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Q1. The speed of a plane at the perihelion is v_p and the radius at perihelion is r_P. At aphelion r_a, what will be the speed?

Q2. Compare the weights of stone when it is 0.5 km above the surface of earth and 1 km below the surface of earth.

Q3. What is the effect of non-spherecity of the earth on the value of '*g*'?

Q4. If you compare the gravitational force on the earth due to the sun to that of due to the moon, you would find that the sun's pull is greater than the moon's pull. However, the effect of the moon's pull is greater than the tidal effect of the sun. Why?

Q5. An astronaut inside a small space ship orbiting around the earth connot detect gravity. If the space-station orbiting around the earth has a large size, can he hope to detect gravity?

Q6. Does the speed increase, decrease or remain constant when a planet comes closer to the sun?

Q7. If it is safe to jump from a height of 2 m on the surface of earth, what is its value on the surface of moon?

Q8. What is the basis for the second Law of Kepler?

Q9. Give the *S.I.* units of '*g*' and '*G*'.

Q10. List one difference and one similarity between gravitational and inertial mass.

Q11. If earth be at half of its present distance from the sun, how many days will there be in a year?

Q12. Answer the following:

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why.

Q13. Explain the law of time period and derive the relation between time period and planets radius.

Q14. The Moon revolves around the Earth and the Earth-Moon system revolves around the sun. If the Earth could be removed suddenly without disturbing the motion of the Moon, what would be the subsequent path of the Moon?

Q15. State Kepler's laws planetary motion.

Q16. State Kepler's laws of planetary motion and deduce Newton's Law of gravitation from them.

Q17. According to Kepler's law of periods, $T^2 = k r^3$. Where k is a constant. Computer the constant k for (a) the Earth and (b) the Venus. Given that orbital radii of the Earth and the Venus are 1.496×10^{11} m and 1.082×10^{11} m; and their respective periods are 1 year and 0.615 year.

Q18. Planet Mars has two Moons – Phobos and Deimos. Phobos has period of 7 hour 39 minutes and orbital radius of 9.4×10^3 km. Calculate the mass of Mars. ($G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}).

Q19. The time period of a satellite of earth is 7 hours. If the separation between the earth and the satellite is increased to two time the previous value, what will be its new time period?

Q20. Prove the kepler's law, that the line joining the sun and the planet sweeps equal areas in equal time, using the angular momentum conservation with the planet.

Q21. The value of acceleration due to gravity at the moon is $\frac{1}{6}$th of the value to gravity at surface of the earth, and the diameter of the moon is $\frac{1}{4}$th of the diameter of the earth. Compare the ratio of the escape velocities.

Q22. The mass and diameter of a planet are twice of those of the earth. What will be the period of oscillation of a pendulum on this planet, if it is second's pendulum on the earth?

Q23. Show graphically how gravitational field strength varies with distance from the centre of earth, outwards. Given the relation also.

Q24. How can you find the mass of earth, starting from the law of gravitation?

Q25. Let the speed of the planet at the perihelion P as shown in the figure be v_P, and the Sun-planet distance SP be r_P. Relate $\{r_P, v_P\}$ to the corresponding quantities at the aphelion $\{r_A, v_A\}$. Will the planet take equal times to traverse BAC and CPB?

Q26. Suppose Earth's orbital motion around the Sun is suddenly stopped, What time the Earth shall take to fall into the Sun?

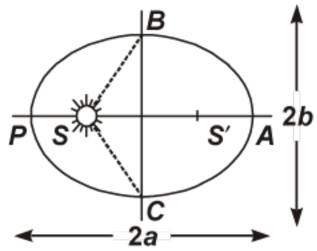
Q27. Calculate the period of revolution of Neptune around the Sun, given that diameter of its orbit is 30 times the diameter of the Earth's orbit around the Sun. Assume both the orbits to be circular.

Q28. How fast (in m^2 s^{-1}) is area swept on by (a) the radius from Sun to Earth? (b) the radius from Earth to moon? Given distance of Sun to Earth = 1.496×10^{11} m distance of Earth Moon = $27\frac{1}{3}$ days.

Q29. A Saturn years is 29.5 times the Earth year. How far is Saturn from the Sun (M) if the Earth is 1.5×10^8 km away from the sun?

Q30. Given $T^2 = k(R_E + h)^3$ where $k = \frac{4\pi^2}{GM_E}$. Here the letters have usual meanings. Express the constant k in days and kilometres. The moon is at a distance of 3.84×10^5 km from Earth. Obtain its time-period of revolution in days.

Q31. Let the speed of the planet at the perihelion *P* in figure be v_P and the Sun-planet distance *SP* be r_P. Relate $\{r_P, v_P\}$ to the corresponding quantities at the aphelion $\{r_A, v_A\}$. Will the planet take equal times to traverse *BAC* and *CPB*?



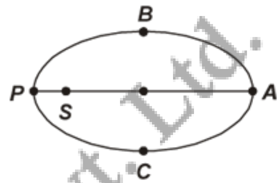
Q32. State the law of orbit and law of area. Derive the Kepler's second law

Q33. Derive the Kepler's third law.

Q34. The planet Mars has two Moons, Phobos and Demos. (a) Phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of Mars. (b) Assume that Earth and Mars move in circular orbits around the Sun, with the Martian orbit being 1.52 times the orbital radius of the Earth. What is the length of the Martian year in days.

Q35. (a) Derive the Kepler's second law.

(b) Let the speed of the planet at perihelion *P* in fig be v_p and Sun planet distance *SP* be r_p Relate (r_p, v_p) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse *BAC* and *CPB*?



Q36. Weighing the Earth. How does one weigh the Earth *i.e.*, estimate its mass? We have seen in text that from the relation $g = GM_e/R_e^2$, M_e can be estimated from the known values of *g*, R_e and *G*. Another way of estimating mass of the Earth is to use data of its satellite, the Moon, The time period of the Moon's revolution around the Earth is 27.3 days and the radius of the lunar orbit *i.e.*, the distance of Moon from Earth is 3.84×10^8 m. Also, $g_E = 9.81\ \text{m s}^{-2}$, $R_E = 6.37 \times 10^6$ m and $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$. Obtain the Mass M_E of the Earth in two different ways.

Q37. Derive the Kepler's third law. The distance of Neptune and Saturn from the sun is nearly 10^{13} m and 10^{12} m respectively. Assuming that they move in circular orbits, then what will be the ratio of their periods.

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S1. Since angular momentum L has to be conserved, $L = mvr$ = constant. For a given planet,

$$v_p r_p = v_a r_a$$

$$v_a = \frac{v_p r_p}{r_a}$$

S2.

$$\frac{W_h}{W_d} = \frac{mg\left(1-\frac{2h}{R}\right)}{mg\left(1-\frac{d}{R}\right)} = \frac{1-\frac{2\left(\frac{1}{2}\right)}{R}}{1-\frac{1}{R}} = 1$$

S3. g is less at equator than at poles, since radius 'h' is more equator than at poles.

S4. Whereas the gravitational force depends inversely on the square of the distance, tidal effect depends inversely on the cube of the distance.

S5. In case the size of the space-station becomes large, the forces of gravity will becomes appreciable, and the astronaut can hope to detect it.

S6. The speed of the planet increases when it comes closer to the sun.

S7. Since g is reduced to 1/6 th, the height will increase to 12 m.

S8. Angular momentum conservation.

S9. $g \to ms^{-2}$, $G \to \text{Nm}^2/\text{kg}^2$

S10. Similarity-Both are equal.

Difference-Gravitational mass is measured by using comparison of mass by force, but inertial mass measured by the acceleration caused by the force.

S11. $T^2 \propto r^3$, Since $r \to \frac{r}{2}, \frac{T_1^2}{T^2} = \left(\frac{1}{2}\right)^3$

$\therefore$ $$T_1 = \left(\frac{1}{8}\right)^{3/2} T; T_1 = \left(\frac{1}{2\sqrt{2}}\right)^3 T$$

i.e., $$T_1 = \frac{T}{16\sqrt{2}} = 16.13 \text{ days}$$

S12. (a) No, gravitational influence of matter on nearby objects cannot be screened by any means. This is because gravitational force unlike electrical forces is independent of the nature of the material medium. Also, it is independent of the status of other objects.

(b) Yes, if the size of the space station is large enough, then the astronaut will detect the change in Earth's gravity (g).

(c) Tidal effect depends inversely upon the cube of the distance while, gravitational force depends inversely on the square of the distance. Since the distance between the Moon and the Earth is smaller than the distance between the Sun and the Earth, the tidal effect of the Moon's pull is greater than the tidal effect of the Sun's pull.

S13. ***Law period:*** The square of the period of any planet about the sun is proportional to the cube of the semi-major axis of the elliptical orbit.

If T is the period of the planet and r, then semi-major axis of elliptical orbit, then

$$T^2 \propto r^3.$$

If follows that smaller the orbit of the planet around the sun; shorter is the time it takes to complete one revolution.

If T_1 and T_2 are the periods of any two planets and r_1 and r_2, the respective semi-major axis of their orbits, then

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3.$$

S14. From Kepler's third law, we **know** that for all the heavenly object moving in a stable **orbit** around the Sun,

$$\frac{T^2}{r^3} = \text{constant} \qquad \text{(independent of mass of the object)}$$

Since r and T are same for both the Moon and Earth (and also for the Earth-Moon system), the present orbit would be an equilibrium orbit for either the Moon or the Earth alone as well as the two together. Therefore, if the Earth could removed, the Moon will revolve around the Sun in the present orbit without any change.

S15. ***law of orbit:*** Each planet revolves around the Sun in an elliptical orbit, with the Sun at one focus of the elliptical path.

law of area: The position vector of the planet from the sun *i.e.*, the line joining the planet to the Sun sweeps out equal areas in equal time *i.e.*, the areal velocity of a planet around the Sun always remains constant.

Law period: The square of the period of any planet about the Sun is proportional to the cube of the semi-major axis of the elliptical orbit.

If T is the period of the planet and r, then semi-major axis of elliptical orbit, then

$$T^2 \propto r^3.$$

S16. (a) The planets including Earth, go around the Sun in elliptical orbits.

(b) The line joining the Sun and the planet sweeps equal areas in equal intervals of time.

(c) The square of the time period of revolution is directly proportional to the cube of the semi-major axis of the elliptical orbit.

Since $T^2 \propto r^3$, we have,

$$\left(\frac{2\pi r}{v}\right)^2 \propto r^3$$

$$v^2 = 4\pi^2 \frac{r^2}{r^3} = \frac{4\pi^2}{r}$$

$$\frac{mv^2}{r} = \frac{4m\pi^2}{r^2}$$

The centripetal force $\frac{mv^2}{r}$ is caused by M – Earth on the planet of mass m.

Thus, $$F \propto \frac{Mm}{r^2}$$

It is the Newton's Universal Law of gravitation.

S17. If T is the period of the planet and r, then semi-major axis of elliptical orbit, then

$$T^2 \propto r^3.$$

Given, $$k = \frac{T^2}{r^3}$$

(a) ***For the earth:*** Given, $r = 1.496 \times 10^{11}$ m

and $$T = 1 \text{ year} = 1 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$k = \frac{(1 \times 365 \times 24 \times 60 \times 60)^2}{(1.496 \times 10^{11})^3}$$

$$= \mathbf{2.97 \times 10^{-19}\ s^2\ m^{-3}}$$

(b) ***For the Venus:*** Given, $r = 1.082 \times 10^{11}$ m

and $$T = 0.615 \text{ year} = 0.615 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$k = \frac{(0.615 \times 365 \times 24 \times 60 \times 60)^2}{(1.082 \times 10^{11})^3}$$

$$= \mathbf{2.97 \times 10^{-19}\ s^2\ m^{-3}}.$$

S18. Given: $r = 9.4 \times 10^3$ km $= 9.4 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}; T = 7hr. 39 min = 27540 sec.

$$M_{mars} = \frac{4\pi^2 r^3}{GT^2}$$

$$M_{mars} = \frac{4 \times \left(\frac{22}{7}\right)^2 \times (9.4 \times 10^6)^3}{6.67 \times 10^{-11} \times (27540)^2}$$

$$= 6.49 \times 10^{23} \text{ kg.}$$

S19. According to Kepler's third law,

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Here, $T_1 = 7\,\text{h}$ and $R_2 = 2R_1$

$$\frac{(7)^2}{T_2^2} = \frac{R_1^2}{(2R_1)^3} \quad \text{or} \quad \frac{T_2^2}{49} = 8$$

or $T_2 = \sqrt{8 \times 49} = \mathbf{19.8\,h}$.

S20. When the planet moves along the line joining the sun and the planet it sweeps some area given by $A = \frac{1}{2}r^2\theta$, where θ is the angular displacement.

$$\therefore \quad \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

S21.

$$E_{go} = \frac{G\frac{4}{3}\pi R^3\rho}{x^2} = \frac{GM}{x^2}$$

$$v_e = \sqrt{\frac{GM}{R}} = \sqrt{2gR} = \sqrt{gD}$$

$$\frac{(v_e)_{moon}}{(v_e)_{earth}} = \sqrt{\frac{(gD)_{moon}}{(gD)_{earth}}}$$

$$\sqrt{\frac{1}{6} \times \frac{1}{4}} = \frac{1}{4.9}$$

S22. We have,

$$g = \frac{GM}{R^2}$$

$\therefore$ $$g_e = \frac{GM_e}{R^2} \quad \text{and} \quad g_p = \frac{GM_p}{R_p^2}$$

Given : $M_p = 2M_e$ and $R_p = 2R_e$

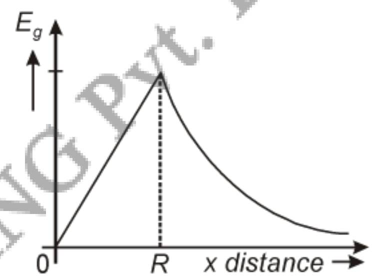
$\therefore$ $$\frac{g_p}{g_e} = \frac{1}{2}$$

The time period of a simple pendulum is given by

$$T_e = 2\pi\sqrt{\frac{l}{g_e}} \quad \text{and} \quad T_p = 2\pi\sqrt{\frac{l}{g_p}}$$

For $T_e = 1\text{s},\ T_p = \sqrt{2}\,s$

S23. For points inside the earth,



$$E_{gi} = \frac{G\frac{4}{3}\pi x^3\rho}{x^2} = \frac{4G}{3}\pi x\rho$$

For point outside the earth,

$$E_{go} = \frac{G\frac{4}{3}\pi R^3\rho}{x^2} = \frac{GM}{x^2}$$

S24. From law of gravitation , $F = \frac{GMm}{R^2}$ If m is a mass on the surface of earth of radius R. The acceleration of the mass m on the surface of earth is g.

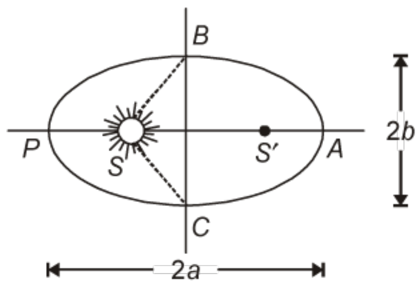
we have, $F = mg$

Using the two forces we have, $M = \frac{gR^2}{G}$.

S25. The magnitude of the angular momentum at P is $L_P = m_P r_P v_P$.

Now, r_P and v_P are mutually perpendicular.

Similarly $L_A = m_P r_A v_A$. From angular momentum conservation



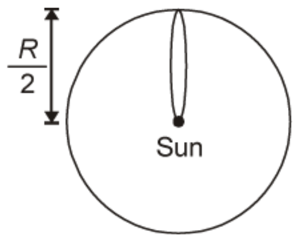
$$m_P r_P v_P = m_P r_A v_A \quad \text{or} \quad \frac{v_P}{v_A} = \frac{r_A}{r_P}$$

Since $r_A > r_P$, $\therefore$ $v_P > v_A$.

The area $SBAC$ bounded by the ellipse and the radius vectors SB and SC is larger than $SBPC$ in the figure. From Kepler's second law, equal areas are swept in equal times. Hence the planet will take a longer time to traverse BAC than CPB.

S26. When the Earth's motion is suddenly stopped, it would fall into the sun and (suppose) it comes back. If the effect of temperature of Sun is ignored, we can say that the Earth would continue to move along a strongly extended flat ellipse whose extreme points are located at the Earth's orbit and at the centre of the Sun.

The major axis is $\frac{R}{2}$.



Now $$\frac{T'^2}{T^2} = \left[\frac{R}{2}\right]^3 \left[\frac{1}{R^3}\right]$$

or $$T'^2 = \frac{T^2}{8} \quad \text{or} \quad T' = \frac{T}{2\sqrt{2}}$$

Now, time required to fall into the Sun,

$$t = \frac{T'}{2} = \frac{T}{4\sqrt{2}} = \frac{365}{4\sqrt{2}} \approx 65 \text{ days}$$

So, the Earth would take **slightly more than 2 months to fall into the Sun**.

S27. If T is the period of the planet and r, then semi-major axis of elliptical orbit, then

$$T^2 \propto r^3.$$

According to Kepler's third law,

$$\left(\frac{T_n}{T_e}\right)^2 = \left(\frac{R_n}{R_e}\right)^3$$

where subscripts n and e refer to Neptune and the Earth respectively.

$\therefore$ $$T_n^2 = T_e^2 \times \left(\frac{R_n}{R_e}\right)^3 \quad \text{or} \quad T_n^3 = 1 \times (30)^3$$

[$\because$ Time period of the Earth's revolution = 1 year and ratio of radii (hence diameters) of the Neptune and the Earth is 30.]

$\therefore$ $$T_n = 1 \times \sqrt{(30)^3} = 30\sqrt{30} = 164.3 \text{ years}$$

$\therefore$ The period of revolution of the Neptune around Sun = **164.3 years**.

S28. (a) The rate, at which the area is swept out by to radius from the Sun to Earth,

$$\frac{dA}{dt} = \frac{\pi R^2}{T}$$

Given; $R = 1.496 \times 10^{11}$ m; $T = 365$ days $= 365 \times 24 \times 60 \times 60$s

$$\frac{dA}{dt} = \frac{\pi \times (1.496 \times 10^{11})^2}{365 \times 24 \times 60 \times 60} = \mathbf{2.23 \times 10^{15}\ m^2\ s^{-1}}$$

(b) Here, $R = 3.845 \times 10^8$ m;

$$T = 27\frac{1}{3} \text{ days} = \frac{82}{3}\ 24 \times 60 \times 60\text{s}$$

Therefore, the rate, at which the area is swept out by rad from the earth to moon,

$$\frac{dA}{dt} = \frac{\pi \times (3.845 \times 10^8)^2}{\frac{82}{3} \times 24 \times 60 \times 60} = \mathbf{1.97 \times 10^{11}\ m^2\ s^{-1}}.$$

S29. It is given that $T_s = 29.5\ T_e$; $R_e = 1.5 \times 10^{11}$ m

Now, according to Kepler's third law

$$\left(\frac{T_S}{T_e}\right)^2 = \left(\frac{R_S}{R_e}\right)^3$$

$$R_S = R_e\left(\frac{T_S}{T_e}\right)^{2/3} = 1.5 \times 10^{11}\left(\frac{29.5\,T_e}{T_e}\right)^{2/3}$$

$$= 1.43 \times 10^{12}\ \text{m} = 1.43 \times 10^9\ \text{km}.$$

S30. If T is the period of the planet and r, then semi-major axis of elliptical orbit, then

$$T^2 \propto r^3.$$

Given: $k = 10^{-13}\ \text{s}^2\,\text{m}^{-3}$

$$= \left[\frac{10^{-13}}{(24 \times 60 \times 60)^2}\, d^2\right]\left[\frac{1}{(1/1000)^3\ \text{km}^3}\right]$$

$$= \mathbf{1.33 \times 10^{-14}\ d^2\ km^{-3}}.$$

Again, using the given equation and the given value of k, the time period of the moon is given by

$$T^2 = (1.33 \times 10^{-14})\,(3.84 \times 10^5)^3$$

$$T = \mathbf{27.3\ d}.$$

Note that the given equation also holds for elliptical orbits if we replace $(R_E + h)$ by the semi-major axis of the ellipse. The Earth will then be at one of the foci of this ellipse.

S31. The magnitude of the angular momentum at P is $L_p = m_p r_p v_p$, since inspection tells us that rp and vp are mutually perpendicular. Similarly, $L_A = m_p r_A v_A$. From angular momentum conservation

$$m_p r_p v_p = m_p r_A v_A$$

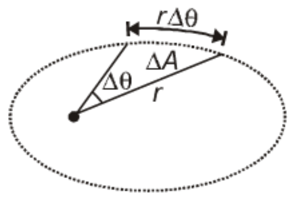
or
$$\frac{v_p}{v_A} = \frac{r_A}{r_p}$$

Since, $r_A > r_p$, $v_p > v_A$.

S32. ***Law of Orbit:*** Each planet revolves around the sun in an elliptical orbit, with the sun at one focus of the elliptical path.

Law of Area: The position vector of the planet from the sun *i.e.*, the line joining the planet to the sun sweeps out equal areas in equal time *i.e.*, the areal velocity of a planet around the sun always remains constant.

Consider a small area ΔA described in a small time interval Δt.



Now $$\Delta A = \frac{1}{2} r (r \Delta\theta)$$

$\therefore$ $$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t}$$

Proceeding to limits as $\Delta t \to 0$, we get

$$\underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta\theta}{\Delta t} \quad \text{or} \quad \frac{dA}{dt} = \frac{1}{2} r^2 \omega \qquad \text{... (i)}$$

Instantaneous angular momentum,

$$L = mr^2 \omega \qquad \text{... (ii)}$$

If follows from equations (i) and (ii) that

$$\frac{dA}{dt} = \frac{L}{2m}$$

The line of action of the gravitational force passes through the axis.

$\therefore$ $\tau = 0$ or L = constant. $\therefore$ $\frac{dA}{dt}$ is constant.

In words, the areal velocity of the planet is constant.

Note: The Kepler's law of areas is identical with the Law of conservation of Angular Momentum.

S33. The orbits of all planets except Mercury, Mars and Pluto are very close to being circular *i.e.*, their elliptical orbits differ little from circles. For example, the ratio of the semi-minor to semi-major axis for our Earth is 0.99986.

Consider the motion of a planet around the Sun. Let m and M represent the masses of the planet and Sun respectively. Let us assume that the planet moves around the Sun in a circular orbit. Let r be the radius of the orbit.

The gravitational force between the planet and the Sun provides the necessary centripetal force to the planet.

$\therefore$ $$\frac{GMm}{r^2} = mr\omega^2 \quad \text{or} \quad \frac{GM}{r^2} = r\left[\frac{2\pi}{T}\right]^2$$

or $$T^2 = \frac{4\pi^2 r^3}{GM} \quad \text{or} \quad T^2 = \left[\frac{4\pi^2}{GM}\right] r^3$$

or $$T^2 \propto r^3$$

A similar result is obtained for elliptical orbits with the radius replaced by the semi-major axis.

S34. (a) The Sun's mass replaced by the Martian mass M_m.

$$T^2 = \frac{4\pi^2}{GM_m} R^3$$

$$M_m = \frac{4\pi^2}{G} \times \frac{R^3}{T^2} = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (459 \times 60)^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times (4.59 \times 6)^2 \times 10^{-5}} = \mathbf{6.48 \times 10^{23}\ kg}$$

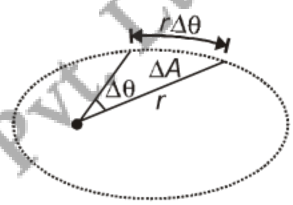
(b) Using Kepler's third law,

$$\left(\frac{T_M}{T_E}\right)^2 = \left(\frac{R_{MS}}{R_{ES}}\right)^3$$

where R_{MS} (R_{ES}) is the Mars (Earth)-Sun distance.

$$T_M = \left(\frac{R_{MS}}{R_{ES}}\right)^{3/2} T_E = (1.52)^{3/2} \times 365 = \mathbf{684\ days}.$$

S35. (a) Consider a small area ΔA described in a small time interval Δt.



Now $\quad \Delta A = \frac{1}{2} r (r \Delta\theta)$

$\therefore \quad \frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t}$

Proceeding to limits as $\Delta t \to 0$, we get

$$\underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta\theta}{\Delta t} \quad \text{or} \quad \frac{dA}{dt} = \frac{1}{2} r^2 \omega \qquad \text{... (i)}$$

Instantaneous angular momentum,

$$L = mr^2 \omega \qquad \text{... (ii)}$$

If follows from equations (i) and (ii) that

$$\frac{dA}{dt} = \frac{L}{2m}$$

The line of action of the gravitational force passes through the axis.

$\therefore \quad \tau = 0$ or L = constant. $\therefore \quad \frac{dA}{dt}$ is constant.

In words, the areal velocity of the planet is constant.

Note: The Kepler's law of areas is identical with the Law of conservation of Angular Momentum.

(b) The magnitude of angular momentum at P is $L_p = m_p r_p v_p$

Similarly magnitude of angular momentum at A is $L_A = m_A r_A v_A$

From conservation of angular momentum

$$m_p r_p v_p = m_A r_A v_A$$

$$\frac{v_P}{v_A} = \frac{r_A}{r_P}$$

$\because \quad r_A > r_P \qquad \therefore \quad v_P > v_A$

Area bound by *SB* & *SC* (*SBAC* > *SBPC*).

$\because$ By 2nd law equal areas are swept in equal intervals of time. Time taken to transverse *BAC* > time taken to traverse *CPB*.

S36. Here,

$$g_E = \frac{GM_E}{R_E^2} \quad \text{or} \quad M_E = \frac{g_E R_E^2}{G}$$

$$= \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}} = \mathbf{5.97 \times 10^{24}\ kg}$$

Again, the Moon is a satellite of the Earth. We know from the derivation of Kelper's third law that

$$T^2 = \frac{4\pi^2 R^3}{GM_E}$$

where *R* is the radius of the lunar orbit, *T* is the time period of the Moon around the Earth and M_E is mass of the Earth.

$\therefore$ Mass of the Earth $= M_E = \dfrac{4\pi^2 R^3}{GT^2}$

Here, $T = 27.3$ days $= 27.3 \times 24 \times 60 \times 60$ second

$R = 3.84 \times 10^8$ m, $\quad G = 6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2}$

Substituting the values, we get

$$M_E = \frac{4 \times (3.14)^2 \times (3.84 \times 10^8)^3}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}\ \text{kg}$$

$$= \mathbf{6.02 \times 10^{24}\ kg}$$

Both the methods yield almost the same answer, the difference between them being less than 1%.

S37. The orbits of all planets except Mercury, Mars and Pluto are very close to being circular *i.e.*, their elliptical orbits differ little from circles. For example, the ratio of the semi-minor to semi-major axis for our Earth is 0.99986.

Consider the motion of a planet around the Sun. Let *m* and *M* represent the masses of the planet and Sun respectively. Let us assume that the planet moves around the Sun in a circular orbit. Let *r* be the radius of the orbit.

The gravitational force between the planet and the Sun provides the necessary centripetal force to the planet.

$$\therefore \qquad \frac{GM_m}{r^2} = mr\omega^2 \quad \text{or} \quad \frac{GM}{r^2} = r\left[\frac{2\pi}{T}\right]^2$$

or $$T^2 = \frac{4\pi^2 r^3}{GM} \quad \text{or} \quad T^2 = \left[\frac{4\pi^2}{GM}\right] r^3$$

or $$T^2 \propto r^3$$

A similar result is obtained for elliptical orbits with the radius replaced by the semi-major axis.

By kepler's IIIrd law

$$\left(\frac{T_n}{T_s}\right)^2 = \left(\frac{R_n}{R_s}\right)^3$$

$$\frac{T_n}{T_s} = \left(\frac{R_n}{R_s}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$$

$$= 10\sqrt{10} = 10 \times 3.16 = 31.6$$

∴ $$T_n : T_s = 31.6 : 1.$$

Date: 13/10/2021

Q1. Why acceleration due to gravity is independent of mass of the body?

Q2. How acceleration due to gravity decreases with increasing depth (assume the earth to be a sphere of uniform density)?

Q3. How acceleration due to gravity decreases with increasing altitude?

Q4. Gravitational force between two bodies is 1 N. If the distance between them is made twice, what will be the force?

Q5. A black hole is a body from whose surface nothing may ever escape. What is the condition for a uniform spherical mass M to be a black hole? What should be the radius of such a black hole if its mass is the same as that of the Earth?

Q6. How the gravitational field at a point inside a solid sphere vary with distance x form its centre, if $x < R$ and $x > R$ (R-radius)?

Q7. What is the gravitational field at a point inside a spherical shell?

Q8. Give two characteristics of gravitational force.

Q9. According to Newton's law of gravitation, the apple and the earth experience equal and opposite forces due to gravitation. But it is the apple that falls towards the earth and not vice-versa. Why?

Q10A mass of 5 kg is first weighed on a balance at the top of a tower 20 m high. The mass is then suspended from a fine wire 20 m long and reweighed. Find the difference in weight. Assume that the radius of the Earth is 6400 km, the mass of the Earth is 6×10^{24} kg and $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Q11. Define gravitational field strength. What is the field at a point distance r from a mass M?

Q12. Why the formula $-G\,Mm(1/r_2 - 1/r_1)$ is more accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth?

Q13. Among the known types of forces in nature, the gravitational force is the weakest. Why then does it play a dominant role for motion of bodies on the terrestrial, astronomical and cosmological scale?

Q14. Find the percentage decrease in the weight of the body when taken to a height of 16 km. above the surface of the earth. Radius of the earth is 6400 km.

Q15. The radius of moon is 1.7×10^6 m and its mass is 7.35×10^{22} kg. What is the acceleration due to gravity on the surface of moon? Given, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Q16. Three masses, each equal to M, are placed at the three corners of a square of side 'a'. Calculate the force of attraction on unit mass at the fourth corner.

Q17. Calculate the force of attraction between two balls each of mass 1 kg, when their centre are 10 cm apart. Given that universal gravitational constant, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Q18. Find the gravitational attraction between the two atoms in a hydrogen molecule. Given that $G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$, mass of a hydrogen atom $= 1.67 \times 10^{-27}$ and distance between the two atoms = 1 A.

Q19. Two stationary particles of masses M_1 and M_2 are a distance d apart. A third particle, lying on the line joining the particles, experiences no resultant gravitational force. What is the distance of this particle from M_1?

Q20. The mass of planet Jupiter is 1.9×10^{27} kg and that of the sun is 7.8×10^{11} m. Calculate the gravitational force which the sun exerts on Jupiter. Assuming that Jupiter moves in a circular orbit around the sun, calculate the speed of the Jupiter.

Q21. At some point between the earth and the moon, the gravitational force on a space ship due to the earth and the moon together is zero. Where is this point? Given that the earth-moon distance is 3.845×10^8 m and the moon has 1.2% of the mass of the earth.

Q22. The acceleration due to gravity on the planet A is 8 times the acceleration due to gravity on plane B. A man jumps to a height of 1.5 m on the surface of A. What is the height of jump by the same person on the planet B?

Q23. Show that the gravitational potential at a point of distance, from the mass M is given by,

$V = -\left(\frac{GM}{r}\right)$.

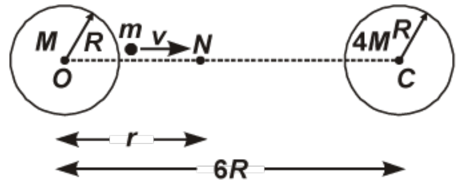
Q24. Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5 ly.

Q25. I_0, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth of the sun.

Q26. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

Q27. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun $= 2 \times 10^{30}$ kg, mass of the earth $= 6 \times 10^{24}$ kg. Neglect the effect of other planets etc. (orbital radius $= 1.5 \times 10^{11}$m).

Q28. Two uniform solid spheres of equal radii R, but mass M and 4 M have a center to centre separation 6 R, as shown in figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.

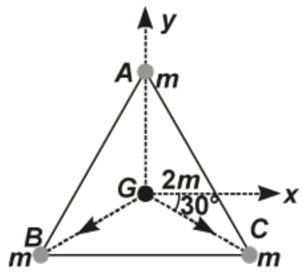


Q29. Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC.

(a) What is the force acting on a mass 2m placed at the centroid G of the triangle?

(b) What is the force if the mass at the vertex A is doubled?

Take $AG = BG = CG = 1$m (see figure)



Q30. Three equal masses of *m* kg each are fixed at the vertices of an equilateral triangle *ABC*.

(a) What is the force acting on a mass 2*m* placed at the centroid *G* of the triangle?

(b) What is the force if the mass at the vertex *A* is doubled? Take *AG* = *BG* = *CG* = 1 m (as shown in the figure)

Date: 13/10/2021

S1. Acceleration due to gravity of body of mass *m* is given by the relation:

$$g = \frac{GM}{R^2}$$

Where, G = Universal gravitational constant

M = Mass of the Earth

R = Radius of the Earth

Hence, it can be inferred that acceleration due to gravity is independent of the mass of the body.

S2. Acceleration due to gravity at depth *d* is given by the relation:

$$g_d = \left(1 - \frac{d}{R_e}\right)g$$

It is clear from the given relation that acceleration due to gravity decreases with an increase in depth.

S3. Acceleration due to gravity at depth *h* is given by the relation:

$$g_h = \left(1 - \frac{2h}{R_e}\right)g$$

Where, R_e = Radius of the Earth

g = Acceleration due to gravity on the surface of the Earth

It is clear from the given relation that acceleration due to gravity decreases with an increase in height.

S4. The gravitational force between the two bodies,

$$F \propto 1/r^2.$$

Therefore, when the distance between the two bodies is made twice, the force between the two bodies will become,

$$F' = \frac{F}{2^2} = \frac{1}{2^2} = \mathbf{0.25\ N}.$$

S5. For a body to be a black hole, the escape velocity should be such that even light cannot escape,

The limiting case for escape velocity is, $\sqrt{\frac{2GM}{R}} \le c$ (speed of light)

For our earth, $M = M_e = 6 \times 10^{24}$ kg,

$$R = \frac{2GM}{c^2}$$

$$= 9 \times 10^{-2} \text{ m or 9 cm.}$$

S6.

$E_g \propto x$ if $x < R$ and $E_g \propto \frac{1}{x^2}$ if $x > R$

S7. Since no mass is enclosed, gravitational field is zero.

S8. (a) Conservative force (b) Acts on the line joining the masses.

S9. Let M and m be the masses of the earth and the apple respectively. If r is the distance between them, then force of attraction between the earth and the apple,

$$F = G\frac{Mm}{r^2}$$

When this force acts on the apple, the acceleration produced in the motion of the apple,

$$a_{\text{apple}} = \frac{F}{m} = \frac{GM}{r^2}$$

On The other hand, when this force acts on the earth, the acceleration produced in the motion of earth,

$$a_{\text{earth}} = \frac{F}{M} = \frac{Gm}{r^2}$$

Since, $m << M$, $a_{\text{apple}} >> a_{\text{earth}}$.

For this reason, it is apple, which falls towards the earth.

S10.

$$\text{Original force} = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 5}{(6400.02 \times 10^3)^2}\text{ N}$$

$$= 48.8522 \text{ N}$$

$$\text{Force at surface} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 5}{(6400 \times 10^3)^2}\text{ N}$$

$$= 48.8525 \text{ N}$$

Therefore change in weight = **0.0003 N**.

If $g = 10 \text{ N kg}^{-1}$, this is equivalent to the weight of a 0.03 g mass at the Earth's surface.

S11. Gravitational field strength is defined as the force experienced by unit mass kept at a point.

At a distance r, the force is $F = \frac{GMm}{r^2}$

The gravitational field = $\frac{F}{m} = \frac{GMm}{r^2 m} = \frac{GM}{r^2}$

S12. Gravitational potential energy of two points r_2 and r_1 distance away from the centre of the Earth is respectively given by:

$$V(r_1) = -\frac{GmM}{r_1}$$

$$V(r_2) = -\frac{GmM}{r_2}$$

$\therefore$ Difference in potential energy,

$$V = V(r_2) - V(r_1) = -GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Hence, this formula is more accurate than the formula $mg(r_2 - r_1)$.

S13. The nuclear forces are short range force. Such forces are effective only over a small distance of the order to 10^{-15} m or 10^{-14} m. On the other hand, electrical forces are long range forces, but such forces can be both attractive as well as repulsive. Therefore, nuclear forces and electrical forces do not play any role for the motion of ***massive neutral bodies*** on the terrestrial, astronomical and cosmological scale. It is due to this reason that the gravitational force (which is always attractive, through weakest) plays a dominant role for the motion of such bodies.

S14. The acceleration due to gravity at a height '*h*' above the surface of the earth is

$$g' = g\left(1 - \frac{2h}{R}\right)$$

$$g - g' = \left(\frac{2hg}{R}\right)$$

$$\frac{mg - mg'}{mg} \times 100 = \frac{g - g'}{g} \times 100$$

$$= \frac{2h}{R} \times 100$$

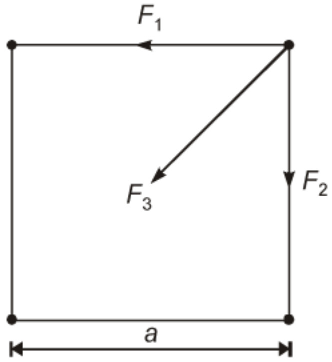
$$= \frac{2 \times 16}{6400} \times 100 = 0.5\%.$$

S15. Given, $R = 1.7 \times 10^6$ m; $M = 7.35 \times 10^{22}$ kg; $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

Now, $$g = \frac{GM}{R^2}$$

$$g = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{-22}}{(1.7 \times 10^6)^2} = \mathbf{1.696\ ms^{-2}}.$$

S16.



$$F_1 = F_2 = \frac{GM}{a^2}$$

Resultant of F_1 and F_2 is $\sqrt{2}\,\frac{GM}{a^2}$ pointing towards the centre.

$$F_3 = \frac{GM}{(\sqrt{2}a)^2} = \frac{GM}{2a^2}$$

Both $\frac{\sqrt{2}GM}{a^2}$ and $\frac{GM}{2a^2}$ act in the same direction (along the diagonal)

Their resultant is

$$\frac{\sqrt{2}GM}{a^2} + \frac{GM}{2a^2} \quad \text{or} \quad \frac{GM}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

S17. Given, $M_1 = M_2 = 1$ kg; $r = 10$ cm $= 0.1$ m;

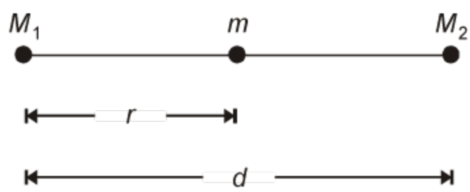
$$G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$$

Now,
$$F = G\,\frac{M_1 M_2}{r^2} = 6.67 \times 10^{-11} \times \frac{1 \times 1}{(0.1)^2}$$
$$= \mathbf{6.67 \times 10^{-9}\ N}.$$

S18. Given,
$$G = 6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2};$$
$$M_1 = M_2 = 1.67 \times 10^{-27}\ \text{kg}; \quad r = 1\text{Å} = 10^{-10}\ \text{m}$$

Now,
$$F = G\,\frac{M_1 M_2}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 1.67 \times 10^{-27}}{(10^{-10})^2}$$
$$= \mathbf{1.86 \times 10^{-44}\ N}.$$

S19.



The force on m towards M_1 is $F = G\,\frac{M_1 m}{r^2}$

The force on ma towards M_2 is $F = G\,\frac{M_2 m}{(d-r)^2}$

Equating the two forces, we have

$$G\,\frac{M_1 m}{r^2} = G\,\frac{M_2 m}{(d-r)^2}$$

$$\Rightarrow \quad \left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1} \qquad \Rightarrow \quad \frac{d}{r} - 1 = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

$\Rightarrow \qquad \dfrac{d}{r} = \dfrac{\sqrt{M_2} + \sqrt{M_1}}{\sqrt{M_1}} \quad \Rightarrow \quad r = \boldsymbol{d}\left(\dfrac{\sqrt{\boldsymbol{M_1}}}{\sqrt{\boldsymbol{M_1}} + \sqrt{\boldsymbol{M_2}}}\right).$

S20. Given, Mass of the sun, $M = 1.99 \times 10^{30}$ kg;

Mass of the Jupiter, $m = 1.9 \times 10^{27}$ kg;

Distance of the Jupiter from the sun, $r = 7.8 \times 10^{11}$ m

Also $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Force exerted by the sun on the Jupiter,

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.9 \times 10^{27}}{(7.8 \times 10^{11})^2}$$

$$= 4.145 \times 10^{23} \text{ N.}$$

The gravitational force of attraction due to the sun provides the necessary centripetal force to the Jupiter to move in the circular orbit. If v is the **tangential** speed of Jupiter, then

$$\frac{mv^2}{r} = F$$

or $$v = \sqrt{\frac{rF}{m}} = \sqrt{\frac{7.8 \times 10^{11} \times 4.145 \times 10^{23}}{1.9 \times 10^{27}}}$$

$$= \mathbf{1.304 \times 10^4 \ m\ s^{-1}}.$$

S21. Let M_1 and M_2 be the masses of the earth and moon respectively. Then,

$$\frac{M_2}{M_1} = 1.2\% = \frac{1.2}{100} = 0.012;$$

The distance of moon from the earth = 3.845×10^8 m

Suppose that force on the spaceship due to the earth and the moon becomes zero at a point, whose distance from the earth is r_1 and from moon is r_2. Then,

$$r_1 + r_2 = \text{earth-moon distance}$$

or $$r_1 + r_2 = 3.845 \times 10^8 \text{ m} \qquad \ldots\text{(i)}$$

At such a point, the force on the space ship due to the earth and moon must be equal in magnitude. If m is mass of the space ship, then

$$\frac{GM_1 m}{r_1^2} = \frac{GM_2 m}{r_2^2}$$

or $$\left(\frac{r_2}{r_1}\right)^2 = \frac{M_2}{M_1}$$

or $$\left(\frac{r_2}{r_1}\right)^2 = 0.012$$

or $$r_2 = \sqrt{0.012} = 0.11\, r_1$$

From the equation (i), we have

$$r_1 + 0.11\, r_1 = 3.845 \times 10^8 \text{ m}$$

or $$1.11\, r_1 = 3.845 \times 10^8 \text{ m}$$

or $$r_1 = \mathbf{3.464 \times 10^8 \text{ m (from the earth)}}.$$

S22. Let u be the speed of the man, while taking the jump and h, the height of the jump.

From the relation: $v^2 - u^2 = 2aS$, we have

$$0^2 - u^2 = 2 \times (-g) \times h$$

or $$h = \frac{u^2}{2g}$$

If h_A and h_B are the height of jump on the planets A and B respectively, then

$$\frac{h_B}{h_A} = \frac{u^2}{2g_B} \times \frac{2g_A}{u^2} = \frac{g_A}{g_B} = \frac{8g_B}{g_B} = 8$$

or $$h_A = 8 \times 1.5 = 12 \text{ m}$$

S23. The gravitational force of attraction between M and m when x is the distance between their centres is given by

$$F = \frac{GMm}{x^2}$$

Suppose the body be moved through a distance dx, therefore, work done is given by.

$$dW = F\,dx = \frac{GMm}{x^2}\, dx$$

When the body is brought from infinity to some distance r,

we write, $$\int dW = \int_{x=\infty}^{x=r} \frac{GMm}{x^2}\, dx$$

or $$W = GMm\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$= -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$= \frac{-GMm}{r}$$

This amount of work done is the change in the potential energy of the body.

$\therefore$ $$\text{P.E. } (U) = \frac{-GMm}{r}$$

Gravitational potential

$$V = \frac{U}{m} = \frac{-GM}{r}.$$

S24. Mass of our galaxy Milky Way, $M = 2.5 \times 10^{11}$ solar mass

Solar mass = Mass of Sun = 2.0×10^{36} kg

Mass of our galaxy, $M = 2.5 \times 10^{11} \times 2 \times 10^{36} = 5 \times 10^{47}$ kg

Diameter of Milky Way, $d = 10^5$ ly

Radius of Milky Way, $r = 5 \times 10^4$ *ly*

$$1\ ly = 9.46 \times 10^{15} \text{ m}$$

$\therefore$ $$r = 5 \times 10^4 \times 9.46 \times 10^{15}$$

$$= 4.73 \times 10^{20} \text{ m}$$

Since a star revolves around the galactic centre of the Milky Way, its time period is given by the relation:

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

$$= \left(\frac{4 \times (3.14)^2 \times (4.73)^3 \times 10^{60}}{6.67 \times 10^{11} \times 5 \times 10^{41}}\right)^{\frac{1}{2}} = \left(\frac{39.48 \times 105.82 \times 10^{30}}{33.35}\right)^{\frac{1}{2}}$$

$$= (125.27 \times 10^{30})^{\frac{1}{2}} = 1.12 \times 10^{16} \text{ s}$$

1 year = $365 \times 24 \times 60 \times 60$ s

$\therefore$ $$1.12 \times 10^{16} \text{ s} = \frac{1.12 \times 10^{16}}{365 \times 24 \times 60 \times 60}$$

$$= 3.55 \times 10^8 \text{ years}$$

S25. Orbital period of I_0, T_{I0} = 1.769 days = $1.769 \times 24 \times 60 \times 60$ s

Orbital radius of I_0, $R_{I0} = 4.22 \times 10^8$ m

Satellite I_0 is revolving around the Jupiter

Mass of the latter is given by the relation:

$$M_J = \frac{4\pi^2 R_{I_0}^2}{G T_{I_0}^2}$$

Where, M_J = Mass of Jupiter

G = Universal gravitational constant

Orbital radius of the Earth,

$$T_e = 365.25 \text{ Days} = 365.25 \times 24 \times 60 \times 60$$

Orbital radius of the Earth,

$$R_e = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

Hence, it can be inferred that the mass of Jupiter is about one.

S26. Time taken by the Earth to complete one revolution around the Sun, T_e = 1 year Orbital radius of the Earth in its orbit,Time taken by the planet to complete one revolution around the Sun,

$T_P = \frac{1}{2} T_e = \frac{1}{2}$ year .

Orbital radius of the planet = R_p

From Kepler's third law of planetary motion, we can write:

$$\left(\frac{R_P}{R_e}\right)^3 = \left(\frac{T_P}{T_e}\right)^2$$

$$\left(\frac{R_P}{R_e}\right) = \left(\frac{T_P}{T_e}\right)^{\frac{2}{3}}$$

$$\left(\frac{\frac{1}{2}}{1}\right)^{\frac{2}{3}} = (0.5)^{\frac{2}{3}} = 0.63$$

Hence, the orbital radius of the plane will be 0.63 times smaller than the Earth.

S27. Mass of the rocket = m

Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton's law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = Gm\frac{M_e}{x^2}$$

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\frac{r-x}{x} = \left(\frac{M_s}{M_e}\right)^{\frac{1}{2}}$$

$$\frac{r\;-\;x}{x} = \left(\frac{2\times10^{30}}{6\;\times10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$1.5\times10^{11} - x = 577.35x$$

$$578\;.35\;x = 1.5 - 10^{11}$$

$$x = \frac{1.5\times10^{11}}{578.35} = 2.59\times10^{8}\text{ m}.$$

S28. The projectile is acted upon by two mutually opposing gravitational forces of the two spheres. The neutral point N (see figure) is defined as the position where the two forces cancel each other exactly. If $ON = r$, we have

$$\frac{GMm}{r^2} = \frac{4GMm}{(6R-r)^2}$$

$$(6R - r)^2 = 4r^2$$

$$6R - r = \pm2r$$

$$r = 2R \quad \text{or} \quad -6R.$$

The neutral point $r = -6R$ does not concern us in this example. Thus $ON = r = 2R$. It is sufficient to project the particle with a speed which would enable it to reach N. Thereafter, the greater gravitational pull of $4M$ would suffice. The mechanical energy (sum of potential and kinetic energy) at the surface of M is

$$E_i = \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R}$$

At the neutral point N, the speed approaches zero. The mechanical energy at N is purely potential.

$$E_N = -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

From the principle of conservation of mechanical energy

$$\frac{1}{2}v^2 - \frac{GM}{R} - \frac{4GM}{5R} = -\frac{GM}{2R} - \frac{GM}{R}$$

or $$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right)$$

$$v = \left(\frac{3GM}{5R}\right)^{1/2}.$$

S29. (a) The angle between GC and the positive x-axis is 30° and so is the angle between GB and the negative x-axis. The individual forces in vector notation are

$$\boldsymbol{F}_{GA} = \frac{Gm(2m)}{1}\,\hat{\boldsymbol{j}}$$

$$\boldsymbol{F}_{GB} = \frac{Gm(2m)}{1}\,(-\,\hat{\boldsymbol{i}}\,\cos 30° - \hat{\boldsymbol{j}}\,\sin 30°)$$

$$\boldsymbol{F}_{GC} = \frac{Gm(2m)}{1}\,(\hat{\boldsymbol{i}}\,\cos 30° - \hat{\boldsymbol{j}}\,\sin 30°)$$

From the principle of superposition and the law of vector addition, the resultant gravitational force $\boldsymbol{F}_R$ on $(2m)$ is

$$\boldsymbol{F}_R = \boldsymbol{F}_{GA} + \boldsymbol{F}_{GB} + \boldsymbol{F}_{GC}$$

$$\boldsymbol{F}_R = 2Gm^2\,\hat{\boldsymbol{j}} + 2Gm^2\,\hat{\boldsymbol{j}}\,(-\,\hat{\boldsymbol{i}}\,\cos 30° - \hat{\boldsymbol{j}}\,\sin 30°) + 2Gm^2\,(\hat{\boldsymbol{i}}\,\cos 30° - \hat{\boldsymbol{j}}\,\sin 30°) = 0$$

Alternatively, one expects on the basis of symmetry that the resultant force ought to be zero.

(b) By symmetry the x-component of the force cancels out. The y-component survives.

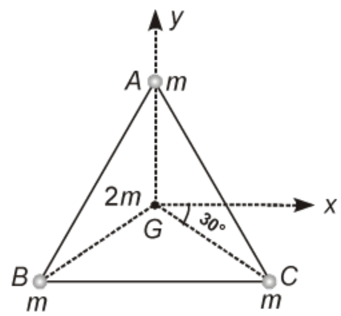
$$F_R = 4Gm^2\,\hat{\boldsymbol{j}} - 2Gm^2\,\hat{\boldsymbol{j}} = 2Gm^2\,\hat{\boldsymbol{j}}\,.$$

S30. (a) We choose the axes as shown in the figure. The angle between GC and the positive x-axis is 30° and so is the angle between GB and the negative x-axis. The individual forces in vector notation are:

$$\vec{F}_{GA} = \frac{Gm(2m)}{1^2}\,\hat{j}$$

$$\vec{F}_{GB} = \frac{Gm(2m)}{1^2}\,(-\,\hat{i}\cos 30° - \hat{j}\cos 60°)$$

$$\vec{F}_{GC} = \frac{Gm(2m)}{1^2}\,(+\,\hat{i}\cos 30° - \hat{j}\cos 60°)$$



From the principle of superposition and the law of vector addition, the resultant gravitational force $\vec{F}_R$ on $(2m)$ is

$$\vec{F}_R = \vec{F}_{GA} + \vec{F}_{GB} + \vec{F}_{GC}$$

$$\vec{F}_R = 2Gm^2\hat{j} + 2Gm^2(-\hat{i}\cos 30^\circ - \hat{j}\cos 60^\circ) + 2Gm^2(\hat{i}\cos 30^\circ - \hat{j}\cos 60^\circ) = 0 .$$

Alternatively, one expects on the basis of symmetry that the resultant force ought to be zero.

(b) By symmetry, the *x*-component of the force cancels out. They *y*-component survives.

$$\vec{F}_R = 4Gm^2\hat{j} - 2Gm^2\hat{j} = \mathbf{2Gm^2\hat{j}} .$$

Date: 13/10/2021

Q1. Acceleration due to gravity increases/decreases with increasing altitude.

Q2. Draw graph showing the variation of acceleration due to gravity with (a) height above the earth's surface (b) depth below the earth's surface.

Q3. Does the force of attraction between two bodies depend upon the presence of other bodies and properties of intervening medium?

Q4. A graph was plotted taking gravitational force (F) along y-axis and the square of the distance (d^2) along x-axis, what is the nature of graph?

Q5. If the earth stops rotating about its own axis, what is the effect on the value of weight of any object on the surface of earth?

Q6. Why weight of a body becomes zero at the centre of Earth?

Q7. What is the effect of acceleration due to gravity?

Q8. Why does the earth have an atmosphere and the moon does not?

Q9. Acceleration due to gravity is independent of mass of the Earth/mass of the body.

Q10. Acceleration due to gravity increases/decreases with increasing depth. (assume the Earth to be a sphere of uniform density).

Q11. The formula $-G\,Mm\,(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg\,(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the Earth.

Q12. Establish the relation: $g = \frac{4}{3}\pi G R \rho$,

where ρ is mean density of the earth, R is the radius of the earth, g is acceleration due to gravity at the surface of the earth and G is gravitational constant.

Q13. Two sphere of masses 64 kg and 298 kg are placed a distance 10 m apart. Find the position of a point on the line joining the centres of the two spheres, where the gravitational field is zero.

Q14. A Planet reduces its radius by 1% with its mass remaining same. How acceleration due to gravity varies?

Q15. While approaching a planet circling a distance star, a space traveler determines the planet's radius to be half that of the earth. After landing on the surface, he finds the acceleration due to gravity to be twice that on the surface of the earth. Find the ratio of the mass of the planet to that of the earth.

Q16. The value of acceleration due to gravity at the surface of the earth is 9.8 ms^{-2} and its mean radius is about 6.4×10^6 m. Assuming that we could get more soil from somewhere, estimate how thick would an added uniform outer layer on the earth have the value of acceleration due to gravity 10 ms^{-2} exactly?

Q17. What is the effect of rotation on the value of 'g'? Derive the relation.

Q18. The value of g on the surface of earth is 9.8 ms^{-2}. Find its value on the surface of Moon. Given the mass of Earth = 6×10^{24} kg; radius of Earth = 6.4×10^{6} m, mass of Moon = 7.4×10^{22} kg and radius of Moon = 1.74×10^{6} m.

Q19. How far away from the surface of the Earth does the acceleration due to gravity reduces by 64% of its value on the surface, Given, radius of the Earth, $R = 6.4 \times 10^{6}$ m.

Q20. A 75 Kg person have weight 735 N on the surface of the earth. How far above the surface of the Earth would he have to go as to lose 10% of his body weight?

Q21. On a planet, whose size is the same and mass three times as that of the Earth, find the amount of work done to lift 5 kg mass vertically upwards through 10 m on the planet. The value of g on the surface of earth is 9.8 ms^{-2}.

Q22. What is the effect of depth on acceleration due to gravity? Explain.

Q23. (a) At what depth below the surface of Earth, value of g is same as that at a height 64 km above the surface of the Earth?

(b) Calculate the value of g (for Earth) at a point $R/2$ (i) below (ii) above the surface of earth where R is the radius of Earth. Given g = 9.8 ms^{-2}.

Q24. (a) Explain the acceleration due to gravity.

(b) What is the effect of height (altitude) on acceleration due to gravity? Explain.

Q25. (a) At what height will a man's weight become half his weight on the surface of Earth? Given R = radius of Earth.

(b) How far away from the surface of the Earth does the acceleration due to gravity reduces by 36% of its value on the surface, Given, radius of the Earth, $R = 6.4 \times 10^{6}$ m.

Q26. (a) At what height from the surface of Earth will the value of 'g' be reduced by 40% from the value at the surface? Radius of Earth = 6400 km.

(b) Object at rest on the earth's surface move in circular paths with a period of 24 hours. Are they in orbit in the same sense that an earth's satellite is in orbit? Why not? What would the length of the day have to be put such objects in a true orbit?

Date: 13/10/2021

S1. Decreases.

Explanation: Acceleration due to gravity at depth *h* is given by the relation:

$$g_h = \left(1 - \frac{2h}{R_e}\right) g$$

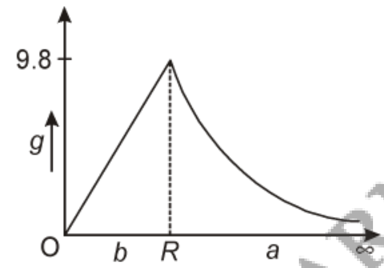
Where,

R_e = Radius of the Earth

g = Acceleration due to gravity on the surface of the Earth

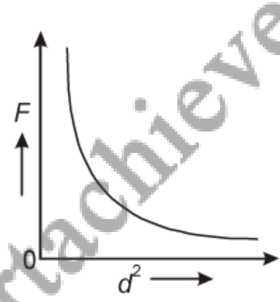
It is clear from the given relation that acceleration due to gravity decreases with an increase in height.

S2.



S3. No, since it acts along the line joining them and is a central force.

S4. The graph is a rectangular hyperbola.



S5. If the earth stops rotating, the weight of the object increases due to absence of centrifugal force. However, the weight at the poles remains the same.

S6. The weight of a body is the force with which the body is attracted towards the centre of Earth. Obviously, when the body reaches the centre of the Earth, it will no longer be attracted and hence its weight becomes zero at the centre of the Earth.

S7. The acceleration due to gravity at a place changes due to the following effects.

(a) The effect of height (or altitude).

(b) The effect of depth.

(c) The effect of the shape of the Earth.

(d) The effect of rotational motion of the Earth.

S8. The escape velocity from the earth surface is about 11.2 km/sec which is greater than the r.m.s velocity of the air molecules. the escape velocity from the moon's surface is about 2.4 km/sec which is less than the r.m.s. velocity of the air molecules. Hence, the earth has an atmosphere and the moon does not.

S9. Mass of the body.

Explanation: Acceleration due to gravity of body of mass *m* is given by the relation:

$$g = \frac{GM}{R^2}$$

Where,

G = Universal gravitational constant

M = Mass of the Earth

R = Radius of the Earth

Hence, it can be inferred that acceleration due to gravity is independent of the mass of the body.

S10. Decreases.

Explanation: Acceleration due to gravity at depth *d* is given by the relation:

$$g_d = \left(1 - \frac{d}{R_e}\right) g$$

It is clear from the given relation that acceleration due to gravity decreases with an increase in depth.

S11. More.

Explanation: Gravitational potential energy of two points r_2 and r_1 distance away from the centre of the Earth is respectively given by:

$$V(r_1) = \frac{GmM}{r_1}$$

$$V(r_2) = \frac{GmM}{r_2}$$

$\therefore$ Difference in potential energy,

$$V = V(r_2) - V(r_1) = -GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Hence, this formula is more accurate than the formula $mg\,(r_2 - r_1)$.

S12. We know,

Mass of earth, $M = \frac{gR^2}{G}$ ... (i)

If the earth is considered as a sphere of radius R and of material of density ρ, then

$$M = \frac{4}{3}\pi R^3 \rho \qquad \ldots \text{(ii)}$$

From the Eq. (i) and (ii), we have

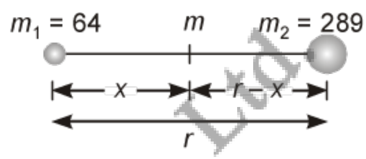
$$\frac{4}{3}\pi R^3 \rho = \frac{gR^2}{G}$$

$$g = \frac{4}{3}\pi G R \rho .$$

S13. Let r be the distance between the two spheres.

Here, $r = 10$ m

Suppose that gravitational field is zero at a distance x from the sphere of mass 64 kg. Then,



$$\frac{G \times 64}{x^2} = \frac{G \times 289}{(r-x)^2}$$

or $8(r - x) = 17x$

or $x = \frac{8}{25} r = \frac{8 \times 10}{25}$

= **3.2 m (from the 64 kg sphere)**.

S14. When mass is same, $g \propto \frac{1}{R^2}$.

$\therefore$ $$\frac{\Delta g}{g} = 2\frac{\Delta R}{R}$$

% variation of g is 2%.

S15. In case of the earth, $\frac{GM_e m}{r_e^2} = mg_e$ … (i)

In case of the planet, $\frac{GM_p m}{r_p^2} = mg_p$ … (ii)

Dividing by (ii) ÷ (i), we get

$$\left(\frac{M_p}{M_e}\right)\left(\frac{r_e^2}{r_p^2}\right) = \frac{g_p}{g_e},$$

but $g_p = 2g_e$ and $r_p = \frac{r_e}{2}$

$\therefore$ $$\frac{M_p}{M_e} = \frac{2}{4} = \frac{1}{2}$$

Thus the ratio of the mass of the planet to the mass of the earth is 1/2.

S16. Given, $R = 6.4 \times 10^6$ m; $g = 9.8$ ms^{-2}; $g' = 10$ ms^{-2} and density of the earth's soil, $\rho = 5{,}520$ kg m^{-3}.

In terms of density of the earth, acceleration due to gravity at the surface of the earth is given by

$$g = \frac{4}{3}\pi G R \rho \qquad \ldots(i)$$

Suppose that the radius of the earth has to be increased to R' by putting up more soil around it, so that the value of acceleration due to gravity will becoming $g' = 10$ ms^{-2} exactly.

Then, $$g' = \frac{4}{3}\pi G R' \rho \qquad \ldots(ii)$$

From the equations (i) and (ii), we get

$$\frac{g'}{g} = \frac{R'}{R}$$

or $$R' = \left(\frac{g'}{g}\right) R = \left(\frac{10}{9.8}\right) \times 6.4 \times 10^6 = 6.53 \times 10^6 \text{ m}$$

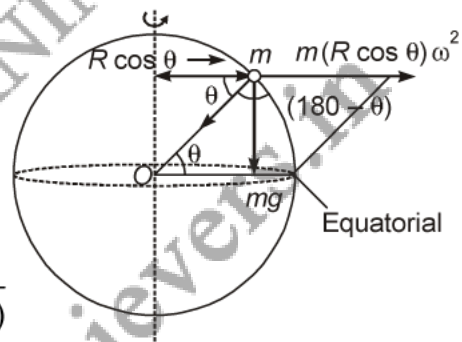
Therefore, thickness of the required layer of the soil,

$$t = R' - R = 6.53 \times 10^6 - 6.4 \times 10^6$$
$$= 0.13 \times 10^6 \text{ m} = \mathbf{130\ km}.$$

S17. Consider a mass m placed at a latitude θ. Two force are experience by it namely.

(a) The gravitational force 'mg' towards centre O.

(b) The centrifugal force trying to lift the mass away.



The net force is given by.

$$Mg_0 = \sqrt{(mg)^2 + (mR\cos\theta\omega^2)^2 + 2(mg)(mR\cos\theta\omega^2)\cos(180^\circ - \theta)}$$

The radius is $R\cos\theta$, since the centrifugal force is due to the rotation about the axis of the Earth.

$$Mg_0 = mg\left(1 + \left(\frac{2R\omega^2\cos^2\theta}{g}\right)^2 - 2\frac{R\omega^2}{g}\cos^2\theta\right)^{\frac{1}{2}}$$

$\therefore$ $$g_0 = g\left(1 - \frac{2R\omega^2}{g}\cos^2\theta\right)$$

Since $\left(\frac{R\omega^2}{g}\right)^2$ is negligibly small

$$g_0 = g\left(1 - \frac{R\omega^2}{g}\cos^2\theta\right).$$

S18. On the surface of Earth:

Here, $M = 6 \times 10^{24}$ kg; $R = 6.4 \times 10^{6}$ m; $g = 9.8$ ms^{-2}

Now, $$g = \frac{GM}{R^2}$$

or $$G = \frac{gR^2}{M}$$

$\therefore$ $$G = \frac{9.8 \times (6.4 \times 10^6)^2}{6 \times 10^{24}}$$

$$= \mathbf{6.69 \times 10^{-11}\ Nm^2\,kg^{-2}}$$

On the surface of Moon: Let g be acceleration due to gravity on the surface of moon.

Now, $$g' = \frac{GM'}{R'^2}$$

Here, $M' = 7.4 \times 10^{22}$ kg; $R = 1.74 \times 10^{6}$ m

$\therefore$ $$g = \frac{6.69 \times 10^{-11} \times 7.4 \times 10^{22}}{(1.74 \times 10^6)^2} = \mathbf{1.635\ ms^{-2}}.$$

S19. Let h be the height at which the value of g will get reduced by 64% *i.e.*, of the value at the surface of the Earth.

Then, $$g' = \frac{36}{100} g \quad \text{... (i)}$$

Such a large decrease in the value of g will occur at a very large height. There, g' is given by

$$g' = g\frac{R^2}{(R+h)^2} \quad \text{... (ii)}$$

From the Eq. (i) and (ii), we get

$$g\frac{R^2}{(R+h)^2} = \frac{36}{100} g$$

or $$\frac{R}{R+h} = \frac{6}{10}$$

or $$h = \frac{2R}{3} = \frac{2 \times 6{,}400}{3} = \mathbf{4266.67\ km}.$$

S20. The value of acceleration due to gravity on the surface of the Earth,

$$g = \frac{\text{Weight of the person}}{\text{Mass of the person}} = \frac{735}{75} = 9.8 \text{ ms}^{-2}$$

So that the he loses 10% of his weight, the acceleration due to gravity should decrease to

$$g' = g \times \frac{90}{100} = 9.8 \times \frac{90}{100} = 8.82 \text{ ms}^{-2}$$

If h is the required height, then

$$g' = g\,\frac{R^2}{(R+h)^2}$$

or $$R + h = R \times \sqrt{\frac{g'}{g}} = R \times \sqrt{\frac{9.8}{8.82}} = 1.054\,R$$

or $$h = 1.054\,R - R = 0.054\,R = 0.054 \times 6{,}400$$

$$= \mathbf{345.6\ km}.$$

S21. Let M and R be the mass and radius of the Earth. The

$$g = \frac{GM}{R^2} \quad \text{... (i)}$$

Now, mass of the planet, $M' = 3$ M

And radius of the planet, $R' = R$

Let g be acceleration due to gravity on the surface of planet. Then,

$$g = \frac{GM'}{R'^2} \quad \text{...(ii)}$$

Dividing the equations (ii) and (i), we have

$$\frac{g'}{g} = \frac{M'}{M} \times \frac{R^2}{R'^2} = \frac{3M}{M} \times \frac{R^2}{R^2} = 3$$

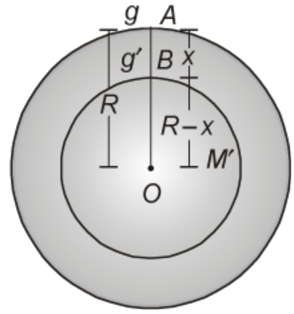
or $$g' = 3\,g = 3 \times 9.8 = 29.4\ \text{ms}^{-2}$$

Now, amount of work done to lift a mass m through distance h on the planet,

$$W = m\,g'\,h = 5 \times 29.4 \times 10 = \mathbf{1470\ J}.$$

S22. Consider that the Earth is a sphere of radius R and mass M as shown in the figure. Then, value of acceleration due to gravity at the point A on the surface of Earth is given by

$$g = \frac{GM}{R^2}$$



If ρ is density of the material of Earth, then

$$M = \frac{4}{3}\pi R^3 \rho$$

∴ $$g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$g = \frac{4}{3}\pi G R \rho \quad \text{... (i)}$$

Let g' be acceleration due to gravity at the point B at a depth x below the surface of Earth. A body at the point B will experience force only due to the portion of the earth of radius OB (= $R - x$). The outer spherical shell, whose thickness is x, will not exert any force on body at point B. If M' is the mass of the portion of the earth, whose radius is $R - x$, then,

$$g' = \frac{GM'}{(R+x)^2}$$

Now, $$M' = \frac{4}{3}\pi(R-x)^3\rho$$

$\therefore$ $$g' = \frac{G \times \frac{4}{3}\pi(R-x)^3\rho}{(R-x)^2}$$

or $$g' = \frac{4}{3}\pi G(R-x)\rho \qquad \text{... (ii)}$$

Dividing the equation (ii) by (i), we get

$$\frac{g'}{g} = \frac{\frac{4}{3}\pi G(R-x)\rho}{\frac{4}{3}\pi G R\rho} = \frac{R-x}{R}$$

or $$g' = g\left(1 - \frac{x}{R}\right) \qquad \text{... (iii)}$$

Therefore, the value of acceleration due to gravity decreases with depth also.

The acceleration due to gravity at the centre of Earth can be found by setting $x = R$ in the equation (iii).

Thus, $$g_{\text{centre}} = g\left(1 - \frac{R}{R}\right) = 0.$$

So, it follows that acceleration due to gravity is maximum at the surface of the Earth.

S23. (a) Let the acceleration due to gravity at a depth x be the same as that at a height h above the surface of earth. Then,

$$g\left(1 - \frac{x}{R}\right) = \left(1 - \frac{2h}{R}\right)g$$

or $$\frac{x}{R} = \frac{2h}{R}$$

or $$x = 2h = 2 \times 64 = \textbf{128 km}.$$

(b) (i) Let g' be the value of acceleration due to gravity at a distance d below the surface of Earth, then

$$g' = g\left(1 - \frac{d}{R}\right)$$

Now $g = 9.8 \text{ ms}^{-2}$ and $d = R/2$

$$\therefore \quad g' = 9.8\left(1 - \frac{R/2}{R}\right) = 9.8 \times \frac{1}{2} = 4.9 \text{ ms}^{-2}$$

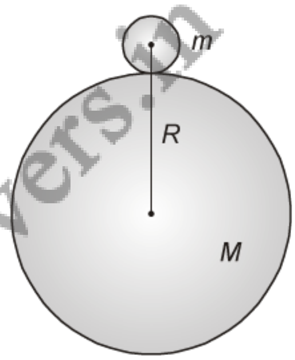
(ii) The value of above the surface of Earth at a height h is given by

$$g' = g\frac{R^2}{(R+h)^2} = g\frac{R^2}{(R+R/h)^2} = \frac{4}{9}g = \frac{4 \times 9.8}{9} = \mathbf{4.36\ ms^{-2}}.$$

S24. (a) The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity. It is denoted by g.

Consider that the Earth is a sphere of mass M and radius R and a body of mass m is lying on the surface of the Earth as shown in the figure. If the size of the body is very small as compared to that of the Earth, then the distance between the centre of the body and that of the Earth will be approximately equal to the radius of the Earth. Therefore, the force of attraction on the body due to the Earth is given by

$$F = \frac{GMm}{R^2} \quad \text{... (i)}$$



The force F on the body of mass m due to the Earth produces acceleration due to gravity (g) in the motion of the body. Therefore, force on the body due to Earth,

$$F = mg \quad \text{... (ii)}$$

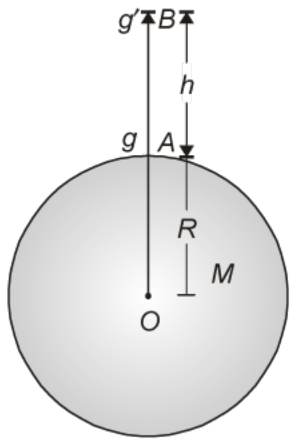
From the equations (i) and (ii), we have

$$g = \frac{GM}{R^2} \quad \text{... (iii)}$$

It gives the value of acceleration due to gravity at the surface of the Earth.

(b) Consider that Earth is a sphere of radius R and mass M. Let g be the value of acceleration due to the gravity at the point A on the surface of Earth as shown in the figure. Then,

$$g = \frac{GM}{R^2} \quad \text{... (i)}$$



If g' is acceleration due to gravity at the point B at height h above the surface of Earth, then

$$g' = \frac{GM}{(R+h)^2} \quad \text{... (ii)}$$

Dividing the equation (ii) by (i), we get

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

or $$\frac{g'}{g} = \frac{R^2}{(R+h)^2} \quad \text{... (iii)}$$

or $$\frac{g'}{g} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

or $$\frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2} \quad \text{... (iv)}$$

Expanding the equation (iv) by using Binomial theorem, we get

$$\frac{g'}{g} = 1-\frac{2h}{R} + \text{terms containing higher powers of } \frac{h}{R}$$

Usually, h is quite small as compared to R and therefore the higher powers of h/R can be neglected. Thus,

$$\frac{g'}{g} = 1-\frac{2h}{R}.$$

S25. (a) Here,

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

or $$\frac{mg_h}{mg} = \frac{R^2}{(R+h)^2} \quad \left[mg_h = \frac{1}{2}\,mg\right]$$

or $$\frac{1}{2} = \frac{R^2}{(R+h)^2}$$

or $$\sqrt{2}R - R = h.$$

$$h = (\sqrt{2}-1)\,R = (1.414-1)\text{R} = 0.414 \text{ x } 6400 \text{ Km}$$

$$h = 26{,}496 \text{ Km}$$

(b) Let h be the height at which the value of g will get reduced by 36% *i.e.*, of the value at the surface of the Earth.

Then, $$g' = \frac{64}{100} g \quad \text{... (i)}$$

Such a large decrease in the value of g will occur at a very large height. There, g' is given by

$$g' = g\frac{R^2}{(R+h)^2} \quad \text{... (ii)}$$

From the Eq. (i) and (ii), we get

$$g\frac{R^2}{(R+h)^2} = \frac{64}{100} g$$

or $$\frac{R}{R+h} = \frac{8}{10}$$

or $$h = \frac{R}{4} = \frac{6,400}{4} = \mathbf{1600\ km}.$$

S26. (a) Here, $$g_h = \frac{60}{100} g = \frac{3}{5} g$$

Now, $$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

or $$\sqrt{\frac{3}{5}} = \frac{R}{R+h}$$

or $$\frac{R+h}{R} = \sqrt{\frac{5}{3}}$$

or $$1 + \frac{h}{R} = \sqrt{\frac{5}{3}}$$

or $$h = \left(\sqrt{\frac{5}{3}} - 1\right) R = 0.29R$$

(b) The objects on the earth's surface are not in orbital motion w.r.t. the earth. In order that an object has an orbital motion close to the earth's surface, its orbital velocity, and period of motion must be

$$v_0 = \sqrt{gR}$$

$$T = \frac{2\pi R}{v_o} = 2\pi\sqrt{\frac{R}{g}} = 1.4 \text{ hrs.}$$

Therefore, the length of the day has to be 1.4 hrs in case in objects on the earth's surface are in the true orbital motion like that of the earth satellite.

Date: 13/10/2021

Q1. What is meant by gravitational field strength?

Q2. What is the gravitational potential energy of two point masses infinite distance away from each other?

Q3. Is the potential energy of a galaxy positive or negative? Give reason in support of your answer.

Q4. A mass thrown up returns to the surface of earth. What is the nature of total energy possessed?

Q5. Define gravitational potential at a point.

Q6. Is potential energy on the surface of earth always zero?

Q7. If the choice of origin is shifted, what is the change in (a) Gravitational potential (b) Potential energy?

Q8. The gravitational Potential energy of a body at a point in gravitational field of another body is $-\frac{GMm}{r}$. What does negative sign means?

Q9. Does the change in gravitational potential energy of a body between two points depend upon the nature of path followed? Explain.

Q10. If the diameter of earth becomes two times its present value and its mass remains unchanged; then how would the weight of an object on the surface of the earth be affected?

Q11. The acceleration due to gravity at a place gives a measure of the gravitational field at that point. Explain.

Q12. A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy? Neglect any mass loss of the comet when comes very close to the Sun.

Q13. Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Q14. What is gravitational potential energy at a point? How much of work is done in shifting a mass from the surface to a height equal to its radius?

Q15. Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also, obtain the potential at the centre of the square.

Q16. The acceleration due to gravity at the moon's surface is 1.67 ms^{-2}. Calculate the mass of the moon, if the radius of the moon is 1.74×10^6 m and $G = 6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$.

Q17. The earth's gravitational field at a certain point out in space accelerates at $1 kgms^{-2}$. How much will it accelerate a 3 kg mass?

Q18. How much energy must be given to a 100 kg rocket missile to carry it from the surface of earth into free space? Given, the gravitational constant $G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$, mass of the earth = 6×10^{24} kg and radius of the earth = 6.4×10^6 m.

Q19. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem?

Q20. What is the binding energy of the earth-sun system neglecting the effect of presence of other planets and satellites. Given that mass of the earth, $M_e = 6 \times 10^{24}$ kg, mass of the sun, $M_s = 2 \times 10^{30}$ kg; distance between the earth and the sun, $r = 1.5 \times 10^{11}$ m and gravitational constant, $G = 6.6 \times 10^{11} Nm^2 kg^{-2}$.

Q21. A body released at a distance far away from the surface of earth. Calculate its speed, when is near the surface of earth. Given, $g = 9.8 ms^{-2}$; radius of earth, $R = 6.37 \times 10^6$ m.

Q22. For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Q23. Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also obtain the potential at the centre of the square.

Q24. Define gravitational potential energy of a body. Derive an expression for the gravitational potential energy of a body of mass '*m*' located at a distance '*r*' from the centre of the earth.

Q25. The potential energy between two atoms (in a molecule) is given by

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

Where a and b are positive constant and x is the distance between the atoms. Find the distance between the atoms, so that the molecule is in stable equilibrium.

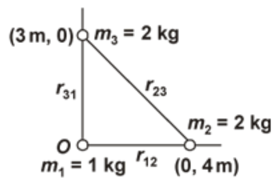
Q26. (a) If g is the acceleration due to gravity on the Earth's surface, what will be the gain in the potential energy of an object of mass m raised from the surface of the Earth to a height equal to twice the radius of the Earth?

(b) Mass of moon is 7.349×10^{22} kg and its radius is 1.738×10^6 m, calculate its mean density and acceleration due to gravity on its surface. Given $G = 6.668 \times 10^{-11} N\text{-}m^2 kg^{-2}$.

Q27. Gravitational potential at a point at distance 25000 km from the surface of Earth is $-4.51 \times 10^7 J kg^{-1}$. Find the gravitational field at this point. Given radius of the Earth, $R = 6,400$ km.

Q28. At a point above the surface of the Earth, the gravitational potential is $-5.02 \times 10^7 J kg^{-1}$ and acceleration due to gravity is $6.27 ms^{-2}$. Find the height of the point above the surface of Earth. Given, radius, $R = 6.4 \times 10^6$ m.

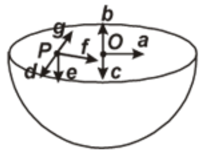
Q29. Three masses are in configuration as shown in figure.



What is the total gravitational potential energy of the configuration?

Q30. As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km.

Q31. Choose the correct answer from among the given ones:
The gravitational intensity at the centre of a hemispherical shell of uniform has the direction indicated by the arrow (as shown figure)
(a) *a*, (b) *b*, (c) *c*, (d) *O*.



Q32. The kinetic energy associated with a satellite is *E*. What is the total energy associated?

Q33. A satellite of mass 1,000 kg moves in a circular orbit of radius 7,000 km round the Earth. Calculate the total energy required to place the satellite in the orbit from the earth's surface. Assuming initially it to be at rest. Take $g = 10\ ms^{-2}$; radius of the earth, R = 6,400 km.

Q34. As you have learnt in the text, a geostationary satellite orbits the Earth at a height of nearly 36,000 km from the surface of the Earth. What is the potential due to Earth's gravity at the site of this satellite? Take the potential energy at infinity to be zero. Given, mass of the Earth = 6.0×10^{24} kg and radius of the Earth = 6,400 km.

Q35. Derive a relation for work done in a gravitational field. Using it, (a) find potential difference between a pair of points. (b) express whether gravitational force is conservative or non-conservative.

Q36. Two stars each of one solar mass (= 2×10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Q37. In Greek mythology, Atlas carried the entire Earth on his shoulders.
(a) How much work did Atlas do in supporting the Earth on his shoulders?
(b) Suppose Atlas decided to rearrange the solar system, so as to relocate the Earth farther from the Sun – out to the orbit of the Mars. What would be the change in Earth's gravitational potential energy relative to the Sun only? Given that mass of the Sun = 2×10^{30} kg, mass of the Earth = 6×10^{24} kg, radius of the Earth's orbit around the Sun = 1.5×10^{11} m and radius of the mars orbit around the Sun = 2.28×10^{11} m. Take $G = 6.67 \times 10^{-11}\ N m^2 kg^{-2}$.

Q38. If the potential at a point due to the Earth's gravitation is taken to approach zero as the distance of the point from the Earth's centre becomes larger and larger (*i.e.*, approaches infinity), what is the gravitational potential and potential energy of a body of mass 0.1 kg
(a) at the Earth's surface,
(b) at a height above the surface of the Earth equal to its mean radius *R*?
Given that $G = 6.67 \times 10^{-11}\ N m^2 kg^{-2}$, mass of the earth = 5.98×10^{24} kg and $R = 6.37 \times 10^6$ m. Assume the Earth to be a perfect sphere of uniform mass density.

Date: 13/10/2021

S1. The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (may be called test mass) placed at that point, provided the unit mass (test mass) itself does not produce any change in the field of the body.

S2. Zero, at infinite distance between masses no gravitational force act on them due to potential energy must be zero.

S3. The force between different galaxies are attractive in nature. Hence, the potential energy of a galaxy is negative.

S4. Total energy has to be negative, since P.E. dominates over kinetic energy.

S5. The work done in carrying unit mass from infinity to a point in gravitational field is gravitational potential.

$$V_g = -\frac{GM}{r}$$

where r is the distance of the point from M.

S6. No. It is $-\frac{GM}{R}$.

S7. (a) Remains on the choice of origin.

(b) Potential energy is independent since it depends only on separation.

S8. Negative sign means that the mass m is bound to M.

S9. The gravitational force is a conservative force. The work done to move a body between two points under the action of a conservative force is independent of the path followed. Therefore, change in gravitational potential energy (work done per unit mass) between two points is independent of the path followed.

S10. Let M and R be the mass and radius of the earth.

Then, the weight of a body of mass m,

$$W = \frac{GMm}{R^2}$$

If the earth becomes of diameter *i.e.*, of radius two times its present value (mass unchanged), the weight of the body of mass m will become

$$W' = \frac{GMm}{(2R)^2} = \frac{1}{4} \times \frac{GMm}{R^2} = \frac{1}{4} \text{W}$$

i.e., weight of the body will become ***one fourth*** of the present value.

S11. From the definition of strength of gravitation field at point, we have

$$E_{\text{grav}} = \frac{F}{m} = \frac{GMm}{mr^2} = \frac{GM}{r^2}$$

But $\frac{GM}{r^2} = g'$,

The value of acceleration due to gravity at distance from the centre of earth.

Hence, acceleration due to gravity at a place gives a measure of gravitational field at that point.

S12. (a) No (b) No (c) Yes (d) No.

Angular momentum and total energy at all points of the orbit of a comet moving in a highly elliptical orbit around the Sun are constant. Its linear speed, angular speed, kinetic, and potential energy varies from point to point in the orbit.

S13. (a) Kinetic energy, total mechanical energy of a satellite is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the gravitational potential energy of the satellite is zero. As the Earth satellite is negative.

Thus, the total energy of an orbiting satellite at infinity is equal to the negative of its kinetic energy.

(b) Less, an orbiting satellite acquires a certain amount of energy that enables it to revolve around the Earth. This energy is provided by its orbit. It requires relatively lesser energy to move out of the influence of the Earth's gravitational field than a stationary object on the Earth's surface that initially contains no energy.

S14. Gravitational potential energy is the work done is shifting *a* mass *m* from one point to the other.

At any distance *x* from the centre of earth (*M*), the gravitational force is, $\frac{GMm}{x^2}$.

∴ Work done in shifting (*m*) from the surface to a height equal to the radius, then,

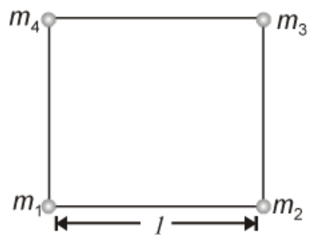
$$W = \int_R^{2R} \frac{GMm}{x^2} dx$$

$$= GMm\left[-\frac{1}{x}\right]_R^{2R} = -\frac{GMm}{2R}$$

S15. In the arrangement of four equal masses *m* at the vertices, the gravitational P.E. is due to all combinations of masses.

∴ $$\text{G.P.E} = \frac{G}{a}(m_1m_2 + m_1m_4 + m_2m_3 + m_3m_4) + \frac{G}{\sqrt{2}l}(m_1m_3 + m_2m_4)$$

$$= \frac{4Gm^2}{a} + \frac{2Gm^2}{\sqrt{2}l}$$



$$\text{G.P.E} = \frac{Gm^2}{a}(4 + \sqrt{2})$$

$$\text{Potential at centre} = \frac{Gm^2}{l\sqrt{2}} \times 4 = \frac{4\sqrt{2}\,Gm}{l}.$$

S16. Let M be the mass of the moon and g, the acceleration due to gravity on its surface.

Here, $G = 6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}$; $R = 1.74 \times 10^6$ m ; g = 1.67 ms^{-2}

Now, $$M = \frac{gR^2}{G} = \frac{1.67 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= \mathbf{7.58 \times 10^{22}\ kg}.$$

S17. The gravitational force a body is proportional to its mass. Hence, if the mass increase by a force of 3 (from 1 kg to 3 kg), the gravitational force in the second case will also become three times. However, the acceleration due to gravity will remain the same. Thus, at a point in space, where the earth's gravitational field accelerates a 1 kg mass at 5 ms^{-2}, a 3 kg mass will also accelerated at 5 ms^{-2}.

S18. The energy required to carry the rocket missile from the surface of the earth into the free space,

E = P.E. of the missile in free space
– P.E. of the missile at the earth's surface

$$= 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

Given, m = 100 kg; $M = 6 \times 10^{24}$ kg; $R = 6.4 \times 10^6$ m;

$$G = 6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}$$

$$E = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 100}{6.4 \times 10^6}$$

$$= \mathbf{6.253 \times 10^9\ J}.$$

S19. **(b)**, **(c)**, and **(d).**

Legs hold the entire mass of a body in standing position due to gravitational pull. In space, an astronaut feels weightlessness because of the absence of gravity. Therefore, swollen feet of an astronaut do not affect him/her in space.

A swollen face is caused generally because of apparent weightlessness in space. Sense organs such as eyes, ears nose, and mouth constitute a person's face. This symptom can affect an astronaut in space.

Headaches are caused because of mental strain. It can affect the working of an astronaut in space.

Space has different orientations. Therefore, orientational problem can affect an astronaut in space.

S20. The binding energy of the earth-sun system,

$$E = -\text{ total energy of the earth revolving around the sun}$$

$$= -\left(-\frac{GM_sM_e}{2r}\right) = \frac{GM_sM_e}{2r}$$

Given, $M_e = 6 \times 10^{24}$ kg; $M_s = 2 \times 10^{30}$ kg; $r = 1.5 \times 10^{11}$ m

and $G = 6.6 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}$

$$E = \frac{6.6 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{2 \times 1.5 \times 10^{11}}$$

$$= \mathbf{2.64 \times 10^{33}\ J}.$$

S21. The sum of P.E. and K.E. of the body on reaching the surface of the earth must be equal to P.E. of the body, when at a distance far away from the surface of the earth i.e.

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$$

or $$v = \sqrt{\frac{2GM}{R}}$$

or $$v = \sqrt{2gr}$$

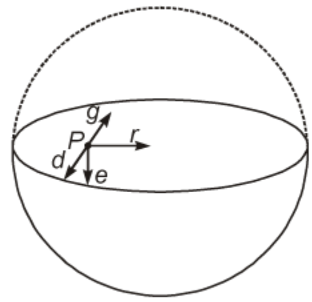
Here, $g = 9.8\ \text{ms}^{-2}$; $R = 6.37 \times 10^6$ m

$$v = \sqrt{2 \times 9.8 \times 6.37 \times 10^8} \qquad (\because\ GM = gR^2)$$

$$= \mathbf{1.117 \times 10^5\ m\,s^{-1}}.$$

S22. (ii).

Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.



If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle at an arbitrary point P will be in the downward direction.

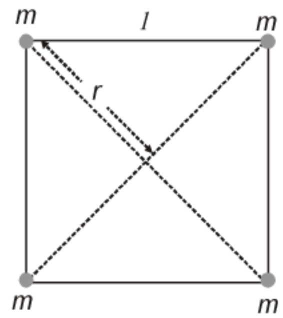
Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at an arbitrary point P of the hemispherical shell has the direction as indicated by arrow **e**.

S23. Consider four masses each of mass m at the corners of a square of side l; (see figure) We have four mass pairs at distance l and two diagonal pairs at distance $\sqrt{2}\,l$.

Work done = Change in potential energy

Hence, $$W(r) = -4\frac{Gm^2}{l} - 2\frac{Gm^2}{\sqrt{2}\,l}$$

$$= -\frac{2Gm^2}{l}\left(2 + \frac{1}{\sqrt{2}}\right) = -5.41\frac{Gm^2}{l}$$



The gravitational potential at the centre of the square ($r = \sqrt{2}\,l/2$) is

$$U(r) = -4\sqrt{2}\frac{Gm}{l}.$$

S24. **Gravitational potential energy:** The work done in carrying a mass '*m*' from infinity to a point at distance *r* is called gravitational potential energy.

$$\text{G.P.E.} = -\frac{GMm}{r}$$

i.e., *G.P.E* = Mass × Gravitational potential It is a scalar quantity measured in joule.

Negative sign means that the mass is bound to *M*.

The gravitational force of attraction between *M* and *m* when *x* is the distance between their centres is given by

$$F = \frac{GMm}{x^2}$$

Suppose the body is moved through a distance *dx*, therefore, work done is given by,

$$dW = Fdx = \frac{GMm}{x^2}\,dx$$

When the body is brought from infinity to some distance *r*,

We write, $$\int dW = \int_{x=\infty}^{x=r}\frac{GMm}{x^2}dx$$

or $$W = GMm\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$= -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right] - \frac{GMm}{r}$$

This amount of work done is the change in the potential energy of the body.

∴ P.E. $$U = -\frac{GMm}{r}.$$

S25. Given, $$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

The molecule will be in stable equilibrium, if potential energy is minimum *i.e.*,

$$\frac{dU}{dx} = 0$$

or $$\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right) = 0$$

or $$\frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0 \quad \text{or} \quad x^6 = \frac{2a}{b}$$

or $$x = \mathbf{(2\,a/b)^{1/6}}.$$

S26. (a) Let M be the mass and R, radius of the earth.

Then, gravitational potential of the object of mass m at the surface of the earth.

$$U_i = -\frac{GMm}{R} \quad \dots \text{(i)}$$

and potential of the object, when taken to a height equal to $2R$,

$$U_f = -\frac{GMm}{R+h} = -\frac{GMm}{R+2R} = -\frac{GMm}{3R} \quad \dots \text{(ii)}$$

Therefore required work done,

$$U_f - U_i = -\frac{GMm}{3R} - \left(-\frac{GMm}{R}\right) = \frac{2GMm}{3R}$$

Since $GM = gR^2$, we have

$$U_f - U_i = \frac{2 \times gR^2 \times m}{3R} = \frac{2}{3}mgR$$

(b) Mass of moon $m = 7.349 \times 10^{22}$ kg, Radius $r = 1.7{,}738 \times 10^6$ m

$$G = 6.668 \times 10^{-11}\ \text{N-m}^2\,\text{kg}^{-2}$$

Volume of moon $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (1.738 \times 10^6)^3\ \text{m}^3 = 21.99 \times 10^{18}\ \text{m}^3$

$\therefore$ Mean density $= \frac{m}{v} = \frac{7.349 \times 10^{22}}{21.99 \times 10^{18}} = 3341.97\ \text{kg m}^{-3}$

Acceleration due to gravity on the surface of moon

$$g_m = \frac{Gm}{r^2} = \frac{6.668 \times 10^{-11} \times 7.349 \times 10^{22}}{(1.738 \times 10^6)^2} = \mathbf{1.622\ ms^{-2}}.$$

S27. Given,

$$V_{grav} = -\frac{GM}{R+x} = -4.51 \times 10^7$$

or $$GM = 4.51 \times 10^7 \times (R + x)$$

Now, $E_{grav} = \dfrac{GM}{(R+x)^2} = \dfrac{4.51 \times 10^7 \times (R+x)}{(R+x)^2}$

$= \dfrac{4.51 \times 10^7}{(R+x)}$

Setting, R = 6,400 KM = 6.4×10^6 m

and x = 2,500 km = 2.5×10^6 m

$\therefore$ $E_{grav} = \dfrac{4.51 \times 10^7}{(6.4 \times 10^6 + 2.5 \times 10^6)}$ = **5.07 N kg^{-1}**.

S28. Given, $V_{grav} = -5.02 \times 10^7$ J kg^{-1} and $g' = 6.27$ ms^{-2}

Let x be the height of the point above the surface of the Earth. Then, gravitational potential at the height x,

$$V_{grav} = -\frac{GM}{R+x} \quad \text{... (i)}$$

Also, acceleration to gravity at this point,

$$g' = \frac{GM}{(R+x)^2} \quad \text{... (ii)}$$

Dividing the equation (i) by (ii), we get

$$\frac{V_{grav}}{g'} = -(R+x)$$

or $x = -\dfrac{V_{grav}}{g'} - R$

$= -\dfrac{-5.02 \times 10^7}{6.27} - 6.4 \times 10^6$

$= 8.0 \times 10^6 - 6.4 \times 10^6$

$= 1.6 \times 10^6$ m = **1,600 km**.

S29. Here, $m_1 = 1$ kg; $m_2 = 2$ kg; $m_3 = 2$kg; $r_{12} = 4$ m; $r_{31} = 3$ m and $r_{23} = \sqrt{r_{12}^2 + r_{31}^2} = \sqrt{4^2 + 3^2} = 5$ m

Therefore, total gravitational potential energy of the configuration,

$$U = -\frac{Gm_1m_2}{r_{12}} + \left(-\frac{Gm_2m_3}{r_{23}}\right) + \left(-\frac{Gm_3m_1}{r_{31}}\right)$$

$$= -G\left(\frac{m_1m_2}{r_{12}} + \frac{m_2m_3}{r_{23}} + \frac{m_3m_1}{r_{31}}\right)$$

$$= -6.67 \times 10^{-11} \times \left(\frac{1\times 2}{4} + \frac{2\times 2}{5} + \frac{1\times 2}{3}\right)$$

$$= -6.67 \times 10^{-11} \times \frac{59}{30} = \mathbf{-1.31 \times 10^{-10}\ J}.$$

S30. Mass of the Earth, $M = 6.0 \times 10^{24}$

Radius of the Earth, $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

Height of a geostationary satellite from the surface of the Earth,

$$h = 36000 \text{ km} = 3.6 \times 10^7 \text{ m}$$

Gravitational potential energy due to Earth's gravity at height

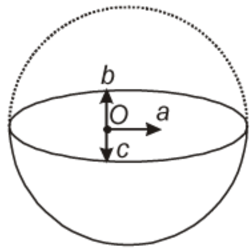
$$= \frac{-GM}{(R+h)}$$

$$= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.6 \times 10^7 + 0.64 \times 10^7}$$

$$= \frac{6.67 \times 6}{4.24} \times 10^{13-7} = 9.4 \times 10^6 \text{ J/kg}.$$

S31. Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dV}{dr}\right)$ is zero every where inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle located at centre O will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at centre O of the given hemispherical shell has the direction as indicated by arrow c.

S32. K.E. with a satellite in an orbit of radius r is

$$E = \frac{1}{2} m v_0^2 = \frac{1}{2} m \left(\sqrt{\frac{Gm}{r}}\right)^2 = \frac{1}{2} m \frac{GM}{r}$$

$$\Rightarrow \quad \frac{GMm}{r} = 2E$$

$$\text{P.E. at the orbit} = -\frac{GMm}{r}$$

Total energy = K.E. + P.E.

$$= +\frac{GMm}{2r} - \frac{GMm}{r}$$

$$= \frac{GMm}{r}\left[-\frac{1}{2}\right] = -\frac{1}{2}2E = -E.$$

S33. Let M be the mass of the Earth. If E is total energy of the satellite in the orbit and U_O is its P.E. on the surface of the Earth, then energy required to place the satellite in the orbit,

$$E' = E - U_O = -\frac{GMm}{2(R+x)} - \left(-\frac{GMm}{R}\right)$$

$$= GMm\left[\frac{1}{R} - \frac{1}{2(R+x)}\right]$$

Now, $g = \frac{GM}{R^2}$

Or $GM = gR^2$

$\therefore$ $E' = mgR^2\left[\frac{1}{R} - \frac{1}{2(R+x)}\right] = mg\left[R - \frac{R^2}{2(R+x)}\right]$

Setting, $m = 1{,}000$ kg; $g = 10$ ms^{-2};

$R = 6{,}400$ km $= 64 \times 10^6$ m; $R + x = 7{,}000$ km $= 7.0 \times 10^6$ m

$\therefore$ $E' = 1{,}000 \times 10\left[6.4 \times 10^6 - \frac{(6.4 \times 10^6)^2}{2 \times 7.0 \times 10^6}\right]$

$= 10{,}000\,(6.4 \times 10^6 - 2.93 \times 10^6)$

$= \mathbf{3.47 \times 10^{10}}$ **J**.

S34. Given, Mass of the Earth, $M = 6.0 \times 10^{24}$ kg;

Radius of the Earth, R = 6,400 km

Height of the satellite from the surface of the Earth,

$x = 36{,}000$ km

Therefore, distance of the satellite from the centre of the Earth,

$r = R + x = 6{,}400 + 36{,}000 = 42{,}400$ km

$= 4.24 \times 10^7$ m

Therefore, potential at the orbit (site) of the satellite due to the Earth's gravity,

$$V_{grav} = -\frac{GM}{r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{4.24 \times 10^{7}}$$

$$= \mathbf{-9.44 \times 10^{6}\ J\ kg^{-1}}.$$

S35. The gravitational force of attraction between M and m when x is the distance between their centres is given by

$$F = \frac{GMm}{x^2}$$

Suppose the body be moved through a distance dx, therefore, work done is given by,

$$dW = \text{F}dx = \frac{GMm}{x^2}\ dx$$

When the body is brought from infinity to some distance r,

we write, $$\int dW = \int_{x=\infty}^{x=r} \frac{GMm}{x^2}\, dx$$

or $$W = GMm\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$= -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$= \frac{-GMm}{r}$$

This amount of work done is the change in the potential energy of the body.

$$\text{P.E. (U)} = \frac{-GMm}{r}$$

Gravitational potential

$$V = \frac{U}{m} = \frac{-GM}{r}$$

The general expression for gravitational potential due to the earth (mass M) at

(a) Potential at distance r is, $V = \frac{-GM}{r}$

Potential at a point $A\,(r_a) = -\frac{GM}{r_a}$

Potential at a point $B\,(r_b) = -\frac{GM}{r_b}$

$\therefore$ Difference in potential between the points

$$= -GM\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(b) Since work done against gravitational force is (i) independent of path and dependent only on the initial and final points and (ii) the work done in a closed path is zero, it is a conservative force.

S36. Given, Mass of each star, $M = 2 \times 10^{30}$ kg

Radius of each star, $R = 10^4$ km = 10^7 m

Distance between the stars, $r = 10^9$ km = 10^{12}m

For negligible speeds, $v = 0$ total energy of two stars separated at distance r

$$= \frac{-GMM}{r} + \frac{1}{2}mv^2$$

$$= \frac{-GMM}{r} + 0 \quad \text{... (i)}$$

Now, consider the case when the stars are about to collide:

Velocity of the stars = v

Distance between the centers of the stars = $2R$

Total kinetic energy of both stars = $\frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$

Total potential energy of both stars = $\frac{-GMM}{2R}$

Total energy of the two stars = $Mv^2 - \frac{-GMM}{2R}$

Using the law of conservation of energy, we can write:

$$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{r}$$

$$v^2 = \frac{-GM}{r} + \frac{GM}{2R} = GM\left(-\frac{1}{r} + \frac{1}{2R}\right)$$

$$= 6.67 \times 10^{-11} \times 2 \times 10^{30}\left[\frac{-1}{10^{12}} + \frac{1}{2 \times 10^7}\right]$$

$$\sim 13.34 \times 10^{19} \times 5 \times 10^{-8}$$

$$\sim 6.67 \times 10^{12}$$

$$v = \sqrt{6.67 \times 10^{12}} = 2.58 \times 10^6 \text{m/s}.$$

S37. (a) Atlas did no work in supporting the earth on his shoulders. It is because, the displacement is zero.

(b) Given, $G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$; $M_s = 2 \times 10^{30}$ kg; $M_e = 6 \times 10^{24}$ kg; $R_{es} = 1.5 \times 10^{11}$ m and $R_{ms} = 2.28 \times 10^{11}$ m

The gravitational potential energy of the Earth in its present position,

$$U_i = -\frac{GM_sM_e}{R_{es}}$$

$$= \frac{6.67 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{1.5 \times 10^{11}}$$

$$= -5.34 \times 10^{33}\,\text{J}$$

The gravitational potential energy of the Earth in its new relocated position,

$$U_f = -\frac{GM_sM_e}{R_{ms}}$$

$$= -\frac{6.67 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{2.28 \times 10^{11}}$$

$$= -3.51 \times 10^{33}\,\text{J}.$$

Therefore, increase in gravitational potential energy of the Earth,

$$U_f - U_i = -3.51 \times 10^{33} - (-5.34 \times 10^{33}) = \mathbf{1.83 \times 10^{33}\,J}.\text{s}$$

S38. Here, $G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$; $M = 5.98 \times 10^{24}$ kg and $R = 6.37 \times 10^6$ m; m = 0.1 kg

(a) V_{grav} (at surface) $= -\frac{GM}{R} = \frac{6.67 \times 10^{11} \times 5.98 \times 10^{24}}{6.37 \times 10^6}$

$= \mathbf{-6.26 \times 10^7\,J\,kg^{-1}}$.

U (at surface) $= -\frac{GMm}{R} = V_{grav} \times m$

$= -6.26 \times 10^7 \times 0.1 = \mathbf{-6.26 \times 10^6\,J}$.

(b) V_{grav} (at height h) $= -\frac{GM}{R+h} = \frac{-6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^6 + 6.37 \times 10^6}$

$= \mathbf{-3.13 \times 10^7\,J\,kg^{-1}}$.

U (at height h) $= -\frac{GMm}{R+h} = V_{grav}(\text{at height}) \times m$

$= -3.13 \times 10^7 \times 0.1 = \mathbf{-3.13 \times 10^6\,J}$.

Date: 13/10/2021

Q1. Define the binding energy of an orbiting satellite.

Q2. What is the acceleration of a particle projected upward at its highest point of motion?

Q3. Why do different planets have different escape velocities?

Q4. What provides the centripetal force to a satellite revolving round the earth?

Q5. Express time period of a satellite in terms of its density and radius of the planets?

Q6. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, what will happen to the satellite?

Q7. What is the sense of rotation of a geostationary satellite?

Q8. What is the time period a geostationary satellite?

Q9. An artificial satellite is at a height of 36,500 km above earth's surface. What is the work done by earth's gravitational force in keeping it in its orbit?

Q10. Show graphically how *g* varies as you move from the centre of earth to great heights above the surface.

Q11. How does the orbital velocity of a satellite depend on the mass of the satellite?

Q12. Does the escape velocity vary with mass?

Q13. How escape velocity is related to orbital velocity for orbits close to earth?

Q14. What is the height at which the satellite is to be parked for it to be geostationary satellite?

Q15. Is geostationary satellites always lie over New Delhi? Why?

Q16. What is the reason for absence of atmosphere in some planets?

Q17. What is parking orbit?

Q18. What is the value of escape velocity from the surface of (a) earth, (b) moon?

Q19. Can a satellite be in an orbit in a plane not passing through the earth's centre? Explain your answer.

Q20. A small mass is released by an astronaut in a satellite in space. Will it fall on the earth?

Q21. A spacecraft consumes more fuel in going from earth to moon than it does on the return trip. Comment on this.

Q22. Two artificial satellites, one close to the surface and other away are revolving around the earth. Which of them has larger speed?

Q23. If the earth's satellite is put into an orbit at a height where resistance due to atmosphere cannot be neglected, how will motion of satellite be affected?

Q24. What are the conditions under which a rocket fired from the earth, launches an artificial satellite of earth?

Q25. The Earth is acted upon by the gravitational attraction of the Sun. Why does not the Earth fall into the Sun?

Q26. If the earth suddenly stops rotating about axis, what would be the effect on *g*? Would this effect same at all places?

Q27. What are the necessary conditions for satellite to appears stationary?

Q28. What are the conditions under which a rocket fired from the Earth becomes a satellite of the Earth and orbits in a circle?

Q29. Why are space rockets usually launched from west to east? Why it is more advantageous to launch rockets in the equatorial plane?

Q30. Air friction increases the velocity of the satellite. Explain.

Q31. What is the difference between ordinary and geostationary satellite?

Q32. Does the escape speed of a body from the Earth depend on:

(a) the mass of the body,
(b) the location from where it is projected,
(c) the direction of projection,
(d) the height of the location from where the body is launched?

Q33. What is escape velocity? Derive an expression for the same.

Q34. The radius of the Earth is reduced by 4%. The mass of the Earth remains unchange. What will be the changed in escape velocity?

Q35. Satellite *A* is in a certain circular orbit about a planet while satellite *B* is in a large circular orbit. Which satellite has (a) the longer period and (b) the greater speed?

Q36. Object at rest on the Earth's surface move in circular paths with a period of 24 hours. Are they in orbit in the same sense that an Earth's satellite is in orbit? Why not? What would the length of the day have to be put such objects in a true orbit?

Q37. A body weighs 64 N on the surface of the Earth. What is the gravitational force on it due to the Earth at a height equal to one-third of the radius of the Earth?

Q38. The escape velocity of the projectile on the Earth's surface is 11.2 km s^{-1}. A body is projected out with twice this speed. What is the speed of the body far away from the Earth? Ignore the presence of the Sun and other planets.

Q39. An artificial satellite is going around the Earth close to its surface. Calculate the orbit velocity and time taken by it to complete one around. The radius of Earth = 64,000 km, acceleration due to gravity = 9.8 ms^{-2}.

Q40. What would be the angular speed of the earth, so that the bodies lying on the equator may appear weightless? Take g = 10 ms^{-2} and radius of the Earth, R = 6,400 km.

Q41. A body of mass 100 kg falls on the Earth from infinity. What will be its velocity and energy on reaching the earth? Given that radius of the earth = 6,400 km and g = 9.8 s^{-2}. Air friction may be neglected.

Q42. A 400 kg satellite is in a circular orbit of radius $2R_E$ about the Earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies?

Q43. Express the constant k of Eq. (8.38) in days and kilometres. Given $k = 10^{-13}\ s^2\ m^{-3}$. The Moon is at a distance of 3.84×10^5 km from the earth. Obtain its time-period of revolution in days.

Q44. A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun = 2×10^{30} kg).

Q45. (a) Define escape velocity.
(b) Derive expression for the escape velocity of an object from the surface of a planet.
(c) Does it depend on location from where it is projected?

Q46. Jupiter has a mass 318 times that of the Earth and its radius is 11.2 times the Earth's radius. Estimate the escape velocity of a body from Jupiter's surface. Given, the escape velocity from the Earth's surface = 11.2 km s^{-1}.

Q47. Find the percentage decrease in weight of a body, when taken 16 km below the surface of the earth. Take radius of the earth as 6,400 km. What happen to the weight of the body, when it is taken to the centre of the Earth?

Q48. (a) Derive an expression for the orbital velocity of a satellite in the orbit. Reduce it to an orbit close to the surface of earth. How is it related to escape velocity?
(b) If the earth has a mass 9 times and radius twice that of the planet Mars, calculate the minimum velocity required by a rocket to pull out of the gravitational force of Mars. Escape velocity on the surface of Earth is 11.2 km/sec.

Q49. (a) For a satellite orbiting in an orbit, close to the surface of earth, to escape, what is the percentage increase in the kinetic energy required?
(b) The escape velocity of a projectile on the surface of earth is 11.2 km s^{-1}. A body is projected out with twice this speed. What is the speed of the body far away from the Earth *i.e.*, at infinity? Ignore the effect of other planets.

Q50. (a) What happens to a body when it is projected vertically upwards from the surface of the Earth with a speed of 11200 m/s, and why? Compare escape speeds for two planets of masses M and $4M$ and radii $2R$ and R respectively.
(b) Calculate the escape velocity of an atmospheric particle 1000 km above surface of Earth. Given $R = 6.4 \times 10^6$ m and $g = 9.8\ ms^{-2}$.

Q51. (a) The escape velocity v of a body depends upon:
(i) The acceleration due to gravity 'g' of the planet.
(ii) The radius of the planet 'R'.
Establish dimensionally the relationship between them.
(b) What should be the percentage increase in the orbital velocity to escape?

Q52. A body weighs 63 N on the surface of the Earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the Earth?

Q53. **The planet Mars has two Moons, Phobos and Delmos. (a) Phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of Mars. (b) Assume that Earth and Mars move in circular orbits around the Sun, with the martian orbit being 1.52 times the orbital radius of the Earth. What is the length of the martian year in days?**

Q54. **The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.**

Q55. **Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the Earth if it weighed 250 N on the surface?**

Q56. **Weighing the Earth : You are given the following data: $g = 9.81$ ms^{-2}, $R_E = 6.37 \times 10^6$ m, the distance to the Moon $R = 3.84 \times 10^8$ m and the time period of the Moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.**

Q57. **A rocket is fired vertically with a speed of 5 km s^{-1} from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^6 m; $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.**

Q58. **A satellite orbits the earth at a height of 400 kms above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.**

Q59. **A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the Sun = 2×10^{30} kg; mass of mars = 6.4×10^{-11} m^2kg; radius of mars = 3395 km; radius of the orbit of mass = 2.28×10^8kg; G = 6.67×10^{-11} m^2kg^{-2}.**

Q60. **A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4×1023 kg; radius of mars = 3395 km; $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.**

Date: 13/10/2021

S1. ***Binding energy of an orbiting satellite:*** It follows that in order to free the satellite from the Earth's gravitational field, an energy equal to – E has to be supplied to it. Thus, the binding energy of the satellite is given by

$$\text{Binding energy} = \frac{GMm}{2(R+x)}.$$

S2. 9.8 ms^{-2} (vertically downward).

S3. The value of mass radius for different planets are different and hence the value of acceleration due to gravity is different for different planets.

Since $v_e = \sqrt{2gR}$,

The different planets have different escape velocities.

S4. The weight of the satellite provides the necessary centripetal force to it, so as to enable it to revolve around the planet.

S5. The time period of a satellite is

$$T = \sqrt{\frac{3(R+x)^3}{GR^3\rho}}$$

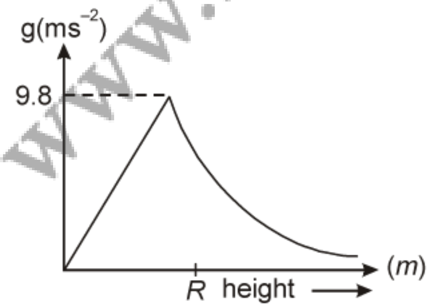
S6. If the gravitational force suddenly becomes zero, the satellite will move tangentially to the original orbit with the velocity, it was revolving around the earth.

S7. Its sense of rotation should be same as that of the earth about its own axis *i.e.*, in anticlockwise direction (from west to east).

S8. Time period of a geostationary satellite = 24 h.

S9. Zero. Because in this state no force is acting on the satellite due to **which** work done of the satellite is zero.

S10.



S11. Independent of mass.

S12. No, because $v_e = \sqrt{2gR}$

S13. $v_e = \sqrt{2}\, v_0$ for near to earth orbits.

S14. Approximately 36,000 km.

S15. No, generally the geo-stationary satellites will be in the equatorial plane. Since New Delhi does not lie in this plane, it is not possible.

S16. Very less escape velocity of 2.38 km/sec.

S17. The orbit at which the satellite will remain static according to earth (as a frame).

S18. (a) $v_e = \sqrt{2gR} = 11.2$ km/s.

(b) $v_m = \sqrt{\dfrac{2g \times R_m}{6}} = 2.38$ km/sec.

S19. The centripetal force required for the orbital motion of the satellite is provided by the gravitational force of attraction. Gravitational force is a central force, *i.e.*, it passes through the centre of mass of the earth and the satellite. Hence, the plane of orbit of the satellite has to pass through the earth's centre.

S20. The orbit of the satellite is independent of its mass. The released mass will continue to orbit the earth as a satellite and hence, will not fall on the earth.

S21. In going from earth to moon, the spacecraft has to do more work against the greater gravitational attraction of the Earth. For the return journey, moon's gravitational force is much less, hence less work is done and less fuel is consumed.

S22. Both have same speed of $\sqrt{gR}$.

S23. It will get slowed down in the orbit and so may fall from its orbit.

S24. (a) Velocity acquired is more than the escape velocity to go out in space.

(b) Provide sufficient velocity to move in a path of its own.

S25. The Earth does not fall into the Sun due to its stable orbit around the Sun. The gravitational pull of the Sun provides the necessary centripetal force to the Earth, so as to make it revolve in a stable orbit. We may say that the Earth is falling freely towards the Sun but always missing it.

Note: If the earth were rest, it would fall straight into the Sun.

S26. The effect of rotation of the earth on acceleration due to gravity is to decrease its value. Therefore, if the earth stops rotating, the value of g will ***increase***.

The effect will not be same at all places. It will be maximum at the equator.

S27. (a) It should revolve in an orbit coplanar and concentric with the equatorial plane.

(b) The sense of its orbital motion should be same as that of the rotational motion of earth *i.e.*, in anticlockwise direction.

(c) Its time period should be exactly 24 hours.

S28. (a) The rocket should be given a velocity, which will take it to the height, at which it is required to revolve around the Earth.

(b) At this height, the rocket should be provided the orbital velocity given by

$$v = \sqrt{\frac{GM}{R+x}} = R\sqrt{\frac{g}{R+x}}$$

(c) There should be no effect of the air resistance at the height, it revolve around the Earth.

S29. The earth rotates about its axis from west to east (anticlockwise direction) with angular speed,

$$\omega = \frac{2\pi}{24 \times 60 \times 60} \text{ rad s}^{-1}$$

When the space rocket is launched from west to east the linear speed of the Earth also gets added to the launching speed of the rocket. Further, as the radius of the Earth in equatorial plane is maximum, its linear speed will also be maximum equatorial plane. For these reasons, the space rockets are launched from west to east in the equatorial plane.

S30. For a satellite of mass *m* revolving in circular orbit force is provided by the gravitational pull *i.e.*,

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

or $$mv^2 r = GM m \quad \text{or} \quad (m v r) v = \text{constant}$$

or $$L v = \text{constant} \qquad \dots \text{(i)}$$

Where $m v r = L$ is the angular momentum of the satellite. As the air friction will constitute a retarding torque, it will, therefore, decrease the angular momentum of the satellite. In turn, the velocity of the satellite increase in order to keep the equation (i) intact.

S31. A satellite put in a circular orbit concentric and coplanar with the equatorial plane of the earth and at a height, such that its period of revolution is just 24 hours, is called a stationary satellite. The calculations show that the height of the orbit of such a satellite should about 36,000 km. Such a satellite hence appears stationary.

As ordinary satellite has time period other than 24 hours and hence appears to be moving w.r.t. Earth.

S32. (a) No (b) No (c) No (d) Yes

Explanation:

Escape velocity of a body from the Earth is given by the relation:

$$v_{ese} = \sqrt{2gR} \quad \text{... (i)}$$

g = Acceleration due to gravity

R = Radius of the Earth

It is clear from equation (i) that escape velocity v_{esc} is independent of the mass of the body and the direction of its projection. However, it depends on gravitational potential at the point from where the body is launched. Since this potential marginally depends on the height of the point, escape velocity also marginally depends on these factors.

S33. The minimum velocity required to escape from the gravitational force of Earth is called escape velocity. Total energy is the sum of P.E. and K.E.

$$\text{T.E.} = \frac{GMm}{R} + \frac{1}{2}mv^2$$

To escape K.E. should greater than P.E., *i.e.*,

$$\frac{1}{2}mv^2 \geq -\frac{GMm}{R}$$

$$v_e = \sqrt{2\frac{GM}{R}} = \sqrt{2gR}$$

S34. Escape velocity, say v, is given by

$$v = \sqrt{\frac{2GM}{R}}$$

or $$v^2 = (2\text{ GM})R^{-1}$$

Differentiate *w.r.t.* 'R', we get,

$$2v\frac{dv}{dR} = -(2\text{GM})\text{R}^{-2}$$

or $$v\frac{dv}{dR} = \frac{-GM}{R^2} \quad \text{... (i)}$$

also $$v^2 = \frac{2GM}{R} \quad \text{... (ii)}$$

Dividing (i) by (ii), we get,

$$\frac{1}{v}\frac{dv}{dR} = \frac{-1}{2R}$$

or $$\frac{dv}{v} = \frac{-dR}{2R}$$

or $\left|\frac{dv}{v}\right| = \left|\frac{-dR}{2R}\right| = \frac{4}{2}\% = 2\%$

Thus, the decrease in the radius by 4% will increase the escape velocity by 2%.

S35.

(a) $T = 2\pi\sqrt{\frac{r^3}{GM}}$ $\therefore$ larger 'r', larger T.

$\therefore$ Satellite B has longer period.

(b) $v_0 = \sqrt{\frac{GM}{r}}$, lesser r, more v_0

$\therefore$ Satellite A has greater speed.

S36. The objects on the Earth's surface are not in orbital motion w.r.t. the Earth. In order that an object has an orbital motion close to the Earth's surface, its orbital velocity, and period of motion must be

$$v_0 = \sqrt{gR}$$

$$T = \frac{2\pi R}{v_0} = 2\pi\sqrt{\frac{R}{g}} = 1.4 \text{ hrs.}$$

Therefore, the length of the day has to be 1.4 hrs in case in objects on the Earth's surface are in the true orbital motion like that of the Earth satellite.

S37. Here, $mg = 64$ N; $h = R/3$

The value of acceleration due to gravity at a height h (when h is not negligible as compared to R),

$$g' = g\frac{R^2}{(R+h)^2}$$

or $$mg' = mg\frac{R^2}{(R+h)^2} = 64 \times \frac{R^2}{(R+R/3)^2}$$

$$= 64 \times \frac{R^2 \times 9}{16R^2} = \mathbf{36\ N}.$$

S38. Here, escape velocity of the projectile,

$$v_e = 11.2 \text{ km s}^{-1}$$

The velocity with which the projectile is thrown.

$$v = 2v_e$$

Let m be the mass of the projectile and v_0, the velocity of the projectile after escaping the gravitational pull. Then, from the principle of conservation of energy, we have

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 - \frac{1}{2} m v_e^2$$

or $$v_0 = \sqrt{v^2 - v_e^2} = \sqrt{(2v_e)^2 - v_e^2}$$

$$= \sqrt{3v_e^2} = \sqrt{3 \times (11.2)^2} = \mathbf{19.4\ km\,s^{-1}}.$$

S39. Here, $R = 64000$ km $= 6.4 \times 10^6$ m; $g = 9.8$ ms^{-2}

For a satellite orbiting close to the surface of the Earth,

$$v = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7.92 \times 10^3 \text{ m s}^{-1}$$

$$= 7.92 \text{ km s}^{-1}$$

Also, $$T = 2\pi \sqrt{\frac{R}{g}} = 2\pi \sqrt{\frac{6.4 \times 10^6}{9.8}}$$

$$= \mathbf{5{,}077.6\,s}.$$

S40. Now, $g_{equa} = g - R\omega^2$

The body lying on equator will appear weightless, when g_{equa} become zero. Let ω' be the angular speed of the earth at which g_{equa} will become zero. Then,

$$0 = g - R\omega'^2 \quad \text{or} \quad R\omega'^2 = g$$

or $$\omega' = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{6{,}400 \times 1{,}000}}$$

$$= \mathbf{1.25 \times 10^{-3}\ rad\ s^{-1}}.$$

S41. When a body is projected up with a velocity equal to the escape velocity, it reaches a point at infinity. It, therefore, follows that if the body falls from infinity, it will acquire a velocity equal to the escape velocity on reaching the surface of the Earth.

Therefore, velocity of the body on reaching the surface of the Earth,

$$v = v_e = \sqrt{2gR}$$

Setting, $g = 9.8$ ms^{-2}; $R = 6{,}400$ km $= 6.4 \times 10^6$ m

$$v = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 \times 10^3 \text{ ms}^{-1}$$

$$= \mathbf{11.2\ km\,s^{-1}}.$$

Also K.E. of the body on reaching the Earth,

$$\frac{1}{2} m v^2 = \frac{1}{2} \times 100 \times (11.2 \times 10^3)^2 = \mathbf{6.272 \times 10^9\ J}.$$

S42. Initially, $$E_i = -\frac{GM_E m}{4R_E}$$

While finally

$$E_f = -\frac{GM_E m}{8R_E}$$

The change in the total energy is

$$\Delta E = E_f - E_i$$

$$= \frac{GM_E m}{8R_E} = \left(\frac{GM_E}{R_E^2}\right)\frac{mR_E}{8}$$

$$\Delta E = \frac{g\,mR_E}{8\,R_E} = \frac{9.81 \times 400 \times 6.27 \times 10^6}{8}$$

$$= 3.13 \times 10^9 \text{ J.}$$

The kinetic energy is reduced and it mimics ΔE, namely,

$$\Delta K = K_f - K_i = -3.13 \times 10^9 \text{ J.}$$

The change in potential energy is twice the change in the total energy, namely

$$\Delta V = V_f - V_i = -6.25 \times 10^9 \text{ J.}$$

S43. Given, $$k = 10^{-13}\,\text{s}^2\,\text{m}^{-3}$$

$$= 1.33 \times 10^{-14}\,d^2\,\text{km}^{-3}$$

Using Eq. (8.38) and the given value of k, the time period of the moon is

$$T^2 = (1.33 \times 10^{-14})(3.84 \times 10^5)^3$$

$$T = 27.3\,d.$$

S44. Yes.

A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $$f_g = \frac{GMm}{R^2}$$

Where,

M = Mass of the star

$= 2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$ kg

m = Mass of the body

R = Radius of the star

$= 12 \text{ km} = 1.2 \times 10^4 \text{ m}$

$$f_c = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30} \times m}{(1.2 \times 10^4)^2}$$

$= 2.31 \times 10^{11}\, m\,\text{N}$

Centrifugal force, $f_c = mr\omega^2$

ω = Angular speed = $2\pi\nu$

ν = Angular frequency = 1.2 rev s^{-1}

$f_c = mR\,(2\pi\nu)^2$

$= m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2$

$= 1.7 \times 10^5\, m\,\text{N}$

Since $f_g > f_c$, the body will remain stuck to the surface of the star.

S45. (a) The minimum speed required for an object to reach infinity (*i.e.*, to get escape from Earth) is called escape velocity.

(b) $$\frac{1}{2} m (v_i^2)_e = \frac{GmM_p}{h + R_p} \qquad \left[\begin{aligned} \text{where} \quad M_p &= \text{Mass of planet} \\ R_p &= \text{Radius of planet} \end{aligned}\right]$$

If the object is thrown from surface of a planet $h = 0$, we get

$$(v_i)_e = \sqrt{\frac{2GM_p}{R_p}}$$

but $g = \dfrac{GM_p}{R_p^2}$, we get

$$(v_i)_e = \sqrt{2gR_p}\,.$$

(c) Depends on location as 'g' varies with location, as most of the celestial bodies are not perfectly spherical.

S46. Let M and R be the mass and radius of the Earth. If m' and R' are the mass and radius of the Jupiter, then

$$M' = 318\,M \quad \text{and} \quad R' = 11.2\,\text{R}$$

Also, the escape velocity on the Earth v_e **and** v_e' be the escape velocity on the Jupiter.

Then, $$\frac{v_e'}{v_e} = \sqrt{\frac{2GM'}{R'}} \times \sqrt{\frac{R}{2GM}} = \sqrt{\frac{R}{R'} \times \frac{M'}{M}}$$

or $$v_e' = v_e \sqrt{\frac{R}{R'} \times \frac{M'}{M}} = 11.2 \times \sqrt{\frac{R}{11.2R} \times \frac{318\,M}{M}}$$

$$= 11.2 \times \sqrt{\frac{318}{11.2}} = \mathbf{59.68\ km\,s^{-1}}.$$

S47. Here, R = 6,400 km; x = 16 km

The value of acceleration due to gravity at a depth x,

$$g' = g\left(1 - \frac{x}{R}\right) = g\left(1 - \frac{16}{6,400}\right) = \frac{399}{400}g$$

$$g - g' = g - \frac{399}{400}g = \frac{1}{400}g = 2.5 \times 10^{-3}\,\text{g}$$

If m is mass of the body, then mg and mg' will be respectively the weight of the body on the surface of earth and at a depth of 16 km below in the surface of the Earth. Then,

% decrease in the weight of the body

$$= \frac{mg - mg'}{mg} \times 100 = \frac{g - g'}{g} \times 100$$

$$= \frac{2.5 \times 10^{-3}\,g}{g} \times 100 = \mathbf{0.25\%}$$

Now, at the centre of the Earth,

$$mg' = mg\left(1 - \frac{x}{R}\right) = mg\left(1 - \frac{R}{R}\right) = \mathbf{0}.$$

S48. (a) In an orbit of radius r, a satellite of mass m moves round a planet of mass M. Then,

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$$

where h is the height at which the satellite is from the surface.

For close to Earth orbits, $h = 0$

$$v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

Since $$v_e = \sqrt{2gR}, \quad v_0 = \frac{v_e}{\sqrt{2}}.$$

(b) Let M and R be the mass and radius of Earth and m and r that of the planet Mars, then

$$M = 9m \quad \text{and} \quad R = 2r$$

Now, $$v_e = \sqrt{\frac{2GM}{R}} \quad \text{and} \quad \sqrt{\frac{2Gm}{r}}$$

$$\therefore \quad \frac{v_m}{v_e} = \sqrt{\frac{2Gm}{r} \times \frac{R}{2GM}} = \sqrt{\frac{m}{M} \times \frac{R}{r}} = \sqrt{\frac{m}{9m} \times \frac{2r}{r}} = \sqrt{\frac{2}{9}} = 0.47$$

$$\therefore \quad v_m = 0.47 \times 11.2 = \mathbf{5.28\ km\ s^{-1}}.$$

S49. (a) Orbital velocity for close to Earth orbits = $\sqrt{gR}$. Escape velocity required = $\sqrt{2gR}$.

% Increase required

$$= \frac{v_e - v_0}{v_0} \times 100$$

v_e = Escape velocity
v_0 = Orbital velocity

$$= \frac{\sqrt{gR}\left(\sqrt{2}-1\right)}{\sqrt{gR}} \times 100 = \mathbf{41.4\%}.$$

(b) If v_e = 11.2 km s^{-1} is the velocity of escape, then

Velocity of projection = 2 v_e

If m is the mass of the projectile v its velocity far away from the Earth and v_e its velocity of escape from the gravitational pull, then from energy consideration

$$\frac{1}{2} m v^2 = \frac{1}{2} m (2v_e)^2 - \frac{1}{2} m v_e^2$$

or $$v^2 = 4 v_e^2 - v_e^2 = 3 v_e^2$$

$$\therefore \quad v = \sqrt{3} \times v_e = 1.73 \times 11.2 = \mathbf{20\ km\ s^{-1}}.$$

S50. (a) Speed of projection = 11200 m/s = 11.2 km/s. Since this is equal to the escape velocity, the mass thrown should escape from the surface of earth. Escape speed on the surface of a planet is given by,

$$v_e = \sqrt{2gR} \quad \Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}}$$

where M and R are the mass and radius of the planet.

$$M_1 : M_2 = 1 : 4$$

$$R_1 : R_2 = 2 : 1$$

$$\therefore \quad \frac{v_{e_1}}{v_{e_2}} = \sqrt{\frac{M_1}{M_2}\frac{R_2}{R_1}} = \sqrt{\frac{1}{4} \cdot \frac{2}{1}}$$

$$= \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

(b) If g' is the acceleration due to gravity 1000 km above the surface of Earth, then escape velocity v_e at that point.

$$v_e = \sqrt{2g'(R+h)}$$

Now, $$g' = \frac{gR^2}{(R+h)^2}$$

$\therefore$ $$v_e = \sqrt{\frac{2gR^2}{R+h}}$$

Now, $g = 9.8\ ms^{-2}$, $R = 6.4 \times 10^6$ m, $h = 100$ km $= 10^6$ m.

$\therefore$ $$v_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{(6.4 \times 10^6 + 10^6)}}$$

$$= 10.42 \times 10^3\ ms^{-1} = \mathbf{10.42\ km\,s^{-1}}.$$

S51. (a) $v \propto g^a R^b$

$$[LT^{-1}] = K[LT^{-2}]^a [L]^b$$

$$a = \frac{1}{2}, \quad b = \frac{1}{2}$$

$$K = \sqrt{2}$$

$$v = \sqrt{2gR}.$$

(b) Orbital velocity $$v_0 = \sqrt{\frac{GM}{R+h}}$$

Escape velocity $$v_e = \sqrt{\frac{2GM}{R+h}}$$

$$\frac{v_e}{v_0} = \sqrt{\frac{2GM}{R+h} \times \frac{R+h}{GM}} = 1.414$$

or $$\frac{v_e - v_0}{v_0} = 0.414$$

$\therefore$ $$\text{\% of increase} = \frac{v_e - v_0}{v_0} \times 100 = 0.414 \times 100 = \mathbf{41.4}.$$

S52. Weight of the body, $W = 63$ N

Acceleration due to gravity at height h from the Earth's surface is given by the relation:

$$g' = \frac{g}{\left(\frac{1+h}{R_e}\right)^2}$$

Where,

g = Acceleration due to gravity on the Earth's surface

R_e = Radius of the Earth

For $$h = \frac{R_e}{2}$$

$$g' = \frac{g}{\left(1+\frac{R_e}{2\times R_e}\right)^2} = \frac{g}{\left(1+\frac{1}{2}\right)^2} = \frac{4}{9}g$$

Weight of a body of mass m at height h is given as:

$$W' = \text{mg}'$$

$$= m\times\frac{4}{9}g = \frac{4}{9}\times\text{mg}$$

$$= \frac{4}{9}W$$

$$= \frac{4}{9}\times 63 = 28\,\text{N}.$$

S53. (a) We employ Eq. (8.38) with the Sun's mass replaced by the martian mass M_m

$$T^2 = \frac{4\pi^2}{GM_m}R^3$$

$$M_m = \frac{4\pi^2}{G}\frac{R^3}{T^2}$$

$$= \frac{4\times(3.14)^2\times(9.4)^3\times10^{18}}{6.67\times10^{-11}\times(4.59\times60)^2}$$

$$M_m = \frac{4\times(3.14)^2\times(9.4)^3\times10^{18}}{6.67\times(4.59\times6)^2\times10^{-5}}$$

$$= 6.48\times10^{23}\,\text{kg}.$$

(b) Once again Kepler's third law comes to our mind, *i.e.*, $T^2 \propto R^3$.

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where R_{MS} is the Mars-Sun distance and R_{ES} is the Earth-Sun distance.

$\therefore$ $$T_M = (1.52)^{3/2}\times 365$$

$$= 684 \text{ days}$$

We note that the orbits of all planets except Mercury, Mars and Pluto are very close to being circular. For example, the ratio of the semiminor to semi-major axis for our Earth is, $b/a = 0.99986$.

S54. Escape velocity of a projectile from the Earth, 11.2 km/s

Projection velocity of the projectile, $v_p = 3v_{esc}$

Mass of the projectile = m

Velocity of the projectile far away from the Earth = v_f

Total energy of the projectile on the Earth = $\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth = $\frac{1}{2}mv_f^2$

From the law of conservation of energy, we have

$$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{v_p^2 - v_{esc}^2}$$

$$= \sqrt{(3v_{esc})^2 - (v_{esc})^2}$$

$$= \sqrt{8}\, v_{esc}$$

$$= \sqrt{8} \times 11.2 = 31.68 \text{ km/s}$$

S55. Weight of a body of mass m at the Earth's surface, $W = mg = 250$ N

Body of mass m is located at depth, $d = \frac{1}{2}R_e$

Where,

R_e = Radius of the Earth

Acceleration due to gravity at depth

$$g' = \left(1 - \frac{d}{R_e}\right)g$$

$$= \left(1 - \frac{R_e}{2 \times R_e}\right)g = \frac{1}{2}g$$

Weight of the body at depth d,

$$W' = mg$$

$$= m \times \frac{1}{2}g = \frac{1}{2}mg = \frac{1}{2}W$$

$$= \frac{1}{2} \times 250 = 125\,N$$

S56. From Eq. (8.12) we have

$$M_E = \frac{g R_E^2}{G}$$

$$= \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= 5.97 \times 10^{24}\,\text{kg.}$$

The moon is a satellite of the Earth. From the derivation of Kepler's third law [see Eq. (8.38)]

$$T^2 = \frac{4\pi^2 R^3}{GM_E}$$

$$M_E = \frac{4\pi^2 R^3}{GT^2}$$

$$= \frac{4 \times 3.14 \times 3.14 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$$= 6.02 \times 10^{24}\,\text{kg}$$

Both methods yield almost the same answer, the difference between them being less than 1%.

S57. Given, 8×10^6 m from the centre of the Earth

Velocity of the rocket, $v = 5$ km/s $= 5 \times 10^3$

Mass of the Earth, $M_e = 6.0 \times 10^{24}$ kg

Radius of the Earth, $R_e = 6.4 \times 10^6$ kg

Height reached by rocket mass, $= h$

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e}\right)$$

At highest point h,

$$v = 0$$

And, potential energy = $\dfrac{GM_e m}{R_e + h}$

$$= 0 + \left(\frac{GM_e m}{R_e + h}\right) = \frac{GM_e m}{R_e + h}$$

Total energy of the rocket

From the law of conservation of energy, we have

Total energy of the rocket at the Earth's surface = Total energy at height *h*

$$\frac{1}{2}mv^2 + \left(-\frac{GM_e m}{R_e}\right) = -\frac{GM_e m}{R_e + h}$$

$$\frac{1}{2}v^2 = GM_e\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$$

$$= GM_e\left(\frac{R_e + h - R_e}{R_e(R_e + h)}\right)$$

$$\frac{1}{2}v^2 = \frac{GM_e h}{R_e(R_e + h)}$$

$$\frac{1}{2}\times v^2 = \frac{gR_e h}{R_e + h} \qquad \{\because\ GM_e = gR_e^2\}$$

$\therefore$ $$v^2(R_e + h) = 2gR_e h$$

$$v^2 R_e = h(2gR_e - v^2)$$

Setting the value $$h = \frac{R_e v^2}{2gR_e - v^2}$$

$$= \frac{6.4\times 10^6 \times (5\times 10^3)^2}{2\times 9.8\times 6.4\times 10^6 - (5\times 10^3)^2}$$

$$h = \frac{6.4\times 25\times 10^{12}}{100.44\times 10^6} = 1.6\times 10^6 \text{m}$$

Height achieved by the rocket with respect to the centre of the Earth

$$= R_e + h$$

$$= 6.4 + 10^6 + 1.6\times 10^6$$

$$= 8.0 \text{ x } 10^6 \text{ m}$$

S58. Given, Mass of the Earth, $M_e = 6.0\times 10^{24}$ kg

Mass of the satellite, $m = 200$ kg

Radius of the Earth, $R_e = 6.4\times 10^6$ m

Universal gravitational constant, $G = 6.67\times 10^{-11}$ Nm^{2}kg^{-2}

Height of the satellite, $h = 400$ km $= 4\times 10^5$ m $= 0.4\times 10^6$ m

Total energy of the satellite at height $h = \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$

Orbital velocity of the satellite, $v = \sqrt{\dfrac{GM_e}{R_e + h}}$

Total energy of height, $h = \dfrac{1}{2}m\left(\dfrac{GM_e}{R_e + h}\right) - \dfrac{GM_e m}{R_e + h} = -\dfrac{1}{2}\left(\dfrac{GM_e m}{R_e + h}\right)$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

Energy required to send the satellite out of its orbit = – (Binding energy)

$$= \frac{1}{2}\frac{GM_e \text{m}}{(R_e + h)}$$

$$= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{2(6.4 \times 10^6 + 0.4 \times 10^6)}$$

$$= \frac{6.67 \times 6 \times 2 \times 10^{15}}{2(6.8 \times 10^6)} = 5.9 \times 10^9 \text{J}$$

$$= 6 \times 10^9 \text{ J.}$$

S59. Given, Mass of the spaceship, m_s = 1000 kg

Mass of the Sun, $M_s = 2 \times 10^{30}$ kg

Mass of Mars, $m_m = 6.4 \times 10^{23}$ kg

Orbital radius of Mars, $R = 2.28 \times 10^8$ kg $= 2.28 \times 10^{11}$m

Radius of Mars, r = 3395 km = 3.395×10^6 m

Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^{2}kg^{-2}

Potential energy of the spaceship due to the gravitational attraction of the Sun = $\dfrac{-\text{G}Mm_s}{R}$

Potential energy of the spaceship due to the gravitational attraction of Mars = $\dfrac{-\text{G}M_m m_s}{R}$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic zero.

Total energy of the spaceship $\dfrac{-\text{G}Mm_s}{R} - \dfrac{-\text{G}M_s m_m}{r}$

$$= -Gm_s\left(\frac{M}{R} + \frac{m_m}{r}\right)$$

The negative sign indicates that the system is in bound state.

Energy required for launching the spaceship out of the solar system

= – (Total energy of the spaceship)

$$= Gm_s\left(\frac{M}{R}+\frac{m_m}{r}\right)$$

$$= 6.67\times10^{-11}\times10^{3}\times\left(\frac{2\times10^{30}}{2.28\times10^{11}}+\frac{6.4\times10^{23}}{3.395\times10^{6}}\right)$$

$$= 6.67\times10^{-8}\,(87.72\times10^{17} + 1.88\times10^{17})$$

$$= 6.67\times10^{-8}\times89.50\times10^{17}$$

$$= 596.97\times10^{9}$$

$$= 6\times10^{11}\text{J}$$

S60. Given, Initial velocity of the rocket, $v = 2$ km/s $= 2\times10^3$ m/s

Mass of Mars, $M = 6.4\times10^{23}$ kg

Radius of Mars, $R = 3395$ km $= 3.395\times10^6$ m

Universal gravitational constant, $G = 6.67\times10^{-11}$ N m^2 kg^{-2}

Mass of the rocket = m

Initial kinetic energy of the rocket = $\frac{1}{2}mv^2$

Initial potential energy of the rocket = $\frac{-GMm}{R}$

Total initial energy = $\frac{1}{2}mv^2 - \frac{GMm}{R}$.

If 20 % of initial kinetic energy is lost due to Martian atmospheric resistance, then only 80 % of its kinetic energy helps in reaching a height.

Total initial energy available = $\frac{80}{100}\times\frac{1}{2}mv^2 - \frac{GMm}{R} = 0.4\,mv^2 - \frac{GMm}{R}$

Maximum height reached by the rocket = h

At this height, the velocity and hence, the kinetic energy of the rocket will become zero.

Total energy of the rocket at height,

$$= \frac{GMm}{(R+h)}$$

Applying the law of conservation of energy for the rocket, we can write:

$$0.4\,mv^2 - \frac{GMm}{R} = \frac{-GMm}{(R+h)}$$

$$0.4v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$= GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= GM\left(\frac{R+h-R}{R(R+h)}\right)$$

$$0.4\,v^2 = \frac{GMh}{R(R+h)}$$

$$\frac{R+h}{h} = \frac{GM}{0.4\,v^2R}$$

$$\frac{R}{h} + 1 = \frac{GM}{0.4v^2R}$$

$$\frac{R}{h} = \frac{GM}{0.4v^2R} - 1$$

$$h = \frac{R}{\frac{GM}{0.4v^2R} - 1}$$

$$= \frac{0.4R^2v^2}{GM - 0.4v^2R}$$

$$= \frac{0.4 \times (3.395 \times 10^6)^2 \times (2 \times 10^3)^2}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)^2}$$

$$= \frac{18.442 \times 10^{18}}{42.688 \times 10^{12} - 5.432 \times 10^{12}} = \frac{18.442}{37.256} \times 10^6$$

$$= 495 \times 10^3 \text{ m} = 495 \text{ km.}$$

Date: 13/10/2021

Q1. **Why does a person in a satellite feels weightlessness?**

Q2. **Explain, why a body weigh's more at poles and less at equator.**

Q3. **Why weight of a body becomes zero at the centre of earth?**

Q4. **A high jumper can jump 1.5 m on earth. With the same effort, how high will he be able to jump on a planet whose density is one-third and radius is one-fourth of that of the earth?**

Q5. **If the radius of the earth were increased by a factor of 3, by what factor would its density have to be change to keep '*g*' the same?**

Q6. **If the earth has a mass nine times and radius twice that of the planet Mars, calculate the maximum velocity required by a rocket to pull out of the gravitational force of Mars. Given escape velocity on the surface of earth is 11.2 km/sec.**

Q7. **Discuss the variation of '*g*'**
(a) due to height (b) due to depth.

Q8. **The radii of two planets are R and $2R$ respectively and their densities ρ and $\rho/2$ respectively. What is the ratio of acceleration due to gravity at their surfaces?**

Q9. **How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.**

Q10. **A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun?**

Q11. **Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centers of the spheres? Is an object placed at that point in equilibrium stable or unstable?**

Date: 13/10/2021

S1. Since no reaction is experienced by the person, he feels weightlessness.

S2. As the value of g is more at the poles than at the equator, the body weighs more at the poles.

S3. The weight of a body is the force with the body is attracted towards the centre of earth. Obviously, when the body reaches the centre of the earth, it will no longer be attracted and hence its weight becomes zero at the centre of the earth.

S4. Let h_e be the height in metre, the man jumps on the earth and h_p on the planet. If the effort is same, the P.E. gained is same. Therefore,

$$mg_p h_p = mg_e h_e$$

$$h_p = \frac{g_e h_e}{g_p}$$

We know,

$$g_e = \frac{GM_e}{R_e^2} = G\frac{4}{3}\pi R_e \rho_e$$

$$\frac{g_e}{g_p} = \frac{R_e \rho_e}{R_p \rho_p} = 12$$

$$h_p = 12 \times h_e = 12 \times 1.5 = 18\text{m}$$

S5.

$$g = \frac{GM}{R^2}$$

Let ρ be the density of earth

$$\rho = \frac{M}{\text{Volume of earth}} = \frac{M}{\frac{4}{3}\pi R^3}$$

$$M = \frac{4}{3}\pi R^3 \rho$$

$$g = G \times \frac{4}{3}\pi G \rho R, \qquad g = \frac{GM}{R^2}$$

Since $\frac{4}{3}\pi, G$ are constants.

For no change in value of 'g', $R \propto \frac{1}{\rho}$ Hence, ρ should be decrease by a factor of 3.

S6.

$$\text{Escape velocity} = \sqrt{\frac{2GM}{R_P^2}}$$

where M_p is the mass of the planet and R_p is the radius of the planet.

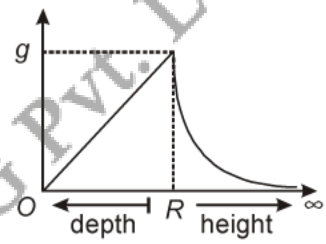
$\therefore$ Escape velocity on Mars

$$= \sqrt{\frac{2 \times G \times 4M_e R_e}{9R_e^2}}$$

$$= \sqrt{\frac{4}{9}}\sqrt{\frac{2GM}{R_e^2}} = \sqrt{\frac{4}{9}}v_e$$

$$= \sqrt{\frac{4}{9}} \times 11.2 = 7.47 \text{ km/sec}$$

S7. (a) $g = g\left(1 - \frac{2k}{R}\right)$ (b) $g' = g\left(1 - \frac{d}{R}\right)$



The value of g decreases both on moving up and on moving down from the surface of earth. This can be shown graphically as above.

S8.

$$g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2} = G\frac{4}{3}\pi R\rho$$

$$\therefore \quad \frac{g_1}{g_2} = \frac{R}{2R} \cdot \frac{\rho}{\rho/2} = 1$$

$$\therefore \quad g_1 : g_2 = 1 : 1$$

S9. Orbital radius of the Earth around the Sun, $r = 1.5 \times 10^{11}$ m

Time taken by the Earth to complete one revolution around the Sun,

$$T = 1 \text{ year} = 365.25 \text{ days}$$

$$= 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Universal gravitational constant, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

Thus, mass of the Sun can be calculated using the relation,

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$= \frac{133.24 \times 10^{33}}{6.64 \times 10^{4}} = 2.0 \times 10^{30} \text{ kg}$$

Hence, the mass of the Sun is 2×10^{30} kg.

S10. Distance of the Earth from the Sun, $r_e = 1.5 \times 10^8$ kg = 1.5×10^{11} m

Time period of the Earth = T_e

Time period of Saturn, $T_s = 29.5\ T_e$

Distance of Saturn from the Sun = r_s

From Kepler's third law of planetary motion, we have

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

For Saturn and Sun, we can write

$$\left(\frac{r_s}{r_e}\right)^3 = \left(\frac{T_s}{T_e}\right)^2$$

$$r_s = r_e \left(\frac{T_s}{T_e}\right)^{\frac{2}{3}}$$

$$= 1.5 \times 10^{11} \left(\frac{29.5\,T_e}{T_e}\right)^{\frac{2}{3}}$$

$$= 1.5 \times 10^{11} (29.5)^{\frac{2}{3}}$$

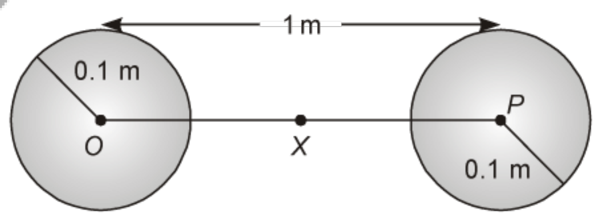
$$= 1.5 \times 10^{11} \times 9.55$$

$$= 14.32 \times 10^{11} \text{ m}$$

S11. Gravitational force is 0; and gravitational potential is -2.7×10^{-8} J/kg;

Yes, it is Unstable.

Explanation: The situation is represented in the given figure:



Mass of each sphere, $M = 100$ kg

Separation between the spheres, $r = 1$ m

X is the mid point between the spheres. Gravitational force at point X will be zero. This is because gravitational force exerted by each sphere will act in opposite directions.

Gravitational potential at point X:

$$\frac{-GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)} = -4\frac{GM}{r}$$

$$= -\frac{4 \times 6.67 \times 10^{-11} \times 100}{1}$$

$$= -2.67 \times 10^{-8}\ \text{J/kg}.$$

Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.