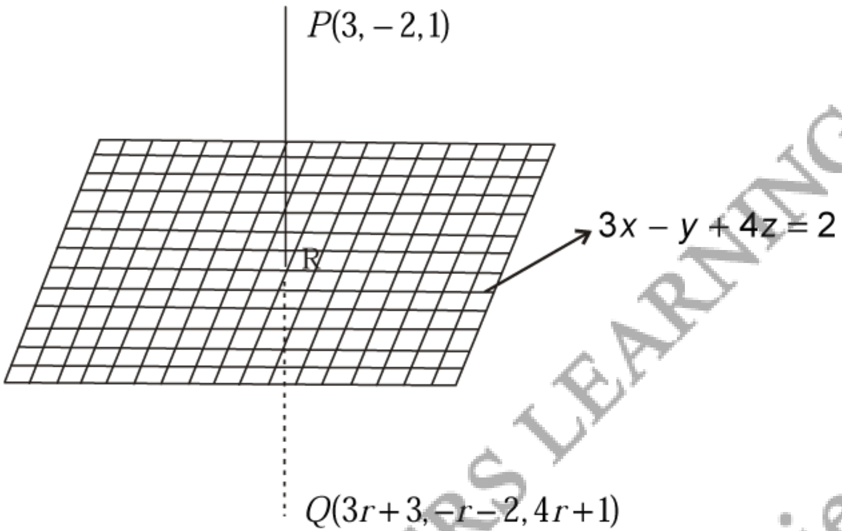
$$(\hat{i} + 3\hat{j} + 4\hat{k}) - 2(2\hat{i} - \hat{j} + \hat{k}) = -3\hat{i} + 5\hat{j} + 2\hat{k}.$$

Which is required Image of given point.

S60. Let Q be the image of the point $P(3, -2, 1)$ in the plane $3x - y + 4z = 2$. Then, PQ is normal to the plane. Therefore, direction ratios of PQ are proportional to $3, -1, 4$. Since PQ passes through $P(3, -2, 1)$ and has direction ratios proportional to $3, -1, 4$. Therefore, equation of PQ is

$$\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} = r(\text{say})$$

Let the coordinates of Q be $(3r + 3, -r - 2, 4r + 1)$. Let R be the mid-point of PQ.



Then, R lies on the plane $3x - y + 4z = 2$. The coordinates of R are

$$\left(\frac{3r+3+3}{2}, \frac{-r-2-2}{2}, \frac{4r+1+1}{2}\right)$$

or $$\left(\frac{3r+6}{2}, \frac{-r-4}{2}, 2r+1\right)$$

Since R lies on $3x - y + 4z = 2$.

$$3\left(\frac{3r+6}{2}\right) - \left(\frac{-r-4}{2}\right) + 4(2r+1) = 2$$

$$\frac{9r}{2} + \frac{r}{2} + 8r + 9 + 2 + 4 = 2$$

$\Rightarrow \quad 13r = -13 \quad \Rightarrow \quad r = -1$

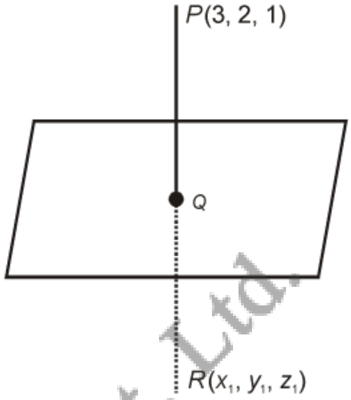
put the value of r in point Q which is required image of given point.

Hence, the coordinates of Q are (0, –1, – 3).

S61. Let Q be the foot of perpendicular and $R(x_1, y_1, z_1)$ be the image of point $P(3, 2, 1)$ on the plane whose equation is

$$2x - y + z + 1 = 0 \qquad \text{... (i)}$$

Here, the line PQ is normal to the given plane. So, DR's of line PQ are proportional to DR's of plane.



Now, DR's of plane (i) are 2, – 1, 1.

$\therefore$ Equation of line PQ where point $P(3, 2, 1)$ is given by

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$$

Let $\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda$ [Say]

$\Rightarrow$ $x - 3 = 2\lambda,\ y - 2 = -\lambda,\ z - 1 = \lambda$

$\Rightarrow$ $x = 2\lambda + 3,\ y = 2 - \lambda,\ z = \lambda + 1$

$\therefore$ Coordinates of Q are $(2\lambda + 3, 2 - \lambda, \lambda + 1)$ ... (ii)

As point Q lies on the plane, so the coordinates must satisfy equation of plane.

$\therefore$ Putting $x = 2\lambda + 3$, $y = 2 - \lambda$, $z = \lambda + 1$ in Eq. (i), we get

$$2(2\lambda + 3) - (2 - \lambda) + (\lambda + 1) + 1 = 0$$

$\Rightarrow$ $4\lambda + 6 - 2 + \lambda + \lambda + 1 + 1 = 0$

$\Rightarrow$ $6\lambda + 6 = 0$

$6\lambda = -6$ or $\lambda = -1$

Putting, $\lambda = -1$ in eq. (ii), we get coordinates of foot of perpendicular on Q as

$$Q(2\lambda + 3, 2 - \lambda, \lambda + 1) = Q(1, 3, 0)$$

Also, perpendicular distance PQ is given by using distance formula as

$$PQ = \sqrt{(1-3)^2 + (3-2)^2 + (0-1)^2}$$

$$\left[\therefore \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{4+1+1} = \sqrt{6} \text{ units}$$

Finally, we find image of point $P(3, 2, 1)$ from the figure, we see that point Q is the mid-point of line PR.

$\therefore$ Using mid-point formula, we have

$$\left(\frac{x_1+3}{2}, \frac{y_1+2}{2}, \frac{z_1+1}{2}\right) = (1, 3, 0) \qquad \left[\therefore \text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)\right]$$

$\Rightarrow$ $\frac{x_1+3}{2} = 1,\ \frac{y_1+2}{2} = 3,\ \frac{z_1+1}{2} = 0$

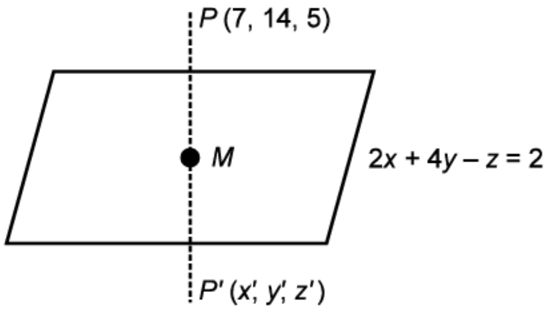
$\Rightarrow$ $x_1 = 2 - 3,\ y_1 = 6 - 2,\ z_1 = 0 - 1$

or $\quad R(x_1, y_1, z_1) = (-1, 4, -1)$

Hence, the image of point $P(3, 2, 1)$ is $(-1, 4, -1)$.

S62. Let $P'(x, y, z)$ be the image of the point $P(7, 14, 5)$. The equation of line PM in plane is given by

$$\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} \quad [\because \text{DR's of a line normal to a plane are same as that of the plane}]$$



Let $\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} = \lambda \quad$ Say

$\Rightarrow \quad x = 2\lambda + 7,\ y = 4\lambda + 14,\ z = -\lambda + 5$

Let coordinates of point M be

$(2\lambda + 7, 4\lambda + 14, -\lambda + 5) \quad \ldots \text{(i)}$

Since, M lies on the given plane $2x + 4y - z = 2$

$\therefore\ 2(2\lambda + 7) + 4(4\lambda + 14) - (-\lambda + 5) = 2$

$\Rightarrow \quad 4\lambda + 14 + 16\lambda + 56 + \lambda - 5 = 2$

$\Rightarrow \quad 21\lambda + 63 = 0 \Rightarrow \lambda = \frac{-63}{21} = -3$

Putting, $\lambda = -3$ in eq. (i) we get, $M = (1, 2, 8)$

$\therefore$ Foot of perpendicular M is $(1, 2, 8)$

Also, Length of perpendicular, PM = Distance between points P and M

$$= \sqrt{(1-7)^2 + (2-14)^2 + (8-5)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{36 + 144 + 9} = \sqrt{189} \text{ units}$$

$\therefore\ M$ = Mid-point of P and P'

$$\Rightarrow \quad (1, 2, 8) = \left(\frac{x'+7}{2}, \frac{y'+14}{2}, \frac{z'+5}{2}\right)$$

$$\Rightarrow \quad \frac{x'+7}{2} = 1,\ \frac{y'+14}{2} = 2 \text{ and } \frac{z'+5}{2} = 8$$

$\Rightarrow \quad x' = 2 - 7,\ y' = 4 - 14 \text{ and } z' = 16 - 5$

$\therefore \quad x' = -5,\ y' = -10,\ z' = 11$

Hence, image of point $P(7, 14, 5)$ is $P'(-5, -10, 11)$.

S63. Let $P(2, 3, 4)$ be the given point and given equation of line be

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Any random point T on the given lines is calculated as

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2} = \lambda \quad \text{[Say]}$$

or $x = 3\lambda - 3$, $y = 6\lambda + 2$, $z = 2\lambda$

$\therefore$ Coordinates of T are $(3\lambda - 3, 6\lambda + 2, 2\lambda)$

Now, DR's of line PT are

$(3\lambda - 3 - 2, 6\lambda + 2 - 3, 2\lambda - 4) = (3\lambda - 5, 6\lambda - 1, 2\lambda - 4)$

Since, the line PT is parallel to the plane

$$3x + 2y + 2z - 5 = 0$$

$$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$$

[$\because$ Line is parallel to the plane, therefore normal to the plane is perpendicular to the line]

where, $a_1 = 3\lambda - 5$, $b_1 = 6\lambda - 1$, $c_1 = 2\lambda - 4$

and $a_2 = 3$, $b_2 = 2$, $c_2 = 2$

[$\because$ a_2, b_2, c_2 are DR's of plane whose equation is $3x + 2y + 2z - 5 = 0$]

$\therefore$ We get,

$$3(3\lambda - 5) + 2(6\lambda - 1) + 2(2\lambda - 4) = 0$$

$$\Rightarrow \quad 9\lambda - 15 + 12\lambda - 2 + 4\lambda - 8 = 0$$

$$\Rightarrow \quad 25\lambda - 25 = 0$$

$$\Rightarrow \quad 25\lambda = 25$$

or $\lambda = 1$

$\therefore$ Coordinates of $T = (3\lambda - 3, 6\lambda + 2, 2\lambda) = (0, 8, 2)$ [Put $\lambda = 1$]

Finally, the required distance between points $P(2, 3, 4)$ and $T(0, 8, 2)$ is given by

$$PT = \sqrt{(0-2)^2 + (8-3)^2 + (2-4)^2}$$

[$\because$ $(x_1, y_1, z_1) = (2, 3, 4)$ and $(x_2, y_2, z_2) = (0, 8, 2)$]

$$= \sqrt{4 + 25 + 4} = \sqrt{33} \text{ units.}$$

S64. Given equation of line and plane are

$$\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$

and $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$

$$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$$

and $\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$ $\left[\text{Put } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}\right]$

Above equation in Cartesian form can be written as

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} \quad \text{... (i)}$$

and $\quad x - y + z = 5 \quad$ ... (ii)

First, we solve Eqs. (i) and (ii) and find their point of intersection. Let the point of intersection be Q.

Let $\quad \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda \quad$ [say]

$\Rightarrow \quad \frac{x-2}{3} = \lambda, \ \frac{y+1}{4} = \lambda, \ \frac{z-2}{2} = \lambda$

$\Rightarrow \quad x = 3\lambda + 2, \ y = 4\lambda - 1, \ z = 2\lambda + 2$

$\therefore$ Any point Q on the given line is $Q(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$

$\because$ Point Q lie on the plane, so coordinates of Q satisfies Eq. (ii).

$\therefore \ (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$

$3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 = 5$

$\Rightarrow \quad \lambda = 0$

Putting $\lambda = 0$ in $Q(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$, we get the point of intersection as $Q(2, -1, 2)$.

Now, using distance formula, the required distance

$$PQ = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{9 + 16 + 144} = \sqrt{169} = 13 \text{ Units}$$

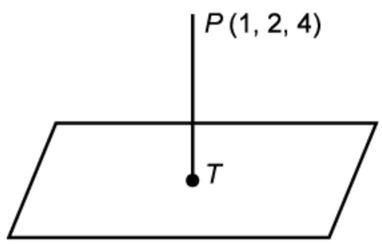
Hence, the required distance = 13 units.

S65. Let PT be the perpendicular drawn from the point $P(1, 2, 4)$ to the plane whose equation is given by

$$2x + y - 2z + 3 = 0 \quad \text{... (i)}$$

We have to find equation, length and coordinates of foot of perpendicular.

$$2x + y - 2z + 3 = 0$$



From Eq. (i), DR's of plane are 2, 1, – 2. Since, the line PT is normal to the plane, so DR's of line normal to plane are 2, 1, – 2.

$\therefore$ Equation of line PT where $P(1, 2, 4)$ is given as

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} \quad \text{... (ii)}$$

$\because$ Equation of line passing through one point

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Let $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = \lambda$ [Say]

$\Rightarrow$ $x = 2\lambda + 1,\ y = \lambda + 2,\ z = -2\lambda + 4$

$\therefore$ Coordinates of any variable point on plane is $T(2\lambda + 1, \lambda + 2, -2\lambda + 4)$ ... (iii)

From the figure, T lies on the given plane, so we put $x = 2\lambda + 1$, $y = \lambda + 2$ and $z = -2\lambda + 4$ in Eq. (i).

$$2(2\lambda + 1) + (\lambda + 2) - 2(-2\lambda + 4) + 3 = 0$$

$$\Rightarrow \quad 4\lambda + 2 + \lambda + 2 + 4\lambda - 8 + 3 = 0$$

$$\Rightarrow \quad 9\lambda - 1 = 0$$

$$\Rightarrow \quad \lambda = \frac{1}{9}$$

Putting value of λ in Eq. (iii), we get the foot of perpendicular.

$$T\left(\frac{2}{9}+1, \frac{1}{9}+2, -\frac{2}{9}+4\right) = T\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$$

Also, length of perpendicular PT = distance between points P and T

$$= \sqrt{\left(1-\frac{11}{9}\right)^2 + \left(2-\frac{19}{9}\right)^2 + \left(4-\frac{34}{9}\right)^2}$$

$$\left[(x_1, y_1, z_1) = (1, 2, 4) \text{ and } (x_2, y_2, z_2) = \left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)\right]$$

$$= \sqrt{\left(-\frac{2}{9}\right)^2 + \left(-\frac{1}{9}\right)^2 + \left(\frac{2}{9}\right)^2}$$

$$= \sqrt{\frac{4}{81} + \frac{1}{81} + \frac{4}{81}} = \sqrt{\frac{9}{81}} = \sqrt{\frac{1}{9}}$$

$$= \frac{1}{3} \text{ unit}$$

Finally, the equation of perpendicular PT where, $P(1, 2, 4)$ and $T\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$ is given as

$$\frac{x-1}{\frac{11}{9}-1} = \frac{y-2}{\frac{19}{9}-2} = \frac{z-4}{\frac{34}{9}-4}$$

$$\left[\text{Using two points from } \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}\right]$$

$$\Rightarrow \quad \frac{x-1}{\left(\frac{2}{9}\right)} = \frac{y-2}{\left(\frac{1}{9}\right)} = \frac{z-4}{\left(-\frac{2}{9}\right)}$$

$$\Rightarrow \quad \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2}.$$

S66. Let $P(-2, 3, -4)$ be the given points and the given line is

$$\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$$

or
$$\frac{x+2}{3} = \frac{2\left(y+\frac{3}{2}\right)}{4} = \frac{3\left(z+\frac{4}{3}\right)}{5}$$

or
$$\frac{x+2}{3} = \frac{y+\frac{3}{2}}{2} = \frac{z+\frac{4}{3}}{\left(\frac{5}{3}\right)} \quad \text{... (i)}$$

Now, any random point T on the given line (i) is calculated as

Let
$$\frac{x+2}{3} = \frac{y+\frac{3}{2}}{2} = \frac{z+\frac{4}{3}}{\left(\frac{5}{3}\right)} = \lambda \ \text{[Say]}$$

$$\Rightarrow \quad \frac{x+2}{3} = \lambda,\ \frac{y+\frac{3}{2}}{2} = \lambda,\ \frac{z+\frac{4}{3}}{\left(\frac{5}{3}\right)} = \lambda$$

$$\Rightarrow \quad x = 3\lambda - 2,\ y = \frac{4\lambda-3}{2},\ z = \frac{5\lambda-4}{3}$$

$\therefore$ Coordinates of T are $\left(3\lambda - 2, \frac{4\lambda-3}{2}, \frac{5\lambda-4}{3}\right)$

Now DR's of line PT are

$$\left(3\lambda - 2 + 2, \frac{4\lambda-3}{2} - 3, \frac{5\lambda-4}{3} + 4\right) = \left(3\lambda, \frac{4\lambda-9}{2}, \frac{5\lambda+8}{3}\right)$$

Since, the line is parallel to the plane

$$4x + 12y - 3z + 1 = 0$$

$\therefore$ We have, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

where, $a_1 = 3\lambda, \quad b_1 = \dfrac{4\lambda - 9}{2}, \quad c_1 = \dfrac{5\lambda + 8}{3}$ [DR's of line]

and $a_2 = 4, \quad b_2 = 12, \quad c_2 = -3$ [DR's of plane]

$\therefore$ We get,

$$4(3\lambda) + 12\left(\frac{4\lambda - 9}{2}\right) - 3\left(\frac{5\lambda + 8}{3}\right) = 0$$

$\Rightarrow$ $12\lambda + 24\lambda - 54 - 5\lambda - 8 = 0$

$\Rightarrow$ $31\lambda - 62 = 0$

$\Rightarrow$ $31\lambda = 62$

or $\lambda = 2$

$\therefore$ Coordinates of T are

$$\left(3\lambda - 2, \frac{4\lambda - 3}{2}, \frac{5\lambda - 4}{3}\right) = \left(4, \frac{5}{2}, 2\right)$$

$\therefore$ The required distance = Distance between the points $P(-2, 3, -4)$ and $T\left(4, \frac{5}{2}, 2\right)$.

$\therefore$ Using distance formula, we have

$$d = \sqrt{(4+2)^2 + \left(\frac{5}{2} - 3\right)^2 + (2+4)^2}$$

$$= \sqrt{36 + \frac{1}{4} + 36} = \sqrt{\frac{144 + 1 + 144}{4}}$$

$$= \sqrt{\frac{289}{4}} = \frac{17}{2} \text{ units}$$

Date: 19/10/2021

Q1. Write the equation of line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through point (1, 2, 3).

Q2. Find the distance of point (2, 3, 4) from x-axis.

Q3. If a unit vector $\hat{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ.

Q4. Find the value of λ, such that the line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{-4}$ is perpendicular to the plane $3x-y-2z=7$.

Q5. If the equation of line AB are $\frac{3-x}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, write the direction ratios of line parallel to above line AB.

Q6. If $P = (1, 5, 4)$ and $Q = (4, 1, -2)$, find the direction ratios of PQ.

Q7. Write the direction cosines of the line parallel to line.

$$\frac{4-x}{2}=\frac{y+3}{5}=\frac{z+2}{6}.$$

Q8. The equation of a line is $\frac{2x-5}{4}=\frac{y+4}{3}=\frac{6-z}{6}$. Find the direction cosines of the line parallel to this line.

Q9. Write the direction cosines of a line parallel to the line $\frac{3-x}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$.

Q10. Write the vector equation of the line given by

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}.$$

Q11. Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

Q12. Find the direction cosines of a line parallel to line.

$$\frac{4-x}{2}=\frac{y+3}{2}=\frac{z+2}{1}.$$

Q13. If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, find the value of λ.

Q14. Show that the lines $\vec{r} = 3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$; $\vec{r} = 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$ are intersecting. Hence, find their point of intersection.

Q15. Show that the following pair of lines do not intersect each other.

$$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5} \quad \text{and} \quad \frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$$

Q16. A line passes through $(2, -1, 3)$ and is perpendicular to the line $\vec{r}=(\hat{i}+\hat{j}+\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$. Obtain its equation.

Q17. Find the direction cosines of the line $\frac{x-2}{2}=\frac{2y-5}{-3}$, $z=-1$. Also, find the vector equation of the line.

Q18. Find the vector equation of a line passing through a point with position vector $2\hat{i}-\hat{j}+\hat{k}$ and parallel to the line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. Also, find the Cartesian equivalent of this equation.

Q19. Show that the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Find their point of intersection.

Q20. Find the angle between the following pair of lines

$$\frac{x-2}{3}=\frac{y+1}{-2},\ z=2 \quad \text{and} \quad \frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}.$$

Q21. Find the angle between following pair of lines

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \quad \text{and} \quad \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$$

and check whether the lines are parallel or perpendicular.

Q22. Find the shortest distance between the lines whose vector equations are

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=(2\hat{i}+4\hat{j}+5\hat{k})+\mu(4\hat{i}+6\hat{j}+8\hat{k})$

Q23. Find the shortest distance between the lines

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$.

Q24. Find the shortest distance between the lines

$$\vec{r}=(4\hat{i}-\hat{j})+\lambda(\hat{i}+2\hat{j}-3\hat{k}) \quad \text{and} \quad \vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}+4\hat{j}-5\hat{k}).$$

Q25. Find the value of λ, so that the lines

$$\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2} \quad \text{and} \quad \frac{x+1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{7}$$

are perpendicular to each other.

Q26. Find the value of λ, so that the lines

$$\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11} \quad \text{and} \quad \frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$$

are perpendicular to each other.

Q27. Find the angle between pair of lines given by

$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\vec{r}=7\hat{i}-6\hat{j}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$.

Q28. Show that the distance *d* from point *P* to the line *l* having equation $\vec{r}=\vec{a}+\lambda\vec{b}$ is given by $d=\frac{|\vec{b}\times\overrightarrow{PQ}|}{|\vec{b}|}$, where *Q* is any point on the line *l*.

Q29. Find the value of λ, so that following lines are perpendicular to each other

$$\frac{x+5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1} \quad \text{and} \quad \frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}.$$

Q30. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$
$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Q31. Find the shortest distance between lines

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$$
$$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$$

Q32. Find the shortest distance between lines whose vector equations are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}, \quad \vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

Q33. Find shortest distance between lines

$$\vec{r} = (1+2\lambda)\hat{i} + (1-\lambda)\hat{j} + \lambda\hat{k}$$
$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Q34. Find shortest distance between lines

$$\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (\lambda+1)\hat{k}$$
$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

Q35. Find the shortest distance between lines l_1 and l_2 whose vector equations are given below

$$l_1 : \vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \qquad l_2 : \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

Q36. Find the shortest distance between the lines

$$l_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$$
$$l_2 : \frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}.$$

Q37. Find the shortest distance between lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

Q38. Find the length of the perpendicular from the point (1, 2, 3) to the line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}.$$

Q39. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P (1, 3, 3).

Q40. Vertices B and C of ΔABC lie along the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4}$. Find the area of the triangle given that A has coordinates (1, – 1, 2) and line segment BC has length 5.

Q41. Find the length and foot of perpendicular drawn from the point (2, – 1, 5) to line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}.$$

Q42. Find the vector and Cartesian equations of line passing through point (1, 2, – 4) and perpendicular to the lines

$$\frac{x-8}{3}=\frac{y-19}{-16}=\frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}.$$

Q43. Find the equation of the perpendicular drawn from the point (3, – 1, 11) to line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also, find the coordinates of foot of perpendicular and the length of perpendicular.

Q44. Find the perpendicular distance of point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$.

Also, find the coordinates of foot of perpendicular and equation of perpendicular.

Q45. Find the equation of line passing through points *A* (0, 6, – 9) and *B* (– 3, – 6, 3). If *D* is the foot of perpendicular drawn from the point *C* (7, 4, – 1) on the line *AB*, then find the coordinates of point *D* and equation of line *CD*.

Q46. Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Also, find the length of the perpendicular.

Q47. Find the perpendicular distance of the point (2, 3, 4) from the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

Also find coordinates of foot of perpendicular.

Q48. Write the vector equations of following lines and hence find the distance between them

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}, \quad \frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}.$$

Q49. Find the image of the point (1, 6, 3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, write the equation of the line joining the given points and its image and find the length of segment joining the given point and its image.

Q50. The points *A* (4, 5, 10), *B* (2, 3, 4) and *C* (1, 2, – 1) are three vertices of parallelogram *ABCD*. Find the vector equations of sides *AB* and *BC* and also find coordinates of point *D*.

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S1. The line passing through (x_1, y_1, z_1) and parallel to the given line is $\frac{x-x_1}{-3}=\frac{y-y_1}{2}=\frac{z-z_1}{6}$. Use this result and simplify it.

Given equation of line is written as

$$\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6} \qquad \text{... (i)}$$

Since, required line is parallel to given line.

So, DR's of required line will be proportional to – 3, 2 and 6.

$\therefore$ Equation of line passing through point (1, 2, 3) and parallel to Eq. (i) is

$$\frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{6}$$

S2. Let any point on x-axis be P (2, 0, 0). Then, distance of point B (2, 3, 4) from P is given as

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Here, $(x_1, y_1, z_1) = (2, 0, 0)$

and $(x_2, y_2, z_2) = (2, 3, 4)$

$$= \sqrt{(2-2)^2+(0-3)^2+(0-4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

S3. $$\cos^2\frac{\pi}{3}+\cos^2\frac{\pi}{4}+\cos^2\theta = 1 \qquad [\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$$

$\Rightarrow$ $\frac{1}{4}+\frac{1}{2}+\cos^2\theta = 1$ $\Rightarrow$ $\cos^2\theta = \frac{1}{4}$

$\Rightarrow$ $\cos\theta = \pm\frac{1}{2}$ $\therefore$ $\cos\theta = \frac{1}{2}$, as θ is an acute angle.

$\Rightarrow$ $\theta = \frac{\pi}{3}$

S4. Given that, line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{-4}$ is perpendicular to plane $3x - y - 2z = 7$.

$\therefore$ $$\frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2}$$

[$\because$ When a line is perpendicular to a plane, their DR's are proportional]

$\Rightarrow$ $2 = -\lambda \Rightarrow -\lambda = 2$

or $\lambda = -2$

S5. Given equation of line can be written as

$$\frac{x-3}{-1} = \frac{y+2}{-2} = \frac{z-5}{4}$$

$\therefore$ DR's of line parallel to above line are – 1, – 2, 4. [Since, parallel lines have same DR's].

S6. Given points are $P(1, 5, 4)$ and $Q(4, 1, -2)$.

$\therefore$ Direction ratios of $PQ = 4-1, 1-5, -2-4$

$= 3, -4, -6$

[$\because$ DR's of a line $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$]

S7. DC's of a line parallel to

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \text{ is}$$

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Given equation of line is written as

$$\frac{x-4}{-2} = \frac{y+3}{5} = \frac{z+2}{6} \qquad \text{... (i)}$$

$\therefore$ Direction cosines of a line parallel to Eq. (i) are

$$\frac{-2}{\sqrt{(-2)^2+(5)^2+(6)^2}}, \frac{5}{\sqrt{(-2)^2+(5)^2+(6)^2}}, \frac{6}{\sqrt{(-2)^2+(5)^2+(6)^2}}$$

$$= \frac{-2}{\sqrt{4+25+36}}, \frac{5}{\sqrt{4+25+36}}, \frac{6}{\sqrt{4+25+36}}$$

$$= \frac{-2}{\sqrt{65}}, \frac{5}{\sqrt{65}}, \frac{6}{\sqrt{65}}$$

S8. DC's of a line parallel to

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \text{ are}$$

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Given equation of line can be written as

$$\frac{x-\left(\frac{5}{2}\right)}{2} = \frac{y+4}{3} = \frac{z-6}{-6}$$

$\therefore$ Direction cosines of the line parallel to above equation are

$$\frac{2}{\sqrt{(2)^2+(3)^2+(-6)^2}}, \quad \frac{3}{\sqrt{(2)^2+(3)^2+(-6)^2}}, \quad \frac{-6}{\sqrt{(2)^2+(3)^2+(-6)^2}}$$

$$= \frac{2}{\sqrt{49}}, \frac{3}{\sqrt{49}}, \frac{-6}{\sqrt{49}}$$

$$= \frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$$

S9.

Given equation of line is written as

$$\frac{x-3}{-3} = \frac{y+2}{-2} = \frac{z+2}{6} \qquad \text{... (i)}$$

$\therefore$ DC's of line parallel to Eq. (i) are given by

$$\frac{-3}{\sqrt{(-3)^2+(-2)^2+(6)^2}}, \quad \frac{-2}{\sqrt{(-3)^2+(-2)^2+(6)^2}}, \quad \frac{6}{\sqrt{(-3)^2+(-2)^2+(6)^2}}$$

$$\left[\begin{array}{l} \because \text{ DC's of a line parallel to } \dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} \\ \quad \text{is } \dfrac{a}{\sqrt{a^2+b^2+c^2}}, \dfrac{b}{\sqrt{a^2+b^2+c^2}}, \dfrac{c}{\sqrt{a^2+b^2+c^2}} \end{array}\right]$$

$$= \frac{-3}{\sqrt{49}}, \frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}} = -\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}$$

S10. Given equation of line in Cartesian form is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$$

The point on the line is (5, –4, 6) and DR's are (3, 7, 2), we know vector equation of a line, if point is *a* and direction of a line is *b*, is

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad \left[\text{Using } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}\right]$$

Here, a = (5, –4, 6) and b = (3, 7, 2)

Its equation in vector form is

$$\vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} + 2\hat{k}\right)$$

S11. If two lines are parallel, then they both have same direction ratios. Use this result and simplify it.

Since, the required line is parallel to the line

$$\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$$

or $$\frac{x+3}{3} = \frac{y-4}{-5} = \frac{z+8}{6}$$

$\therefore$ DR's of both lines are proportional to each other.

The required equation of the line passing through $(-2, 4, -5)$ is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$.

S12. Given equation of line can be written as

$$\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$$

$\therefore$ DC's of line parallel to above line given by

$$\frac{-2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{1}{\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$$= \frac{-2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{1}{\sqrt{4+4+1}}$$

or $$\frac{-2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}} = -\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$$

Hence, required DC's of a line parallel to the given line is $\left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$.

Note: Before we can use the DR's of a line, first we ensure that coefficient of x, y and z are unity with positive sign.

S13. The equation of the line passing through $A(-1, 3, 2)$ and $B(-4, 2, -2)$ is

$$\frac{x+1}{-4+1} = \frac{y-3}{2-3} = \frac{z-2}{-2-2}$$

$$\Rightarrow \quad \frac{x+1}{-3} = \frac{y-3}{-1} = \frac{z-2}{-4}$$

$$\Rightarrow \quad \frac{x+1}{3} = \frac{y-3}{1} = \frac{z-2}{4} \quad \text{... (i)}$$

If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, then the coordinates of C must satisfy equation (i). Therefore,

$$\frac{5+1}{3} = \frac{5-3}{1} = \frac{\lambda-2}{4}$$

$$\Rightarrow \quad \frac{\lambda-2}{4} = 2$$

$$\Rightarrow \quad \lambda = 10.$$

S14. Given, vector lines are

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \quad \text{and} \quad \vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

It's Cartesian form are

$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = r \text{ [Say]} \quad \text{... (i)}$$

and
$$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z-0}{6} = p \text{ [Say]} \quad \text{... (ii)}$$

Let $(r+3, 2r+2, 2r-4)$ and $(3p+5, 2p-2, 6p)$ are the any points on the lines Eq. (i) and Eq. (ii) respectively.

If this line intersect each other, then

$$r + 3 = 3p + 5 \Rightarrow r - 3p = 2 \quad \text{... (iii)}$$

$$2r + 2 = 2p - 2 \Rightarrow r - p = -2 \quad \text{... (iv)}$$

and
$$2r - 4 = 6p \Rightarrow 2r - 6p = 4 \Rightarrow r - 3p = 2 \quad \text{... (v)}$$

Now, subtracting Eq. (v) from Eq. (iv),

$$2p = -4 \Rightarrow p = -2$$

Putting $p = -2$ in Eq. (iv), we get

$$r - (-2) = -2 \Rightarrow r = -4$$

Using Eq. (iii), we get

$$-4 - 3p = 2 \Rightarrow -3p = 6 \Rightarrow p = -2$$

$\therefore$ Any point on line Eq. (i) is

$(-4 + 3, -8 + 2, -8 - 4) = (-1, -6, -12)$

and any point on line Eq. (ii) is

$(-6 + 5, -4 - 2, -12) = (-1, -6, -12)$

$\because$ Both point are same *i.e.*, both lines intersect each other at point $(-1, -6, -12)$.

S15. We have,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \quad \text{(Say)}$$

$\Rightarrow$
$$x = 3\lambda + 1, \quad y = 2\lambda - 1, \quad z = 5\lambda + 1$$

So, the coordinates of a general point on this line are $(3\lambda + 1, 2\lambda - 1, 5\lambda + 1)$.

The equation of the second line is

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \quad \text{(Say)}$$

$\Rightarrow \quad x = 4\mu - 2,\ y = 3\mu + 1,\ z = -2\mu - 1.$

So, the coordinates of a general point on this line are $(4\mu - 2, 3\mu + 1, -2\mu - 1)$.

If the line intersect, then they have a common point. So, for some values of λ and μ, we must have

$$3\lambda + 1 = 4\mu - 2,\ 2\lambda - 1 = 3\mu + 1 \text{ and } 5\lambda + 1 = -2\mu - 1$$

$\Rightarrow \quad 3\lambda - 4\mu = -3 \quad \ldots \text{(i)}$

$2\lambda - 3\mu = 2 \quad \ldots \text{(ii)}$

and $\quad 5\lambda + 2\mu = -2 \quad \ldots \text{(iii)}$

Solving Eq. (i) and Eq.(ii), we obtain $\lambda = -17$ and $\mu = -12$.

These values of λ and μ do not satisfy the third equation.

Hence, the given lines do not intersect.

S16. The required line is perpendicular to the lines which are parallel to vectors $\vec{b_1} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$ respectively.

So, it is parallel to the vector $\vec{b} = \vec{b_1} \times \vec{b_2}$

Now, $$\vec{b} = \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 6\hat{k}$$

Thus, the required line passes through the point $(2, -1, 3)$ and is parallel to the vector $\vec{b} = -6\hat{i} - 3\hat{j} + 6\hat{k}$.

So, its vector equation is

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(-6\hat{i} - 3\hat{j} + 6\hat{k}\right)$$

or $$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + \hat{j} - 2\hat{k}\right), \text{ where } \mu = -3\lambda.$$

S17. The given line is $\quad \frac{x-2}{2} = \frac{2y-5}{-3}, \quad z = -1$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{2y-5}{-3} = \frac{z+1}{0}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-\frac{5}{2}}{\frac{-3}{2}} = \frac{z+1}{0}$$

This shows that the given line passes through the point $\left(2, \frac{5}{2}, -1\right)$ and has direction ratios proportional to $2, -\frac{3}{2}, 0$. So, its direction cosines are

$$\frac{2}{\sqrt{2^2+\left(-\frac{3}{2}\right)^2+0^2}}, \frac{-\frac{3}{2}}{\sqrt{2^2+\left(-\frac{3}{2}\right)^2+0^2}}, \frac{0}{\sqrt{2^2+\left(-\frac{3}{2}\right)^2+0^2}}$$

or $$\frac{2}{\frac{5}{2}}, \frac{-\frac{3}{2}}{\frac{5}{2}}, 0 \quad \text{or} \quad \frac{4}{5}, \frac{-3}{5}, 0$$

Thus given line passes through the point having position vector $\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}$ and is parallel to the vector $\vec{b} = 2\hat{i} - \frac{3}{2}\hat{j} + 0\hat{k}$.

So, it's vector equation is $\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - \frac{3}{2}\hat{j} + 0\hat{k}\right)$.

S18. Let A, B, C be the points with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$ respectively. We have to find the equation of a line passing through the point A and parallel to $\vec{BC}$. We have,

$$\vec{BC} = \text{Position vector of } C - \text{Position vector of } B$$

$$\Rightarrow \quad \vec{BC} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right) - \left(-\hat{i} + 4\hat{j} + \hat{k}\right)$$

$$= 2\hat{i} - 2\hat{j} + \hat{k}$$

We know that the equation of a line passing through a point $\vec{a}$ and parallel to $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Here, $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

So, the equation of the required line is

$$\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right) \quad \ldots \text{(i)}$$

Reduction to Cartesian form:

Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in (i), we obtain

$$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (1 + \lambda)\hat{k}$$

$$\Rightarrow \quad x = 2 + 2\lambda,\ y = -1 - 2\lambda,\ z = 1 + \lambda$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$$ which is the Cartesian form of equation (i).

S19. We have

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \quad \text{(Say)}$$

$\Rightarrow$ $$x = 2\lambda + 1,\ y = 3\lambda + 2 \text{ and } z = 4\lambda + 3$$

So, the coordinates of a general point on this line are $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$.

The equation of second line is

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu \quad \text{(Say)}$$

$\Rightarrow$ $$x = 5\mu + 4,\ y = 2\mu + 1,\ z = \mu$$

So, the coordinates of a general point on this line are $(5\mu + 4, 2\mu + 1, \mu)$.

If the lines intersect, then they have a common point. So, for some values of λ and μ, we must have

$$2\lambda + 1 = 5\mu + 4, \quad 3\lambda + 2 = 2\mu + 1 \quad \text{and} \quad 4\lambda + 3 = \mu$$

or $$2\lambda - 5\mu = 3, \quad 3\lambda - 2\mu = -1, \quad 4\lambda - \mu = -3.$$

Solving first two of these two equations, we get

$$\lambda = -1 \quad \text{and} \quad \mu = -1.$$

Since $\lambda = -1$ and $\mu = -1$ satisfy the third equation. So the given lines intersect.

Putting $\lambda = -1$ in $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$, the coordinates of the required point of intersection are $(-1, -1, -1)$.

S20. The given equations are not in the standard form. The equations of the given lines can be written as

$$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0} \qquad \text{... (i)}$$

and $$\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2} \qquad \text{... (ii)}$$

Let $\vec{b_1}$ and $\vec{b_2}$ be vectors parallel to Eq. (i) and Eq. (ii) respectively. Then,

$$\vec{b_1} = 3\hat{i} - 2\hat{j} + 0\hat{k} \text{ and } \vec{b_2} = \hat{i} + \frac{3}{2}\hat{j} + 2\hat{k}.$$

If θ is the angle between the given lines, then

$$\cos\theta = \frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}||\vec{b_2}|} = \frac{(3)(1) + (-2)\left(\frac{3}{2}\right) + (0)(2)}{\sqrt{3^2 + (-2)^2 + 0^2}\sqrt{1^2 + \left(\frac{3}{2}\right)^2 + 2^2}}$$

$$= \frac{3-3+0}{\sqrt{13}\frac{\sqrt{29}}{2}} = 0$$

$\Rightarrow$ $$\theta = \frac{\pi}{2}$$

S21. Above equations can be written as

$$\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3} \quad \text{... (i)}$$

and $$\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4} \quad \text{... (ii)}$$

Comparing Eqs. (i) and (ii) with one point form of equation of line

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

we get

$$a_1 = 2, \quad b_1 = 7, \quad c_1 = -3$$

and $$a_2 = -1, \quad b_2 = 2, \quad c_2 = 4$$

Now, we know that angle between two lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\cdot\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$\therefore$ $$\cos\theta = \frac{(2)(-1)+(7)(2)+(-3)(4)}{\sqrt{(2)^2+(7)^2+(-3)^2}\cdot\sqrt{(-1)^2+(2)^2+(4)^2}}$$

$\Rightarrow$ $$\cos\theta = \frac{-2+14-12}{\sqrt{62}\times\sqrt{21}} = \frac{0}{\sqrt{62}\times\sqrt{21}} = 0$$

$\Rightarrow$ $$\cos\theta = \cos\frac{\pi}{2} \quad \left[\because 0 = \cos\frac{\pi}{2}\right]$$

$\Rightarrow$ $$\theta = \frac{\pi}{2}$$

Hence, angle between them is $\frac{\pi}{2}$. Since, angle between the two lines is $\frac{\pi}{2}$, therefore the above pair of lines are perpendicular to each other.

S22. The given lines are

$$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(2\hat{i}+3\hat{j}+4\hat{k}) \quad \text{...(i)}$$

and $$\vec{r} = (2\hat{i}+4\hat{j}+5\hat{k}) + 2\mu(2\hat{i}+3\hat{j}+4\hat{k}) \quad \text{...(ii)}$$

Equation (ii) can re-written as

$$\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu'\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \quad ...(iii)$$

where $\mu' = 2\mu$.

These two lines passes through the points having position vectors $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k}$ respectively and both are parallel to the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

$$\text{Shortest distance} = \frac{\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|}{|\vec{b}|} \quad ... (iv)$$

We have, $\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right) \times \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$

$$\Rightarrow \quad \left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix} = (8-6)\hat{i} - (4-4)\hat{j} + (3-4)\hat{k} = 2\hat{i} - 0\hat{j} - \hat{k}$$

$$\Rightarrow \quad |\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}| = \sqrt{4+0+1} = \sqrt{5} \text{ and } |\vec{b}| = \sqrt{4+9+16} = \sqrt{29}$$

Substituting the values of $\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|$ and $|\vec{b}|$ in (iv), we get

$$\text{Shortest distance} = \frac{\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|}{|\vec{b}|}$$

$$\therefore \quad \text{Shortest distance} = \frac{\sqrt{5}}{\sqrt{29}}.$$

S23. The equations of two given lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad ... (i)$$

and

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \quad ... (ii)$$

Line (i) passes through (1, 2, 3) and has direction ratios proportional to 2, 3, 4. So, its vector equation is

$$\vec{r} = \vec{a_1} + \lambda\vec{b_1} \quad ... (iii)$$

where, $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Line (ii) passes through (2, 4, 5) and has direction ratio proportional to 3, 4, 5. So, its vector equation is

$$\vec{r} = \vec{a_2} + \mu\vec{b_2} \qquad \text{... (iv)}$$

where, $\vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$.

The shortest distance between the lines (iii) and (iv) is given by :

We have, $\vec{a_2} - \vec{a_1} = (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} + 2\hat{j} + 2\hat{k}$

and $$\vec{b_2} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$\therefore$ $$|\vec{b_1} \times \vec{b_2}| = \sqrt{1+4+1} = \sqrt{6}$$

and $$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = -1 + 4 - 2 = 1.$$

$\therefore$ $$\text{Shortest distance} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| = \frac{1}{\sqrt{6}}$$

S24. We know that the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

Comparing the given equations with the equations $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ respectively, we have

$$\vec{a_1} = 4\hat{i} - \hat{j},\ \vec{a_2} = \hat{i} - \hat{j} + 2\hat{k},\ \vec{b_1} = \hat{i} + 2\hat{j} - 3\hat{k} \text{ and } \vec{b_2} = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

Now, $$\vec{a_2} - \vec{a_1} = (-4+1)\hat{i} + (-1+1)\hat{j} + (2-0)\hat{k} = -3\hat{i} + 0\hat{j} + 2\hat{k}$$

and $$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} + 0\hat{k}$$

$$\therefore\ (\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (-3\hat{i} + 0\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + 0\hat{k})$$
$$= -6 + 0 + 0 = -6$$

and, $$|\vec{b_1} \times \vec{b_2}| = \sqrt{4+1+0} = \sqrt{5}$$

$$\therefore\ \text{Shortest Distance} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| = \left| \frac{-6}{\sqrt{5}} \right| = \frac{6}{\sqrt{5}}.$$

S25. Given equations of lines are

$$\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2} \quad \dots \text{(i)}$$

and $$\frac{x+1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7} \quad \dots \text{(ii)}$$

So, Eqs. (i) and (ii) can be written as

$$\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2} \quad \dots \text{(iii)}$$

and $$\frac{x+1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-7} \quad \dots \text{(iv)}$$

Comparing (iii) and (iv), with $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

$$a_1 = -3, \quad b_1 = 2\lambda, \quad c_1 = 2$$

and $$a_2 = 3\lambda, \quad b_2 = 1, \quad c_2 = -7$$

Now, since the two lines are perpendicular.

$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow \quad -3(3\lambda) + 2\lambda(1) + 2(-7) = 0$

$\Rightarrow \quad -9\lambda + 2\lambda - 14 = 0$

$\Rightarrow \quad -7\lambda - 14 = 0$

$\Rightarrow \quad -7\lambda = 14$

$\Rightarrow \quad \lambda = -\frac{14}{7} = -2$

Hence, $\lambda = -2$.

S26. Given equations of lines are

$$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11} \quad \dots \text{(i)}$$

and $$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5} \quad \dots \text{(ii)}$$

First, we convert the above equations of lines in the symmetrical form

Eqs. (i) and (ii) can be written as

$$\frac{x-1}{-3} = \frac{7(y-2)}{2\lambda} = \frac{5(z-2)}{11}$$

or $$\frac{x-1}{-3} = \frac{y-2}{\left(\frac{2\lambda}{7}\right)} = \frac{z-2}{\left(\frac{11}{5}\right)} \quad \text{... (iv)}$$

and $$\frac{7(1-x)}{3\lambda} = \frac{y-5}{1} = \frac{(z-6)}{-5}$$

or $$\frac{x-1}{\left(-\frac{3\lambda}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5} \quad \text{... (v)}$$

Comparing (iv) and (v), with $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

$$a_1 = -3, \quad b_1 = \frac{2\lambda}{7}, \quad c_1 = \frac{11}{5}$$

and $$a_2 = -\frac{3\lambda}{7}, \quad b_2 = 1, \quad c_2 = -5$$

Since, the two lines are perpendicular so, we have

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\therefore \quad -3\left(-\frac{3\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right)(1) + \left(\frac{11}{5}\right)(-5) = 0$$

$$\Rightarrow \quad \frac{9\lambda}{7} + \frac{2\lambda}{7} - 11 = 0$$

$$\Rightarrow \quad \frac{9\lambda + 2\lambda - 77}{7} = 0$$

$$\Rightarrow \quad 11\lambda - 77 = 0$$

$$\Rightarrow \quad \lambda = \frac{77}{11} = 7$$

Hence, $\lambda = 7$.

S27. The given equation of lines are

$$\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right) \quad \text{... (i)}$$

and $$\vec{r} = \left(7\hat{i} - 6\hat{j} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right) \quad \text{... (ii)}$$

Comparing Eqs. (i) and (ii) with vector form of equations of line which is

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \quad \text{and} \quad \vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$a_1 = 2\hat{i} - 5\hat{j} + \hat{k}, \; b_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

and $a_2 = 7\hat{i} - 6\hat{j} - 6\hat{k},\ b_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

Now, we know that angle between two lines is given by

$$\cos\theta = \frac{b_1 \cdot b_2}{|b_1||b_2|}$$

$$\Rightarrow \quad \cos\theta = \frac{(3\hat{i} + 2\hat{j} + 6\hat{k})\cdot(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{(3)^2 + (2)^2 + (6)^2}\cdot\sqrt{(1)^2 + (2)^2 + (2)^2}}$$

$$\Rightarrow \quad \cos\theta = \frac{3 + 4 + 12}{\sqrt{49} \times \sqrt{9}}$$

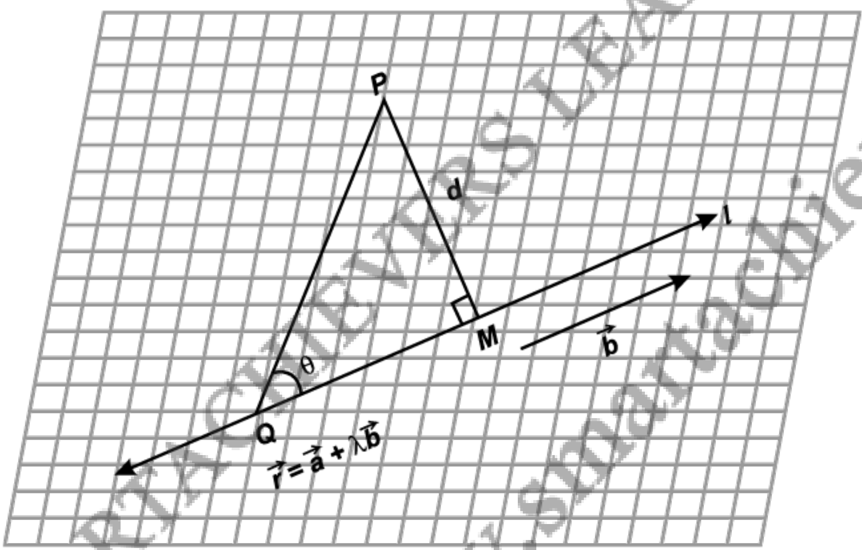
$$\therefore \quad \cos\theta = \frac{19}{7 \times 3}$$

or $$\cos\theta = \frac{19}{21}$$

Hence, angle between two lines is

$$\theta = \cos^{-1}\left(\frac{19}{21}\right).$$

S28. Let PM be perpendicular from P to line l and Q be point on it such that PQ makes an angle θ with l.



In right triangle PMQ, we have

$$\sin\theta = \frac{PM}{PQ}$$

$$\Rightarrow \quad \sin\theta = \frac{d}{PQ}$$

$$\Rightarrow \quad d = PQ\sin\theta$$

$$\Rightarrow \quad d|\vec{b}| = |\overrightarrow{PQ}||\vec{b}|\sin\theta$$

$$\Rightarrow \quad d|\vec{b}| = |\vec{b}\times\overrightarrow{PQ}|$$

$\left[\because \vec{b}\text{ is parallel to the line } l. \text{ So, angle between } \vec{b} \text{ and } \overrightarrow{PQ} \text{ is also } \theta\right]$

$$\Rightarrow \quad d = \frac{|\vec{b}\times\overrightarrow{PQ}|}{|\vec{b}|}$$

S29. Given equations of lines are

$$\frac{x+5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$$

and $$\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$$

Above equations can be written as

$$\frac{x+5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1} \qquad \text{... (i)}$$

and $$\frac{x}{1} = \frac{2\left(y+\frac{1}{2}\right)}{4\lambda} = \frac{z-1}{3}$$

or $$\frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3} \qquad \text{... (ii)}$$

Comparing Eqs. (i) and (ii) with $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, we get

$$a_1 = 5\lambda + 2,\ b_1 = -5,\ c_1 = 1$$

and $$a_2 = 1,\ b_2 = 2\lambda,\ c_2 = 3$$

Since, the two lines are perpendicular,

$$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \quad 1 \times (5\lambda + 2) + 2\lambda(-5) + 3(1) = 0$$

$$\Rightarrow \quad 5\lambda + 2 - 10\lambda + 3 = 0$$

$$\Rightarrow \quad -5\lambda + 5 = 0$$

$$\Rightarrow \quad 5\lambda = 5$$

$$\Rightarrow \quad \lambda = 1.$$

S30. Given equation of lines are

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \qquad \text{... (i)}$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}) \qquad \text{... (ii)}$$

We know that vector form of equation of line is

$$\vec{r} = \vec{a} + \lambda \vec{b} \quad \text{... (iii)}$$

Comparing Eqs. (i) and (ii) with Eq (iii), we get

$$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}, \quad \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}, \quad \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$

We know that distance between the lines are given by

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \quad \text{... (iv)}$$

Now, $$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}\,(-2-1) - \hat{j}\,(2-2) + \hat{k}\,(1+2)$$

$\Rightarrow$ $$\vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{k} \quad \text{... (v)}$$

and $$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2} \quad \text{... (vi)}$$

and $$\vec{a}_2 - \vec{a}_1 = (2\hat{i} - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k} \quad \text{... (vii)}$$

$\therefore$ From Eqs. (iv), (v) (vi) and (vii), we get

$$d = \left| \frac{(-3\hat{i} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} - 2\hat{k})}{3\sqrt{2}} \right|$$

$$= \left| \frac{-3+0-6}{3\sqrt{2}} \right| = \left| \frac{-9}{3\sqrt{2}} \right|$$

$$= \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

Hence, shortest distance = $\frac{3\sqrt{2}}{2}$ units.

S31. Given equations of lines are

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \quad \text{... (i)}$$

$$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}) \quad \text{... (ii)}$$

We know that vector form of equation of line is

$$\vec{r} = \vec{a} + \lambda \vec{b} \quad \text{... (iii)}$$

Comparing Eqs. (i) and (ii) with Eq. (iii), we get

$$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}, \ \vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = -4\hat{i} - \hat{k}, \ \vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

We know that the shortest distance between two lines is given by

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \quad \text{... (iv)}$$

Now, $$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6)$$

$\Rightarrow$ $$\vec{b}_1 \times \vec{b}_2 = 8\hat{i} + 8\hat{j} + 4\hat{k} \quad \text{... (v)}$$

and $$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(8)^2 + (8)^2 + (4)^2}$$

$$= \sqrt{64 + 64 + 16} = \sqrt{144} = 12 \quad \text{... (vi)}$$

Also, $$\vec{a}_2 - \vec{a}_1 = \left(-4\hat{i} - \hat{k}\right) - \left(6\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

$$\vec{a}_2 - \vec{a}_1 = -10\hat{i} - 2\hat{j} - 3\hat{k} \quad \text{... (vii)}$$

So, from Eqs. (iv), (v), (vi) and (vii), we get

$$d = \left| \frac{\left(8\hat{i} + 8\hat{j} + 4\hat{k}\right) \cdot \left(-10\hat{i} - 2\hat{j} - 3\hat{k}\right)}{12} \right|$$

$\Rightarrow$ $$d = \left| \frac{-80 - 16 - 12}{12} \right| = \left| \frac{-108}{12} \right| = \frac{108}{12} = 9$$

$\therefore$ Shortest distance = 9 units.

S32. Given equations are $$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \quad \text{... (i)}$$

and $$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k} \quad \text{... (ii)}$$

First we convert both equations in the vector form of

$$\vec{r} = \vec{a} + \lambda \vec{b} \quad \text{... (iii)}$$

Now, Eq. (i) can be written as

$$\vec{r} = \hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$$

or $$\vec{r} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right) \quad \text{... (iv)}$$

and Eq. (ii) can be written as

$$\vec{r} = s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k}$$

or $$\vec{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + s\left(\hat{i} + 2\hat{j} - 2\hat{k}\right) \quad \text{... (v)}$$

Now, we know that the shortest distance between the lines is given as

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| \quad \text{... (vi)}$$

Now, from Eqs. (iii), (iv) and (v), we get

$$\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k},\ b_1 = -\hat{i} + \hat{j} - 2\hat{k}$$
$$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k},\ b_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$\therefore$ $$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{i}(-2+4) - \hat{j}\,(2+2) + \hat{k}\,(-2-1)$$

$\Rightarrow$ $$\vec{b}_1 \times \vec{b}_2 = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

and $$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$
$$= \sqrt{4 + 16 + 9} = \sqrt{29}$$

Also, $$\vec{a}_2 - \vec{a}_1 = \left(\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) = \hat{j} - 4\hat{k}$$

Therefore, from Eq. (vi) shortest distance is given by

$$d = \left|\frac{\left(2\hat{i} - 4\hat{j} - 3\hat{k}\right) \cdot \left(\hat{j} - 4\hat{k}\right)}{\sqrt{29}}\right|$$

$$= \left|\frac{0 - 4 + 12}{\sqrt{29}}\right| = \frac{8}{\sqrt{29}}$$

$\therefore$ $$d = \frac{8}{\sqrt{29}} \text{ units.}$$

S33. Given equations of lines are

$$\vec{r} = (1 + 2\lambda)\hat{i} + (1 - \lambda)\hat{j} + \lambda\hat{k} \quad \text{... (i)}$$

and $$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}) \quad \text{... (ii)}$$

Eq. (i) can be written as

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \text{... (iii)}$$

Now, we know that shortest distance between two lines is given by

$$\vec{d} = \left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right| \qquad \ldots \text{(iv)}$$

Now, from Eqs. (ii) and (iii), we get

$$\vec{a}_1 = 2\hat{i} - \hat{j} - \hat{k}, \quad \vec{b}_1 = 2\hat{i} + \hat{j} + 2\hat{k}$$

and $\vec{a}_2 = \hat{i} + \hat{j}, \quad \vec{b}_2 = 2\hat{i} - \hat{j} + \hat{k}$

$$\therefore \qquad \vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & j & \hat{k} \\ 2 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(1+2) - \hat{j}(2-4) + \hat{k}(-2-2)$$

$$\Rightarrow \qquad \vec{b}_1\times\vec{b}_2 = 3\hat{i} + 2\hat{j} - 4\hat{k} \qquad \ldots \text{(v)}$$

and $|\vec{b}_1\times\vec{b}_2| = \sqrt{(3)^2+(2)^2+(-4)^2} = \sqrt{9+4+16} = \sqrt{29}$ ... (vi)

and $\vec{a}_2 - \vec{a}_1 = \left(\hat{i}+\hat{j}\right) - \left(2\hat{i} - \hat{j} - \hat{k}\right) = -\hat{i} + 2\hat{j} + \hat{k}$ ... (vii)

So, from Eqs. (iv), (v), (vi) and (vii), we get

$$d = \left|\frac{\left(3\hat{i}+2\hat{j}-4\hat{k}\right)\cdot\left(-\hat{i}+2\hat{j}+\hat{k}\right)}{\sqrt{29}}\right|$$

$$= \left|\frac{-3+4-4}{\sqrt{29}}\right| = \frac{3}{\sqrt{29}} \text{ units.}$$

S34. Given equations of lines are

$$\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (\lambda+1)\hat{k} \qquad \ldots \text{(i)}$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right) \qquad \ldots \text{(ii)}$$

Eq. (i) can be written as

or $\vec{r} = \left(\hat{i} + 2j + \hat{k}\right) + \lambda\left(\hat{i} - j + \hat{k}\right)$ ... (iii)

Comparing Eqs. (ii) and (iii) with vector form of line

$$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1, \text{ and } \vec{r} = \vec{a}_2 + \mu\vec{b}_2$$

We get, $\vec{a}_1 = 2\hat{i} - j - \hat{k}, \quad \vec{b}_1 = 2\hat{i} + j + 2\hat{k}$

and $\vec{a}_2 = \hat{i} + 2\hat{j} + \hat{k}, \quad \vec{b}_2 = \hat{i} - \hat{j} + \hat{k}$

Now, we know that shortest distance between two lines is given by

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| \quad \text{... (iv)}$$

Now, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix}$

$$= \hat{i}(1+2) - \hat{j}(2-2) + \hat{k}(-2-1)$$

$\Rightarrow$ $\vec{b}_1 \times \vec{b}_2 = 3\hat{i} - 3\hat{k}$ ... (v)

and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(3)^2 + (-3)^2}$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \quad \text{... (vi)}$$

Also, $\vec{a}_2 - \vec{a}_1 = \left(\hat{i} + 2\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} - \hat{k}\right)$

$$= -\hat{i} + 3\hat{j} + 2\hat{k} \quad \text{... (vii)}$$

From Eqs. (iv), (v), (vi) and (vii) we get,

$$d = \left|\frac{(3\hat{i} - 3\hat{k})\cdot(-\hat{i} + 3\hat{j} + 2\hat{k})}{3\sqrt{2}}\right|$$

$$= \left|\frac{-3+0-6}{3\sqrt{2}}\right| = \left|\frac{-9}{3\sqrt{2}}\right|$$

$$= \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ units.}$$

S35. Given equation of lines are

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \quad \text{... (i)}$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right) \quad \text{... (ii)}$$

Comparing above equations with vector equation

$$\vec{r} = \vec{a} + \lambda\vec{b},$$

we get $\vec{a}_1 = \hat{i} + \hat{j}$, $\vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

and $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Now, we know that the shortest distance between two lines is given by

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| \quad \text{... (iii)}$$

Now, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$$\vec{b}_1 \times \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k} \quad \text{... (iv)}$$

and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(3)^2 + (-1)^2 + (-7)^2} = \sqrt{9+1+49} = \sqrt{59}$ ... (v)

Also, $\vec{a}_2 - \vec{a}_1 = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j}) = \hat{i} - \hat{k}$ ... (vi)

∴ From Eqs. (iii), (iv), (v) and (vi) we get

$$d = \left|\frac{(3\hat{i} - \hat{j} - 7\hat{k})\cdot(\hat{i} - \hat{k})}{\sqrt{59}}\right|$$

∴ $$d = \left|\frac{3-0+7}{\sqrt{59}}\right| = \frac{10}{\sqrt{59}}$$

∴ Required shortest distance = $\frac{10}{\sqrt{59}}$ units.

S36. The shortest distance between the lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

and $$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is $$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2 + (c_1a_2-c_2a_1)^2 + (a_1b_2-a_2b_1)^2}}$$

using this result and simplify it.

Given equations of lines are

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1} \quad \text{... (i)}$$

and $$\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2} \quad \text{... (ii)}$$

Comparing Eqs. (i) and (ii) with the one point form of equation of line

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

We get, $x_1 = 1,\ y_1 = 2,\ z_1 = 1$

$a_1 = 1,\ b_1 = -1,\ c_1 = 1$

and $x_2 = 2,\ y_2 = -1,\ z_2 = -1$

$a_2 = 2,\ b_2 = 1,\ c_2 = 2$

Now, we know that the shortest distance between two lines is given by

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

$\therefore$ We get,
$$d = \frac{\begin{vmatrix} 2-1 & -1-2 & -1-1 \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}}{\sqrt{(-2-1)^2+(2-2)^2+(1+2)^2}}$$

$$\Rightarrow \quad d = \frac{\begin{vmatrix} 1 & -3 & -2 \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}}{\sqrt{9+0+9}}$$

$$= \left|\frac{1(-2-1)+3(2-2)-2(1+2)}{\sqrt{18}}\right|$$

$$= \left|\frac{-3+0-6}{\sqrt{18}}\right| = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$$

Hence, the required shortest distance $= \frac{3}{\sqrt{2}}$ units.

S37. Given equations of lines are

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{... (i)}$$

and $$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad \text{... (ii)}$$

Comparing above equations with the one point form of equation of line which is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

we get, $a_1 = 1,\ b_1 = -2,\ c_1 = 1,\ x_1 = 3,\ y_1 = 5,\ z_1 = 7$

$a_2 = 7,\ b_2 = -6,\ c_2 = 1,\ x_2 = -1,\ y_2 = -1,\ z_2 = -1$

We know that the shortest distance between two lines is given by

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

$\therefore$ We get,

$$d = \frac{\begin{vmatrix} -4 & -6 & -8 \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}}{\sqrt{(-2+6)^2 + (7-1)^2 + (-6+14)^2}}$$

$$= \left|\frac{-4(-2+6) + 6(1-7) - 8(-6+14)}{\sqrt{(-4)^2 + (6)^2 + (8)^2}}\right|$$

$$= \left|\frac{-4(4) + 6(-6) - 8(8)}{\sqrt{16+36+64}}\right| = \left|\frac{-16-36-64}{\sqrt{116}}\right|$$

$$= \left|\frac{-116}{\sqrt{116}}\right| = \frac{116}{\sqrt{116}} = \frac{\left(\sqrt{116}\right)^2}{\sqrt{116}} = \sqrt{116}$$

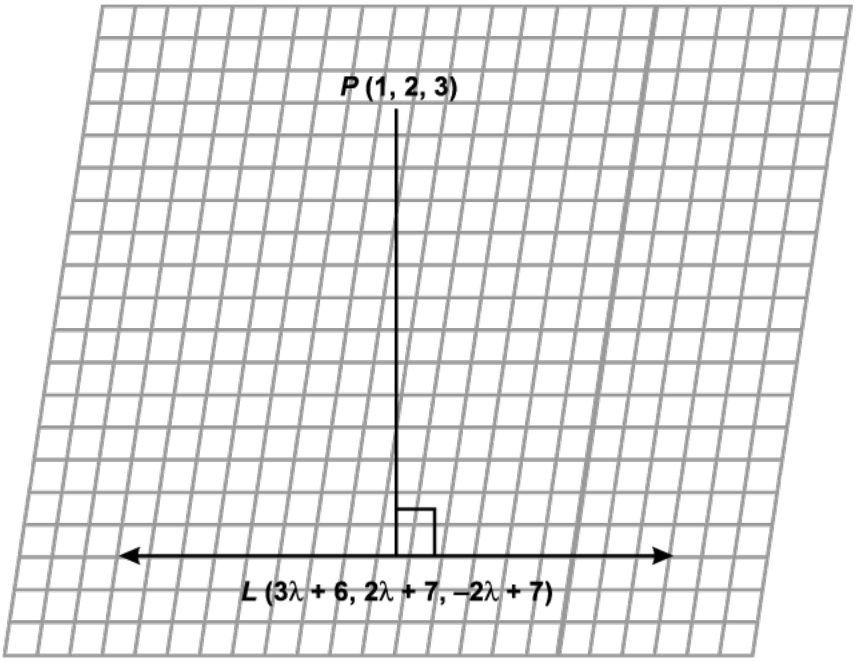
Hence, the required shortest distance = $\sqrt{116}$ units.

S38. Let L be the foot of the perpendicular drawn from the point $P(1, 2, 3)$ to the given line.

The coordinates of a general point on $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ are given by

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$$

i.e., $x = 3\lambda + 6,\ y = 2\lambda + 7,\ z = -2\lambda + 7$



Let the coordinates of L be $(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$

$\therefore$ Direction ratios of PL are proportional to

$$3\lambda + 6 - 1, \quad 2\lambda + 7 - 2, \quad -2\lambda + 7 - 3$$

i.e.,

$$3\lambda + 5, \quad 2\lambda + 5, \quad -2\lambda + 4.$$

Direction ratios of the given line are proportional to 3, 2, –2.

Since PL is perpendicular to the given line. Therefore,

$$\therefore \quad 3(3\lambda + 5) + 2(2\lambda + 5) + (-2)(-2\lambda + 4) = 0 \Rightarrow 17\lambda + 17 = 0 \Rightarrow \lambda = -1$$

Putting $\lambda = -1$ in (i), we obtain the coordinates of L as (3, 5, 9)

$$\therefore \quad PL = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7 \text{ units}$$

Hence, the required length of the perpendicular is 7 units.

S39. Given equation of line is

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$$

We find any random point on the line is as follows

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$$

$$\Rightarrow \quad x = 3\lambda - 2, \quad y = 2\lambda - 1, \quad z = 2\lambda + 3$$

$\therefore$ We have the point

$$Q\,(3\lambda - 2, 2\lambda - 1, 2\lambda + 3) \qquad \text{... (i)}$$

Now, given that distance between point P (1, 3, 3) and Q $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$, is 5 units. *i.e.*,

$$PQ = 5$$

$\Rightarrow \quad \sqrt{(3\lambda-2-1)^2+(2\lambda-1-3)^2+(2\lambda+3-3)^2} = 5$

$$\left[\because \text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\right]$$

$\Rightarrow \quad \sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2} = 5$

Squaring both sides, we get

$$(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2 = 25$$

$\Rightarrow \quad 9\lambda^2+9-18\lambda+4\lambda^2+16-16\lambda+4\lambda^2 = 25$

$\Rightarrow \quad 17\lambda^2-34\lambda = 0$

$\Rightarrow \quad 17\lambda(\lambda-2) = 0$

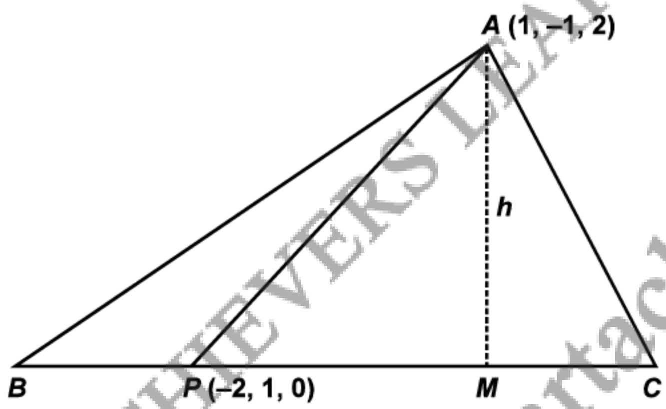
$\Rightarrow$ Either $\quad 17\lambda = 0$ or $\lambda-2 = 0$

$\Rightarrow \quad \lambda = 0 \quad \text{or} \quad 2$

Putting $\lambda = 0$ and $\lambda = 2$ in Eq. (i), we get the required points as $(-2, -1, 3)$ or $(4, 3, 7)$.

S40. Clearly, height h of ΔABC is the length of perpendicular from $A(1, -1, 2)$ to the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4}$ which passes through $P(-2, 1, 0)$ and is parallel to $\vec{b} = 2\hat{i}+\hat{j}+4\hat{k}$.

$\therefore \quad h = \frac{|\overrightarrow{PA}\times\vec{b}|}{|\vec{b}|}$



Now, $\quad \overrightarrow{PA} = (-2-1)\hat{i}+(1+1)\hat{j}+(0-2)\hat{k} = -3\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}+4\hat{k}$

$\therefore \quad \overrightarrow{PA}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -2 \\ 2 & 1 & 4 \end{vmatrix} = 10\hat{i}+8\hat{j}-7\hat{k}$

and $\quad |\vec{b}| = \sqrt{4+1+16} = \sqrt{21}$

$\Rightarrow \quad |\overrightarrow{PA}\times\vec{b}| = \sqrt{100+64+49} = \sqrt{213}$

$\therefore \qquad h = \dfrac{|\vec{PA} \times \vec{b}|}{|\vec{b}|} = \sqrt{\dfrac{213}{21}} = \sqrt{\dfrac{71}{7}}$

It is given that the length of BC is 5 units.

$\therefore \qquad \text{Area of } \Delta ABC = \dfrac{1}{2}(BC \times h)$

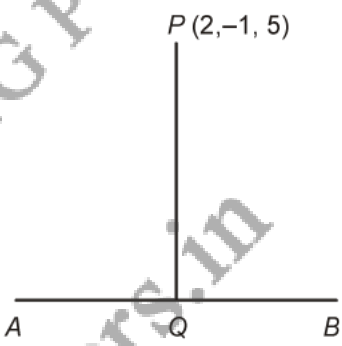
$$= \frac{1}{2} \times 5 \times \sqrt{\frac{71}{7}}$$

$$= \sqrt{\frac{1775}{28}} \text{ sq. units.}$$

S41. Let AB be the line whose equation is

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

Let Q is the foot of perpendicular and PQ is the length of perpendicular.



Any point Q on the given line is given as

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda \quad \text{(Say)}$$

$\Rightarrow \qquad x = 10\lambda + 11,\ y = -4\lambda - 2,\ z = -11\lambda - 8$

$\therefore$ Coordinates of Q are $(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$.

Now, DR's of the line PQ are

$$(10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5)$$

$\therefore$ DR's of $\quad PQ = (10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$

$\because \quad PQ \perp AB$

$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$

where, $\quad a_1 = 10\lambda + 9,\ b_1 = -4\lambda - 1,\ c_1 = -11\lambda - 13$ [DR's of PQ]

and $\quad a_2 = 10,\ b_2 = -4,\ c_2 = -11$ [DR's of AB]

$\therefore$ We get

$10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$

$\Rightarrow \qquad 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$

$\Rightarrow \qquad 237\lambda + 237 = 0$

$\Rightarrow \qquad 237\lambda = -237$

$\Rightarrow \qquad \lambda = -1$

Putting $\lambda = -1$ in coordinates of point Q, we get

Foot of perpendicular = $Q(-10 + 11, 4 - 2, 11 - 8)$

$$= Q\,(1, 2, 3)$$

Also, length of perpendicular PQ is given as

$$PQ = \sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2}$$

$$= \sqrt{1+9+4} = \sqrt{14} \text{ units}$$

S42. Let the required equation of line passing through (1, 2, –4) is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \qquad \dots \text{(i)}$$

Given that line (i) is perpendicular to lines

$$\frac{x-8}{3} = \frac{y-19}{-16} = \frac{z-10}{7} \qquad \dots \text{(ii)}$$

and $$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \qquad \dots \text{(iii)}$$

We know that when two lines are perpendicular, then we have $a_1a_2 + b_1b_2 + c_1c_2 = 0$, where a_1, b_1, c_1 and a_2, b_2, c_2 are the DR's of two lines.

Using property in Eqs. (i) and (ii) and then in Eqs. (i) and (iii), we get

$$3a - 16b + 7c = 0 \qquad [\because\ a_1a_2 + b_1b_2 + c_1c_2 = 0] \qquad \dots \text{(iv)}$$

and $$3a + 8b - 5c = 0 \qquad \dots \text{(v)}$$

Subtracting Eq. (v) from Eq. (iv), we get

$$\begin{array}{r} 3a - 16b = -\ 7c \\ -3a -\ 8b = -\ 5c \\ \hline -24b = -12c \end{array}$$

or $$b = \frac{c}{2}$$

Putting $b = \frac{c}{2}$ in Eq. (iv), we get

$$3a - 16\left(\frac{c}{2}\right) + 7c = 0$$

$$\Rightarrow \quad 3a - 8c + 7c = 0$$

$$\Rightarrow \quad 3a - c = 0 \Rightarrow a = \frac{c}{3}$$

Putting $a = \frac{c}{3}$ and $b = \frac{c}{2}$ in Eq. (i), we get the required equation of line as

$$\frac{x-1}{\left(\frac{c}{3}\right)} = \frac{y-2}{\left(\frac{c}{2}\right)} = \frac{z+4}{c}$$

$\Rightarrow \qquad \dfrac{x-1}{2c} = \dfrac{y-2}{3c} = \dfrac{z+4}{6c}$

Multiply denominator by 6

or $\qquad \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+4}{6}$

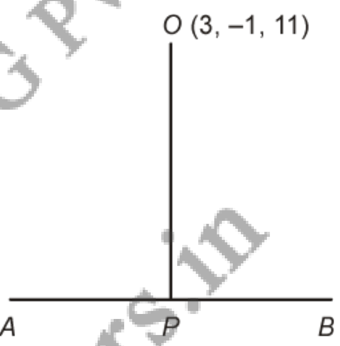
Also, the vector equation of line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Note: Vector equation of line is $\vec{r} = \vec{a} + \lambda\vec{b}$ where $\vec{a}$ is a point on a line and $\vec{b}$ is a direction ratios of a line.

S43. Given equation of line AB is $\dfrac{x}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$

we find any arbitrary point on the given line as follows



$$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \quad \text{[Say]}$$

$\Rightarrow \qquad \dfrac{x}{2} = \lambda, \ \dfrac{y-2}{3} = \lambda$ and $\dfrac{z-3}{4} = \lambda$

$\Rightarrow \qquad x = 2\lambda, \quad y = 3\lambda + 2$

and $\qquad z = 4\lambda + 3$

$\therefore$ Any point P on the given lines = $(2\lambda, 3\lambda + 2, 4\lambda + 3)$

Let P be the foot of perpendicular drawn from point $O(3, -1, 11)$ on line AB.

Now, DR's of line $\qquad OP = (2\lambda - 3, 3\lambda + 2 + 1, 4\lambda + 3 - 11)$

$\Rightarrow$ DR's of line $\qquad OP = (2\lambda - 3, 3\lambda + 3, 4\lambda - 8)$

Since, $OP \perp AB$

$\therefore$ We have $\quad a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad$ ...(i)

where a_1, b_1, c_1 are DR's of line OP and a_2, b_2, c_2 are DR's of line AB.

$\therefore$ We have $\qquad a_1 = 2\lambda - 3, \quad b_1 = 3\lambda + 3, \quad c_1 = 4\lambda - 8$

and $\qquad a_2 = 2, \quad b_2 = 3, \quad c_2 = 4$

$\therefore$ From Eq. (i),

$2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0$

$\Rightarrow \qquad 4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$

$\Rightarrow \qquad 29\lambda - 29 = 0$

$\Rightarrow$ $\quad 29\lambda = 29$ or $\lambda = 1$

$\therefore$ Coordinates of point, $P = (2\lambda, 3\lambda + 2, 4\lambda + 3)$

$\therefore$ Foot of perpendicular, $P = (2, 3 + 2, 4 + 3)$

$= (2, 5, 7)$

Now, equation of perpendicular OP where $O(3, -1, 11)$ and $P(2, 5, 7)$ is

$$\frac{x-3}{2-3} = \frac{y+1}{5+1} = \frac{z-7}{7-11}$$

$$\left[\because \text{Using two points form } \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \text{ of equation of line}\right]$$

$$\frac{x-3}{-1} = \frac{y+1}{6} = \frac{z-7}{-4}$$

Finally, length of perpendicular OP = Distance between points $O(3, -1, 11)$ and $P(2, 5, 7)$.

$$= \sqrt{(2-3)^2 + (5+1)^2 + (7-11)^2}$$

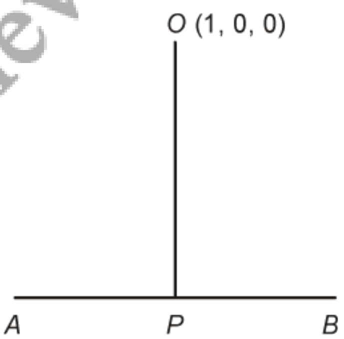
$$\left[\because \text{Distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}\right]$$

$$= \sqrt{1+36+16} = \sqrt{53}$$

$\therefore$ Length of perpendicular $= \sqrt{53}$

S44. Let AB be the line whose equation is given as

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$



Let P be any variable point on line AB.

Let $\quad \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$ [Say]

$\Rightarrow$ $\quad \frac{x-1}{2} = \lambda, \ \frac{y+1}{-3} = \lambda, \ \frac{z+10}{8} = \lambda$

$\Rightarrow$ $\quad x = 2\lambda + 1, y = -3\lambda - 1, z = 8\lambda - 10$

$\therefore$ Coordinates of P are $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

$\Rightarrow$ DR's of line $OP = (2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0)$

$= (2\lambda, -3\lambda - 1, 8\lambda - 10)$

Since $OP \perp AB$

$\therefore$ $\quad a_1a_2 + b_1b_2 + c_1c_2 = 0$ ...(i) where a_1, b_1, c_1 are DR's of line OP and a_2, b_2, c_2 are DR's of line AB.

$\therefore$ We get, $a_1 = 2\lambda,\ b_1 = -3\lambda - 1,\ c_1 = 8\lambda - 10$

and $a_2 = 2,\ b_2 = -3,\ c_2 = 8$

$\therefore$ From Eq. (i)

$2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10) = 0$

$\Rightarrow\quad 4\lambda + 9\lambda + 3 + 64\lambda - 80 = 0$

$\Rightarrow\quad 77\lambda - 77 = 0$

$\Rightarrow\quad 77\lambda = 77$

$\Rightarrow\quad \lambda = 1$

$\Rightarrow$ Coordinates of point $P = (2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

$= (3, -4, -2)$

Now, since P is the foot of perpendicular, so coordinates of foot of perpendicular

$= (3, -4, -2)$.

$\therefore$ Using distance formula, $O(1, 0, 0)$, $P(3, -4, -2)$

$$OP = \sqrt{(3-1)^2 + (-4-0)^2 + (-2-0)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6} \text{ units}$$

$\therefore\quad OP = 2\sqrt{6}$ units

Finally equation of perpendicular OP, where $O(1, 0, 0)$ and $P(3, -4, -2)$ is given by

$$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$$

$$\left[\because \text{Using two point form of line } \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}\right]$$

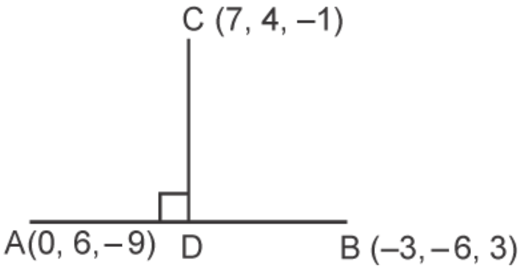
$\Rightarrow\quad \frac{x-1}{2} = \frac{y}{-4} = \frac{z}{-2}$ is the required equation.

S45. We know that equation of line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad \ldots (i)$$

Here, $(x_1, y_1, z_1) = (0, 6, -9)$ and $(x_2, y_2, z_2) = (-3, -6, 3)$

$\therefore$ Equation of line *AB* is given by



$$\frac{x-0}{-3-0} = \frac{y-6}{-6-6} = \frac{z+9}{3+9}$$

$$\Rightarrow \quad \frac{x}{-3} = \frac{y-6}{-12} = \frac{z+9}{12}$$

$$\Rightarrow \quad \frac{x}{-1} = \frac{y-6}{-4} = \frac{z+9}{4}$$

Next, we have to find coordinates of foot of perpendicular *D*.

Now, let $\frac{x}{-1} = \frac{y-6}{-4} = \frac{z+9}{4} = \lambda$ [Say]

$\Rightarrow$ $x = -\lambda,\ y - 6 = -4\lambda$ and $z + 9 = 4\lambda$

$\therefore$ $x = -\lambda,\ y = -4\lambda + 6$ and $z = 4\lambda - 9$

Let coordinates of $D = (-\lambda, -4\lambda + 6, 4\lambda - 9)$ ... (ii)

Now, DR's of line *CD* are

$$(-\lambda - 7, -4\lambda + 6 - 4, 4\lambda - 9 + 1) = (-\lambda - 7, -4\lambda + 2, 4\lambda - 8)$$

$\because$ *D* is the foot of perpendicular.

Now, $CD \perp AB$

$\Rightarrow$ $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Where, $a_1 = -\lambda - 7,\ b_1 = -4\lambda + 2,\ c_1 = 4\lambda - 8$ [DR's of line *CD*]

$a_2 = -1,\ b_2 = -4,\ c_2 = 4$ [DR's of line *AB*]

$\therefore -1(-\lambda - 7) - 4(-4\lambda + 2) + 4(4\lambda - 8) = 0$

$\Rightarrow$ $\lambda + 7 + 16\lambda - 8 + 16\lambda - 32 = 0$

$\Rightarrow$ $33\lambda - 33 = 0$

$\Rightarrow$ $33\lambda = 33$

$\Rightarrow$ $\lambda = 1$

Putting $\lambda = 1$ in Eq. (ii), we get required foot of perpendicular,

$$D = (-1, 2, -5)$$

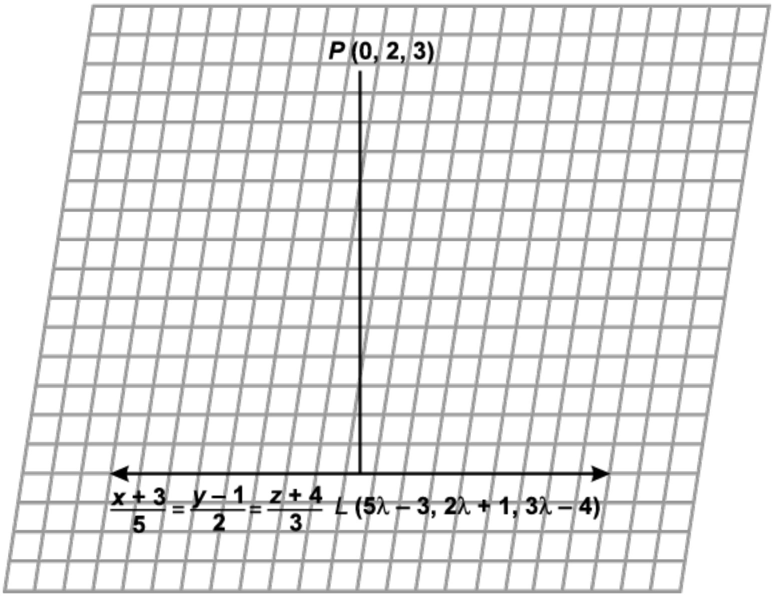
Also, we have to find equation of line *CD*, where $C(7, 4, -1)$ and $D(-1, 2, -5)$.

$\therefore$ Required equation is $\frac{x-7}{-1-7} = \frac{y-4}{2-4} = \frac{z+1}{-5+1}$ [Using Eq. (i)]

$$\Rightarrow \quad \frac{x-7}{-8} = \frac{y-4}{-2} = \frac{z+1}{-4}$$

$$\Rightarrow \quad \frac{x-7}{-4} = \frac{y-4}{-1} = \frac{z+1}{-2}.$$

S46. Let L be the foot of the perpendicular drawn from the point $P(0, 2, 3)$ to the given line.



The coordinates of a general point on

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} \text{ are given by } \frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$$

i.e., $x = 5\lambda - 3,\ y = 2\lambda + 1,\ z = 3\lambda - 4$

Let the coordinates of L be $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$ ... (i)

$\therefore$ Direction ratios of PL are proportional to

$5\lambda - 3 - 0,\ 2\lambda + 1 - 2,\ 3\lambda - 4 - 3$

i.e., $5\lambda - 3,\ 2\lambda - 1,\ 3\lambda - 7$.

Direction ratios of the given line are proportional to 5, 2, 3.

Since PL is perpendicular to the given line.

$\therefore$ $5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0 \Rightarrow \lambda = 1$.

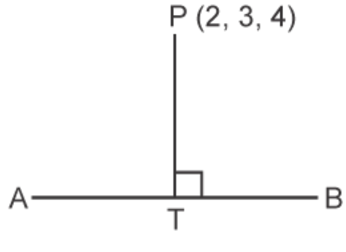
Putting $\lambda = 1$ in (i), the coordinates of L are $(2, 3, -1)$

$\therefore$ Length of the perpendicular from P on the given line

$$PL = \sqrt{(2-0)^2 + (3-2)^2 + (-1-3)^2} = \sqrt{4+1+16} = \sqrt{21} \text{ units.}$$

S47. The given equation of line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ which can be written in standard form as

$$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} \quad \text{... (i)}$$



Let T be any variable point on Eq (i). Then, its coordinates are calculated as follows

$$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda \text{ [Say]}$$

$\Rightarrow$ $$\frac{x-4}{-2} = \lambda,\ \frac{y}{6} = \lambda \text{ and } \frac{z-1}{-3} = \lambda$$

$\Rightarrow$ $$x = -2\lambda + 4,\ y = 6\lambda \text{ and } z = -3\lambda + 1$$

$\therefore$ Coordinates of point T are $(-2\lambda + 4, 6\lambda, -3\lambda + 1)$

Now, DR's of line

$$PT = (-2\lambda + 4 - 2, 6\lambda - 3, -3\lambda + 1 - 4)$$

$$= (-2\lambda + 2, 6\lambda - 3, -3\lambda - 3)$$

$\because PT \perp AB$

$\therefore$ $$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

where, $a_1 = -2\lambda + 2,\ b_1 = 6\lambda - 3,\ c_1 = -3\lambda - 3$

and $a_2 = -2,\ b_2 = 6,\ c_2 = -3$

We get,

$$-2(-2\lambda + 2) + 6(6\lambda - 3) - 3(-3\lambda - 3) = 0$$

$\Rightarrow$ $$4\lambda - 4 + 36\lambda - 18 + 9\lambda + 9 = 0$$

$\Rightarrow$ $$49\lambda - 13 = 0 \Rightarrow \lambda = \frac{13}{49}$$

$\therefore$ Coordinates of T are

$$\left(-\frac{26}{49} + 4, \frac{78}{49}, -\frac{39}{49} + 1\right) = \left(\frac{170}{49}, \frac{78}{49}, \frac{10}{49}\right)$$

$\therefore$ The required perpendicular distance = Distance PT

$$= \sqrt{\left(2 - \frac{170}{49}\right)^2 + \left(3 - \frac{78}{49}\right)^2 + \left(4 - \frac{10}{49}\right)^2} \quad \left[\because \text{Distance } \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{\left(-\frac{72}{49}\right)^2 + \left(\frac{69}{49}\right)^2 + \left(\frac{186}{49}\right)^2} = \sqrt{\frac{5184 + 4761 + 34596}{2401}}$$

$$= \sqrt{\frac{44541}{2401}} = \sqrt{18.55} = 4.31 \text{ units.}$$

S48. Given equation of lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

and $$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Now, the vector equation of lines are

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \quad \text{... (i)}$$

[$\because$ Vector form of equation of line is $r = a + \lambda b$]

and $$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k}) \quad \text{... (ii)}$$

Here, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$

and $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b}_2 = 4\hat{i} + 6\hat{j} + 12\hat{k}$ ... (iii)

Now,
$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 4 & 6 & 12 \end{vmatrix}$$

$$= \hat{i}(36 - 36) - \hat{j}(24 - 24) + \hat{k}(12 - 12)$$

$$= 0\hat{i} - 0\hat{j} + 0\hat{k} = 0$$

$\because$ $\vec{b}_1 \times \vec{b}_2 = 0 \Rightarrow$ Vector $\vec{b}_1$ parallel to $\vec{b}_2$ [$\because$ If $\vec{b}_1 \times \vec{b}_2 = 0$, then $\vec{a} \parallel \vec{b}$]

$\therefore$ The two lines are parallel.

$\therefore$ $\vec{b} = (2\hat{i} + 3\hat{j} + 6\hat{k})$ [Since, DR's of given lines are proportional]

Since, the two lines are parallel, we use the formula for shortest distance between two parallel lines :

We know that
$$d = \left| \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right| \quad \text{... (iv)}$$

$\therefore$ From Eqs. (iii) and (iv), we get

$$d = \left| \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) \times (2\hat{i} + \hat{j} - \hat{k})}{\sqrt{(2)^2 + (3)^2 + (6)^2}} \right| \quad \text{... (v)}$$

Now,
$$(2\hat{i} + 3\hat{j} + 6\hat{k}) \times (2\hat{i} + \hat{j} - \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(-3 - 6) - \hat{j}(-2 - 12) + \hat{k}(2 - 6)$$

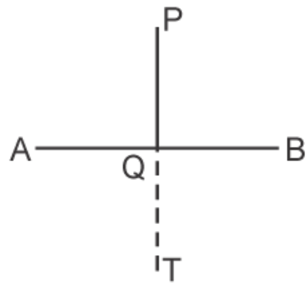
$$= -9\hat{i} + 14\hat{j} - 4\hat{k}$$

$\therefore$ From Eq. (v), we get

$$d = \left| \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{\sqrt{49}} \right| = \frac{\sqrt{(-9)^2 + (14)^2 + (-4)^2}}{7}$$

$$= \frac{\sqrt{81+196+16}}{7} = \frac{\sqrt{293}}{7} \text{ units.}$$

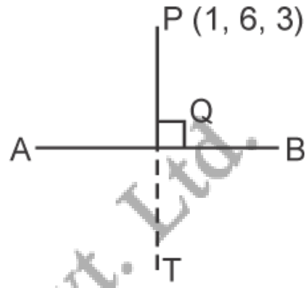
S49. First find the coordinates of foot of perpendicular Q. Then, find the image which is point T by using the fact that Q is the mid-point of line PT.



Here, T is the image of the point P (1, 6, 3). Q is the foot of perpendicular PQ on the line AB. First, we find Q.

Equation of line AB is given by

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} \quad \text{... (i)}$$



Let $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$ [Say]

$\Rightarrow$ $x = \lambda,\ y - 1 = 2\lambda,\ z - 2 = 3\lambda$

$\Rightarrow$ $x = \lambda,\ y = 2\lambda + 1,\ z = 3\lambda + 2$

Let coordinates of $Q = (\lambda, 2\lambda + 1, 3\lambda + 2)$ ... (ii)

Now DR's of line $PQ = (\lambda - 1, 2\lambda + 1 - 6, 3\lambda + 2 - 3)$

$\therefore$ DR's of $PQ = (\lambda - 1, 2\lambda - 5, 3\lambda - 1)$

Now, line $PQ \perp AB$.

$\therefore$ $a_1a_2 + b_1b_2 + c_1c_2 = 0$

where, $a_1 = \lambda - 1,\quad b_1 = 2\lambda - 5,\quad c_1 = 3\lambda - 1$

and $a_2 = 1,\quad b_2 = 2,\quad c_2 = 3$

$\therefore$ $1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow$ $\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow$ $14\lambda - 14 = 0$

$\Rightarrow$ $\lambda = 1$

Putting $\lambda = 1$ in Eq. (ii), we get

$$Q\,(1, 2 + 1, 3 + 2) = (1, 3, 5)$$

Now, as discussed earlier Q is the mid-point of PT.

Let coordinates of $T = (x, y, z)$

$\therefore$ Using by mid-point formula,

Q = Mid-point of P (1, 6, 3) and T (x, y, z)

$$= \left(\frac{x+1}{2}, \frac{y+6}{2}, \frac{z+3}{2}\right) \qquad \left[\because \text{Mid-point is } \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)\right]$$

But $Q = (1, 3, 5)$

$$\therefore \left(\frac{x+1}{2}, \frac{y+6}{2}, \frac{z+3}{2}\right) = (1, 3, 5)$$

$$\Rightarrow \quad \frac{x+1}{2} = 1, \ \frac{y+6}{2} = 3, \ \frac{z+3}{2} = 5$$

$$\Rightarrow \quad x = 2 - 1, \quad y = 6 - 6, \quad z = 10 - 3$$

$$\Rightarrow \quad x = 1, \quad y = 0, \quad z = 7$$

$\therefore$ Coordinates of $T = (x, y, z) = (1, 0, 7)$

Hence, coordinates of image of point

$$P(1, 6, 3) = T(1, 0, 7).$$

$\therefore$ line joining the point P and T is

$$\frac{x-1}{1-1} = \frac{y-6}{0-6} = \frac{z-3}{7-3}$$

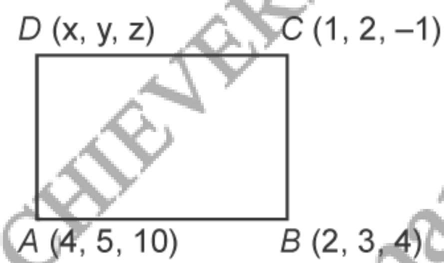
$$\frac{x-1}{0} = \frac{y-6}{-6} = \frac{z-3}{4}$$

and length of $PT = \sqrt{(1-1)^2 + (0-6)^2 + (7-3)^2}$

$$= \sqrt{0 + 36 + 16} = \sqrt{52}$$

$$\Rightarrow \quad = 2\sqrt{13}.$$

S50. First, we find vector equation of AB where A (4, 5, 10) and B (2, 3, 4).



We know that two points vector form of line is given by

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) \quad \text{... (i)}$$

where, $\vec{a}$ and $\vec{b}$ are the position vector of points through which the line is passing through.

Here, $\vec{a} = \overrightarrow{OA} = 4\hat{i} + 5\hat{j} + 10\hat{k}$

and $\vec{b} = \overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore$ Using Eq. (i), the required equation of line AB is

$$\vec{r} = (4\hat{i}+5\hat{j}+10\hat{k})+\lambda\left[(2\hat{i}+3\hat{j}+4\hat{k})-(4\hat{i}+5\hat{j}+10\hat{k})\right]$$

$$\vec{r} = (4\hat{i}+5\hat{j}+10\hat{k})+\lambda(-2\hat{i}-2\hat{j}-6\hat{k})$$

Similarly, vector equation of line *BC*, where *B* (2, 3, 4) and *C* (1, 2, –1) is

$$\vec{r} = (2\hat{i}+3\hat{j}+4\hat{k})+\lambda\left[(\hat{i}+2\hat{j}-\hat{k})-(2\hat{i}+3\hat{j}+4\hat{k})\right]$$

$$\vec{r} = (2\hat{i}+3\hat{j}+4\hat{k})+\lambda(-\hat{i}-\hat{j}-5\hat{k})$$

Next, we have to find coordinates of point *D*. From figure parallelogram *ABCD*, we have

Mid-point of diagonal *BD* = Mid-point of diagonal *AC*

[$\because$ Diagonals of parallelogram bisect each other]

$$\therefore \quad \left(\frac{x+2}{2}, \frac{y+3}{2}, \frac{z+4}{2}\right) = \left(\frac{4+1}{2}, \frac{5+2}{2}, \frac{10-1}{2}\right)$$

Comparing corresponding coordinates

$$\Rightarrow \quad \frac{x+2}{2} = \frac{5}{2}, \quad \frac{y+3}{2} = \frac{7}{2} \quad \text{and} \quad \frac{z+4}{2} = \frac{9}{2}$$

$$\Rightarrow \quad x = 3, \quad y = 4, \quad z = 5$$

$\therefore$ Coordinates of point *D* (*x*, *y*, *z*) = (3, 4, 5).