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Q1. At the end of each year the value of a certain machine has depreciated by 20% of its value at the beginning of that year. If its initial value was Rs. 1250, find the value at the end of 5 years.

Q2. We know the sum of the interior angles of a triangle is 180°. Show that the sums of the interior angles of polygons with 3, 4, 5, 6, ... sides form an arithmetic progression. Find the sum of the interior angles for a 21 sided polygon.

Q3. If a, b, c, d are four distinct positive quantities in A.P., then show that $bc > ad$.

Q4. Show that $(x^2 + xy + y^2)$, $(z^2 + xz + x^2)$ and $(y^2 + yz + z^2)$ are consecutive terms of an AP., if x, y and z are in A.P.

Q5. Find the sum of first 24 terms of the A.P., a_1, a_2, a_3, ... if it is known that

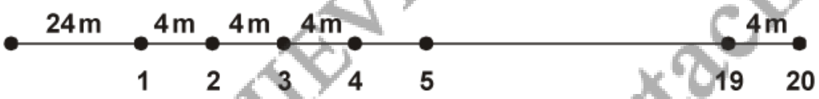
$$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225.$$

Q6. The product of three numbers in A.P., is 224, and the largest number is 7 times the smallest. Find the numbers.

Q7. A man saved \$ 66000 in 20 years. In each succeeding year after the first year he saved \$ 200 more than what he saved in the previous year. How much did he save in the first year?

Q8. If there are $(2n + 1)$ terms in an A.P., then prove that the ratio of the sum of odd terms and the sum of even tems is $(n + 1) : n$.

Q9. In a potato race 20 potatoes are placed in a line at intervals of 4 metres with the first potato 24 metres from the starting point. A contestant is required to bring the potatoes back to the starting place one at a time. How far would he run in bringing back all the potatoes?



Q10. A carpenter was hired to build 192 window frames. The first day he made five frames and each day, thereafter he made two more frames than he made the day before. How many days did it take him to finish the job?

Q11. A man accepts a position with an initial salary of \$ 5200 per month. It is understood that he will receive an automatic increase of \$ 320 in the every next month and each month thereafter.

(i) Find his salary for the tenth month.

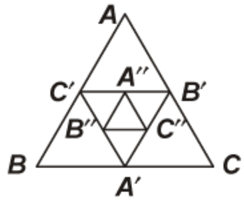
(ii) What is his total earnings during the first twelve year?

Q12. If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P. Then prove that $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$.

Q13. If a, b, c, d are in G.P., prove that $a^2 - b^2$, $b^2 - c^2$, $c^2 - d^2$ are in G.P.

Q14. In a cricket tournament 16 school teams participated. A sum of \$ 8000 is to be awarded among themselves as prize money. If the last placed team is awarded \$ 275 in prize money and the award increases by the same amount for successive finishing places, how much amount will the first place team receive?

Q15. A side of an equilateral triangle is 20 cm long. A second equilateral triangle is inscribed in it by joining the mid-points of the sides of the first triangle. The process is continued as shown in the accompanying diagram. Find the perimeter of the sixth inscribed equilateral triangle.



Q16. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers, then prove that: $2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$.

Q17. If a, b, c, d are four distinct positive quantities in G.P., then show that $a + d > b + c$.

Q18. If the p^{th} and q^{th} terms of a G.P. are q and p respectively, show that its $(p + q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.

Q19. Find the natural number a for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function f satisfies $f(x + y) = f(x) \cdot f(y)$ for all natural numbers x, y and further $f(1) = 2$.

Q20. Find the sum of the series: $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$ to (i) n terms (ii) 10 terms.

Q21. If $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in A.P., whose common difference is d, show that

$$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}.$$

Q22. If the sum of m terms of an A.P. is equal to the sum of either the next n terms or the next p terms, then prove that

$$(m+n)\left(\frac{1}{m} - \frac{1}{p}\right) = (m+p)\left(\frac{1}{m} - \frac{1}{n}\right).$$

Q23. If the sum of p terms of an A.P. is q and the sum of q terms is p, show that the sum of $p + q$ terms is $-(p + q)$. Also, find the sum of first $p - q$ terms, $(p > q)$.

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S1. After end of each year the value of the machine is 80% of its value the previous year so at the end of 5 years the machine will depreciate as many times as 5.

Hence, we have to find the 6^{th} term of the G.P., whose first term a_1 is 1250 and common ratio r is .8.

Hence, value at the end 5 years = $t_6 = a_1 r^5 = 1250\,(.8)^5$ = **Rs. 409.6**.

S2. Interior angles of polygon with sides 3, 4, 5, 6, ... are 180°, 360°, 540°, ...

Now, $a = 180°$

and $d = 360° - 180° = 180°$ [Let a be the first term and d be the common difference of the A.P.]

Thus, it forms an A.P.

Now, $a_n = a + (n-1)d$

$$a_{19} = 180 + (19-1) \times 180$$
$$= 180 + 18 \times 180$$
$$= 19 \times 180°$$
$$= \mathbf{3420°}.$$

S3. Since a, b, c, d are in A.P., then A.M. > G.M., for the first three terms.

Therefore, $b > \sqrt{ac}$ $\left(\text{Here } \frac{a+c}{2} = b\right)$

Squaring, we get $b^2 > ac$ ... (i)

Similarly, for the last three terms

A.M. > G.M.

$c > \sqrt{bd}$ $\left(\text{Here } \frac{b+d}{2} = c\right)$

$c^2 > bd$ ... (ii)

Multiplying Eq. (i) and (ii), we get

$$b^2c^2 > (ac)(bd)$$

$\Rightarrow$ $bc > ad$. **Proved.**

S4. The terms $(x^2 + xy + y^2)$, $(z^2 + xz + x^2)$ and $(y^2 + yz + z^2)$ are consecutive terms of an A.P., if

$$(z^2 + xz + x^2) - (x^2 + xy + y^2) = (y^2 + yz + z^2) - (z^2 + xz + x^2)$$

i.e., $z^2 + xz - xy - y^2 = y^2 + yz - xz - x^2$

i.e., $x^2 + z^2 + 2xz - y^2 = y^2 + yz + xy$

i.e., $(x + z)^2 - y^2 = y(x + y + z)$

i.e., $x + z - y = y$

i.e., $x + z = 2y$

which is true, since x, y, z are in A.P. Hence, $x^2 + xy + y^2$, $z^2 + xz + x^2$, $y^2 + yz + z^2$ are in A.P.

S5. We know that in an A.P., the sum of the terms equidistant from the beginning and end is always the same and is equal to the sum of first and last term.

Therefore, $d = b - a$

i.e., $a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$

It is given that $(a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow$ $(a_1 + a_{24}) + (a_1 + a_{24}) + (a_1 + a_{24}) = 225$

$\Rightarrow$ $3(a_1 + a_{24}) = 225$

$\Rightarrow$ $a_1 + a_{24} = 75$

We know that $S_n = \frac{n}{2}[a + l]$, where a is the first term and l is the last term of an AP.

Thus, $S_{24} = \frac{24}{2}[a_1 + a_{24}] = 12 \times 75 = 900.$

S6. Let the three numbers in AP., be $a - d$, a, $a + d$, $(d > 0)$

Now, $(a - d)\,a\,(a + d) = 224$

$\Rightarrow$ $a(a^2 - d^2) = 224$ ... (i)

Now, since the largest number is 7 times the smallest, *i.e.*, $a + d = 7(a - d)$

Therefore, $d = \frac{3a}{4}$

Substituting this value of d in Eq. (i), we get

$$a\left(a^2 - \frac{9a^2}{16}\right) = 224, \qquad (a \neq 0)$$

$$a = 8$$

and $d = \frac{3a}{4} = \frac{3}{4} \times 8 = 6$

Hence, the three numbers are 2, 8, 14.

S7. Let First term $= a$, $d = 200$, $n = 20$

$\therefore$ $S_{20} = \frac{20}{2}[2a + (20 - 1)d]$

$\Rightarrow$ $10[2a + 19 \times 200] = 66000$

$\Rightarrow$ $2a + 3800 = 6600$

$\Rightarrow$ $2a = 6600 - 3800$

$\Rightarrow \qquad 2a = 2800$

$\Rightarrow \qquad a = 1400$

Hence, he saved **$ 1400** in the first year.

S8. Let a be the first term and d the common difference of the A.P. Also let S_1 be the sum of odd terms of A.P. having $(2n + 1)$ terms. Then

$$S_1 = a_1 + a_3 + a_5 + \dots + a_{2n+1}$$

$$S_1 = \frac{n+1}{2}(a_1 + a_{2n+1})$$

$$S_1 = \frac{n+1}{2}[a + a + (2n + 1 - 1)d]$$

$$= (n + 1)(a + nd)$$

Similarly, if S_2 denotes the sum of even terms, then

$$S_2 = \frac{n}{2}[2a + 2nd] = n(a + nd)$$

Hence, $$\frac{S_1}{S_2} = \frac{(n+1)(a+nd)}{n(a+nd)} = \frac{n+1}{n}$$

S9. Distance travelled to bring the first potato

$$= 24 + 24 = 48 \text{ cm.}$$

Distance travelled to bring the second potato

$$= 28 + 28 = 56 \text{ cm.}$$

Thus, respective distances are 48, 56, 64, ...

$\therefore \qquad a = 48, \quad d = 8, \quad n = 20$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}[2 \times 48 + (20 - 1) \times 8]$$

$$= 10[96 + 19 \times 8]$$

$$= 10[96 + 152]$$

$$= 10 \times 248$$

$$= \mathbf{2480\ m}.$$

S10. Let $\qquad a = 5, \quad d = 2, \quad n = ?$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\Rightarrow \quad 192 = \frac{n}{2}[2 \times 5 + (n-1) \times 2]$

$\Rightarrow \quad 192 = n[5 + (n-1)]$

$\Rightarrow \quad 192 = n(n + 4)$

$\Rightarrow \quad n^2 + 4n - 192 = 0$

$\Rightarrow \quad (n + 16)(n - 12) = 0$

$\Rightarrow \quad n + 16 = 0 \quad \text{or} \quad n - 12 = 0$

$\Rightarrow \quad n = 12, \qquad n = -16 \qquad \text{(not possible)}$

Hence, number of days = **12**.

S11. Let $a = \$\,5200, \quad d = \$\,320$

(i) $n = 10$

$$a_n = a + (n-1)\,\text{d}$$

$$a_{10} = 5200 + (9) \times 320$$

$$= \$\,8080.$$

(ii) $n = 12$

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$S_{12} = \frac{12}{2}[2 \times 5200 + (12 - 1) \times 320]$$

$$= 6[10400 + 3520]$$

$$= 6[13920]$$

$$= \mathbf{\$\,83{,}520}.$$

S12. We have a, b, c as three consecutive terms of A.P. Then

$$b - a = c - b = d \quad \text{(say)}$$

$$c - a = 2d$$

$$a - b = -d$$

Now, $x^{b-c} \,.\, y^{c-a} \,.\, z^{a-b} = x^{-d} \,.\, y^{2d} \,.\, z^{-d}$

$= x^{-d}\,(\sqrt{xz}\,)^{2d} \,.\, z^{-d}$ (since $y = (\sqrt{xz}\,)$) as x, y, z are in G.P.)

$= x^{-d} \,.\, x^{d} \,.\, z^{d} \,.\, z^{-d}$

$= x^{-d+d} \,.\, z^{d-d}$

$= x^{o}z^{o} = 1.$ **Proved.**

S13. Let r be the common ratio of the given G.P. Then

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$$

$\Rightarrow$ $$b = ar, \quad c = br = ar^2, \quad d = cr = ar^3$$

Now, $$a^2 - b^2 = a^2 - a^2r^2 = a^2(1 - r^2)$$

$$b^2 - c^2 = a^2r^2 - a^2r^4 = a^2r^2(1 - r^2)$$

and $$c^2 - d^2 = a^2r^4 - a^2r^6 = a^2r^4(1 - r^2)$$

Therefore, $$\frac{b^2 - c^2}{a^2 - b^2} = \frac{c^2 - d^2}{b^2 - c^2} = r^2$$

Hence, $a^2 - b^2$, $b^2 - c^2$, $c^2 - d^2$ are in G.P.

S14. Let First prize money = a

$$n = 16, \quad a_{16} = 275$$

Let Common difference = d

$$a_n = a + (n - 1)\,d$$

$\Rightarrow$ $$a_{16} = a + (16 - 1) \times d$$

$\Rightarrow$ $$275 = a + 15d \qquad \ldots \text{(i)}$$

Now, $$S_n = \frac{n}{2}\,[2a + (n - 1)\,d]$$

$\Rightarrow$ $$8000 = \frac{16}{2}\,[2a + (16 - 1)\,d]$$

$\Rightarrow$ $$1000 = 2a + 15d \qquad \ldots \text{(ii)}$$

From Eq. (i) and (ii), we get

$$a = 1000 - 275$$

$\Rightarrow$ $$a = \$\,725$$

Hence, first place team receives **$ 725**.

S15. Let Perimeter of 1st triangle = $20 \times 3 = 60$ cm.

Perimeter of 2nd triangle = $10 \times 3 = 30$ cm.

Perimeter of 3rd triangle = $5 \times 3 = 15$ cm.

Now, $$a = 60 \text{ cm}$$

$$r = \frac{30}{60} = \frac{1}{2}$$

$$a_n = a\,r^{n-1}$$

$$a_6 = 60\left(\frac{1}{2}\right)^{6-1}$$

$$= 60 \times \left(\frac{1}{2}\right)^5 = \frac{60}{32} = \mathbf{\frac{15}{8}\ cm}.$$

S16. Let the numbers be a and b.

$\therefore$ $$A = \frac{a+b}{2}$$

Now, a, G_1, G_2, b are in G.P.

Let Common ratio = r

$\therefore$ $$T_4 = a\,.\,r^3 = b \quad \Rightarrow \quad r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$\therefore$ $$G_1 = a\left(\frac{b}{a}\right)^{\frac{1}{3}}, \quad G_2 = a\left(\frac{b}{a}\right)^{\frac{2}{3}}$$

$$\text{R.H.S.} = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

$$= \frac{\left[a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right]^2}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\left[a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right]^2}{a\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

$$= \frac{a^2}{a} + \frac{a^2}{a}\left(\frac{b}{a}\right)^{\frac{4}{3}-\frac{1}{3}}$$

$$= a + a \cdot \frac{b}{a} = a + b$$

$$= 2A = \text{L.H.S.}$$

S17. Since a, b, c, d are in G.P., then A.M. > G.M., for the first three terms.

Therefore, $$\frac{a+c}{2} > b \qquad \left(\text{Since } \sqrt{ac} = b\right)$$

$\Rightarrow$ $$a + c > 2b \qquad \ldots \text{(i)}$$

Similarly, for the last three terms

$$\frac{b+d}{2} > c \qquad \left(\text{Since } \sqrt{bd} = c\right)$$

$$b + d > 2c \qquad \ldots \text{(ii)}$$

Adding Eq. (i) and (ii), we get

$$(a + c) + (b + d) > 2b + 2c$$

$\Rightarrow$ $$a + d > b + c.$$

S18. Let $T_p = q, \quad T_q = p$

Let first term is a and common ratio = r

$\therefore$ $a\,r^{p-1} = q$ and $a\,r^{q-1} = p$

$\therefore$ $$\frac{a\,r^{p-1}}{a\,r^{q-1}} = \frac{q}{p}$$

$\Rightarrow$ $$r^{p-q} = \frac{q}{p}$$

$\Rightarrow$ $$r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

Now, $a\,r^{p-1} = q$

or $$a\left(\frac{q}{p}\right)^{\frac{p-1}{p-q}} = q$$

$$a = q \times \left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}$$

Now $T_{p+q} = a\,r^{p+q-1}$

$$= q \times \left(\frac{p}{q}\right)^{\frac{p-1}{p-q}} \cdot \left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}}$$

$$= q\,\frac{q^{\frac{p+q-1}{p-q} - \frac{p-1}{p-q}}}{p^{\frac{p+q-1}{p-q} - \frac{p-1}{p-q}}}$$

$$= q\left(\frac{q^{\frac{q}{p-q}}}{p^{\frac{q}{p-q}}}\right) = \frac{q^{\frac{q}{p-q}+1}}{p^{\frac{q}{p-q}}}$$

$$= \left[\frac{q^p}{p^q}\right]^{\frac{1}{p-q}}$$ **(Hence proved.)**

S19. Given that $f(x + y) = f(x) \,.\, f(y)$ and $f(1) = 2$

Therefore, $f(2) = f(1 + 1) = f(1) \,.\, f(1) = 2^2$

$f(3) = f(1 + 2) = f(1) \,.\, f(2) = 2^3$

$f(4) = f(1 + 3) = f(1) \,.\, f(3) = 2^4$

and so on. Continuing the process, we obtain

$$f(k) = 2^k \quad \text{and} \quad f(a) = 2^a$$

Hence,
$$\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a) \, . \, f(k)$$

$$= f(a) \sum_{k=1}^{n} f(k)$$

$$= 2^a (2^1 + 2^2 + 2^3 + \dots + 2^n)$$

$$= 2^a \left\{ \frac{2 \, . \, (2^n - 1)}{2 - 1} \right\} = 2^{a+1}(2^n - 1)$$

But, we are given
$$\sum_{k=1}^{n} f(a+k) = 16\,(2^n - 1)$$

$$2^{a+1}(2^n - 1) = 16\,(2^n - 1)$$

$$\Rightarrow \quad 2^{a+1} = 2^4 \quad \Rightarrow \quad a + 1 = 4$$

$$\Rightarrow \quad a = 3.$$

S20. Let
$$S_n = (3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \dots$$

$$S_n = (3^3 + 5^3 + \dots) - (2^3 + 4^3 + 6^3 + \dots)$$

(i) Let $S_{n_1} = 3^3 + 5^3 + \dots$ 3, 5, 7, ... are in A.P.

$\therefore$ $t_{n_1} = 3 + (n-1)2 = 2n + 1$

Again let $S_{n_2} = 2^3 + 4^3 + \dots$ 2, 4, 6, ... are in A.P.

$\therefore$ $t_{n_2} = 2 + (n-1) \times 2 = 2n$

$\therefore$ For given sequence,

$$t_n = (2n+1)^3 - (2n)^3$$

$$t_n = 8n^3 + 1 + 6n\,(2n + 1) - (8n^3)$$

$$t_n = 12n^2 + 6n + 1$$

$$\therefore \quad S_n = 12\sum n^2 + 6\sum n + n$$

$$= 12\,\frac{n\,(n+1)(2n+1)}{6} + \frac{6n\,(n+1)}{2} + n$$

$$= 2\,(n^2 + n)(2n + 1) + 3\,(n^2 + n) + n$$

$$= 2\,[2n^3 + 3n^2 + n] + 3n^2 + 3n + n$$

$$= 4n^3 + 9n^2 + 6n$$

(ii) For $n = 10$

$$S_{10} = 4\,(10)^3 + 9\,(10)^2 + 6 \times 10$$

$$= 4 \times 1000 + 9 \times 100 + 60$$

$$= \mathbf{4960}.$$

S21. As $\theta_1, \theta_2, \theta_3, ..., \theta_n$ are in A.P.

$\therefore$ $\theta_2 - \theta_1 = \theta_3 - \theta_2 = ... = \theta_n - \theta_{n-1} = d$

L.H.S. = $\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + ... + \sec\theta_{n-1} \sec\theta_n$

$$= \frac{1}{\sin d}\left[\frac{\sin d}{\cos\theta_1 \cos\theta_2} + \frac{\sin d}{\cos\theta_2 \cos\theta_3} + ... + \frac{\sin d}{\cos(\theta_{n-1})\cos\theta_n}\right]$$

$$= \frac{1}{\sin d}\left[\frac{\sin(\theta_2 - \theta_1)}{\cos\theta_1 \cos\theta_2} + \frac{\sin(\theta_3 - \theta_2)}{\cos\theta_2 \cos\theta_3} + ...\right]$$

$$= \frac{1}{\sin d}\left[\frac{\sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1}{\cos\theta_1 \cdot \cos\theta_2} + \frac{\sin\theta_3 \cos\theta_2 - \cos\theta_3 \sin\theta_2}{\cos\theta_2 \cos\theta_3} + ...\right]$$

$$= \frac{1}{\sin d}\left[\left(\frac{\sin\theta_2}{\cos\theta_2} - \frac{\sin\theta_1}{\cos\theta_1}\right) + \left(\frac{\sin\theta_3}{\cos\theta_3} - \frac{\sin\theta_2}{\cos\theta_2}\right) + ... + \left(\frac{\sin\theta_n}{\cos\theta_n} - \frac{\sin\theta_{n-1}}{\cos\theta_{n-1}}\right)\right]$$

$$= \frac{1}{\sin d}[(\tan\theta_2 - \tan\theta_1) + (\tan\theta_3 - \tan\theta_2) + ... + (\tan\theta_n - \tan\theta_{n-1})]$$

$$= \frac{1}{\sin d}[\tan\theta_n - \tan\theta_1] = \text{R.H.S.}$$

S22. Let the A.P. be $a, a + d, a + 2d, ...$. We are given

$$a_1 + a_2 + ... + a_m = a_{m+1} + a_{m+2} + ... + a_{m+n} \quad ... \text{(i)}$$

Adding $a_1 + a_2 + ... a_m$ on both sides of Eq. (i), we get

$$2[a_1 + a_2 + ... + a_m] = a_1 + a_2 + ... + a_m + a_{m+1} + a_{m+2} + ... + a_{m+n}$$

$$2S_m = S_{m+n}$$

Therefore, $2\frac{m}{2}\{2a + (m-1)d\} = \frac{m+n}{2}\{2a + (m+n-1)d\}$

Putting $2a + (m - 1)d = x$, in the above equation, we get

$$mx = \frac{m+n}{2}(x + nd)$$

$$(2m - m - n)x = (m + n)nd$$

$\Rightarrow$ $(m - n)x = (m + n)nd$ ... (ii)

Similarly, if $a_1 + a_2 + ... + a_m = a_{m+1} + a_{m+2} + ... + a_{m+p}$

Adding $a_1 + a_2 + ... + a_m$ on both sides, we get

$$2(a_1 + a_2 + \ldots + a_m) = a_1 + a_2 + \ldots + a_{m+1} + \ldots + a_{m+p}$$

$$2\,S_m = S_{m+p}$$

$\Rightarrow$ $$2\left[\frac{m}{2}\{2a+(m-1)d\}\right] = \frac{m+p}{2}\{2a+(m+p-1)d\}$$ which gives

i.e., $$(m-p)\,x = (m+p)\,pd \qquad \ldots \text{(iii)}$$

Dividing Eq. (ii) by (iii), we get

$$\frac{(m-n)\,x}{(m-p)\,x} = \frac{(m+n)\,nd}{(m+p)\,pd}$$

$\Rightarrow$ $$(m-n)(m+p)\,p = (m-p)(m+n)\,n$$

$$(m+p)\left(\frac{1}{n}-\frac{1}{m}\right) = (m+n)\left(\frac{1}{p}-\frac{1}{m}\right)$$

$$(m+n)\left(\frac{1}{m}-\frac{1}{p}\right) = (m+p)\left(\frac{1}{m}-\frac{1}{n}\right)$$

S23. Let $$S_p = q \quad \text{and} \quad S_q = p$$

To prove: $$S_{p+q} = -(p+q)$$

Let first term = a and common difference = d.

$\therefore$ $$S_p = \frac{p}{2}\,[2a + (p-1)\,d] = q \qquad \ldots \text{(i)}$$

$$S_q = \frac{q}{2}\,[2a + (q-1)\,d] = p \qquad \ldots \text{(ii)}$$

$\therefore$ $$2a + (p-1)\,d = \frac{2q}{p} \qquad \ldots \text{(iii)}$$

$$2a + (q-1)\,d = \frac{2p}{q} \qquad \ldots \text{(iv)}$$

Subtracting Eq. (iv) from (iii), we get

$$(p-q)\,d = \frac{2q}{p} - \frac{2p}{q}$$

$\Rightarrow$ $$(p-q)\,d = \frac{2(q^2-p^2)}{pq}$$

$\Rightarrow$ $$d = \frac{2(p+q)}{pq}$$

Subtracting Eq. (ii) from (i), we get

$$2a\,(p-q) + [p\,(p-1) - q\,(q-1)]\,d = 2p - 2q$$

$\Rightarrow \quad 2a(p-q) + [(p^2 - q^2) - (p-q)]\, d = 2(p-q)$

$\Rightarrow \quad 2a + (p+q-1)\, d = 2$

Now, $$S_{p+q} = \frac{p+q}{2}\,[2a + (p+q-1)\, d]$$

$$= \frac{p+q}{2}\,(2) = (p-q)$$ **(Proved.)**

Again $$S_{p-q} = \frac{p-q}{2}\,[2a + (p-q-1)\, d]$$

$$= \frac{p-q}{2}\,[2a + (p-1)\, d - qd]$$

$$= \frac{p-q}{2}\left[\frac{2q}{p} + q \times \frac{2(q+p)}{pq}\right]$$

$$= \frac{p-q}{2}\left[\frac{2q}{p} + \frac{2(q+p)}{p}\right]$$

$$= \frac{(p-q)[q+q+p]}{p}$$

$$= \frac{(\boldsymbol{p}-\boldsymbol{q})(\mathbf{2}\boldsymbol{q}+\boldsymbol{p})}{\boldsymbol{p}}.$$