Date: 13/10/2021

Q1. A unit vector is represented by $a\hat{i} + b\hat{j} + c\hat{k}$. If the values of *a* and *b* are 0.6 and 0.8 respectively, find the value of *c*.

Q2. Find the angle between $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$.

Q3. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, what is the angle between $\vec{A}$ and $\vec{B}$?

Q4. If $\vec{A} \times \vec{B} = \vec{A} . \vec{B}$, what is the angle between $\vec{A}$ and $\vec{B}$?

Q5. What is the value of *m* in $\hat{i} + m\hat{j} + \hat{k}$ to be unit vector?

Q6. A river 200 m wide is flowing at the rate of 1 km h^{-1}. A swimmer who can swim at the rate of 2 km h^{-1} wants to reach a point just opposite to his starting point. In what time the swimmer will cross the river?

Q7. What is the magnitude of the component of $9\hat{i} - 7\hat{j} + 13.9\hat{k}$ along *x*-axis?

Q8. For what angle between $\vec{P}$ and $\vec{Q}$, the value of $\vec{Q} + \vec{P}$ is maximum?

Q9. What is the minimum number of coplanar vectors of different magnitudes which can give zero resultant?

Q10. What is the angle between $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$?

Q11. Two vectors $5\hat{i} + 7\hat{j} - \hat{k}$ and $2\hat{i} + 2\hat{j} - a\hat{k}$ are mutually perpendicular. What is the value of *a*?

Q12. Under what condition $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$, holds good?

Q13. Define the unit vector.

Q14. Given : $\vec{A} + \vec{B} = \vec{C}$. The magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 3, 4 and 5 respectively. What is the angle between $\vec{A}$ and $\vec{B}$? Also what is the angle between $\vec{A}$ and $\vec{C}$?

Q15. The magnitude of the resultant of two vectors of magnitudes 4 and 3 is 1. What is the angle between the two vectors?

Q16. Given: Four forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_4}$ such that $\vec{F_1} = 3\hat{i} - \hat{j} + 9\hat{k}$, $\vec{F_2} = 2\hat{i} - 2\hat{j} + 16\hat{k}$, $\vec{F_3} = 9\hat{i} + \hat{j} + 18\hat{k}$ and $\vec{F_4} = \hat{i} + 2\hat{j} - 18\hat{k}$. If all the forces simultaneously act on a particle at rest at the origin of the co-ordinate system, then the particle would begin to move in plane. Identify that plane.

Q17. Two vectors of magnitudes 2 and 3 give a resultant of magnitude 5. What is the angle between the two vectors?

Q18. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$ and its magnitude is half that of $\vec{Q}$. What is the angle between $\vec{P}$ and $\vec{Q}$?

Q19. Find the change in velocity of a yatch if it changes its velocity from 5 ms^{-1} due north to 3 ms^{-1} due west.

Q20. To a man walking due east at the rate of 2 km h^{-1}, rain appears to fall vertically. When he increases his speed to 4 km h^{-1}, it appears to meet him at an angle of 45°. Find the real direction and speed of rain.

Q21. One of the rectangular components of an acceleration of 8 ms^{-2} is 4 ms^{-2}. Calculate the other component.

Q22. The greatest and least resultant of two forces acting at a point is 10 N and 6 N respectively. If each force is increased by 3 N, find the resultant of new forces when acting at a point at an angle of 90° with each other.

Q23. Two forces whose magnitudes are in the ratio of 3 : 4 give a resultant of 35 N. If the angle of inclination be 60°, calculate the magnitude of each force.

Q24. Two forces of 5 kgf and 10 kgf are acting at an angle of 120°. Calculate the magnitude and direction of the resultant force.

Q25. Show that the vectors $2\hat{i} - 3\hat{j} + 3\hat{k}$ and $-6\hat{i} + 9\hat{j} + 13\hat{k}$ are perpendicular to each other.

Q26. Given: $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$. What is the vector component of $\vec{A}$ in the direction of $\vec{B}$?

Q27. Given: $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} + 6\hat{k}$. Calculate the angle made by $(\vec{A} + \vec{B})$ with x-axis?

Q28. If $\vec{A} = 4\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{B} = -2\hat{i} - 5\hat{j} + 7\hat{k}$, find the angle between the vectors $\vec{A}$ and $\vec{B}$.

Q29. A force $\vec{F} = \hat{i} + 5\hat{j} + 7\hat{k}$ acts on a particle and displaces it through $\vec{S} = 6\hat{i} + 9\hat{k}$. Calculate the work done if the force is in newton and distance is in metre.

Q30. A plane is travelling eastward at a speed of 500 km h^{-1}. But a 90 km h^{-1} wind is blowing southward. What is the speed of the plane relative to the ground?

Q31. Vector addition is different from scalar addition. Explain.

Q32. Is a quantity which has a magnitude and direction always a vector? Give examples.

Q33. If two vectors of equal magnitude add to either of them by magnitude, what is the angle between them?

Q34. Given $\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$. Which of the following statements are correct?

(a) $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ must each be a null vector.

(b) The magnitude of $(\vec{a} + \vec{c})$ equals the magnitude of $(\vec{b} + \vec{d})$.

(c) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$.

(d) $\vec{b} + \vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$, $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear?

Q35. Which of the following quantities are independent of the choice of orientation of the co-ordinate axes: $a + b$, $3a_x + 2b_y$, $|a + b + c|$, angle between b and c.

Q36. State polygon law of vectors and show that it can be deduced from triangle law of vectors.

Q37. State parallelogram law of vector addition. What is the effect on the magnitude of the resultant of two vectors when the angle between two vectors is increased from 0° to π?

Q38. If $\vec{R} = \vec{A} - \vec{B}$, show that $R^2 = A^2 + B^2 - 2AB\cos\theta$,

Where θ is the angle between $\vec{A}$ and $\vec{B}$.

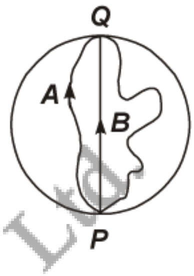
Q39. What is the essential condition for adding the vectors?

Q40. (a) Which of the following quantities are independent of the choice of orientation of the coordinate axes: $\vec{A}+\vec{B}$, $|\vec{A}+\vec{B}-\vec{C}|$ angle between $\vec{A}$ and $\vec{C}$ and $\lambda\vec{A}$, where λ is scalar.

(b) Giving reasons state whether the following statement is true or not:

The average speed of a particle is always greater than or equal to the magnitude of the average velocity, when both are measured in the same time interval.

Q41. Three girls skating on a circular ice ground of radius 200 m start from a point *P* on the edge of the ground and reach a point *Q* diametrically opposite to *P* following different paths as shown in figure. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of the path skated?



Q42. Are the commutative and associative laws applicable to vector subtraction?

Q43. Discuss the problem of a swimmer who wants to cross the river in the shortest time.

Q44. State parallelogram law of vector addition. Show that resultant of two vectors *A* and *B* inclined at an angle θ is $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$.

Q45. We know that dot product of two equal vectors is equal to the square of the magnitude of either of the two vectors.
Use this property to derive the relation $v^2 - u^2 = 2\vec{a}\cdot\vec{S}$ where the letters have usual meanings.

Q46. The position of a particle is given by $\vec{r} = 9t\hat{i} + 6t^2\hat{j} + 8\hat{k}$, where *t* is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres (a) Find velocity $\vec{v}(t)$ of particle and $\vec{a}(t)$ acceleration of the particle. (b) Find the magnitude and direction of $\vec{v}(t)$ at $t = 2$ sec.

Q47. Define the dot product of two vectors. Determine a unit vector which is perpendicular to both $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

Q48. What is unit vector? Find a unit vector parallel to the vector $3\hat{i} + 7\hat{j} + 4\hat{k}$.

Q49. There are two displacement vectors, one of magnitude 3 m and other of magnitude 4 m. How should the two vectors be added so that the magnitude of resultant vector be (a) 7 m (b) 1 m and (c) 5 m?

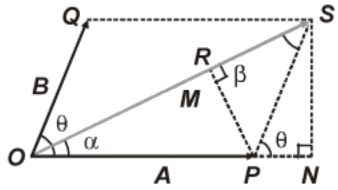
Q50. (a) Define dot product of two vectors and give its geometrical interpretation.

(b) The magnitude of the resultant of two vectors of magnitudes 4 and 3 is 1. What is the angle between the two vectors?

Q51. Write parallelogram law of vector addition. Define the terms resulant or equivalent of two forces. Two forces F_1 and F_2 acting at an angle θ on a body simultaneously have a resultant *F*. Show that

$$\theta = \cos^{-1}[(F_1^2 - F_2^2)/2F_1F_2]$$

Q52. Find the magnitude and direction of the resultant of two vectors *A* and *B* in terms of their magnitudes and angle θ between them.



Q53. Read each statement below carefully and state, with reasons and examples, if it is TRUE or FALSE:

A scalar quantity is one that

(b) is conserved in a process.

(c) can never take negative values.

(d) must be dimensionless.

(d) does not vary from one point to another in space.

(e) has the same value for observers with different orientations of axes.

Q54. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Q55. State the triangle law. Determine the direction and magnitude of the resultant of the following velocities impressed on a particle, 8 ms^{-1} to south, 12 ms^{-1} to east and $3\sqrt{2}$ ms^{-1} to north-east.

Q56. Establish the following vector inequalities geometrically or otherwise:

(a) $|\boldsymbol{a} - \boldsymbol{b}| \le |\boldsymbol{a}| + |\boldsymbol{b}|$;

(b) $|\boldsymbol{a} - \boldsymbol{b}| \ge ||\boldsymbol{a}| - |\boldsymbol{b}||$

When does the equality sign above apply?

Q57. Establish the following vector inequalities geometrically or otherwise:

(a) $|\boldsymbol{a} + \boldsymbol{b}| \le |\boldsymbol{a}| + |\boldsymbol{b}|$;

(b) $|\boldsymbol{a} + \boldsymbol{b}| \ge ||\boldsymbol{a}| - |\boldsymbol{b}||$

When does the equality sign above apply?

Q58. State the parallelogram law. The diagonals of a parallelogram are given by the vectors $(3\hat{i} + \hat{j} + 2\hat{k})$ and $(\hat{i} - 3\hat{j} + 4\hat{k})$. Find the area of the parallelogram.

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S1. Given, a unit vector; $a\hat{i} + b\hat{j} + c\hat{k}$, a = 0.6 and b = 0.8

$$\sqrt{a^2 + b^2 + c^2} = 1$$

$$\sqrt{(0.6)^2 + (0.8)^2 + c^2} = 1$$

$$1 + c^2 = 1$$

$$c = 0.$$

S2.

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1+2+2}{\sqrt{6}\sqrt{6}} = \frac{5}{6}$$

$\therefore$ $$\theta = \cos^{-1}\left(\frac{5}{6}\right).$$

S3.

$$\sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$4AB\cos\theta = 0, \qquad \cos\theta = 0,$$

$$\theta = \pi/2.$$

S4.

$$AB\sin\theta = AB\cos\theta$$

$\therefore$ $$\tan\theta = 1 \qquad \Rightarrow \qquad \theta = 45°.$$

S5. For unit vector

$$\left|\hat{i} + m\hat{j} + \hat{k}\right| = \sqrt{1 + m^2 + 1} = 1$$

i.e., $$m^2 + 2 = 1$$

$\therefore$ $$m^2 = -1 \text{ (} m \text{ is imaginary)}.$$

S6. Given: width of river b = 200 m

$$v_r = 1\,\text{km/h} \quad \text{and} \quad v_s = 2\,\text{km/h}$$

$$v = \sqrt{v_s^2 - v_r^2} = \sqrt{2^2 - 1^2} = \sqrt{3}\,\text{km/h},$$

$$t = \frac{\text{Displacement}}{\text{Velocity}} = \frac{0.2\,\text{km}}{\sqrt{3}\,\text{km h}^{-1}} = \mathbf{0.115\ hr}.$$

S7. $9\hat{i}$ represent x-axis component vector.

Hence magnitude is 9.

S8.
$$|\vec{P}+\vec{Q}| = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

Clearly $|\vec{P}+\vec{Q}|$ is maximum when $\cos\theta = 1$ *i.e.*, when $\theta = 0°$.

S9. Three. If three vectors can be represented by the three sides of a triangle taken in the same order, then the resultant is zero.

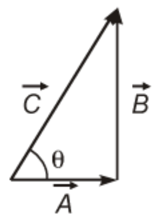
S10. Let,
$$\vec{A} = \hat{i}+\hat{j} \quad \text{and} \quad \vec{B} = \hat{i}-\hat{j}$$
$$\cos\theta = \frac{A.B}{|A||B|}$$
$$= \frac{(\hat{i}+\hat{j})(\hat{i}-\hat{j})}{(\sqrt{2})(\sqrt{2})} = \frac{0}{2} = 0$$
$$\theta = 90°.$$

S11. Let
$$\vec{A} = 5\hat{i}+7\hat{j}-\hat{k}$$
and
$$\vec{B} = 2\hat{i}+2\hat{j}-a\hat{k}$$
$$(\vec{A}).(\vec{B}) = 0.$$
$$(5\hat{i}+7\hat{j}-\hat{k})(2\hat{i}+2\hat{j}-a\hat{k}) = 0$$
$$10 + 14 + a = 0$$
$$a = -24.$$

S12. When $\vec{A}$ and $\vec{B}$ are both parallel.

S13. ***Unit vector*** is vector having unit magnitude. It is used to denote the direction of a given vector.

S14. Given: $\vec{A} = 3$, $\vec{B} = 4$ and $\vec{C} = 5$



$$C = \sqrt{A^2+B^2} = \sqrt{3^2+4^2}$$
$$\therefore \quad \vec{A} \perp \vec{B}$$
Also,
$$\cos\theta = \frac{3}{5} = 0.6$$
or
$$\theta = \cos^{-1}(0.6) = 53.13°$$

S15. Let
$$\vec{A} = 4 \quad \text{and} \quad \vec{B} = 3, \quad R = 1$$
$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$
$$1 = \sqrt{(4)^2+(3)^2+2\times 4\times 3\cos\theta}$$

$$1 = 25 + 24\cos\theta$$

$$24\cos\theta = -24$$

$$\cos\theta = -1$$

$$\theta = 180°.$$

S16. Resultant force,

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$= (3\hat{i} - \hat{j} + 9\hat{k}) + (2\hat{i} - 2\hat{j} + 16\hat{k}) + (9\hat{i} + \hat{j} + 18\hat{k}) + (\hat{i} + 2\hat{j} - 18\hat{k})$$

$$= 15\hat{i} + 0\hat{j} + 25\hat{k}$$

Clearly, the particle shall move in the $X - Z$ plane.

S17. Let $\vec{A} = 2$, $\vec{B} = 3$ and $R = 5$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$5 = \sqrt{(2)^2 + (3)^2 + 2 \times 2 \times 3\cos\theta}$$

$$25 = 13 + 12\cos\theta$$

$$12 = 12\cos\theta$$

$$\cos\theta = 1$$

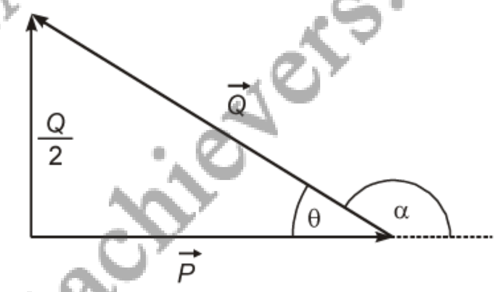
$$\theta = 0°.$$

S18.

$$Q^2 = P^2 + \frac{Q^2}{4}$$

or $$P^2 = \frac{3}{4}Q^2$$

or $$P = \frac{\sqrt{3}}{2}Q$$



Now, $\tan\theta = \dfrac{Q/2}{\frac{\sqrt{3}}{2}Q} = \dfrac{1}{\sqrt{3}}$ or $\theta = 30°$

Required angle $\alpha = 180° - 30° = 150°$.

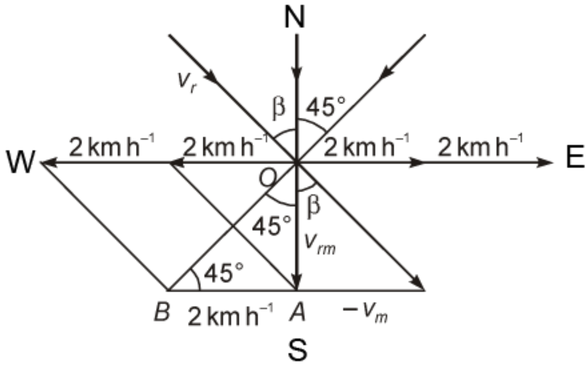
S19.

$$\vec{v} = \vec{v_A} - \vec{v_B}$$

Change in velocity $= \sqrt{(5)^2 - (3)^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$\therefore$ $v = 4$ m/sec North-East.

S20. Given: $v_m = 2\ \text{km h}^{-1}$, $v'_m = 4\ \text{km h}^{-1}$



In ΔOAB, $OA = AB = 2\text{km h}^{-1}$

$$v_m = 2\ \text{km h}^{-1};\quad v_r^2 = 2^2 + 2^2 = 8$$

$$v_r = \sqrt{8}\ \text{km h}^{-1} = 2.828\ \text{km h}^{-1}$$

$$\tan\beta = \frac{2}{2} = 1,\quad \beta = 45°.$$

S21. Given: Resultant acceleration $a = 8\ \text{m sec}^{-2}$

Horizontal component $a_x = 4\ \text{m sec}^{-2}$

Now,
$$a = \sqrt{(a_x)^2 + (a_y)^2}$$

$$8^2 = 4^2 + a_y^2$$

$$a_y = \sqrt{\mathbf{48}}\ \mathbf{ms^{-2}}.$$

S22. Greatest resultant force is given by

$$P + Q = 10\,\text{N} \qquad \text{... (i)}$$

Smallest resultant force is given by

$$P - Q = 6\,\text{N} \qquad \text{... (ii)}$$

Now solve the Eq. (i) and (ii), we get

$$P = 8\ \text{N} \quad \text{and} \quad Q = 2\,\text{N}$$

Now new forces after increasing by 3 N

$$P' = 8 + 3 = 11\,\text{N},$$

$$Q' = 2 + 3 = 5\,\text{N},\quad \theta = 90°$$

$$R' = \sqrt{121+25}\,\text{N} = \sqrt{146}\,\text{N} = 12.01\,\text{N} \simeq 12\,\text{N}$$

$$\tan\beta = \frac{5\sin 90°}{11+5\cos 90°} = \frac{5}{11} = 0.545$$

or
$$\beta = \tan^{-1}(0.4545) = 14°26'.$$

S23. Given: Resultant force $R = 35$ N and angle between forces is $(\theta) = 60°$

Let common factor between two forces is x

Now, Force $F_1 = 3x$, $F_2 = 4x$

$$R^2 = (F_1)^2 + (F_2)^2 + 2F_1F_2\cos\theta$$

$$35^2 = 9x^2 + 25x^2 + 30x^2\cos 60°$$

$$35^2 = 49x^2 \quad\Rightarrow\quad x = \frac{35}{7} = 5.$$

Force $F_1 = 3\times 5 =$ **15 N** and $F_2 = 4\times 5 =$ **20 N**.

S24. Let force $F_1 = 5$ kgf, $F_2 = 10$ kgf and $\theta = 120°$

Now, resultant force $R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2\cos\theta}$

$$R^2 = 5^2 + 10^2 + 2 \times 5 \times 10 \cos 120° = 75$$

$$R = \sqrt{75} \text{ kgf} = 8.66 \text{ kgf}$$

$$\tan\beta = \frac{10\sin 120°}{5 + 10\cos 120°}$$

$$= \frac{10\sin 120°}{0} = \infty \;;\quad \beta = 90°.$$

S25. Let $\vec{A} = 2\hat{i} - 3\hat{j} + 3\hat{k}$ and $\vec{B} = -6\hat{i} + 9\hat{j} + 13\hat{k}$

We know $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

$$= \frac{(2\hat{i} - 3\hat{j} + 3\hat{k})(-6\hat{i} + 9\hat{j} + 13\hat{k})}{\sqrt{(2)^2 + (-3)^2 + (-3)^2}\sqrt{(-6)^2 + (9)^2 + (13)^2}}$$

$$= \frac{-12 - 27 + 39}{\sqrt{22}\sqrt{286}}$$

$$\cos\theta = 0$$

$$\theta = 90°.$$

S26. Component of $\vec{A}$ in the direction of B **is** $A\cos\theta$

$$\vec{B} = A\cos\theta = \frac{AB\cos\theta}{B} = \frac{\vec{A}\cdot\vec{B}}{B}$$

Vector component of $\vec{A}$ in the direction of $\vec{B}$

$$= \frac{\vec{A}\cdot\vec{B}}{B}\hat{B} = \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\frac{\vec{B}}{B}.$$ **[Multilpying and dividing by $|\vec{B}|$]**

S27. Given: $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} + 6\hat{k}$

$$\vec{A} + \vec{B} = (\hat{i} - 2\hat{j} - 3\hat{k}) + (4\hat{i} - 2\hat{j} + 6\hat{k})$$

$$= 5\hat{i} - 4\hat{j} + 3\hat{k}$$

$$(\vec{A} + \vec{B})\cdot\hat{i} = |\vec{A} + \vec{B}||\hat{i}|\cos\theta$$

or $\cos\theta = \dfrac{(\vec{A} + \vec{B})\cdot\hat{i}}{|\vec{A} + \vec{B}||\hat{i}|}$

or $\cos\theta = \dfrac{5}{\sqrt{25+16+9}} = \dfrac{1}{\sqrt{2}}$, $\theta = 45^\circ$

S28. Given: $\vec{A} = 4\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{B} = -2\hat{i} - 5\hat{j} + 7\hat{k}$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

$$= \frac{(4\hat{i} + 6\hat{j} - 3\hat{k})\cdot(-2\hat{i} - 5\hat{j} + 7\hat{k})}{\sqrt{4^2 + 6^2 + (-3)^2}\sqrt{(-2)^2 + (-5)^2 + 7^2}}$$

or $\cos\theta = \dfrac{-8 - 30 - 21}{\sqrt{61}\sqrt{78}} = -\dfrac{59}{68.978} = -0.855$

or $\theta =$ **148.8°**.

S29. Given: Force $\vec{F} = (\hat{i} + 5\hat{j} + 7\hat{k})$

Displacement $\vec{S} = (6\hat{i} + 9\hat{k})$

Now work done $W = \vec{S}\,.\,\vec{F}$

$= (\hat{i} + 5\hat{j} + 7\hat{k})\,.\,(6\hat{i} + 9\hat{k}) = 6 + 63 =$ **69 J**.

S30. Given: $v_p = 500$ kmh^{-1} and $v_r = 90$ km h^{-1}

$$v_{pw} = \sqrt{(v_p)^2 + (v_r)^2}\ \text{km h}^{-1}$$

$$v_{pw} = \sqrt{500^2 + 90^2}\ \text{km h}^{-1} = 508\ \text{km h}^{-1}.$$

S31. In scalar addition, one has to add the magnitudes of scalars. Since the vectors possess direction in addition to their magnitudes, the scalar addition cannot be used to add two vectors. For this reason, the vector addition is different from the scalar addition

S32. No. A quantity which possesses both magnitude and direction may not be a vector. For example, electric current, finite angular displacement, etc. For a quantity having magnitude and direction to be a vector, it must obey the commutative law for addition.

S33. $\sqrt{p^2 + p^2 + 2p^2\cos\theta} = p$

Squaring, $2p^2(1 + \cos\theta) = p^2$

$$1 + \cos\theta = \frac{1}{2}$$

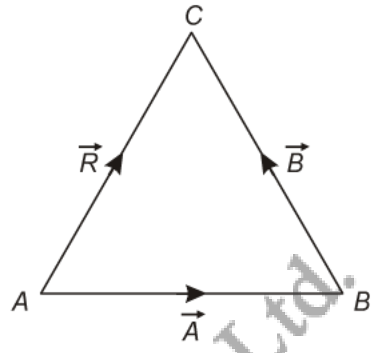
$$\cos\theta = \frac{-1}{2},\quad \theta = 120^\circ.$$

S34. (a) Incorrect (b) Correct (c) Correct (d) Correct

S35. All the quantities except $(3a_x + 2b_y)$ are independent of the choice of orientation of coordinate axes.

S36. According to polygon law of vectors, the sum of all vectors representing the sides of a regular polygon is zero.

According to triangle law, if $\vec{A}$ and $\vec{B}$ are two vectors on the two sides of a triangle, their sum is represented by AC, *i.e.*, $\vec{R}$.



i.e., $$\vec{A} + \vec{B} = \vec{R}$$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

If $\overrightarrow{CA}$ is a vector, then

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA} = 0$$

Therefore the sum of three vectors representing the three sides of a triangle is also zero.

So, the polygon law is proved.

S37. It state that "if two vectors acting simultaneously at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram, the resultant is represented completely (both in magnitude and direction) by the diagonal of the parallelogram passing through that point".

If $\vec{A}$ and $\vec{B}$ be the two given vectors, then magnitude of their resultant is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}.$$

When θ is increased from 0 to π, $\cos\theta$ goes on decreasing. So, the magnitude of the resultant will also go on decreasing.

S38. Given, $\vec{R} = \vec{A} - \vec{B}$

Taking the dot product of $\vec{R}$ with itself, we have

$$\vec{R}\cdot\vec{R} = (\vec{A} - \vec{B})\cdot(\vec{A} - \vec{B}) = \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B}$$

Now, the dot product of a vector with itself is equal to the square of its magnitude.

$\therefore$ $$R^2 = A^2 - AB\cos\theta - BA\cos\theta + B^2$$

or $$\boldsymbol{R^2 = A^2 + B^2 - 2AB\cos\theta}.$$

S39. The essential condition for adding the vectors is that the quantities represented by them should be of the same physical nature. For example, all the vectors to be added should be displacements or velocities or forces. Forces cannot be added to velocities.

S40. (a) A vector, its magnitude and angle between two vectors do not depend upon the choice of orientation of the coordinate axes. Therefore, $\vec{A}+\vec{B}$, $|\vec{A}+\vec{B}-\vec{C}|$ angles between $\vec{B}$ and $\vec{C}$ and $\lambda\vec{A}$ are independent of the orientation of the coordinate axes.

(b) The statement is true. Since the distance covered by an object between two points may be equal to or greater than the magnitude of the displacement between the two points, average speed is always greater than or equal to the average velocity.

S41. Displacement is given by the minimum distance between the initial and final positions of a particle. In the given case, all the girls start from point P and reach point Q. The magnitudes of their displacements will be equal to the diameter of the ground.

Radius of the ground = 200 m

Diameter of the ground = 2 × 200 = 400 m

Hence, the magnitude of the displacement for each girl is 400 m. This is equal to the actual length of the path skated by girl B.

S42. The difference of two vectors $\vec{A}$ and $\vec{B}$ can be obtained by adding $(-\vec{B})$ to $\vec{A}$

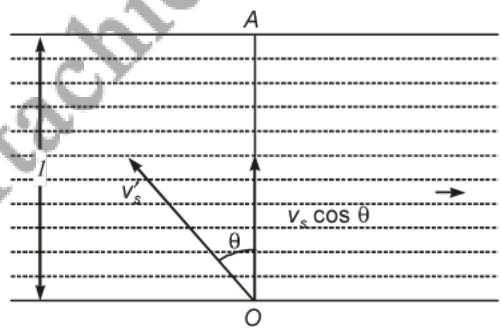
$$\therefore \quad \vec{A} - \vec{B} = \vec{A} + (-\vec{B}).$$

So, subtraction of vectors can be regarded as one form of addition of vectors. Thus, since the commutative and associative laws are applicable to vector addition therefore they must be applicable to vector subtraction also.

$$\therefore \quad \vec{A} + (-\vec{B}) = (-\vec{B}) + \vec{A}$$

and $$(\vec{A} + \vec{B}) + (-\vec{C}) = \vec{A} + [\vec{B} + (-\vec{C})].$$

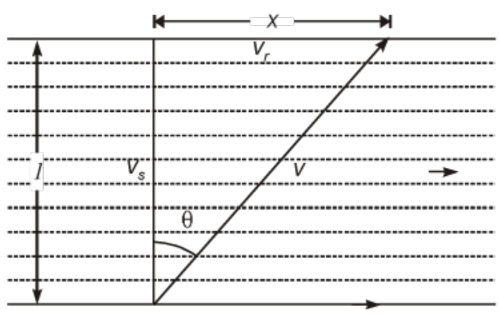
S43. Let $\vec{v}_s$ and $\vec{v}_r$ represent the velocities of swimmer and river respectively. Let $\vec{v}$ represent the resultant of $\vec{v}_s$ and $\vec{v}_r$.



(a) Suppose the swimmer begins to swim at an angle θ with the line OA. OA is perpendicular to the flow of river.

Time to cross the river, $t = \dfrac{l}{v_s \cos\theta}$.

For t to be minimum, cos θ should be maximum. This is possible if θ = 0°. So the swimmer should swim in a direction perpendicular to the direction of flow of river.



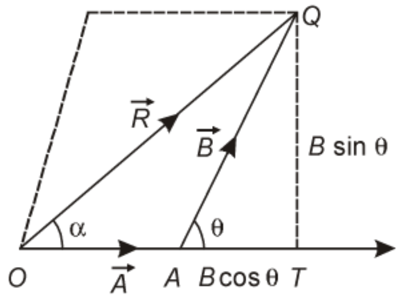
(b) $v = \sqrt{v_s^2 + v_r^2}$

(c) $\tan\theta = \dfrac{v_r}{v_s} = \dfrac{x}{l}$ or $x = \dfrac{v_r l}{v_s}$

(d) Time taken to cross = $\frac{l}{v_s}$.

S44. **Parallelogram law of vector addition:** If two vectors $\vec{A}$ and $\vec{B}$ present two adjacent sides of a parallelogram, the sum of the vectors is represented by the diagonal of the parallelogram.

Let $\vec{A}$ and $\vec{B}$ be two vectors at an angle θ between them. According to the law of parallelogram of vectors, the diagonal of the parallelogram indicates the sum of the other two sides vector $\vec{A}$ and $\vec{B}$.



$$|\overrightarrow{OQ}| = |\vec{A}+\vec{B}| = \sqrt{OT^2 + TQ^2}$$

$$|\vec{R}| = \sqrt{(A+B\cos\theta)^2 + (B\sin\theta)^2}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

The resultant R is at an angle α to $\vec{A}$ given by,

$$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right).$$

S45. We know that $\vec{v} = \vec{u} + \vec{a}t$

Now, $\vec{v}\cdot\vec{v} = (\vec{u}+\vec{a}t)\cdot(\vec{u}+\vec{a}t)$

or $v^2 = \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{a}t + \vec{a}t\cdot\vec{u} + \vec{a}t\cdot\vec{a}t$

or $v^2 = u^2 + \vec{a}\cdot\vec{u}t + \vec{a}\cdot\vec{u}t + \vec{a}\cdot\vec{a}t^2$ $\quad$ ∵ Dot product is commutative]

or $v^2 = u^2 + 2\vec{a}\cdot\vec{u}t + \vec{a}\cdot\vec{a}t^2$

or $v^2 = u^2 + 2\vec{a}\cdot\left(\vec{u}t + \frac{1}{2}\vec{a}t^2\right)$

or $v^2 = u^2 + 2\vec{a}\cdot\vec{S}$ $\quad$ $\left[\because\ \vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2\right]$

∴ $v^2 - u^2 = 2\vec{a}\cdot\vec{S}$.

S46. (a) $\vec{r} = 9t\hat{i} + 6t^2\hat{j} + 8\hat{k}$

$$\vec{v}(t) = \frac{\overrightarrow{dr}}{dt} = 9\hat{i} + 12t\hat{j}$$

$$\vec{a}(t) = \frac{\overrightarrow{dv}}{dt} = 12\hat{j}$$

$= 12$ m/s along y-axis

(b) At $t = 2s.\vec{v} = 9\hat{i} + 12(2)\hat{j}$

or, $\vec{v} = 9\hat{i} + 24\hat{j}$

magnitude, $|\vec{v}| = \sqrt{(9)^2 + (24)^2} = \sqrt{657}$

$= 25.6$ m/s

direction $\theta = \tan^{-1}\left(\frac{24}{9}\right)$ with x-axis.

S47. The dot product of two vectors $\vec{a}$ and $\vec{b}$ inclined to each other at an angle θ is equal to the product of the magnitudes of the two vectors and the cosine of the smaller angle θ between the two vectors.

$$\vec{a} \cdot \vec{b} = ab\cos\theta.$$

Unit vector perpendicular to both $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is given by

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$= \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}[2 - (-1)] - \hat{j}(4 - 1) + \hat{k}(-2 - 1)$$

$$= 3\hat{i} - 3\hat{j} - 3\hat{k}$$

Unit vector is $\hat{n} = \frac{3\hat{i} - 3\hat{j} - 3\hat{k}}{\sqrt{9 + 9 + 9}} = \frac{3\hat{i} - 3\hat{j} - 3\hat{k}}{\sqrt{27}} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$.

S48. ***Unit vector:*** Unit vector is a vector having unit magnitude. It is used to denote the direction of a given vector.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

Let $3\hat{i} + 7\hat{j} + 4\hat{k} = \vec{a}$.

$$|\vec{a}| = \sqrt{3^2 + 7^2 + 4^2}$$

$$= \sqrt{9 + 49 + 16} = \sqrt{74}$$

Unit vector in the direction of

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{3\hat{i} + 7\hat{j} + 4\hat{k}}{\sqrt{74}}.$$

S49. (a) $\theta = 0$, in same direction (or) parallel.

(b) $\theta = 180°$, in same line but opposite or anti parallel.

(c) $\theta = 90°$, in perpendicular direction.

S50. (a) Dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined as, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$.

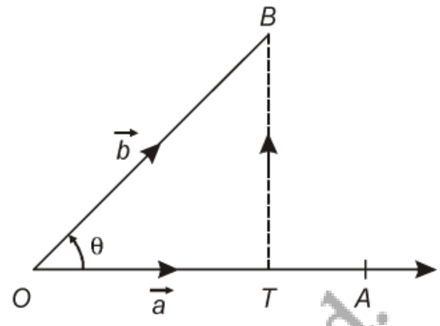
Let $\overline{OA} = \vec{a}$, $\overline{OB} = \vec{b}$ and $\angle AOB = \theta$

Then, $OT = |\vec{b}|\cos\theta$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$= OA + OT$$

$$\vec{a} \cdot \vec{b} = \text{OA}.$$



Projection of $\vec{b}$ on $\vec{a}$.

(b) Let $\vec{A} = 4$ and $\vec{B} = 3$, $R = 1$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$1 = \sqrt{(4)^2 + (3)^2 + 2 \times 4 \times 3\cos\theta}$$

$$1 = 25 + 24\cos\theta$$

$$24\cos\theta = -24$$

$$\cos\theta = -1$$

$$\theta = 180°.$$

S51. It state that "if two vectors acting simultaneously at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram, the resultant is represented completely (both in magnitude and direction) by the diagonal of the parallelogram passing through that point".

The resultant vector of two or more vectors is defined as that single vector which produces the same effect as is produced by individual vectors together. If F_1 and F_2 are the two forces the magnitude of the resultant F is given by,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

The angle θ between them is,

$$\theta = \cos^{-1}\left(\frac{F^2 - F_1^2 - F_2^2}{2F_1F_2}\right).$$

S52. Let ***OP*** and ***OQ*** represent the two vectors ***A*** and ***B*** making an angle θ (see figure). Then, using the parallelogram method of vector addition, ***OS*** represents the resultant vector ***R***:

$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B}$$

SN is normal to *OP* and *PM* is normal to *OS*.

From the geometry of the figure,

$$OS^2 = ON^2 + SN^2$$

but $ON = OP + PN = A + B\cos\theta$

$$SN = B\sin\theta$$

$$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

or, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$R = A^2 + B^2 + 2AB\cos\theta \quad \text{... (i)}$$

In $\Delta\, OSN$, $SN = OS\sin\alpha = R\sin\alpha$, and

In $\Delta\, PSN$, $SN = PS\sin\theta = B\sin\theta$

Therefore, $R\sin\alpha = B\sin\theta$

or, $$\frac{R}{\sin\theta} = \frac{R}{\sin\alpha} \quad \text{... (ii)}$$

Similarly, $PM = A\sin\alpha = \mathrm{B}\sin\beta$

or, $$\frac{A}{\sin\beta} = \frac{B}{\sin\alpha} \quad \text{... (iii)}$$

Combining Eqs. (ii) and (iii), we get

$$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha} \quad \text{... (iv)}$$

Using Eq. (iv), we get: $$\sin\alpha = \frac{B}{R}\sin\theta \quad \text{... (v)}$$

where *R* is given by Eq. (i).

or, $$\tan\alpha = \frac{SN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta} \quad \text{... (vi)}$$

Equation (i) gives the magnitude of the resultant and Eqs. (v) and (vi) its direction. Equation (i) is known as the ***Law of cosines*** and Eq. (iv) as the ***Law of sines***.

S53. (a) False

Despite being a scalar quantity, energy is not conserved in inelastic collisions.

(b) False

Despite being a scalar quantity, temperature can take

(c) False

Total path length is a scalar quantity. Yet it has the dimension of length.

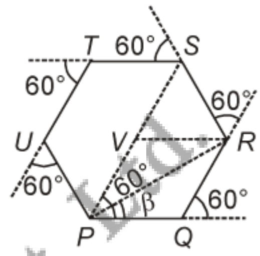
(d) False

A scalar quantity such as gravitational potential can vary from one point to another in space.

(e) True

The value of a scalar does not vary for observers with different orientations of axes.

S54. The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure



Let the motorist start from point P.

The motorist takes the third turn at S.

$\therefore$ Magnitude of displacement = $PS = PV + VS = 500 + 500 = 1000$ m

Total path length = $PQ + QR + RS$

= 500 + 500 +500 = 1500 m

The motorist takes the sixth turn at point P, which is the starting point.

$\therefore$ Magnitude of displacement = 0

Total path length = $PQ + QR + RS + ST + TU + UP$

= 500 + 500 + 500 + 500 + 500 + 500 = 3000 m

The motorist takes the eight turn at point R

$\therefore$ Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + 2(PQ)\cdot(QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2 \times 500 \times 500 \times \cos 60^\circ)}$$

$$= \sqrt{250000 + 250000 + \left(500000 \times \frac{1}{2}\right)}$$

= 866.03 m

$$\beta = \tan^{-1}\left(\frac{500 \sin 60^\circ}{500 + 500 \cos 60^\circ}\right) = 30^\circ$$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30° with PR.

Total path length = Circumference of the hexagon + $PQ + QR$

$= 6 \times 500 + 500 + 500 = 4000$ m

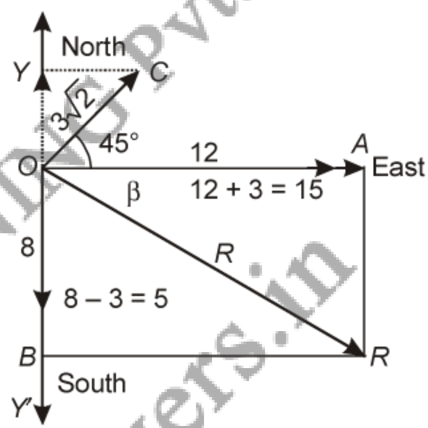
The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table

Turn	Magnitude of displacement (m)	Total path length (m)
Third	1000	1500
Sixth	0	3000
Eighth	866.03; 30°	4000

S55. ***Triangle law:*** If two vectors can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.

Let *OA*, *OB* and *OC* represent the velocities of 12 ms^{-1} to east, 8 ms^{-1} to south and $3\sqrt{2}$ ms^{-1} to north-east respectively.

The velocity $3\sqrt{2}$ acting along *OC* makes an angle of 45° with *OA*. Its rectangular component along



$$OY = 3\sqrt{2}\cos 45° = 3\ ms^{-1}$$

Its component along

$$OA = 3\sqrt{2}\cos 45° = 3\ ms^{-1}$$

∴ The resultant velocity along

$$OA = 12 + 3 = 15\ ms^{-1}$$

and resultant velocity along

$$OB = 8 - 3 = 5\ ms^{-1}$$

The angle between these two velocities is 90°

∴ $$R = \sqrt{15^2 + 5^2} = 15.81\ ms^{-1}$$

Let this resultant make an angle β with the direction *OA*, then

$$\tan\beta = \frac{AR}{OA} = \frac{5}{15} = 0.3333$$

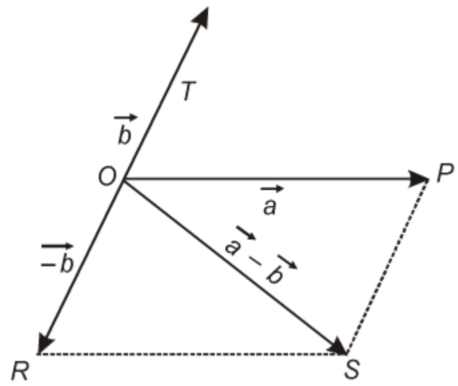
∴ $$\beta = 18°\ 26' \text{ south of east.}$$

S56. (a) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram *PORS*, as shown in the given figure.

Here, we can write:

$$|\overrightarrow{OR}| = |\overrightarrow{PS}| = |\vec{b}| \quad \text{... (i)}$$

$$|\overrightarrow{OP}| = |\vec{a}| \quad \text{... (ii)}$$



In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOPS, we have:

$$OS < OP + PS$$

$$|\vec{a} - \vec{b}| < |\vec{a}| + |-\vec{b}|$$

$$|\vec{a} - \vec{b}| < |\vec{a}| + |\vec{b}| \quad \text{... (iii)}$$

If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line but in opposite direction, then we can write:

$$|\vec{a} - \vec{b}| = |\vec{a}| + |\vec{b}| \quad \text{... (iv)}$$

Combining equations (iii) and (iv), we get:

$$|\vec{a} - \vec{b}| \le |\vec{a}| + |\vec{b}|$$

(b) As shown in the above figure, the quantity on the LHS is always positive and that on the RHS can be positive or negative. To make both quantities positive, we take modulus on both sides as:

$$||\vec{a} - \vec{b}|| > ||\vec{a}| - |\vec{b}||$$

$$|\vec{a} - \vec{b}| > ||\vec{a}| - |\vec{b}|| \quad \text{... (v)}$$

If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line but in the opposite direction, then we can write:

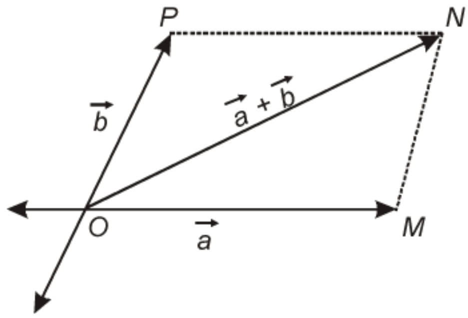
$$|\vec{a} - \vec{b}| = ||\vec{a}| - |\vec{b}|| \quad \text{... (vi)}$$

Combining equations (v) and (vi), we get:

$$|\vec{a} - \vec{b}| \ge ||\vec{a}| - |\vec{b}||.$$

S57. (a) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram *OMNP*, as shown in the given figure. particle is moving in a straight line.

Here, we can write:



$$|\overrightarrow{OM}| = |\vec{a}| \quad \text{... (i)}$$

$$|\overrightarrow{MN}| = |\overrightarrow{OP}| = |\vec{b}| \quad \text{... (ii)}$$

$$|\overrightarrow{ON}| = |\vec{a}+\vec{b}| \quad \text{... (iii)}$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

$$ON < (OM + MN)$$

$$|\vec{a}+\vec{b}| < |\vec{a}| + |\vec{b}| \quad \text{... (iv)}$$

If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then we can write:

$$|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}| \quad \text{... (v)}$$

Combining equations (iv) and (v), we get:

$$|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$$

(b) As shown in the above figure, in a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

$$ON + MN > OM$$

$$ON + OM > MN$$

$$|\overrightarrow{ON}| > |\overrightarrow{OM} - \overrightarrow{OP}| \quad (\because\ OP = MN)$$

$$|\vec{a}+\vec{b}| > ||\vec{a}| - |\vec{b}|| \quad \text{... (vi)}$$

If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then we can write:

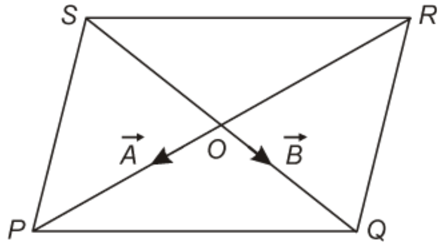
$$|\vec{a}+\vec{b}| = ||\vec{a}| - |\vec{b}|| \quad \text{... (vii)}$$

Combining equations (vi) and (vii), we get:

$$|\vec{a}+\vec{b}| \ge ||\vec{a}| - |\vec{b}||.$$

S58. ***Parallelogram law:*** If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram passing through that point.

Consider a parallelogram $PQRS$ such that the diagonal PR is represented by the vector $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and the diagonal SQ by the vector $\vec{B} = \hat{i} - 3\hat{j} + 4\hat{k}$. Now the diagonal PR and SQ bisect each other at O. Hence



$$\overrightarrow{PO} = \frac{1}{2}\vec{A} = \frac{1}{2}(3\hat{i} + \hat{j} + \hat{k})$$

and $$\overrightarrow{OQ} = \frac{1}{2}\vec{B} = \frac{1}{2}(\hat{i} - 3\hat{j} + 4\hat{k})$$

Area of parallelogram

$$PQRS = 4 \times \text{areas } \Delta POQ$$

$$= 4 \times \frac{1}{2}\overrightarrow{PO} \times \overrightarrow{OQ}$$

$$= 2\left[\frac{1}{2}(3\hat{i} + \hat{j} + 2\hat{k}) \times \frac{1}{2}(\hat{i} - 3\hat{j} + 4\hat{k})\right]$$

$$= \frac{1}{2}(3\hat{i} + \hat{j} + 2\hat{k})(\hat{i} - 3\hat{j} + 4\hat{k})$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & -3 & 4 \end{vmatrix} = (10\hat{i} - 10\hat{j} - 10\hat{k})$$

$$= \frac{1}{2}(10\hat{i} - 10\hat{j} - 10\hat{k}) = (5\hat{i} - 5\hat{j} - 5\hat{k})$$

$$= \sqrt{5^2 + 5^2 + 5^2} = \sqrt{75} = 8.66 \text{ sq. units.}$$

Date: 13/10/2021

Q1. What is the angle between the directions of velocity and acceleration at the highest point of the trajectory of a projectile?

Q2. Can there be a two dimensional motion even though acceleration is present in only one direction?

Q3. In long jump, does it matter how high you jump? What factors determine the span of the jump?

Q4. If the net external force acting on a body is zero, then the body at rest continues to remain at rest and a body in motion continues to move with uniform motion. What is the name given to this property of the body?

Q5. A foot ball is thrown in a parabolic path. Is there any point at which the acceleration is perpendicular to the velocity?

Q6. Is it possible for a body to change the direction of its velocity vector when the body is experiencing constant acceleration?

Q7. An aeroplane moving horizontally at 20 ms^{-1} drops a bag. What is the displacement of the bag after 5 second? Given: $g = 10\ ms^{-2}$.

Q8. A hiker stands on the edge of a cliff 490 m above the ground and throwns a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground, and speed with which it hits the ground. Given: $g = 9.8\ ms^{-2}$.

Q9. What will be the effect on the horizontal range of a projectile when its initial speed is doubled keeping its angle of projection same?

Q10. Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere? Explain.

Q11. What is the distance travelled by a point during the time, if it moves in $x - y$ plane, according to the relation $x = a \sin \omega t$ any $y = a(1 - \cos \omega t)$?

Q12. Suppose you have two forces $\vec{F}$ and $\vec{F}$. How would you combine them in order to have resultant force of magnitudes (a) zero, (b) $2\vec{F}$ and (c) $\vec{F}$?

Q13. Water from a sprinkler comes out with a constant velocity u in all the directions. What is the maximum area of the grass-land that can be watered at any time?

Q14. What is meant by resolving a force into rectangular components? Resolve a force of 10 N into two components, if it acts at an angle 30° with the horizontal.

Q15. A particle starts from origin at $t = 0$ with a velocity $5.0\,\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3.0i + 2.0\,\hat{j})$ m/s^2. (a) What is the y-coordinate of the particle at the instant its x-coordinate is 84 m? (b) What is the speed of the particle at this time?

Q16. The position of a particle is given by

$$r = 3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$$

where t is in seconds and the coefficients have the proper units for r to be in metres. (a) Find v(t) and a(t) of the particle. (b) Find the magnitude and direction of v(t) at t = 1.0 s.

Q17. A cyclist moves along a circular path of radius 70 m. If he completes one round in 11 s, calculate (a) total length of path, (b) magnitude of the displacement, (c) average speed and (d) magnitude of average velocity.

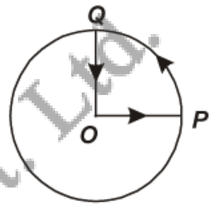
Q18. The position of a particle is given by $\vec{r} = 3t\hat{i} + 2t^2\hat{j} + 5\hat{k}$

Where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find $\vec{v}(t)$ and $\vec{a}(t)$ of the particle.

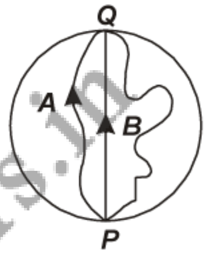
(b) Find the magnitude and direction $\vec{v}(t)$ at t = 3 s.

Q19. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in figure. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



Q20. (a) From the top of a tower 100 m in height, a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m^{-1}. Find when and where the two balls will meet? Given g = 9.8 ms^{-2}.

(b) Define displacement vector. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of the path skated?



Q21. Define the instantaneous velocity and instantaneous acceleration. The position of a particle is given by: $\vec{r} = 3.0t\hat{i} - 2.0t^2\hat{j} + 4\hat{k}$ m

where t is in seconds, $\vec{r}$ is in metres and the coefficients have the proper units.

(a) Find the velocity v and acceleration a.

(b) What is the magnitude of velocity of the particle at t = 2s?

Q22. A particle starts from origin at t = 0 with a velocity $5\hat{i}$ ms^{-1} and moves in XY-plane under the action of a force, which produces a constant acceleration of $3\hat{i} + 2\hat{j}$ ms^{-2}. (a) What is the y-coordinate of the particle at the instant its x-coordinate is 84 m? (b) What is the speed of the particle at this time.

Q23. $\hat{i}$ and $\hat{j}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

Q24. A particle starts from the origin at t = 0 s with a velocity of 10.0 $\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})$ ms^{-2}.

At what time is the x-coordinate of the particle 16 m? What is the particle at that time? What is the speed of the particle at the time?

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S1. At the highest point, the velocity is horizontal. The acceleration is vertical. So, angle is 90°.

S2. Yes, the motion of projectile is an example for the same.

S3. In order to have long jump, it is necessary to have the horizontal component of velocity more than the vertical component. The height does not matter.

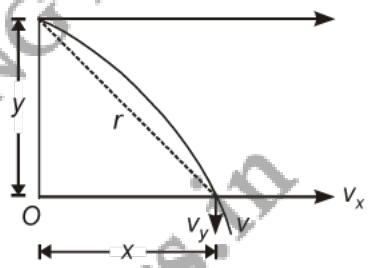
The angle of elevation at the time of jumping and the magnitude of the speed will determine the span of the jump.

S4. Inertia.

S5. At the highest point. **Since velocity has horizontal component only while g is acting downward.**

S6. Yes. The motion of the projectile is an example.

S7. Horizontal distance $x = v_x \times t = 20 \times 5 \text{ m} = 100 \text{ m}$



and vertical distance $y = v_y t + \frac{1}{2}gt^2$

$= 0 \times 5 + \frac{1}{2} \times 10 \times (5)^2$

$= 125$

Now, resultant distance $r = \sqrt{(x)^2 + (y)^2}$ m

$= \sqrt{(100)^2 + (125)^2}$ m $=$ **160.1 m**.

S8. $S_y = \frac{1}{2}gt^2$

Time taken $t = \sqrt{\frac{2 \times 490}{9.8}}\ s = 10\ s;$

Vertical speed $v_y = 9.8 \times 10 \text{ ms}^{-1} = 98 \text{ ms}^{-1}$

Horizontal speed $v_x = 15 \text{ ms}^{-1}$

Now, resultant speed $v^2 = v_x^2 + v_y^2$

$= \sqrt{15 \times 15 + 98 \times 98}\ \text{ms}^{-1}$ $=$ **99.1 ms^{-1}**.

S9. Let initial speed u and angle of projection θ and horizontal range R

$$R = \frac{u^2 \sin 2\theta}{g} \quad \text{... (i)}$$

Now, initial speed u' and range R' and same angle of projection

$$R' = \frac{u'^2 \sin 2\theta}{g}$$

$$R' = \frac{(2u)^2 \sin 2\theta}{g}$$

$$= 4\left(\frac{u^2 \sin 2\theta}{g}\right)$$

$$R' = \mathbf{4R}.$$

S10. (a) No.

Explanation: One cannot associate a vector with the length of a wire bent into a loop.

(b) Yes.

Explanation: One can associate an area vector with a plane area. The direction of this vector is normal, inward or outward to the plane area.

(c) No.

Explanation: One cannot associate a vector with the volume of a sphere. However, an area vector can be associated with the area of a sphere.

S11. The motion will be Simple Harmonic with amplitude of $\sqrt{a^2 + a^2} = a\sqrt{2}$ if in the same direction.

Because directions are perpendicular to X and Y, it may be interpreted for a circular motion with radius a.

S12. (a) If they act at opposite direction, resultant is zero.

(b) If they act in same direction,

$$R = 2F.$$

(c) If they acts at angle 120°

$$F^2 = F^2 + F^2 + 2F^2 \cos\theta$$

$$\cos\theta = -1/2 \quad \text{or} \quad \theta = 120^\circ$$

S13. Let $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j}$

Projection of

$$\vec{A} \text{ on } \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|} = 0, \qquad (1 = 90^\circ)$$

$$\theta = 90^\circ \qquad (\therefore \text{ Projection} = 0)$$

S14. Let F be the force at 30° to horizontal.

The horizontal component is,

$$F\cos 30° = \frac{\sqrt{3}F}{2} = 5\sqrt{3}.$$

The vertical component is,

$$F\sin 30° = \frac{F}{2} = 5$$

S15. The position of the particle is given by

$$r(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$= 5.0\hat{\boldsymbol{i}}\, t\,(1/2)\,(3.0\hat{\boldsymbol{i}} + 2.0\hat{\boldsymbol{j}})\,t^2$$

$$= (5.0t + 1.5t^2)\,\hat{\boldsymbol{i}} + 1.0t^2\,\hat{\boldsymbol{j}}$$

Therefore, $x(t) = 5.0t + 1.5t^2$

$y(t) = +1.0t^2$

Given $x(t) = 84$ m, $t = ?$

$$5.0t + 1.5t^2 = 84 \quad \Rightarrow \quad t = 6\text{ s}$$

At $t = 6$ s, $y = 1.0\,(6)^2 = 36.0$ m

Now, the velocity $\boldsymbol{v} = \dfrac{d\boldsymbol{r}}{dt} = (5.0 + 3.0t)\,\hat{\boldsymbol{i}} + 2.0t\,\hat{\boldsymbol{j}}$

At $t = 6$ s, $v = 23.0\,\hat{\boldsymbol{i}} + 12.0\,\hat{\boldsymbol{j}}$

$$\text{speed} = |\boldsymbol{v}| = \sqrt{23^2 + 12^2} \cong 26 \text{ m s}^{-1}.$$

S16.

$$\mathbf{v}(t) = \frac{d\boldsymbol{r}}{dt} = \frac{d}{dt}(3.0t\,\hat{\boldsymbol{i}} + 2.0t^2\,\hat{\boldsymbol{j}} + 5.0\,\hat{\boldsymbol{k}})$$

$$= 3.0\,\hat{\boldsymbol{i}} + 4.0t\,\hat{\boldsymbol{j}}$$

$$\boldsymbol{a}(t) = \frac{d\mathbf{v}}{dt} = +4.0\,\hat{\boldsymbol{j}}$$

$a = 4.0$ m s^{-2} along y-direction

At $t = 1.0$ s, $\boldsymbol{v} = 3.0\,\hat{\boldsymbol{i}} + 4.0\,\hat{\boldsymbol{j}}$

It's magnitude is $v = \sqrt{3^2 + 4^2}$ 5.0 m s^{-1} and direction is $\theta = \tan^{-1}\left(\dfrac{v^y}{v_x}\right) = \tan^{-1}\left(\dfrac{4}{3}\right) \cong 53°$ with x-axis.

S17. Here, radius of circular path, $R = 70$ m

Timer taken to complete one round, $t = 11$ s

(a) Total length of the path,

$$S = 2\pi R = 2 \times \frac{22}{7} \times 70 = \mathbf{440\ m}.$$

(b) Since the cyclist returns to the starting point on completing one round,

Displacement = **0**.

(c) $$\text{Average speed} = \frac{S}{t} = \frac{440}{11} = \mathbf{40\ ms^{-1}}.$$

(d) $$\text{Average velocity} = \frac{\text{Displacement}}{t} = \mathbf{0}.$$

S18. Here, $\vec{r} = 3t\hat{i} + 2t^2\hat{j} + 5\hat{k}$

(a) Now, $$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[3t\hat{i} + 2t^2\hat{j} + 5\hat{k}\right] = \mathbf{3\hat{i} + 4t\hat{j}}$$

Also, $$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[3\hat{i} + 4t\hat{j}\right] = \mathbf{4\hat{j}}$$

(b) At t = 3 s, velocity of the particle,

$$\vec{v} = 3\hat{i} + 4 \times 3\hat{j} = 3\hat{i} + 12\hat{j}$$

$\therefore$ $$|\vec{v}| = \sqrt{3^2 + 12^2} = \sqrt{153} = \mathbf{12.37\ ms^{-1}}.$$

If the velocity of the particle is inclined at angle θ with the X-axis, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{12}{3}$$

or $$\theta = \mathbf{76^\circ}.$$

S19. Displacement is given by the minimum distance between the initial and final positions of a body. In the given case, the cyclist comes to the starting point after cycling for 10 minutes. Hence, his net displacement is zero.

Average velocity is given by the relation:

$$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Total time}}$$

Since the net displacement of the cyclist is zero, his average velocity will also be zero.

Average speed of the cyclist is given by the relation:

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Total time}}$$

$$\text{Total path length} = OP + PQ + QO$$

$$= 1 + \frac{1}{4}(2\pi \times 1) + 1$$

$$= 2 + \frac{1}{4} 2\pi = 3.570 \text{ km}$$

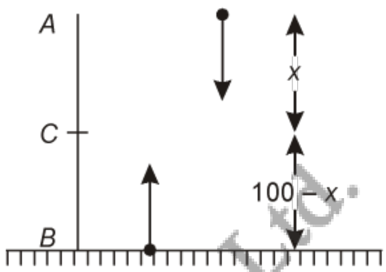
$$\text{Time taken} = 10 \text{ min} = \frac{10}{60} = \frac{1}{6} \text{ h}$$

$$\therefore \quad \text{Average speed} = \frac{3.570}{\frac{1}{6}} = 21.42 \text{ km/h}.$$

S20. (a)

$$x = 0 + \frac{1}{2} \times 9.8\, t^2 = 4.9\, t^2 \quad \text{... (i)}$$

$$100 - x = 25t + \frac{1}{2} \times (-9.8) t^2 \quad \text{... (ii)}$$



Solving Eq. (i) and (ii), we get

$t = 4$ second

$x = 4.9 \times 16 =$ **78.4 m**.

(b) ***Displacement vector:*** The displacement of a particle is defined as the change in position of the particle in a particular direction and is given by a vector drawn from its initial position to its final position.

Displacement is given by the minimum distance between the initial and final positions of a particle. In the given case, all the girls start from point *P* and reach point *Q*. The magnitudes of their displacements will be equal to the diameter of the ground.

Radius of the ground = 200 m

Diameter of the ground = 2 × 200 = 400 m

Hence, the magnitude of the displacement for each girl is 400 m. This is equal to the actual length of the path skated by girl *B*.

S21. ***Instantaneous velocity:*** The velocity of an object at a particular instant or at a particular point of its path is called ***instantaneous velocity***.

Instantaneous acceleration: The instantaneous acceleration of an object is defined as the limiting value of the average acceleration of the object in a small time interval around that instant, when time interval approaches zero.

$$\vec{r} = 3.0 t\hat{i} - 2.0 t^2 \hat{j} + 4\hat{k} \text{ m}$$

(a) $\vec{v} = \text{velocity} = \frac{d\vec{r}}{dt} = 3.0\, \hat{i} - 4.0\, t\, \hat{j}$ m/s

$\vec{a} = \text{acceleration} = \frac{d\vec{v}}{dt} = -4.0\, \hat{j}$ m/s^2

(b) Magnitude of velocity at $t = 2$ s

$$\vec{v} = 3.0\,\hat{i} - 8.0\,\hat{j}$$

$$|\vec{v}| = \sqrt{(3)^2 + (-8)^2}$$

$$= \sqrt{9 + 64} = \sqrt{\mathbf{73}}\ \mathbf{m/s}\ .$$

S22. Here, $\vec{u} = 5\hat{i}\ \text{ms}^{-1}$; $\vec{a} = 3\hat{i} + 2\hat{j}\ \text{ms}^{-2}$

The position of particle at any time t is given by

$$\vec{r} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$$

Substituting for $\vec{u}$ and $\vec{a}$, we have

$$\vec{r} = (5\hat{i})\,t + \frac{1}{2}(3\hat{i} + 2\hat{j})\,t^2$$

or $$\vec{r} = (5t + 1.5t^2)\,\hat{i} + t^2\,\hat{j}$$

(a) If (x, y) are the coordinates of the particle, then

$$x = 5\,t + 1.5\,t^2 \qquad \text{... (i)}$$

and $$y = t^2 \qquad \text{... (ii)}$$

The time, when x-coordinate of the particle is 84, can be found by setting x = 84 m in the equation (i). Thus,

$$84 = 5\,t + 1.5\,t^2$$

or $$1.5\,t^2 + 5\,t - 84 = 0$$

On solving, we get

$$t = 6\ \text{s}$$

Therefore, at t = 6 s,

$$y = (6)^2 = \mathbf{36\ m}$$

(b) Now, $$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[(5\,t + 1.5\,t^2)\,\hat{i} + t^2\,\hat{j}\right] = (5 + 3\,t)\,\hat{i} + 2t\hat{j}$$

At t = 6 s,

$$\vec{v} = (5 + 3 \times 6)\,\hat{i} + 2 \times 6\,\hat{j} = 23\,\hat{i} + 12\,\hat{j}$$

Therefore, speed of the particle at t = 6 s,

$$\vec{v} = \sqrt{23^2 + 12^2} = \mathbf{25.94\ m\,s^{-1}}$$

S23. Consider a vector $\bar{P}$, given as:

$$\bar{P} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$$

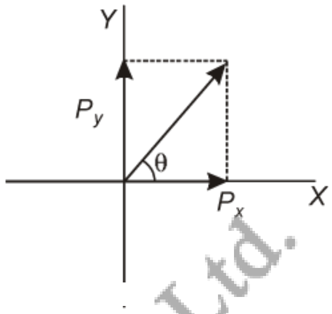
$$P_x\hat{\mathbf{i}} + P_y\hat{\mathbf{j}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$$

On comparing the components on both sides, we get:

$$P_x = P_y = 1$$

$$|\vec{P}| = \sqrt{P_x^2 + P_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \quad \text{... (i)}$$

Hence, the magnitude of the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is $\sqrt{2}$.



Let θ be the angle made by the vector $\bar{P}$, with the x-axis, as shown in the following figure.

$$\therefore \quad \tan\theta = \left(\frac{P_y}{P_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45° \quad \text{... (ii)}$$

Hence, the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ makes an angle of 45° with the x-axis.

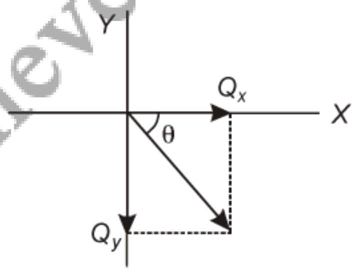
Let $\vec{Q} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

$$Q_x\hat{\mathbf{i}} - Q_y\hat{\mathbf{j}} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$$

$$|\vec{Q}_x| = |\vec{Q}_y| = 1$$

$$|\vec{Q}| = \sqrt{Q_x^2 + Q_y^2} = \sqrt{2} \quad \text{... (iii)}$$

Hence, the magnitude of the vector $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ is $\sqrt{2}$.



Let θ be the angle made by the vector $\vec{Q}$, with the x- axis, as shown in the following figure.

$$\therefore \quad \tan\theta = \left(\frac{Q_y}{Q_x}\right)$$

$$\theta = -\tan^{-1}\left(-\frac{1}{1}\right) = -45° \quad \text{... (iv)}$$

Hence, the vector $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ makes an angle of –45° with the x-axis. It is given that:

$$\vec{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$$

$$A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$$

On comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we have:

$$A_x = 2 \quad \text{and} \quad A_y = 3$$

$$\vec{A} = \sqrt{2^2 + 3^2} = \sqrt{13}.$$

S24. Velocity of the particle, $\vec{v} = 10.0\hat{\mathbf{j}}$ m/s

Acceleration of the particle $\vec{a} = (8.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$

Also, $\vec{a} = \frac{d\vec{v}}{dt} = 8.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}$

But, $d\vec{v} = (8.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\,dt$

Integrating both sides:

$$\vec{v}(t) = 8.0t\hat{\mathbf{i}} + 2.0t\hat{\mathbf{j}} + \vec{u}$$

Where,

$\vec{u}$ = Velocity vector of the particle at $t = 0$

$\vec{v}$ = Velocity vector of the particle at time t

But, $\vec{v} = \frac{d\vec{r}}{dt}$

$$d\vec{r} = \vec{v}\,dt = (8.0t\hat{\mathbf{i}} + 2.0t\hat{\mathbf{j}} + \vec{u})\,dt$$

Integrating the equations with the conditions: at $t = 0$; $r = 0$ and at $t = t$; $r = r$

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\hat{\mathbf{i}} + \frac{1}{2}\times 2.0t^2\hat{\mathbf{j}}$$

$$= \vec{u}t + 4.0t^2\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}$$

$$= (10.0\hat{\mathbf{j}})\,t + 4.0t^2\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}$$

$$x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = 4.0t^2\hat{\mathbf{i}} + (10t + t^2)\hat{\mathbf{j}}$$

Since the motion of the particle is confined to the x-y plane, on equating the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get:

$$x = 4t^2$$

$$t = \left(\frac{x}{4}\right)^{\frac{1}{2}}$$

When $x = 16$ m:

$$t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2\text{ s}$$

and $y = 10t + t^2$

$\therefore$ $y = 10 \times 2 + (2)^2 = 24$ m

Velocity of the particle is given by:

$$\vec{v}(t) = 8.0t\,\hat{\mathbf{i}} + 2.0t\,\hat{\mathbf{j}} + \vec{u}$$

at $t = 2\,s$

$$\vec{v}(t) = 8.0 \times 2\,\hat{\mathbf{i}} + 2.0 \times 2\,\hat{\mathbf{j}} + 10\,\hat{\mathbf{j}}$$

$$= 16\,\hat{\mathbf{i}} + 14\,\hat{\mathbf{j}}$$

$\therefore$ Speed of the particle:

$$|\vec{v}| = \sqrt{(16)^2 + (14)^2} = \sqrt{256 + 196} = \sqrt{452} = \mathbf{21.26\ m/s}.$$

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Q1. A steel ball is dropped from the window of a train moving an horizontal rails. What is the nature of the path followed by the ball during its journey to the ground?

Q2. A body is projected so that it has maximum range R. What is the maximum height reached during the flight?

Q3. If the greatest height to which you can throw a ball is h, then what is the maximum possible horizontal distance to which you can throw the ball?

Q4. A projectile of mass m is thrown with velocity u from the ground at an angle of 45° with the horizontal. What is the magnitude of change in momentum between leaving and arriving back at the ground?

Q5. The velocity of a projectile is 10 ms^{-1}. At what angle to the horizontal should it be projected so that it covers maximum horizontal distance?

Q6. The maximum range of a projectile is 3 km. What would be its range when launched at an angle of 15° with the horizontal?

Q7. From a certain height above the ground, a stone P is dropped gently. Simultaneously, another stone Q is thrown horizontally. Which of the two stones will arrive at the ground earlier?

Q8. A railway carriage moves over a straight level track with an acceleration 'a'. A passenger in carriage drops a stone. What is the acceleration of the stone with respect to carriage and the earth?

Q9. Define relative velocity.

Q10. Rain appears to a man walking at 3 km/h towards east to be falling vertically downward at 4 km/h. Find the actual velocity of the rain in magnitude and direction.

Q11. A ball is thrown horizontally and at the same time another ball is dropped from the top of a tower.
(a) Will both the balls hit the ground with the same velocity?
(b) Will both the balls reach the ground at the same time?

Q12. Determine the acceleration due to gravity from the following data of an oblique projectile:

$$y = 8t - 5t^2.$$

Q13. From the same point, two balls A and B are thrown simultaneously. A is thrown vertically up with a velocity of 20 ms^{-1}. B is thrown with a velocity of 20 ms^{-1} at an angle of 60° with the vertical. Determine the separation between the two balls at t = 1 second.

Q14. A projectile is fired horizontally with a velocity of 98 ms^{-1} from a cliff 490 m high. Calculate (a) time taken to reach the ground (b) distance of the target from the cliff (c) the velocity with which the projectile hits the ground.

Q15. A hose lying on the ground shoots a stream of water upwards at an angle of 40° to the horizontal. The speed of water is 20 ms^{-1} as it leaves the hose. How high up will it strike a wall which is 8 m away?

Q16. A swimmer can swim with velocity of 10 km/h. w.r.t. the water flowing in a river with velocity of 5 km/h. In what direction should he swim to reach the point on the other bank just opposite to his starting point?

Q17. For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = (1/2)\,[v(t_1) + v(t_2)]$

(b) $v_{average} = [r(t_2) - r(t_1)]\,(t_2 - t_1)$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)\,t + (1/2)\,at^2$

(e) $a_{average} = [v(t_2) - v(t_1)]\,(t_2 - t_1)$

Q18. A shell is fired with a speed of 30 ms^{-1} at an angle of 37° with the horizontal. After 1 second, the shell is moving at an angle θ with the horizontal. Calculate θ. Given: $\sin 37° = \frac{3}{5}$.

Q19. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft?

Q20. An accelerating train is passing over a high bridge. A stone is dropped from the train at an instant when its speed is 10 m/s and acceleration is 1 m/s^2. Find the horizontal and vertical components of the velocity and acceleration of the stone one second after it is dropped. Take $g = 10\ m/s^2$.

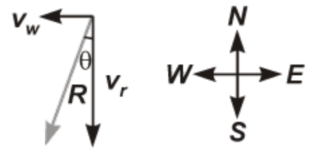
Q21. A lady walking towards east on a road with velocity of 10 m/s encounters rain falling vertically with a velocity of 30 m/s. At what angle she should hold her umbrella to protect herself from the rain?

Q22. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 $m\,s^{-1}$ can go without hitting the ceiling of the hall?

Q23. A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Q24. A passenger arriving in a new town whishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity. Are the two equal?

Q25. Rain is falling vertically with a speed of 35 $m\,s^{-1}$. Winds starts blowing after sometime with a speed of 12 $m\,s^{-1}$ in East to West direction. In which direction should a boy waiting at a bus stop hold his umbrella?



Q26. Rain is falling vertically with a speed of 35 $m\,s^{-1}$. A woman rides a bicycle with a speed of 12 $m\,s^{-1}$ in east to west direction. What is the direction in which she should hold her umbrella?

Q27. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 $m\,s^{-1}$. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take $g = 9.8\ m\,s^{-2}$).

Q28. Galileo, in his book Two new sciences, stated that "for elevations which exceed or fall short of 45° by equal amounts, the ranges are equal". Prove this statement.

Q29. A cricket ball is thrown at a speed of $28\ \text{m s}^{-1}$ in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.

Q30. If a shower of rain appears to be falling vertically downward with a speed at 12 km/h to a person walking due west with a speed of 5 km/h, what is the actual velocity of the rain?

Q31. Rain is falling vertically downward with a velocity of 4 km/h. In what direction and with what velocity does it appear to fall to a man travelling at 2 km/h due east.

Q32. A ship is sailing due east velocity at 30 km/h and another is going due south with a velocity of 40 km/h. Find the velocity of the second ship as it appears to an observer on the first.

Q33. In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

Q34. Rain is falling vertically with a speed of $30\ \text{m s}^{-1}$. A woman rides a bicycle with a speed of $10\ \text{m s}^{-1}$ in the north to south direction. What is the direction in which she should hold her umbrella?

Q35. A man capable of swimming with a velocity u in still water, wants to cross a river of width 'd' flowing with a velocity v. Find the angle in which he should be directed to reach at the exactly opposite point? To cross by the shortest time, in which direction, he should swim? What is the value of shortest time?

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S1. The ball will follow a parabolic path.

S2. When R_{max} if $\theta = 45°$

$$R = \frac{u^2}{g} \quad \dots (i)$$

Now, height attained by the body is

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{u^2 \sin^2(45)}{2g} = \frac{u^2\left(\frac{1}{\sqrt{2}}\right)^2}{2g} = \frac{u^2}{4g}$$

From Eq. (i), we get

$$H = \frac{R}{4}.$$

S3. We know,
$$H_{max} = \frac{u^2}{2g}$$

Now,
$$R_{max} = \frac{u^2}{g}$$

$$= 2\left(\frac{u^2}{2g}\right) = 2H_{max}.$$

S4. Magnitude of change in momentum = $mg \times \frac{2u\sin\theta}{g}$

$$= 2mu\sin 45° = \sqrt{2}\,mu$$

S5. We know
$$R = \frac{u^2 \sin 2\theta}{g}$$

when R_{max} if
$$\sin 2\theta = 1$$

$$2\theta = 90°$$

$$\theta = 45°$$

At an angle of 45°.

S6. Given: $R_{max} = 3$ km; $\theta = 15°$

We know $$R_{max} = \frac{u^2}{g} = 3 \text{ km}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

Setting the values, we get

$$R = 3 \sin 30° = 1.5 \text{ km.}$$

S7. Both the stones will reach the ground at the same time. This result follows from the physical independence of horizontal and vertical motions.

S8. The acceleration of the stone with respect to earth is 'g'. In the carriage, the stone possesses two accelerations: (a) horizontal acceleration 'a', due to motion of the carriage and (b) vertical accelerations are perpendicular to each other. Resultant is $\sqrt{a^2 + g^2}$. So, acceleration of stone with respect to carriage and the earth is $\sqrt{a^2 + g^2}$.

S9. ***Relative velocity:*** The relative velocity of an object w.r.t. another moving object is the effective velocity with which the object will appear to move, when the other object is considered to be at rest.

S10. Rest and motion are relative terms. A body at rest w.r.t. another body may be in motion w.r.t. some other body. For example, an electric pole is a rest w.r.t. a tree on the surface of the earth. But the same electric pole is in motion w.r.t. an observer in a running train.

S11. (a) When the balls hit the ground, their vertical velocity are equal. However the horizontal velocities will be different. Consequently, the resultant velocities are different. Thus, the balls would hit the ground with different velocities.

(b) Both the balls would reach the ground at the same time because their initial vertical velocities, acceleration and distances covered are all equal.

S12.

$$v_y = \frac{d}{dt}(8t - 5t^2) = 8 - 10t$$

$$a_y = \frac{d}{dt}(8 - 10t) = -10 \text{ units}; \quad g = 10 \text{ units.}$$

S13. Let the horizontal separation between the two balls be denoted by x.

Then, $$x = 20 \sin 60° \times 1 = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ m}.$$

Let the vertical separation between the two balls be denoted by y.

Then, $$y = \left[20 \times 1 - \frac{1}{2} g \times 1^2\right] - \left[20 \cos 60° \times 1 - \frac{1}{2} g \times 1^2\right]$$

or $$y = 20 - 20 \cos 60° = 20 - 10 = 10 \text{ m}$$

Separation between two balls

$$= \sqrt{x^2 + y^2} = \sqrt{(10\sqrt{3})^2 + (10)^2} = \sqrt{400} = \mathbf{20\,m}.$$

S14.

(a) $t = \sqrt{\dfrac{2 \times 490}{9.8}}\ s = 10\,\text{s},$

(b) $x = 98 \times 10\ \text{m} = 980\ \text{m}$

(c) $v_x = 98\ \text{ms}^{-1}$

$$v_y = 9.8 \times 10\ \text{ms}^{-1} = 98\ \text{ms}^{-1}$$

$$v^2 = v_x^2 + v_y^2$$

$$= 98^2 + 98^2 = 2 \times 98^2 = 19208$$

$$v = \mathbf{138.59\ m\ sec^{-1}}.$$

S15. Given, $\theta = 40°$, $u_w = 20\ \text{ms}^{-1}$, $x = 8$ m

We know $y = x \tan\theta - \dfrac{gx^2}{2u_w^2 \cos^2\theta}$

$$= 8 \tan 40° - \frac{9.8 \times 8 \times 8}{2 \times 20 \times 20 \times \cos^2 40°} = \mathbf{4.6\ m}.$$

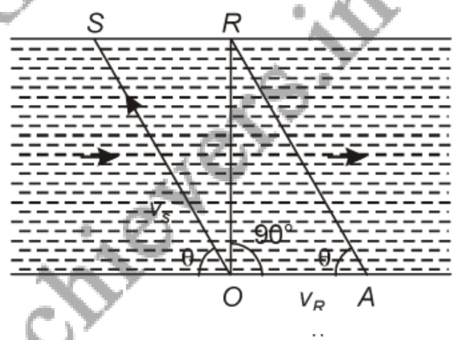
S16.

$$v_R = 5\ \text{km/h}$$

$$v_s = 10\ \text{km/h}$$

$$\cos\theta = \frac{5}{10} = \frac{1}{2}$$

$$\theta = 60°$$



$\Rightarrow$ with the direction of river, the angle would be 120° to reach the opposite point.

S17. Since the motion is arbitrary, the acceleration may not be uniform. Therefore relation (c) and (d) cannot be correct, because acceleration in these equations cannot be constant.

Again, for an arbitrary motion, the average velocity cannot be defined as in equation (a). Therefore, the relation (a) also not correct. Hence, only relations (b) and (e) are correct relations.

S18.

$$\tan\theta = \frac{v \sin\alpha - gt}{v \cos\alpha}$$

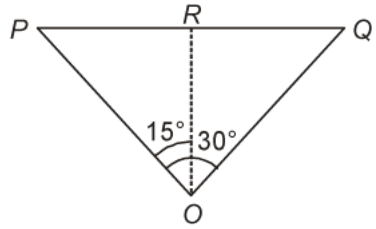
$$= \frac{30 \sin 37° - 9.8 \times 1}{30 \cos 37°}$$

or $$\tan\theta = \frac{30\times\frac{3}{5}-9.8}{30\times\frac{4}{5}} = \frac{8.2}{24} = \mathbf{0.3417}$$

$$\theta = 20.96°.$$

S19. The positions of the observer and the aircraft are shown in the given figure.

Height of the aircraft from ground, OR = 3400 m



Angle subtended between the positions, $\Delta POQ = 30°$

Time = 10 s

In ΔPRO:

$$\tan 15 = \frac{PR}{OR}$$

$$PR = OR \tan 15° = 3400 \times \tan 15°$$

ΔPRO is similar to ΔRQO.

$\therefore$ $$PR = RQ$$

$$PQ = PR + RQ$$

$$= 2PR = 2 \times 3400 \tan 15°$$

$$= 6800 \times 0.268 = 1822.4 \text{ m}$$

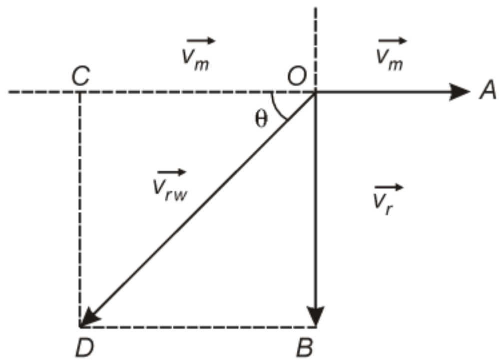
$\therefore$ Speed of the aircraft $= \frac{1822.4}{10}$ = 182.24 m/s.

S20. Horizontal acceleration of the train is not carried by the stone. horizontal velocity of the stone will remain constant, during the fall of the stone.

	Horizontal component	*Vertical component*
Velocity	10 m/s	0 m/s
Acceleration	1 m/s^2	10 m/s^2

S21. $$\overrightarrow{v_w} = 10 \text{ m/s} = \overrightarrow{OA}$$

$$\overrightarrow{v_r} = 30 \text{ m/s} = \overrightarrow{OB}$$



$\therefore$ $$\overrightarrow{v_{rw}} = \sqrt{v_r^2 + v_w^2}$$

$$= \sqrt{(10)^2 + (30)^2}$$

$$= 31.6 \text{ ms}$$

$$\tan\theta = \frac{BD}{OB} = \frac{10}{30}$$

$$= 0.33 \ (BD = OA)$$

$\theta = 18°10'$ with vertical direction.

S22. Speed of the ball, $u = 40$ m/s

Maximum height, $h = 25$ m

In projectile motion, the maximum height reached by a body projected at an angle θ, is given by the relation:

$$h = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{(40)^2 \sin^2\theta}{2 \times 9.8}$$

$$\sin^2\theta = 0.30625$$

$$\sin\theta = 0.5534$$

$\therefore$ $$\theta = \sin^{-1}(0.5534) = 33.60°$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

Horizontal range, $$R = \frac{(40)^2 \times \sin(2 \times 33.60)}{9.8}$$

$$= \frac{1600 \times \sin 67.2}{9.8}$$

$$= \frac{1600 \times 0.922}{9.8} = \mathbf{150.53\ cm}.$$

S23. Speed of the man, $v_m = 4$ km/h

Width of the river = 1 km

$$\text{Time taken to cross the river} = \frac{\text{Width of the river}}{\text{Speed of the man}}$$

$$= \frac{1}{4}\text{h} = \frac{1}{4} \times 60 = 15 \text{ min}$$

Speed of the river, $v_r = 3$ km/h

Distance covered with flow of the river = $v_r \times t$

$$= 3 \times \frac{1}{4} = \frac{3}{4}\text{ km}$$

$$= \frac{3}{4} \times 1000 = 750\text{ m}.$$

S24. (a) Average speed of taxi = Total distance travelled/Total Time taken.

$$= \frac{23\text{ km}}{28 \times (1/60)\text{h}}$$

$$= 23 \times \frac{15}{7}\text{ km/h} = 49.285\text{ km/h}$$

(b) Average velocity of taxi

$$= \frac{\text{Net displacement}}{\text{Total time taken}}$$

$$= \frac{10 \times 15}{7} = 21.4\text{ km/h}.$$

No, they are not equal.

S25. The velocity of the rain and the wind are represented by the vectors $\boldsymbol{v_r}$ and $\boldsymbol{v_w}$ in figure and are in the direction specified by the problem. Using the rule of vector addition, we see that the resultant of $\boldsymbol{v_r}$ and $\boldsymbol{v_w}$ is R as shown in the figure. The magnitude of R is

$$R = \sqrt{v_r^2 + v_w^2}$$

$$= \sqrt{35^2 + 12^2}\text{ m s}^{-1} = 37\text{ m s}^{-1}$$

The direction θ that R makes with the vertical is given by

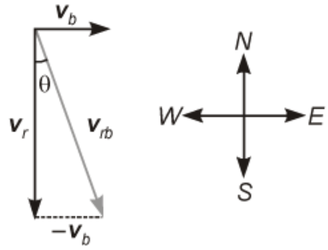
$$\tan\theta = \frac{v_w}{v_r} = \frac{12}{35} = 0.343$$

Or, $$\theta = \tan^{-1}(0.343) = 19°$$

Therefore, the boy should hold his umbrella in the vertical plane at an angle of about 19° with the vertical towards the East.

S26. In figure $\boldsymbol{v_r}$ represents the velocity of rain and $\boldsymbol{v_b}$, the velocity of the bicycle, the woman is riding. Both these velocities are with respect to the ground. Since the woman is riding a bicycle, the velocity of rain as experienced by her is the velocity of rain relative to the velocity of the bicycle she is riding. That is $\boldsymbol{v_{rb}} = \boldsymbol{v_r} - \boldsymbol{v_b}$

This relative velocity vector as shown in figure makes an angle θ with the vertical. It is given by



$$\tan\theta = \frac{v_b}{v_r} = \frac{12}{35} = 0.343$$

Or, $\theta \cong 19°$

Therefore, the woman should hold her umbrella at an angle of about 19° with the vertical towards the west.

Note carefully the difference between this Example and the Example 4.1. In Example 4.1, the boy experiences the resultant (vector sum) of two velocities while in this example, the woman experiences the velocity of rain relative to the bicycle (the vector difference of the two velocities).

S27. We choose the origin of the x-, and y-axis at the edge of the cliff and t = 0 s at the instant the stone is thrown. Choose the positive direction of x-axis to be along the initial velocity and the positive direction of y-axis to be the vertically upward direction. The x-, and y-components of the motion can be treated independently. The equations of motion are:

$$x(t) = x_0 + v_{0x}t$$
$$y(t) = y_0 + v_{0y}t + (1/2)\, a_y t^2$$

Here, $x_0 = y_0 = 0$, $voy = 0$, $ay =$
$-g = -9.8 \text{ m s}^{-2}$,
$v_{0x} = 15 \text{ m s}^{-1}$.

The stone hits the ground when

$$y(t) = -490 \text{ m}.$$
$$-490 \text{ m} = -(1/2)(9.8)\, t^2.$$

This gives $t = 10$ s.

The velocity components are $v_x = v_{0x}$ and $v_y = v_{0y} - gt$ so that when the stone hits the ground:

$$v_{0x} = 15 \text{ m s}^{-1}$$
$$v_{0y} = 0 - 9.8 \times 10 = -98 \text{ m s}^{-1}$$

Therefore, the speed of the stone is

$$\sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = 99 \text{ m s}^{-1}.$$

S28. For a projectile launched with velocity $\boldsymbol{v}_0$ at an angle θ_o, the range is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

Now, for angles, (45° + α) and (45° – α), $2\theta_0$ is (90° + 2α) and (90° – 2α), respectively. The values of sin (90° + 2α) and sin (90° – 2α) are the same, equal to that of cos 2α. Therefore, ranges are equal for elevations which exceed or fall short of 45° by equal amounts α.

S29. (a) The maximum height is given by

$$h_m = \frac{(v_0 \sin \theta_0)^2}{2g} = \frac{(28 \times \sin 30°)^2}{2(9.8)} \text{ m}$$

$$= \frac{14 \times 14}{2 \times 9.8} = 10.0 \text{ m}$$

(b) The time taken to return to the same level is or time of flight

$$T_f = (2v_0 \sin \theta_0)/g = (2 \times 28 \times \sin 30°)/9.8$$

$$= 28/9.8 \text{ s} = 2.9 \text{ s}$$

(c) The distance from the thrower to the point where the ball returns to the same level is

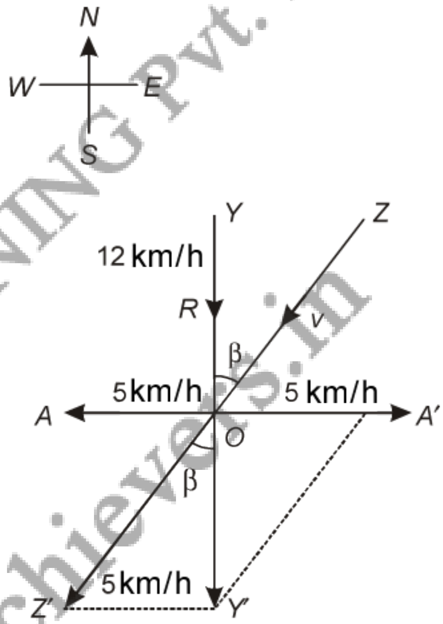
$$R = \frac{(v_0^2 \sin 2\theta_0)}{g} = \frac{28 \times 28 \times \sin 60°}{9.8} = 69 \text{ m.}$$

S30. Since the rain appears to fall down vertically.

$\therefore$ Resultant velocity R = 12 km/h along YO.

Velocity of man = 5 km/h due west along OA.

Let v be the velocity of rain falling along ZO in a direction making an angle β with OY.



Impress a velocity OA' equal and opposite to the velocity of the man on both these velocities. The man is brought to rest and rain has two velocities one along $OZ' = v$ and the other along OA' = 5 km/h . The resultant of these velocities R = 12 km/h is acting along OY.

$\therefore$ Actual velocity of rain $v = \sqrt{12^2 + 5^2} = 13$ km/h

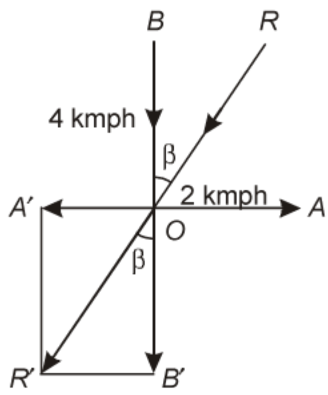
Also, $\tan \beta = \frac{5}{12} = 0.4167$

or $\beta = 22° - 37'$

Hence the rain has an actual velocity of 13 kmph along the direction ZO making an angle of $22° - 37'$ east to vertical.

S31. Velocity of rain = 4 km h^{-1} along BO

Velocity of man = 2 km h^{-1} along OA



To find the velocity of rain relative to man impress upon both a velocity OA' equal and opposite to that of the man. The resultant of OB' and OA' will be the relative velocity of rain with respect to the man.

Produce BO to B' so that both the velocities acting at O are directed away from it. The resultant is represented by OR' and is given by

$$OR' = \sqrt{2^2 + 4^2} = 2\sqrt{5} \text{ km/h}$$

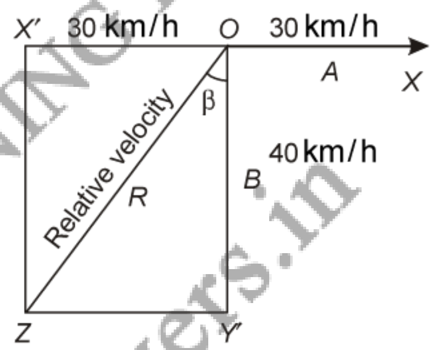
The rain will thus appear to fall along RO making an angle β east of north which is given by

$$\tan\beta = \frac{B'R'}{B'O} = \frac{2}{4} = 0.5000$$

$\therefore$ $\beta = 26° - 34'$.

S32. Velocity of ship A = 30 km/h along OX

Velocity of ship B = 40 km/h along OY.



To find the relative velocity of B with respect to A, impress upon both a velocity equal and opposite to that of A. The A ship is brought to a stationary position and B possesses two velocities one of 40 km/h along OY' and the other of 30 km/h along OX'.

$\therefore$ Relative velocity of B with respect to A

$$= \sqrt{30^2 + 40^2} = 50 \text{ km/h}$$

Suppose the resultant makes an angle β with OY', then

$$\tan\beta = \frac{YZ}{OY'} = \frac{30}{40} = 0.7500$$

or $\beta = 36° - 52'$

Hence the ship B will appear to move with a velocity of 50 km/h in a direction making an angle of 36° – 52′ due west of south.

S33. Velocity of the boat, v_b = 51 km/h

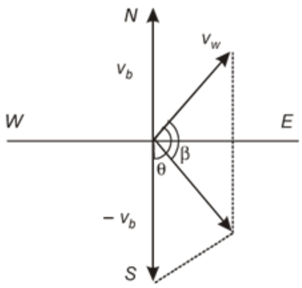
Velocity of the wind, v_w = 72 km/h

The flag is fluttering in the north-east direction. It shows that the wind is blowing toward the north-east direction. When the ship begins sailing toward the north, the flag will move along the direction of the relative velocity (v_{wb}) of the wind with respect to the boat.

The angle between v_w and $(-v_b)$ = 90° + 45°

$$\tan\beta = \frac{v_b \sin(90+45)}{v_w + v_b\cos(90+45)} = \frac{51\sin(90+45)}{72+51\cos(90+45)}$$

$$= \frac{51\sin 45}{72+51(-\cos 45)} = \frac{51\times\frac{1}{\sqrt{2}}}{72-51\times\frac{1}{\sqrt{2}}}$$



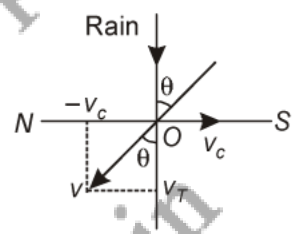
$$= \frac{51}{72\sqrt{2}-51} = \frac{51}{72\times1.414-51} = \frac{51}{50.800}$$

$\therefore\quad \beta = \tan^{-1}(1.0038) = 45.11°$

Angle with respect to the east direction = 45.11° – 45° = 0.11°

Hence, the flag will flutter almost due east.

S34. The described situation is shown in the given figure.



Here,

v_c = Velocity of the cyclist

v_r = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity (v) of the rain with respect to the woman.

$$v = v_r + (-v_c)$$
$$= 30 + (-10) = 20 \text{ m/s}$$

$$\tan\theta = \frac{v_c}{v_r} = \frac{10}{30}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

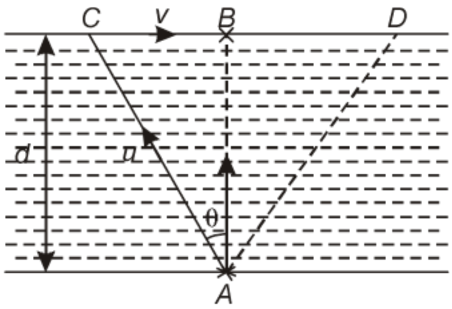
$$= \tan^{-1}(0.333) \approx 20°$$

Hence, the woman must hold the umbrella toward the south, at an angle of nearly 20° with the vertical.

S35. ***To opposite point (A to B):*** Since the river is flowing, to reach the opposite point the man should direct himself to C, *i.e.*, at angle θ with AB.

We know, $\sin\theta = \frac{CB}{AC} = \frac{v}{u}$

$\therefore$ $\theta = \sin^{-1}\left(\frac{v}{u}\right)$



i.e., at an angle $\frac{\pi}{2} + \theta$ with the direction of river.

Shortest time: To travel by shortest time, he should swim perpendicular to river current. The river will drag him to the point *D*.

The time taken

$$t = \frac{AB}{u} \text{ or } \frac{BD}{v} \text{ or } \frac{AD}{\sqrt{u^2 + v^2}}.$$

Date: 13/10/2021

Q1. What are the possible angles of projection with same velocity to have same range?

Q2. What is the angle of projection at which the h_{max} and Range are equal?

Q3. If two bodies have circular path of radius r_1 and r_2 and the time taken are the same, find the ratio of the angular speed.

Q4. A mass is projected horizontally with a velocity u from a tower. Find the horizontal length it will cover from the foot of the tower?

Q5. Prove the following statement "For Elevations which exceed or fall short of 45° by equal amounts, the ranges are equal".

Q6. Find the angle of projection for a projectile motion whose range R is n times the maximum height H.

Q7. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Q8. A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.

Q9. What is the maximum height reached by an oblique projectile if its time of flight is T?

Q10. What is the angle of projection of an oblique projectile if its range is $\frac{\sqrt{3}\,u^2}{2g}$?

Q11. A projectile is fired from ground level with velocity 500 ms^{-1} at 30° to the horizontal. Find its horizontal range, the greatest vertical height to which it rises and the time to reach the greatest height. What is the least speed with which it could be projected in order to achieve the same horizontal range? The resistance of air to the motion of the projectile may be neglected. Given: g = 10 ms^{-2}.

Q12. Prove that the path of a projectile is a parabola.

Q13. Define horizontal range and time of flight in the parabolic motion. If the time of flight of a projectile projected with a velocity u at an angle θ is $\frac{2u\sin\theta}{g}$, find the condition for maximum range and its value?

Q14. Derive a relation for the time taken by a projectile to reach the highest point and the maximum height attained?

Q15. (a) What is the change in momentum between the initial and final points of the projectile path, if the range is maximum?

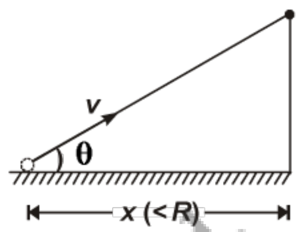
(b) Two bodies are thrown with the same initial velocity at angles θ and $(90° - \theta)$ to the horizontal. Determine the ratio of the maximum heights reached by the bodies.

Q16. A projectile is thrown at an angle of 60° with he horizontal. After how much time will its inclination with the horizontal be 45°? Give $|\vec{v}|$ = 147 m/s.

Q17. A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the *muzzle* speed to the fixed, and neglect air resistance.

Q18. A ball is thrown with an initial velocity of 100 ms^{-1} at an angle of 30° above the horizontal. How far from the throwing point will the ball attain its original level? Solve the problem without using formula for horizontal range.

Q19. A ball (shown by dotted circle) is projected directly towards a second ball (shown by dark circle). The horizontal distance x of the second ball is less than the horizontal range R of the first ball. The second ball is released from rest at the instant the first is projected. Will the two balls collide?



Q20. From the top of a tower 156.8 m high, a projectile is thrown up with a velocity of 39.2 $m\,s^{-1}$ making an angle 30° with the horizontal direction. Find the distance from the foot of tower, where it strikes the ground and the time taken by it to do so.

Q21. A cricket ball is thrown at a speed of 28 m/s in a direction 30° above the horizontal. Calculate:
(i) the maximum height.
(ii) the time taken by the ball to return to the same level.
(iii) the horizontal distance from the point of projection to the point where the ball returns to the same level.

Q22. Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right).$$

Show that the projection angle θ_0 for a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right).$$

Where the symbols have their usual meaning.

Q23. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 $m\,s^{-1}$ to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 $m\,s^{-2}$)

Q24. A projectile shot at an angle of 60° above the horizontal ground strikes a vertical wall 30 m away at a point 15 m above the ground. Find the speed with which the projectile was launched and the speed with which it strikes the wall.

Q25. Prove that the velocity at the end of flight of an oblique projectile is the same in magnitude as at the beginning but the angle that it makes with the horizontal is negative of the angle of projection.

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S1. θ and $(90 - \theta)$ are two values of projection angle at which range is same, for same velocity of projection.

S2. $\frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \sin 2\theta}{g} \Rightarrow \theta = \tan^{-1}(4) \Rightarrow \theta = 84.40°$.

S3. Since time taken T is same and $\omega = \frac{2\pi}{T}$, $\omega_1 : \omega_2 = 1 : 1$.

S4. If h is the height of the tower, the time taken to reach the ground is $t = \sqrt{\frac{2h}{g}}$. Since the horizontal velocity u is same everywhere, the distance covered is $ut = u\sqrt{\frac{2h}{g}}$.

S5. **Case (i):** When angle of projection

$$\theta = 45° + \alpha$$

and range is R

$$R = \frac{u^2 \sin 2(45° + \alpha)}{g} = \frac{u^2 \cos 2\alpha}{g}$$

Case (ii): When angle of projection

$$\theta = 45° - \alpha$$

then range is R'

$$R' = \frac{u^2 \sin 2(45° - \alpha)}{g} = \frac{u^2}{g} \sin (90° - 2\alpha) = \frac{u^2 \cos 2\alpha}{g} = R$$

$$R' = \mathbf{R}.$$

S6. Given $R = nH$

$$\Rightarrow \quad \frac{u^2 \sin 2\theta}{g} = n \times \frac{u^2 \sin^2 \theta}{2g}$$

$$\Rightarrow \quad \theta = \tan^{-1}(4/n).$$

S7. Maximum horizontal distance, $R = 100$ m

The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is 45°, *i.e.*, $\theta = 45°$.

The horizontal range for a projection velocity v, is given by the relation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$100 = \frac{u^2}{g} \sin 90^\circ$$

$$\frac{u^2}{g} = 100 \qquad \dots \text{(i)}$$

The ball will achieve the maximum height when it is thrown vertically upward. For such motion, the final velocity v is zero at the maximum height H.

Acceleration, $a = -g$

Using the third equation of motion:

$$v^2 - u^2 = -2gH$$

$$H = \frac{1}{2} \times \frac{u^2}{g} = \frac{1}{2} \times 100 = \mathbf{50\ m}.$$

S8. Given speed $u = 28$ m sec^{-1}; $\theta = 30^\circ$

(a) The maximum height is given by

$$h_{max} = \frac{u^2 \sin^2 \theta}{2g} = \frac{(28^2 \sin^2 30^\circ)}{2(9.8)} \text{m}$$

$$= \frac{28 \times 28}{2 \times 9.8 \times 4} = \mathbf{10.0\ m}.$$

(b) The time taken to return to the same level is

$$= (2u \sin \theta) / g = (2 \times 28 \times \sin 30^\circ) / 9.8$$

$$= \frac{2 \times 28}{2 \times 9.8} = \mathbf{2.9\ s}.$$

(c) The distance from the thrower to the point where the ball returns to the same level is

$$\text{Range} = \frac{u^2 \sin 2\theta}{g} = \frac{28 \times 28 \times \sin 60^\circ}{9.8} = \mathbf{69.3\ m}.$$

S9.

$$T = \frac{2u \sin \theta}{g} \quad \text{or} \quad u \sin \theta = \frac{gT}{2}$$

$$\text{Maximum height} = \frac{u^2 \sin^2 \theta}{2g} = \frac{g^2 T^2}{8g} = \frac{gT^2}{8}.$$

S10. Given

$$R = \frac{\sqrt{3}\,u^2}{2g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{\sqrt{3}\,u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$2\theta = \sin\left(\frac{\sqrt{3}}{2}\right)$$

or $\theta = \mathbf{30°}$.

S11. Given: u = 500 ms^{-1}; $\theta = 30°$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{500 \times 500 \sin 60°}{10}\text{ m} = 21650 \text{ m}$$

$$h_{max} = \frac{500 \times 500 \sin^2 30°}{2 \times 10}\text{ m}$$

$$= \frac{500 \times 500}{2 \times 10 \times 4}\text{ m} = 3125 \text{ m}$$

$$t = \frac{u \sin\theta}{g} = \frac{500 \times \sin 30°}{10} = 25\text{ s}$$

For calculation of least speed, take $\theta = 45°$.

Now, $21650 = \frac{v^2}{10}$

$$v^2 = 216500$$

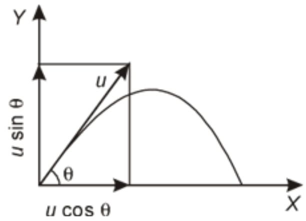
$$v = 465.30 \text{ m s}^{-1}.$$

S12. Consider a projectile thrown at an angle θ with a velocity u. The components of velocity horizontal and vertical are $u \cos\theta$ and $u \sin\theta$.

After time t, the horizontal displacement

$$x = u \cos\theta t. \quad \text{... (i)}$$

The vertical displacement



$$y = ut - \frac{1}{2}gt^2$$

$$y = u\sin\theta t - \frac{1}{2}gt^2$$

Put the value t from Eq. (i), we get

$$\therefore \qquad y = u\sin\theta \cdot \left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$y \propto x^2$, the path of a projectile is a parabola.

S13. ***Horizontal range:*** It is the distance covered by a projectile along the horizontal between the point of projection to the point on ground, where the projectile returns again. It is denoted by R.

Time of flight: It is the time taken by the projectile to return to ground or the time for which the projectile remains in air above the horizontal plane from the point of projection. It is denoted by T.

$$\text{Time of flight} = \frac{2u\sin\theta}{g}$$

$$\text{Range} = u\cos\theta \, \text{T}$$

$$\text{Range} = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$= \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$$

Range is maximum when,

$$\sin 2\theta = 1, \qquad \textit{i.e.,} \quad \theta = 45°$$

$$\text{Maximum range} = \frac{u^2}{g}.$$

S14. Consider a projectile projected at an angle θ to the horizontal with velocity u. The horizontal and vertical components initially with velocity are $u\cos\theta$ and $u\sin\theta$ respectively. Vertical velocity at highest point is zero, due to acceleration due to gravity acting vertically downwards.

Using $\qquad v = u + at,$

we have, $\qquad 0 = u\sin\theta - gt$

$$\Rightarrow \qquad t = \frac{u\sin\theta}{g}$$

The time to reach topmost point

$$t = \frac{u \sin \theta}{g}$$

Using $v^2 = u^2 + 2as$

we have, $0 = u^2 \sin \theta - 2g\, h_{max}$

$$h_{max} = \frac{u^2 \sin^2 \theta}{2g}.$$

S15. (a) Range is maximum. Therefore, the angle of projection is 45°.

Since, the initial velocity, horizontally remains same throughout, there is no change in momentum in the horizontal. The vertical velocity of $u \sin 45°$ will be equal in magnitude but opposite in direction at these points.

∴ Change in momentum

$$= 2\, mu \sin 45° = \frac{2mu}{\sqrt{2}} = \sqrt{2}\, mu.$$

(b) Let the two bodies be projected with the same velocity v. Further, suppose that H_1 and H_2 are the heights reached by the two objects thrown at angles θ and $90° - \theta$ respectively.

Then, $$H_1 = \frac{u^2 \sin^2 \theta}{2g}$$

and $$H_2 = \frac{u^2 \sin^2 (90° - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

∴ $$\frac{H_1}{H_2} = \frac{u^2 \sin^2 \theta}{2g} \times \frac{2g}{u^2 \cos^2 \theta} = \mathbf{\tan^2 \theta}.$$

S16. $$v_x = v \cos 60°$$

$$v_y = v \sin 60°$$

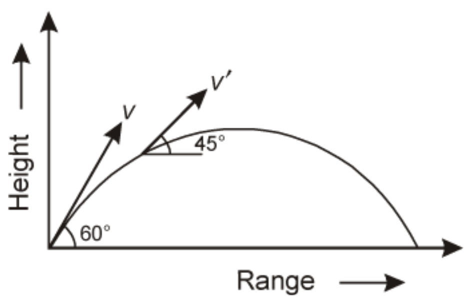
Let v' be the velocity of projection when the inclination is 45°. Since horizontal velocity is same everywhere,

$$v \cos 60° = v' \cos 45°$$

$$v \frac{1}{2} = v' \frac{1}{\sqrt{2}}$$

$$v' = \frac{147}{\sqrt{2}} \qquad (v = 147 \text{ m/s})$$

$$v_y = v \sin 60°$$



$$= \frac{147\sqrt{3}}{2} = 127.3 \text{ m/s}$$

$$v_y' = v' \sin 45° = \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{147}{2} = 73.5 \text{ m/s}$$

$$v_y' - v_y = gt$$

$$t = \frac{v_y' - v_y}{g}$$

$$= \frac{73.5 - 127.3}{-9.8} = 5.9 \text{ sec.}$$

S17. No.

Range, $R = 3$ km

Angle of projection, $\theta = 30°$

Acceleration due to gravity, g = 9.8 m/s

Horizontal range for the projection velocity u_0, is given by the relation:

$$R = \frac{u_0^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2}{g} \sin 60°$$

$$\frac{u_0^2}{g} = 2\sqrt{3} \qquad \text{... (i)}$$

The maximum range (R_{max}) is achieved by the bullet when it is fired at an angle of 45° with the horizontal, that is,

$$R_{max} = \frac{u_0^2}{g} \qquad \text{... (ii)}$$

On comparing equations (i) and (ii), we get:

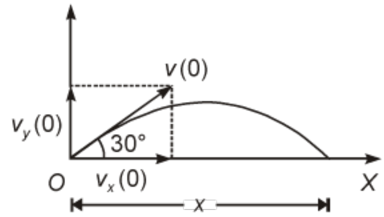
$$R_{max} = 2\sqrt{3} = 2 \times 1.732 = \mathbf{3.46 \text{ km}}$$

Hence, the bullet will not hit a target 5 km away.

S18. Let the point O from where the ball is thrown be chosen as the origin (x = 0 and y = 0). Let the time 't' when the projectile is thrown be taken as zero.

The initial velocity $v(0)$ of the projectile may be resolved into two rectangular components $v_x(0)$

and $v_y(0)$ such that



$$v_x(0) = v(0)\cos 30° = 100 \times 0.866\ \text{ms}^{-1} = 86.6\ \text{ms}^{-1}$$

and $$v_y(0) = v(0)\sin 30° = 100 \times 0.5\ \text{ms}^{-1} = 50\ \text{ms}^{-1}.$$

The motion of the projectile can be divided into two parts – horizontal motion and vertical motion. The horizontal motion is uniform motion while the vertical motion is an accelerated motion.

Consider the vertical motion.

$$y(t) = y(0) + v_y(0)\,t + \frac{1}{2}\,a_y t^2$$

Now, $$y(t) = 0$$

$$v_y(0) = 50\ \text{ms}^{-1}, \quad a_y = -g = -9.8\ \text{ms}^{-2}, \quad y(0) = 0$$

$\therefore$ $$0 = 0 + 50\,t + \frac{1}{2}(-9.8)\,t^2$$

or $$4.9\,t^2 - 50\,t = 0 \quad \text{or} \quad t(4.9\,t - 50) = 0$$

or $$t = 0 \quad \text{and} \quad 4.9\,t - 50 = 0$$

The value $t = 0$ represents the initial situation.

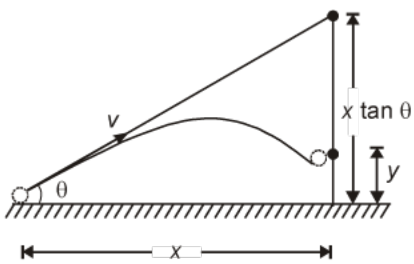
$\therefore$ $$4.9\,t - 50 = 0 \quad \text{or} \quad t = \frac{50}{4.9}\ \text{s} = 10.2\ \text{s}$$

Let us now consider the horizontal motion

$$x(t) = v_x(0)\,t = 86.6 \times 10.2\ \text{m} = \mathbf{883.32\ m}.$$

S19. Yes. Initial elevation of second ball = $x \tan\theta$. During time t, the second ball covers a distance $\frac{1}{2}gt^2$. If y is the elevation at the instant of collision, then

$$y = x\tan\theta - \frac{1}{2}gt^2 \qquad \text{... (i)}$$



For the second ball $$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

Also, $$x = v\cos\theta \times t \quad \text{or} \quad t = \frac{x}{v\cos\theta}$$

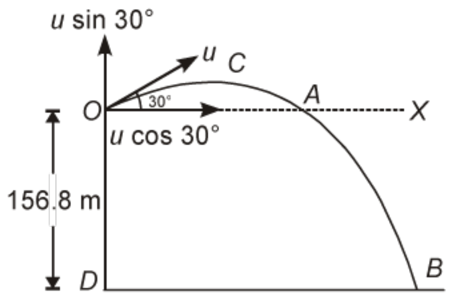
$\therefore$ $$y = x\tan\theta - \frac{1}{2}gt^2 \qquad \text{... (ii)}$$

It follows from (i) and (ii) that both the balls attain the same elevation at time t. So, the two must collide, irrespective of the initial velocity of the first ball.

S20. The Projectile is thrown at an angle of 30° with horizontal as shown in the figure.

Here, height of the tower *i.e.*, *OD* is equal to 156.8 m.

Also, $u = 39.2\ \text{m s}^{-1}$; $\theta = 30°$ and $g = 9.8\ \text{m s}^{-2}$



For motion along vertical: Let us consider *OD* as the positive direction for motion along *Y*-axis.

Initial velocity, $a_y = g = 9.8\ \text{m s}^{-2}$ $u_y = -19.6$

Now, $$y = u_y \times t + \frac{1}{2} a_y t^2$$

or $$156.8 = -19.6 \times t + \frac{1}{2} 9.8 \times t^2$$

or $$t^2 - 4t - 32 = 0$$

On solving, we obtain

$$t = -4\ \text{s} \quad \text{or} \quad 8\ \text{s}$$

Hence, the distance from the foot of the tower, where the projectile strikes against the ground,

$$x = BD = u \cos 30° \times t$$

$$= 39.2 \times 0.866 \times 8 = \mathbf{271.58\ m}.$$

S21. Here $u = 28$ m/s, $\theta = 30°$

(i) Maximum height:

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{(28)^2 \sin^2 30°}{2 \times 9.8} = \mathbf{10.0\ m}.$$

(ii) Time of flight:

$$T = \frac{2u \sin\theta}{g} = \frac{2 \times 28 \times \sin 30°}{9.8} = \mathbf{2.9\ sec}.$$

(ii) Horizontal range:

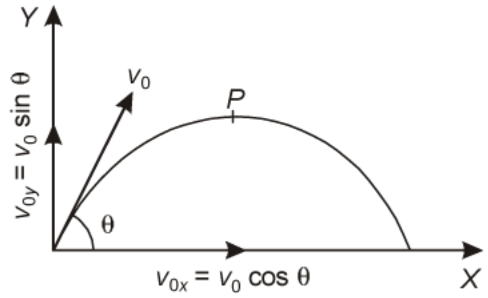
$$R = \frac{u^2 \sin 2\theta}{g} = (28)^2 \times \frac{\sin(2 \times 30°)}{9.8} = \mathbf{69.28\ m}.$$

S22. Let v_{0x} and v_{0y} respectively be the initial components of the velocity of the projectile along horizontal (*x*) and vertical (*y*) directions.

Let v_x and v_y respectively be the horizontal and vertical components of velocity at a point *P*.

Time taken by the projectile to reach point $P = t$.

Applying the first equation of motion along the vertical and horizontal directions, we get:



$$v_y = v_{0y} - gt$$

and $$v_x = v_{0x}$$

$\therefore$ $$\tan\theta = \frac{v_y}{v_x} = \frac{v_{0y} - gt}{v_{0x}}$$

$$\theta = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

Maximum vertical height,

$$h_m = \frac{u_0^2 \sin^2\theta}{2g} \quad \text{... (i)}$$

Horizontal range,

$$R = \frac{u_0^2 \sin 2\theta}{g} \quad \text{... (ii)}$$

Solving equations (i) and (ii), we get:

$$\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin^2\theta} = \frac{\sin\theta \times \sin\theta}{2 \times 2\sin\theta\cos\theta} = \frac{1}{4}\frac{\sin\theta}{\cos\theta} = \frac{1}{4}\tan\theta$$

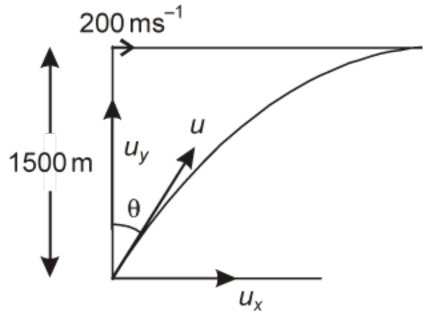
$$\tan\theta = \left(\frac{4h_m}{R}\right)$$

$$\theta = \tan^{-1}\left(\frac{4h_m}{R}\right).$$

S23. Height of the fighter plane = 1.5 km = 1500 m

Speed of the fighter plane, v = 720 km/h = 200 m/s

Let θ be the angle with the vertical so that the shell hits the plane. The situation is shown in the given figure.



Muzzle velocity of the gun, u = 600 m/s

Time taken by the shell to hit the plane = t

Horizontal distance travelled by the shell = $u_x t$

Distance travelled by the plane = vt

The shell hits the plane. Hence, these two distances must be equal.

$$u_x t = vt$$

$$u \sin\theta = v$$

$$\sin\theta = \frac{v}{u}$$

$$\theta = \sin^{-1}(0.33) = \mathbf{19.5°}.$$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (H) higher than the maximum height achieved by the shell. $u = 600$ m/s

$$\therefore \quad H = \frac{u^2 \sin^2(90-\theta)}{2g}$$

$$= \frac{(600)^2 \cos^2\theta}{2g}$$

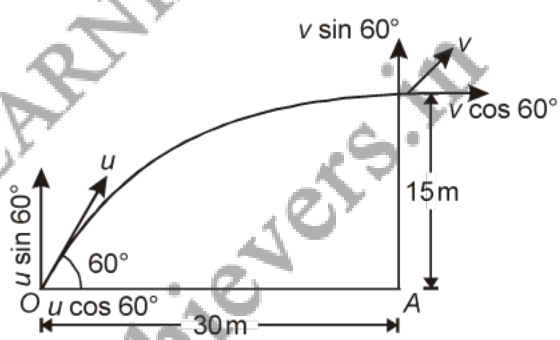
$$= \frac{360000 \times \cos^2 19.5}{2g}$$

$$= 18000 \times (0.942)^2$$

$$= 16006.482 \text{ m}$$

$$\approx \mathbf{16\ km}.$$

S24. Let the projectile be shot at angle 60° with velocity u. Velocity 'u' will have two components : Horizontal component.



$$u_x = u\cos 60° = \frac{30}{t}$$

$$t = \frac{60}{u}$$

Vertical component

$$u_y = u\sin 60°$$

Distance travelled by the projectile

$$S = u_y t + \frac{1}{2}gt^2$$

$$15 = u\sin 60° t - \frac{1}{2} \times 10 t^2$$

$$15 = \frac{\sqrt{3}}{2}ut - 5t^2$$

$$15 = 30\sqrt{3} - 5\left(\frac{60}{u}\right)^2$$

$$u = \frac{18000}{(30\sqrt{3} - 15)} = 22.07 \text{ m/s}$$

∴ Initial velocity of projectile = 22.07 m/s. Let the projectile strike the wall at point B above horizontal and vertical components. Horizontal component.

$$u_H = u \cos 60°$$

$$= 22.07 \times \frac{1}{2} = 11.03 \text{ m/s}$$

Vertical component

$$u_V = u - gt$$

$$= 22.07 - 10 \times 2.72$$

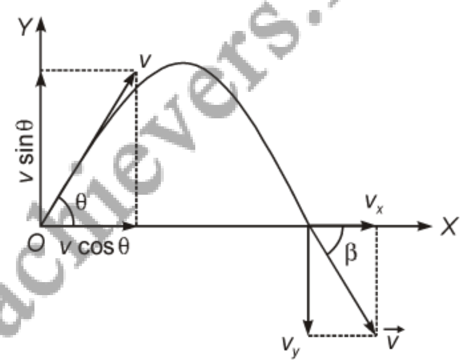
$$= -5.13 \text{ m/s} \qquad \left[\because \; t = \frac{60}{22.07} = 2.72 \text{ sec}\right]$$

Resultant velocity

$$v = \sqrt{v_V^2 + v_H^2} = \sqrt{(-5.13)^2 + (11.03)^2}$$

v = **12.16 m/s**.

S25. Let $\vec{v}$ be the velocity of the projectile at the end of flight. Let v_x and v_y be the horizontal and vertical components respectively. Since horizontal motion is uniform motion,



∴ $\quad v_x = v \cos \theta$

Again, $\quad v_y = v \cos \theta - gT$

where T is the time of flight

$$v_y = v \sin \theta - g \times \frac{2v \sin \theta}{g}$$

or $\quad v_y = -v \sin \theta$

Now, $\quad v = \sqrt{v_x^2 + v_y^2}$

or $\quad v = \sqrt{(v \cos\theta)^2 + (-v \sin \theta)^2} = \sqrt{v^2(\cos^2 \theta + \sin^2 \theta)} = v$.

So, the magnitude of the velocity at the end of the flight is equal to the magnitude of the velocity of projection.

If β is the angle which the velocity $\vec{v}$ makes with the horizontal, then

$$\tan \beta = \frac{v_y}{v_x} = \frac{-v \sin \theta}{v \cos \theta}$$

or $\tan\beta = -\tan\theta$ or $\tan\beta = \tan(-\theta)$

or $\beta = -\theta$

So, the velocity vector at the end of the flight makes an angle '$-\theta$' with the horizontal. This angle is negative of the angle of projection.

Date: 13/10/2021

Q1. What is the angle between velocity vector and acceleration vector in uniform circular motion?

Q2. Is uniform circular motion an example of uniform acceleration. Why?

Q3. Name the physical quantity which remains same in an uniform circular motion.

Q4. If both speed of a body and radius of the circular path are doubled, what will be the change in centripetal force?

Q5. Electrons revolve round the nucleus of the atom. What is the source for centripetal force?

Q6. Define the angular acceleration.

Q7. What is the angular acceleration of a particle moving in a circle of radius 'r' with a angular speed 'ω'?

Q8. Why are the passengers of a car rounding a curve thrown outward?

Q9. Two cars are going in two concentric circular orbits of radius r_1 and r_2 with angular velocities ω_1 and ω_2. What is the ratio of their linear velocities?

Q10. A cyclist has to bend a little inwards from his vertical position while turning. Why?

Q11. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Q12. Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point

(c) The acceleration vector of a particle in uniform circular averaged over one cycle is a null vector.

Q13. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Q14. Define a uniform circular motion. For uniform circular motion, prove that : Linear velocity $v = r\omega$.

Q15. Define the angular velocity and it dimensions. Establish a relation between linear velocity and angular velocity in a uniform circular motion and explain the direction of linear velocity.

Q16. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector? What is its magnitude?

Q17. Find an expression for the maximum speed of circular motion of a car in a circular horizontal track of radius '*R*'. The coefficient of static friction between the car tyres and the road along the surfaces is μ_s.

Q18. Drive an expression for the acceleration of a body of mass '*m*' moving with a uniform speed '*v*' in a circular path of radius '*r*'.

Q19. Read each statement below carefully and state, with reasons, if it is true or false:
- (a) The net acceleration of a particle in circular motion is *always* along the radius of the circle towards the centre
- (b) The velocity vector of a particle at a point is *always* along the tangent to the path of the particle at that point
- (c) The acceleration vector of a particle in *uniform* circular motion averaged over one cycle is a null vector

Q20. Define the centripetal force and centrifugal (pseudo) force. What is the angular velocity in rad s^{-1} of the hour and minute hand of a clock?

Q21. Derive the relation between linear velocity and angular velocity. Two particles are moving with common speed *v* such that they are always at a constant distance '*d*' apart and their velocities are always equal and opposite. After what time, they turn to their initial positions?

Q22. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Q23.
- (a) A wheel is 0.60 m in radius is moving with a speed of 10 m s^{-1}. Find the angular speed.
- (b) The radius of Earth's orbit around the sum is 1.5×10^{11} m. Calculate the speed of the moon in km per day.

Q24. Define the circular motion. A stone of mass 0.3 kg tied to the end of string in a horizontal plane is whirled round in a circle of radius 1 m with a frequency of 40 r.p.m. What is the tension in the string? What is the maximum speed with which the stone can be whirled around, if the string can withstand a maximum tension of 200 N?

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S1. Angle between velocity vector and acceleration vector in uniform circular motion is 90°.

S2. No, since acceleration is always towards the centre, it is different vectorically

S3. Kinetic energy and speed.

S4.

The centripetal force $F = \frac{mv^2}{r}$.

If speed and radius are doubled it gets doubled.

S5. The electrostatic force of the protons on electrons.

S6. ***Angular acceleration:*** If the angular velocity of a rotating body is not uniform, it is said to have an angular acceleration. It is defined as the rate of change of angular velocity with time. If the angular velocity increases from ω_1 to ω_2 in a time t.

S7. Since ω is constant, angular acceleration will be zero.

S8. When the car turns round a curve, the passengers sitting in the car experience an outward force, *i.e.*, centrifugal force due to the absence of the necessary centripetal force.

S9. We know $v = r\omega$

$$\therefore \quad \frac{v_1}{v_2} = \frac{r_1\,\omega_1}{r_2\,\omega_2}.$$

S10. By bending, a component of normal reaction of the ground is spared to provide him the necessary centripetal force for turning.

S11. Length of the string, l = 80 cm = 0.8 m

Number of revolutions = 14

Time taken = 25 s

$$\text{Frequency, } \nu = \frac{\text{number of revolutions}}{\text{Time taken}} = \frac{14}{25}\,Hz$$

$$\text{Angular frequency, } \omega = 2\pi\nu$$

$$= 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25}\ \text{rad s}^{-1}$$

Centripetal acceleration, $a_c = \omega^2 r$

$$= \left(\frac{88}{25}\right)^2 \times 0.8 = \mathbf{9.91\ m/s}.$$

The direction of centripetal acceleration is always directed along the string, toward the centre, at all points.

S12. (a) **It is false.** The acceleration of a particle moving along a circular path is along the radius only if the particle moves with a uniform speed.

(b) **It is true**, if the particle is released at a point it will move along the tangent to the path at the point. Since velocity is always tangentially.

(c) **It is true**, because over a complete cycle, for an acceleration at any point of circular path, there is an equal and opposite acceleration vector at a point diametrically opposite to the first point, resulting in a null net acceleration vector.

S13. Radius of the loop, $r = 1\text{ km} = 1000\text{ m}$

Speed of the aircraft, $v = 900\text{ km/h} = 900 \times \frac{5}{18} = 250\text{ m/s}$

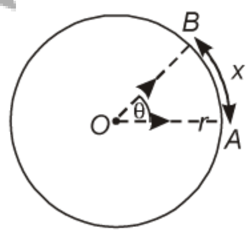
Centripetal acceleration, $a_c = \frac{v^2}{r}$

$$= \frac{(25)^2}{1000} = 62.5\text{ m/s}^2$$

Acceleration due to gravity, $g = 9.8\text{ m/s}^2$

$$= \mathbf{9.8\ m/s^2}.$$

S14. If the speed of the particle in circular path remains constant, the motion is uniform circular motion. We know, the arc length x covered with an angular displacement θ is



$$x = r\theta \quad \text{... (i)}$$

Differentiating, Eq. (i), we get

$$\frac{dx}{dt} = r\frac{d\theta}{dt} \qquad (r \text{ is constant})$$

$$v = r\omega \qquad \left\{v = \frac{dx}{dt} \text{ and } \omega = \frac{d\theta}{dt}\right\}$$

S15. ***Angular velocity:*** It is defined as the ratio of angular displacement to the time taken by the object to undergo the displacement. It is denoted by ω.

It's dimensions is $[T^{-1}]$.

We know that $s = r\theta$ if a body covers an arc of length s in a radius r, turning its radial line by θ. Differentiating both sides with respect to time, we have

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \qquad i.e., \quad v = r\omega.$$

Linear velocity = radius $\times$ angular velocity.

At each point the body moves along the tangent. The presence of centripetal force mv^2/r makes it to pass in the circular path. Thus the direction of velocity is always along the tangent at any point in the circular path.

S16. This is an example of uniform circular motion. Here $R = 12$ cm. The angular speed ω is given by

$$\omega = 2\pi/T = 2\pi \times 7/100 = 0.44 \text{ rad/s}$$

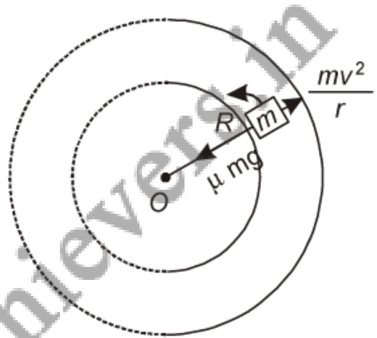
The linear speed v is:

$$v = \omega R = 0.44\,\text{s}^{-1} \times 12\,\text{cm} = 5.3\text{ cm s}^{-1}$$

The direction of velocity $\boldsymbol{v}$ is along the tangent to the circle at every point. The acceleration is directed towards the centre of the circle. Since this direction changes continuously, acceleration here is *not* a constant vector. However, the magnitude of acceleration is constant:

$$a = \omega^2 R = (0.44\text{ s}^{-1})^2\,(12\text{ cm})$$
$$= 2.3\text{ cm s}^{-2}.$$

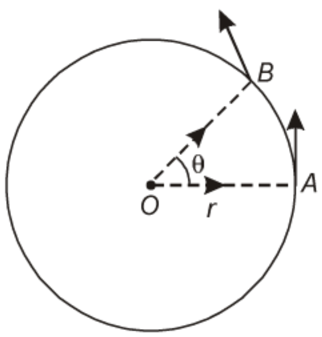
S17. Let a car of mass m move in a circular orbit of radius R as shown. The centrifugal force trying to take it away from the circular path is overcome by the force of friction.



$$\therefore \qquad \frac{mv^2}{R} = \mu_s\,\text{mg}$$
$$v^2 = \mu_s\, gR$$

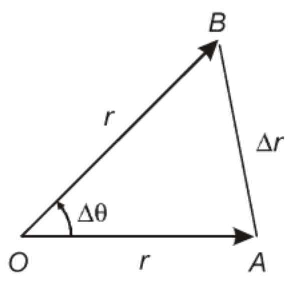
$\therefore$ Maximum speed $v = \sqrt{Rg\mu_s}$.

S18. Consider a body in a circular path of radius r, with a speed v. The velocity direction is tangential at any point in the path.



The position vectors at A and B are represented by two sides of an isosceles triangle first.

The change in position vector is indicated by $AB = \Delta r$. The velocity at A and B are along the tangents at these points and the change in velocity will complete an isosceles triangle of velocities.



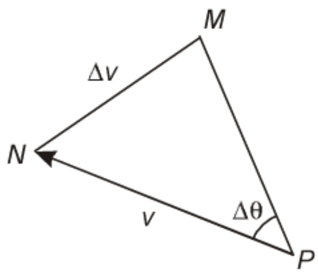
$$\overline{MN} = \overline{\Delta v}$$

Since the triangles are similar,

$$\frac{\Delta v}{\Delta r} = \frac{v}{r} \quad \Rightarrow \quad \Delta v = \frac{v}{r}.\Delta r$$

$$\underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta v}{\Delta r} = \underset{\Delta t \to 0}{\text{Lt}} \frac{v}{r}\frac{\Delta r}{\Delta t}$$

$$\therefore \quad \frac{dv}{dt} = \frac{v}{r}.v \quad \Rightarrow \quad a = \frac{v^2}{r}$$



S19. (a) False

Explanation: The net acceleration of a particle in circular motion is not always directed along the radius of the circle towards the centre. It happens only in the case of uniform circular motion.

(b) True

Explanation: At a point on a circular path, a particle appears to move tangentially to the circular path. Hence, the velocity vector of the particle is always along the tangent at a point.

(c) True

Explanation: In uniform circular motion (UCM), the direction of the acceleration vector points toward the centre of the circle. However, it constantly changes with time. The average of these vectors over one cycle is a null vector.

S20. *Centripetal force:* An external force required to make a body move along circular path with uniform speed is called ***centripetal force***.

Centrifugal force: This outward radial force experienced by an object, when in circular motion, is called ***centrifugal force***.

The hour hand of a clock completes one rotation in 12 hours *i.e.*, it covers an angle 2π in $12 \times 60 \times 60$ s.

Therefore, angular speed of the hour hand

$$= \frac{2\pi}{12 \times 60 \times 60} = \frac{\pi}{\mathbf{21,600}}\ \mathbf{rad\,s^{-1}}$$

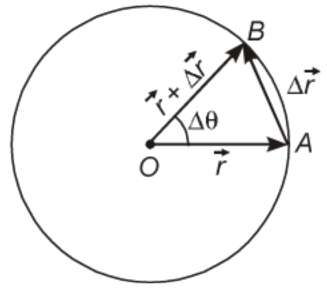
The minute hand of a watch completes one rotation in 1 hour *i.e.*, it covers an angle 2π in 60×60 s.

Therefore, angular speed of the minute hand

$$= \frac{2\pi}{60 \times 60} = \frac{\pi}{\mathbf{1,800}}\ \mathbf{rad\,s^{-1}}.$$

S21. Consider that an object is moving with uniform angular speed ω along a circular path, whose centre is O. Suppose that at any time t, the object is at point A such that $\overrightarrow{OA} = \vec{r}$ is its position vector at that time. Further, suppose that at time $t + \Delta t$, the object reaches point B (as shown in the figure). Let $\overrightarrow{OB} = \vec{r} + \Delta\vec{r}$ be the position vector of the object at time $t + \Delta t$. Then, in small time interval Δt, the object undergoes the linear displacement $\overrightarrow{AB} = \Delta\vec{r}$ and the angular displacement equal to $\angle AOB$ *i.e.*, equal to $\Delta\theta$.

If w is the uniform angular speed of the object, then



$$\omega = \frac{\Delta\theta}{\Delta t}$$

or $$\Delta\theta = \omega\Delta t \qquad \text{... (i)}$$

Also, $$\Delta\theta = \frac{\overset{\frown}{AB}}{OA}$$

Since Δt is very small, circular arc AB can be taken as to be equal to $|\overrightarrow{AB}|$ or $|\Delta\vec{r}|$. If the circular path is of radius $OA = OB = r$, then

$$\Delta\theta = \frac{|\Delta\vec{r}|}{r}$$

or $$|\Delta\vec{r}| = (\Delta\theta)\, r$$

Using the Eq. (i), the above relation becomes

$$|\Delta\vec{r}| = (\omega\Delta t)\, r$$

or $$\frac{|\Delta\vec{r}|}{\Delta t} = \omega r$$

$$v = \underset{\Delta t \to 0}{\text{Lt}} \frac{|\Delta\vec{r}|}{\Delta t} = \underset{\Delta t \to 0}{\text{Lt}} (\omega r)$$

or $$v = \omega r$$

In vector form $$\vec{v} = \vec{\omega} \times \vec{r}\,.$$

S22. Speed of the cyclist,

$$v = 27 \text{ km/h} = 7.5 \text{ m/s}$$

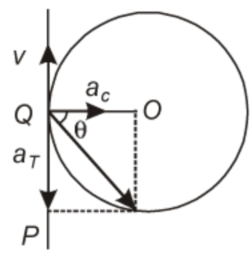
Radius of the circular turn,

$$r = 80 \text{ m}$$

Centripetal acceleration is given as:

$$a_c = \frac{v^2}{r} = \frac{(7.6)^2}{80} = 0.7 \text{ m/s}^2$$

The situation is shown in the given figure:



Suppose the cyclist begning cycling from point P and moves toward point Q. At point Q, he applies the breaks and decelerates the speed of the bicycle by 0.5 m/s^2.

This acceleration is along the tangent at Q and opposite to the direction of motion of the cyclist.

Since the angle between a_c and a_T is 90°, the resultant acceleration a is given by:

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(0.7)^2 + (0.5)^2} = \sqrt{0.74} = 0.86 \text{ m/s}^2$$

$$\tan\theta = \frac{a_c}{a_T}$$

Where θ is the angle of the resultant with the direction of velocity

$$\tan\theta = \frac{0.7}{0.5} = \mathbf{1.4}$$

$$\theta = \tan^{-1}(1.4) = \mathbf{54.46°}.$$

S23. (a) Here, $r = 0.60$ m; $v = 10 \text{ m s}^{-1}$

We know, $v = \omega r$ (ω = **Angular speed**)

∴ $\omega = \frac{v}{r} = \frac{10}{0.60}$ = **16.67 rad s⁻¹**.

(b) Here, radius of Earth's orbit $r = 1.5 \times 10^{11}$ m.

We know, period of revolution of the Earth,

$$T = 1 \text{ year} = 365 \times 24 \times 60 \times 60 = 3.154 \ \ 10^7 \text{ s}$$

(i) Therefore, angular velocity of the Earth,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.154 \times 10^7} = \mathbf{1.99 \times 10^{-7} \text{ rad s}^{-1}}$$

(ii) Linear velocity of the Earth,

$$v = r\omega = 1.5 \times 10^{11} \times 1.99 \times 10^{-7} = \mathbf{2.99 \times 10^4 \text{ m s}^{-1}}$$

(iii) The angle turned by the Earth in time t,

$$\theta = \omega t$$

Here, $\omega = 1.99 \times 10^{-7} \text{ rad s}^{-1}$

and $t = 1 \text{ day} = 24 \times 60 \times 60 \text{ s}$

∴ $\theta = 1.99 \times 10^{-7} \times 24 \times 60 \times 60 = \mathbf{1.72 \times 10^{-2} \text{ rad}}$.

S24. ***Circular motion:*** The motion of an object along a circular path with constant angular speed is called ***uniform circular motion***.

Here, $M = 0.3$ kg; $r = 1$ m;

$$v = 40 \text{ rev. min}^{-1} = 50 \text{ rev} \times (60 \text{ s}) = \frac{2}{3} \text{ r.p.s.}$$

∴ $\omega = 2\pi v = 2\pi \times \frac{2}{3} = \frac{4\pi}{3} \text{ rad s}^{-1}$

The tension in the string provides the necessary centripetal force. Therefore,

$$T = \frac{Mv^2}{r} = Mr\omega^2 = 0.3 \times 1 \times \left(\frac{4\pi}{3}\right)^2 = \mathbf{5.264 \text{ N}}.$$

Let v_{max} be the maximum speed at which tension would become 200 N.

Then, $\frac{Mv_{max}^2}{r} = 200$

or $v_{max} = \sqrt{\frac{200 \times r}{M}} = \sqrt{\frac{200 \times 1}{0.3}}$ = **25.82 m s^{-1}**.