Date: 18/10/2021

Q1. Given function $f: Z \to Z$, defined as f (x) = 4x. Is function "f" onto? Give reasons.

Q2. Let $f: N \to N$ be defined by $f(x) = 3x$. Show that f is not an onto function.

Q3. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. State whether f is one-one or not.

Q4. Let $f: R \to R$ defined by

$$f(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$$

. Examine the function f for one-one.

Q5. Prove that $f: R \to R$, given by $f(x) = x^3 + 1$ is one-one function.

Q6. Is the function $f: N \to N$, defined by $f(x) = 4 + 3x$ one-one? Check with reasons.

Q7. Let $f: R \to R$ be defined by $f(x) = x^2$. Is f one-to-one?

Q8. Let $f: R \to R$ is defined by $f(x) = |x|$. Is function f onto? Give reasons.

Q9. Let $A = \{-1, 0, 1\}$ and $B = \{0, 1\}$. State whether the function 'f' : $A \to B$ defined as $f(x) = x^2$, is a bijective.

Q10. Prove that the greatest integer function $f: R \to R$, given by $f(x) = [x]$ is neither one-one nor onto.

Q11. Let $f: N \to N$ be defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

for all $n \in N$. Find whether the function f is bijective.

Q12. Show that the function $f: R \to R$ defined by $f(x) = \dfrac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto.

Q13. Let $A = \{x \in R : -1 \le x \le 1\} = B$. Show that $f: A \to B$ given by $f(x) = x|x|$ is a bijection.

Q14. Show that the function $f: R \to R$ given by $f(x) = 2x$, is one-one and onto.

Q15. Show that the absolute value function $f: R \to R$, given $f(x) = |x|$ is neither one-one nor onto.

Q16. Let the function $f: R \to R$ be defined by $f(x) = \cos x \ \forall x \in R$. Show that f is neither one-one nor onto.

Q17. Show that $f: N \to N$, given by $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$, is both one-one and onto.

Q18. Let $f: N - \{1\} \to N$ be defined by $f(n)$ = the highest prime factor of n
Show that f is neither one-one nor onto. Find the range of f.

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Q19. Show that the function $f : R \to R$ given by $f(x) = x^3 + x$ is a bijection.

Q20. Let $A = R - \{2\}$ and $B = R - \{1\}$ If $f : A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.

Q21. Show that the function $f : R \to R$ given by $f(x) = ax + b$, where $a, b \in R$, $a \neq 0$ is a bijection.

Q22. Show that the function $f : \{0\} \to R - \{0\}$ defined by $f(x) = \frac{1}{x}$ is one-one. is the result true, if the domain $R - \{0\}$ is replaced by N?

Q23. Consider function $f : \left[0, \frac{\pi}{2}\right] \to R$ given by $f(x) = \sin x$ and $g : \left[0, \frac{\pi}{2}\right] \to R$ given by $g(x) = \cos x$. Show that the 'f' and 'g' are one-one but '$f + g$' is not one-one.

Q24. Let A be the set of all 46 students of class XII in a school. Let $f : N \to N$ be a function defined $f(x)$ = Roll number of the student x. Show that 'f' is one-one but not onto.

Q25. Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

Q26. If $f : R \to R$ be the function defined by $f(x) = 4x^3 + 7$, show that f is a bijection.

Q27. Show that the function $f : R - \{3\} \to R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is an onto function.

Q28. Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one function.

Q29. Show that the function $f : N \to N$ given by $f(n) = n - (-1)^n$ for all $n \in N$ is a bijection.

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S1. Let for $y \in Z$ (co-domain), there exists $x \in Z$ (domain), such that

$y = f(x) \Rightarrow y = 4x \Rightarrow x = \frac{y}{4} \neq Z$, for y may not be a multiple of 4, hence not onto.

S2. $f : N \to N$ defined by $f(x) = 3x$.

Let for $y \in N$, there exists, $x \in N$ of domain such that

$f(x) = y \Rightarrow 3x = y \Rightarrow x = \frac{y}{3}$, which may not be a natural number, hence not onto.

S3. One - one as for $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

S4. Not one-one, as for $x > 0$, $f(x) = 1$, *e.g.*, $f(2) = 1$ and $f(3) = 1$.

S5. Given: $f(x) = x^3 + 1$

For $x_1 \neq x_2 \Rightarrow x_1^3 \neq x_2^3 \Rightarrow x_1^3 + 1 \neq x_2^3 + 1 \Rightarrow f(x_1) \neq f(x_2)$

Hence one-one

S6. Given $f(x) = 4 + 3x$

$f(x_1) = f(x_2) \Rightarrow 4 + 3x_1 = 4 + 3x_2 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2 \Rightarrow$ One-one function.

S7. No, as $f(-2) = (-2)^2 = 4$ and $f(2) = (2)^2 = 4$, *i.e.*, $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$. Not one-one.

S8. 'f' is not onto, as for $y \in R^-$ from co-domain, there is no $x \in R$ from domain such that $y = f(x)$, *e.g.*, for $-2 \in R$ (co-domain) there is no $x \in R$ (domain) such that $f(x) = -2$ *i.e.*, $|x| = -2$. Hence not onto.

S9. Given $f : A \to B$, *i.e.*, $f : \{-1, 0, 1\} \to \{0, 1\}$.

We notice $f(-1) = (-1)^2 = 1$, $f(0) = (0)^2 = 0$ and $f(1) = (1)^2 = 1$

i.e., $R_f = B$. Also $f(x_1) = f(x_2) \not\Rightarrow x_1 = x_2$, as for $x_1 = -1$ and $x_2 = 1$, i.e., $x_1 \neq x_2$ but $f(x_1) = f(x_2)$

This function is onto but not one-one. So it is not a bijective function.

S10. Given $f : R \to R$ defined by $f(x) = [x]$

For one-one: We know by definition that

for $a \leq x < a + 1$, $f(x) = a$, a is integer

i.e., for $x_1, x_2 \in [a, a + 1)$, $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2) = a$

Hence not one-one.

Similarly for y (not integer) $\in R$ in co-domain there does not exist $x \in R$ in domain such that $f(x) = y$. Hence not onto.

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S11. Let $x_1 = 5,\ x_2 = 6$

Then $f(x_1) = \dfrac{5+1}{2} = 3,\quad f(x_2) = \dfrac{6}{2} = 3$

As $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$. Not one-one

So, not bijective.

S12. For $x_1, x_2 \in R$

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1} \Rightarrow x_1x_2^2 + x_1 = x_1^2x_2 + x_2 \Rightarrow x_1x_2(x_2 - x_1) + (x_1 - x_2) = 0$$

$$\Rightarrow \quad (x_2 - x_1)(x_1x_2 - 1) = 0 \Rightarrow x_2 - x_1 = 0 \quad \text{or} \quad x_1x_2 - 1 = 0 \Rightarrow x_1 = x_2 \quad \text{or} \quad x_1x_2 = 1$$

Let $x_1 = 2,\ x_2 = \dfrac{1}{2}$, then we notice $f(x_1) = f(x_2)$; $\therefore$ Not one-one.

Here we notice $f(x_1) \neq 1$ for any $x \in R$.

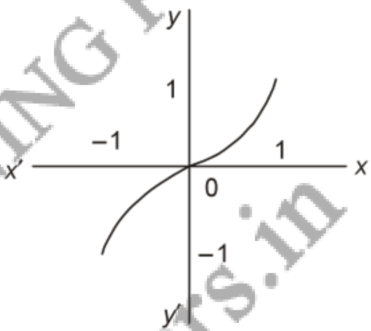
Therefore, $1 \in R$ from co-domain does not have pre-image in domain. So not onto.

S13. Show that the function is one-one, onto.

i.e., $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for one-one and

for $y \in B$, there is $x \in A$ such that $y = f(x)$.

From graph



$$f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$ hence f is bijection.

S14. Given $f : R \to R$ such that $f(x) = 2x$.

For one-one: $f(x) = f(y) \Rightarrow 2x = 2y \Rightarrow x = y$. Hence one-one.

For onto: For $y \in R$, there exists, $x \in R$

$f(x) = y \Rightarrow 2x = y \Rightarrow x = \dfrac{y}{2}$. This is true as for $y \in R$, there exists, $\dfrac{y}{2} \in R$

Such that $f\left(\dfrac{y}{2}\right) = y$. Hence onto.

S15. Given $f : R \to R$, given by $f(x) = |x|$

We can proceed as above

Let $x \in R^+$ then $-x \in R^-$. Let $x_1 = x$ and $x_2 = -x$

For $x_1 \neq x_2$, $f(x_1) = |x| = x$ and $f(x_2) = |-x| = x$.

$\therefore \quad x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$. Hence not one-one.

Conversely for $y \in R^-$, we do not have unique $x \in R$ from domain such that $f(x) = y$.

Hence not onto.

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S16. If $x_1 = \frac{\pi}{3}$, $x_2 = \frac{5\pi}{3}$, then $f(x_1) = \cos\frac{\pi}{3} = \frac{1}{2}$, and $f(x_2) = \cos\frac{5\pi}{3} = \frac{1}{2}$.

As $x_1 \neq x_2$ but $f(x_1) = f(x_2)$. Therefore, $f(x)$ is not one-one.
Again $f(x) = \cos x \in [-1, 1] \neq$ codomain, hence $f(x)$ is not onto.

S17. Given function $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$

For one-one: (i) Let $x_1, x_2 \in N$ and x_1, x_2 are both even.

$x_1 \neq x_2 \Rightarrow x_1 - 1 \neq x_2 - 1 \Rightarrow f(x_1) \neq f(x_2)$

(ii) Let $x_1, x_2 \in N$ and x_1, x_2 are both odd.

$x_1 \neq x_2 \Rightarrow x_1 + 1 \neq x_2 + 1 \Rightarrow f(x_1) \neq f(x_2)$

(ii) Let $x_1, x_2 \in N$ and x_1 is even, x_2 is odd $x_1 \neq x_2$

Also, $f(x_1) = x_1 - 1$ (odd) and $f(x_2) = x_2 + 1$ (even) $\Rightarrow f(x_1) \neq f(x_2)$

In all the three cases $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$. Hence function is one-one.

For onto: Let $y \in N$ (co-domain)

If y is even $\Rightarrow$ $y = x + 1 \Rightarrow x = y - 1 \in N$ (domain)

If y is odd $\Rightarrow$ $y = x - 1 \Rightarrow x = y + 1 \in N$ (domain)

$\therefore$ For every $y \in N$ (co-domain), there exists $x \in N$ (domain)

Such that $y = f(x)$. Hence onto.

$\therefore$ 'f' is both one-one and onto.

S18. We have

$f(6)$ = (the highest prime factor of 6) = 3

$f(9)$ = (the highest prime factor of 9) = 3

and, $f(12)$ = (the highest prime factor of 12) = 3.

So, f is a many-one function.
Clearly, image of any $n \in N - \{1\}$ is the largest prime number that divides n. So, the range of f consist of prime numbers only. Consequently, range of $f \neq N$ (co-domain).

So, f is not function.
Hence, f is neither one-one nor onto. The range of f is the set of all prime numbers.

S19. Injectivity: Let $x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow \quad x^3 + x = y^3 + y$

$\Rightarrow \quad x^3 - y^3 + (x - y) = 0$

$\Rightarrow \quad (x - y)(x^2 + xy + y^2 + 1) = 0$

$\Rightarrow \quad x - y = 0$ $\left[\begin{array}{l} \because \ x^2 + xy + y^2 \geq 0 \ \text{ for all } x, y \in R \\ \because \ x^2 + xy + y^2 + 1 \geq 1 \ \text{ for all } x, y \in R \end{array}\right]$

$\Rightarrow \quad x = y$

Thus, $\quad f(x) = f(y) \Rightarrow x = y$ for all $x, y \in R$.

So, f is an injective map.

Surjectivity: Let y be an arbitary element of R. Then,

$$f(x) = y \Rightarrow x^3 + x = y \Rightarrow x^3 + x - y = 0.$$

We know that an odd degree equation has at least one real root. Therefore, for every real value of y, the equation $x^3 + x - y = 0$ has a real root α such that

$$\alpha^3 + \alpha - y = 0$$

$$\Rightarrow \quad \alpha^3 + \alpha = y$$

$$\Rightarrow \quad f(\alpha) = y$$

Thus, for every $y \in R$ there exists $\alpha \in R$ such that $f(\alpha) = y$.

So, f is a surjective map.

Hence, $f : R \to R$ is a bijection.

S20. Injectivity: Let x, y be any two elements of A. Then, $f(x) = f(y)$

$$\Rightarrow \quad \frac{x-1}{x-2} = \frac{y-1}{y-2}$$

$$\Rightarrow \quad (x-1)(y-2) = (x-2)(y-1)$$

$$\Rightarrow \quad xy - y - 2x + 2 = xy - x - 2y + 2 \Rightarrow x = y$$

Thus, $\quad f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$.

So, f is an injective map.

Surjectivity: Let y be an arbitary element of B. Then, $f(x) = y$

$$\Rightarrow \quad \frac{x-1}{x-2} = y$$

$$\Rightarrow \quad (x-1) = y(x-2)$$

$$\Rightarrow \quad x = \frac{1-2y}{1-y}$$

Clearly, $x = \dfrac{1-2y}{1-y}$ is a real number for all $y \neq 1$.

Also, $\dfrac{1-2y}{1-y} \neq 2$ for any y, if we take $\dfrac{1-2y}{1-y} = 2$, then we get 1 = 0, which is wrong.

Thus, every element of y in B has its pre-image x in A given by $x = \dfrac{1-2y}{1-y}$.

So, f is a surjective map.

Hence, f is a bijective map.

S21. Injectivity: Let x, y be any two real numbers. Then,

$$f(x) = f(y) \Rightarrow ax + b = ay + b \Rightarrow ax = ay \Rightarrow x = y$$

Thus, $\quad f(x) = f(y) \Rightarrow x = y$ for all $x, y \in R$ (domain)

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So, f is an injection.

Surjectivity: Let y be an arbitary elelment of R (co-domain). Then,

$$f(x) = y \Rightarrow ax + b = y \Rightarrow x = \left(\frac{y-b}{a}\right).$$

Clearly, $x = \frac{y-b}{a} \in R$ (domain) for all $y \in R$ (co-domain)

Thus, for all $y \in R$ (co-domain) there exists $x = \frac{y-b}{a} \in R$ (domain) such that

$$f(x) = f\left(\frac{y-b}{a}\right) = a\left(\frac{y-b}{a}\right) + b = y$$

This shows that every element in co-domain has its pre-image in domain.
So, f is surjection. Hence, f is a bijection.

S22. Given $f : \{0\} \to R - \{0\}$, defined as $f(x) = \frac{1}{x}$.

For one-one: $f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$, Hence one-one.

For onto: Let $y \in R - \{0\}$, then there must exists $x \in R - \{0\}$

such that $f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$, hence for $y \in R$, there exists unique $\frac{1}{y} \in R - \{0\}$.
Hence onto.

If domain is replaced by N.
For any $x, y \in N$, we find that

$$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$$

So, $f : N \to R - \{0\}$ is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc., in co-domain $R - \{0\}$ do not have their pre-image in domain N.

So, $f : N \to R - \{0\}$ is onto.
Hence, $f : N \to R - \{0\}$ is one-one but not onto.

S23. Given functions $f(x) = \sin x$ and $g(x) = \cos x$.
For $x_1, x_2 \in R$, $f(x_1) = f(x_2) \Rightarrow \sin x_1 = \sin x_2 \Rightarrow x_1 = x_2$

and $\cos x_1 = \cos x_2 \Rightarrow x_1 = x_2$. Hence one-one.

But for $f + g : \sin x + \cos x$.

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Let $x_1 = \frac{\pi}{3}$ and $x_2 = \frac{\pi}{6}$

$$(f+g)\left(\frac{\pi}{3}\right) = f\left(\frac{\pi}{3}\right) + g\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} + \frac{1}{2}$$

and $$(f+g)\left(\frac{\pi}{6}\right) = f\left(\frac{\pi}{6}\right) + g\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

We notice $(f + g)(x_1) = (f + g)(x_2) \not\Rightarrow x_1 = x_2$

Hence '$f + g$' is not one-one.

S24. $f : N \to N$ such that $f(x)$ = roll number of the student x.

For one-one: $f(x) = f(y)$

$\Rightarrow$ Roll number of student x and y is same. This is possible, if $x = y$, *i.e.*, student is same.

$f(x) = f(y) \Rightarrow x = y$. Hence. f is one one.

For onto: Let $b\,(> 46) \in N$, then there does not exist $a \in A$ such that $f(a) = b$. Hence not onto.

S25. We have, $f(x) = x^2 + x + 1$

Injectivity: Let $x, y \in N$ be such that $f(x) = f(y)$

$\Rightarrow \quad x^2 + x + 1 = y^2 + y + 1$

$\Rightarrow \quad x^2 - y^2 + x - y = 0$

$\Rightarrow \quad (x - y)(x + y + 1) = 0$

$\Rightarrow \quad x - y = 0 \quad [\because\ x + y + 1 \neq 0$ for any $x, y \in N]$

$\Rightarrow \quad x = y$

So, f is a one-one function.

Clearly, $f(x) = x^2 + x + 1 \geq 3$ for all $x \in N$.

So, $f(x)$ does not assume values 1 and 2. Therefore, $f : N \to N$ is not an onto function.

S26. **Injectivity:** Let $x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow \quad 4x^3 + 7 = 4y^3 + 7$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad x = y$

Thus, $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in R$.

So, f is an injective map.

Surjectivity: Let y be an arbitary element of R. Then,

$$f(x) = y \Rightarrow 4x^3 + 7 = y \Rightarrow 4x^3 + 7 - y = 0.$$

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We know that an odd degree equation has at least one real root. Therefore, for every real value of y, the equation $4x^3 + 7 - y = 0$ has a real root α such that

$$4\alpha^3 + 7 - y = 0$$

$$\Rightarrow \quad 4\alpha^3 + 7 = y$$

$$\Rightarrow \quad f(\alpha) = y$$

Thus, for every $y \in R$ there exists $\alpha \in R$ such that $f(\alpha) = y$.

So, f is a surjective map.

Hence, $f : R \to R$ is a bijection.

S27. $f : R - \{3\} \to R - \{1\}$ is given by $f(x) = \dfrac{x-2}{x-3}$

Injectivity: Let $x, y \in R - \{3\}$ be such that $f(x) = f(y)$

$$\Rightarrow \quad \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow \quad 1 + \frac{1}{x-3} = 1 + \frac{1}{y-3}$$

$$\Rightarrow \quad \frac{1}{x-3} = \frac{1}{y-3} \Rightarrow x - 3 = y - 3 \Rightarrow x = y$$

So, f is a one-one function.

Surjectivity: Let y be an arbitary element of $R - \{1\}$. Then,

$$f(x) = y \Rightarrow \frac{x-2}{x-3} = y \Rightarrow \frac{2-3y}{1-y} = x$$

Also, $x = 3 \Rightarrow 1 = 0$ which is an absurd result. Therefore, $x \neq 3$.
Clearly, $x \in R - \{3\}$ for all $y \in R - \{1\}$.

Thus, for each $y \in R - \{1\}$ there exists $x = \dfrac{2-3y}{1-y} \in R - \{3\}$ such that $f(x) = y$.

So, f is an onto function.

S28. We have,

$$f(x) = \frac{x}{1+|x|} = \begin{cases} \dfrac{x}{1+x}, & \text{if } \ x \geq 0 \\ \dfrac{x}{1-x}, & \text{if } \ x < 0 \end{cases}$$

So, following cases arise:
Case I: When $x \geq 0$

In this case, we have $f(x) = \dfrac{x}{1+x}$

Injectivity: Let $x, y \in R$ such that $x \geq 0$, $y \geq 0$. Then, $f(x) = f(y)$

$$\Rightarrow \quad \frac{x}{1+x} = \frac{y}{1+y}$$

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$\Rightarrow \quad x + xy = y + xy \Rightarrow x = y$

So, f is an injective map.

Surjectivity: When $x \geq 0$, we have

$$f(x) = \frac{x}{1+x} \geq 0 \quad \text{and} \quad f(x) < 1$$

Let $y \in [0, 1)$ be any real number. Then,

$$f(x) = y \Rightarrow \frac{x}{1+x} = y \Rightarrow x = \frac{y}{1-y}$$

Clearly, $x \geq 0$ for all $y \in [0, 1)$.

So, f is an onto function from $[0, \infty)$ to $[0, 1)$.

Case II: When $x < 0$:

In this case, we have $f(x) = \dfrac{x}{1-x}$

Injectivity: Let $x, y \in R$ such that $x < 0$, $y < 0$. Then, $f(x) = f(y)$

$$\Rightarrow \quad \frac{x}{1-x} = \frac{y}{1-y}$$

$\Rightarrow \quad x - xy = y - xy \Rightarrow x = y$

So, f is an injective map.

Surjectivity: When $x < 0$, we have

$$f(x) = \frac{x}{1-x} < 0$$

Also, $$f(x) = \frac{x}{1-x} = -1 + \frac{1}{1-x} > -1$$

$\therefore \quad -1 < f(x) < 0$

Let $y \in (-1, 0)$ be an arbitary real number such that $f(x) = y$. Then,

$$f(x) = y \Rightarrow \frac{x}{1-x} = y \Rightarrow x = \frac{y}{1+y}$$

Clearly, $x < 0$ for $y \in (-1, 0)$.

So, f is an onto function from $(-\infty, 0)$ to $(-1, 0)$.

Hence, $f : R \to \{x \in R : -1 < x < 1\}$ is a one-one onto function.

S29. We have,

$$f(n) = n - (-1)^n \quad \text{for all} \quad n \in N$$

$$f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$$

Injectivity: Let n, m be any two even natural numbers. Then

$$f(n) = f(m) \Rightarrow n - 1 = m - 1 \Rightarrow n = m$$

If n, m be any two odd natural numbers. Then

$$f(n) = f(m) \Rightarrow n + 1 = m + 1 \Rightarrow n = m.$$

Thus in both the cases, $f(n) = f(m) \Rightarrow n = m$.

If n is even and m is odd, then $n \neq m$. Also $f(n)$ is odd and $f(m)$ is even. So, $f(n) \neq f(m)$.

Thus, $n \neq m \Rightarrow f(n) \neq f(m)$.

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So, f is an injective map.

Surjectivity: Let n be an arbitary natural number.

If n is an odd natural number, then there exists an even natural number $n + 1$ such that

$$f(n + 1) = n + 1 - 1 = n.$$

If n is an even natural number, then there exists an odd natural number $(n - 1)$ such that

$$f(n - 1) = n - 1 + 1 = n.$$

Thus, every $n \in N$ has its pre-image in N.

So, $f : N \to N$ is a surjection.

Hence, $f : N \to N$ is a bijection.

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Q1. If $f(x) = x + 7$ and $g(x) = 2x - 7$, $x \in R$, find $(fog)(7)$.

Q2. Write fog, if $f : R \to R$ and $g : R \to R$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

Q3. Write fog, if $f : R \to R$ and $g : R \to R$ are given by: $f(x) = |x|$ and $g(x) = |5x - 2|$.

Q4. If $f : R \to R$ is defined by $f(x) = 3x + 2$, define $f[f(x)]$.

Q5. If $f(x) = 27x^3$ and $g(x) = x^{1/3}$, find $gof(x)$.

Q6. If $f : R \to R$ and $g : R \to R$ are given by $f(x) = \sin x$ and $g(x) = 5x^2$, find $gof(x)$.

Q7. If $f : R \to R$ be defined by $f(x) = (1 - x^5)^{1/5}$, then find $fof(x)$.

Q8. Let $f : \{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g = \{3, 4, 5, 9\} \to \{7, 11, 15\}$ be functions defined as $f(2) = 3$, $f(3) = 4$, $f(4) = f(5) = 5$ and $g(3) = g(4) = 7$ and $g(5) = g(9) = 11$, find gof.

Q9. Find fog and gof if, $f(x) = [x]$, $g(x) = \sin x$.

Q10. Let $f : \{1, 3, 4\} \to \{1, 2, 5\}$ and $g : \{1, 2, 5\} \to \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

Q11. Find fog and gof giving their domains, where $f(x) = e^x$ and $g(x) = \log x$.

Q12. Let 'f' and 'g' be two real functions defined as $f(x) = 2x - 3$; $g(x) = \frac{3+x}{2}$. Find fog and gof. Can you say one is inverse of the other?

Q13. Le $f(x) = |x|$ and $g(x) = [x]$. Evaluate: $(fog)\left(-\frac{5}{3}\right) - (gof)\left(-\frac{5}{3}\right)$.

Q14. Let $f : R \to R$ be defined by $f(x) = 2x - 3$ and $g : R \to R$ be defined by $g(x) = \frac{x+3}{2}$. Show that $fog = I_R = gof$.

Q15. If $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = 2x$, 'f', 'g' 'h' are real valued functions, show that $ho(fg) = foh$.

Q16. Find fog and gof if, $f(x) = x^2 + 2$, $g(x) = 1 - \frac{1}{1-x}$, $x \neq 1$

Q17. Find fog and gof if, $f(x) = \alpha$, $(\alpha \in R)$, $g(x) = \sin x^2$.

Q18. Let $f : R \to R$ and $g : R \to R$ be two functions such that $fog(x) = \sin x^{2/7}$ and $gof(x) = \sin^{2/7} x$. Then, find $f(x)$ and $g(x)$.

Q19. Give examples of two functions $f : N \to N$ and $g : N \to N$ such that gof is injective but g is not injective.

Q20. Give examples of two functions $f : N \to N$ and $g : N \to N$ such that gof is onto but f is not onto.

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Q21. Let $f: R \to R$ be the function defined

as $f(x)=\begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$ and $g: R \to R$ be the

greatest integer function given by $g(x)=[x]$. Then, prove that *fog* and *gof* coincide in $[-1, 0)$.

Q22. Let '*f*' be real valued function defined by $f(x) = 4x + 3$. Find the real valued function '*g*' such that $gof = fog = I_R$.

Q23. Let $A = \{1, 2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$. Let $f: A \to B$; $g: B \to C$ be defined by $f(1) = 4$, $f(2) = 5$, $f(3) = 4$, $g(4) = 5$, $g(5) = 6$. Find $gof: A \to C$.

Q24. If functions '*f*' and '*g*' are given by $f = \{(1, 2), (3, 5), (4, 1), (2, 6)\}$ and $g = \{(2, 6), (5, 4), (1, 3), (6, 1)\}$, find the range of '*f*' and '*g*' and write down the functions *fog* and *gof*.

Q25. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

Q26. Let *f*, *g* and *h* be functions from *R* to *R*. Show that:

(i) $(f + g)oh = foh + goh$ (ii) $(fg)\,oh = (foh)(goh)$

Q27. Let $f: R \to R$ be a function given by $f(x) = ax + b$ for all $x \in R$. Find the constants *a* and *b* such that $fof = I_R$.

Q28. Let $A = \{x \in R : 0 \le x \le 1\}$. If $f: A \to A$ is defined by

$f(x)=\begin{cases} x, & \text{if } x \in Q \\ 1-x, & \text{if } x \notin Q \end{cases}$

Then prove that $fof(x) = x$ for all $x \in A$.

Q29. Consider $f: N \to N$, $g: N \to N$ and $h: N \to N$ defined as $f(x) = 2x$, $g(y) = 3y + 4$ and $h(z) = \sin z$ for all $x, y, z \in N$. Show that $ho(gof) = (hog)of$.

Q30. If $f: R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g: R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$. Show that $gof = I_A$ and $fog = I_B$, where $B = R - \left\{\frac{3}{5}\right\}$ and $A = R - \left\{\frac{7}{5}\right\}$.

Q31. Let, $f: Z \to Z$ be defined by $f(n) = 3n$ for all $n \in Z$ and $g: Z \to Z$. be defined by

$g(n)=\begin{cases} \frac{n}{3}, & \text{if } n \text{ is a multiple of 3} \\ 0, & \text{if } n \text{ is not a multiple of 3} \end{cases}$ for all $n \in Z$.

Show that $gof = I_Z$ and $fog \ne I_Z$.

Q32. If $f, g: R \to R$ are defined respectively by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$, find (i) *fog* (ii) *gof* (iii) *fof* (iv) *gog*.

Q33. Let $f(x) = \frac{x}{\sqrt{1+x^2}}$. Then, show that $(fofof)(x) = \frac{x}{\sqrt{1+3x^2}}$.

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S1. $(fog)\,(7) = f\{g(7)\} = f\{(2 \times 7) - 7\} = f(7) = 7 + 7 = 14$

S2. $(fog)\,(x) = f\{g\,(x)\} = f\{x^{1/3}\} = 8\,(x^{1/3})^3 = 8x$.

S3. $(fog)\,(x) = f[g\,(x)] = f\{|5x - 2\,|\} = ||\,5x - 2\,|| = |\,5x - 2\,|$.

S4. $f[f\,(x)] = f(3x + 2) = 3\,(3x + 2) + 2 = 9x + 8$.

S5. $(gof)\,(x) = g\,[f\,(x)] = g\,(27\,x^3) = (27\,x^3)^{1/3} = 3\,x$.

S6. $(gof)\,(x) = g\,[f\,(x)] = g\,(\sin\,x) = 5\,\sin^2\,x$.

S7. $(fof)\,(x) = f\,[f\,(x)] = f[\{(1 - x^5)^{1/5}\} = [1 - \{(1 - x^5)^{1/5}\}^5]^{1/5} = (x^5)^{1/5} = x$.

S8. Given $f: \{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g = \{3, 4, 5, 9\} \to \{7, 11, 15\}$

Now $(gof)\,(2) = g\{f(2) = g\,(3) = 7$; $(gof)\,(3) = g\,\{f(3)\} = g\,(4) = 7$

$(gof)\,(4) = g\{f(4)\} = g\,(5) = 11$; $(gof)\,(5) = g\,\{f(5)\} = g\,(5) = 11$

$\therefore$ $gof = \{(2, 7), (3, 7), (4, 11), (5, 11)\}$

S9. $(fog)\,(x) = f\{g\,(x)\} = f(\sin\,x) = [\sin\,x]$

$(gof)\,(x) = g\,\{f\,(x)\} = g\,\{[x]\} = \sin\,[x]$.

S10. $(gof)\,(1) = g\,\{f\,(1)\} = g\,(2) = 3$.

$(gof)\,(3) = g\,\{f\,(3)\} = g\,(5) = 1$

$(gof)\,(4) = g\,\{f\,(4)\} = g\,(1) = 3$

$\therefore$ $gof = \{(1, 3), (3, 1), (4, 3)\}$.

S11. Given $f\,(x) = e^x$, $g\,(x) = \log\,x$

$(fog)\,(x) = f\,\{g\,(x)\} = f\,(\log\,x) = e^{\log\,x} = x$ Domain : $x > 0$

$(gof)\,(x) = g\,\{f\,(x)\} = g\,(e^x) = \log\,e^x = x$ Domain : Real numbers

S12. $(fog)\,(x) = f\,\{g\,(x)\} = f\left(\frac{3+x}{2}\right) = 2\left(\frac{3+x}{2}\right) - 3 = 3 + x - 3 = x$

$(gof)\,(x) = g\,\{f\,(x)\} = g\,(2x - 3) = \frac{3 + 2x - 3}{2} = \frac{2x}{2} = x$

As '*fog*' and '*gof*' are identity functions, so one is inverse of the other.

S13. $$(fog)\left(-\frac{5}{3}\right) = f\left\{g\left(-\frac{5}{3}\right)\right\} = f\left\{\left[-\frac{5}{3}\right]\right\} = f(-2) = |-2| = 2$$

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$$(gof)\left(-\frac{5}{3}\right) = g\left\{f\left(-\frac{5}{3}\right)\right\} = g\left\{\left|-\frac{5}{3}\right|\right\} = g\left(\frac{5}{3}\right) = \left[\frac{5}{3}\right] = 1$$

$$\therefore \qquad (fog)\left(-\frac{5}{3}\right) - (gof)\left(-\frac{5}{3}\right) = 2 - 1 = 1$$

S14. $(fog)(x) = f\{g(x)\} = f\left(\frac{x+3}{2}\right) = 2\left(\frac{x+3}{2}\right) - 3 = x + 3 - 3 = x$

$(gof)(x) = g\{f(x)\} = g(2x - 3) = \frac{2x - 3 + 3}{2} = x$.

S15. $\{ho(fg)\}(x) = h\{(fg)(x)\} = h\{f(x)\, g(x)\} = h\{\sin x \cdot \cos x\}$

$= 2 \sin x \cos x = \sin 2x$.

$foh\,(x) = f\{h(x)\} = f(2x) = \sin 2x$

Therefore, $ho(fg) = foh$.

S16. $(fog)(x) = f\{g(x)\} = f\left\{1 - \frac{1}{1-x}\right\} = \left(1 - \frac{1}{1-x}\right)^2 + 2$

$(gof)(x) = g\{f(x)\} = g(x^2 + 2) = 1 - \frac{1}{1-(x^2+2)} = 1 + \frac{1}{x^2+1}$.

S17. $(fog)(x) = f\{g(x)\} = f(\sin x^2) = \alpha$

$(gof)(x) = g\{f(x)\} = g(\alpha) = \sin \alpha^2$.

S18. We have,

$fog\,(x) = \sin x^{2/7}$ and $gof(x) = \sin^{2/7} x$

$\Rightarrow \quad f(g(x)) = \sin(x^{2/7})$ and $g(f(x)) = (\sin x)^{2/7}$

$\Rightarrow \quad f(x) = \sin x$ and $g(x) = x^{2/7}$.

S19. Let $f : N \to N$ and $g : N \to N$ be given by $f(x) = x$ and $g(x) = |x|$. Then, g is not injective as $g(-2) = g(2) = 2$.

Now, $\quad gof : N \to N$ is given by

$gof(x) = g(f(x)) = g(x) = |x| = x \qquad [\because\ x \in N]$

Therefore, gof is injective but g is not injective.

S20. If $f(x) = x + 1$ and $g(x) = \begin{cases} x - 1, & \text{if } x > 1 \\ x, & \text{if } x = 1 \end{cases}$, then $f : N \to N$ is not onto because range $(f) = N - \{1\}$ $\neq$ Co-domain of f

Now, $gof(x) = g(f(x)) = g(x + 1) = x + 1 - 1 = x \qquad [\because\ x + 1 > 1]$

Therefore, gof, being identity function, is onto.

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S21. For any $x \in [-1, 0)$, we have

$$fog(x) = f(g(x)) = f([x]) = f(-1) = -1$$

and, $$gof(x) = g(f(x)) = g(-1) = [-1] = -1$$

$\therefore$ $$gof(x) = fog(x) \text{ for all } x \in [-1, 0)$$

Hence, *gof* and *fog* coincide in $[-1, 0)$.

S22. Given $f(x) = 4x + 3$.

Let $f(x) = a \Rightarrow 4x + 3 = a \Rightarrow x = \frac{a-3}{4}$

We define $g : R \to R$ as $g(x) = \frac{x-3}{4}$

Now, $(fog)(x) = f\{g(x)\} = f\left(\frac{x-3}{4}\right) = 4\left(\frac{x-3}{4}\right) + 3 = x$

and $(gof)(x) = g\{f(x)\} = g\{4x+3\} = \frac{4x+3-3}{4} = x$

Hence $g(x) = \frac{x-3}{4}$

S23. $(gof)(1) = g\{f(1)\} = g(4) = 5$

$(gof)(2) = g\{f(2)\} = g(5) = 6$

$(gof)(3) = g\{f(3)\} = g(4) = 5$

$\therefore$ $gof : \{(1, 5), (2, 6), (3, 5)\}$.

S24. $\text{Range}_f = \{1, 2, 5, 6\}$; $\text{Range}_g = \{1, 3, 4, 6\}$

$(fog)(2) = f\{g(2)\} = f(6)$, not defined.

fog is not defined.

$(gof)(1) = g\{f(1)\} = g(2) = 6$

$(gof)(3) = g\{f(3)\} = g(5) = 4$

$(gof)(4) = g\{f(4)\} = g(1) = 3$

$(gof)(2) = g\{f(2)\} = g(6) = 1$

gof = $\{(1, 6), (3,4), (4, 3), (2, 1)\}$

S25. We have,

$$f(f(x)) = f(x^2 - 3x + 2)$$

$$f(f(x)) = f(y), \text{ where } y = x^2 - 3x + 2.$$

$$f(f(x)) = y^2 - 3y + 2 \qquad [\because f(x) = x^2 - 3x + 2]$$

$$f(f(x)) = (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2 = x^4 - 6x^3 + 10x^2 - 3x$$

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S26. (i) Since f, g and h are functions from R to R. Therefore,

$$(f+g)oh : R \to R \quad \text{and} \quad foh + goh : R \to R$$

Now

$$((f+g)oh)(x) = (f+g)(h(x))$$

$$\Rightarrow \quad ((f+g)oh)(x) = f(h(x)) + g(h(x))$$

$$\Rightarrow \quad ((f+g)oh)(x) = foh(x) + goh(x) \text{ for all } x \in R$$

$$\therefore \quad (f+g)oh = foh + goh$$

(ii) Since f, g and h are functions from R to R. Therefore,

$$(fg)oh : R \to R \text{ and } (foh)(goh) : R \to R$$

We have,

$$\{(fg)oh\}(x) = (fg)(h(x))$$

$$\Rightarrow \quad \{(fg)oh\}(x) = f(h(x))\,g(h(x))$$

$$\Rightarrow \quad \{(fg)oh\}(x) = (foh)(x)\,(goh)(x))$$

$$\Rightarrow \quad \{(fg)oh\}(x) = \{(foh).(goh)\}(x) \text{ for all } x \in R$$

$$\therefore \quad (fg)oh = (foh).(goh).$$

S27. We have, $fof = I_R$

$$\Rightarrow \quad fof(x) = I_R \text{ for all } x \in R$$

$$\Rightarrow \quad f(f(x)) = x \text{ for all } x \in R \qquad [\because I_R(x) = x \text{ for all } x \in R]$$

$$\Rightarrow \quad f(ax+b) = x \text{ for all } x \in R$$

$$\Rightarrow \quad a(ax+b) + b = x \text{ for all } x \in R$$

$$\Rightarrow \quad a^2x + ab + b = x \text{ for all } x \in R$$

Compairing the coefficient of x and constant term

$$\Rightarrow \quad a^2 - 1 = 0 \text{ and } ab + b = 0$$

$$\Rightarrow \quad a = \pm 1 \text{ and } b(a+1) = 0$$

When $a = 1$,

$$b(a+1) = 0 \Rightarrow 2b = 0 \Rightarrow b = 0$$

$$\therefore \quad a = 1 \text{ and } b = 0.$$

When $a = -1$,

$$b(a+1) = 0 \text{ for all } b \in R$$

$$\therefore \quad a = -1 \text{ and } b \text{ can take any real value.}$$

Hence, either $a = 1$ and $b = 0$ or $a = -1$ and b can take any real value.

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S28. Let $x \in A$. Then, either x is rational or x is irrational. So two cases arise.

Case I: When $x \in Q$:

In this case, we have $f(x) = x$.

$\therefore \quad fof(x) = f(f(x))$

$\Rightarrow \quad fof(x) = f(x) \qquad [\because f(x) = x]$

$\Rightarrow \quad fof(x) = x$

Case II: When $x \notin Q$:

In this case, we have $f(x) = 1 - x$.

$\therefore \quad fof(x) = f(f(x))$

$\Rightarrow \quad fof(x) = f(1 - x) \qquad [\because x \notin Q \quad \therefore f(x) = 1 - x]$

$\Rightarrow \quad fof(x) = 1 - (1 - x) = x \qquad [\because x \notin Q \Rightarrow 1 - x \notin Q \Rightarrow f(1 - x) = 1 - (1 - x)]$

Thus $\quad \Rightarrow \quad fof(x) = x$ whether $x \in Q$ or $x \notin Q$.

Hence, $\quad fof(x) = x$ for all $x \in A$.

S29. We have,

$f(x) = 2x$, $g(y) = 3y + 4$ and $h(z) = \sin z$ for all $x, y, z \in N$

$\therefore \quad gof(x) = g(f(x)) = g(2x) = 3(2x) + 4 = 6x + 4$

$\Rightarrow \quad \{ho(gof)\}(x) = h\{(gof)(x)\} = h(6x + 4) = \sin(6x + 4) \qquad \dots \text{(i)}$

$(hog(x) = h(g(x)) = h(3x + 4) = \sin(3x + 4)$

$\therefore \quad \{(hog)of)\}(x) = (hog)(f(x)) = (hog)(2x) = \sin 2(3x + 4) = \sin(6x + 4) \quad \dots \text{(ii)}$

From (i) and (ii), we get

$ho(gof) = (hog)of$

S30. We have,

$f : A \to B$ and $g : B \to A$

$\therefore \quad gof : A \to A$ and $fog : B \to B$

Now,

$gof(x) = g(f(x))$

$\Rightarrow \quad gof(x) = g\left(\frac{3x + 4}{5x - 7}\right)$

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$$\Rightarrow \quad gof(x) = \frac{7\left(\frac{3x+4}{5x-7}\right)+4}{5\left(\frac{3x+4}{5x-7}\right)-3} = \frac{21x+28+20x-28}{15x+20-15x+21} = \frac{41x}{41} = x$$

$\therefore$ $gof = A \to A$ such that $gof(x) = x$ for all $x \in A$.

So, $gof = I_A$.

We have,

$$fog(x) = f(g(x))$$

$$\Rightarrow \quad fog(x) = f\left(\frac{7x+4}{5x-3}\right)$$

$$\Rightarrow \quad fog(x) = \frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7} = \frac{21x+12+20x-12}{35x+20-35x+21} = \frac{41x}{41} = x$$

$\therefore$ $fog = B \to B$ such that $fog(x) = x$ for all $x \in B$.

So, $fog = I_B$.

S31. Since $f : Z \to Z$ and $g : Z \to Z$. Therefore, $gof : Z \to Z$ and $fog : Z \to Z$.

For any $n \in Z$, we have

$$gof(n) = g(f(n))$$

$$\Rightarrow \quad gof(n) = g(3n)$$

$$\Rightarrow \quad gof(n) = \frac{3n}{3} = n \qquad \left[\because \ 3n \text{ is a multiple of 3} \quad \therefore \ f(3n) = \frac{3n}{3}\right]$$

$$\Rightarrow \quad gof(n) = n \text{ for all } n \in Z.$$

$$\Rightarrow \quad gof = I_Z.$$

Now, for any $n \in Z$, we have

$$fog(n) = f(g(n))$$

$$\Rightarrow \quad fog(n) = \begin{cases} f\left(\frac{n}{3}\right), & \text{if } n \text{ is a multiple of 3} \\ f(0), & \text{if } n \text{ is not a multiple of 3} \end{cases}$$

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$$\Rightarrow \quad fog(n) = \begin{cases} 3\left(\frac{n}{3}\right), & \text{if } n \text{ is a multiple of 3} \\ 3 \times 0, & \text{if } n \text{ is not a multiple of 3} \end{cases}$$

$$\Rightarrow \quad fog(n) = \begin{cases} n, & \text{if } n \text{ is a multiple of 3} \\ 0, & \text{if } n \text{ is not a multiple of 3} \end{cases}$$

Clearly, $fog(n) \neq n$ for all $n \in Z$. In fact, $fog(n) = n$ only for multiple of 3.

So, $fog \neq I_Z$.

S32. Since range f = domain g and range g = domain f. Therefore,

fog, gof, fof and gog all exists.

(i) For any $x \in R$, we have

$$(fog)(x) = f(g(x)) = f(2x-3) = (2x-3)^2 + 3(2x-3) + 1 = 4x^2 - 6x + 1$$

$\therefore$ $fog : R \to R$ is defined by $(fog)(x) = 4x^2 - 6x + 1$, for all $x \in R$.

(ii) For any $x \in R$, we have

$$(gof)(x) = g(f(x)) = g(x^2 + 3x + 1) = 2(x^2 + 3x + 1) - 3 = 2x^2 + 6x - 1$$

$\therefore$ $gof : R \to R$ is defined by $(gof)(x) = 2x^2 + 6x - 1$, for all $x \in R$.

(iii) For any $x \in R$, we have

$$(fof)(x) = f(f(x)) = f(x^2 + 3x + 1) = (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$$

$\Rightarrow$ $(fof)(x) = x^4 + 6x^3 + 14x^2 + 15x + 5$

$\therefore$ $fof : R \to R$ is defined by $(fof)(x) = x^4 + 6x^3 + 14x^2 + 15x + 5$

(iv) For any $x \in R$, we have

$$(gog)(x) = g(g(x)) = g(2x-3) = 2(2x-3) - 3 = 4x - 9$$

$\therefore$ $gog : R \to R$ is defined by $(gog)(x) = 4x - 9$.

S33. We have, $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

Clearly, domain $(f) = R$.

In order to find the range of f, we proceed as follows:

Let $f(x) = y$. Then,

$$\Rightarrow \quad \frac{x}{\sqrt{1+x^2}} = y$$

$$\Rightarrow \quad \frac{x^2}{1+x^2} = y^2$$

$\Rightarrow \qquad x = \pm \dfrac{y}{\sqrt{1-y^2}}$

Since *x* takes real values. Therefore,

$1 - y^2 > 0$

$\Rightarrow \qquad y^2 - 1 < 0$

$\Rightarrow \qquad y \in (-1, 1),$

Hence, range $(f) = (-1, 1)$

Clearly, range $(f) \subset$ co-domain *f*. Therefore, $fof : R \to R$ and $fofof : R \to R$.

Now,

$(fofof)\,(x) = ((fof)\,of)\,(x) = (fof)\,(f(x))$

$$\Rightarrow \qquad (fofof)(x) = (fof)\left(\frac{x}{\sqrt{1+x^2}}\right) = f\left(f\left(\frac{x}{\sqrt{1+x^2}}\right)\right)$$

$$\Rightarrow \qquad (fofof)(x) = f\left(\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}\right) = f\left(\frac{x}{\sqrt{1+2x^2}}\right) = \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}} = \frac{x}{\sqrt{1+3x^2}}$$

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Q1. If 'f' is an invertible function, defined as $f(x) = \frac{3x-4}{5}$, write $f^{-1}(x)$.

Q2. Show that the function $f: R \to R$ is given by $f(x) = x^2 + 1$ is not invertible.

Q3. If $f(x)$ is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$.

Q4. Let $f: A \to B$ be defined by $f(x) = x^2 + 1$. Find the pre-image of (i) 10; (ii) –5.

Q5. Let $f: A \to B$, where set $A = \{1, 2, 3\}$ and $B = \{a, c\}$ defined as $f(1) = a$, $f(2) = c$, $f(3) = a$. Find f^{-1}, if exists.

Q6. Let $f: R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \to R$ such that $gof = fog = I_R$.

Q7. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse (f^{-1}) of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

Q8. Consider $f: \{1, 2, 3\} \to \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Show that $(f^{-1})^{-1} = f$.

Q9. Let $f: N \to A$ be a function defined as $f(x) = 5x + 4$, where $A = \{a \in N) \mid a = 5x + 4$ for some $x \in N\}$. Show that 'f' is invertible. Find the inverse.

Q10. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f?

Q11. Consider $f: \{1, 2, 3\} \to \{a, b, c\}$ and $g: \{a, b, c\} \to$ {apple, ball, cat} defined as $f(1) = a$, $f(2) = b$, $f(3) = c$, $g(a)$ = Apple, $g(b)$ = ball and $g(c)$ = cat. Show that f, g and gof are invertible. Find f^{-1}, g^{-1} and $(gof)^{-1}$ and show that $(gof)^{-1} = f^{-1} o\, g^{-1}$.

Q12. Consider $f: R \to R$ given by $f(x) = 8x + 5$. Show that f is invertible. Find the inverse of f.

Q13. Show that $f: [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one, Find the inverse of the function $f: [-1, 1] \to Range(f)$.

Q14. Let $Y = \{n^2 : n \in N\} \subset N$. Consider $f: N \to Y$ given by $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

Q15. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \frac{x-2}{x-3}$. Show that 'f' is one-one and onto and hence find f^{-1}.

Q16. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$. Show that f is invertible. Find its inverse.

Q17. Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to Range(f)$ is invertible. Find the inverse of f.

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S1. $y = \frac{3x-4}{5} \Rightarrow 5y = 3x - 4 \Rightarrow x = \frac{5y+4}{3} \Rightarrow f^{-1}(x) = \frac{5x+4}{3}$

S2. We have,

$$f(x) = x^2 + 1.$$

Clearly, $-2 \neq 2$ but, $f(-2) = f(2) = 5$.

So, f is not a one-one function. Hence, f is not invertible.

S3. $y = f(x) \Rightarrow \; y = \frac{3x-2}{5} \Rightarrow 5y = 3x - 2 \Rightarrow x = \frac{5y+2}{3} \Rightarrow f^{-1}(y) = \frac{5y+2}{3}$

$\Rightarrow \quad f^{-1}(x) = \frac{5x+2}{3}$.

S4. (i) $10 = x^2 + 1 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

(ii) $-5 = x^2 + 1 \Rightarrow x^2 = -6$. Pre-image does not exist.

S5. We notice that function is not one-one as $f(1) = a$ and $f(3) = a$.

Hence not bijective. So f^{-1} does not exist.

S6. We have,

$fog = I_R \Rightarrow fog(x) = I_R(x)$ for all $x \in R$

$\Rightarrow \quad f(g(x)) = x$

$\Rightarrow \quad 10g(x) + 7 = x$ for all $x \in R$

$\Rightarrow \quad g(x) = \frac{x-7}{10}$ for all $x \in R$

Aliter: We have, $fog = gof = I_R$

g is the inverse of f

Let $f(x) = y$. Then,

$$10x + 7 = y \Rightarrow x = \frac{y-7}{10} \Rightarrow f^{-1}(y) = \frac{y-7}{10} \Rightarrow f^{-1}(x) = \frac{x-7}{10}$$

Hence, $g(x) = \frac{x-7}{10}$.

S7. $f(x_1) = f(x_2) \Rightarrow x_1^2 + 4 = x_2^2 + 4 \Rightarrow x_1 = x_2$ ($\because \; x_1, x_2 \in R_+$). So one-one.

For $y \in [4, \infty)$, Let $x \in R_+$. Such that $f(x) = y$

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$\Rightarrow \quad x^2 + 4 = y \Rightarrow x = \sqrt{y-4} \in R_+$. Hence onto

$\therefore \quad f$ is invertible, with $f^{-1}(y) = \sqrt{y-4}$, *i.e.*, $f^{-1}(x) = \sqrt{x-4}$.

S8. We have,

$$f = \{(1, a), (2, b), (3, c)\} \quad \text{... (i)}$$

Clearly, f is bijection and hence invertible

$\therefore \quad f^{-1} = \{(a, 1), (b, 2), (c, 3)\}$

$\Rightarrow \quad (f^{-1})^{-1} = \{(1, a), (2, b), (3, c)\} \quad \text{... (ii)}$

From (i) and (ii), we have $(f^{-1})^{-1} = f$.

S9. Consider some element $a \in A$,
By definition $a = 5x + 4$, for some $x \in N$

$\Rightarrow \quad x = \frac{a-4}{5}$, we define $g : A \to N$ as $g(a) = \frac{a-4}{5}$

Now $\quad (gof)(x) = g\{f(x)\} = g\{5x + 4\} = \frac{5x+4-4}{5} = x$

and $\quad (fog)(a) = f\{g(a)\} = f\left(\frac{a-4}{5}\right) = 5\left(\frac{a-4}{5}\right) + 4 = a$

We notice $gof = I_N$ and $fog = I_A$, which implies that 'f' is invertible and 'g' is inverse of 'f'.
Hence $f^{-1}(x) = \frac{x-4}{5}$.

S10. Given

$$f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}, (f o f)(x) = f\{f(x)\} = f\left(\frac{4x+3}{6x-4}\right) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x.$$

$\Rightarrow \quad f o f(x) = x \quad \Rightarrow \quad f(x) = f^{-1}(x)$, hence inverse of f is f itself.

S11. $f : \{1, 2, 3\} \to \{a, b, c\}$ is given by $f(1) = a$, $f(2) = b$, $f(3) = c$. Clearly, it is a bijection. Similarly, $g : \{a, b, c\} \to$ {apple, ball, cat} given by $g(a)$ = Apple, $g(b)$ = ball and $g(c)$ = cat is also a bijection. Since composition of two bijection is a bijection. So, gof : {1, 2, 3} $\to$ {apple, ball, cat} is a bijection.

It is given that

$f = \{(1, a), (2, b), (3, c)\}$ and g = {(a, apple), (b, ball), (c, cat)}

$\therefore \quad$ gof = {(1, apple), (2, ball), (3, cat)}

Clearly, $f^{-1} = \{(a, 1), (b, 2), (c, 3)\}$ and g^{-1} = {(apple, a), (ball, b), (cat, c)}

$\therefore \quad (gof)^{-1}$ = {(apple, 1), (ball, 2), (cat, 3)} $\quad$ ... (i)

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and $f^{-1}o\,g^{-1} = \{(\text{apple}, 1), (\text{ball}, 2), (\text{cat}, 3)\}$ ... (ii)

From (i) and (ii) $(gof)^{-1} = f^{-1}o\,g^{-1}$.

S12. It is given that $f : R \to R$ such that $f(x) = 8x + 5$.

***f* is an injection:** Let $x, y \in R$ be such that

$$f(x) = f(y) \Rightarrow 8x + 5 = 8y + 5 \Rightarrow x = y$$

So, f is an injection.

***f* is an surjection:** Let y be an arbitary element of R (Co-domain) such that $f(x) = y$. Then,

$$f(x) = y \Rightarrow 8x + 5 = y \Rightarrow x = \frac{y-5}{8}$$

Thus, for any $y \in R$ there exists $x = \frac{y-5}{8} \in R$ such that

$$f(x) = f\left(\frac{y-5}{8}\right) = 8\left(\frac{y-5}{8}\right) + 5 = y.$$

So, $f : R \to R$ is a bijection and hence invertible.

Let f^{-1} denote the inverse of f. Then,

$$fof^{-1}(x) = x \text{ for all } x \in R.$$

$$\Rightarrow \quad f(f^{-1}(x)) = x \text{ for all } x \in R$$

$$\Rightarrow \quad 8f^{-1}(x) + 5 = x \text{ for all } x \in R$$

$$\Rightarrow \quad f^{-1}(x) = \frac{x-5}{8} \text{ for all } x \in R.$$

S13. Let x, y be any two elements of $[-1, 1]$. Then,

$$f(x) = f(y)$$

$$\Rightarrow \quad \frac{x}{x+2} = \frac{y}{y+2}$$

$$\Rightarrow \quad xy + 2x = xy + 2y$$

$$\Rightarrow \quad x = y$$

So, $f : [-1, 1] \to Range(f)$ is one-one.

Obviously, $f : [-1, 1] \to Range(f)$ is onto and hence invertible. Let f^{-1} denote the inverse of f. Then,

$$fof^{-1}(x) = x \text{ for all } x \in Range(f)$$

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$\Rightarrow \quad f(f^{-1}(x)) = x$

$$\Rightarrow \quad \frac{f^{-1}(x)}{f^{-1}(x)+2} = x$$

$\Rightarrow \quad f^{-1}(x) = xf^{-1}(x) + 2x$

$$\Rightarrow \quad f^{-1}(x) = \frac{2x}{1-x}$$

Hence, $f^{-1} : Range(f) \rightarrow [-1, 1]$ is given $f^{-1}(x) = \frac{2x}{1-x}$.

S14. In order to prove that *f* is invertible, it is sufficient to show that it is a bijection.

***f* is one-one:** For any $n, m \in N$, we find that

$f(n) = f(m)$

$\Rightarrow \quad n^2 = m^2$

$\Rightarrow \quad n = m$ $\quad [\because n, m \in N]$

So, $f : N \rightarrow Y$ is one-one.

***f* is onto:** Let *y* be an arbitary element of *Y*. Then there exists $n \in N$ such that

$y = n^2$ $\quad$ (By def. of *Y*]

$\Rightarrow \quad y = f(n)$

Thus, for each $y \in Y$ there exists $n \in N$ such that $y = f(n)$.

So, $f : N \rightarrow Y$ is onto.

Hence, $f : N \rightarrow Y$ is a bijection. Consequently, it is invertible.

Let f^{-1} denote the inverse of *f*. Then,

$fof^{-1}(x) = x$ for all $x \in Y$

$\Rightarrow \quad f(f^{-1}(x)) = x$ for all $x \in Y$

$\Rightarrow \quad \{f^{-1}(x)\}^2 = x$ for all $x \in Y$ $\quad$ [Using def. of *f*]

$\Rightarrow \quad f^{-1}(x) = \sqrt{x}$ for all $x \in Y$

Hence, $f^{-1} : Y \rightarrow N$ is given by $f^{-1}(x) = \sqrt{x}$ for all $x \in Y$.

S15. Given $A = R - \{3\}$ and $B = R - \{1\}$ and $f(x) = \frac{x-2}{x-3}$

For one-one: Let for $x_1, x_2 \in A$, $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6 \Rightarrow x_2 = x_1$$

As $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence, function is one-one.

For onto: Let for $y \in B$, there exists $x \in A$ such that $y = f(x) \Rightarrow y = \frac{x-2}{x-3}$

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$\Rightarrow \quad xy - 3y = x - 2 \Rightarrow xy - x = 3y - 2$

$\Rightarrow \quad x(y - 1) = 3y - 2$

$\Rightarrow \quad x = \dfrac{3y-2}{y-3} \in A$. Hence onto

For inverse: $x = \dfrac{3y-2}{y-1} \Rightarrow f^{-1}(x) = \dfrac{3x-2}{x-1}$.

S16. In order to prove that f is invertible, it is sufficient to show that it is bijection

f is one-one: For any $x, y \in N$, we find that

$f(x) = f(y)$

$\Rightarrow \quad 4x + 3 = 4y + 3$

$\Rightarrow \quad x = y$

So, $f: N \to Y$ is one-one.

f is onto: Let y be an arbitary element of Y. Then, there exists $x \in N$ such that

$y = 4x + 3$ (By def. of Y]

$\Rightarrow \quad y = f(x)$

Thus, for each $y \in Y$ there exists $x \in N$ such that $f(x) = y$.

So, $f: N \to Y$ is onto.

Thus, $f: N \to Y$ is both one-one and onto. Consequently, it is invertible.

Now,

$fof^{-1}(x) = x$ for all $x \in Y$

$\Rightarrow \quad f(f^{-1}(x)) = x$ for all $x \in Y$

$\Rightarrow \quad 4f^{-1}(x) + 3 = x$ for all $x \in Y$ [Using def. of f]

$\Rightarrow \quad f^{-1}(x) = \dfrac{x-3}{4}$ for all $x \in Y$

Hence, $f^{-1}: Y \to N$ is given by $f^{-1}(x) = \dfrac{x-3}{4}$ for all $x \in Y$.

S17. In order to prove that f is invertible, it is sufficient to show that

$f: N \to Range(f)$ is a bijection.

f is one-one: For any $x, y \in N$, we find that

$f(x) = f(y)$

$\Rightarrow \quad 4x^2 + 12x + 15 = 4y^2 + 12y + 15$

$\Rightarrow \quad 4(x^2 - y^2) + 12(x - y) = 0$

$\Rightarrow \quad (x-y)(4x+4y+3)=0$

$\Rightarrow \quad x-y=0 \qquad [\because\ 4x+4y+3 \neq 0 \text{ for any } x, y \in N]$

$\Rightarrow \quad x=y$

So, $f: N \rightarrow Range(f)$ is one-one.

Obviously, $f: N \rightarrow Range(f)$ is onto.

Hence, $f: N \rightarrow Range(f)$ is invertible.

Let f^{-1} denote the inverse of f. Then,

$fof^{-1}(x) = x$ for all $x \in Range(f)$

$\Rightarrow \quad f(f^{-1}(x)) = x$ for all $x \in Range(f)$

$\Rightarrow \quad 4\{f^{-1}(x)\}^2 + 12f^{-1}(x) + 15 = x$ for all $x \in Range(f)$

$\Rightarrow \quad 4\{f^{-1}(x)\}^2 + 12f^{-1}(x) + 15 - x = 0$

$$\Rightarrow \quad f^{-1}(x) = \frac{-12 \pm \sqrt{144-16(15-x)}}{8}$$

$$\Rightarrow \quad f^{-1}(x) = \frac{-12 \pm \sqrt{16x-96}}{8} = \frac{-3 \pm \sqrt{x-6}}{2}$$

$$\Rightarrow \quad f^{-1}(x) = \frac{-3 + \sqrt{x-6}}{2} \qquad [\because\ f^{-1}(x) \in N,\ f^{-1}(x) > 0]$$

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Q1. If the binary operation * on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation * in Z.

Q2. The binary operation $* : R \times R \to R$ is defined as $a * b = 2a + b$. Find $(2 * 3) * 5$.

Q3. Let * be a 'binary' operation on N given by $a * b$ = l.c.m. (a, b) for all $a, b \in N$. Find $3 * 7$.

Q4. If the binary operation *, defined on Q, is defined as $a * b = 2a + b - ab$, for all $a, b \in Q$, find the value of $3 * 5$.

Q5. Let * be a binary operation on N given by $a * b$ = H.C.F (a, b), $a, b \in N$. Write the value of $22 * 4$.

Q6. If the binary operation * on the set of integers Z, is defined by $a * b = a + 3b^2$, then find the value of $2 * 3$.

Q7. Let * be a binary operation, defined by $a * b = 3a + 4b - 2$, find $3 * 4$.

Q8. Show that binary operation $* : N \times N \to N$ given by $a * b = a + b - ab$ is not a valid operation.

Q9. Show that binary operation $* : N \times N \to N$ given by $a * b = a + b + ab$ is a valid operation.

Q10. R is a relation in set A, where $A = \{2, 4, 6\}$. Show that the relation

$R = \{(a, b) \in A \times A$ such that $|a - b| \geq 0\}$ is universal relation.

Q11. R is a relation in set $A = \{1, 2, 3, 4\}$ given by $R = \{(a, b) \in A \times A \mid a - b = 5\}$. FInd the relation R.

Q12. If $A = \{2, 5, 7, 9\}$, write the relation aRb such that 'a is less than b' $a, b \in A$.

Q13. If $A = \{1, 2, 3, 4, 5, 6\}$, write the relation aRb such that $a + b = 8$ $a, b \in A$.

Q14. Let A be a set, having more than one element. Let o be a binary operation on A defined by $aob = a$, $(a, b) \in A$. Is operation o associative?

Q15. Show that division is not a binary operation on N.

Q16. An operation * on Z^+ is defined as $a * b = a - b$. Is operation * a binary operation? Justify your answer.

Q17. If * is a binary operation defined on the set of natural number N, defined by $a * b = a^b$. Find
(a) $2 * 4$ (b) $3 * 5$.

Q18. If * is a binary operation defined on Q, given by $a * b = a + ab$, $a, b \in Q$. Is * commutative?

Q19. Let * be a binary operation on N, given by $a * b$ = l.c.m (a, b), $a, b \in N$. Find $(2 * 3) * 6$.

Q20. If * is a binary operation on Z, given by $a * b = a - b$, $a, b \in Z$. Can we say that $a * b = b * a$? Give reasons.

Q21. Show that binary operation $* : R \times R \to R$ defined as $a * b = a \div b$ exists, is not commutative.

Q22. Show that binary operation $* : N \times N \to N$ defined as $a * b$ = l.c.m (a, b) is associative.

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Q23. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by $a * b = \min.\{a, b\}$. Write the operation table of the operation *.

Q24. Check whether, if the binary operation *, given by $a * b = \frac{a+b}{2}$, in the set of real numbers, is operation associative.

Q25. A binary operation * over the set of rational numbers Q, is given by $a * b = \frac{ab}{4}$, Evaluate

(i) $5 * 4$ (ii) $3 * 5$ (iii) $-2 * 5$ (iv) $0 * 7$

Q26. Let * the binary operation on N given by $a * b$ = l.c.m. (a, b). Find:

(i) $2 * 3$ (ii) $5 * 15$ (iii) Is * commutative? (iv) Find the identity of operation * in N. (v) which elements of N are invertible? Find them.

Q27. A binary operation * over $R - \{-1\}$ is defined as $a * b = \frac{a}{b+1}$. Is the operation * commutative, associative?

Q28. Consider the binary operations* : $R \times R \to R$ and $o : R \times R \to R$ defined as $a * b = |a - b|$ and $a\ o\ b = a$ for all, $a, b \in R$. Show that '*' is commutative but not associative, 'o' is associative but not commutative.

Q29. Let * be a binary operation defined on Q. Find which of the binary operations are associative.

(i) $a * b = a - b$ (ii) $a * b = \frac{ab}{4}$ (iii) $a * b = a - b + ab$ (iv) $a * b = ab^2$.

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S1. Let e be identity element, then $a * e = e * a = a \Rightarrow a + e - 5 = a \Rightarrow e = 5$.

S2. Given $a * b = 2a + b \Rightarrow (2 * 3) * 5 = (2 \times 2 + 3) * 5 = (4 + 3) * 5 = 7 * 5 = (2 \times 7 + 5) = 14 + 5 = 19$.

S3. $3 * 7 =$ l.c.m $(3, 7) = 21$.

S4. $3 * 5 = 2 \times 3 + 5 - 3 \times 5 = 6 + 5 - 15 = 11 - 15 = -4$.

S5. $22 * 4 =$ H.C.F $(22, 4) = 2$.

S6. $2 * 3 = 2 + 3\,(3)^2 = 2 + 3 \times 9 = 2 + 27 = 29$.

S7. $a * b = 3a + 4b - 2 \Rightarrow 3 * 4 = 3 \times 3 + 4 \times 4 - 2 = 9 + 16 - 2 = 23$.

S8. As $a, b \in N$, $a + b$ is a natural number and ab is also a natural number but $a + b - ab$ may or may not be a natural number.

e.g., Let $a = 2$, $b = 3$ then $a * b = a + b - ab$

$\Rightarrow \quad 2 * 3 = 2 + 3 - 2 \times 3 = 5 - 6 = -1 \notin N$

Therefore, $a * b$ is not a valid operation in N.

S9. As $a, b \in N$, $a + b$ is a natural number and ab is also a natural number. Therefore, $a + b + ab \in N$ for all $a, b \in N$.

Hence, $a * b$ is a valid binary operation in N.

S10. Given $R : A \to A$, where $A = \{2, 4, 6\}$.

We conclude that, $|2 - 2| = 0$, $|2 - 4| > 0$, $|2 - 6| > 0$, $|4 - 6| > 0$, $|4 - 2| > 0$, $|6 - 2| > 0$. etc.

$\therefore \quad R = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\} = A \times A$.

Therefore, R is a universal relation.

S11. $R : A \to A$, we conclude that $1 - 2 \neq 5$, $1 - 3 \neq 5$, $4 - 3 \neq 5$, $2 - 1 \neq 5$ etc.

There is no values of $a, b \in A$, $a - b = 5$.

Hence R has no element. Therefore, R is an empty relation.

S12. Given $A = \{2, 5, 7, 9\}$

Relation $R \subseteq A \times A$, such that $R = \{(a, b) \in A \times A \mid a < b\}$

Now, $2 < 5$, $2 < 7$, $2 < 9$, $5 < 7$, $5 < 9$, $7 < 9$, and $5 \not< 2$, $7 \not< 2$ etc.

$\therefore \quad R = \{(2, 5), (2, 7), (2, 9), (5, 7), (5, 9), (7, 9)\}$

S13. Given $A = \{1, 2, 3, 4, 5, 6\}$

Relation R, such that $R = \{(a, b) \in A \mid a + b = 8\}$

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Now, 2 + 6 = 8, 3 + 5 = 8, 4 + 4 = 8, 5 + 3 = 8, 6 + 2 = 8

$\therefore \quad R = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$.

S14. Given $\quad aob = a, \; a, b \in A,$

$(aob)\, oc = aoc = a$ and $ao\,(boc) = aob = a$.

As $\quad (aob)\, oc = ao\,(boc)$. Hence associative.

S15. Let * be a binary operation on N defined as $a * b = a \div b$, for $a, b \in N$.

If b is not a factor of a then $a * b \notin N$, Therefore, division is not a binary operation on N.

S16. * is defined on Z^+ as $a * b = a - b$,

If $a < b$, then $a - b \notin Z^+$

i.e., $\quad a * b \notin Z^+$. Hence * is not binary operaion on Z^+.

S17. (a) $2 * 4 = 2^4 = 16$ (b) $3 * 5 = 3^5 = 243$.

S18. $a * b = a + ab$ and $b * a = b + ba$

As $\quad a + ab \neq b + ab$, for $a, b \in Q$

Not commutative.

S19. Given a * b = l.c.m (a, b)

$\therefore \quad (2 * 3) * 6 = \{\text{l.c.m. } (2, 3)\} * 6 = 6 * 6 = \text{l.c.m. } (6 * 6) = 6$.

S20. $a * b = a - b$ and $b * a = b - a$

But $a - b \neq b - a$, for $a, b \in Z$. Hence $a * b \neq b * a$.

S21. Given $a * b = a \div b$

Consider $\quad 2, 7 \in R$.

Then $\quad 2 * 7 = 2 \div 7 = \dfrac{2}{7}$

and $\quad 7 * 2 = 7 \div 2 = \dfrac{7}{2}$

but $\quad \dfrac{2}{7} \neq \dfrac{7}{2} \Rightarrow 2 * 7 \neq 7 * 2$. Hence binary operation * is not commutative.

S22. Consider $a, b, c \in N$.

Then $\quad a * (b * c) = a * \text{l.c.m. } (b, c) = \text{l.c.m. } \{a, \text{l.c.m. } (b, c)\} = \text{l.c.m. } \{a, b, c\}$

Also $\quad (a * b) * c = \text{l.c.m. } (a, b) * c = \text{l.c.m. } \{\text{l.c.m. } (a, b), c\} = \text{l.c.m. } \{a, b, c\}$

Hence $\quad a * (b * c) = (a * b) * c$

Let us unerstand by an example also

Consider $2, 3, 5 \in N$.

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Then $a * (b * c) = 2 * (3 * 5) = 2 * 15 = 30$

and $(a * b) * c = (2 * 3) * 5 = 6 * 5 = 30$

$\Rightarrow$ $2 * (3 * 5) = (2 * 3) * 5.$

Hence operation * is associative in N.

S23. Given $a * b = \text{min.} \{a, b\}$.

Operation table for * is

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

S24.

$$a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{a + \frac{b+c}{2}}{2} = \frac{2a+b+c}{4} \quad \ldots \text{(i)}$$

and $$(a * b) * c = \left(\frac{a+b}{2}\right) * c = \frac{\frac{a+b}{2} + c}{2} = \frac{a+b+2c}{4} \quad \ldots \text{(ii)}$$

From (i) and (ii), we get

$a * (b * c) \neq (a * b) * c$. hence not associative.

S25. Given $a * b = \frac{ab}{4}$

(i) $5 * 4 = \frac{5 \times 4}{4} = 5$

(ii) $3 * 5 = \frac{3 \times 5}{4} = \frac{15}{4}$

(iii) $-2 * 5 = \frac{-2 \times 5}{4} = \frac{-5}{2}$

(v) $0 * 7 = \frac{0 \times 7}{4} = 0$

S26. Given $a * b = \text{l.c.m.} (a, b)$

(i) $2 * 3 = \text{l.c.m.} (2, 3) = 6$

(ii) $5 * 15 = \text{l.c.m.} (5, 15) = 15$

(iii) As, a * b = l.c.m. of a and b

and $b * a$ = l.c.m. of b and a

for, $a, b \in N$

l.c.m. of a and b = l.c.m of b and a

i.e., $a * b = b * a$. Hence commutative.

(iv) Let for $e \in N$, we get

$x * e = e * x = x$ for all $x \in N$

$\Rightarrow$ l.c.m. (x, e) = l.c.m. $(e, x) = x \Rightarrow e = 1$

$\therefore$ 1 is identity element in N.

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(v) Let a be an invertible element in N. Then, there exists $b \in N$ such that

$a * b = 1$

$\Rightarrow$ l.c.m. $(a, b) = 1$

$\Rightarrow$ $a = b = 1$.

Thus, 1 is the invertible element of N.

S27. Given: $a * b = \dfrac{a}{b+1}$.

For commutative: $a * b = \dfrac{a}{b+1}$ and $b * a = \dfrac{b}{a+1}$

As $\dfrac{a}{b+1} \neq \dfrac{b}{a+1}$

$\therefore$ $a * b \neq b * a$

Therefore, not commutative

For associative $a * (b * c) = a * \left(\dfrac{b}{c+1}\right) = \dfrac{a}{\dfrac{b}{c+1}+1} = \dfrac{a}{\dfrac{b+c+1}{c+1}} = \dfrac{a(c+1)}{b+c+1}$

and $(a * b) * c = \left(\dfrac{a}{b+1}\right) * c = \dfrac{\dfrac{a}{b+1}}{c+1} = \dfrac{a}{(b+1)(c+1)}$

$\dfrac{a(c+1)}{b+c+1} \neq \dfrac{a}{(b+1)(c+1)}$, therefore, $a * (b * c)$ = $(a * b) * c$. Not associative

S28. (i) Given: $a * b = |a - b|$, for all $a, b \in R$

and $b * a = |b - a| = |-(a - b)| = |a - b|$

As $a * b = b * a$ for all $a, b \in R$. Hence * is commutative.

Let $a = 2,\ b = 3,\ c = 4$

$(a * b) * c = (2 * 3) * 4 = |2 - 3| * 4 = 1 * 4 = |1 - 4| = 3.$

$a * (b * c) = 2 * (3 * 4) = 2 * |3 - 4| = 2 * 1 = |2 - 1| = 1.$

As $(a * b) * c \neq a * (b * c)$. Hence * is not associative.

(ii) Given: $a\ o\ b = a$, for all $a, b \in R$

and $b\ o\ a = b$

As $a\ o\ b \neq b\ o\ a$. Hence o is not commutative.

Also consider $(a\,o\,b)\,o\,c = a\,o\,c = a$ and $a\,o\,(b\,o\,c) = a\,o\,b = a$

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As $(a \, o \, b) \, o \, c = a \, o \, (b \, o \, c)$ for all $a, b, c \in R$

Hence o is associative.

S29. (i) $a * b = a - b$

$(a * b) * c = (a - b) * c \quad \Rightarrow \quad (a - b) - c = a - b - c$

$a * (b * c) = a * (b - c) \quad \Rightarrow \quad a - (b - c) = a - b + c$

As $(a * b) * c \neq a * (b * c)$. Hence not associative.

(ii) $$(a * b) * c = \left(\frac{ab}{4}\right) * c = \frac{\frac{ab}{4} \times c}{4} = \frac{abc}{16}$$

$$a * (b * c) = a * \left(\frac{bc}{4}\right) = \frac{a \times \frac{bc}{4}}{4} = \frac{abc}{16}$$

As $(a * b) * c = a * (b * c)$. Hence associative.

(iii) $(a * b) * c = (a - b + ab) * c = (a - b + ab) - c + (a - b + ab)\, c$

$a * (b * c) = a * (b - c + bc) = a - (b - c + bc) + a\,(b - c + bc)$

As $(a * b) * c \neq a * (b * c)$, Hence not associative.

(iv) $a * b = ab^2$

$(a * b) * c = (ab^2) * c = (ab^2)\, c^2 = ab^2c^2$

$a * (b * c) = a * (bc^2) = a\,(bc^2)^2 = ab^2c^4$

As $(a * b) * c \neq a * (b * c)$, Hence not associative.

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Q1. For any relation R in a set A, we can define the inverse relation R^{-1} by $a R^{-1} b$ if and only if bRa. Prove that R is symmetric if and only if $R = R^{-1}$.

Q2. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, which of the following two is a partition giving rise to an equivalence relation? Why?

(i) $A_1 = \{1, 3, 5\}$, $A_2 = \{2\}$, $A_3 = \{4, 7\}$. (ii) $B_1 = \{1, 2, 5, 7\}$, $B_2 = \{3\}$, $B_3 = \{4, 6\}$.

Q3. For the set $A = \{1, 2, 3\}$, define a relation R in the set A as follows:
$R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Write the ordered pairs to be added to R to make it the smallest equivalence relation.

Q4. Let $A = \{a, b, c\}$ and relation R in set A be given by $R = \{(a, c), (c, a)\}$. Is relation R symmetric? Give reasons.

Q5. Let A be any non-empty set and $P(A)$ be the power set of A. A relation R defined on $P(A)$ by $XRY \Leftrightarrow X \cap Y = X$, $(X, Y) \in P(A)$. Examine whether R is symmetric.

Q6. Let R be a relation in the set of natural numbers N, defined by $R = \{(a, b) \in N \times N : a < b\}$. Is relation R reflexive? Give reasons.

Q7. Let $A = \{a, b, c\}$ and R is a relation in A given by $R = \{(a, a), (a, b), (a, c), (b, a), (c, c)\}$. Is R symmetric?

Q8. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

Q9. Show that the relation R in the set of natural numbers N given by $R = \{(a, b) \in N \times N \mid a$ is divisible by $b\}$ is reflexive and transitive but not symmetric.

Q10. Check whether the following relation is equivalence relation:
aRb iff a "is perpendicular to" b, $a, b \in$ set of lines.

Q11. Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive, nor symmetric and nor transitive.

Q12. Let T be the set of all triangles in a plane with R as a relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.

Q13. Let $A = N \times N$ and * be binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Find the identity element for * on A, if any.

Q14. Show that the relation $R : \{1, 2, 3\} \to \{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric nor transitive

Q15. Show that the relation $R = \{(a, b) \in N \times N \mid a > b\}$ is neither reflexive, nor symmetric but transitive.

Q16. Show that the relation R defined by $R = \{(a, b) : a - b$ is divisible by 3; $a, b \in N\}$ is an equivalence relation.

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Q17. Show that the relation R defined by $(a, b) R (c, d) \Rightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

Q18. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation.

Show that all the elements of {1, 2, 3} are related to each other and all the elements of {2, 4} are related to each other, But, no element of {1, 3, 5} is related to any element of {2, 4}.

Q19. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b) : a, b \in Z, (a - b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.

Q20. Let $f : A \to B$ be a given function. A relation R in set A is given by $R = \{(a, b) \in A \times A \mid f(a) = f(b)\}$. Check, if R is an equivalence relation.

Q21. Let P be the set of all the points in a plane and the relation R in set P be defined as $R = \{(A, B) \in P \times P \mid$ distance between points A and B is less than 3 units$\}$. Show that the relation R is not an equivalence relation.

Q22. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b) : a, b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric nor transitive.

Q23. Let $A = \{1, 2, 3, ..., 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$, for $(a, b), (c, d) \in A \times A$. Prove that R is an equivalence relation, also obtain the equivalence class $[(2, 5)]$.

Q24. If R_1 and R_2 are two equivalence relations in a given set A, show that $R_1 \cap R_2$ is also an equivalence relation.

Q25. In the set of natural numbers N, define a relation R as follows:

For every $m, n \in N$, nRm if on division by 5 each of the integers n and m leave the remainder less than 5, *i.e.*, one of the numbers 0, 1, 2, 3, 4. Show that R is an equivalence relation. Also obtain the pairwise disjoint subsets determined by R.

Q26. Check the relation R for reflexivity, symmetry and transitivity, given as aRb, if b is divisible by a, $\forall\ a, b \in N$.

Q27. Let N be the set of all natural numbers and let R be a relation on $N \times N$, defined by $(a, b) R (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show the R is an equivalence relation on $N \times N$.

Q28. Show that the relation R defined on the set A of all triangles in a plane as

$R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ is an equivalence relation.

Consider three right angle triangles T_1 with sides 3, 4, 5; T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Q29. Show that the relation R on the set A of all the books in a library of a college, given by $R = \{(x, y) : x \text{ and } y \text{ have the same number of pages}\}$, is an equivalence relation.

Q30. Let R be a relation on the set A of ordered pairs of integers defined by $(x, y) R (u, v)$ if $xv = yu$. Show that R is an equivalence relation.

Q31. Given a non-empty set X, consider $P(X)$ which is the set of all subsets of X. Define a relation in $P(X)$ as follows:

For subsets A, B in $P(X)$, $A R B$ if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify your answer.

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Q32. Show that the relation R, defined on the set A of all polygons as

$R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$,

is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Q33. Let R be the relation defined on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b)$: both a and b are either odd or even$\}$. Show that R is an equivalence relation. Further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.

Q34. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\, R\, (c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Check whether R is an equivalence relation on $N \times N$.

Q35. Show that the relation R on the set A of points in a plane, given by

$R = \{(P, Q)$: Distance of the point P from the origin is same as the distance of the point Q from the origin$\}$,

is an equivalence relation.

Further show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

Q36. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation on X given by $R_1 = \{(x, y) : x - y$ is divisible by 3$\}$ and R_2 another relation on X given by $R_2 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\}$ or $\{x, y\} \subset 2, 5, 8\}$ or $(x, y\} \subset \{3, 6, 9\}$. Show that $R_1 = R_2$.

Q37. Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1.

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S1. Let R is symmetric $\Rightarrow (a, b) \in R \Rightarrow (b, a) \in R \Rightarrow (a, b) \in R^{-1}$, Therefore, $R = R^{-1}$

Let $R = R^{-1}$, consider $(a, b) \in R \Rightarrow (a, b) \in R^{-1} \Rightarrow (b, a) \in R$. Hence symmetric.

S2. (i) Not equivalence relation because A_1, A_2, A_3 are disjoint in pairs but $A_1 \cup A_2 \cup A_3 \neq A$.

(ii) Equivalence relation because B_1, B_2, B_3 are disjoint in pairs and $B_1 \cup B_2 \cup B_3 = A$.

S3. As $(1, 3) \in R$, for symmetry (3, 1) should belong to R. Ordered pair (3, 1) should be added.

S4. Yes, as for $(a, c) \in R$, $(c, a) \in R$.

S5. $XRY \Rightarrow X \cap Y = X \Rightarrow Y \cap X = X \Rightarrow YRX$. Hence symmetric.

S6. Given $R = \{(a, b) \in N \times N : a < b\}$. Not reflexive as for $(a, a) \in R$, $a < a$, not true.

S7. Not symmetric as for $(a, c) \in R$, but $(c, a) \notin R$.

S8. $(1, 2) \in R$, $(2, 1) \in R$, but $(1, 1) \notin R$. Therefore, R is not transitive.

S9. Given $R : N \to N$ such that $R = \{(a, b) \in N \times N \mid a \text{ is divisible by } b\}$

We notice aRa for all $a \in N$, i.e., a natural number is divisible by itself. Hence R is reflexive.

Again we notice $aRb \not\Rightarrow bRa$, for all $a, b \in N$, i.e., if a divisible by b then b may not be divisible by a.

For example, 15 is divisible by 5 but 5 is not divisible by 15 in N. Hence R is not symmetric.

Also we notice, aRb, $bRc \Rightarrow aRc$, for all $a, b, c \in N$, if a is divisible by b and b is divisible by c then a is divisible by c.

For example 18 is divisible by 9 and 9 is divisible by 3, then 18 is divisible by 3. Hence relation R is transitive.

S10. Given $R = \{(m, n) \in L \times L \mid m \text{ is perpendicular to } n\}$.

For reflexive: As line m is not perpendicular to itself.
Hence $(m, m) \notin R$. Not reflexive.

For symmetric: $(m, n) \in R \Rightarrow$ line m is perpendicular to n
$\Rightarrow$ line n is perpendicular to $m \Rightarrow (n, m) \in R$. Hence symmetric.

For transitive: Let $(m, n) \in R$ and $(n, p) \in R \Rightarrow m$ is perpendicular to n and n is perpendicular to p
$\Rightarrow$ m is parallel to p $\Rightarrow$ $(m, p) \notin R$. Hence not transitive.
hence not an equivalence relation.

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S11. **For reflexive:** Let $a = \frac{1}{2}, (a, a) \in R \Rightarrow \frac{1}{2} \le \left(\frac{1}{2}\right)^2$ false. Not reflexive.

For symmetric: As $-1 \le (2)^2$ true, but $2 \le (-1)^2$. False, i.e., $(-1, 2) \in R$ but $(2, -1) \notin R$.

Hence R is not Symmetric.

For transitive: As $2 \le (-3)^2$ and $-3 \le 1^2$ but $2 \not\le 1^2$

i.e., $(2, -3) \in R$ and $(-3, 1) \in R$ but $(2, 1) \notin R$. Hence, R is not transitive.

From above R is not reflexive, symmetric, transitive, therefore R is not an equivalence relation.

S12. Given $R = \{T_1, T_2) \in T \times T \mid T_1 \cong T_2\}$

For reflexive: $(T, T) \in R$ is true as $T \cong T$ (i.e., triangle is congruent to itself). Hence R is reflexive.

For symmetric: $(T_1, T_2) \in R \Rightarrow T_1 \cong T_2 \Rightarrow T_2 \cong T_1 \Rightarrow (T_2, T_1) \in R$. Hence R is Symmetric.

For transitive: Let $(T_1, T_2) \in R$ and $(T_2, T_3) \in R \Rightarrow T_1 \cong T_2$ and $T_2 \cong T_3$

$\Rightarrow T_1 \cong T_3 \Rightarrow (T_1, T_3) \in R$. Hence R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

S13. For $(a, b) \in N \times N$, if $(c, d) \in N \times N$ is identity element, then

$(a, b) * (c, d) = (c, d) * (a, b) = (a, b)$

$\Rightarrow$ $(a + c, b + d) = (c + a, d + b) = (a, b)$

$\Rightarrow$ $a + c = a, \quad b + d = b \Rightarrow c = 0, \quad d = 0$

$\Rightarrow$ $(0, 0) \in N \times N$ is identity element. But $(0, 0) \notin N \times N$

Hence no identity element.

S14. Given $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ defined on $R : \{1, 2, 3\} \to \{1, 2, 3\}$

For reflexive: As $(1, 1), (2, 2), (3, 3) \in R$. Hence reflexive.

For symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$. Hence not symmetric.

For transitive: $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$. Hence not transitive.

S15. Consider the relation $R : N \to N$ given by $R = \{(a, b) \in N \times N \mid a > b\}$

We notice aRa, for $a \in N$ is not true as $a > a$ is false. Hence relation is not reflexive.

Again $aRb \not\Rightarrow bRa$, for all $a, b \in N$ as $a > b \not\Rightarrow b > a$. Hence relation is not symmetric.

Also $aRb, bRc \Rightarrow aRc$, for all $a, b, c \in N$ as $a > b, b > c \Rightarrow a > c$.

Therefore, relation R is transitive.

S16. $R = \{(a, b) : a - b$ is divisible by 3, $a, b \in N\}$

For reflexive: $(a, a) \in R \Rightarrow a - a$ is divisible by 3. Hence R is reflexive.

For symmetric: $(a, b) \in R \Rightarrow a - b$ divisible by $3 \Rightarrow b - a$ is divisible by $3 \Rightarrow (b, a) \in R$

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Hence R is Symmetric.

For transitive: Let for $(a, b), (b, c) \in R$

$(a, b) \in R \Rightarrow a - b$ divisible by 3

$(b, c) \in R \Rightarrow b - c$ divisible by 3

As $a - b$ divisible by 3 and $b - c$ divisible by 3. Hence $a - c$ is also divisible by 3

i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$. Hence R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

S17. $(a, b) R (c, d) \Rightarrow a + d = b + c$

For reflexive: $(a, b) R (a, b) \Rightarrow a + b = b + a$, true in N. Hence R is reflexive.

For symmetric: $(a, b) R (c, d) \Rightarrow a + d = b + c \Rightarrow c + b = d + a$

$\Rightarrow (c, d) R (a, b)$ for $(a, b), (c, d) \in N \times N$. Hence R is Symmetric.

For transitive: Let for $(a, b), (c, d), (e, f) \in N \times N$

$(a, b) R (c, d)$ and $(c, d) R (e, f)$

$(a, b) R (c, d) \Rightarrow a + d = b + c$ ... (i)

$(c, d) R (e, f) \Rightarrow c + f = d + e$ ... (ii)

Adding (i) + (ii)

$a + d + c + f = b + c + d + e \Rightarrow a + f = b + e \Rightarrow (a, b) R (e, f)$. Hence transitive

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

S18. **For reflexive:** $(a, a) \in R \Rightarrow |a - a|$ is even, true. Hence R is reflexive.

For symmetric: $(a, b) \in R \Rightarrow |a - b|$ is even $\Rightarrow |b - a|$ is even $\Rightarrow (b, a) \in R$. Hence R is symmetric.

For transitive: $(a, b) \in R, (b, c) \in R \Rightarrow |a - b|$ is even, $|b - c|$ is even ... (i)

Consider $|a - c| = |(a - b) + (b - c)|$ is even, true [from (i)]. Hence R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

As the difference of any two odd (even) natural numbers is always an even natural number. Therefore, all the elements of set $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other.

Similarly, the difference of an even natural numbers and odd natural number is always an odd natural number. Therefore, no element of set $\{1, 3, 5\}$ is related to any elements of $\{2, 4\}$.

S19. $R = \{(a, b) : a - b$ is divisible by 5, $a, b \in Z\}$

For reflexive: $(a, a) \in R \Rightarrow a - a$ is divisible by 5, true. Hence R is reflexive.

For symmetric: $(a, b) \in R \Rightarrow a - b$ divisible by 5 $\Rightarrow b - a$ is divisible by 5 $\Rightarrow (b, a) \in R$, Hence R is symmetric.

For transitive: Let for $(a, b), (b, c) \in R$

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$(a, b) \in R \Rightarrow a - b$ divisible by 5

$(b, c) \in R \Rightarrow b - c$ divisible by 5

As $a - b$ divisible by 5 and $b - c$ divisible by 5. Hence $a - c$ is also divisible by 5

i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$. Hence R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

S20. Given $R = \{(a, b) \in A \times A \mid f(a) = f(b)\}$ and $f : A \to B$.

For reflexive: For $a \in A$, $f(a) = f(a) \Rightarrow (a, a) \in R$, Hence reflexive.

For symmetric: For $a, b \in A$, $f(a) = f(b) \Rightarrow f(b) = f(a)$

$\Rightarrow (a, b) \in R \Rightarrow (b, a) \in R$. Hence symmetirc.

For transitive: Let $a, b, c \in A$

For $a, b \in A$, $f(a) = f(b)$ and for $b, c \in A$, $f(b) = f(c) \Rightarrow f(a) = f(c)$.

Hence transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

S21. Given $R = \{(A, B) \in P \times P \mid$ distance between points A and B is less than 3 units$\}$

For reflexive: Let $(A, A) \in R$ is true as distance between points A and A is 0, which is less than 3. Hence R is reflexive.

For symmetric: Let $(A, B) \in R \Rightarrow$ distance between points A and B is less than 3 units $\Rightarrow$ distance between B and A is less than 3 units hence $(B, A) \in R$.

Hence R is symmetric.

For transitive: Let points A, B, C are collinear. B is the point between A and C such that distance between A and B is 2 units and between B and C is also 2 units, *i.e.*, $(A, B) \in R$ and $(B, C) \in R$, we notice distance between A and C is 4 units.

Hence R is not transitive.

Hence R is not an equivalence relation.

From above R is reflexive, symmetric, but not transitive, therefore R is not an equivalence relation.

S22. Given $R_1 = \{(a, b) \in R \times R \mid a \le b^3\}$

We can consider counter example.

For reflexive: Let $(-2, -2) \in R_1 \Rightarrow -2 \le (-2)^3 \Rightarrow -2 \le -8$, False. Hence not reflexive.

For symmetric: As $1 \le (3^{1/3})^3$ true, but $3^{1/3} \le 1$. False, i.e., $(1, 3^{1/3}) \in R$ but $(3^{1/3}, 1) \notin R$.

Hence R is not symmetric.

For transitive: As $5 \le (2)^3$ and $2 \le (3^{1/3})^3$ but $5 \not\le (3^{1/3})^3$.

i.e., $(5, 2) \in R$ and $(2, 3^{1/3}) \in R$ but $(5, 3^{1/3}) \notin R$. Hence, R is not transitive.

From above R is not reflexive, symmetric, transitive, therefore R is not an equivalence relation.

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S23. **For reflexive:** $(a, b) R (a, b) \Rightarrow a + b = b + a$, true. Hence reflexive.

For symmetric: $(a, b) R (c, d) \Rightarrow a + d = b + c$

$\Rightarrow c + b = d + a \Rightarrow (c, d) R (a, b)$. Hence symmetric.

For transitive: Let for $(a, b), (c, d), (e, f) \in A \times A$

$(a, b) R (c, d)$ and $(c, d) R (e, f)$

$(a, b) R (c, d) \Rightarrow a + d = b + c$ ... (i)

$(c, d) R (e, f) \Rightarrow c + f = d + e$ ... (ii)

Adding (i) + (ii)

$a + d + c + f = b + c + d + e \Rightarrow a + f = b + e \Rightarrow (a, b) R (e, f)$. Hence transitive

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

For equivalence class

$[(2, 5)] = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6,9)\}$.

S24. We have $R_1 \cap R_2 = \{(a, b) \mid (a, b) \in R_1 \text{ and } (a, b) \in R_2\}$

For reflexive: Let $a \in A$, then

$(a, a) \in R_1$ and $(a, a) \in R_2 \Rightarrow (a, a) \in R_1 \cap R_2$. Hence reflexive.

For symmetric: Let $(a, b) \in R_1 \cap R_2$

$\Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2$

$\Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2$ [As R_1 and R_2 are equivalence relations]

$\Rightarrow (b, a) \in R_1 \cap R_2$. Hence symmetric.

Similarly we can show transitivity for $R_1 \cap R_2$. Hence the problem.

S25. Given $nRm \Rightarrow$ on division by 5 each of the integers n and m leaves remainder less than 5.

For reflexive: $aRa \Rightarrow a, a$ on division by 5 leave remainder less than 5, true so reflexive.

For symmetric: $aRb \Rightarrow a, b$ on division by 5 leave remainder less than 5 $\Rightarrow b, a$ on division by 5 leave remainder less than 5. Hence symmetric.

For transitive: $aRb \Rightarrow a, b$ on division by 5 leave remainder less than 5

$bRc \Rightarrow b, c$ on division by 5 leave remainder less than 5

$aRc \Rightarrow a, c$ on division by 5 leave remainder less than 5. Hence transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

Pairwise disjoint subsets on R

$[0] = (5, 10, 15, 20, ...\}$; $[1] = \{1, 6, 11, 16, ...\}$, $[2] = \{2, 7, 12, 22, ...\}$; $[3] = \{3, 8, 13, 18, ...\}$,

$[4] = \{4, 9, 14, 19, ...\}$

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S26. Given $aRb \Rightarrow b$ is divisible by a, for $a, b \in N$.

For reflexive: $aRa \Rightarrow a$ is divisible by a, true hence reflexive.

For symmetric: $aRb \Rightarrow b$ is divisible by a

and $bRa \Rightarrow a$ is divisible by b may not be true. Hence not symmetric.

For transitive: Consider aRb and bRc

$aRb \Rightarrow b$ is divisible by a ... (i)

$bRc \Rightarrow c$ is divisible by b ... (ii)

From (i) and (ii) we observe, c is divisible by a, i.e., aRc

Hence aRb, $bRc \Rightarrow aRc$. Hence transitive

S27. Relation R is defined by $(a, b)\,R\,(c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$.

For reflexive: $(a, b)\,R\,(a, b) \Leftrightarrow ab = ba$, which is true, Hence reflexive.

For symmetric: $(a, b)\,R\,(c, d) \Leftrightarrow ad = bc \Leftrightarrow cb = da$

$\Leftrightarrow \quad (c, d)\,R\,(a, b)$. Hence symmetric.

For transitive: Consider $(a, b)\,R\,(c, d)$ and $(c, d)\,R\,(e, f)$

$\Leftrightarrow \quad ad = bc$ and $cf = de \Leftrightarrow ad \cdot cf = bc \cdot de$

$\Leftrightarrow \quad af = be \Leftrightarrow (a, b)\,R\,(e, f)$. Hence transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

S28. We observe the following properties of relation R:

For reflexive: We know that every triangle is similar to itself. Therefore,

$(T, T) \in R$ for all $T \in A$

$\Rightarrow \quad R$ is reflexive.

For symmetry: Let $(T_1, T_2) \in R$. Then, $(T_2, T_1) \in R$

$\Rightarrow \quad T_1$ is similar to T_2

$\Rightarrow \quad T_2$ is similar to T_1

$\Rightarrow \quad (T_2, T_1) \in R$.

So, R is symmetric.

For transitivity: Let $T_1, T_2, T_3 \in A$ such that $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$. Then,

$\Rightarrow \quad T_1$ is similar to T_2 and T_2 is similar to T_3

$\Rightarrow \quad T_1$ is similar to T_3

$\Rightarrow \quad (T_1, T_3) \in R$.

So, R is transitive.

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From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

In triangles T_1 and T_3, we observe that the corresponding angles are equal and the corresponding sides are proportional *i.e.*, $\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$

Hence, T_1 and T_3 are related.

S29. We observe the following properties of relation R:

For reflexivity: For any book x in set A, we observe that x and x have the same number of pages.

$\Rightarrow \quad (x, x) \in R$

Thus, $(x, x) \in R$ for all $x \in A$

So, R is reflexive.

For symmetric: Let $(x, y) \in R$. Then, $(x, y) \in R$

$\Rightarrow$ x and y have the same number of pages

$\Rightarrow$ y and x have the same number of pages

$\Rightarrow \quad (y, x) \in R$

Then, $(x, y) \in R \Rightarrow (y, x) \in R$

So, R is symmetric.

For transitive: Let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow$ (x and y have the same number of pages) and (y and z have the same number of pages)

$\Rightarrow$ x and z have the same number of pages.

$\Rightarrow \quad (x, z) \in R$

So, R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

S30. The relation R on $Z \times Z$ is defined by

$$(x, y) R (u, v) \iff xv = yu \text{ for all } (x, y), (u, v) \in Z \times Z$$

For reflexive: For any $(x, y) \in Z \times Z$

$xy = yx$ [$\because$ Multiplication is commutative on Z]

$\Rightarrow \quad (x, y) R (x, y)$

Thus, $(x, y) R (x, y)$ for all $(x, y) \in Z \times Z$.

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So, R is a reflexive relation on $Z \times Z$.

For symmetric: Let $(x, y), (u, v) \in Z \times Z$ such that

$(x, y) R (u, v)$. Then,

$(x, y) R (u, v)$

$\Rightarrow \quad xv = yu \Rightarrow uy = vx \Rightarrow (u, v) R (x, y)$

Thus, $(x, y) R (u, v) \Rightarrow (u, v) R (x, y)$ for all $(x, y), (u, v) \in Z \times Z$.

So, R is a symmetric relation on Z.

For transitive: Let $(x, y), (u, v), (a, b) \in Z \times Z$ be such that $(x, y) R (u, v)$ and $(u, v) R (a, b)$.

Then,

$$\text{and} \quad \left.\begin{matrix}(x, y)R(u, v) \Rightarrow xv = yu \\ (u, v)R(a, b) \Rightarrow ub = va\end{matrix}\right\} \Rightarrow (xv)(ub) = (yu)(va) \Rightarrow xb = ya \Rightarrow (x, y)R(a, b)$$

So, R is a transitive relation on $Z \times Z$

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation on $Z \times Z$.

S31. It is given that for any A, B in $P(X)$

$ARB \Leftrightarrow A \subset B$

We observe the following properties of R:

For reflexive: For any A in $P(X)$, we have

$A \subset A \Leftrightarrow ARA$

So, R is reflexive on $P(X)$

For symmetric: Let A, B in $P(X)$ be such that ARB. Then,

$ARB \Leftrightarrow A \subset B$

This need not imply that $B \subset A$. In fact it is possible only when $A = B$.

e.g., $\{1, 2\} \subset \{1, 2, 3\}$, but $\{1, 2, 3\} \not\subset \{1, 2\}$

So, R is not a symmetric relation on $P(X)$.

For transitive: Let A, B, C be in $P(X)$ such that

ARB and BRC

$\Rightarrow \quad A \subset B$ and $B \subset C$

$\Rightarrow \quad A \subset C$

$\Rightarrow \quad ARC$

So, R is a transitive relation on $P(X)$.

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Thus, R is reflexive and transitive relation on $P(X)$ but it is not symmetric.

Hence, R is not an equivalence relation on $P(X)$.

S32. The relation R on the set of A of all polygons is defined as

$R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$

For reflexive: Let P be any polygon in A. Then,

P and P have same number of sides.

$\Rightarrow \quad (P, P) \in R$

Thus, $(P, P) \in R$ for all $P \in A$. So, R is a reflexive relation on A.

For symmetric: Let P_1, P_2 be two polygons in A such that $(P_1, P_2) \in R$.

$\Rightarrow \quad P_1$ and P_2 have same number of sides

$\Rightarrow \quad P_2$ and P_1 have same number of sides

$\Rightarrow \quad (P_2, P_1) \in R$

So, R is symmetric on A.

For transitive: Let P_1, P_2, P_3 be three polygons in A such that $(P_1, P_2) \in R$ and $(P_2, P_3) \in R$. Then,

$(P_1, P_2) \in R \Rightarrow P_1$ and P_2 have same number of sides

$(P_2, P_3) \in R \Rightarrow P_2$ and P_3 have same number of sides

$\therefore \quad P_1$ and P_3 have same number of sides.

$\Rightarrow \quad (P_1, P_3) \in R$

Thus, $(P_1, P_2) \in R$ and $(P_2, P_3) \in R \Rightarrow (P_1, P_3) \in R$

So, R is transitive relation on A.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation on A.

Let P be a polygon in A such that $(P, T) \in R$, where T is a right angle triangle with sides 3, 4 and 5. Then,

$(P, T) \in R$ Polygon P and triangle T have same number of sides

$\Rightarrow \quad P$ is any triangle in A.

Hence, the set of all elements in A related to T is the set of all triangles in A.

S33. The relation R on set $A = \{1, 2, 3, 4, 5, 6, 7\}$ is defined by

$R = \{(a, b)$: both a and b are either odd or even$\}$

For reflexive: As, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7) \in R$. So, R is a reflexive relation on A.

For symmetric: Let $a, b \in A$ be such that $(a, b) \in R$. Then,

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$(a, b) \in R$

$\Rightarrow$ both a and b are either odd or even

$\Rightarrow$ both b and a are either odd or even

$\Rightarrow$ $(b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$. So, R is symmetric relation on A.

For transitive: Let $a, b, c \in Z$ be such that $(a, b) \in R$, $(b, c) \in R$. Then,

$(a, b) \in R \Rightarrow$ both a and b are either odd or even

$(b, c) \in R \Rightarrow$ both b and c are either odd or even

If both a and b are even, then

$(b, c) \in R \Rightarrow$ both b and c are even

$\therefore$ both a and c are even

If both a and b are odd, then

$(b, c) \in R \Rightarrow$ both b and c are odd

$\therefore$ both a and c are odd

Thus, both a and c are even or odd

$\therefore$ $(a, c) \in R$

So, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

Consequently, R is a transitive relation on A.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation on A.

We observe that two elements in A are related if both are odd or both are even. Since {1, 3, 5, 7} has all odd elements of A. So, all the elements of {1, 3, 5, 7} are related to each other. Similarly, all the elements of {2, 4, 6} are related to each other as it contains all even elements of set A. An even(odd) number in A is related to an even (odd) number in A. So, no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

S34. For reflexivity: Let (a, b) be an arbitary element of $N \times N$. Then,

$(a, b) \in N \times N$

$\Rightarrow$ $a, b \in N$

$\Rightarrow$ $ab(b + a) = ba(a + b)$ [by comm. of add. and mult. on N]

$\Rightarrow$ $(a, b) R (a, b)$

Thus, $(a, b) R (a, b)$ for all $(a, b) \in N \times N$. So R is reflexive on $N \times N$.

For symmetry: Let $(a, b), (c, d) \in N \times N$ be such that $(a, b) R (c, d)$. Then,

$(a, b) R (c, d)$

$\Rightarrow$ $ad(b + c) = bc(a + d)$

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$\Rightarrow \quad cb(d + a) = da(c + b)$ [by comm. of add. and mult. on N]

$\Rightarrow \quad (c, d)\, R\, (a, b)$

Thus, $(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for all $(a, b), (c, d) \in N \times N$.

So. R is symmetric on $N \times N$.

For transitive: Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$. Then,

$$(a, b)\, R\, (c, d) \Rightarrow ad(b + c) = bc(a + d) \Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d} \quad \text{(i)}$$

and, $$(c, d)\, R\, (e, f) \Rightarrow cf(d + e) = de(c + f) \Rightarrow \frac{d+e}{de} = \frac{c+f}{cf} \Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f} \quad \text{(ii)}$$

Adding (i) and (ii) we get

$$\left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{d} + \frac{1}{e}\right) = \left(\frac{1}{a} + \frac{1}{d}\right) + \left(\frac{1}{c} + \frac{1}{f}\right)$$

$$\Rightarrow \quad \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow \frac{b+e}{be} = \frac{a+f}{af}$$

$\Rightarrow \quad af(b + e) = be(a + f) \Rightarrow (a, b)\, R\, (e, f)$

Thus, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f) \Rightarrow (a, b)\, R\, (e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$.

So. R is transtive on $N \times N$.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation on $N \times N$.

S35. Let O denote the origin in the given plane. Then,

$R = \{(P, Q) : OP = OQ\}$.

We observe the following properties of relation R:

For reflexivity: For any point P in set A, we have

$OP = OP$

$\Rightarrow \quad (P, P) \in R$

Thus, $(P, P) \in R$ for all $P \in A$

So, R is reflexive.

For symmetric: Let P and Q be two points in set A such that

$(P, Q) \in R$

$\Rightarrow \quad OP = OQ$

$\Rightarrow \quad OQ = OP$

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$\Rightarrow \quad (Q, P) \in R$

Thus, $(P, Q) \in R \Rightarrow (Q, P) \in R$ for all $P, Q \in A$

So, R is symmetric.

For transitive: Let P, Q and S be three points in set A such that

$(P, Q) \in R$ and $(Q, S) \in R$

$\Rightarrow \quad OP = OQ$ and $OQ = OS$

$\Rightarrow \quad OP = OS$

$\Rightarrow \quad (P, S) \in R$

So, R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

Let P, Q be a fixed point in set A such that $(P, Q) \in R$. Then,

$(P, Q) \in R$

$\Rightarrow \quad OP = OQ$

$\Rightarrow$ Q moves in the plane in such a way that its distance from the origin O (0, 0) is always same and is equal to OP.

$\Rightarrow$ Locus of Q is a circle with centre at the origin and radius OP.

Hence, the set of all points related to P is the circle passing through P with origin O as centre.

S36. Clearly, R_1 and R_2 are subsets of $X \times X$. In order to prove that $R_1 = R_2$, it is sufficient to show that $R_1 \subset R_2$ and $R_2 \subset R_1$.

We observe that the difference between any two elements of each of the sets {1, 4, 7}, {2, 5, 8} and {3, 6, 9} is a multiple of 3.

Let (x, y) be an arbitary element of R_1. Then, $(x, y) \in R_1$

$\Rightarrow \quad x - y$ is divisible by 3.

$\Rightarrow \quad x - y$ is a multiple of 3.

$\Rightarrow \quad \{x, y\} \subset \{1, 4, 7\}$ or $\{x, y\} \subset \{2, 5, 8\}$ or $\{x, y\} \subset \{3, 6, 9\}$

$\Rightarrow \quad (x, y) \in R_2$

Thus, $(x, y) \in R_1 \Rightarrow (x, y) \in R_2$

So, $R_1 \subset R_2$ ... (i)

Now, let (a, b) be an arbitary element of R_2. Then, $(a, b) \in R_2$

$\Rightarrow \quad \{a, b\} \subset \{1, 4, 7\}$ or $\{a, b\} \subset \{2, 5, 8\}$ or $\{a, b\} \subset \{3, 6, 9\}$

$\Rightarrow \quad a - b$ is divisible by 3

$\Rightarrow \quad (a, b) \in R_1$

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Thus, $(a, b) \in R_2 \Rightarrow (a, b) \in R_1$.

So, $R_2 \subset R_1$ ... (ii)

From (i) and (ii), we get

$$R_1 = R_2.$$

S37. Given $A = \{x \in Z : 0 \le x \le 12\}$

$S = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$

For reflexive: For $a \in A$

$(a, a) \in S \Rightarrow |a - a|$ is divisible by 4 $\Rightarrow$ 0 is divisible by 4, true. Hence reflexive.

For symmetric: For $a, b \in A$

$(a, b) \in S \Rightarrow |a - b|$ is divisible by 4 $\Rightarrow$ $|b - a|$ is divisible by 4

$(a, b) \in S \Rightarrow (b, a) \in S$. Hence symmetric.

For transitive: For $a, b, c \in A$

$(a, b) \in S$ and $(b, c) \in S$

$\Rightarrow$ $|a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4

$\Rightarrow$ $(a - b)$ is divisible by 4 and $(b - c)$ is divisible by 4

$\Rightarrow$ $|a - c| = |(a - b) + (b - c)|$ is divisible by 4

$\Rightarrow$ $(a, c) \in S$. Hence transitive.

From above S is reflexive, symmetric, transitive, therefore S is an equivalence relation.

Let x be an equivalence relation.

$|x - 1|$ is a multiple of 4

$\Rightarrow$ $|x - 1| = 0, 4, 8, 12$

$\Rightarrow$ $x - 1 = 0, 4, 8, 12$

$\Rightarrow$ $x = 1, 5, 9$ $[\because\ 13 \notin A]$

Hence, the set of all elements of A which are related to 1 is $\{1, 5, 9\}$.

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