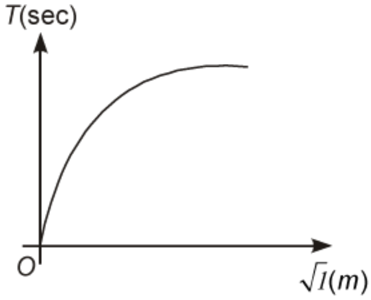
Date: 14/10/2021

S1. $T = 2\pi/\omega$

S2. No effect on time period when amplitude of pendulum is increased or decreased.

S3. Yes, e.g., circular motion is periodic but not oscillatory.

S4. $T \propto \sqrt{l}$



S5. At height as we move up 'g' decreases. Since $T \propto \dfrac{1}{\sqrt{g}}$ time period increases.

S6. Acceleration $\propto$ displacement is not sufficient since it does not refer the direction of these quantities. As you know acceleration is always against displacement.

S7. Let 1st pendulum length is l_1 and second is length l_2 and time period T_1 and T_2 respectably

$$\left(\frac{T_2}{T_1}\right)^2 = \frac{l_2}{l_1}$$

$$\left(\frac{T_2}{T_1}\right)^2 = \frac{1}{3} \Rightarrow (T_2)^2 = \frac{(2)^2}{3} \Rightarrow T_2 = \frac{2}{\sqrt{3}}$$

S8. For S.H.M $a = -\omega^2 x$ ... (i)

Comparing with $a = -16x$ ... (ii)

Compare Eq. (ii) w.r.t. (i)

$\because \quad \omega^2 = 16 \Rightarrow \quad \omega = \dfrac{2\pi}{T} = \sqrt{16} = 4$

$\therefore \quad T = \dfrac{\pi}{2}$ second

S9. The motion of a satellite around a planet is periodic. For S.H.M., the motion has to be both periodic and oscillatory.

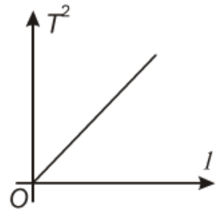
S10. The simple harmonic motion is defined as such an oscillatory motion about fixed point in which restoring force is always proportional from point and is always director towards that points.

S11. No.

S12. A motion that repeats itself over and over again after a regular interval of time about its mean position within two extreme position on the two sides of the mean position is called an ***oscillatory motion***.

S13. A motion which repeats itself over and over again after a regular interval is called a ***periodic motion***.

S14. $T^2 \propto l$.



S15.

$$T_2^2 / T_1^2 = \frac{l_3}{l_1} = \frac{l_2}{l_1} \Big/ l = \frac{1}{3}$$

or

$$T_2^2 = \frac{(2)^2}{3} \quad \text{or} \quad T_2 = \frac{2}{\sqrt{3}} \text{sec}$$

S16. We know,

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}.$$

When a stout spring is loaded with a mass m, the extension l will be lesser. Accordingly, frequency of oscillation of the ***stout spring*** will be more.

S17. No. Since acceleration is proportional to negative of displacement. Time period is independent of displacement.

S18. The time period is increased by a factor of $\sqrt{2}$.

S19.

$$\text{Maximum acceleration} = \omega^2 a = \frac{4\pi^2 \times 49}{121} \times 0.025 \, \text{m s}^{-2} = 0.4 \, \text{m s}^{-2}.$$

S20. The motion of a simple pendulum will be simple harmonic when the angular displacement θ of the bob is small.

S21. A harmonic wave function is a periodic function whose functional form is sine or cosine.

S22. Swinging through small angles.

S23.

$$T = 2\pi\sqrt{\frac{20}{980}} = \frac{2\pi}{7} \text{ second}.$$

S24. $$a = -4\pi^2 x = -\omega^2 x \Rightarrow \omega = 2\pi$$

$$\frac{2\pi}{T} = 2\pi \qquad \Rightarrow T = 1 \text{ sec.}$$

S25. No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

S26. It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π.

$$\omega = 2\pi n, \quad \text{S.I. unit is rad s}^{-1}$$

S27. In the y-z plane or in plane perpendicular to x-axis.

S28. Maximum velocity is ωA.

S29. The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.

S30. (a)
$$\sin \omega t - \cos \omega t = \sin \omega t - \sin (\pi/2 - \omega t)$$
$$= 2 \cos (\pi/4) \sin (\omega t - \pi/4)$$
$$= \sqrt{2} \sin (\omega t - \pi/4)$$

This function represents a simple harmonic motion having a period $T = 2\pi/\omega$ and a phase angle $(-\pi/4)$ or $(7\pi/4)$

(b)
$$\sin^2 \omega t = ½ - ½ \cos 2\omega t$$

The function is periodic having a period $T = \pi/\omega$. It also represents a harmonic motion with the point of equilibrium occurring at ½ instead of zero.

S31. The beat frequency of heart $= 75/(1 \text{ min})$

$= 75/(60 \text{ s})$

$= 1.25 \text{ s}^{-1}$

$= 1.25$ Hz

The time period $T = 1/(1.25 \text{ s}^{-1})$ $[\because \nu = 1/T]$

$= 0.8$ s.

S32.
When $t = 2$ s, $y = \frac{\sqrt{3}}{2} a$

Now, $y = a \sin \omega t$

$$\therefore \quad \frac{\sqrt{3}}{2} a = a \sin (\omega \times 2) \quad \text{or} \quad \sin 2\omega = \frac{\sqrt{3}}{2}$$

$$\text{or} \quad 2\omega = \frac{\pi}{3} \quad \text{or} \quad \omega = \frac{\pi}{6}$$

Hence, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/6} = \mathbf{12\,s}$.

S33. Given, $x = 0.24 \cos (400\,t - 0.5)$ …(i)

The standard equation for S.H.M. is

$$x = a \cos (2\,\pi\,\nu\,t - \phi) \quad \text{…(ii)}$$

Comparing the equations (i) and (ii), we have

$$a = 0.24 \text{ m}$$

Also, $2\,\pi\,\nu = 400$

or $\nu = \frac{400}{2\pi} = \frac{200}{\pi}$ Hz

Also, $T = \frac{1}{\nu} = \frac{\pi}{200}$ s.

S34. The simple harmonic motion may be defined as that periodic motion in which acceleration is directly proportional to the displacement from mean position and it always towards the mean position.

Characteristics of S.H.M.

1. The displacement of a S.H.M. varies sinusoidal with time *i.e.*,

$$y = r \sin \omega t.$$

2. The velocity of S.H.M. also varies sinusoidal with time *i.e.*,

$$v = r \cos \omega t = r \sin (\omega t + \pi/2).$$

3. The acceleration of S.H.M. is always directly proportional to the displacement and is directed toward the mean position.

S35. ***Periodic Motion:*** A motion which repeats itself over and over again after a regular interval is called a ***periodic motion***.

The orbital motion of the earth around the sun is periodic in nature.

Oscillatory Motion: A motion that repeats itself over and over again after a regular interval of time about its mean position with in two extreme position on the two sides of the mean position is called an ***oscillatory motion***.

The motion of a loaded spring is oscillatory in nature.

S36. Given, $T = 20$ s;

Also, when $t = 2$ s, $v = 5$ cm s^{-1}

Now, $v = a\omega \cos \omega t = a \times \frac{2\pi}{T} \cos \frac{2\pi}{T} t$

$\Rightarrow$ $5 = a \times \frac{2\pi}{20} \cos \frac{2\pi}{20} \times 2$ or $\frac{\pi a}{10} \cos \frac{\pi}{5} = 5$

or $\frac{\pi a}{10} \cos 36° = 5$ or $\frac{\pi a}{10} \times 0.8090 = 5$

or $a =$ **19.67 cm**.

S37. Since initial position at $t = 0$ is $x = 0$.

We represent S.H.M.

by $x = a \sin \omega t$ ... (i)

When $x = \frac{a}{2}$

From Eq. (i), we get

$$\frac{a}{2} = a \sin \omega t$$

$\therefore$ $\omega t = \frac{\pi}{6}$, $t = \frac{T}{12}$.

S38. Angular frequency of the piston, $\omega = 200$ rad/min.

Stroke = 1.0 m

Amplitude, $a = \frac{1.0}{2} = 0.5$ m

The maximum speed (v_{max}) of the piston is give by the relation:

$$v_{max} = a\omega$$
$$= 200 \times 0.5 = 100 \text{ m/min}$$

S39. **(b)** and **(c)** are represent the periodic motion

Explanation:

(a) The swimmer's motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

S40. Here, at $t = 0$; $x = 1$ cm and $v = \pi$ cm s^{-1};

Also, $\omega = \pi$ s^{-1}

The displacement function of the S.H.M. at any time t is given by

$$x = B \sin(\omega t + \alpha)$$

Since at $t = 0$, $x = 1$, we have

$$1 = B \sin(\omega \times 0 + \alpha) \quad \ldots \text{(i)}$$

or $B \sin \alpha = 1$

Now, $$v = \frac{dx}{dt} = \frac{d}{dt}[B \sin(\omega t + \alpha)]$$

or $$v = B\omega \cos(\omega t + \alpha)$$

Since at $t = 0$, $v = \pi$ cm s^{-1}, we have

$$\pi = B(\pi) \cos(\omega \times 0 + \alpha)$$

or $$B \cos \alpha = 1 \quad \ldots \text{(ii)}$$

Squaring and adding the equations (i) and (ii), we get

$$B^2 \sin^2 \alpha + B^2 \cos^2 \alpha = 1^2 + 1^2 \quad \text{or} \quad B^2 = 2$$

or $$B = \sqrt{2} \text{ cm}$$

Dividing the equation (i) by (ii), we have

$$\frac{B \sin \alpha}{B \cos \alpha} = \frac{1}{1} \quad \text{or} \quad \tan \alpha = 1$$

$$\alpha = \pi/4 \quad \text{or} \quad 5\pi/4.$$

S41. Given, $y = \sin^2 \omega t$

Now differentiate w.r.t. t

$$\frac{dy}{dt} = 2 \sin \omega t \times \cos \omega t \times \omega = \omega \sin 2\omega t$$

2nd derivative w.r.t. t

$$\frac{d^2y}{dt^2} = \omega \times (\cos 2\omega t) \times 2\omega = 2\omega^2 \cos 2\omega t$$

or $$a = 2\omega^2 \cos 2\omega t.$$

Since acceleration is not proportional to displacement (y), the function *does not represent a S.H.M.*

Again, $$y = \sin^2 \omega t = 1 - \cos 2\omega t$$

Thus, the given function is a periodic function of angular frequency 2ω. If T is period of the function, then

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}.$$

S42. Girl can be considered as an extended body. As the girl stands up on the swing so, the separation 'd' between the point of suspension and the centre of gravity decreases. Since time period is inversely proportional to $\sqrt{d}$, time period increases and frequency decreases.

S43. (c) ***a* = – 10 *x* is showing SHM**.

A motion represents simple harmonic motion if it is governed by the force law:

$$F = -kx$$
$$ma = -kx$$

Where, F is the force, m is the mass (a constant for a body), x is the displacement, a is the acceleration, k is a constant.

Among the given equations, only equation $a = -10\,x$ is written in the above form with $\frac{k}{m}$. Hence, this relation represents SHM.

S44. The velocity and acceleration of a body executing S.H.M. are given by

$$v = A\omega \cos \omega t = A\omega \sin (\omega t + \pi/2) \quad \text{... (i)}$$

and $$y = A \sin \omega t \quad \text{... (ii)}$$

From the Eqns. (i) and (ii), it follows that velocity and displacement of a body executing S.H.M. differ in phase by $\pi/2$.

S45. Let A be the displacement amplitude and ω, the angular frequency of the simple harmonic oscillator.

We know,

maximum acceleration, $a_0 = \omega^2 A$

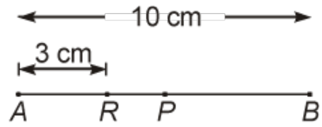
and maximum velocity, $v_0 = \omega A$

Now , $$\frac{v_0^2}{a_0} = \frac{(\omega A)^2}{\omega^2 A} = A$$

$\therefore$ $$A = \frac{v_0^2}{a_0}.$$

S46. (a) At the end B velocity is zero. Here acceleration and force are negative as they are directed along BR, *i.e.*, along negative direction.

(b) At 3 cm away from A going towards B, the particle is at R, with a tendency to move along RP which is positive direction, here velocity, acceleration are all positive.



S47. Given $x = 5 \sin \pi t$

$\because$ $x = 5$

$\cancel{5} = \cancel{5} \sin \pi t$

$1 = \sin \pi t$

$\pi t = \sin^{-1}(1)$

$$\pi t = \frac{\pi}{2}$$

$$t = \frac{1}{2} \text{ sec.}$$

S48.

Frequency of a mass attached to a spring is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$. It is independent of acceleration due to gravity. So, the frequency is not affected on the surface of moon.

S49. Here, $\nu = 0.5 \text{ s}^{-1}$; $g = 9.8 \text{ m s}^{-2}$

When displacement is y, the acceleration of S.H.M. is given by

$$a = \omega^2 y = (2\pi\nu)^2 y = 4\pi^2 \nu^2 y$$

The acceleration will be maximum at the extreme position ($y = r$) i.*e.*,

$$a_{max} = 4\pi^2 \nu^2 r$$

The block will remain in contact with the piston, if a_{max} does not exceed the acceleration due to gravity *i.e.*, a_{max} is at the most equal to g, *i.e.*,

$$4\pi^2\nu^2 r = g$$

or $$r = \frac{g}{4\pi^2 v^2} = \frac{9.8}{4\pi^2 \times (0.5)^2} = \mathbf{0.993\ m}.$$

S50. Here, $y_1 = 0.1 \sin(100\pi t + \pi/3)$ … (i)

and $y_2 = 0.1 \cos \pi t$ … (ii)

Now, $$v_1 = \frac{dy_1}{dt} = 0.1 \cos(100\pi t + \pi/3) \times 100\pi$$

$$= 10\pi \cos(100\pi t + \pi/3)$$

and $$v_2 = \frac{dy_2}{dt} = 0.1 \times (-\sin \pi t) \times \pi = -0.1\pi \sin t$$

$$= 0.1 \cos(\pi t + \pi/2)$$

Hence, phase difference of the velocity of the particle 1 with respect to the velocity of particle 2,

$$\Delta\phi = \phi_1 - \phi_2 = \pi/3 - \pi/2 = \mathbf{-\pi/6}.$$

S51. Let a be the amplitude of the two simple harmonic motions.

Since ω is angular frequency of S.H.M. along X-axis,

$$X = a \sin \omega t \qquad \dots \text{(i)}$$

Now, S.H.M. along Y-axis is of angular frequency $2\,\omega$ and it has phase difference of $\pi/2$ with S.H.M. along X-axis.

$$\therefore \qquad y = a \sin(2\,\omega t + \pi/2) = a \cos 2\,\omega t$$

$$= a\,[1 - 2\sin^2 \omega t]$$

From the question (i), substituting for $\sin \omega t$, we have

$$y = a\left[1 - 2\frac{x^2}{a^2}\right]$$

$$y = a - 2\frac{x^2}{a}$$

$$x^2 = \frac{1}{2}(a^2 - ay) \text{ (parabola)}$$

$$2x^2 = a^2 - ay.$$

S52. The displacement of the particle at time t is given by

$$y = a \sin \frac{2\pi}{T} t \qquad \dots \text{(i)}$$

Let t be the time taken by the particle to move from the mean position to a point 12.5 cm from it.

Setting $y = 12.5$ cm, $a = 25$ cm and $T = 3$ s, the equation (i) becomes

$$12.5 = 25 \sin \frac{2\pi}{3} t$$

$$\text{or} \qquad \sin \frac{2\pi}{3} t = \frac{12.5}{25}$$

$$= \frac{1}{2}$$

$$= \sin \frac{\pi}{6}$$

or $\frac{2\pi}{3} t = \frac{\pi}{6}$

or $t = \mathbf{0.25\ s}$.

Therefore, the minimum time taken by the particle to move between two points 12.5 cm on either side of the mean position,

$$2t = 2 \times 0.25 = \mathbf{0.5\ s}.$$

S53. Let the particle be at R when its velocity $v = v_{max}/2 = A\omega/2$ and its displacement from the mean position O be y.

As $v = \omega\sqrt{A^2 - y^2}$

So, $y = \sqrt{A^2 - v^2/\omega^2}$

Given $v = A\omega/2$,

then $y = \sqrt{A^2 - \frac{A^2\omega^2}{4\omega^2}} = \frac{\sqrt{3}}{2}\boldsymbol{A}$.

S54. (i) $\sin \omega t + \cos \omega t$ is a periodic function, it can also be written as $\sqrt{2}\sin(\omega t + \pi/4)$.

Now, $\sqrt{2}\sin(\omega t + \pi/4) = \sqrt{2}\sin(\omega t + \pi/4 + 2\pi)$

$$= \sqrt{2}\sin[\omega(t + 2\pi/\omega) + \pi/4]$$

The periodic time of the function is $2\pi/\omega$.

(ii) This is an example of a periodic motion. It can be noted that each term represents a periodic function with a different angular frequency. Since period is the least interval of time after which a function repeats its value, $\sin \omega t$ has a period $T_0 = 2\pi/\omega$; $\cos 2\omega t$ has a period $\pi/\omega = T_0/2$; and $\sin 4\omega t$ has a period $2\pi/4\omega = T_0/4$. The period of the first term is a multiple of the periods of the last two terms. Therefore, the smallest interval of time after which the sum of the three terms repeats is T_0, and thus the sum is a periodic function with a time period $2\pi/\omega$.

(iii) The function $e^{-\omega t}$ is not periodic, it decreases monotonically with increasing time and tends to zero as $t \to \infty$ and thus, never repeats its value.

(iv) The function $\log(\omega t)$ increases monotonically with time t. It, therefore, never repeats its value and is a non-periodic function. It may be noted that as $t \to \infty$, $\log(\omega t)$ diverges to ∞. It, therefore, cannot represent any kind of physical displacement.

S55. Given equation is

$$x = 10 \sin(10\pi t + \pi/4) \quad \dots \text{(i)}$$

$$x = a \sin(\omega t + \phi) \quad \dots \text{(ii)}$$

Compare the Eq. (i) w.r.t (ii), we have

Amplitude, **$a = 10\ m$.**

Angular frequency, **$\omega = 10\pi$.**

Epoch = initial phase $\pi/4$.

Time period, **$T = 1/5$ sec.**

Frequency, **$f = 5$ Hz.**

Maximum velocity, **$\omega a = 100\ \pi\ m\,s^{-1}$.**

S56.

$$x = 2\sin 20t$$
$$x = 2\cos 20t$$
$$x = -2\cos 20t$$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, $A = 2.0$ cm

Force constant of the spring, $k = 1200\ N\,m^{-1}$

Mass, $m = 3$ kg

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = \sqrt{400} = 20\ rad\,s^{-1}$$

When the mass is at the mean position, initial phase is 0.

Displacement, $x = A\sin\omega t$

$$= 2\sin 20t$$

At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

Displacement, $x = A\sin\left(\omega t + \frac{\pi}{2}\right)$

$$= 2\sin\left(20t + \frac{\pi}{2}\right) = 2\cos 20t$$

At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

Displacement, $x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$

$$= 2\sin\left(20t + \frac{3\pi}{2}\right) = 2\cos 20t$$

The functions have the same frequency $\left(\frac{20}{2\pi}\text{Hz}\right)$ and amplitude (2 cm), but different initial phases $\left(0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$.

S57. Here,

$$\text{Inertia factor} = \text{Oscillating mass}$$

$$= (m + m_1) = 1 + 0.5 = 1.5 \text{ kg}$$

$$\text{Spring factor} = \text{Force constant}$$

$$= K = 600 \text{ Nm}^{-1}$$

Frequency of oscillation,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertial factor}}} = \frac{10}{\pi} \text{ Hz}$$

Let v_1 be the velocity of the mass $(m + m_1)$ after collision.

According to law of conservation of linear momentum, we have

$$m_1 v = (m + m_1) v_1$$

or

$$v_1 = \frac{m_1 v}{m + m_1}$$

$$\frac{m_1 v}{m + m_1} = \frac{0.5 \times 3}{1 + 0.5} = 1 \text{ ms}^{-1}$$

Here the collision is inelastic. According to the law of conservation of mechanical energy, we have

$$(\text{K.E.})_{max} = (\text{P.E.})_{max}$$

i.e.,

$$\frac{1}{2}(m + m_1) v_1^2 = \frac{1}{2} kA^2$$

$$A = v_1 \sqrt{\frac{m + m_1}{k}}$$

$$= \sqrt{\frac{1.5}{600}} = \frac{1}{20} = \textbf{5 cm}.$$

S58. Here, $a = 0.07$ m, $T = 5.5$ sec

$\therefore$ Angular velocity $\omega = \frac{2\pi}{T} = \frac{2 \times 22}{5.5 \times 7} = \frac{8}{7} \text{ rad s}^{-1}$

(a) At the mid-point displacement $y = 0$

$\therefore$ Velocity $= \omega\sqrt{a^2 - y^2} = \frac{8}{7}\sqrt{0.0049 - 0} = \frac{8}{7} \times 0.07$

$$= 0.08 \text{ m s}^{-1}$$

$$\text{Acceleration} = -\omega^2 y = -\omega^2 \times 0 = \mathbf{0}.$$

(b) At the end of the path $\quad y = a = 0.07$ m

$$\therefore \quad \text{Velocity} = \omega\sqrt{a^2 - y^2} = \frac{8}{7}\sqrt{0.0049 - 0.0049}$$

$$= 0 \text{ m s}^{-1}$$

$$\text{Acceleration} = -\omega^2 y = -\frac{8}{7} \times \frac{8}{7} \times 0.07 = \mathbf{-0.0914 \ m\,s^{-2}}.$$

(c) $\quad a = 0.07$ m; $\quad y = 0.05$ m

$$\text{Velocity} = \omega\sqrt{a^2 - y^2}$$

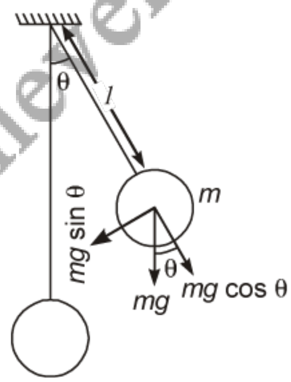
$$= \frac{8}{7}\sqrt{(0.07)^2 - (0.05)^2}$$

$$= \mathbf{0.6 \ m\,s^{-1}}$$

$$\text{Acceleration} = -\omega^2 y = \left(\frac{8}{7}\right)^2 \times 0.05 \text{ m}$$

$$= \mathbf{0.07 \ m\,s^{-2}}.$$

S59. Restoring force is provided by the portion $mg \sin\theta$ of gravitational force. Since, it acts perpendicular to length l, the restoring torque $= -$mg l



Also $\quad \tau = I\alpha = ml^2\alpha$

$$ml^2\alpha = -mg \sin\theta \,.\, l$$

$$\alpha = -\frac{g \sin\theta}{l}$$

For small angles of oscillation. $\sin\theta \cong \theta$.

$$\alpha = -\frac{g}{l}\,.\,\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} \quad i.e., \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0. \qquad \left\{\because \ \frac{d^2\theta}{dt^2} = \alpha\right\}$$

Giving $\quad \omega = \sqrt{\frac{g}{l}}$

$$T = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{l}{g}}.$$

S60. **(b)** and **(c)** are SHMs and **(a)** and **(d)** are periodic, but not SHMs

Explanation:

(a) During its rotation about its axis, earth comes to the same position again and again in equal intervals of time. Hence, it is a periodic motion. However, this motion is not simple harmonic. This is because earth does not have a to and fro motion about its axis.

(b) An oscillating mercury column in a U-tube is simple harmonic. This is because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.

(c) The ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to its initial position in the same period of time, again and again. Hence, its motion is periodic as well as simple harmonic.

(d) A polyatomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic.

S61. (a) SHM

The given function is:

$$\sin \omega t - \cos \omega t = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$$

$$= \sqrt{2}\left[\sin\omega t \times \cos\frac{\pi}{4} - \cos\omega t \times \sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

This function represents SHM as it can be written in the form: $a\sin(\omega t + \phi)$

Its period is: $\frac{2\pi}{\omega}$.

(b) Periodic, but not SHM

The given function is:

$$\sin^3 \omega t = \frac{1}{2}[3\sin\omega t - \sin 3\omega t]$$

The terms $\sin \omega t$ and $\sin 3\omega t$ individually represent simple harmonic motion (SHM).

However, the superposition of two SHM is periodic and not simple harmonic.

(c) SHM

The given function is:

$$3\cos\left[\frac{\pi}{4} - 2\omega t\right] = 3\cos\left[2\omega t - \frac{\pi}{4}\right]$$

This function represents simple harmonic motion because it can be written in the form:

$$a\cos(\omega t + \phi)$$

Its period is: $\frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Periodic, but not SHM

(d) The given function is $\cos\omega t + \cos 3\omega t + \cos 5\omega t$. Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic.

Non-periodic motion

(e) The given function $\exp(-\omega^2 t^2)$ is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion.

(f) The given function $1 + \omega t + \omega^2 t^2$ is non-periodic.

S62. (a) Zero, Positive, Positive (b) Zero, Negative, Negative

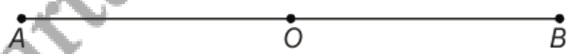
(c) Negative, Zero, Zero (d) Negative, Negative, Negative

(e) Zero, Positive, Positive (f) Negative, Negative, Negative

Explanation:

The given situation is shown in the following figure. Points *A* and *B* are the two end points, with *AB* = 10 cm. O is the midpoint of the path.

A particle is in linear simple harmonic motion between the end points.



(a) At the extreme point *A*, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is positive as it is directed along *AO*.

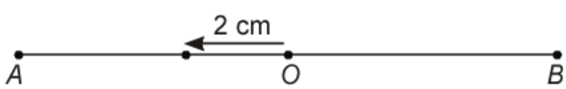
Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point *B*, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is negative as it is directed along *B*.

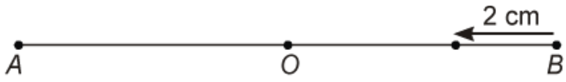
Force is also negative in this case as the particle is directed leftward.

(c) The particle is executing a simple harmonic motion. *O* is the mean position of the particle. Its velocity at the mean position *O* is the

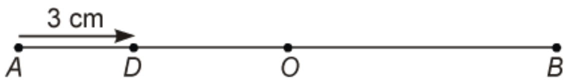


maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d) The particle is moving toward point *O* from the end *B*. This direction of motion is opposite to the conventional positive direction, which is from *A* to *B*. Hence, the particle's velocity and acceleration, and the force on it are all negative.

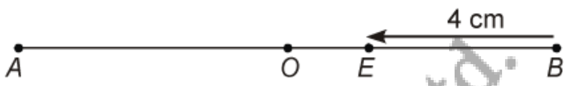


(e) The particle is moving toward point *O* from the end *A*. This direction of motion is from *A* to *B*, which is the conventional positive direction.



Hence, the values for velocity, acceleration, and force are all positive.

(f) This case is similar to the one given in **(d)**.



S63. **(b)** and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

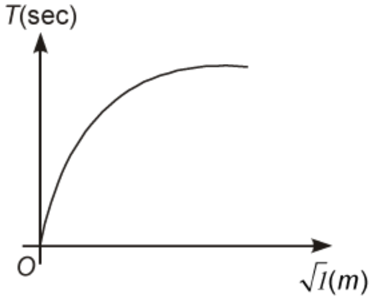
Date: 14/10/2021

S1. $T = 2\pi/\omega$

S2. No effect on time period when amplitude of pendulum is increased or decreased.

S3. Yes, e.g., circular motion is periodic but not oscillatory.

S4. $T \propto \sqrt{l}$



S5. At height as we move up 'g' decreases. Since $T \propto \frac{1}{\sqrt{g}}$ time period increases.

S6. Acceleration $\propto$ displacement is not sufficient since it does not refer the direction of these quantities. As you know acceleration is always against displacement.

S7. Let 1st pendulum length is l_1 and second is length l_2 and time period T_1 and T_2 respectably

$$\left(\frac{T_2}{T_1}\right)^2 = \frac{l_2}{l_1}$$

$$\left(\frac{T_2}{T_1}\right)^2 = \frac{1}{3} \Rightarrow (T_2)^2 = \frac{(2)^2}{3} \Rightarrow T_2 = \frac{2}{\sqrt{3}}$$

S8. For S.H.M $\quad a = -\omega^2 x \quad$ ... (i)

Comparing with $\quad a = -16x \quad$ ... (ii)

Compare Eq. (ii) w.r.t. (i)

$\because \quad \omega^2 = 16 \Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{16} = 4$

$\therefore \quad T = \frac{\pi}{2}$ second

S9. The motion of a satellite around a planet is periodic. For S.H.M., the motion has to be both periodic and oscillatory.

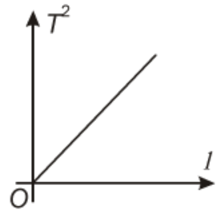
S10. The simple harmonic motion is defined as such an oscillatory motion about fixed point in which restoring force is always proportional from point and is always director towards that points.

S11. No.

S12. A motion that repeats itself over and over again after a regular interval of time about its mean position within two extreme position on the two sides of the mean position is called an ***oscillatory motion***.

S13. A motion which repeats itself over and over again after a regular interval is called a ***periodic motion***.

S14. $T^2 \propto l$.



S15.

$$T_2^2 / T_1^2 = \frac{l_3}{l_1} = \frac{l_2}{l_1} \Big/ l = \frac{1}{3}$$

or

$$T_2^2 = \frac{(2)^2}{3} \quad \text{or} \quad T_2 = \frac{2}{\sqrt{3}} \text{ sec}$$

S16. We know,

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}.$$

When a stout spring is loaded with a mass m, the extension l will be lesser. Accordingly, frequency of oscillation of the ***stout spring*** will be more.

S17. No. Since acceleration is proportional to negative of displacement. Time period is independent of displacement.

S18. The time period is increased by a factor of $\sqrt{2}$.

S19.

$$\text{Maximum acceleration} = \omega^2 a = \frac{4\pi^2 \times 49}{121} \times 0.025\,\text{m s}^{-2} = 0.4\,\text{m s}^{-2}.$$

S20. The motion of a simple pendulum will be simple harmonic when the angular displacement θ of the bob is small.

S21. A harmonic wave function is a periodic function whose functional form is sine or cosine.

S22. Swinging through small angles.

S23.

$$T = 2\pi\sqrt{\frac{20}{980}} = \frac{2\pi}{7} \text{ second}.$$

S24.
$$a = -4\pi^2 x = -\omega^2 x \Rightarrow \omega = 2\pi$$
$$\frac{2\pi}{T} = 2\pi \qquad \Rightarrow T = 1 \text{ sec.}$$

S25. No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

S26. It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π.

$$\omega = 2\pi n, \quad \text{S.I. unit is rad s}^{-1}$$

S27. In the y-z plane or in plane perpendicular to x-axis.

S28. Maximum velocity is ωA.

S29. The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.

S30. (a)
$$\sin \omega t - \cos \omega t = \sin \omega t - \sin (\pi/2 - \omega t)$$
$$= 2 \cos (\pi/4) \sin (\omega t - \pi/4)$$
$$= \sqrt{2} \sin (\omega t - \pi/4)$$

This function represents a simple harmonic motion having a period $T = 2\pi/\omega$ and a phase angle $(-\pi/4)$ or $(7\pi/4)$

(b)
$$\sin^2 \omega t = \tfrac{1}{2} - \tfrac{1}{2} \cos 2\omega t$$

The function is periodic having a period $T = \pi/\omega$. It also represents a harmonic motion with the point of equilibrium occurring at ½ instead of zero.

S31. The beat frequency of heart $= 75/(1 \text{ min})$

$= 75/(60 \text{ s})$

$= 1.25 \text{ s}^{-1}$

$= 1.25 \text{ Hz}$

The time period $T = 1/(1.25 \text{ s}^{-1})$ $[\because \; \nu = 1/T]$

$= 0.8 \text{ s}.$

S32.
When $t = 2 \text{ s}, \quad y = \frac{\sqrt{3}}{2} a$

Now, $y = a \sin \omega t$

$$\therefore \quad \frac{\sqrt{3}}{2} a = a \sin (\omega \times 2) \quad \text{or} \quad \sin 2\omega = \frac{\sqrt{3}}{2}$$

$$\text{or} \quad 2\omega = \frac{\pi}{3} \quad \text{or} \quad \omega = \frac{\pi}{6}$$

Hence, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/6}$ = **12 s**.

S33. Given, $x = 0.24 \cos (400\,t - 0.5)$ ...(i)

The standard equation for S.H.M. is

$$x = a \cos (2\,\pi\,\nu\,t - \phi) \quad \text{...(ii)}$$

Comparing the equations (i) and (ii), we have

$$a = 0.24 \text{ m}$$

Also, $2\,\pi\,\nu = 400$

or $\nu = \frac{400}{2\pi} = \frac{200}{\pi}$ Hz

Also, $T = \frac{1}{\nu} = \frac{\pi}{200}$ s.

S34. The simple harmonic motion may be defined as that periodic motion in which acceleration is directly proportional to the displacement from mean position and it always towards the mean position.

Characteristics of S.H.M.

1. The displacement of a S.H.M. varies sinusoidal with time *i.e.*,

$$y = r \sin \omega t.$$

2. The velocity of S.H.M. also varies sinusoidal with time *i.e.*,

$$v = r \cos \omega t = r \sin (\omega t + \pi/2).$$

3. The acceleration of S.H.M. is always directly proportional to the displacement and is directed toward the mean position.

S35. ***Periodic Motion:*** A motion which repeats itself over and over again after a regular interval is called a ***periodic motion***.

The orbital motion of the earth around the sun is periodic in nature.

Oscillatory Motion: A motion that repeats itself over and over again after a regular interval of time about its mean position with in two extreme position on the two sides of the mean position is called an ***oscillatory motion***.

The motion of a loaded spring is oscillatory in nature.

S36. Given, $T = 20$ s;

Also, when $t = 2$ s, $v = 5$ cm s^{-1}

Now, $v = a\omega \cos \omega t = a \times \frac{2\pi}{T} \cos \frac{2\pi}{T} t$

$\Rightarrow$ $5 = a \times \frac{2\pi}{20} \cos \frac{2\pi}{20} \times 2$ or $\frac{\pi a}{10} \cos \frac{\pi}{5} = 5$

or $\frac{\pi a}{10} \cos 36° = 5$ or $\frac{\pi a}{10} \times 0.8090 = 5$

or $a =$ **19.67 cm**.

S37. Since initial position at $t = 0$ is $x = 0$.

We represent S.H.M.

by $x = a \sin \omega t$ ... (i)

When $x = \frac{a}{2}$

From Eq. (i), we get

$$\frac{a}{2} = a \sin \omega t$$

$\therefore$ $\omega t = \frac{\pi}{6}$, $t = \frac{T}{12}$.

S38. Angular frequency of the piston, $\omega = 200$ rad/min.

Stroke = 1.0 m

Amplitude, $a = \frac{1.0}{2} = 0.5$ m

The maximum speed (v_{max}) of the piston is give by the relation:

$$v_{max} = a\omega$$
$$= 200 \times 0.5 = 100 \text{ m/min}$$

S39. **(b)** and **(c)** are represent the periodic motion

Explanation:

(a) The swimmer's motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

S40. Here, at $t = 0$; $x = 1$ cm and $v = \pi$ cm s^{-1};

Also, $\omega = \pi$ s^{-1}

The displacement function of the S.H.M. at any time t is given by

$$x = B \sin(\omega t + \alpha)$$

Since at $t = 0$, $x = 1$, we have

$$1 = B \sin(\omega \times 0 + \alpha) \quad \ldots \text{(i)}$$

or $B \sin \alpha = 1$

Now, $$v = \frac{dx}{dt} = \frac{d}{dt}[B \sin(\omega t + \alpha)]$$

or $$v = B\omega \cos(\omega t + \alpha)$$

Since at $t = 0$, $v = \pi$ cm s^{-1}, we have

$$\pi = B(\pi)\cos(\omega \times 0 + \alpha)$$

or $$B \cos \alpha = 1 \quad \ldots \text{(ii)}$$

Squaring and adding the equations (i) and (ii), we get

$$B^2 \sin^2 \alpha + B^2 \cos^2 \alpha = 1^2 + 1^2 \quad \text{or} \quad B^2 = 2$$

or $B = \sqrt{2}$ cm

Dividing the equation (i) by (ii), we have

$$\frac{B \sin \alpha}{B \cos \alpha} = \frac{1}{1} \quad \text{or} \quad \tan \alpha = 1$$

$$\alpha = \pi/4 \quad \text{or} \quad 5\pi/4.$$

S41. Given, $y = \sin^2 \omega t$

Now differentiate w.r.t. t

$$\frac{dy}{dt} = 2 \sin \omega t \times \cos \omega t \times \omega = \omega \sin 2\omega t$$

2nd derivative w.r.t. t

$$\frac{d^2y}{dt^2} = \omega \times (\cos 2\omega t) \times 2\omega = 2\omega^2 \cos 2\omega t$$

or $$a = 2\omega^2 \cos 2\omega t.$$

Since acceleration is not proportional to displacement (y), the function *does not represent a S.H.M.*

Again, $$y = \sin^2 \omega t = 1 - \cos 2\omega t$$

Thus, the given function is a periodic function of angular frequency 2ω. If T is period of the function, then

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}.$$

S42. Girl can be considered as an extended body. As the girl stands up on the swing so, the separation 'd' between the point of suspension and the centre of gravity decreases. Since time period is inversely proportional to $\sqrt{d}$, time period increases and frequency decreases.

S43. (c) **$a = -10x$ is showing SHM**.

A motion represents simple harmonic motion if it is governed by the force law:

$$F = -kx$$
$$ma = -kx$$

Where, F is the force, m is the mass (a constant for a body), x is the displacement, a is the acceleration, k is a constant.

Among the given equations, only equation $a = -10x$ is written in the above form with $\frac{k}{m}$. Hence, this relation represents SHM.

S44. The velocity and acceleration of a body executing S.H.M. are given by

$$v = A\omega \cos \omega t = A\omega \sin (\omega t + \pi/2) \quad \text{... (i)}$$

and $$y = A \sin \omega t \quad \text{... (ii)}$$

From the Eqns. (i) and (ii), it follows that velocity and displacement of a body executing S.H.M. differ in phase by $\pi/2$.

S45. Let A be the displacement amplitude and ω, the angular frequency of the simple harmonic oscillator.

We know,

maximum acceleration, $a_0 = \omega^2 A$

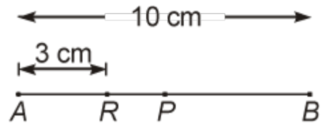
and maximum velocity, $v_0 = \omega A$

Now, $$\frac{v_0^2}{a_0} = \frac{(\omega A)^2}{\omega^2 A} = A$$

$\therefore$ $$A = \frac{v_0^2}{a_0}.$$

S46. (a) At the end B velocity is zero. Here acceleration and force are negative as they are directed along BR, *i.e.*, along negative direction.

(b) At 3 cm away from A going towards B, the particle is at R, with a tendency to move along RP which is positive direction, here velocity, acceleration are all positive.



S47. Given $x = 5 \sin \pi t$

$\because$ $x = 5$

$\cancel{5} = \cancel{5} \sin \pi t$

$1 = \sin \pi t$

$\pi t = \sin^{-1}(1)$

$$\pi t = \frac{\pi}{2}$$

$$t = \frac{1}{2} \text{ sec.}$$

S48.

Frequency of a mass attached to a spring is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. It is independent of acceleration due to gravity. So, the frequency is not affected on the surface of moon.

S49. Here, $\nu = 0.5 \text{ s}^{-1}$; $g = 9.8 \text{ m s}^{-2}$

When displacement is y, the acceleration of S.H.M. is given by

$$a = \omega^2 y = (2\pi\nu)^2 y = 4\pi^2\nu^2 y$$

The acceleration will be maximum at the extreme position ($y = r$) i.*e.*,

$$a_{max} = 4\pi^2\nu^2 r$$

The block will remain in contact with the piston, if a_{max} does not exceed the acceleration due to gravity *i.e.*, a_{max} is at the most equal to g, *i.e.*,

$$4\pi^2\nu^2 r = g$$

or $r = \frac{g}{4\pi^2 v^2} = \frac{9.8}{4\pi^2 \times (0.5)^2}$ = **0.993 m**.

S50. Here, $y_1 = 0.1 \sin(100\pi t + \pi/3)$ … (i)

and $y_2 = 0.1 \cos \pi t$ … (ii)

Now, $v_1 = \frac{dy_1}{dt} = 0.1 \cos(100\pi t + \pi/3) \times 100\pi$

$= 10\pi \cos(100\pi t + \pi/3)$

and $v_2 = \frac{dy_2}{dt} = 0.1 \times (-\sin \pi t) \times \pi = -0.1\pi \sin t$

$$= 0.1 \cos (\pi t + \pi/2)$$

Hence, phase difference of the velocity of the particle 1 with respect to the velocity of particle 2,

$$\Delta\phi = \phi_1 - \phi_2 = \pi/3 - \pi/2 = \mathbf{-\pi/6}.$$

S51. Let a be the amplitude of the two simple harmonic motions.

Since ω is angular frequency of S.H.M. along X-axis,

$$X = a \sin \omega t \quad \text{... (i)}$$

Now, S.H.M. along Y-axis is of angular frequency $2\,\omega$ and it has phase difference of $\pi/2$ with S.H.M. along X-axis.

$$\therefore \quad y = a \sin (2\,\omega t + \pi/2) = a \cos 2\,\omega t$$

$$= a\,[1 - 2 \sin^2 \omega t]$$

From the question (i), substituting for $\sin \omega t$, we have

$$y = a\left[1 - 2\frac{x^2}{a^2}\right]$$

$$y = a - 2\frac{x^2}{a}$$

$$x^2 = \frac{1}{2}(a^2 - ay) \text{ (parabola)}$$

$$2x^2 = a^2 - ay.$$

S52. The displacement of the particle at time t is given by

$$y = a \sin \frac{2\pi}{T}\, t \quad \text{... (i)}$$

Let t be the time taken by the particle to move from the mean position to a point 12.5 cm from it.

Setting $y = 12.5$ cm, $a = 25$ cm and $T = 3$ s, the equation (i) becomes

$$12.5 = 25 \sin \frac{2\pi}{3}\, t$$

or
$$\sin \frac{2\pi}{3}\, t = \frac{12.5}{25}$$

$$= \frac{1}{2}$$

$$= \sin \frac{\pi}{6}$$

or $\frac{2\pi}{3} t = \frac{\pi}{6}$

or $t = \mathbf{0.25\ s}$.

Therefore, the minimum time taken by the particle to move between two points 12.5 cm on either side of the mean position,

$$2t = 2 \times 0.25 = \mathbf{0.5\ s}.$$

S53. Let the particle be at R when its velocity $v = v_{max}/2 = A\omega/2$ and its displacement from the mean position O be y.

As $v = \omega\sqrt{A^2 - y^2}$

So, $y = \sqrt{A^2 - v^2/\omega^2}$

Given $v = A\omega/2$,

then $y = \sqrt{A^2 - \frac{A^2\omega^2}{4\omega^2}} = \frac{\sqrt{3}}{2} \boldsymbol{A}$.

S54. (i) $\sin \omega t + \cos \omega t$ is a periodic function, it can also be written as $\sqrt{2}\sin(\omega t + \pi/4)$.

Now, $\sqrt{2}\sin(\omega t + \pi/4) = \sqrt{2}\sin(\omega t + \pi/4 + 2\pi)$

$= \sqrt{2}\sin[\omega(t + 2\pi/\omega) + \pi/4]$

The periodic time of the function is $2\pi/\omega$.

(ii) This is an example of a periodic motion. It can be noted that each term represents a periodic function with a different angular frequency. Since period is the least interval of time after which a function repeats its value, $\sin \omega t$ has a period $T_0 = 2\pi/\omega$; $\cos 2\omega t$ has a period $\pi/\omega = T_0/2$; and $\sin 4\omega t$ has a period $2\pi/4\omega = T_0/4$. The period of the first term is a multiple of the periods of the last two terms. Therefore, the smallest interval of time after which the sum of the three terms repeats is T_0, and thus the sum is a periodic function with a time period $2\pi/\omega$.

(iii) The function $e^{-\omega t}$ is not periodic, it decreases monotonically with increasing time and tends to zero as $t \to \infty$ and thus, never repeats its value.

(iv) The function $\log(\omega t)$ increases monotonically with time t. It, therefore, never repeats its value and is a non-periodic function. It may be noted that as $t \to \infty$, $\log(\omega t)$ diverges to ∞. It, therefore, cannot represent any kind of physical displacement.

S55. Given equation is

$$x = 10\sin(10\pi t + \pi/4) \quad \dots \text{(i)}$$

$$x = a\sin(\omega t + \phi) \quad \dots \text{(ii)}$$

Compare the Eq. (i) w.r.t (ii), we have

Amplitude, **$a = 10\ m$.**

Angular frequency, **$\omega = 10\pi$.**

Epoch = initial phase $\pi/4$.

Time period, **$T = 1/5$ sec.**

Frequency, **$f = 5$ Hz.**

Maximum velocity, **$\omega a = 100\ \pi\ m\,s^{-1}$.**

S56.

$$x = 2\sin 20t$$
$$x = 2\cos 20t$$
$$x = -2\cos 20t$$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, $A = 2.0$ cm

Force constant of the spring, $k = 1200\ N\,m^{-1}$

Mass, $m = 3$ kg

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = \sqrt{400} = 20\ rad\ s^{-1}$$

When the mass is at the mean position, initial phase is 0.

Displacement, $x = A \sin \omega t$

$= 2 \sin 20t$

At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

Displacement, $x = A \sin\left(\omega t + \frac{\pi}{2}\right)$

$$= 2 \sin\left(20t + \frac{\pi}{2}\right) = 2 \cos 20t$$

At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

Displacement, $x = A \sin\left(\omega t + \frac{3\pi}{2}\right)$

$$= 2 \sin\left(20t + \frac{3\pi}{2}\right) = 2 \cos 20t$$

The functions have the same frequency $\left(\frac{20}{2\pi}\text{Hz}\right)$ and amplitude (2 cm), but different initial phases $\left(0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$.

S57. Here,

$$\text{Inertia factor} = \text{Oscillating mass}$$

$$= (m + m_1) = 1 + 0.5 = 1.5 \text{ kg}$$

$$\text{Spring factor} = \text{Force constant}$$

$$= K = 600 \text{ Nm}^{-1}$$

Frequency of oscillation,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertial factor}}} = \frac{10}{\pi} \text{ Hz}$$

Let v_1 be the velocity of the mass $(m + m_1)$ after collision.

According to law of conservation of linear momentum, we have

$$m_1 v = (m + m_1) v_1$$

or

$$v_1 = \frac{m_1 v}{m + m_1}$$

$$\frac{m_1 v}{m + m_1} = \frac{0.5 \times 3}{1 + 0.5} = 1 \text{ ms}^{-1}$$

Here the collision is inelastic. According to the law of conservation of mechanical energy, we have

$$(\text{K.E.})_{max} = (\text{P.E.})_{max}$$

i.e.,

$$\frac{1}{2}(m + m_1) v_1^2 = \frac{1}{2} kA^2$$

$$A = v_1\sqrt{\frac{m + m_1}{k}}$$

$$= \sqrt{\frac{1.5}{600}} = \frac{1}{20} = \textbf{5 cm}.$$

S58. Here, $a = 0.07$ m, $T = 5.5$ sec

$\therefore$ Angular velocity $\omega = \frac{2\pi}{T} = \frac{2 \times 22}{5.5 \times 7} = \frac{8}{7} \text{ rad s}^{-1}$

(a) At the mid-point displacement $y = 0$

$\therefore$ Velocity $= \omega\sqrt{a^2 - y^2} = \frac{8}{7}\sqrt{0.0049 - 0} = \frac{8}{7} \times 0.07$

$$= 0.08 \text{ m s}^{-1}$$

$$\text{Acceleration} = -\omega^2 y = -\omega^2 \times 0 = \mathbf{0}.$$

(b) At the end of the path $y = a = 0.07$ m

$$\therefore \quad \text{Velocity} = \omega\sqrt{a^2 - y^2} = \frac{8}{7}\sqrt{0.0049 - 0.0049}$$

$$= 0 \text{ m s}^{-1}$$

$$\text{Acceleration} = -\omega^2 y = -\frac{8}{7}\times\frac{8}{7}\times 0.07 = \mathbf{-0.0914\ m\,s^{-2}}.$$

(c) $a = 0.07$ m; $y = 0.05$ m

$$\text{Velocity} = \omega\sqrt{a^2 - y^2}$$

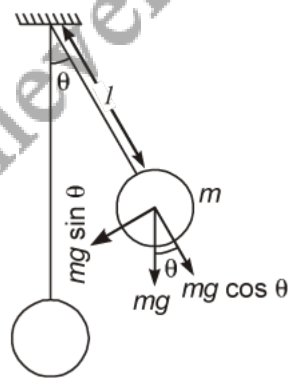
$$= \frac{8}{7}\sqrt{(0.07)^2 - (0.05)^2}$$

$$= \mathbf{0.6\ m\,s^{-1}}$$

$$\text{Acceleration} = -\omega^2 y = \left(\frac{8}{7}\right)^2 \times 0.05 \text{ m}$$

$$= \mathbf{0.07\ m\,s^{-2}}.$$

S59. Restoring force is provided by the portion $mg\sin\theta$ of gravitational force. Since, it acts perpendicular to length l, the restoring torque = – mg l



Also $\tau = I\alpha = ml^2\alpha$

$$ml^2\alpha = -mg\sin\theta \,.\, l$$

$$\alpha = -\frac{g\sin\theta}{l}$$

For small angles of oscillation. $\sin\theta \cong \theta$.

$$\alpha = -\frac{g}{l}.\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} \quad i.e., \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0. \qquad \left\{\because \ \frac{d^2\theta}{dt^2} = \alpha\right\}$$

Giving $\omega = \sqrt{\frac{g}{l}}$

$$T = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{l}{g}}.$$

S60. **(b)** and **(c)** are SHMs and **(a)** and **(d)** are periodic, but not SHMs

Explanation:

(a) During its rotation about its axis, earth comes to the same position again and again in equal intervals of time. Hence, it is a periodic motion. However, this motion is not simple harmonic. This is because earth does not have a to and fro motion about its axis.

(b) An oscillating mercury column in a U-tube is simple harmonic. This is because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.

(c) The ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to its initial position in the same period of time, again and again. Hence, its motion is periodic as well as simple harmonic.

(d) A polyatomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic.

S61. (a) SHM

The given function is:

$$\sin \omega t - \cos \omega t = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$$

$$= \sqrt{2}\left[\sin\omega t \times \cos\frac{\pi}{4} - \cos\omega t \times \sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

This function represents SHM as it can be written in the form: $a\sin(\omega t + \phi)$

Its period is: $\frac{2\pi}{\omega}$.

(b) Periodic, but not SHM

The given function is:

$$\sin^3 \omega t = \frac{1}{2}[3\sin\omega t - \sin 3\omega t]$$

The terms $\sin \omega t$ and $\sin 3\omega t$ individually represent simple harmonic motion (SHM).

However, the superposition of two SHM is periodic and not simple harmonic.

(c) SHM

The given function is:

$$3\cos\left[\frac{\pi}{4} - 2\omega t\right] = 3\cos\left[2\omega t - \frac{\pi}{4}\right]$$

This function represents simple harmonic motion because it can be written in the form:

$$a\cos(\omega t + \phi)$$

Its period is: $\frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Periodic, but not SHM

(d) The given function is $\cos \omega t + \cos 3\omega t + \cos 5\omega t$. Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic.

Non-periodic motion

(e) The given function $\exp(-\omega^2 t^2)$ is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion.

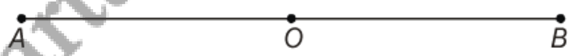
(f) The given function $1 + \omega t + \omega^2 t^2$ is non-periodic.

S62. (a) Zero, Positive, Positive
(b) Zero, Negative, Negative
(c) Negative, Zero, Zero
(d) Negative, Negative, Negative
(e) Zero, Positive, Positive
(f) Negative, Negative, Negative

Explanation:

The given situation is shown in the following figure. Points *A* and *B* are the two end points, with *AB* = 10 cm. O is the midpoint of the path.

A particle is in linear simple harmonic motion between the end points.



(a) At the extreme point *A*, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is positive as it is directed along *AO*.

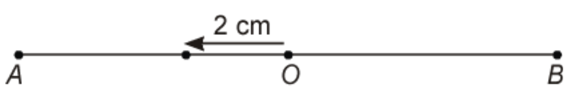
Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point *B*, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is negative as it is directed along *B*.

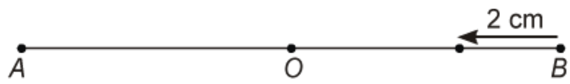
Force is also negative in this case as the particle is directed leftward.

(c) The particle is executing a simple harmonic motion. *O* is the mean position of the particle. Its velocity at the mean position *O* is the

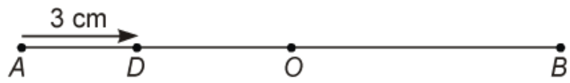


maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d) The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A to B. Hence, the particle's velocity and acceleration, and the force on it are all negative.

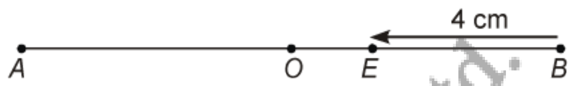


(e) The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction.



Hence, the values for velocity, acceleration, and force are all positive.

(f) This case is similar to the one given in **(d)**.



S63. **(b)** and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

Date: 14/10/2021

Q1. A Pendulum-controlled clock is transferred from earth to moon. What would be the effect on the clock?

Q2. Why a simple pendulum vibrating in air eventually stops?

Q3. Why the pitch of an organ pipe on a hot summer day is higher?

Q4. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), then time period of the pendulum increased or decrease.

Q5. How is the length of seconds pendulum related with acceleration due gravity of any planet?

Q6. State force law for a simple harmonic motion.

Q7. What is the frequency of variation of potential or kinetic energy when the frequency of the oscillation is f?

Q8. What is a second's pendulum? What is the length of a second's pendulum?

Q9. How would the period of spring mass system change when it is made to oscillate horizontally and then vertically?

Q10. Why the amplitude of the vibrating pendulum should be small?

Q11. Define force constant and give its dimensional formula.

Q12. A particle is in SHM of amplitude 2 cm. At the extreme position, the force is 4 N. What is the force at a point mid-way between mean and extreme positions?

Q13. A particle of mass is attached to a spring (of spring constant k) and has a natural angular frequency ω_o. An external force. $F(t) \propto \cos \omega t$ $(\omega \neq \omega_0)$ is applied to the oscillator. How does the time displacement of the oscillator vary?

Q14. The bob of a simple pendulum is a ball full of water. If a final hole is made in the bottom of the ball, what will be its effect on the time period of the pendulum?

Q15. Define restoring force and force constant. Give the SI unit of force constant.

Q16. (a) The shortest distance travelled by a particle moving in S.H.M. from its mean position in 2 sec. is $\sqrt{3}/2$ of its amplitude, find the period?

(b) A S.H.M. is given by $y = 5 \sin (2\pi t + \pi/4)$. What is the value of y at $t = 0$.

Q17. In what time after its motion began, will a particle oscillating according to the equation $X = 7 \sin (0.5\,\pi t)$ move from the mean position to maximum displacement.

Q18. What is the length of a simple pendulum, which ticks seconds ?

Q19. What is a second's pendulum? How much is its length on the surface of moon?

Q20. In a gasoline engine the motion of piston is a S.H.M. The piston has a mass of 2 kg and stroke of 0.1 m which is twice the amplitude. Find the maximum acceleration and maximum unbalanced force on the piston, if it vibrates 50 times per minute.

Q21. A bob executes S.H.M. of period 20 seconds. Its velocity is found to be 0.05 ms^{-1} after 2 seconds when it has passed through its mean position. Find the amplitude of the bob.

Q22. A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0. [Hint: Start with the equation $x = a \cos(\omega t + \theta)$ and note that the initial velocity is negative.]

Q23. As shown in the figure depicts two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated on the figures. Obtain the simple harmonic motions of the x-projection of the radius vector of the rotating particle P in each case.

Q24. A body oscillates with S.H.M. according to the equation (in SI units),

$$x = 5 \cos [2\pi t + \pi/4].$$

At $t = 1.5$ s, calculate the (a) displacement, (b) speed and (c) acceleration of the body.

Q25. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Q26. A body is executing SHM with an amplitude of 0.15 m and frequency 4 Hz compute (a) maximum velocity of the particle (b) acceleration when displacement is 0.09 m and (c) time required to move from mean position to a point 0.12 m away from it.

Q27. A body describes simple harmonic motion with amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm, (b) 3 cm, (c) 0 cm.

Q28. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Date: 14/10/2021

S1. Due to decrease in the value of g, T shall increase. So, the clock shall slow down.

S2. This is because its energy is dissipated in the form of heat.

S3. On a hot day, the velocity of sound will be more since (frequency proportional to velocity) the frequency of sound increases and hence its pitch increases.

S4. Increased

S5. Length of the seconds pendulum proportional to (acceleration due to gravity).

S6. Force $F \propto -x \Rightarrow F = -kx \Rightarrow F = -m\omega^2 x$.

S7. Since K.E. or P.E. $\propto \cos^2 \omega t$ or $\sin^2 \omega t$, the frequency of variation is $2f$.

S8. A pendulum a time period of 2 seconds is called second's pendulum.

Since $$T = 2\pi\sqrt{\frac{l}{g}},$$

$$l = \frac{T^2 g}{4\pi^2} = \frac{4g}{4\pi^2} = 1\,\text{m (approx)}.$$

S9. Time period is independent of g. So no change.

S10. For S.H.M., restoring force should always be pointing towards the mean position which is not possible at large angles.

S11. ***Force Constant:*** Spring constant (or ***force constant***) of a spring is defined as the restoring force setup per unit extension in the spring.

Its S.I. unit is Nm^{-1}.

The dimensional formula of force constant is $[ML^0 T^{-2}]$.

S12. 2 N.

S13. Here, natural angular frequency of the particle executing S.H.M. = ω_0

Therefore, acceleration of the particle at displacement y.

$$a_0 = \omega_0^2 y$$

Now, external force applied $\propto \cos \omega t$

Thus, external force has angular frequency ω. Therefore, forced acceleration of the particle at displacement y,

$$a' = \omega^2 y$$

Therefore, resultant acceleration of the particle at displacement y.

$$a = a_0 + a'$$

$$= -\omega_0^2 y + \omega^2 y = -(\omega_0^2 - \omega^2)\, y$$

Hence, resultant force on the particle at displacement y,

$$\frac{F}{m} = (\omega_0^2 - \omega^2)\, y$$

or $$y = \frac{F}{m(\omega_0^2 - \omega^2)}$$

or $$y \propto \frac{\mathbf{1}}{\boldsymbol{m(\omega_0^2 - \omega^2)}}.$$

S14. As water drips out, the concentration of mass becomes more on the lower portion. The centre of mass shifts down as water completely drop out then it reaches back to the original point i.e., centre of ball. So, the length increases and decreases to the original value. Time period increases first and then decreases back to the original value.

S15. ***Restoring Force:*** An equal and opposite force, which comes into play, when an external force is applied to change the configuration (shape or size) of a body.

Force Constant: The force constant of a spring may be defined as the restoring force set up per unit extension in the spring.

SI unit of force constant. It is Nm^{-1}.

S16. (a) If a is the amplitude and ω the angular velocity, then displacement at any instant t is given by

$$y = a \sin \omega t$$

or $$\sqrt{3}/2\,a = a \sin \omega t \qquad \left(\because\ y = \frac{\sqrt{3}}{2}a\right)$$

Hence $$\omega t = \pi/3$$

Now for $t = 2$ sec, we get

$$\frac{2\pi}{T} \times 2 = \frac{\pi}{3}$$

or Time period $T = 12$ s

(b) Substituting $t = 0$ in the equation $y = 5 \sin(2\pi t + \pi/4)$, we get

$$y = 5 \sin(2\pi \times 0 + \pi/4)$$

$$= 5 \sin (\pi/4)$$

$$= 5 \times \frac{1}{\sqrt{2}} = \mathbf{3.536\ cm}.$$

S17. The equation of S.H.M. is

$$X = 7 \sin (0.5\ \pi t)$$

$$\text{Amplitude (a)} = 7 \text{ cm}$$

and Maximum displacement X = amplitude = 7

$\therefore$ $$7 = 7 \sin (0.5\ \pi t)$$

or $$\sin (0.5\ \pi t) = 1$$

or $$0.5\ \pi t = \sin^{-1} (1)$$

$$0.5\ \pi t = \pi/2$$

$\Rightarrow$ $$\boldsymbol{t} = \mathbf{1\ sec}.$$

S18.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, $$l = \frac{gT^2}{4\pi^2}$$

$$l = \frac{9.8 \times (2)^2}{4 \times \frac{22}{7} \times \frac{22}{7}} = 1\text{ m}.$$

S19. A second's pendulum is one whose time period of oscillation is 2 seconds. On the surface of moon,

$$a = \frac{g}{6}$$

$\therefore$ Using $$T = 2\pi\sqrt{\frac{l}{g}}$$

We have $$l = \frac{4g}{6 \times 4\pi^2} = \frac{1}{6}\text{m}$$

S20. Given, $m = 2$ kg, $a = 0.1/2 = 0.05$ m, $\nu = 50$ r.p.m. $= 5/6$ r.p.s.

$\therefore$ $$\omega = 2\pi\nu = \frac{2\pi \times 5}{6}$$

$$= 5.236 \text{ rad s}^{-1}$$

Maximum acceleration $= a\omega^2 = 0.05 \times (5.236)^2 = \mathbf{1.37\ ms^{-2}}$

$\therefore$ Maximum force $= m \times \text{acc.} = 2 \times 1.37 = \mathbf{2.74\ N}.$

S21. Given, $T = 20$ sec., $a = ?$; $\omega = 2\pi/20 = \pi/10$ rad s^{-1} and velocity after 2s = 0.05 ms^{-1}

$$y = a \sin \omega t \quad \text{... (i)}$$

Now, differentiate Eq. (i), w.r.t. t, we get

$$v = \frac{dy}{dt} = a\omega \cos \omega t$$

$$v = a\omega \cos \omega t \quad \text{... (ii)}$$

Setting the values in Eq. (ii), we get

$$0.05 = \frac{a\pi}{10} \cos\left(\frac{\pi}{10} \times 2\right)$$

$$= \frac{a\pi}{10} \cos \frac{\pi}{5}$$

or $$0.05 = \text{a} \times \pi/10 \times 0.809$$

$$= 0.2543$$

$\therefore$ $$a = \frac{0.05}{0.2543}$$

$$= \mathbf{0.197\ m}.$$

S22. The displacement equation for an oscillating mass is given by:

$$x = A \cos (\omega t + \theta)$$

Where, A is the amplitude

x is the displacement

θ is the phase constant

Velocity, $$v = \frac{dx}{dt} = -A\omega \sin (\omega t + \theta)$$

At $t = 0$, $x = x_0$

$$x_0 = A \cos \theta = x_0 \quad \text{... (i)}$$

And, $$\frac{dx}{dt} = -v_0 = A\omega \sin \theta$$

$$A \sin \theta = \frac{v_0}{\omega} \quad \text{... (ii)}$$

Squaring and adding equations (i) and (ii), we get:

$$A^2(\cos^2 \theta + \sin^2 \theta) = x_0^2 + \left(\frac{v_0^2}{\omega^2}\right)$$

$\therefore \qquad A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$

Hence, the amplitude of the resulting oscillation is $\sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$.

S23. (a) At $t = 0$, OP makes an angle of $45° = \pi/4$ rad with the (positive direction of) x-axis. After time t, it covers an angle $\frac{2\pi}{T}t$ in the anticlockwise sense, and makes an angle of $\frac{2\pi}{T}t + \frac{\pi}{4}$ with the x-axis.

The projection of OP on the x-axis at time t is given by,

$$x(t) = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{4}\right)$$

For $T = 4$ s,

$$x(t) = A\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$$

which is a S.H.M. of amplitude A, period 4 s, and an initial phase $= \frac{\pi}{4}$.

(b) In this case at $t = 0$, OP makes an angle of $90° = \frac{\pi}{2}$ with the x-axis. After a time t, it covers an angle of $\frac{2\pi}{T}t$ in the clockwise sense and makes an angle of $\left(\frac{\pi}{2} - \frac{\pi}{T}t\right)$ with the x-axis. The projection of OP on the x-axis at time t is given by

$$x(t) = B\cos\left(\frac{\pi}{2} - \frac{\pi}{T}t\right)$$

$$= B\sin\left(\frac{2\pi}{T}t\right)$$

For $T = 30$ s,

$$x(t) = B\sin\left(\frac{\pi}{15}t\right)$$

Writing this as $x(t) = B\cos\left(\frac{\pi}{15}t - \frac{\pi}{2}\right)$, and comparing with Eq. (14.4). We find that this represents a S.H.M. of amplitude B, period 30 s, and an initial phase of $-\frac{\pi}{2}$.

S24. The angular frequency ω of the body $= 2\pi\ s^{-1}$ and its time period $T = 1$ s.

At $t = 1.5$ s

(a) $$\text{displacement} = (5.0\ \text{m})\cos[(2\pi\ s^{-1}) \times 1.5\ \text{s} + \pi/4]$$

$$= (5.0\ \text{m})\cos[(3\pi + \pi/4)]$$

$= -5.0 \times 0.707$ m

$= -3.535$ m

(b) Using Eq., the speed of the body $= -A\,\omega \sin(\omega t + \phi)$

$= -(5.0\text{ m})(2\pi\text{ s}^{-1}) \sin[(2\pi\text{ s}^{-1}) \times 1.5\text{ s} + \pi/4]$

$= -(5.0\text{ m})(2\pi\text{ s}^{-1}) \sin[(3\pi + \pi/4)]$

$= 10\pi \times 0.707\text{ m s}^{-1}$

$= 22\text{ m s}^{-1}$

(c) Using Eq., the acceleration of the body $= -\omega^2 x$

$= -(2\pi\text{ s}^{-1})^2 \times \text{displacement}$

$= -(2\pi\text{ s}^{-1})^2 \times (-3.535\text{ m})$

$= 140\text{ m s}^{-2}$.

S25. Maximum mass that the scale can read, $M = 50$ kg

Maximum displacement of the spring = Length of the scale, $l = 20$ cm = 0.2 m

Time period, $T = 0.6$ s

Maximum force exerted on the spring, $F = Mg$

Where, g = acceleration due to gravity = 9.8 m/s

$F = 50 \times 9.8 = 490$

$\therefore$ Spring constant, $k = \dfrac{F}{l} = \dfrac{490}{0.2} = 2450\text{ Nm}^{-1}$

Mass m, is suspended from the balance.

Time period, $T = 2\pi\sqrt{\dfrac{m}{k}}$

$\therefore$ $m = \left(\dfrac{T}{2\pi}\right)^2 \times k = \left(\dfrac{0.6}{2 \times 3.14}\right)^2 \times 2450 = \mathbf{22.36\ kg}$

$\therefore$ Weight of the body $= mg = 22.36 \times 9.8 =$ **219.167 N**

Hence, the weight of the body is about **219 N**.

S26. Here amplitude $a = 0.15$ m, $\nu = 4$ Hz

$\therefore$ Time period $T = \dfrac{1}{\nu} = \dfrac{1}{4} = 0.25$ sec

Angular velocity $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.25} = 8\pi\text{ rad s}^{-1}$.

(a) Maximum velocity is at O the mid-point where y = 0

$\therefore$ Max. velocity = $\omega a = 8\pi \times 0.15 =$ **3.77 m s^{-1}**.

(b) Acceleration when displacement y = 0.09 m

$\therefore$ Acceleration = $-\omega^2 y = -8\pi \times 8\pi \times 0.09 =$ **$-$56.85 m s^{-2}**.

Negative sign indicates that acceleration is directed towards mean position.

(c) Displacement $y = 0.12$ m. Substituting in $y = a \sin \omega t$, we get

$$0.12 = 0.15 \sin 8\pi t$$

or $$\sin 8\pi t = \frac{0.12}{0.15} = 0.80$$

or $$8\pi t = 53° - 8' = 0.295\pi \text{ rad}$$

$\therefore$ Time $$t = \frac{0.295\pi}{8\pi} = \mathbf{0.0369\ sec}.$$

S27. Amplitude, $A = 5$ cm = 0.05 m

Time period, $T = 0.2$ s

For displacement, $x = 5$ cm = 0.05 m

Acceleration is given by: $a = -\omega^2 x$

$$= -\left(\frac{2\pi}{T}\right)^2 x$$

$$= -\left(\frac{2\pi}{0.2}\right)^2 \times 0.05$$

$$= -5\pi^2 \text{ m/s}^2$$

Velocity is given by: $v = \omega\sqrt{A^2 - x^2}$

$$= \frac{2\pi}{T}\sqrt{(0.05)^2 - (0.05)^2}$$

$$= 0$$

When the displacement of the body is 5 cm, its acceleration is $-5\pi^2$ m/s^2 and velocity is 0.

For displacement, $x = 3$ cm = 0.03 m

Acceleration is given by: $a = -\omega^2 x$

$$= -\left(\frac{2\pi}{T}\right)^2 x$$

$$= -\left(\frac{2\pi}{0.2}\right)^2 \times 0.03$$
$$= -3\pi^2 \text{ m/s}^2$$

Velocity is given by: $v = \omega\sqrt{A^2 - x^2}$

$$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{T}\sqrt{(0.05)^2 - (0.03)^2}$$

$$= \frac{2\pi}{0.2} \times 0.04$$
$$= 0.4\pi \text{ m/s}$$

When the displacement of the body is 3 cm, its acceleration is -3π m/s^2 and velocity is 0.4π m/s.

For displacement, $x = 0$

Acceleration is given by: $a = -\omega^2 x = 0$

Velocity is given by: $v = \omega\sqrt{A^2 - x^2}$

$$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{0.2}\sqrt{(0.05)^2 - 0}$$
$$= 0.5\pi \text{ m/s}$$

When the displacement of the body is 0, its acceleration is 0 and velocityis 0.5π m/s.

S28. Mass of the automobile, $m = 3000$ kg

Displacement in the suspension system,

$$x = 15 \text{ cm} = 0.15 \text{ m}$$

There are 4 springs in parallel to the support of the mass of the automobile.

The equation for the restoring force for the system:

$$F = -4kx = mg$$

Where, k is the spring constant of the suspension system

Time period, $T = 2\pi\sqrt{\frac{m}{4k}}$

And $\qquad k = \dfrac{mg}{4x} = \dfrac{3000 \times 10}{4 \times 0.15} = 5000 = 5 \times 10^4$ N/m

Spring constant, $\qquad k = 5 \times 10^4$ N/m

Each wheel supports a mass, $M = \dfrac{3000}{4}$

For damping factor b, the equation for displacement is written as:

$$x = x_0 e^{-bt/2M}$$

The amplitude of oscillation decreases by 50%.

$\therefore \qquad x = \dfrac{x_0}{2}$

$$\frac{x_0}{2} = x_0 e^{-bt/2M}$$

$$\log_e 2 = \frac{bt}{2M}$$

$\therefore \qquad b = \dfrac{2M \log_e 2}{t}$

Where,

Time period, $\qquad t = 2\pi\sqrt{\dfrac{m}{4k}} = 2\pi\sqrt{\dfrac{3000}{4 \times 5 \times 10^4}} = 0.7691$ s

$\therefore \qquad b = \dfrac{2 \times 750 \times 0.693}{0.7691} = 1351.58$ kg/s

Therefore, the damping constant of the spring is 1351.58 kg/s.

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Q1. What fraction of the total energy is potential energy when the displacement is one-half of the amplitude?

Q2. When a particle oscillates simple harmonically, its potential energy varies periodically. If ν is the frequency of oscillation of the particle, then what is the frequency of variation of potential energy?

Q3. The total energy of a particle executing SHM is *E*. What is the kinetic energy when the displacement is equal to one-half the amplitude?

Q4. Two exactly similar simple pendulum are vibrating with amplitudes 1 cm and 3 cm. What is the ratio of their energies of vibration?

Q5. What is the frequency of total energy of a particle in S.H.M.?

Q6. How is the frequency of oscillation related with the frequency of change in the of K.E and P.E of the body in S.H.M.?

Q7. At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy?

Q8. What is the maximum value of the kinetic energy in the case of S.H.M.?

Q9. Why does the amplitude of an oscillating pendulum go on decreasing?

Q10. When is the potential energy and kinetic energy of a harmonic oscillator maximum and what are these maximum values?

Q11. Why can't we use a pendulum to work as a clock in a satellite?

Q12. A man with a wrist watch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

Q13. At what points, the energy of simple harmonic oscillator will be entirely potential? Explain.

Q14. A particle of mass 10 g is describing simple harmonic motion along a straight line a period of 2 s and of amplitude of 10 cm. What is the kinetic energy, when it is (a) 2 cm from its equilibrium position, (b) 5 cm from its equilibrium position?

Q15. Derive an expression for the potential energy of an elastic stretched spring.

Q16. What is meant by potential energy of loaded spring?

Q17. At a time when the displacement is half the amplitude, what fraction of the total energy is kinetic and what fraction is potential in S.H.M.? At what displacement is the energy half kinetic and half potential?

Q18. At what distance from mean position is the kinetic energy in simple harmonic oscillator equal to potential energy?

Q19. The time period of oscillation of spring is 1.57 sec when a mass of 100 gm is suspended from its lower end. Calculate (a) the force constant of the spring (b) the K.E. of the mass when its displacement is equal to the amplitude. (Take $\pi = 3.14$).

Q20. The total energy of a particle executing S.H.M. of period 2π second is 10,240 erg. The displacement of the particle at $\pi/4$ second is $8\sqrt{2}$ cm. Calculate the amplitude of motion and mass of the particle.

Q21. A particle is executing S.H.M. Identify the positions of the particle where,

(a) K.E. of the particle is zero. (b) P.E. of zero.

(c) P.E. is one fourth of the total energy. (d) P.E. and K.E. are equal.

Q22. A body weighing 10 g has a velocity of 6 cm s^{-1} after one second of its starting from mean position. If the time period is 6 seconds, find the kinetic energy, potential energy and the total energy.

Q23. What is meant by potential energy of loaded spring? A spring of force constant 800 Nm^{-1} has an extension of 5 cm. What is the work done in increasing the extension from 5 cm to 15 cm?

Q24. A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 $N m^{-1}$. The block is pulled to a distance $x = 10$ cm from its equilibrium position at $x = 0$ on a frictionless surface from rest at $t = 0$. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.

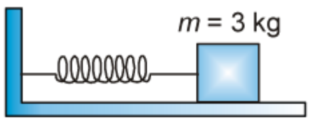
Q25. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation $J = -\alpha\theta$, where J is the restoring couple and θ the angle of twist).

Q26. Show that for a particle in linear S.H.M. the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Q27. Derive the expression potential energy for oscillatory motion determine the (a) potential energy at mean position (b) potential energy at extreme position.

Q28. Derive expression of kinetic for oscillatory motion and also find the K.E. at mean position and extreme position

Q29. A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.



Determine (a) the frequency of oscillations, (b) maximum acceleration of (c) the maximum speed of the mass.

Q30. Show that for a particle in linear S.H.M., the average K.E. over a period of oscillation equals the average P.E. over the same period.

Q31. A horizontal coiled spring is found to be stretched 3 cm from its equilibrium position, when a force of 4 dyne acts on it. Then, a bob of 2 g is attached to the end of the spring which is pulled 4 cm along a horizontal frictionless table and released. Find (a) force constant of the spring, (b) time period and (c) K.E., P.E. and total energy of the bob, when it is half way from its mean position.

Q32. A body of mass 1 kg is executing S.H.M. given by $y = 6\cos(100t + \pi/4)$ cm. What is the (a) amplitude of displacement (b) frequency (c) initial phase (d) velocity (e) acceleration (f) maximum kinetic energy.

Q33. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right)$ (b) $x = \cos\left(\frac{\pi}{6} - t\right)$ (c) $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$ (d) $x = 2\cos\pi t$

Q34. (a) Find the total energy of the particle executing S.H.M. What is the frequency of these energies with respect to the frequency of the particle executing S.H.M.

(b) A 2 kg body panel of a car oscillates with a frequency of 2 Hz and amplitude 2.5 cm. If the oscillations are assumed to be simple harmonic and undamped, calculate (i) the maximum velocity of the panel, (ii) the total energy of the panel, (iii) the maximum potential energy of the panel, and (iv) the kinetic energy of the panel 1.0 cm from its equilibrium position.

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S1. Given, $x = a/2$.

$$\frac{P.E.}{\text{Total energy}} = \frac{\frac{1}{2}m\omega^2 a^2/4}{\frac{1}{2}m\omega^2 a^2} = \frac{1}{4}.$$

S2. 2ν.

S3. Given, $x = a/2$.

$$\frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right) = \frac{3}{4}\times\frac{1}{2}m\omega^2 a^2 = \frac{3}{4}E.$$

S4. Given, a_1 = 1 cm and a_2 = 3 cm.

$$\frac{E_1}{E_2} = \frac{a_1^2}{a_2^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}.$$

S5. The frequency of total energy of particle is S.H.M is zero because it remains constant.

S6. P.E. or K.E. completes two vibrations in a time during which S.H.M completes one vibration or the frequency of P.E. or K.E. is double than that of S.H.M

S7. Not at the mid point, between mean and extreme position. it will be at

$$x = a/\sqrt{2}.$$

S8. Maximum value of K.E. is total energy. *i.e.*, $\frac{1}{2}m\omega^2 A^2$.

S9. As the pendulum oscillates, it drags air along with it. Therefore, its kinetic energy is dissipated in overcoming viscous a drag due to air and hence its amplitude goes on decreasing.

S10. Potential energy of a harmonic oscillator is maximum at extreme positions, while kinetic energy is maximum at mean position. The maximum value of both P.E. and K.E is $\frac{1}{2}mr^2\omega^2$.

S11. Time period of an oscillating pendulum changes with acceleration due to gravity. It is zero in a satellite. So, only clocks with spring can be used.

S12. Since time period of the wrist watch working on oscillation of spring is independent of acceleration due to gravity, it will give correct time during free fall also.

S13. In SHM the kinetic energy (K.E.)

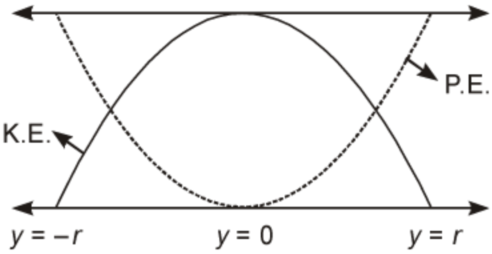
$$\text{K.E.} = \frac{1}{2} m\omega^2 (a^2 - y^2) \qquad \text{... (i)}$$

In SHM the potential energy (P.E.)

$$\text{P.E.} = \frac{1}{2} m\omega^2 y^2$$

Graph between displacement *P* energy graph

It is clear that at extreme position the PE is maximum



S14. Here, $m = 10\,\text{g}; \quad T = 2\,\text{s}; \quad a = 10\,\text{cm}$

$$\therefore \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \alpha \text{ rad s}^{-1}$$

(a) When $y = 2\,\text{cm}$:

$$\text{K.E.} = \frac{1}{2} m\omega^2 (a^2 - y^2) = \frac{1}{2} \times 10 \times (\pi)^2 \times (10^2 - 2^2)$$

$= \mathbf{4,737.6\ erg}$.

(b) When $y = 5\,\text{cm}$:

$$\text{K.E.} = \frac{1}{2} m\omega^2 (a^2 - y^2) = \frac{1}{2} \times 10 \times (\pi)^2 \times (10^2 - 5^2)$$

$= \mathbf{3,701.25\ erg}$.

S15. Consider a spring attached with a mass *m* stretching by a length *y* after *t* sec.

Restoring force

$$F = \text{mass} \times \text{acceleration}$$

$$= -m\omega^2 y = -ky$$

where, k = spring constant = $m\omega^2$

Work done for an additional displacement *dy* against restoring force is

$$dW = -F\,dy$$

$$= -(-ky)\,dy = ky\,dy$$

Total work done $\qquad W = \int_0^y ky\,dy = \frac{1}{2}ky^2$

This work done appears as a P.E. '*U*' of the particle.

$$U = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2 y^2 \qquad [\because \; y = a \sin \omega t]$$

$$= \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t .$$

S16. When a spring is loaded, the length of the spring increases and the restoring force is set up in the spring. Therefore, load does work against the restoring force in stretching the spring. The work done by the load is stored in the spring in the form of its potential energy.

S17. Given,

$$y = \frac{1}{2}\, a$$

Now,

$$\text{K.E.} = \frac{1}{2}\, m\,\omega^2(a^2 - y^2)$$

$$= \frac{1}{2} m\omega^2 \left[a^2 - \left(\frac{a}{2}\right)^2\right] = \frac{3}{8}\, m\,\omega^2 a^2$$

$$\text{P.E.} = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 \left(\frac{a}{2}\right)^2 = \frac{1}{8}\, m\,\omega^2 a^2$$

Total energy,

$$E = \frac{1}{2}\, m\,\omega^2\, a^2$$

$\therefore$

$$\frac{\text{K.E.}}{E} = \frac{\frac{3}{8} m\omega^2 a^2}{\frac{1}{2} m\omega^2 a^2} = \frac{3}{4}$$

Also

$$\frac{\text{P.E.}}{E} = \frac{\frac{1}{8} m\omega^2 a^2}{\frac{1}{2} m\omega^2 a^2} = \frac{1}{4}$$

Let y be the displacement, when energy is half and half potential. It means that at the displacement y, P.E. and K.E. of the S.H.M. are equal

i.e.,

$$\frac{1}{2}\, m\,\omega^2 y^2 = \frac{1}{2}\, m\,\omega^2(a^2 - y^2)$$

or

$$y^2 = a^2 - y^2$$

or

$$2y^2 = a^2$$

or

$$y = a/\sqrt{2} .$$

S18. When displacement is y, K.E. and P.E. of a simple harmonic oscillator are given by

$$\text{K.E.} = \frac{1}{2}\, m\,\omega^2(a^2 - y^2)$$

and $\quad \text{P.E.} = \frac{1}{2} m\omega^2 y^2$

If the kinetic energy of the oscillator becomes equal to its potential energy, when the displacement is y, then

$$\frac{1}{2} m\,\omega^2 (a^2 - y^2) = \frac{1}{2} m\omega^2 y^2$$

or $\quad 2y^2 = a^2$

$$y = \frac{a}{\sqrt{2}}.$$

S19. Here, mass m = 100 gm = 0.1 kg and T = 1.57 sec.

(a) Now, $\quad T = 2\pi\sqrt{\frac{m}{k}}$

or $\quad k = \frac{4\pi^2 m}{T^2} = \frac{4 \times (3.14)^2 \times 0.1}{(1.57)^2} = 1.6\ \text{Nm}^{-1}$

(b) When the displacement is equal to the amplitude, the body is in the extreme position and velocity is zero. Hence K.E. is zero.

S20. Here, $\quad T = 2\pi\text{s}; \quad E = 10{,}240$ erg

Now, $\quad y = a \sin \omega t = a \sin \frac{2\pi}{T} t$

When $\quad t = \frac{\pi}{4} s, \quad y = 8\sqrt{2}$ cm

$\therefore \quad 8\sqrt{2} = a \sin \frac{2\pi}{2\pi} \times \frac{\pi}{4} \quad \text{or} \quad a \sin \frac{\pi}{4} = 8\sqrt{2}$

or $\quad a \times \frac{1}{\sqrt{2}} = 8\sqrt{2}$

or $\quad a = 16$ cm

Also, $\quad E = \frac{2\pi^2 m a^2}{T^2}$

or $\quad m = \frac{ET^2}{2\pi^2 a^2} = \frac{10{,}240 \times (2\pi)^2}{2\pi^2 \times (16)^2} = \mathbf{80\,g}.$

S21. (a) At extreme position ($x = A$)

(b) Ate mean position ($x = 0$)

(c) $$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{4} \times m\omega^2 A^2$$

$$x = \frac{\sqrt{3}\,A}{2}$$

(d) $$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$$x = \frac{A}{\sqrt{2}}$$

S22. Here, $m = 10\text{ g}; \quad T = 6\text{ s} \quad \text{and} \quad v = 6\text{ cm s}^{-1}$

$\therefore$ $$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}\text{ rad s}^{-1}$$

At any time t, the velocity of S.H.M. is given by

$$v = a\,\omega \cos \omega t$$

When $t = 1\text{ s}, \quad v = 6\text{ cm s}^{-1}$

$\therefore$ $$6 = a \times \frac{\pi}{3} \cos \frac{\pi}{3} \times 1$$

$$= a \times \frac{\pi}{3} \cos 60°$$

$$= a \times \frac{\pi}{3} \times \frac{1}{2} = \frac{\pi a}{6}$$

or $$a = \frac{36}{\pi}\text{ cm}$$

Now, Total energy, $$E = \frac{1}{2} m\, a^2\, \omega^2$$

$$= \frac{1}{2} \times 10 \times \left(\frac{36}{\pi}\right)^2 \times \left(\frac{\pi}{3}\right)^2 = \textbf{720 erg}$$

Kinetic energy $$= \frac{1}{2}\, m v^2 = \frac{1}{2} \times 10 \times 6^2 = \textbf{180 erg}$$

and Potential energy = Total energy – Kinetic energy

$$= 720 - 180 = \textbf{540 erg}.$$

S23. When a spring is loaded, the length of the spring increases and the restoring force is set up in the spring. Therefore, load does work against the restoring force in stretching the spring. The work done by the load is stored in the spring in the form of its potential energy.

Here, $k = 800 \text{ N m}^{-1}$

When the length of a spring is stretched by x, work done is given by

$$W = \frac{1}{2} k x^2$$

When $x = 5$ cm: Here, $x = 5 \text{ cm} = 0.05 \text{ m}$

$$\therefore \quad W_1 = \frac{1}{2} \times 800 \times (0.05)^2 = 1 \text{ J}$$

When $x = 15$ cm: Here, $x = 15 \text{ cm} = 0.15 \text{ m}$

$$\therefore \quad W_2 = \frac{1}{2} \times 800 \times (0.15)^2 = 9 \text{ J}$$

Therefore, work done in extending the length of the spring from 5 cm to 15 cm,

$$W = W_2 - W_1 = 9 - 1 = \mathbf{8\ J}.$$

S24. The block executes S.H.M., its angular frequency, as given by Eq. (14.14b), is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50 \text{ N m}^{-1}}{1 \text{ kg}}} = 7.07 \text{ rad s}^{-1}$$

Its displacement at any time t is then given by, $x(t) = A \cos \omega t$

$$x(t) = 0.1 \cos (7.07t)$$

Therefore, when the particle is 5 cm away from the mean position, we have

$$0.05 = 0.1 \cos (7.07t)$$

Or $\quad \cos (7.07t) = 0.5$ and hence

$$\sin (7.07t) = \frac{\sqrt{3}}{2} = 0.866$$

Then, the velocity of the block at $x = 5$ cm is

$$= 0.1\ 7.07\ 0.866 \text{ m s}^{-1}$$

$$= 0.61 \text{ m s}^{-1}$$

Hence the K.E. of the block, $= \frac{1}{2} m v^2$

$$= ½ [1 \text{kg} \times (0.6123 \text{ m s}^{-1})^2] = 0.19 \text{ J}$$

The P.E. of the block, $= \frac{1}{2} k x^2$

$$= ½ (50 \text{ N m}^{-1} \times 0.05 \text{ m} \times 0.05 \text{ m})$$

$$= 0.0625 \text{ J}$$

The total energy of the block at $x = 5$ cm,

$$= \text{K.E.} + \text{P.E.}$$
$$= 0.25\ \text{J}$$

we also know that at maximum displacement, K.E. is zero and hence the total energy of the system is equal to the P.E. Therefore, the total energy of the system,

$$= (50\ \text{N m}^{-1} \times 0.1\ \text{m} \times 0.1\ \text{m})$$
$$= 0.25\ \text{J}$$

which is same as the sum of the two energies at a displacement of 5 cm. This is in conformity with the principle of conservation of energy.

S25. Mass of the circular disc, $m = 10$ kg

Radius of the disc, $r = 15\ \text{cm} = 0.15\ \text{m}$

The torsional oscillations of the disc has a time period, $T = 1.5$ s

The moment of inertia of the disc is:

$$I = \frac{1}{2} mr^2$$
$$= \frac{1}{2} \times (10) \times (0.15)^2$$
$$= 0.1125\ \text{kg m}^2$$

Time period, $T = 2\pi\sqrt{\frac{I}{\alpha}}$

α is the torsional constant.

$$\alpha = \frac{4\pi^2 I}{T^2}$$
$$= \frac{4 \times (\pi)^2 \times 0.1125}{(1.5)^2}$$
$$= 1.972\ \text{Nm/rad}$$

Hence, the torsional spring constant of the wire is 1.972 Nm rad^{-1}.

S26. The equation of displacement of a particle executing S.H.M. at an instant t is given as:

$$x = A \sin \omega t$$

Where,

A = Amplitude of oscillation

ω = Angular frequency = $\sqrt{\frac{k}{M}}$

The velocity of the particle is:

$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

The kinetic energy of the particle is:

$$E_k = \frac{1}{2} Mv^2 = \frac{1}{2} MA^2\omega^2 \cos^2 \omega t$$

The potential energy of the particle is:

$$E_p = \frac{1}{2} kx^2 = \frac{1}{2} M\omega^2 A^2 \sin^2 \omega t$$

For time period T, the average kinetic energy over a single cycle is given as:

$$(E_k)_{\text{avg}} = \frac{1}{T}\int_0^T E_k \, dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2} MA^2\omega^2 \cos^2 \omega t \, dt$$

$$= \frac{1}{2T} MA^2\omega^2 \int_0^T \frac{(1+\cos 2\omega t)}{2} dt$$

$$= \frac{1}{4T} MA^2\omega^2 \left[t + \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{1}{4T} MA^2\omega^2 (T)$$

$$= \frac{1}{4} MA^2\omega^2 \qquad \text{... (i)}$$

And, average potential energy over one cycle is given as:

$$(E_p)_{\text{avg}} = \frac{1}{T}\int_0^T E_p \, dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2} M\omega^2 A^2 \sin^2 \omega t \, dt$$

$$= \frac{1}{2T} M\omega^2 A^2 \int_0^T \frac{(1-\cos 2\omega t)}{2} dt$$

$$= \frac{1}{4T} M\omega^2 A^2 \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{1}{4T} M\omega^2 A^2 (T)$$

$$= \frac{M\omega^2 A^2}{4} \quad \text{... (ii)}$$

It can be inferred from equations (i) and (ii) that the average kinetic energy for a given time period is equal to the average potential energy for the same time period.

S27. At any instant the displacement of a particle executing S.H.M. may be given by

$$y = A \sin \omega t \quad \text{...(i)}$$

Now differentiate Eq. (i) w.r.t. "t"

$$v = \frac{dy}{dt} = A \cos \omega t\, (\omega)$$

$$= a\omega \cos \omega t$$

$$a = \frac{d^2y}{dt^2} = A\omega^2 \sin \omega t$$

Restoring force at any instant

$$F = -\, m\, \omega^2 y$$

Let work done by applied force to displace is given particle through distance dy away from mean position

$$dW = m\, \omega^2 y\, dy$$

$$\int dW = \int_0^y m\omega^2 y\, dy$$

$$W = m\omega^2 \left[\frac{y^2}{2}\right]_0^y$$

$$= \frac{1}{2}\, m\, \omega^2 y^2$$

This work done is stored in the potential energy

$$E_P = \frac{1}{2}\, m\, \omega^2 y^2$$

(a) At mean position $\quad y = 0$

$$E_P = 0$$

(b) At extreme position $\quad y = a$

$$E_P = \frac{1}{2}\, m\, \omega^2 a^2$$

S28. Let the particle has execute S.H.M.

$$y = a \sin \omega t \quad \dots \text{(i)}$$

Now differentiate Eq. (i) w.r.t. "t"

$$v = \frac{dy}{dt}$$

$$v = a\omega \cos \omega t$$

K.E. of the particle whose mass m

$$E_K = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m (a\omega \cos \omega t)^2$$

$$= \frac{1}{2} m a^2 \omega^2 \cos^2 \omega t$$

$$= \frac{1}{2} m a^2 \omega^2 (1 - \sin^2 \omega t)$$

From Eq. (i), we get

$$\sin \omega t = y/a$$

$$E_K = \frac{1}{2} ma^2\omega^2 \left\{1 - \left(\frac{y}{a}\right)^2\right\}$$

$$E_K = \frac{1}{2} m \omega^2 (a^2 - y^2).$$

At mean position $y = 0$

$$E_K = \frac{1}{2} m \omega^2 a^2$$

This is the maximum possible K.E.

At extreme position $y = a$

$$E_K = \frac{1}{2} m \omega^2 (a^2 - a^2)$$

$$E_K = 0.$$

S29. Given, Spring constant, $k = 1200 \text{ N m}^{-1}$

Mass, $m = 3 \text{ kg}$

Displacement, $A = 2.0 \text{ cm} = 0.02 \text{ m}$

(a) Frequency of oscillation ν, is given by the relation:

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Where, T is the time period

$\therefore$ $$\nu = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3}} = 3.18 \text{ s}^{-1}$$

Hence, the frequency of oscillations is 3.18 m/s.

(b) Maximum acceleration (a) is given by the relation:

$$a = \omega^2 A$$

Where, ω = Angular frequency = $\sqrt{\frac{k}{m}}$

A = Maximum displacement

$$a = \left(\frac{k}{m}\right) A$$

$$= \frac{1200}{3} \times 0.02$$

$$a = 8 \text{ m/s}^2$$

Hence, the maximum acceleration of the mass is 8.0 m/s

(c) Maximum velocity, $v_{max} = A\omega$

$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$= 0.02\sqrt{\frac{1200}{3}}$$

$$= 0.02 \times 20 = \mathbf{0.4 \text{ m/s}}$$

Hence, the maximum velocity of the mass is 0.4 m/s.

S30. Consider a particles of mass m executing S.H.M. along a straight path with O as mean position. Let r be the amplitude of oscillation of the particle and ω be its angular frequency of vibration. Let at an instant the displacement of the particle from the mean position by y.

Then $y = r \sin \omega t$.

$\therefore$ Velocity, $v = \frac{dy}{dt} = r\omega \cos \omega t$

and acceleration,

$$a = \frac{dv}{dt} = -\omega^2 r \sin \omega t = -\omega^2 y$$

P.E. of the particle for displacement y is

$$E_p = \int_0^y m\omega^2 y dy$$

$$= \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 r^2 \sin^2 \omega t$$

Average potential energy over the period of oscillation is

$$E_{p_{av}} = \frac{1}{T}\int_0^T \frac{1}{2} m^2 r^2 \omega^2 \sin^2 \omega t \, dt \qquad \left[\because \int_0^T \sin^2 \omega t \, dt = \frac{T}{2}\right]$$

$$= \frac{1}{2T} m\omega^2 r^2 \left(\frac{T}{2}\right) = \frac{1}{4} m\omega^2 r^2 \qquad \text{... (i)}$$

K.E. of the particle

$$= \frac{1}{2} mv^2 = \frac{1}{2} mr^2\omega^2 \cos^2 \omega t.$$

Average kinetic energy over the period of oscillation

$$E_{k_{av}} = \frac{1}{T}\int_0^T \frac{1}{2} mr^2 \omega^2 \cos^2 \omega t \, dt \qquad \left[\because \int_0^T \cos^2 \omega t \, dt = \frac{T}{2}\right]$$

$$= \frac{1}{2T} m\omega^2 r^2 \left(\frac{T}{2}\right) = \frac{1}{4} m\omega^2 r^2 \qquad \text{... (ii)}$$

From Eq. (i) and (ii),

$$E_{p_{av}} = E_{k_{av}}.$$

S31. (a) Here, $F = 4$ dyne; $l = 3$ cm

Now, $F = kl$

$\therefore$ $k = \frac{F}{l} = \frac{4}{3} = 1.33$ dyne cm^{-1}

(b) Also, $T = 2\pi\sqrt{\frac{m}{k}}$

Here, $m = 2$ g; $k = 1.33$ dyne cm^{-1}

$\therefore$ $$T = 2\pi\sqrt{\frac{2}{1.33}} = \mathbf{7.7\ s}$$

(c) Here, $r = 4$ cm; $y = \frac{4}{2} = 2$ cm;

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{7.7} = 0.816 \text{ rad s}^{-1}$$

Now, $$\text{K.E.} = \frac{1}{2} m\omega^2(r^2 - y^2)$$

$$= \frac{1}{2} \times 2 \times (0.816)^2 \times (4^2 - 2^2) = \mathbf{7.99\ erg}$$

$$\text{P.E.} = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} \times 2 \times (0.816)^2 \times 2^2 = \mathbf{2.66\ erg}.$$

S32. Comparing the given S.H.M. equation with the general displacement equation $y = a \cos(\omega t + \theta)$, we get

(a) amplitude $a = 6.0$ cm

(b) frequency $n = \omega/2\pi = 100/2\pi = 15.91$ Hz

(c) initial phase $\theta = \pi/4$ rad.

(d) velocity $\frac{dy}{dt} = 600 \sin(100t + \pi/4) \text{ cm}^{-1}$

maximum velocity $v = 600 \text{ cm s}^{-1} = 6 \text{ ms}^{-1}$

(e) acceleration $\frac{d^2y}{dt^2} = -60000 \cos(100t + \pi/4) \text{ cm}^{-2}$

(f) maximum K.E. $= \frac{1}{2} mv^2 = \frac{1}{2} \times 1 \times 6 \times 6 = 18$ J.

S33. (a) $$x = -2\sin\left(3t + \frac{\pi}{3}\right)$$

$$= +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$= 2\cos\left(3t + \frac{5\pi}{6}\right)$$

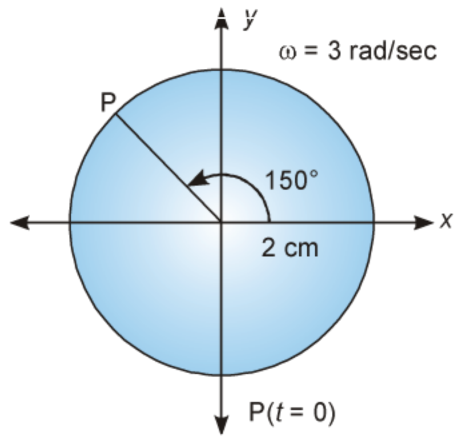
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, $A = 2$ cm

Phase angle, $\phi = \frac{5\pi}{6} = 150°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3$ rad/sec.

The motion of the particle can be plotted as shown in the figure.



(b) $$x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$$

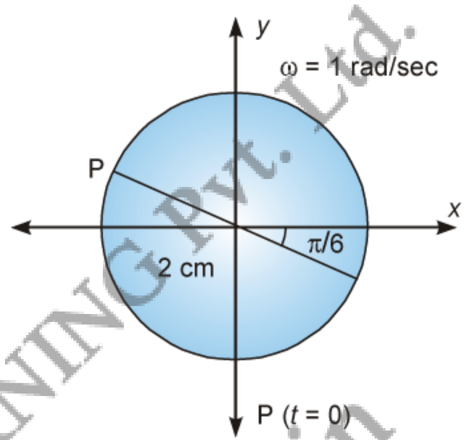
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, $A = 1$

Phase angle, $\phi = -\frac{\pi}{6} = -30°$

Angular velocity $\omega = \frac{2\pi}{T} = 1$ rad/s

The motion of the particle can be plotted as shown in the figure.



(c) $$x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right]$$

$$= -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$$

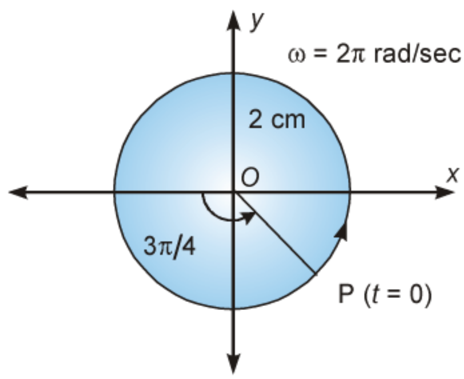
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, $A = 3$ cm

Phase angle, $\phi = \frac{3\pi}{4} = 135°$

Angular velocity $\omega = \frac{2\pi}{T} = 2\pi$ rad/s

The motion of the particle can be plotted as shown in the figure.



(d) $$x = 2\cos \pi t$$

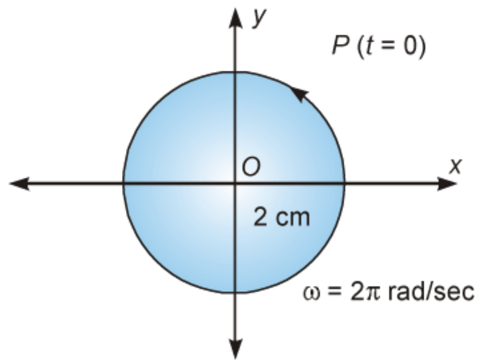
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, we get:

Amplitude, $A = 2$ cm

Phase angle, $\Phi = 0$

Angular velocity, $\omega = \pi$ rad/s

The motion of the particle can be plotted as shown in the figure.



S34. (a) In a S.H.M., with $y = A\sin\omega t$,

$$\text{P.E.} = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2y^2$$

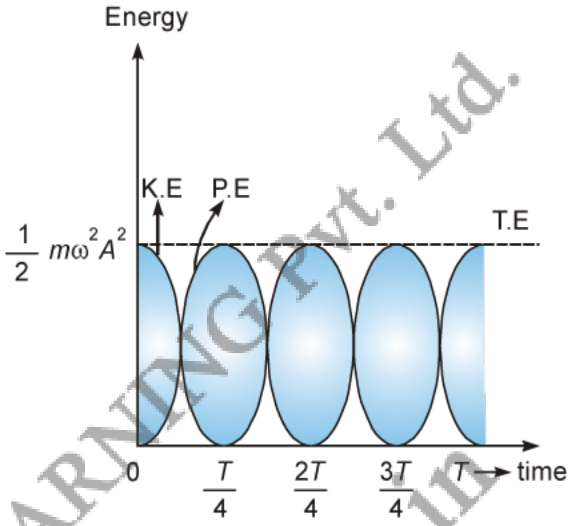
$$= \frac{1}{2}m\omega^2A^2\sin^2\omega t$$

$$\text{K.E} = \frac{1}{2}m\omega^2 = \frac{1}{2}m\left(\frac{dy}{dt}\right)^2$$

$$= \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

Total energy = P.E. + K.E.

$$= \frac{1}{2}m\omega^2A^2$$



As $\sin^2\omega t + \cos^2\omega t = 1$

Since, both P.E. and K.E. are square sinusoidal functions, their frequency will be double of a normal simple harmonic function. So, P.E. and K.E. will have a frequency $2f$ for a S.H.M. with frequency f. Total energy = $\frac{1}{2}m\omega^2A^2$ is a constant and so, there is no variation.

(b) (i) $v_{max} = a\omega$

$$= 2.5\times10^{-2}\times2\times\frac{22}{7}\times2\text{ ms}^{-1}$$

$$= 0.314\text{ m s}^{-1}$$

(ii) $E = \frac{1}{2}m\omega^2a^2 = \frac{1}{2}mv_{max}^2$

$$= \frac{1}{2}\times2\times0.314\times0.314\text{ J} = 0.1\text{ J}$$

(iii) Maximum potential energy

$= 0.1$ J.

(iv) Kinetic energy $= \frac{1}{2} m\omega^2(a^2 - y^2)$.

$= \frac{1}{2} \times 2 \times \frac{88}{7} \times \frac{88}{7} [(2.5)^2 - (1)^2] \times (10^{-2})^2$

$= 0.0829$J

$= 0.083$J

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Q1. What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Q2. A restoring force is a must for a body to execute S.H.M., Explain, why.

Q3. How is the time period of the pendulum effected when pendulum is taken to hills or in mines?

Q4. A simple pendulum is inside a space craft. What should be its time period of vibration?

Q5. Two unequal spring of same material are loaded with same load. Which one will have a larger value of time period?

Q6. Two simple pendulum of equal length cross each other at mean position. What is their phase difference?

Q7. A pendulum is making one oscillation in every two seconds. What is the frequency of oscillation?

Q8. Water in a U-tube executes S.H.M. Will the time period for mercury filled up to the same height in the U-tube be lesser or greater than that in case of water?

Q9. A small body of mass 0.1 kg is undergoing S.H.M. of amplitude 1.0 m and period 0.2 s.

(a) What is the maximum value of the force acting on it?

(b) If the oscillation are produced by a spring, what is the force constant of the spring?

Q10. A vertical *U*-tube of uniform cross-section contains water upto a height of 2.45 cm. When the water on one side is depressed and then released, its up and down motion in the tube is S.H.M. Calculate its time period. Given, $g = 980\ \text{cm s}^{-2}$.

Q11. If the earth were a homogeneous sphere and a straight was bored in it through its centre, then a body dropped in the hole executes S.H.M. Calculate the time period of its vibration. Radius of earth is 6.4×10^6 m and $g = 9.8\ \text{m s}^{-2}$.

Q12. A spring of force constant *k* is cut into two pieces, such that one piece is double the length of the other. What is the force constant of longer piece of the spring?

Q13. A spring has time period *T*. It is cut into n equal parts. What will be the time period of each part of the spring?

Q14. Does a simple pendulum in a lift moving downwards with an acceleration '*g*' execute simple harmonic motion? Explain.

Q15. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillation bob gets suddenly unplugged. How would the time period of oscillation of the pendulum change, till water is coming out?

Q16. An uncelebrated spring balance is found to have a period of oscillation of 0.314 s, when a 1 kg weight is suspended from it? How much does the spring elongate, when a 1 kg weight is suspended from it? Take $\pi = 3.14$.

Q17. A body of mass 2 kg is made to oscillate using a spring of force constant 8 $N\,m^{-1}$. Find (a) angular frequency, (b) frequency of vibration and (c) time period of vibration.

Q18. The acceleration due to gravity on the surface of the moon is 1.7 $m\,s^{-2}$. What is the time period of a simple pendulum on the surface of the moon, if its time period on the surface of the earth is 3.5 s? Take g = 9.8 $m\,s^{-2}$ on the surface of the earth.

Q19. A wooden cylinder of mass 20 g and area of cross-section 1 cm^2, having a piece of leas of mass 60 g attached to its bottom floats in water. The cylinder is depressed and then released. Show that it execute S.H.M. Find the frequency of oscillations. (Density of water 1 $g\,cm^{-3}$).

Q20. A cubical body (side 0.01 m and mass 0.002 kg) floats in water. It is pressed and then released, so that it oscillates vertically, is simple harmonic motion, find the time period.

Q21. What provides the restoring force for simple harmonic oscillations in the following cases?
(a) Simple pendulum (b) Spring (c) Column of mercury in U-tube.

Q22. A simple pendulum performs S.H.M. about $x = 0$ with an amplitude a and time period T. What is the speed of the pendulum at $x = A/2$?

Q23. A 5 kg collar is attached to a spring of spring constant 500 $N\,m^{-1}$. It slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate (a) the period of oscillation, (b) the maximum speed and (c) maximum acceleration of the collar.

Q24. Two particle A and B of equal masses are suspended from two mass less spring of spring constant k_1 and k_2 respectively. If the maximum velocities during oscillations are equal, what is the ratio of the amplitudes of A and B?

Q25. The length of a simple pendulum executing simple harmonic motion in increased by 21%. What is the percentage increase in the time period of the pendulum?

Q26. The time taken by a simple pendulum to perform 100 vibrations is 8 minutes 9 second in Mumbai and 8 minutes 20 second in Pune. Calculate the ratio of acceleration due to gravity in Mumbai and Pune.

Q27. A mass M attached to a spring oscillates with a period of 2 s. If the mass is increased by 2 kg, the period increase by 1 s. Find the initial mass M, assuming that Hook's a law is obeyed.

Q28. (a) A simple pendulum with a brass bob has a time period T. The bob is now immersed in a non-viscous liquid and oscillated. If the density of the liquid is $1/9$ that of brass, find the time of the same pendulum.

(b) The motion of the mass suspended by a coil spring

(i) What is the net force on the suspended mass at its lowermost position?

(ii) What is the elastic restoring force on the mass due to the spring at its uppermost position? (Given : g = 10 $m\,s^{-2}$)

Q29. A cylinder of length l, cross-sectional area A is floating on a liquid of density σ. If the cylinder (material density ρ) is depressed by a length x further by an external force acting for a short while, estimate the time period of S.H.M.

Q30. A uniform U-tube has a liquid of density ρ and a length L. The cross-sectional area is A. If it is made to oscillate, show that it will be S.H.M. and find its frequency.

Q31. A spring of constant k is attached with a mass m and is made to oscillate. What is its time period?

Q32. Gravity at poles exceeds gravity at the equator in the ratio of 301 : 300. A pendulum regulated for poles is taken to the equator. Calculate how many seconds a day it will gain or lose?

Q33. If the length of a second's pendulum is decreased by 2% find the gain or loss in time per day.

Q34. (a) A particle executes S.H.M. of period 8 sec. After what time of its passing through the mean position the energy will be half kinetic and half potential.

(b) If the earth were a homogeneous sphere and a straight was bored in it through its centre, then a body dropped in the hole executes S.H.M. Calculate the time period of its vibration. Radius of earth is 6.4×10^6 m and g = 9.8 m s^{-2}.

Q35. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

Q36. The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 ms^{-2})

Q37. Derive the expression for resultant spring constant when two springs having constant k_1 and k_2 are connected (a) in parallel (b) in series.

Q38. (a) A 0.2 kg. of mass hangs at the end of a spring. When 0.02 kg more mass is added to the end of the spring, it stretches 7 cm more. If the 0.02 kg mass is removed, what will be the period of vibration of the system?

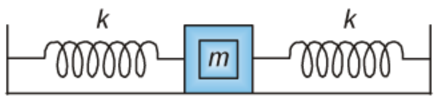
(b) A mercury column of mass m oscillates in a U-tube. One centimetre of the mercury column weights 15 g. Calculate (i) spring constant of motion and (ii) period of oscillation.

Q39. (a) Calculate the percentage change in time period of a simple pendulum if its length is increased by 8%.

(b) A vertical U-tube of uniform cross-section contains water up to a height of 0.3 m. Show that if water on one side is depressed and then released, its motion up and down the two sides of the tube is simple harmonic motion. Calculate its time period.

Q40. Two identical springs of spring constant k each are attached to a block of mass m as shown in figure:

Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations.



Q41. The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\cos(\omega t + \phi)$.

If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi\ s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM: $x = B\sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions?

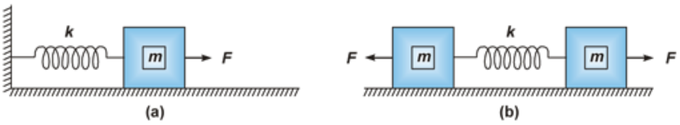
Q42. (a) A simple pendulum of length *l* and having a bob of mass *M* is suspended in a car. The car is moving on a circular track of radius *R* with a uniform speed *v*. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

(b) A small body of mass 0.1 kg is undergoing S.H.M. of amplitude 1.0 m and period 0.2 s.

(i) What is the maximum value of the force acting on it?

(ii) If the oscillation are produced by a spring, what is the force constant of the spring?

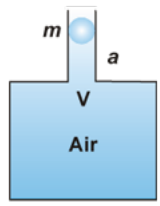
Q43. Figure (a) shows a spring of force constant *k* clamped rigidly at one end and a mass m attached to its free end. A force *F* applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force *F*.



(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (b) are released, what is the period of oscillation in each case?

Q44. An air chamber of volume *V* has a neck area of cross section *a* into which a ball of mass *m* just fits and can move up and down without any friction. Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal (see the given figure)



Q45. Cylindrical piece of cork of density ρ of base area *A* and height *h* floats in a liquid of density ρ_l. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_t g}}$$

where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

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S1. $f = 0$.

S2. For a body to execute S.H.M., it should oscillate about its mean position. When the body is at mean position, it possesses kinetic energy and by virtue of it, the body moves from the mean position to extreme position. The body can return to the mean position only, if it is acted upon by a restoring force.

S3. As $T \propto \frac{1}{\sqrt{g}}, T$ will increase because g decreases.

S4. Infinity or it does not oscillate.

S5. When a longer spring is loaded with weight *mg*, the extension l will more. Therefore, according to the expression,

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

The ***longer spring*** will have a larger value of the time period.

S6. The phase difference between pendulum is π radians.

S7. Given, t = 2 sec for one oscillation

$$f = \frac{\text{No. of oscillation}}{t} = \frac{1}{2} = 0.5\,\text{Hz}.$$

S8. The period of the liquid executing S.H.M. in a U-tube does not depend upon the density of the liquid. Therefore, time period will period will be the same, when mercury filled up to the same height as the water in the U-tube.

S9. Given, Mass of the body, $m = 0.1$ kg;

Amplitude, $a = 1.0$ m;

Time period, $T = 0.2$ s

(a) Maximum force acting on the body

$$= m \times (\omega^2 a) = m \times \left(\frac{2\pi}{T}\right)^2 \times a$$

$$= 0.1 \times \left(\frac{2\pi}{0.2}\right)^2 \times 1.0 = 0.1 \times (10\,\pi)^2 = \mathbf{98.7\ N}$$

(b) Time period of a loaded spring, $T = 2\pi\sqrt{\frac{m}{k}}$

$$\therefore \qquad k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 \times 0.1}{(0.2)^2} = 10\,\pi^2 = \mathbf{98.7\ N\,m^{-1}}.$$

S10. Given, $h = 2.45$ cm; $g = 980$ cm s^{-2}

The Time period of oscillation executed by a liquid in a *U*-tube,

$$T = 2\pi\sqrt{\frac{h}{g}}$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{2.45}{980}} = 2\,\pi \times 0.05$$

$$= \mathbf{0.314\,s}.$$

S11. Given, $R = 6.4 \times 10^6$ m; $g = 9.8$ m s^{-2}

The time period of oscillations executed by the body dropped in the hole along the diameter of the earth,

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{6.4 \times 10^6}{9.8}} = 2\,\pi \times 808.1$$

$$= \mathbf{5{,}077.4\,s}.$$

S12. Let the original length of the spring be *L*. If *y* is the extension produced, when the spring is stretched by a force *F*, then

$$F = -ky \qquad \dots \text{(i)}$$

If the spring is cut into two pieces, such that one piece is double the length of the other, the length of the two pieces will be $2l/3$ and $l/3$. When the longer pieces is stretched by the same force *F*, extension in the longer piece will by $2y/3$. If k' is force constant of the longer piece, then

$$F = -k'\left(\frac{2y}{3}\right) \qquad \dots \text{(ii)}$$

From the equations (i) and (ii), we have

$$k'\left(\frac{2y}{3}\right) = k\,y$$

or $$k' = \frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{k}.$$

S13. Let the spring be of length L. When a mass m is attached to the spring, its period of oscillation will be

$$T = 2\pi\sqrt{\frac{m}{k}}$$

When the spring is cut into n equal parts, the force constant of each part of the spring will be

$$k' = nk$$

When the same mass m is attached, the period of oscillation of each part of the spring will be

$$T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}.$$

S14. When a lift moves downwards with acceleration a, the apparent weight of the bob,

$$mg' = m(g - a)$$

When the lift moves downwards with acceleration g, then ($a = g$)

$$mg' = m(g - g) = 0$$

or $\quad g' = 0$

Time period of the pendulum in the lift moving downwards,

$$T = 2\pi\sqrt{\frac{L}{g'}} = 2\pi\sqrt{\frac{L}{0}} = \infty$$

$$\boldsymbol{T = \infty}.$$

i.e., the pendulum will *not execute S.H.M.* in the lift moving downwards with an acceleration g.

S15. We know, $\quad T = 2\pi\sqrt{\frac{L}{g}}$

i.e., $\quad T \propto \sqrt{L}$

As the hollow sphere (of negligible mass) filled with water is made to vibrate, the length of the pendulum (distance sphere) goes on increasing as the water flows out and hence time period of the pendulum also increases. But as soon as the follow *i.e.*, length of the pendulum decreases and hence time period of the pendulum also decreases.

S16. Here, $\quad T = 0.314\text{ s}; \quad m = 1\text{ kg}$

Now, $\quad T = 2\pi\sqrt{\frac{m}{k}} \quad \text{or} \quad k = \frac{4\pi^2 m}{T^2}$

or $\quad k = \frac{4\pi^2 \times 1}{(0.314)^2} = \frac{4 \times (3.14)^2 \times 1}{(0.314)^2} = 400\text{ N m}^{-1}$

When spring is loaded with a weight mg,

$mg = kl$

or $l = \dfrac{mg}{k}$

or $l = \dfrac{1 \times 9.8}{400}$ = 0.0245 m = **2.45 cm**.

S17. Given, $m = 2\,\text{kg}; \quad k = 8\ \text{N m}^{-1}$

(a) We know, $\omega = \dfrac{2\pi}{T}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{2}{8}} = \boldsymbol{\pi}\ \textbf{sec}$$

$$\omega = \frac{2\pi}{\pi} = 2\ \text{rad s}^{-1}$$

(b) $\nu = \dfrac{\omega}{2\pi}\ \mathbf{s^{-1}}$

$$= \frac{2}{2\pi} = \boldsymbol{\frac{1}{\pi}}\ \mathbf{s^{-1}}.$$

(c) $T = \dfrac{1}{\nu} = \dfrac{1}{1/\pi} = \boldsymbol{\pi}\ \textbf{sec}.$

S18. Let g and g' be the values of acceleration due to gravity at the surface of the earth and the moon respectively. If T and T' are time period of the pendulum at the surface of the earth and the moon respectively, then

Given, $T = 3.5$ sec; $g = 9.8\ \text{m s}^{-2}$; $g' = 1.7\ \text{m s}^{-2}$

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad T' = 2\pi\sqrt{\frac{L}{g'}}$$

$\therefore$ $$\frac{T'}{T} = 2\pi\sqrt{\frac{L}{g'}} \times \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \sqrt{\frac{g}{g'}}$$

Setting the values

or $T' = T\sqrt{\dfrac{g}{g'}}$

$$= 3.5\sqrt{\frac{9.8}{1.7}} = \textbf{8.4 s}.$$

S19. Given, $A = 1\ \text{cm}^2$; $M = 20$ g; $m = 60$ g; $\sigma = 1\ \text{g cm}^{-3}$

Suppose that the loaded wooden block sinks upto a height h. Then,

Weight of water displaced by the block

= weight of the block with lead

If a is area of cross-section of the block and A, the density of water, then

$$A\,h\sigma g = (M + m)\,g$$

or
$$h = \frac{M+m}{A\sigma}$$

$$= \frac{20+60}{1\times 1} = 80\,\text{cm} = 80\ \text{cm} \quad (\because \text{ density of water} = 1\ \text{g cm}^{-3})$$

Now,
$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{h}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{980}{80}} = \mathbf{0.557\ s^{-1}}.$$

S20. Given, Mass of the cubical body, $m = 0.002$ kg

Each side of the cubical body, $L = 0.01$ m

Volume of the cubical body,

$$V = 0.01 \times 0.01 \times 0.01 = 10^{-6}\,\text{m}^3$$

Therefore, density of the material of body,

$$\rho = \frac{m}{V} = \frac{0.002}{10^{-6}} = 2{,}000\ \text{kg m}^{-3}$$

Also, Density of water, $\sigma = 1{,}000\ \text{kg m}^{-3}$

Now,
$$T = 2\pi\sqrt{\frac{\rho L}{\sigma g}}$$

$$= 2\pi\sqrt{\frac{2{,}000\times 0.01}{1{,}000\times 9.8}} = \mathbf{0.284\ s}.$$

S21. (a) Parts of the force of gravity.

(b) Elastic restoring force.

(c) Force due to difference in column of mercury or pressure difference between the levels on the two limbs.

S22. Velocity of oscillating body (v) = $\omega\sqrt{A^2 - x^2}$

$$x = \frac{A}{2}$$

$$v = \omega\sqrt{A^2 - \left(\frac{A^2}{4}\right)}$$

$$= \frac{\sqrt{3}\,A\omega}{2} = \frac{\sqrt{3}\,A\pi}{T}. \qquad \left[\because \quad \omega = \frac{2\pi}{T}\right]$$

S23. (a) The period of oscillation as given by Eq. (14.21) is,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5.0\text{ kg}}{500\text{ Nm}^{-1}}}$$

$$= (2\pi/10)\text{ s} = 0.63\text{ s}$$

(b) he velocity of the collar executing S.H.M. is given by,

$$v(t) = -A\omega\sin(\omega t + \phi)$$

The maximum speed is given by,

$$v_m = A\omega = 0.1\times\sqrt{\frac{k}{m}} \qquad [\because \quad v = \omega\sqrt{A^2 - x^2} \text{ and } x = \text{displacement}]$$

$$= 0.1\times\sqrt{\frac{500\text{ Nm}^{-1}}{5\text{ kg}}} = 1\text{ ms}^{-1}$$

and it occurs at $x = 0$.

(c) The acceleration of the collar at the displacement $x(t)$ from the equilibrium is given by,

$$a(t) = -\omega^2 x(t) = -\frac{k}{m}x(t)$$

Therefore the maximum acceleration is,

$$a_{\max} = \omega^2 A = \frac{500\text{ Nm}^{-1}}{5\text{ kg}}\times 0.1\text{m} = 10\text{ m s}^{-2}$$

and it occurs at the extremities.

S24. The maximum velocity during S.H.M.,

$$v_{\max} = a\omega = a\times\frac{2\pi}{T} \qquad \dots \text{(i)}$$

Also, the time period of oscillation of a loaded spring,

$$T = 2\pi\sqrt{\frac{M}{k}}$$

Put the value T in Eq. (i), we get

$$\therefore \qquad v_{max} = a \times 2\pi/2\pi\sqrt{\frac{M}{k}} = a\sqrt{\frac{k}{M}}.$$

Hence, maximum velocities of the two particles A and B suspended from the two spring of force constant k_1 and k_2 given by

$$v_{max}(A) = a_1\sqrt{\frac{k_1}{M}} \quad \text{and} \quad v_{max}(B) = a_2\sqrt{\frac{k_2}{M}}$$

Here, a_1 and a_2 are the amplitudes of the two particles.

Since $v_{max}(A) = v_{max}(B)$, we have

$$a_1\sqrt{\frac{k_1}{M}} = a_2\sqrt{\frac{k_2}{M}}$$

or

$$\frac{a_1}{a_2} = \sqrt{\frac{\boldsymbol{k_2}}{\boldsymbol{k_1}}}.$$

S25. Let L be the length of the simple pendulum. Then its time period is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{... (i)}$$

On increasing that time period of the pendulum becomes ,

$$L' = \frac{L \times 121}{100} = 1.21\,L$$

Suppose that time period of the pendulum becomes T'.

Then,

$$T' = 2\pi\sqrt{\frac{1.21L}{g}} \qquad \text{... (ii)}$$

Dividing the equation (ii) by (i), we have

$$\frac{T'}{T} = 2\pi\sqrt{\frac{1.21L}{k}} \times \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 1.1$$

or

$$T' = 1.1\,T$$

Therefore, percentage increase in the time period of the pendulum is

$$\frac{T' - T}{T} \times 100 = \frac{1.1T - T}{T} \times 100 = \mathbf{10\%}.$$

S26. Let g_1 and g_2 be the values of acceleration due to gravity in Mumbai and Pune and T_1 and T_2 be the values of the time-period of the pendulum at the respective places. Then,

$$T_1 = 2\pi\sqrt{\frac{L}{g_1}} \quad \text{or} \quad T_2 = 2\pi\sqrt{\frac{L}{g_2}}$$

or $$\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$$

or $$\frac{g_1}{g_2} = \frac{T_2^2}{T_1^2}$$

Now, $$T_1 = \frac{8 \text{ min } 9 \text{ s}}{100} = \frac{489 \text{ s}}{100} = 4.89 \text{ s}$$

And $$T_2 = \frac{8 \text{ min } 20 \text{ s}}{100} = \frac{500 \text{ s}}{100} = 5 \text{ s}$$

$$\frac{g_1}{g_2} = \frac{(5)^2}{(4.89)^2} = \mathbf{1.0455}.$$

S27. ***When the spring is loaded with mass M:***

Here, $T = 2$ s

Now, $$T = 2\pi\sqrt{\frac{m}{k}}$$

or $$k = \frac{4\pi^2 m}{T^2}$$

or $$k = \frac{4\pi^2 M}{2^2} = \pi^2 M \text{ (in N m}^{-1}\text{)} \qquad \ldots \text{(i)}$$

When the attached mass is increased by 2 kg:

Now, $m = (M + 2)$ kg; $T = 2 + 1 = 3$ s; $k = \pi^2 M$

Again $$T = 2\pi\sqrt{\frac{m}{k}}$$

$\therefore$ $$3 = 2\pi\sqrt{\frac{M+2}{\pi^2 M}} \quad \text{or} \quad \frac{M+2}{M} = (1.5)^2$$

or $M = \mathbf{1.6\ kg.}$

S28. (a) Let V be the volume and ρ be the density of the brass bob.

Mass of the bob $M = V\rho$ and weight of *bob* $= V\rho g$.

Buoyancy force of liquid on *bob* $= V(\rho/9)\, g = V\rho g/9$.

So, the effective weight of *bob* in liquid = $V\rho g - V\rho g/9 = 8\,V\rho g/9$,

$$\therefore \qquad \text{Acceleration } g' = \frac{8V\rho g/9}{M}$$

$$= \frac{8V\rho g/9}{V\rho} = \frac{8g}{9}$$

Time period of the bob $= 2\pi\sqrt{\dfrac{l}{g}} = 2\pi\sqrt{\dfrac{l}{(8g/9)}}$

$$= 2\pi\sqrt{\frac{l}{g}} \times \frac{3}{\sqrt{8}} = \frac{3T}{\sqrt{8}} \qquad \left(\because \; T = 2\pi\sqrt{\frac{l}{g}}\right)$$

(b) (i) When the mass is in the lowermost position, extension

$$= 9 \text{ cm} = 9 \times 10^{-2} \text{ m}$$

Restoring force acing upwards

$$= 2 \times 10^{3} \times 9 \times 10^{-2} \text{ N} = 180 \text{ N}$$

Weight acting downwards

$$12 \times 10 \text{ N} = 120 \text{ N}$$

Net force $= (180 - 120)$ N = 60 N

It acts upwards.

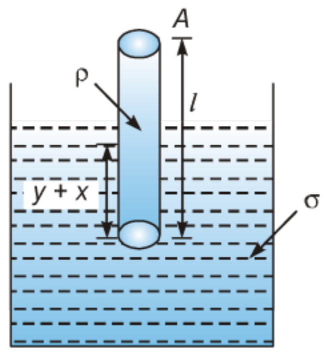
(ii) For the uppermost position, restoring force

$$= 2 \times 10^{3} \times 3 \times 10^{-2} \text{ N} = 60 \text{ N (upwards)}$$

Note that the equilibrium length is 56 cm and amplitude is 3 cm.

S29. Let y be the length immersed in the liquid as it floats. The weight was balanced by upthrust (Ayog) while floating. If further displacement x is brought, the upthrust increases and so the oscillation is made possible, *i.e.*,

$$\text{Restoring force = excess upthrust} = -x A \sigma g.$$



Also, $\qquad ma = Al\rho a$

$$\therefore \qquad Al\rho a = -A\sigma g x$$

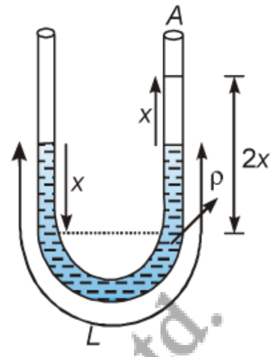
$$a = \left(\frac{-\sigma g}{\rho l}\right)(x)$$

$$a \propto -x$$

$\therefore$ Acceleration is directly proportional displacement, hence it show S.H.M.

$$T = 2\pi\sqrt{\frac{\rho l}{\sigma g}}.$$

S30. Let L be the length of the liquid column in the U-tube of uniform cross-section A. The liquid density be ρ. If by tilting, the level of liquid in the limbs differ by $2x$, the excess pressure on the higher level side from the same height as the other equaling $2x\rho g$ provides restoring force. The entire liquid oscillates as a result.



Since, Restoring force = Mass × Acceleration

$$-2x\rho gA = AL\rho a, \quad a = -\frac{2gx}{L}$$

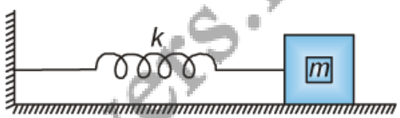
$$a \propto -x$$

$\because$ Acceleration is directly proportional displacement, hence it show S.H.M.

$$\text{The frequency} = f = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2g}{L}}$$

S31. The mass when displaced will stretch or compress the spring. If x is the displacement, the restoring force will be $F = -kx$. This makes the mass to oscillate.



$$ma = -kx, \quad a = \frac{-k}{m}x$$

we know, $$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

or $$= 2\pi\sqrt{\frac{\text{Inertial factor}}{\text{Spring factor}}}$$

$$T = 2\pi\sqrt{-\frac{x}{a}} \quad \text{or} \quad 2\pi\sqrt{\frac{m}{k}}$$

S32. Let g and g' be the values of acceleration due to gravity at poles and equator respectively. Then,

$$\frac{g}{g'} = \frac{301}{300}$$

Let L be the length of the pendulum. If and T and T' are time period of the pendulum at the poles and equator respectively, then

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad T' = 2\pi\sqrt{\frac{L}{g'}}$$

$$\therefore \qquad \frac{T'}{T} = 2\pi\sqrt{\frac{L}{g'}} \times \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{301}{300}}$$

$$= 1.001665$$

$$\therefore \qquad T' = 1.001665\,T$$

$$\therefore \qquad \frac{T'-T}{T} = \frac{1.001665\,T - T}{T} = 0.001665$$

Thus, increase in time period per second at the equator is 0.001665 s *i.e.*, at the equator, the pendulum will lose 0.001665 s per second. Therefore,

Loss in time per day = $0.001665 \times 24 \times 60 \times 60$ = **144 s**.

S33. Let L be the length of the second's pendulum

From the relation: $T = 2\pi\sqrt{\frac{L}{g}}$, we have

$$2 = 2\pi\sqrt{\frac{L}{g}} \qquad \text{... (i)}$$

On decreasing by 2% length of the pendulum becomes,

$$L' = \frac{L \times 98}{100} = 0.98\text{ L}$$

Suppose that time period of the pendulum becomes T'

Then, $$T' = 2\pi\sqrt{\frac{0.98\,L}{g}} \qquad \text{... (ii)}$$

Dividing the equation (ii) by (i), we have

$$\frac{T'}{2} = 2\pi\sqrt{\frac{0.98\,L}{g}} \times \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \sqrt{0.98}$$

or $$T' = 2 \times \sqrt{0.98} = 1.98\text{ s}$$

As the time period of the pendulum decreases, it will gain in time. The clock having such a pendulum will run faster. Therefore, the time gained by the pendulum in 2 s

$$= 2 - 1.98 = 0.02\text{ s}$$

Hence, the time gained by the pendulum per day

$$= \frac{0.02}{2} \times 24 \times 60 \times 60 = \mathbf{864\ S}.$$

S34. Given, $T = 8$ sec, $\therefore \quad \omega = 2\pi/8 = \pi/4$ rad s^{-1}

K.E. = $\frac{1}{2}$ Total energy = $\frac{1}{2} \cdot \frac{1}{2} m a^2 \omega^2 = \frac{1}{4} m a^2 \omega^2$

Let y be the displacement at which K.E. = P.E.

Now $\qquad$ P.E. $= \frac{1}{2} m y^2 \omega^2$

$\therefore \qquad \frac{1}{2} m y^2 \omega^2 = \frac{1}{4} m a^2 \omega^2$

or $\qquad y = \frac{a}{\sqrt{2}}$

Also $\qquad y = a \sin \omega t = a \sin (\pi/4 \times t)$

$\therefore \qquad \frac{a}{\sqrt{2}} = a \sin (\pi/4 \times t)$

$$\frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4} t\right) = \sin\left(\frac{\pi}{4}\right) \qquad \because \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

or $\qquad \frac{\pi}{4} t = \frac{\pi}{4}$

$\therefore \qquad t =$ **1 sec**.

S35. Area of cross-section of the U-tube = A; Density of the mercury column = ρ; Acceleration due to gravity = g

Restoring force, $\qquad F$ = Weight of the mercury column of a certain height

$F = -(\text{Volume} \times \text{Density} \times g)$

$F = -(A \times 2h \times \rho \times g) = -2A\rho g h$

$= -k \times$ Displacement in one of the arms (h)

Where,

$2h$ is the height of the mercury column in the two arms

k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

Time period, $2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where,

m is the mass of the mercury column

Let l be the length of the total mercury in the U

Mass of mercury, m = Volume of mercury $\times$ Density of mercury

$= Al\rho$

$$T = 2\pi\sqrt{\frac{Al\rho}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

Hence, the mercury column executes simple harmonic motion with time period $2\pi\sqrt{\frac{l}{2g}}$.

S36. Acceleration due to gravity on the surface of moon, $g' = 1.7 \text{ m s}^{-2}$

Acceleration due to gravity on the surface of earth, $g = 9.8 \text{ m s}^{-2}$

Time period of a simple pendulum on earth, $T = 3.5$ s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where, l is the length of the pendulum

$$\therefore \qquad l = \frac{T^2}{(2\pi)^2} \times g = \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8 \text{ m}$$

The length of the pendulum remains constant.

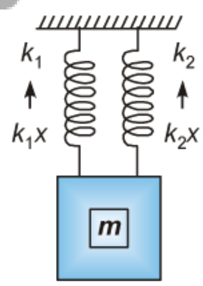
On moon's surface, time period,

$$T' = 2\pi\sqrt{\frac{l}{g'}}$$

$$= 2\pi\sqrt{\frac{\frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8}{1.7}} = 8.4 \text{ s}$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

S37. (a) When the spring are connected in parallel, the extension in them will be same and the total restoring force is the sum of their restoring force.

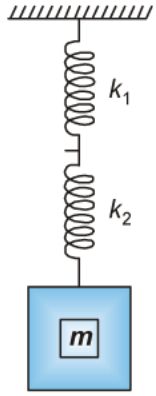


$$F = F_1 + F_2$$

$$-k_{eq}\,x = -k_1\,x - k_2\,x$$

or $$k_{eq} = k_1 + k_2$$

(b) When the spring are connected in series, the restoring force is same in both the springs and the extension will be different so the net extension



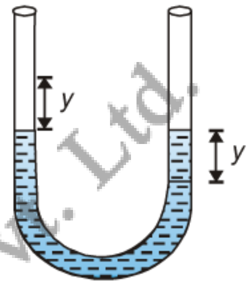
i.e., $x = x_1 + x_2$

$$\frac{F}{-k_{eq}} = \frac{-F}{k_1} - \frac{F}{k_2}$$

$$\therefore \quad \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

when connected in series.

S38. (a) When 0.02 kg is added, there is a stretch of 7 cm. Using $mg = kx$, we have



$$K = \frac{0.02 \times 10}{7 \times 10^{-2}} = \frac{20}{7}$$

$$= 2.86\ \text{N/m}$$

$$\text{Time period} = T = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{0.2}{2.86}} = \mathbf{1.66\ sec}.$$

(b) (i) Let the liquid in the right arm be depressed through a distance y metre. Then the liquid in the left arm will rise through a distance y metre.

Restoring force,

$F = -$ weight of unbalanced column of length $2y$ metre

or $$F = \frac{15 \times 10^{-3} \times 9.8}{1 \times 10^{-2}}$$

or $$F = -29.4\, y$$

Comparing with $F = -hy$, we get

$$h = 29.4\ \text{N m}^{-1}$$

(ii) $$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{m}{29.4}}\ \text{s}$$

$$= \frac{6.286}{5.422}\sqrt{m}\ \text{s} = \mathbf{1.159\sqrt{m}\ s}.$$

S39. (a) As $T \propto \sqrt{l}$

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100$$

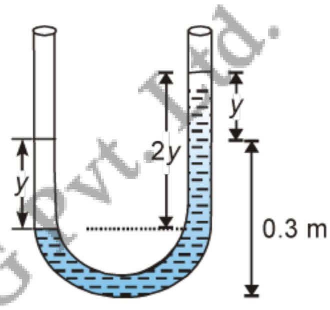
$\therefore$ % change in $T = \frac{1}{2} \times 8 = 4\%$

(b) Length of water column = 2×0.3 metre = 0.6 metre

Mass of water, $m = 0.6 \times A \times 1000\,\text{kg} = 600\,A$ kg

where $A\,m^2$ is the cross-sectional area of the U-tube and 1000 kg m^{-3} is the density of water.

Restoring force,



$F = -$ (weight of water column of length $2y$)

$= -(2y \times A \times 1000 \times 9.8)$ N

Acceleration, $\frac{d^2y}{dt^2} = \frac{2y \times A \times 1000 \times 9.8}{66600\,A}$

or $\frac{d^2y}{dt^2} = -\frac{98}{3}y$

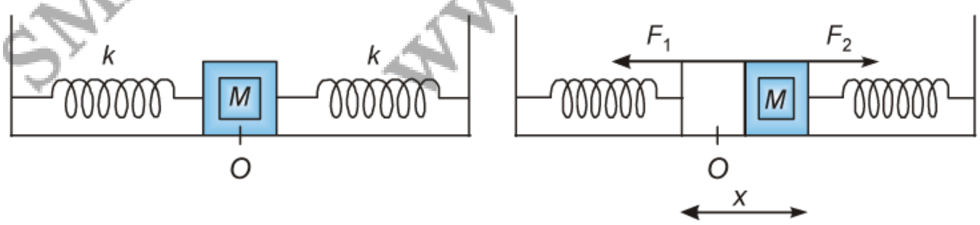
So, the acceleration is directly proportional to displacement and is directed towards the mean position. Hence, the motion of the water column is sinple harmonic.

Time period, $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$

$= 2\pi\sqrt{\frac{y}{98\,y/3}}$

$= 2\pi\sqrt{\frac{3}{98}}$ s = **1.1 s**.

S40. Let the mass be displacement by a small distance x to the right of equilibrium position. Due to this, spring on left gets elongated by length equal to x and that on the right side gets compressed by same length. Then force acting on masses are



$F_1 = -kx$ (force acting on left side and trying to pull the mass towards the mean position.)

$F_2 = -kx$ (force exerted by spring on right side trying to push the mass towards mean position.)

Net force F, acting on the mass

$$F = F_1 + F_2$$
$$F = -2\,kx$$
$$F \propto -x$$

$\therefore$ Force acting on mass is directly proportional to displacement and it directed towards mean position.

$\therefore$ Motion is simple harmonic and time period of oscillation is

$$T = \mathbf{2}\pi\sqrt{\frac{\boldsymbol{m}}{\mathbf{2}\boldsymbol{k}}}.$$

S41. Initially, at $t = 0$:

Displacement, $x = 1$ cm

Initial velocity, $\nu = \omega$ cm/sec.

Angular frequency, $\omega = \pi$ rad/s^{-1}

It is given that:

$$x(t) = A\cos(\omega t + \phi)$$
$$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$$
$$A\cos\phi = 1 \qquad \text{... (i)}$$

Velocity, $\nu = \dfrac{dx}{dt}$

$$\omega = -A\,\omega\sin(\omega t + \phi)$$
$$1 = -A\sin(\omega \times 0 + \phi) = -A\sin\phi$$
$$A\sin\phi = -1 \qquad \text{... (ii)}$$

Squaring and adding equations (i) and (ii), we get:

$$A^2(\sin^2\phi + \cos^2\phi) = 1 + 1$$
$$A^2 = 2$$

$\therefore$ $A = \sqrt{2}$ cm

Dividing equation (ii) by equation (i), we get:

$$\tan\phi = -1$$

$\therefore \qquad \phi = \frac{3p}{4}, \frac{7\pi}{4}, \ ....$

SHM is given as:

$$x = B\sin(\omega t + \alpha)$$

Putting the given values in this equation, we get:

$$1 = B\sin[\omega \times 0 + \alpha]$$

$$B\sin\alpha = 1 \qquad \text{... (iii)}$$

$$\text{Velocity, } \nu = \omega B\cos(\omega t + \alpha)$$

Substituting the given values, we get:

$$\pi = \pi B\cos\alpha$$

$$B\cos\alpha = 1 \qquad \text{... (iv)}$$

Squaring and adding equations (iii) and (iv), we get:

$$B^2[\sin^2\alpha + \cos^2\alpha] = 1 + 1$$

$$B^2 = 2$$

$\therefore \qquad B = \sqrt{2}$ cm

Dividing equation (iii) by equation (iv), we get:

$$\frac{B\sin\alpha}{B\cos\alpha} = \frac{1}{1}$$

$$\tan\alpha = 1 = \tan\frac{\pi}{4}$$

$\therefore \qquad \alpha = \frac{\pi}{4}, \frac{5\pi}{4}, \ ......$

S42. (a) The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity = g

Centripetal acceleration = $\frac{v^2}{R}$

Where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (a_{eff}) is given as:

$$a_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

Time period, $T = 2\pi\sqrt{\frac{l}{a_{eff}}}$

Where, l is the length of the pendulum

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$

(b) Given, Mass of the body, m = 0.1 kg; Amplitude, a = 1.0 m; Time period, T = 0.2 s

(i) Maximum force acting on the body

$$= m \times (\omega^2 a) = m \times \left(\frac{2\pi}{T}\right)^2 \times a$$

$$= 0.1 \times \left(\frac{2\pi}{0.2}\right)^2 \times 1.0 = 0.1 \times (10\,\pi)^2 = \mathbf{98.7\ N}$$

(ii) Time period of a loaded spring, $T = 2\pi\sqrt{\frac{m}{k}}$

$\therefore$ $$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 \times 0.1}{(0.2)^2} = 10\,\pi^2 = \mathbf{98.7\ N\,m^{-1}}.$$

S43. (a) If l is the extension produced in the spring then force developed in the spring is balanced by applied force F.

i.e., $F = -Kl$

or $l = -\frac{F}{k}$ ... (i)

Again, $F = ma = \frac{md^2y}{dt^2}$ ... (ii)

Hence, $\frac{md^2y}{dt^2} = -ky$ [$\because$ disp. of mass $\Rightarrow y = l$]

[Negative sign has been introduced as the direction of elastic force is opposite to direction of displacement y].

or $\frac{d^2y}{dt^2} = -\left(\frac{k}{m}\right)y = -\omega^2 y$ ... (iii)

We have, $\omega^2 = \frac{k}{m}$

= $\omega = \sqrt{\frac{k}{m}}$

Equation (iii) shows that motion of mass *m* simple harmonic whose time period is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \dots \text{(v)}$$

Further, from Eq. (i) extension produced in the spring,

$$l = \frac{F}{k}$$

(b) In this case displacement of each mass,

$$y = \frac{l}{2}$$

or displacement $y = \frac{F/k}{2} = \frac{F}{2k}$

or $F = 2\,ky$ ... (vi)

Also as $F = m \cdot \frac{d^2y}{dt^2}$ … (vii)

Hence, from Eq. (vi) and Eq. (vii)

$$m\frac{d^2y}{dt^2} = -2ky$$

[Again negative sign is introduced as the direction of electric force in opposite direction of displacement *y*].

or, $\frac{d^2y}{dt^2} = -\left(\frac{2k}{m}\right)y = -\omega^2 y$ … (viii)

Where $\omega^2 = \frac{2k}{m}$ or $\omega = \sqrt{\frac{2k}{m}}$

Thus, we again find from equation (viii) that here also masses executive simple harmonic motion, whose time period is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2k}{m}}} = 2\pi\sqrt{\frac{m}{2k}}.$$

S44. Volume of the air chamber = V

Area of cross-section of the neck = a

Mass of the ball = m

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\Delta V = ax$

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$\Rightarrow \qquad \frac{\Delta V}{V} = \frac{ax}{V}$$

$$B = \frac{\text{Stress}}{\text{Strain}} = \frac{-p}{\frac{ax}{V}}$$

Bulk Modulus of air,

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$$p = \frac{-Bax}{V}$$

The restoring force acting on the ball,

$$F = p \times a$$

$$= \frac{-Bax}{V} \times a = \frac{-Ba^2x}{V} \qquad \text{... (i)}$$

In simple harmonic motion, the equation for restoring force is:

$$F = -kx \qquad \text{... (ii)}$$

Where, k is the spring constant

Comparing equations (i) and (ii), we get:

$$k = \frac{Ba^2}{V}$$

Time period, $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

S45. Given, Base area of the cork = A

Height of the cork = h

Density of the liquid = ρ_t

Density of the cork = ρ

In equilibrium:

Weight of the cork = Weight of the liquid displaced by the floating cork

Let the cork be depressed slightly by x. As a result, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.

Up-thrust = Restoring force,

F = Weight of the extra water displaced

F = – (Volume × Density × g)

Volume = Area × Distance through which the cork is depressed

Volume = Ax

$$\therefore \quad F = -Ax\rho_t g \quad \ldots \text{(i)}$$

According to the force law: $F = kx$

Where, k is a constant
$$k = \frac{F}{x} = -A\rho_t g \quad \ldots \text{(ii)}$$

The time period of the oscillations of the cork:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \ldots \text{(iii)}$$

Where, m = Mass of the cork

= Volume of the cork × Density

= Base area of the cork × Height of the cork × Density of the cork

= $Ah\rho$

Hence, the expression for the time period becomes:

$$T = 2\pi\sqrt{\frac{Ah\rho}{A\rho_t g}} = 2\pi\sqrt{\frac{h\rho}{\rho_t g}}.$$

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Q1. Why a point on a rotating wheel cannot be considered as executing S.H.M.?

Q2. When is the tension maximum in the string of a simple pendulum?

Q3. Is the damping force constant on a system executing S.H.M?

Q4. How is the path difference related to phase difference?

Q5. What is an epoch? Name the unit in which it is measured.

Q6. What forces keep the simple pendulum in simple harmonic motion?

Q7. What is the time period of oscillation of a pendulum in a satellite?

Q8. If the displacement is represented by $x = 3 \sin \omega t + 4 \cos \omega t$, what is the amplitude?

Q9. If the total energy with an oscillating system is E, what is the kinetic energy at $x = \frac{A}{3}$?

Q10. If the motion of revolving particle is periodic in nature, give the nature of motion or projection of the revolving particle along the diameter.

Q11. A simple pendulum is transferred from Earth to Moon. Will it go faster or slower?

Q12. Can a simple pendulum be used in an artificial satellite?

Q13. The bob of a vibrating simple pendulum is made of ice. How will the period of swing change when the ice starts melting?

Q14. With the help of examples differentiate between free oscillations and forced oscillations.

Q15. For an oscillating simple pendulum, (a) what is the direction of acceleration of the bob at (i) the mean position, (ii) the end points ? (b) is the tension in the string constant throughout the oscillation? If not, when is it (i) the least, (ii) the greatest?

Q16. A body describes S.H.M. in a line 0.04 m long. Its velocity through the centre of the line is 0.12 ms^{-1}, find the period. Also find the velocity at a distance $10^{-2}\sqrt{3}$ m from the central position.

Q17. If two S.H.Ms acting in the same straight line have amplitudes 0.4 and 0.7 m respectively and their difference in phase in $\pi/2$, find the amplitude and the phase of the resultant.

Q18. If the length of a second's pendulum is decreased by 2% find the gain or loss per day.

Q19. A simple pendulum of length l suspended from a roof of a trolley which moves in a horizontal direction with an acceleration a. Find its time period of oscillation.

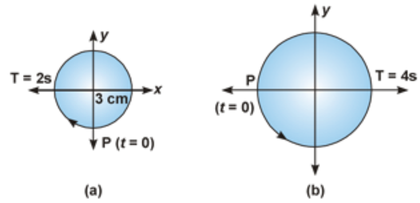
Q20. Two simple harmonic motions are represented by:

$$x_1 = 10 \sin \left(4\pi t + \frac{\pi}{4}\right) \quad \text{and} \quad x_2 = 5 (\sin 4\pi t + \sqrt{3} \cos 4\pi t)$$

What is the ratio of the amplitudes?

Q21. The bottom of a dip on a road has a radius of curvature R. A rickshaw of mass M left a little away from the bottom oscillates about this dip. Deduce an expression for the period of oscillation.

Q22. Figures shows that, correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (*i.e.*, clockwise or anti-clockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Q23. A faulty second's pendulum loses 5 seconds in a day. By how much its length must be shortened to keep correct time?

Q24. For the damped oscillator shown in figure, the mass m of the block is 200 g, $k = 90\ \text{N m}^{-1}$ and the damping constant b is $40\ \text{g s}^{-1}$. Calculate (a) the period of oscillation, (b) time taken for its amplitude of vibrations to drop to half of its initial value and (c) the time taken for its mechanical energy to drop to half its initial value.

Q25. Answer the following questions:

(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

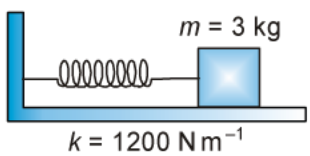
A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Q26. In the figure, let us take the position of mass when the spring is unstretched as $x = 0$, and the direction from left to right as the positive direction of x-axis.



Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch ($t = 0$), the mass is at the mean position, at the maximum stretched position, and at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

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S1. It is only periodic and not oscillatory.

S2. At the lower-most point or mean position.

S3. No. It is directly proportional to velocity which is a variable with time.

S4. Path difference = $\frac{\lambda}{2\pi} \times$ phase difference.

S5. The initial difference in position from mean position is called epoch.
It is measured in radians.

S6. Restoring force $mg \sin \theta$ and proper tension maintain simple harmonic motion of simple pendulum.

S7. Infinite.

S8. Phase difference between 3 sin ωt and 4 cos ωt is $\pi/2$.

$\therefore$ The amplitude is $\sqrt{3^2 + 4^2}$, *i.e.*, 5 units.

S9.

$$\text{K.E.} = \frac{1}{2} m\omega^2 (A^2 - x^2).$$

Total energy, $$E = \frac{1}{2} m\omega^2 A^2$$

$$\text{K.E.} = \frac{1}{2} m\omega^2 \left(A^2 - \frac{A^2}{9}\right)$$

$$= \frac{8}{9}\left(\frac{1}{2} m\omega^2 A^2\right) = \frac{8}{9} E$$

S10. Both amplitude and energy of the particle can be maximum only in the case of resonance, for resonance to occur $\omega_1 = \omega_2$.

S11. Due to decrease in value of g, T shall increase. So, pendulum will vibrate slower.

S12. No. This is because there exists a state of weightlessness in a satellite.

S13. As ice melts the centre of gravity raises. So, the time period reduces. As complete ice melts, the centre of gravity retains its original position. So, time period decreases. and Increases back to the same value.

S14. In the absence of air resistance, a pendulum oscillates freely but another pendulum dipped in a liquid oscillates only when external force exists. Oscillations which exist by the use of an external force overcoming any loss of energy are called forced oscillations.

S15. Case (a): When a simple pendulum oscillates, the bob moves along a circular arc.

(i) At the mean position, the tension in the string is greatest

And acts vertically upwards i.e. along the radius of the circular arc. Hence, at mean position, the acceleration of the simple pendulum is directed radially towards the point of suspension.

(ii) At the end point, tension in the string is the least. The simple pendulum moves back to the mean position under the effect of tangential component of its weight. Hence, at end points, the acceleration is directed tangentially towards mean position.

Case (b): As discussed in case (i), the tension in the string is not constant throughout the oscillation.

(i) It is the least at the end point.

(ii) It is the greatest at the mean position.

S16. Given, Length of line = 0.04 m

$\therefore$ Amplitude $a = 0.02$ m

Now, $v_{max} = 0.12\ ms^{-1}$; $\omega = \frac{2\pi}{T}$

$\therefore$ $a\omega = a \times \frac{2\pi}{T} = 0.12$

or $T = \frac{0.02 \times 2\pi}{0.12} = \frac{\pi}{3} = 1.047$ s

Now, $y = 10^{-2}\sqrt{3}$ m

Velocity $v = \omega\sqrt{a^2 - y^2}$

$= \omega\sqrt{0.0004 - 0.0003} = \omega \times 10^{-2}$

$= \frac{2\pi}{T} \times 10^{-2}$

$= \frac{2\pi}{\pi} \times 3 \times 10^{-2} = \mathbf{0.06\ ms^{-1}}$.

S17. Given, $a = 0.4$ m, $b = 0.7$ m and $\phi = \pi/2$ rad

$\therefore$ Resultant amp. $A = \sqrt{a^2 + b^2 + 2ab\cos\phi}$

$= \sqrt{0.16 + 0.49} = 0.806$ m

If θ is the phase angle of the resultant, then

$$\tan\theta = \frac{b\sin\phi}{a + b\cos\phi} = \frac{0.7\times 1}{0.4} = 1.7500$$

$$\theta = \mathbf{66.95°}.$$

S18. Time period of the second's pendulum = 2sec.

Let t be the time period when the length of the pendulum is decreased by 2%, then

$$\frac{t}{2} = \sqrt{\frac{98}{100}} = 0.98995 \qquad \because \; t \propto \sqrt{l}$$

or $$t = 0.98995 \times 2 = 1.9799$$

Decreases in time period = 2 – 1.9799 = 0.0201 sec

As the time period decreases, therefore the pendulum gains 0.0201 sec in 2 seconds. Hence

$$\text{Gain in one day} = \frac{0.0201\times 24\times 60\times 60}{2} = \mathbf{868.32\ sec}.$$

S19. The trolley is accelerated horizontally by a. So, there will be two accelerations, g vertically down and horizontal acceleration a. The net acceleration is $\sqrt{g^2 + a^2}$. The time period of oscillation:

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$

S20. Given,

$$x_1 = 10\sin\left(4\pi t + \frac{\pi}{4}\right)$$

$$x_2 = 5\,(\sin 4\pi t + \sqrt{3}\,\cos 4\pi t)$$

Let amplitude a_1 and a_2 S.H.M. x_1 and x_2 respectively

$$a_1 = 10$$

Amplitude of

$$a_2 = \sqrt{5^2 + (5\sqrt{3})^2} = 10$$

Since the sin $4\pi t$ and cos $4\pi t$ functions are out of phase by $\pi/2$.

Amplitude of a_2 = 10

$\therefore$ Ratio of amplitudes is $\frac{a_1}{a_2} = \frac{10}{10} = 1 : 1.$

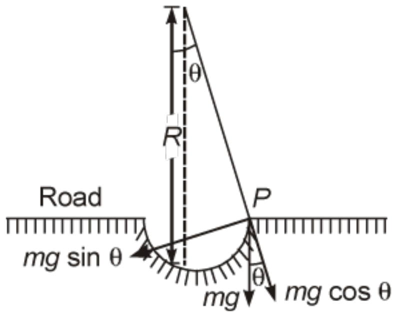
S21. Let R be the radius of the dip, and O be its centre. Let the rickshaw of mass M be at P at any instant. This case is similar to that of a simple pendulum.

The force that produces oscillations in the rickshaw is $F = Mg\sin\theta$.

If θ is small and is measured in radian then, $\sin\theta = \theta$,

$\therefore$ $$F = -Mg\theta$$ ($\because$ force acts to reduce θ)

Displacement of the rickshaw $OP = y = R\theta$



$\therefore$ Force constant, $$k = \frac{-\text{Force}}{\text{Displacement}}$$

$$= -\left(\frac{-Mg\theta}{R\theta}\right) = \frac{Mg}{R}$$

$\therefore$ Time period, $$T = 2\pi\sqrt{\frac{\text{Inertial factor}}{\text{Spring factor}}}$$

$$= 2\pi\sqrt{\frac{MR}{Mg}} = 2\pi\sqrt{\frac{R}{g}}$$

$$T = 2\pi\sqrt{\frac{R}{g}}.$$

S22. Given, Time period, $T = 2$ s

Amplitude, $A = 3$ cm

At time, $t = 0$, the radius vector OP makes an angle $\frac{\pi}{2}$ with the positive x-axis, *i.e.*, phase angle $\phi = \frac{\pi}{2}$.

Therefore, the equation of simple harmonic motion for the x-projection of OP, at time t, is given by the displacement equation:

$$x = A\cos\left[\frac{2\pi t}{T} + \phi\right]$$

$$= 3\cos\left(\frac{2\pi t}{2} + \frac{\pi}{2}\right) = -3\sin\left(\frac{2\pi t}{2}\right)$$

$\therefore$ $$x = -3\sin\pi t \text{ cm}$$

Time period, $T = 4$ s

Amplitude, $a = 2$ m

At time $t = 0$, OP makes an angle π with the x-axis, in the anticlockwise direction. Hence, phase angle, $\Phi = \pi$.

Therefore, the equation of simple harmonic motion for the x-projection of OP, at time t, is given as:

$$x = a\cos\left(\frac{2\pi t}{T} + \phi\right) = 2\cos\left(\frac{2\pi t}{4} + \pi\right)$$

$\therefore$
$$x = -2\cos\left(\frac{\pi}{4}t\right).$$

S23. Let the length of the pendulum be l and x be the decrease in length so that it keeps correct time. A pendulum which beats seconds has a time period of 2 seconds. In a day of 86400 seconds his pendulum loses 5 seconds. Hence

Number of vibrations executed by this pendulum per day

$$= \frac{86400 - 5}{2} = \frac{86395}{2}$$

$\therefore$ Time of one vibration $= \dfrac{86400 \times 2}{86395}$

Time period of faulty pendulum $= \dfrac{86400 \times 2}{86395} = 2\pi\sqrt{\dfrac{l}{g}}$

or
$$\left(\frac{86400}{86395}\right)^2 = \pi^2 \frac{l}{g} \quad \text{... (i)}$$

If the faulty pendulum is corrected its length is (1 – x) and has a time period of 2 seconds.

$\therefore$
$$2 = 2\pi\sqrt{\frac{l-x}{g}} \quad \text{or} \quad 1 = \pi^2 \frac{(l-x)}{g} \quad \text{... (ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$\pi^2 \frac{x}{g} = \left(\frac{86400}{86395}\right)^2 - 1 = 0.0001156$$

$$x = \frac{0.0001156 \times 9.8}{\pi^2} = \mathbf{0.0001147\ m}.$$

S24. (a) We see that $km = 90 \times 0.2 = 18$ kg N m^{-1} = kg^2 s^{-2}; therefore $\sqrt{km} = 4.243$ kg s^{-1}, and $b = 0.04$ kg s^{-1}. Therefore b is much less than $\sqrt{km}$. Hence the time period T from Eq. (14.34) is given by

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2\ \text{kg}}{90\ \text{N m}^{\pm 1}}} = 0.3\ \text{s}$$

(b) Now, from Eq. (14.33), the time, $T_{1/2}$, for the amplitude to drop to half of its initial value is given by,

$$T_{1/2} = \frac{\ln(1/2)}{b/2m} = \frac{0.693}{40} \times 2 \times 200\ s\ = 6.93\ \text{s}$$

(c) For calculating the time, $t_{1/2}$, for its mechanical energy to drop to half its initial value we make use of Eq. (14.35). From this equation we have,

$$E(t_{1/2})/E(0) = \exp(-bt_{1/2}/m)$$

Or $$1/2 = \exp(-bt_{1/2}/m)$$

$$\ln(1/2) = -(bt_{1/2}/m)$$

Or $$t_{1/2} = \frac{0.693}{40\ \text{g s}^{-1}} \times 200\ \text{g}\ = 3.46\ \text{s}$$

This is just half of the decay period for amplitude. This is not surprising, because, according to Eqs. (14.33) and (14.35), energy depends on the square of the amplitude. Notice that there is a factor of 2 in the exponents of the two exponentials.

S25. (a) The time period of a simple pendulum, $T = 2\pi\sqrt{\frac{m}{k}}$

For a simple pendulum, k is expressed in terms of mass m, as:

$$k \propto \text{m}$$

$$\frac{m}{k} = \text{Constant}$$

Hence, the time period T, of a simple pendulum is independent of the mass of the bob.

(b) In the case of a simple pendulum, the restoring force acting on the bob of the pendulum is given as:

$$F = -mg \sin\theta$$

Where,

F = Restoring force

m = Mass of the bob

g = Acceleration due to gravity

θ = Angle of displacement

For small θ, $\sin\theta \simeq \theta$

For large θ, $\sin\theta$ is greater than θ.

This decreases the effective value of g.

Hence, the time period increases as:

$$T = 2\pi\sqrt{\frac{I}{g'}}$$

Where, l is the length of the simple pendulum.

(c) The time shown by the wristwatch of a man falling from the top of a tower is not affected by the fall. Since a wristwatch does not work on the principle of a simple pendulum, it is not affected by the acceleration due to gravity during free fall. Its working depends on spring action.

(d) When a simple pendulum mounted in a cabin falls freely under gravity, its acceleration is zero. Hence the frequency of oscillation of this simple pendulum is zero.

S26.

$$x = 2 \sin 20t$$

$$x = 2 \cos 20t$$

$$x = -2 \cos 20t$$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, $A = 2.0$ cm

Force constant of the spring, $k = 1200 \text{ N m}^{-1}$

Mass, $m = 3$ kg

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = \sqrt{400} = 20 \text{ rad s}^{-1}$$

When the mass is at the mean position, initial phase is 0.

Displacement, $x = A \sin \omega t = 2 \sin 20t$

At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

Displacement,

$$x = A \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= 2 \sin\left(20t + \frac{\pi}{2}\right)$$

$$= 2 \cos 20t.$$

At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

Displacement,

$$x = A \sin\left(\omega t + \frac{3\pi}{2}\right)$$

$$= 2 \sin\left(20t + \frac{3\pi}{2}\right)$$

$$= -2 \cos 20t$$

The functions have the same frequency $\left(\frac{20}{2\pi}\text{Hz}\right)$ and amplitude (2 cm), but different initial phases $\left(0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$.

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Q1. A bus begins to make a loud rattling sound at a certain speed on the road. Explain, why.

Q2. What are isochronous vibrations?

Q3. In a forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. What is the relation between ω_1 and ω_2?

Q4. Sound waves from a point source are propagating in all directions. What will be the ratio of amplitudes at distances of x meter and y meter from the source?

Q5. What do you mean by resonance in oscillation?

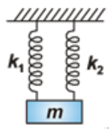
Q6. In the arrangement shown in the figure, the block of mass m is displaced, what is the frequency of oscillation?

Q7. A spring of force constant k is broken into n equal parts ($n > 0$). What will be the spring factor of each part?

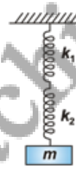
Q8. Define resonance.

Q9. If $y = \frac{1}{\sqrt{a}} \sin \omega t - \frac{1}{\sqrt{b}} \cos \omega t$, find the amplitude of motion.

Q10. Find the frequency in the case given below:



Q11. Find the frequency in the case given below:



Q12. A particle executes S.H.M. of period 8 sec. After what time of its passing through the mean position will the energy be half kinetic and half potential?

Q13. The amplitude of an oscillating simple pendulum is doubled. What will be its effect on the (a) Periodic time: (b) Total energy; (c) Maximum velocity?

Q14. The frequency of oscillations of a mass m suspended by a spring is 'ν_1' If the length of spring is cut to one half, the same mass oscillates with frequency 'ν_2'. Find the ratio of frequencies.

Q15. A mass of 0.50 kg stretches a spring through 20 cm. Find the force constant of the spring. If the mass is pulled down through an additional 5 cm and then released, find the time period. Given $g = 9.8$ m/sec^2.

Q16. The masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2.

Q17. Two identical springs *A* and *B* each having spring constant *k* are connected to a body *P* of mass *m* in three different ways as shown. Find the spring factor for the oscillation of the body *P* in each case.

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S1. The frequency of vibration of the structure of the bus becomes equal to its natural frequency.

S2. When the time period is independent of amplitude, the oscillation is called isochronous.

S3. Both amplitude and energy of the particle can be maximum only in the case of resonance, for resonance to occur $\omega_1 = \omega_2$.

S4.
$$\text{Intensity} = \text{amplitude}^2 \propto \frac{1}{(\text{distance})^2}$$

$\therefore$ required ratio = y/x

S5. When the natural frequency of oscillation and the frequency of the force oscillating it are same then there is said to be resonance in oscillation.

S6. Since extension is of equal amount acting in the springs, the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

S7. The spring factor of each equal part is nk.

S8. The phenomenon of setting a body into oscillations of large amplitude by the influence of another vibrating body having the same natural frequency is called resonance.

S9. Given function the phase difference between them $\pi/2$

$$\text{Amplitude } a_1 = \frac{1}{\sqrt{a}}$$

and $$a_2 = \frac{1}{\sqrt{b}}$$

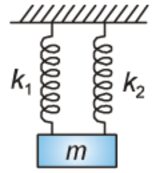
$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

$$\text{Amplitude of the displacement} = \sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2}$$

$$= \sqrt{\frac{1}{a} + \frac{1}{b}}.$$

S10. Restoring force in the springs will be in the same direction and the displacement is same in both.

$\therefore$ Restoring force $= -k_1x - k_2x$

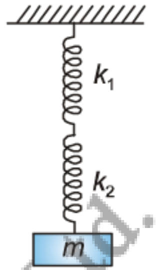


$$k_{eq} = k_1 + k_2$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

S11. Restoring force is in same direction but the extensions are different even though the force is same.

$$\therefore \quad x = x_1 + x_2$$



$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\frac{1}{k_{eq}} = \frac{k_2 + k_1}{k_1 k_2}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}.$$

S12. Given P.E. = K.E.

$$\frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2(a^2 - y^2)$$

$$y^2 = a^2 - y^2$$

i.e., $y = \dfrac{a}{\sqrt{2}}$

Now $y = a \sin \omega t$

or $y = a \sin(2\pi/T)\, t$

$$\frac{a}{\sqrt{2}} = a \sin\left(\frac{2\pi}{8}\right)t \qquad [T = 8 \text{ sec.}]$$

$$\sin\frac{\pi t}{4} = \frac{1}{\sqrt{2}}$$

$$\frac{\pi t}{4} = \frac{\pi}{4} \quad \text{or} \quad t = 1 \text{ sec.}$$

S13. (a) Time period in independent of amplitude. So no change in T with amplitude.

(b) Total energy $= 1/2m\omega^2A^2$

If A is doubled, total energy becomes four times.

(c) Maximum velocity $v = \omega A$

If A is doubled, maximum velocity becomes double.

S14. Frequency of oscillation of mass *m* suspended by a spring of constant *k* is,

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

If the length is cut into one half, the force constant will becomes $2k$ for each portion. The frequency with same mass *m* is,

$$\nu_2 = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

Ratio of frequencies $\qquad \frac{\nu_1}{\nu_2} = \frac{1}{\sqrt{2}}$.

S15. Here mass $\qquad m = 0.50$ kg

$\therefore$ Stretching force $\qquad F = mg = 0.5 \times 9.8 = 4.9$ N

Stretching produced $\qquad x = 20$ cm $= 0.2$ m

$\therefore$ Force constant $\qquad k = \frac{F}{x} = \frac{4.9}{0.2} = 24.5\ \text{N m}^{-1}$

Time period $\qquad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.5}{24.5}}$ = **0.898 sec**.

S16. Let x_2 be the extension with m_1 and m_2 in position, and *k* the force constant, then

$$(m_1 + m_2)\,g = kx_2 \qquad \text{... (i)}$$

If x_1 is the extension with m_2 alone, then

$$m_2 g = kx_1 \qquad \text{... (ii)}$$

Subtracting Eq (ii) from Eq. (i), we get

$$m_1 g = k(x_2 - x_1)$$

or $$(x_2 - x_1) = \frac{m_1 g}{k}$$

As $(x_2 - x_1)$ is the amplitude of the oscillation

$\therefore$ $$\text{Amplitude} = \frac{m_1 g}{k}$$

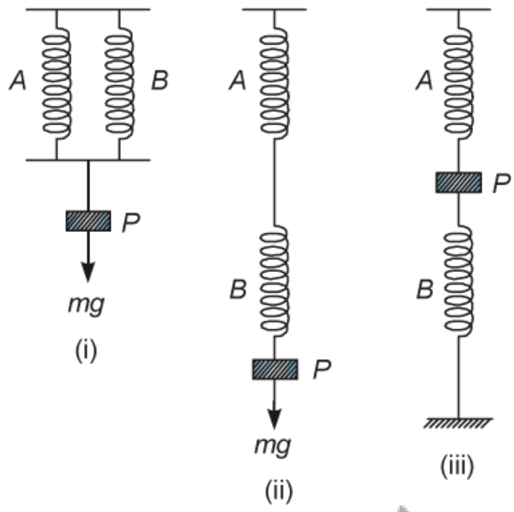
If *T* is the time period with m_2 alone, then

$$T = 2\pi\sqrt{\frac{m_2}{k}}$$

$\therefore$ $$\text{Angular frequency } \omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{m_2/k}} = \sqrt{\frac{k}{m_2}}.$$

S17. Here k is the spring constant of each spring. If F represents the restoring force produced for an extensions x, then

$$F = -kx \quad \text{... (i)}$$



(a) The weight mg causes an extension x in each spring. Let F be the restoring force produced in each spring. If k_1 is the spring factor of the combined system, then

$$2F = -k_1 x \quad \text{or} \quad F = -\frac{k_1}{2}x \quad \text{... (ii)}$$

Comparing Eq. (i) and Eq. (ii), we get

$$k_1/2 = k$$

or $$k_1 = 2k$$

(b) As shown in the figure (ii), spring length is doubled and the weight mg will produce double the extension. If k_2 is the spring factor of the combination, then

$$F = -k_2(2x) = -2k_2 x \quad \text{... (iii)}$$

Comparing Eq. (i) and Eq. (ii), we get

$$2k_2 = k$$

or $$k_2 = k/2$$

(c) As shown in the figure (iii), the weight mg stretches the spring A and compresses the spring B as the lower end is fixed to a rigid support and so the body P is pulled down by x and the upper spring is extended further by x while the lower spring is compressed. The restoring force in both the springs is acting in the same direction. If k_3 is the spring factor of the combination, then

$$2F = -k_3 x \quad \text{or} \quad F = -\frac{k_3}{2}x \quad \text{... (iv)}$$

Comparing Eq. (i) and Eq. (iii), we get

$$k_3/2 = k$$

or $$k_3 = 2k$$

Hence spring constants in the three cases are $2k$, $k/2$ and $2k$.